

# Weekly Report of Interference Alignment

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## I. INTRODUCTION

Interference Alignment (IA) is a revolutionary wireless transmission strategy that reduces the impact of interference.

## II. PRIORI KNOWLEDGE

### A. Paper

In the paper [1], a centralized scheme is present in the form of closed form expressions for the transmit precoding matrices. However, these solutions suffer from expressions require global channel knowledge which can be an overwhelming overhead in practice. Secondly, closed form solutions have only been found in certain cases. In general, analytical solutions to interference alignment problem are difficult to obtain and even the feasibility of interference alignment over a limited number of signalling dimension is an open problem. Thirdly, the result of [1] are meaningful only at high SNR, and to the best of our knowledge, their performance at low/moderate SNR is unknown.

In the paper [2], two distributed strategies are present. The idea for strategies are that each iteration, users try to minimize the extent to which their signal leaks into the other users' desired signal subspaces. After the algorithm converges, the IA condition should ideally be satisfied and the desired signal spaces be free of interference. Relying on reciprocity for precoding over the interference channel, however, has a number of potential drawbacks. Iterating over the air may incur a non-negligible overhead due to the recurring pilot transmissions.

### B. Based System Model

1) *Channel Transmission:* Consider the  $K$ -user MIMO interference where the  $k^{th}$  transmitter and receiver are equipped with  $M$  and  $N$  antennas respectively. Note that the channel matrices will have a diagonal structure. The channel is defined as:

$$Y_{[k]} = \sum_{l=1}^K H_{[k,l]} X_{[l]} + Z_{[k]}, \quad \forall k \in \mathcal{K} \quad (1)$$

where  $Y_{[k]}, Z_{[k]}$  are the  $N \times 1$  received signal vector and the zero mean unit variance circularly symmetric Additive White Gaussian Noise vector(AWGN) at receiver  $k$ ,  $X_{[l]}$  is the  $M \times 1$  signal vector transmitted by transmitter  $l$ , and  $H_{[k,l]}$  is the  $N \times M$  matrix of channel coefficients between transmitter  $l$  and receiver  $k$ . The transmit power at transmitter  $l$  is  $E[||X_{[l]}||^2] = P_{[l]}$ .  $\mathcal{K} \triangleq \{1, 2, \dots, K\}$  is the index set of  $K$  users.

2) *Precoding at Transmitter:* Let  $V_{[k]}$  be an  $M \times d$  matrix whose columns are the orthonormal basis of of the transmitterd signal space of user  $k$ . The degree of freedom can be obtain by  $d \leq \min(M, N)$ . Mathematically, the transmitted signal vector of user  $k$  is given by:

$$X_{[k]} = V_{[k]} \bar{X}_{[k]}, \quad \bar{X}_{[k]} \sim \mathcal{N}(0, \frac{P_{[k]}}{d} \mathbf{I}). \quad (2)$$

Each element of the  $d \times 1$  vector  $\bar{X}_{[k]}$  represents an independently encode Gaussian codebook symbol with power  $\frac{P_{[k]}}{d}$  that is beamformed with the corresponding vector of  $V_{[k]}$ .

3) *Interference Suppression at Receiver:* Let  $U_{[k]}$  be an  $N \times d$  matrix whose columns are the orthonormal basis of the interference-free desired signal subspace at receiver  $k$ . The receiver  $k$  filters its received signal to obtain:

$$\bar{Y}_{[k]} = U_{[k]}^\dagger Y_{[k]} \quad (3)$$

Then we rewrite the Eq.(3) from Eq.(1) and Eq.(2):

$$\bar{Y}_{[k]} = U_{[k]}^\dagger H_{[k,k]} V_{[k]} \bar{X}_{[k]} + \sum_{l=1, l \neq k}^K U_{[k]}^\dagger H_{[k,l]} V_{[l]} \bar{X}_{[l]} + U_{[k]}^\dagger Z_{[k]}. \quad (4)$$

If interference is aligned into the null space of  $U_{[k]}$  then the following condition must be satisfied:

$$U_{[k]}^\dagger H_{[k,l]} V_{[l]} = \mathbf{0}, \quad \forall l \neq k \quad (5)$$

$$\text{rank}(U_{[k]}^\dagger H_{[k,k]} V_{[k]}) = d. \quad (6)$$

In other words the desired signals are received through a  $d \times d$  full rank channel matrix:

$$\bar{H}_{[k,k]} \triangleq U_{[k]}^\dagger H_{[k,k]} V_{[k]}, \quad (7)$$

while the interference is completely eliminated. The effective channel for user  $k$  is then expressed as:

$$\bar{Y}_{[k]} = \bar{H}_{[k,k]} \bar{X}_{[k]} + \bar{Z}_{[k]}, \quad (8)$$

where  $\bar{Z}_{[k]}$  is the effective  $d \times 1$  AWGN vector at receiver  $k$ . The rate  $R_{[k]}$  achieve on this channel is:

$$R_{[k]} = \log(\mathbf{I}_d + \frac{P_{[k]}}{d} \bar{H}_{[k,k]} \bar{H}_{[k,k]}^\dagger). \quad (9)$$

### C. Objective of Interference Alignment

Given the channel matrices  $H_{[k,l]}$ , the degrees of freedom  $d$  is feasible id there exist transmit precoding matrices  $V_{[k]}$  and receive interference suppression matrices  $U_{[k]}$  which can satisfy constraint condition Eq.(5) and Eq.(6), such that

$$\begin{cases} U_{[k]}^\dagger H_{[k,l]} V_{[l]} = \mathbf{0}, & \forall l \neq k \\ \text{rank}(U_{[k]}^\dagger H_{[k,k]} V_{[k]}) = d. \end{cases} \quad (10)$$

### D. Scheme of Interference Alignment

- Centralized Scheme
- Iterative Interference Alignment
- Max-SINR Aligorithm

### III. SIMULATION WORK

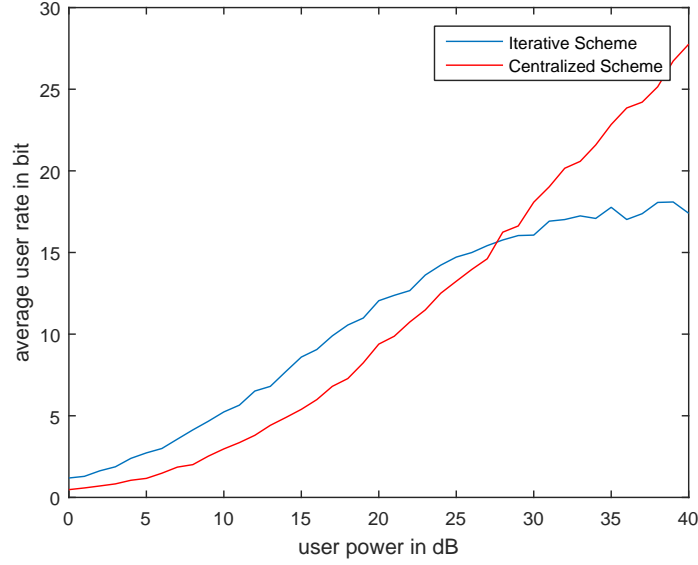


Fig. 1: Centralized and Distributed Scheme(6,6,3)

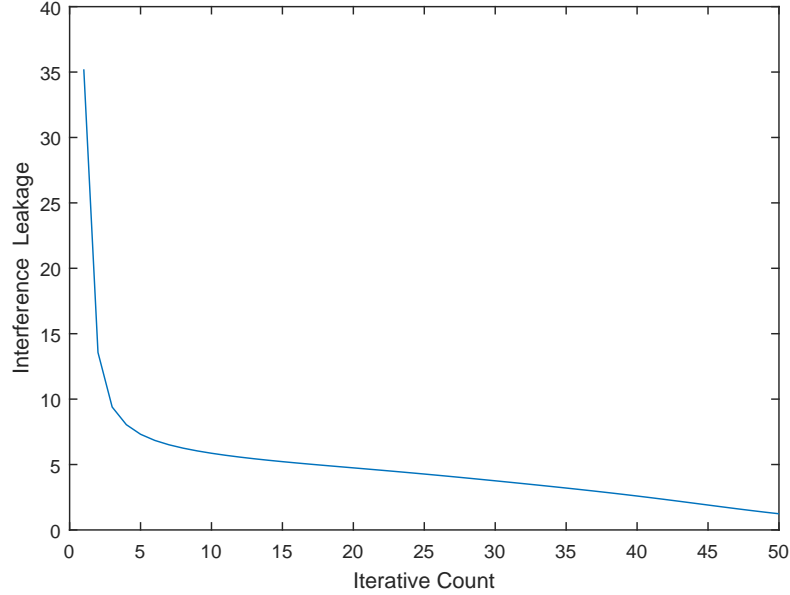


Fig. 2: Centralized and Distributed Scheme

For evaluate the performance of Centralized Scheme [1] and Distributed Iterative Scheme [2], we do some numerical simulations with MATLAB. Consider the 3-user MIMO interference where both of transmitter and receiver are equipped with 6 antennas. transmission code stream is 3. The channel matrix  $H_{[i,j]}$  is chosen to be independent zero-mean unit-variance i.i.d Gaussian matrix. Meanwhile the noise is zero mean unit variance AWGN. As we can see from Fig. 1, the performance of Centralized Scheme will better than Distributed Iterative Scheme with high user power. It is because that Iterative Scheme can not eliminate interference from other users completely. Fig. 2 shows interference leakage will decrease with a increased iterative count in Distributed Iterative Scheme .

Similar to [2], we also consider a 3 user interference channel where each node is equipped with 2 antennas and all channel coefficients are i.i.d. zero mean unit variance circularly symmetric complex user achieves 1 degree of freedom. As shown in Fig. 3, we compare the performance of the distributed iterative scheme and centralized scheme.

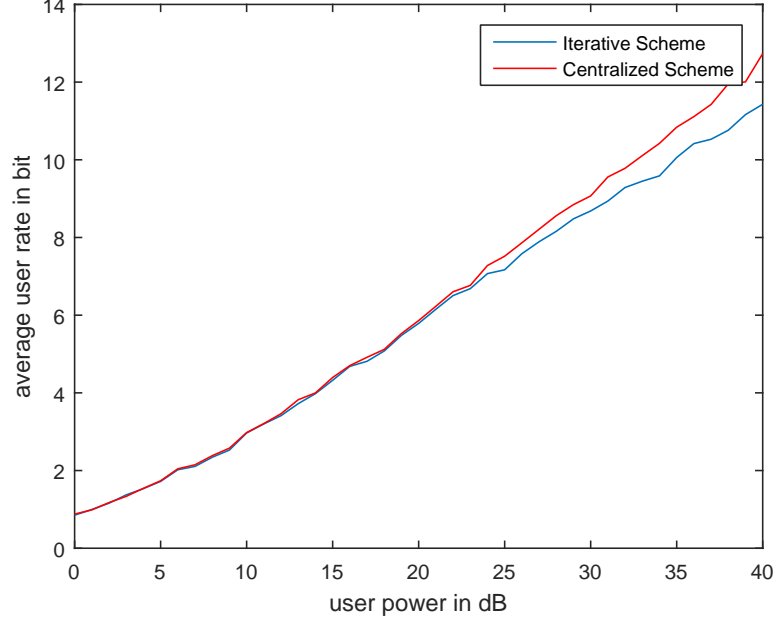


Fig. 3: Centralized and Distributed Scheme(2,2,1)

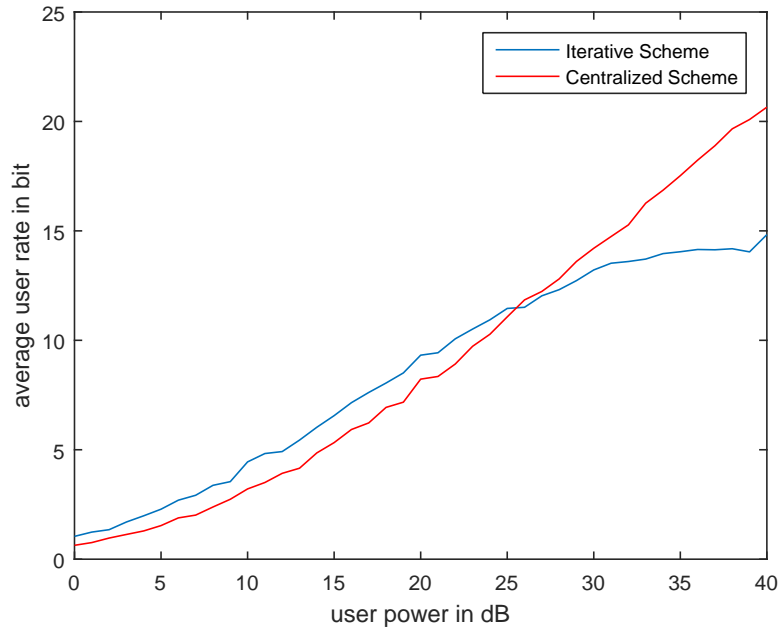


Fig. 4: Centralized and Distributed Scheme(4,4,2)

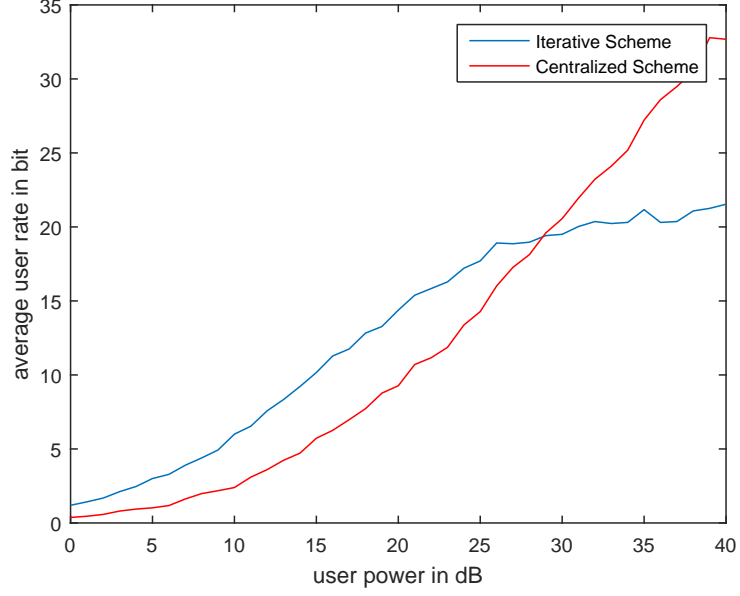


Fig. 5: Centralized and Distributed Scheme(8,8,4)

#### REFERENCES

- [1] V. R. Cadambe and S. A. Jafar, "Interference alignment and degrees of freedom of the  $k$ -user interference channel," *IEEE Transactions on Information Theory*, vol. 54, no. 8, pp. 3425–3441, 2008.
- [2] K. Gomadam, V. R. Cadambe, and S. A. Jafar, "A distributed numerical approach to interference alignment and applications to wireless interference networks," *IEEE Transactions on Information Theory*, vol. 57, no. 6, pp. 3309–3322, 2011.