争取未来开挖掘机

姜圣的追随者

2024.7.12

摘要

沉迷游戏的我无意间看见关于姜圣的新闻。深感愧疚，幼儿班的我就已经熟练的掌握了的九九乘法表。而现在我却每天沉迷于提瓦特大陆，天天只知道打丘丘人。

从今天开始我也要努力学习数学，希望姜圣以后当上院士的时候能带我一起开发挖掘机。

(本书内容：仅有公式，定理及证明)

(作者文凭：中专学历，混的文凭，简单理解就是初中学历（-。-）!)

(公式及证明出处：公式及证明都是在别的书里参考过来的，极个别公式证明是我自己瞎写的。)

目录

1 三角函数

1.1 三角恒等式

$$
\begin{aligned}
\sin(A+B) &= \sin A\cos B+\cos A\sin B \\
\sin(A-B) &= \sin A\cos B-\cos A\sin B \\
\cos(A+B) &= \cos A\cos B-\sin A\sin B \\
\cos(A-B) &= \cos A\cos B+\sin A\sin B
\end{aligned}
$$

1.2 双曲函数

定义

$$
\begin{aligned}
\sinh x &= \frac{e^x-e^{-x}}{2} & \cosh x &= \frac{e^x+e^{-x}}{2} \\
\tanh x &= \frac{e^x-e^{-x}}{e^x+e^{-x}} & \coth x &= \frac{e^x+e^{-x}}{e^x-e^{-x}}
\end{aligned}
$$

恒等式

$$\sinh(2x) = 2\sinh x\cosh x \tag{1.2.1}$$

$$\cosh^2 x-\sinh^2 x = 1 \tag{1.2.2}$$

$$\cosh^2 x+\sinh^2 x = cosh(2x) \tag{1.2.3}$$

$$\cosh x = 1+2\sinh^2\frac{x}{2} \tag{1.2.4}$$

2 不等式

$$\frac{x_1 + x_2 + \cdots + x_n}{n} \geqslant \sqrt[n]{x_1 + x_2 + \cdots + x_n} \tag{2.0.1}$$

$$|x + y| < |x| + |y| \tag{2.0.2}$$

$$\sin x < x < \tan x \tag{2.0.3}$$

伯努利不等式

$$(1 + x)^n \leqslant 1 + nx \tag{2.0.4}$$

3 排列组合

3.1 定义

$$\mathbb{A}_n^k = \frac{n!}{(n-k)!} \tag{3.1.1}$$

$$\mathbb{C}_n^k = \frac{\mathbb{A}_n^k}{k!} = \frac{n!}{k!(n-k)!} \tag{3.1.2}$$

3.2 运算

4 区间与映射

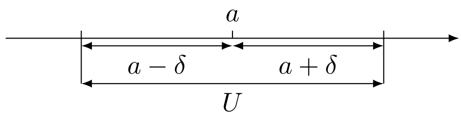
4.1 区间定义

$$
\text{区间定义}\begin{cases}(a,b)=\{x|a<x<b\}\\ [a,b]=\{x|a\leqslant x\leqslant b\}\\ (a,b]=\{x|a<x\leqslant b\}\\ (a,+\infty)=\{x|a<x\}\end{cases}
$$

4.2 领域定义

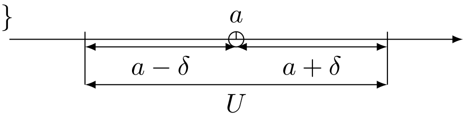
点 a 的领域

$$
U(a,\delta)=\begin{cases}\{x|a-\delta<x<a+\delta\}\\ \{x|\ |x-a|<\delta\}\end{cases}
$$



点 a 的去心领域

$$
\mathring{U}(a,\delta)=\begin{cases}\{x|a-\delta<x<a+\delta\wedge x\neq 0\}\\ \{x|0<|x-a|<\delta\}\end{cases}
$$



点 a 的左领域　　$(a-\delta,a)$

点 a 的右领域　　$(a,a+\delta)$

4.3 映射定义

定义:X 与 Y 是两个非空集合, 如果存在一个法则对任一 $x\in X$, 都有确定的 y 与之对应。则称 f 为从 X 到 Y 的一个映射。

$$
\text{记作}\qquad f:X\to Y
$$

$$
f(x)=y\qquad\begin{cases}\text{定义域 }(D_f)=X & x\text{-原像}\\ \text{值域 }(R_f)=\{f(x)|x\in X\} & y\text{-像}\end{cases}
$$

$$
\text{映射类型}\begin{cases}
\text{满射:} & R_f = Y \\
\text{单射:} & x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2) \\
\text{一一映射:} & \text{即使满射又是单射} \Leftrightarrow \text{逆映射:} \begin{cases} f^{(}x) & = y \\ f^{-1}(y) & = x \end{cases} \\
\text{复合映射:} & g \circ f \Leftrightarrow g\left[f\left(x\right)\right] \begin{cases} f : X \to Y_1 \\ g : Y_2 \to Z \\ g \circ f : X \to Z \qquad (Y_1 \subset Y_2) \end{cases}
\end{cases}
$$

5 函数与图像

5.1 函数的定义

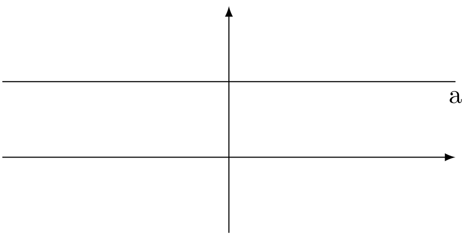
设数集 $D \in R$ 的映射

$$f : D \to R$$

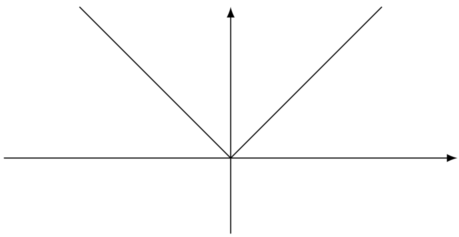
称 f 为定义在 D 上的函数, 记为

$$y = f(x)\ \{x \in D\}$$

5.2 函数的类型

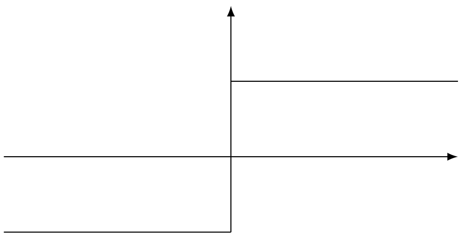


常函数 $f(x) = a\{a \in R\}$



$f(x) = |x|$

$$f(x) = sgn\ x = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$$



$$|x| = x\ sgnx$$

5.3 函数的性质

5.3.1 函数的有界性

$$f : D \to R\{D \subset R\} \begin{cases} 有界 \begin{cases} 有上界 \left\{ \exists k_1,\ 使 f(x) \leqslant k_1,\ \forall x \in D \right. \\ 有下界 \left\{ \exists k_1,\ 使 f(x) \geqslant k_1,\ \forall x \in D \right. \end{cases} \\ 无界 \begin{cases} 无上界 \left\{ \forall K_1,\ \exists x \in D\ 使,\ f(x) \geqslant k_1 \right. \\ 无下界 \left\{ \forall K_1,\ \exists x \in D\ 使,\ f(x) \leqslant k_1 \right. \end{cases} \end{cases}$$

5.3.2 函数的单调性

单调增加 若$\{x_1, x_2 \in D\}$ $x_1 < x_2 \Rightarrow \begin{cases} f(x_1) < f(x_2)\text{称}f(x)\text{在 D 上单调增加} \\ f(x_1) > f(x_2)\text{称}f(x)\text{在 D 上单调减少} \\ f(x_1) \leqslant f(x_2)\text{称}f(x)\text{在 D 上单调非降} \\ f(x_1) \geqslant f(x_2)\text{称}f(x)\text{在 D 上单调非增} \end{cases}$

5.3.3 函数的奇偶性

定义域

$\forall x \in D \qquad f(-x) = \begin{cases} f(x) & \text{偶函数} \\ -f(x) & \text{奇函数} \end{cases}$

奇偶性运算

$$\text{奇函数} \times \text{奇函数} = \text{偶函数} \tag{5.3.1}$$

$$\text{奇函数} \times \text{偶函数} = \text{奇函数} \tag{5.3.2}$$

$$\text{偶函数} \times \text{偶函数} = \text{偶函数} \tag{5.3.3}$$

5.3.4 周期性

$Def:$ $f(x+L) = f(x)\{L > 0\text{常数}, \forall x \in D\} \Rightarrow f(x)$ 为 L 的周期函数

6 并集，交集

6.1 定义

$$
\begin{gathered}
(\vee \text{ 或}, \wedge \text{ 与}) \\
A \cup B = \{x \in A \vee x \in B\} \\
A \cap B = \{x \in A \wedge x \in B\}
\end{gathered}
$$

6.2 运算

$$
\text{运算满足}\begin{cases}
\text{交换律}\begin{cases} A \cup B & = B \cup A \\ A \cap B & = B \cap A \end{cases} \\
\text{结合律}\begin{cases} (A \cup B) \cup C = A \cup (B \cup C) \\ (A \cap B) \cap C = A \cap (B \cap C) \end{cases} \\
\text{分配律}\begin{cases} (A \cup B) \cap C = (A \cap C) \cup (B \cap C) \\ (A \cap B) \cup C = (A \cup C) \cap (B \cup C) \end{cases} \\
\text{对偶律}\begin{cases} (A \cup B)^C = A^C \cap B^C \\ (A \cap B)^C = A^C \cup B^C \end{cases}
\end{cases}
$$

$$
\begin{gathered}
A \cup A = A = A \cap A \\
A = B \Leftrightarrow A \subset B \wedge A \supset B \\
A \cup \varnothing = A \qquad A \cap \varnothing = \varnothing
\end{gathered}
$$

6.3 性质

性质 1.

$$A \subset (A \cup B) \qquad A \supset (A \cap B) \tag{6.3.1}$$

性质 2.

$$A \cup B = B \Leftrightarrow A \subset B \tag{6.3.2}$$

性质 3.

$$A \cap B = A \Leftrightarrow A \subset B \tag{6.3.3}$$

性质 4. ($n \in N$)

$$A \cup (B_1 \cap B_2 \cap \cdots \cap B_n) = (A \cup B_1) \cap (A \cup B_2) \cap \cdots \cap (A \cup B_n) \tag{6.3.4}$$

性质 5. ($n \in N$)

$$A \cap (B_1 \cup B_2 \cup \cdots \cup B_n) = (A \cap B_1) \cup (A \cap B_2) \cup \cdots \cup (A \cap B_n) \tag{6.3.5}$$

6.4 gustus De Morgan 定理

$$\neg(A \vee B) \Leftrightarrow (\neg A) \wedge (\neg B)$$

$$\neg(A \wedge B) \Leftrightarrow (\neg A) \vee (\neg B)$$

6.5 德摩根律 定理

$$\left(\bigcup_{\alpha} E_{\alpha}\right)^{C} = \bigcap_{\alpha}(E_{\alpha}^{C})$$

$$\left(\bigcap_{\alpha} E_{\alpha}\right)^{C} = \bigcup_{\alpha}(E_{\alpha}^{C})$$

7 群，环，域

7.1 群

7.1.1 M1

7.1.2 M2

7.1.3 M3

7.1.4 M4

7.1.5 sdas

7.2 环

7.3 域

8 极限

8.1 数列极限

8.1.1 数列的定义

$Def:\ \{x_n\}: N^+ \to R$

$$x_n = f(n)$$

8.1.2 数列极限的定义

$Def:\ \{x_n\},\ n \in N^+, \exists a,\ \forall \varepsilon > 0, \exists N,\ n > N \Rightarrow |x_n - a| < \varepsilon$

$\lim\limits_{n\to\infty} x_n = a$

极限存在，为收敛，不存在为发散

8.1.3 有界数列

若$\exists M > 0, \{M \in$ 正数$\}$

使得$\forall n, \quad |x_n| \leqslant M$

则称数列 $\{x_n\}$ 为有界数列

8.1.4 收敛数列的有界性

$$\text{收敛数列必有界} \tag{8.1.1}$$

8.1.5 收敛数列的保号性

$$\text{如果 } \lim n \to \infty x_n = a \text{ 存在，且 } a > 0, \text{ 则 } \exists N > 0\{N \in N^+\} \text{ 当 } n > N \text{ 时，都有 } x_n > 0 \tag{8.1.2}$$

8.2 函数极限

8.2.1 性质

$$\text{极限的唯一性} \tag{8.2.1}$$

8.3 运算

$$\lim_{n\to\infty}(x_n \pm y_n) = \lim_{n\to\infty} x_n \pm \lim_{n\to\infty} y_n \tag{8.3.1}$$

$$\lim_{n\to\infty}(x_n y_n) = \lim_{n\to\infty}(x_n) \lim_{n\to\infty}(y_n) \tag{8.3.2}$$

$$\lim_{n\to\infty}\frac{x_n}{y_n} = \frac{\lim\limits_{n\to\infty} x_n}{\lim\limits_{n\to\infty} y_n} \tag{8.3.3}$$

8.3.1 夹逼定理 (三明治定理)

$$\begin{aligned} & x_n \leqslant z_n \leqslant y_n \qquad \forall b > N_0 \\ & \text{若} \lim_{n\to\infty} x_n = \lim_{n\to\infty} y_n = a \text{则} \lim_{n\to\infty} z_n = a \end{aligned} \tag{8.3.4}$$

9 导数

9.1 幂数，指数，对数

$$\frac{d}{\mathrm{d}x}x^a = ax^{a-1} \tag{9.1.1}$$
$$\frac{d}{\mathrm{d}x}b^x = b^x \ln b \tag{9.1.2}$$
$$\frac{d}{\mathrm{d}x}e^x = e^x \tag{9.1.3}$$
$$\frac{d}{\mathrm{d}x}\ln x = \frac{1}{x} \tag{9.1.4}$$
$$\tag{9.1.5}$$

9.2 三角函数

$$\frac{d}{\mathrm{d}x}\sin x = \cos x \tag{9.2.1}$$
$$\frac{d}{\mathrm{d}x}\arcsin x = \frac{1}{\sqrt{1-x^2}} \tag{9.2.2}$$
$$\frac{d}{\mathrm{d}x}\csc x = -\csc x \cot x \tag{9.2.3}$$
$$\frac{d}{\mathrm{d}x}\cos x = -\sin x \tag{9.2.4}$$
$$\frac{d}{\mathrm{d}x}\sec x = \sec x \tan x \tag{9.2.5}$$
$$\frac{d}{\mathrm{d}x}\operatorname{arcsec} x = \frac{1}{|x|\sqrt{x^2-1}} \tag{9.2.6}$$
$$\frac{d}{\mathrm{d}x}\tan x = \sec^2 x \tag{9.2.7}$$
$$\frac{d}{\mathrm{d}x}\arctan = \frac{1}{1+x^2} \tag{9.2.8}$$
$$\frac{d}{\mathrm{d}x}\cot x = -\csc^2 x \tag{9.2.9}$$
$$\frac{d}{\mathrm{d}x}\sinh x = \cosh x \tag{9.2.10}$$
$$\frac{d}{\mathrm{d}x}\cosh x = \sinh x \tag{9.2.11}$$

9.3 倒数运算

$$A = B \tag{9.3.1-1}$$

$$C = D \tag{9.3.1-2}$$

10 积分

10.1 幂数，指数，对数

$$\int x^a \mathrm{d}x = \frac{1}{a-1} x^{a-1} + C \tag{10.1.1}$$

$$\int b^x \mathrm{d}x = \frac{b^x}{\ln b} + C \tag{10.1.2}$$

$$\int e^x \mathrm{d}x = e^x + C \tag{10.1.3}$$

$$\int \frac{1}{x} \mathrm{d}x = \ln x + C \tag{10.1.4}$$

10.2 三角函数

$$\int \sin x \mathrm{d}x = -\cos x + C \tag{10.2.1}$$

$$\int \frac{1}{\sqrt{1-x^2}} \mathrm{d}x = \arcsin x + C \tag{10.2.2}$$

$$\int \csc x \cot x \mathrm{d}x = -\csc x + C \tag{10.2.3}$$

$$\int \cos x \mathrm{d}x = \sin x + C \tag{10.2.4}$$

$$\int \sec x \tan x \mathrm{d}x = \sec x + C \tag{10.2.5}$$

$$\int \sec^2 x \mathrm{d}x = \tan x + C \tag{10.2.6}$$

$$\int \csc^2 x \mathrm{d}x = -\cot x + C \tag{10.2.7}$$

$$\int \frac{1}{|x|\sqrt{x^2-1}} \mathrm{d}x = \operatorname{arcsec} x + C \tag{10.2.8}$$

$$\int \frac{1}{1+x^2} \mathrm{d}x = \tan x + C \tag{10.2.9}$$

$$\int \sinh x \mathrm{d}x = \cosh x + C \tag{10.2.10}$$

$$\int \cosh x \mathrm{d}x = \sinh x + C \tag{10.2.11}$$

10.3 积分运算

11 零散的一些

$$\sum_{k=0}^{n} q^k = \frac{1-q^{n+1}}{1-q} \tag{11.0.1}$$

$$A_N = \sum_{k=0}^{n} q^k \qquad qA_N = \sum_{k=1}^{n+1} q^k$$

$$A_N - qA_N = \sum_{k=0}^{n} q^k - \sum_{k=1}^{n+1} q^k = 1 - q^{n+1}$$

$$A_N = \frac{1-q^{n+1}}{1-q}$$

$$\log_{10} x = \lg_x \tag{11.0.2}$$

$$\log_e x = \ln_x \tag{11.0.3}$$

$$\log_b xy = \log_b x + \log_b y \tag{11.0.4}$$

$$\log_{(b^n)} x = \frac{1}{n} \log_b x \tag{11.0.5}$$

$$\log_b x^n = n \log_b x \tag{11.0.6}$$

$$\log_b x = \frac{\log_c x}{\log_c b} \tag{11.0.7}$$

$$b^n = x \qquad b^m = y$$

$$b^{n+m} = xy$$

$$\log_b xy = n + m = \log_b x + \log_b y$$

$$b^n = x$$

$$\log_b x = n$$

$$\frac{1}{n} \log_b x = 1 = \log_{(b^n)} x$$

$$b^1 = x^n \qquad b^{\frac{1}{n}} = x$$

$$n \log_b x = 1 = \log_b x^n$$

$$\log_b x = log_{c^{(\log_c b)}} c^{(\log_c x)} = \frac{\log_c x}{\log_c b}$$

12 证明

12.1 1.2.1

$$\begin{aligned}
\sinh x\cosh x &= \left(\frac{e^x - e^{-x}}{2}\right)\left(\frac{e^x + e^{-x}}{2}\right) \\
&= \left(\frac{1}{2}\right)\left(\frac{e^{2x} - e^{-2x}}{2}\right) \\
&= \frac{1}{2}\sinh(2x) \\
\sinh(2x) &= 2\sinh x\cosh x
\end{aligned}$$

12.2 1.2.2

$$\begin{aligned}
\cosh^2 x - \sinh^2 x &= \left(\frac{e^x + e^{-x}}{2} + \frac{e^x - e^{-x}}{2}\right)\left(\frac{e^x + e^{-x}}{2} - \frac{e^x - e^{-x}}{2}\right) \\
&= e^x \times e^{-x} \\
&= 1
\end{aligned}$$

12.3 1.2.3

$$\begin{aligned}
\cosh^2 x + \sinh^2 x &= \left(\frac{e^x + e^{-x}}{2}\right)^2 + \left(\frac{e^x - e^{-x}}{2}\right)^2 \\
&= \frac{2e^{2x} + 2e^{-2x}}{4} \\
&= \frac{e^{2x} + e^{-2x}}{2} \\
&= \cosh(2x)
\end{aligned}$$

12.4 1.2.4

$$\begin{aligned}
\cosh(2x) &= \cosh^2 x + \sinh^2 x \\
&= \sinh^2 x + 1 + \sinh^2 x \\
&= 2\sinh^2 x + 1 \\
\cosh x &= 2\sinh^2 \frac{x}{2} + 1
\end{aligned}$$

12.5 8.1.1

$$
\begin{aligned}
&\varepsilon = 1,\ \exists N > 0,\ \text{当 n>N 时} \left|X_n - a\right| < 1 \\
&\left|X_n\right| = \left|(X_n - a) + a\right| \\
&\leqslant \left|x_n - a\right| + \left|a\right| \\
&\leqslant 1 + \left|a\right| \\
&M = \max\{\left|X_n\right|, \left|X_2\right|, \ldots, \left|X_n\right|, 1 + \left|a\right|\} \\
&\forall n,\ \left|X_n\right| \leqslant M
\end{aligned}
$$

12.6 8.1.2

12.7 8.2.1

$$
\begin{aligned}
&\text{反设} \lim_{n\to\infty} x_n = a,\ \lim_{n\to\infty} x_n = b, \text{且} a < b \\
&\varepsilon = \frac{b-a}{3} \begin{cases} \exists N_1,\ n > N_1,\ \left|x_n - a\right| < \frac{b-a}{3} \\ \exists N_2,\ n > N_2,\ \left|x_n - b\right| < \frac{b-a}{3} \end{cases} \\
&N = \max\{N_1, N_2\},\ n > N \Rightarrow \begin{cases} n > N_1 \\ n > N_2 \end{cases} \\
&b - a = \left|(x_n - a) - (x_n - b)\right| \\
&\leqslant \left|x_n - a\right| + \left|x_n - b\right| \\
&< \frac{b-a}{3} + \frac{b-a}{3} \\
&< \frac{2(b-a)}{3}
\end{aligned}
$$

12.8 8.3.4

$$
\begin{aligned}
&\forall \varepsilon > 0 \\
&\left|x_n - a\right| < \varepsilon \qquad \forall n > N_1 \\
&\left|y_n - a\right| < \varepsilon \qquad \forall n > N_2 \\
&\text{令} N = \max\left\{N_1, N_2, N_0\right\}, \text{则当} n > N \text{时有} \\
&a - \varepsilon < x_n \leq z_n \leq y_n < a + \varepsilon \\
&\left|z_n - a\right| < \varepsilon \\
&\lim_{n\to\infty} z_n = a
\end{aligned}
$$