Markov chain on a finite state set $\mathcal{X}$.

$$\Phi = \{ \underbrace{K(X_{t+1} = i \mid X_t = j)}_{\text{conditional probability.}} \}_{i,j \in \mathcal{X}}$$

where

X_t : r.v. at time t.

Then, given a prob. dist $P(X_t)$ at time t,

$$P(X_{t+1} = i) = \sum_{j \in \mathcal{X}} \underbrace{K(X_{t+1} = i \mid X_t = j)\, P(X_t = j)}_{\substack{\shortparallel \\ P(X_t = j \,\wedge\, X_{t+1} = i)}}$$

$$\boxed{\underline{\text{Fact}} \quad D(P(X_t) \| Q(X_t)) \geq D(P(X_{t+1}) \| Q(X_{t+1}))}$$

$\because$ $D(P(X_t, X_{t+1}) \| Q(X_t, X_{t+1}))$

$$= D(P(X_t) \| Q(X_t)) + \overset{\overset{0}{\shortparallel}}{\boxed{D(P(X_{t+1} \mid X_t) \| Q(X_{t+1} \mid X_t))}}$$

$$= D(P(X_{t+1}) \| Q(X_{t+1})) + D(P(X_t \mid X_{t+1}) \| Q(X_t \mid X_{t+1}))$$

$\blacksquare$

<u>NB</u> Equality $\Leftrightarrow$ $P(X_t \mid X_{t+1}) = Q(X_t \mid X_{t+1})$