

$$1. (a) f_Z(z) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{z^2}{2}}$$

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$$\begin{aligned} (b) P(-1 \leq Z \leq 1) &= 1 - 2 \cdot (1 - P(Z \leq 1)) \\ &= 1 - 2 \cdot (1 - 0.8413) \\ &= 1 - 0.3174 \\ &= 0.6826 \end{aligned}$$

$$(c) 2(1 - y) = 0.05 \Rightarrow y = 0.975$$

$$P(Z \leq x) = 0.975$$

$$x = 1.96 \text{ (查表)}$$

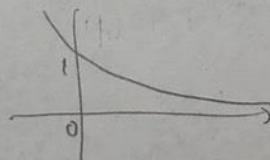
$$\begin{aligned} (d) f_Q(q) &= \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{\pi}} \cdot q^{-\frac{1}{2}} \cdot e^{-\frac{q^2}{2}} \\ &= \frac{1}{\sqrt{2\pi}} q^{-\frac{1}{2}} \cdot e^{-\frac{q^2}{2}} \end{aligned}$$

$$(e) E(Q) = V = 1$$

$$(f) \text{std}[Q] = \sqrt{\sigma^2} = \sqrt{20} = \sqrt{2}$$

$$\begin{aligned} (g) P(Q \leq 1) &= 1 - P(Q \geq 1) \\ &= 1 - 0.242 = 0.758 \end{aligned}$$

$$2. (a) f_T(t) = \begin{cases} e^{-t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



$$(b) E(T) = \beta = 1$$

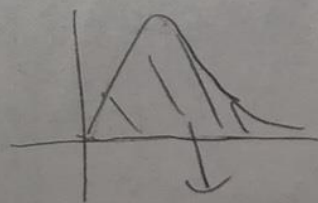
$$(c) \text{std}[T] = \sqrt{\sigma^2} = \sqrt{1^2} = 1$$

$$(d) P(T > 1) = e^{-1} = 0.3679$$

$$(e) f_{T^3}(t) = \frac{t^3 e^{-t}}{\Gamma(4)} = \frac{t^3 \cdot e^{-t}}{6}, \quad t > 0$$

$$(f) E[T^3] = 3! = 6$$

$$(g) \text{std}[T^3] = \sqrt{\sigma^2} = \sqrt{3!^2} = \sqrt{36} = 6$$



總面積  $\Rightarrow 1$

$$\begin{aligned}
 2. (b) \quad P(T_3 > 3) &= 1 - \sum_{t=1}^3 P(T_3 = t) \\
 &= 1 - 0.6787 \\
 &= 0.3213
 \end{aligned}$$

$$\begin{aligned}
 (i) \quad &\text{使用七年以上} \Rightarrow P(T_3 > 7) \\
 P(T_3 > 7) &= 1 - \sum_{t=1}^7 P(T_3 = t) \\
 &= 1 - 0.9764 \\
 &= 0.0236
 \end{aligned}$$

在「該產品開始使用之後平均壽命為1年」之假設下，3個產品總共使用七年機率以上之機率為0.0236，而  $0.0236 < 0.05$ ，可將0.0236視為不太可能發生之機率，但如今發生了，可見原「平均壽命為1年」之假設不為真。