Parametrization and Applications of Precoding Reuse and Downlink Interference Alignment

Eduardo Castañeda, *Member, IEEE,* Daniel Castanheira, *Member, IEEE,* Adão Silva, *Member, IEEE,* and Atílio Gameiro

***Abstract*—In this paper, we consider the downlink communication of an arbitrary number of transmitters over broadcast interference channels. In order to suppress interference we investigate a network-level linear precoding scheme dubbed precoding reuse. Such a technique is leveraged on the interference alignment (IA) principle, where the undesired signals overlap in a reduced-dimension subspace at the receivers. Precoding reuse relies on two cascaded precoders to suppress intra- and inter-cell interference and requires feedback only within a cell. We designed precoding reuse schemes for which interference suppression can be attained in a coordinated or uncoordinated fashion over a fixed number of quasi-static channel uses. We provide expressions for the parametrization of the precoders and investigate the advantages, limitations, suitable scenarios and applications of the proposed IA scheme. Finally, we pointed out open problems where the proposed IA scheme and parametrization can be implemented with little effort, demanding minimum changes to an existing cellular system supporting multi-user MIMO.**

***Index Terms*—Interference alignment, Kronecker product, precoding reuse, spatial multiplexing.**

I. INTRODUCTION

INTERFERENCE management is a major challenge in wireless networks and complementary approaches are necessary to increase the robustness of conventional methods that allocate resources at the network-level, e.g., frequency reuse, sectoring, spread spectrum, cooperative transmission, or time/frequency multiplexing [1]. Contemporary interference management techniques for wireless communications make use of multiple-input multiple-output (MIMO), a key technology for enabling significant throughput and link range gains by making use of spatial signal spaces without an additional increase in bandwidth or transmit power [2]. However, in cellular networks the MIMO processing gains are limited by inter-cell interference (ICI) as the operational signal to noise ratio (SNR) increases [1]. In scenarios with multiple transmitters, different approaches for ICI coordination rely on cooperative transmission, which can be performed according to the network configuration, e.g., adaptive transmission based on knowledge of the channel state information (CSI) or predefined settings of the network parameters (designed off-line), cf. [3]. Recent multi-cell *coordinated beamforming* schemes attempt to suppress inter-user interference (IUI) and ICI simultaneously with minimum coordination or control signaling between transmitters. This kind of *hierarchical* precoders rely on instantaneous or statistical CSI, whose objective is to maximize the system capacity [2], [3], [4]. The interference is canceled through the sequential construction of outer and inner precoders that suppress ICI and IUI, respectively. The ICI cancellation is attained by overlapping interference subspaces at the receivers, whilst the IUI is canceled at the transmitters using linear precoding and local CSI, e.g., [5], [6]. Although these works efficiently enhance system performance, they require the exchange of control parameters through the backhaul network in order to fully mitigate interference.

Other hierarchical precoding schemes, constructed upon the interference alignment (IA) principles [7], can operate without message exchange between transmitters, e.g., [8], [9]. In IA the signaling dimensions provided by MIMO (or any other orthogonalizable resource) can be used by the transmitters in such a way that the interference collapses into a reduced subspace at the receivers. Several IA schemes have been developed using channel extensions, where additional dimensions can be attained by signaling over different frequency bands [10], [11], [12]. Conventional IA schemes (cf. [7]) require global CSI knowledge, in addition to other requirements in practical scenarios, e.g., synchronization and limited backhaul/feedback rates.

Nonetheless, distributed IA schemes have been developed to address the issues described above for the downlink of cellular networks under different CSI conditions at the transmitters. The authors in [8] extended the concept of IA to non-cooperative multi-cell scenarios,

This work was supported by the Portuguese Fundação para a Ciência e Tecnologia (FCT) PURE-5GNET (UID/EEA/50008/2013) project.

The authors are with the Department of Electronics, Telecommunications and Informatics at the Aveiro University, and the Instituto de Telecomunicações (IT), Aveiro, 3810-193, Portugal (e-mail: {ecastaneda, dcastanheira, asilva}@av.it.pt; amg@ua.pt).

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TWC.2017.2670540, IEEE Transactions on Wireless Communications

where ICI was mitigated by means of subspace alignment using local CSI. The transmitters precode their signals in two phases. In the first phase the users align their direct channels to a subspace orthogonal to the subspace spanned by their precoded cross channels, i.e., the spatial directions of the ICI. In the second phase each transmitter implements conventional precoding schemes to mitigate IUI, so that all signal spaces remain orthogonal. The transmission scheme relies on two cascades precoders to cancel ICI and IUI. The authors in [13] designed IA algorithms using a cascaded precoding structure for cellular networks. By removing partially or totally the cross channel information fed back to the transmitters, it was shown the feasibility of the proposed iterative precoder optimization with limited CSI. Along the same lines others works [14], [15], attempted to minimize the cross channel knowledge required to align signal subspaces. This process requires that for a given antenna configuration all intended data streams can be transmitted over interference-free channels, i.e., IA feasibility. Further bound analysis of the capacity achieved by IA having partial CSI was conducted in [16], where it was shown that even if all transmitters share CSI, there is a degradation in performance due to noisy feedback channels.

The work by Medra and Davidson in [9] extended the scheme in [8] to scenarios where data streams are spread over space and time, increasing all signals subspaces by means of special structures for the ICI and IUI precoders. The authors introduced the notion of *spatial reuse precoding* (SRP): an off-line network-level ICI precoding scheme, where the signals from interfering sources arrive in the same reduced dimensional subspace. SRP is based on ICI and IUI precoders with decomposable structures, which are applied over temporally extended wireless channel, where the transmitted signals are spread over the spatial and temporal dimensions. SRP requires the channels to be quasi-static over a certain number of realizations to align interference subspaces. The ICI is mitigated by allocating at adjacent transmitters distinct outer precoders, which can provide the required signal subspace for every user. In [6], [8] it was shown that the IUI management can be performed independently at each transmitter using local CSI for any particular objective function, e.g., maximizing the weighted sum-rate [17], [18] or providing quality of service [19].

At a high level, SRP resembles CDMA systems in which each transmitter uses a signature (the ICI precoder in SRP), that enables suppression of interference imposed on users at other cells [9]. SRP is also analogous to frequency reuse [20], since the function of the ICI precoders is to isolate neighboring cells from one another so that their transmissions are carried out over orthogonal resources. There are no parametrization framework or implementation guidelines to design or characterize SRP in the current literature. The work in [9] investigated the degrees of freedom achieved by SRP assuming static channels over an asymptotically large number of channel uses. The lack of any structure of the ICI and IUI precoders in [9] makes hard to optimize the overall performance, set independent design variables, and provides no insights regarding the capabilities and limitations of SRP for more practical implementations.

A. Research Scope

It should be stressed that the main motivation of SRP is to mitigate ICI generated by the dominant sources of interference and complement other techniques that improve the effective signal-to-interference-plus-noise ratio (SINR) at the receivers. SRP does not require significantly more feedback or coordination between transmitters than current systems that manage interferences on a cell-by-cell basis. There are important advantages of SRP relative to traditional cooperative transmission schemes (e.g., network MIMO [3] or CoMP [18]): several transmissions from different sources are supported over the same frequency band, on-line cooperation between transmitters is not required to mitigate ICI, and resource allocation can be applied in a single-cell fashion. In this paper, we address several problems regarding SRP design and performance optimization. The aim of this work is manifold: to develop pragmatic transmission schemes based on the principles of IA that can be scaled with the size of the network; to propose several applications at the signal processing and network planning levels; and to assess if this sort of IA schemes can provide gains for general MIMO configurations. Our contributions are the following:

- In contrast to [9] where all design parameters are coupled, we have derived a parametrization framework to construct the cascaded precoders where the number of quasi-static channel uses is a free design variable. This parametrization allows practical implementation of the precoding reuse scheme, e.g., the number of channel uses might represent time-slots within a frame in standard wireless technologies. Moreover, the proposed approach only relies on local CSI to construct the precoders.
- We defined the conditions to implement precoding reuse, and established the operative values for all system parameters, i.e., number of deployed transmitters, number of transmit and receive antennas, co-scheduled users per transmitter, and required

number of channel uses. Furthermore, our approach generalizes the work in [9] by allowing more than one data stream per user.

- It was introduced a general methodology to construct structured cascaded precoders so that the proposed scheme can be implemented for both coordinated or uncoordinated transmission, attaining interference suppression without on-line cooperation between an arbitrary number of transmitters.
- We analyzed and assessed the role of each parameter affecting the ICI precoders and presented several design guidelines. We also investigated the limitations and advantages of the proposed transmission scheme, and presented a discussion regarding suitable scenarios and applications.

B. Organization and Notation

The rest of the paper is organized as follows. Section II presents the conventional and extended signal models and the setting characteristics. Section III introduces the sequential procedure to construct the cascaded precoders and the proposed parametrization. The parametrization for uncoordinated and coordinated transmission are presented in Section IV and Section V, respectively. Numerical simulations are presented in Section VI and conclusions are provided in Section VII. We use the following notation: matrices and vectors are set in upper and lower boldface, respectively. $Tr(\cdot)$, $\lfloor \cdot \rfloor$, $(\cdot)^T$, $(\cdot)^H$, $\| \cdot \|$ denote the trace, floor, transpose, hermitian, and norm operations, respectively. $\mathcal{V}(\mathbf{H})$ designates the subspace spanned by the matrix $\mathbf{H} \in \mathbb{C}^{m \times n}$, whose left and right null spaces are defined as $\mathcal{N}_L(\mathbf{H}) = \{\mathbf{x} \in \mathbb{C}^{m \times 1} : \mathbf{x}^H \mathbf{H} = \mathbf{0}\}$ and $\mathcal{N}_R(\mathbf{H}) = \{\mathbf{y} \in \mathbb{C}^{n \times 1} : \mathbf{H}\mathbf{y} = \mathbf{0}\}$, respectively. $\dim(\cdot)$ denotes the dimension of the operated subspace, $\mathrm{rank}(\cdot)$ denotes the rank of the operated matrix, and $\otimes$ denotes the Kronecker product. $\mathbf{I}_n$ is the identity matrix of size n. $\mathbb{E}[\cdot]$ denotes expectation and $\mathbb{Z}_+$ is the set of nonnegative integer numbers. Throughout the paper we say that a matrix $\mathbf{X}$ is drawn from a continuous distribution if its element $x_{ij} \sim \mathcal{CN}(0,1)\ \forall i,j$, is circularly symmetric complex Gaussian distributed with zero mean and unit variance.

II. System Model

A. Linear Signal Model

Consider a scenario with B transmitters each equipped with M antennas, and each transmitter sends narrowband signals to its own set of $\tilde{K}$ distinct users. Every user is equipped with N receive antennas and the k-th user served by the i-th transmitters, denoted as (i,k), receives d_{ik} data streams. Throughout the paper we consider $M \geq N$. The signal received by the (i,k)-th user at time t is modeled as

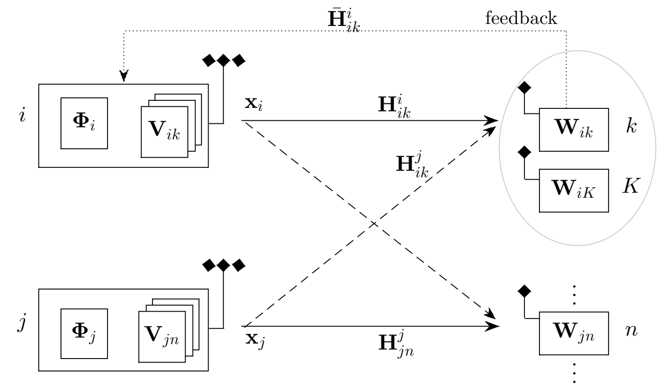

Fig. 1. The structured outer precoder $\mathbf{\Phi}_i$ confine the local channels at the i-th transmitter to a reduce signal subspace, which allow distributed ICI suppression. The inner precoder $\mathbf{V}_{ik}$ provide multiplexing gains and mitigate IUI for each user k served by the i-th transmitter.

$$\tilde{\mathbf{y}}_{ik}[t] = \tilde{\mathbf{H}}_{ik}^i[t]\tilde{\mathbf{x}}_i[t] + \sum_{j=1, j\neq i}^{B} \tilde{\mathbf{H}}_{ik}^j[t]\tilde{\mathbf{x}}_j[t] + \tilde{\mathbf{n}}_{ik}[t], \quad (1)$$

where $\tilde{\mathbf{y}}_{ik}[t] \in \mathbb{C}^{N \times 1}$ and $\tilde{\mathbf{H}}_{ik}^j[t] \in \mathbb{C}^{N \times M}$ is a full rank channel matrix between the (i,k)-th user and the j transmitter. The elements of $\tilde{\mathbf{H}}_{ik}^j[t]$ are Gaussian distributed with zero mean and unit variance. The information signal vector $\mathbf{s}_{ik}[t] \in \mathbb{C}^{d_{ik} \times 1}$ is linearly processed (subject to $\sum_{k=1}^{\tilde{K}} d_{ik} \leq M$ cf. [17]), which yields the precoded transmitted signal $\tilde{\mathbf{x}}_i[t]$ with power constraint $E[\|\tilde{\mathbf{x}}_i[t]\|^2] \leq P$. The noise vector $\tilde{\mathbf{n}}_{ik}[t]$ is AWGN with unit variance. Hereafter we set the same number of data streams per user $d_{ik} = d$ for all i,k with $d \in \{1, \ldots, N\}$, i.e., all transmitters send d streams to each of their scheduled users. If zero forcing processing is used, cf. [17], then the i-th transmitter can serve up to $\tilde{K} \leq \left\lfloor \frac{M}{d} \right\rfloor$ simultaneous users.

B. Kronecker Structured Signal Model

Hereafter, consider block-fading channels [21], which means that the MIMO CSI is constant for a number of consecutive channel uses before changing independently. Throughout, it will be assumed that the coherence time of the multiplicative fading process, T_{coh}, exceeds the minimum required transmission time. Consider a signaling scheme based in blocks of size T_c, where the channels are quasi-static in the sense that they vary slowly enough that can be modeled as being constant over $T_c < T_{coh}$ channel uses. Under this assumption let

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TWC.2017.2670540, IEEE Transactions on Wireless Communications

$\tilde{\mathbf{H}}_{ik}^{j} = \tilde{\mathbf{H}}_{ik}^{i}[1] = \ldots = \tilde{\mathbf{H}}_{ik}^{i}[T_c]$ be a constant matrix during T_c channel uses, and define the block received signal as

$$\mathbf{y}_{ik} = [\tilde{\mathbf{y}}_{ik}[1], \ldots, \tilde{\mathbf{y}}_{ik}[t], \ldots, \tilde{\mathbf{y}}_{ik}[T_c]]^T, \tag{2}$$

where $\tilde{\mathbf{y}}_{ik}[t]$ is the received signal at user (i, k) at the t-th channel use. The block noise $\mathbf{n}_{ik}$ vector is defined analogously. The receive signal of user (i, k) over T_c channel uses can be expressed as

$$\mathbf{y}_{ik} = \sum_{j=1}^{B} \mathbf{H}_{ik}^{j} \mathbf{x}_j + \mathbf{n}_{ik}, \tag{3}$$

where $\mathbf{H}_{ik}^{j} \in \mathbb{C}^{T_c N \times T_c M}$ is a 2-*level decomposable* block diagonal matrix (in the sense of [22]), with diagonal blocks $\tilde{\mathbf{H}}_{ik}^{j}$ defined mathematically as [9]

$$\mathbf{H}_{ik}^{j} = \mathbf{I}_{T_c} \otimes \tilde{\mathbf{H}}_{ik}^{j}. \tag{4}$$

In order to suppress interference the proposed scheme relies on two cascaded precoders (cf. [8], [9]), as illustrated in Fig. 1. Denote the inner precoder that cancels IUI as $\mathbf{V}_{ik}$, and the outer precoder to suppress ICI as $\mathbf{\Phi}_i$. The spatio-temporal precoder matrix of the user (i, k) can be defined as $\mathbf{F}_{ik} = \mathbf{\Phi}_i \mathbf{V}_{ik}$, and let the transmitted signal be expressed as

$$\mathbf{x}_i = \sum_{k=1}^{K} \mathbf{F}_{ik} \mathbf{s}_{ik}, \tag{5}$$

where $\mathbf{s}_{ik} = \mathbf{s}_{ik}[1] = \ldots = \mathbf{s}_{ik}[T_c]$ is the transmitted vector of symbols over a block of T_c channel uses. It is worth pointing out that the total number of co-scheduled users, K, at the i-th transmitter is not only a function of M and N (as $\tilde{K}$ in Section II-A), but also depends on B and T_c. The range of values for K will be elaborated upon Section III-C.

The columns of $\mathbf{V}_{ik}$ are the orthonormal basis of the transmitted signal space of user (i, k) and we will present a method to construct them in Section IV-B. The matrices $\mathbf{V}_{ik}$, $\forall k$, suppress IUI for the K simultaneously co-scheduled users at the i-th transmitter only using local CSI. Let us define unitary matrix $\mathbf{T} \in \mathbb{C}^{T_c \times T_c}$, and the outer precoder $\mathbf{\Phi}_i$ to confine the signal subspace of the receivers in cell i is defined as [9]:

$$\mathbf{\Phi}_i = [\mathbf{T}_i^1 \otimes \mathbf{T}_i^2]. \tag{6}$$

where the Kronecker structure above is built off-line. To construct the matrices comprising $\mathbf{\Phi}_i$, consider that each temporal processing matrix $\mathbf{T}_i^1 \in \mathbb{C}^{T_c \times \beta}$ comprises β distinct columns of $\mathbf{T}$. In Section IV it will be shown that the spatial processing matrix $\mathbf{T}_i^2 \in \mathbb{C}^{M \times M}$ can be defined as a unitary matrix. The proposed design provides a well defined structure for $\mathbf{\Phi}_i$, which can be used to improve performance and simplify implementation. Our design contrasts to the scheme proposed in [9], where the matrices comprising $\mathbf{\Phi}_i$ are random and lack of any special structure. The following sections will explain the role of $\mathbf{T}_i^1$ and $\mathbf{T}_i^2$ in the system performance, as well as the operative values of the parameters T_c and β.

C. Downlink Interference Alignment

The receivers use linear processing to suppress interference and perform separate decoding of their data streams. The input at the decoder of user (i, k) can be modeled as

$$\begin{aligned}\hat{\mathbf{y}}_{ik} = &\underbrace{\mathbf{W}_{ik}^H \mathbf{H}_{ik}^i \mathbf{F}_{ik} \mathbf{s}_{ik}}_{\text{desired signal}} + \underbrace{\mathbf{W}_{ik}^H \mathbf{H}_{ik}^i \sum_{l=1, l \neq k}^{K} \mathbf{F}_{il} \mathbf{s}_{il}}_{\text{intra-cell interference}} + \\ &\underbrace{\mathbf{W}_{ik}^H \sum_{j=1, j \neq i}^{B} \mathbf{H}_{ik}^j \sum_{n=1}^{K} \mathbf{F}_{jn} \mathbf{s}_{jn}}_{\text{inter-cell interference}} + \mathbf{W}_{ik}^H \mathbf{n}_{ik},\end{aligned} \tag{7}$$

where $\mathbf{W}_{ik} \in \mathbb{C}^{T_c N \times d_{ik}}$ is a receive beamforming matrix calculated by the user (i, k). The characteristics and methods to construct the receive filters depend on the values of T_c and β, which will be elaborated upon in Section IV and Section V. To recover the desired signal, the receive filters and spatial-temporal precoders must be designed under the IA principle [7]. The objective of the proposed scheme is to find the signal subspaces at the transmitters and the interference subspaces at the receivers to satisfy simultaneously the following [9]:

$$\text{rank}(\mathbf{W}_{ik}^H \mathbf{H}_{ik}^i \mathbf{F}_{ik}) = d_{ik}, \tag{8}$$

$$\mathbf{W}_{ik}^H \mathbf{H}_{ik}^i \mathbf{F}_{il} = \mathbf{0}, \quad \forall l \neq k, \tag{9}$$

$$\mathbf{W}_{ik}^H \mathbf{H}_{ik}^j \mathbf{F}_{jn} = \mathbf{0}, \quad \forall j \neq i. \tag{10}$$

where the condition in (8) implies that the signal subspace has dimension d_{ik}, whilst (9) and (10) provide the conditions to suppress IUI and ICI, respectively. Authors in [23] showed that in the absence of a special structure in the channel matrices $\mathbf{H}_{ik}^j$ and for arbitrary large values of M and N, it is an NP-hard problem to maximize the number of simultaneous data streams, i.e., $\sum_{j=1}^{B} \sum_{k=1}^{K} d_{jk}$, and determine whether the equations above have a joint solution. Nevertheless, recent works have proposed different approaches to solve the IA problem under certain network configurations relying only on local CSI feedback, cf. [8], [9].

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TWC.2017.2670540, IEEE Transactions on Wireless Communications

III. General Kronecker Structured Linear Spatial-Temporal Precoding

This section presents the sequential mechanism used to generate the outer and inner precoders, and hereafter define $d_{ik} = d$, $\forall i, k$. We will present a general parametrization that establishes the relationship between T_c and B. We also define the maximum number of co-scheduled users per transmitter given the parameters M and N.

A. Sequential Cascaded Precoding Design

The proposed downlink IA scheme (IAS) requires several phases to cancel ICI and IUI, by using training sequences for the CSI acquisition, as illustrated in Fig. 2. This scheme does not require on-line coordination between transmitters and is analogous to the approaches developed in [8], [9]. The transmitters design off-line their temporal precoders $\mathbf{\Phi}_i$ independently of the MIMO channel. In the training phase, user (i, k) learns its direct channel $\mathbf{H}^i_{ik}\mathbf{\Phi}_i$, the cross channel from the j-th transmitter, $\forall j \neq i$, defined as

$$\mathbf{Z}^j_{ik} = \mathbf{H}^j_{ik}\mathbf{\Phi}_j, \tag{11}$$

and the matrix of cross channels defined as

$$\mathbf{Z}_{ik} = [\mathbf{Z}^1_{ik}, \ldots, \mathbf{Z}^{i-1}_{ik}, \mathbf{Z}^{i+1}_{ik}, \ldots, \mathbf{Z}^B_{ik}]. \tag{12}$$

Depending on the parametrization of $\mathbf{\Phi}_i$, which will be explained in more detail in the following sections, the (i, k)-th user designs its receive filter at the end of a training phase, so that ICI is canceled, i.e., $\mathbf{W}^H_{ik}\mathbf{Z}_{ik} = \mathbf{0}$ or similarly $\mathbf{W}_{ik} \in \mathcal{N}_L(\mathbf{Z}_{ik})$.

The (i, k)-th user feeds back its effective channel $\bar{\mathbf{H}}^i_{ik} = \mathbf{W}^H_{ik}\mathbf{H}^i_{ik}\mathbf{\Phi}_i \in \mathbb{C}^{d\times\beta M}$ to transmitter i, and the inner precoder will be computed so that $\mathbf{V}_{ik} \in \mathcal{N}_R(\bar{\mathbf{H}}_{ik})$, where $\bar{\mathbf{H}}_{ik}$ is the matrix of effective channels of the co-scheduled users excluding the k-th user, defined as

$$\bar{\mathbf{H}}_{ik} = [\ldots, (\bar{\mathbf{H}}^i_{ik-1})^T, (\bar{\mathbf{H}}^i_{ik+1})^T, \ldots]^T. \tag{13}$$

The spatial precoder can be designed to meet a particular performance objective, such as weighted sum-rate maximization with optimal (e.g. [23]) or suboptimal (e.g. [17]) linear processing. Finally, to perform coherent detection and recover $\mathbf{s}_{ik}$, the (i, k)-th user determines $\mathbf{V}_{ik}$ during a dedicated training phase (in the sense of [24]) at the single-cell level. The subspace and vector operations that are performed at the transmitter and receiver sides are illustrated in Fig. 3.

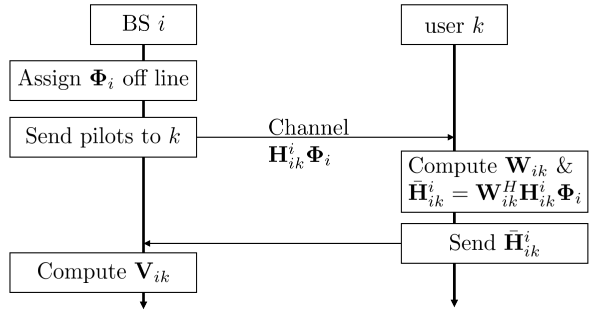

Fig. 2. Sequential Cascaded Precoding Design and CSI acquisition.

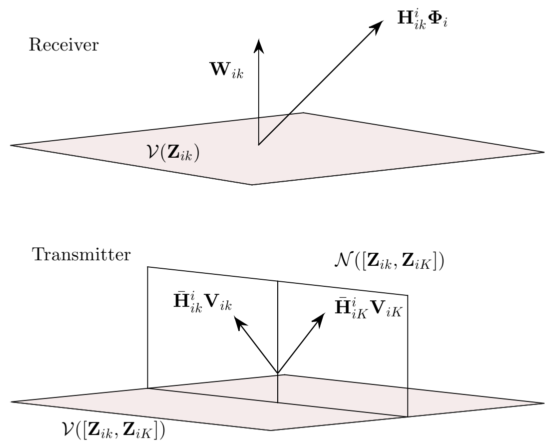

Fig. 3. The desired signal and interference span different subspaces and the IA is achieved in two phases, *i*) mitigating ICI at the receiver and *ii*) suppressing IUI over a dimension-reduced subspace at the transmitter.

B. Outer Precoder Parametrization

Based on the sequential interference mitigation scheme presented above, we need to define the operative values of T_c, B, and β. We have considered block-fading channels (4), with blocks of T_c transmission intervals, where independent data streams are spread over the spatial and temporal domains. Practical values of T_c are expected to be within a small range of symbol intervals that guarantees $T_c < T_{coh}$. The coherence time depends on the Doppler spread, the operative frequency, the characteristics of the employed wireless technology and the user mobility [21], [25]. The number of quasi-static channel uses is a design parameter that should be fixed since its value modifies the characteristics of the cascaded precoders, $\mathbf{\Phi}_i$, $\mathbf{V}_{ik}$, and received filter $\mathbf{W}_{ik}$. This is in contrast to the scheme presented in [9] where all variable are coupled.

By taking T_c as a multiple of B, i.e., $T_c = \beta B$ and $\beta \in \mathbb{Z}_+$, it can be guaranteed that the outer precoders are constructed with β distinct columns of $\mathbf{T} \in \mathbb{C}^{T_c\times T_c}$. To verify that for such parametrization the ICI can

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TWC.2017.2670540, IEEE Transactions on Wireless Communications

be eliminated, it is necessary to satisfy the following constraint

$$\dim(\mathcal{N}_L(\mathbf{Z}_{ik})) \geq d, \qquad \forall i, k \tag{14}$$

which guarantees the existence of $\mathbf{W}_{ik}$, [cf. Section III-A]. This means that the subspace spanned by the effective channel $\bar{\mathbf{H}}_{ik}^{i}$ can be a subspace of dimension at least equal to d. Express (11), the precoded cross channel from the j-th transmitter to user (i,k), $\mathbf{Z}_{ik}^{j} \in \mathbb{C}^{T_c N \times \beta M}$, as [cf. Appendix A]:

$$\mathbf{Z}_{ik}^{j} = [\mathbf{I}_{T_c} \otimes \tilde{\mathbf{H}}_{ik}^{j}][\mathbf{T}_j^1 \otimes \mathbf{T}_j^2] = [\mathbf{T}_j^1 \otimes \tilde{\mathbf{H}}_{ik}^{j} \mathbf{T}_j^2], \tag{15}$$

whose rank can be computed as [cf. Appendix A]:

$$\begin{aligned} \operatorname{rank}(\mathbf{Z}_{ik}^{j}) &= \operatorname{rank}(\mathbf{T}_j^1) \min\{\operatorname{rank}(\tilde{\mathbf{H}}_{ik}^{j}), \operatorname{rank}(\mathbf{T}_j^2)\} \\ &= \beta N. \end{aligned} \tag{16}$$

Consider $\mathbf{Z}_{ik} \in \mathbb{C}^{T_c N \times [(B-1)(\beta M)]}$ defined in (12), and generalizing (16) for $B-1$ sources of interference yields

$$\operatorname{rank}(\mathbf{Z}_{ik}) = (B-1)(\beta N), \tag{17}$$

and the interference subspaces at all receivers have the same dimensions such that

$$\dim(\mathcal{N}_L(\mathbf{Z}_{ik})) = T_c N - \operatorname{rank}(\mathbf{Z}_{ik}), \qquad \forall i, k. \tag{18}$$

Recall that the number of supported streams d is the same for all users and using (18), it can be verified that (14) is satisfied for all $d \leq N$. Therefore, the proposed IAS is feasible for all $T_c \geq B \geq 2$, and the parametrization accepts any valid antenna configuration, $M \geq N$, and do not restrict the maximum number of channel uses T_c. In contrast to the scheme presented in [9], in the IAS T_c is a free design variable and the scheme is valid for all $1 \leq d \leq N$, which provides a flexible system parametrization for an arbitrary number of transmitters B.

C. Degrees of Freedom

The number of supported streams by the i-th transmitter is defined by the dimensions of the matrix that collects all local effective channels, $\bar{\mathbf{H}}_i = [(\bar{\mathbf{H}}_{i1}^{i})^T, \ldots, (\bar{\mathbf{H}}_{iK}^{i})^T]^T \in \mathbb{C}^{Kd \times \beta M}$. Observe that the columns of $\bar{\mathbf{H}}_i$ are determined by $\mathbf{\Phi}_i$, and the role of β is to scale the dimension of the signal subspace per transmitter according to T_c. To guarantee that each user can receive d streams we need to satisfy the following constraint

$$\dim(\mathcal{N}_R(\bar{\mathbf{H}}_{ik})) \geq d, \qquad \forall i, k \tag{19}$$

which provides the condition to cancel IUI generated by $K-1$ co-scheduled users, so that $\mathcal{V}(\bar{\mathbf{H}}_{ik}^{i} \mathbf{V}_{ik})$ can be a subspace of dimension at least equal to d. From (19) and (13) we have

$$\beta M - (K-1)d \geq d. \tag{20}$$

Assuming linear processing to construct the inner precoders the number of simultaneous users supported per transmitter is upper bounded as

$$K \leq \left\lfloor \frac{\beta M}{d} \right\rfloor. \tag{21}$$

Observe that the bound of K grows with T_c, however, the derived results presented in the following sections show that the operational values of T_c must be confined to limited ranges for practical scenarios. Operating within the coherence time, i.e., $T_c < T_{coh}$, is a critical design constraint that must be satisfied [26], which also impose a physical constraint on the maximum value of clustered transmitters B.

The system performance in the interference-limited regime can be captured by the notion of degrees-of-freedom (DoF) [8], [10]. The DoF can be interpreted as the first-order approximation of the maximum sum-rate capacity of a MIMO system at high SNR [23], or as the number of sent data streams over the total number of dimensions required to transmit them [6]. The proposed IAS uses T_c channel uses to allow B transmitters to send simultaneously Kd streams each, resulting in DoF $= \frac{BKd}{T_c}$ in the whole network.

IV. Uncoordinated IAS

A. Receive Filter Design

The construction of the outer precoders $\mathbf{\Phi}_i = [\mathbf{T}_i^1 \otimes \mathbf{T}_i^2]$ presented in Section II-B highly simplifies the construction of the receive filters $\mathbf{W}_{ik}$ that remove ICI. The principle behind a distributed IAS is that the temporal matrices $\mathbf{T}_i^1$ are constructed in such a way that the users only need to learn their direct channel $\mathbf{H}_{ik}^{i}\mathbf{\Phi}_i$ to mitigate ICI, but not the cross channels $\mathbf{H}_{ik}^{j}\mathbf{\Phi}_j$, $\forall j \neq i$. The following result summarizes the advantages of the parametrization defined above.

Theorem 1. *Consider that the parameters $M \geq N \geq d \geq 1$, $B \geq 2$, $T_c = \beta B$, and $\beta \geq 1$. Let $\mathbf{T}_i^1 \in \mathbb{C}^{T_c \times \beta}$, $\forall i \in \{1, \ldots, B\}$, be constructed from distinct columns of the unitary matrix $\mathbf{T} \in \mathbb{C}^{T_c \times T_c}$. The receive filter $\mathbf{W}_{ik}$ for the (i,k)-th user that maximizes the received signal strength and suppresses ICI without knowledge of the cross channels, $\mathbf{H}_{ik}^{j}\mathbf{\Phi}_j$, $\forall j \neq i$, is given by the d left-eigenvectors associated to the largest eigenvalues of the direct channel $\mathbf{H}_{ik}^{i}\mathbf{\Phi}_i$.*

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TWC.2017.2670540, IEEE Transactions on Wireless Communications

Proof. Let us define the direct channel of the (i,k)-th user from Appendix A, (4) and (6) as:

$$\mathbf{D}_{ik}^{i} = \mathbf{H}_{ik}^{i}\mathbf{\Phi}_i = [\mathbf{T}_i^1 \otimes \tilde{\mathbf{H}}_{ik}^{i}\mathbf{T}_i^2], \tag{22}$$

and bear in mind the cross channel from the j-th BS defined in (15). Consider that two temporal precoding matrices $\mathbf{T}_i^1$ and $\mathbf{T}_j^1$, $\forall j \neq i$, are constructed from distinct columns of the unitary matrix $\mathbf{T}$ so that $(\mathbf{T}_i^1)^H\mathbf{T}_j^1 = \mathbf{0}$. Consequently, by construction $(\mathbf{D}_{ik}^i)^H\mathbf{Z}_{ik}^j = \mathbf{0}$, regardless of the channel realization and spatial processing matrices $\mathbf{T}_i^2$ and $\mathbf{T}_j^2$

$$\begin{aligned}(\mathbf{D}_{ik}^{i})^H\mathbf{Z}_{ik}^{j} &= [\mathbf{T}_i^1 \otimes \tilde{\mathbf{H}}_{ik}^{i}\mathbf{T}_i^2]^H[\mathbf{T}_j^1 \otimes \tilde{\mathbf{H}}_{ik}^{j}\mathbf{T}_j^2], \\ &= [(\mathbf{T}_i^1)^H\mathbf{T}_j^1 \otimes (\tilde{\mathbf{H}}_{ik}^{i}\mathbf{T}_i^2)^H\tilde{\mathbf{H}}_{ik}^{j}\mathbf{T}_j^2],\end{aligned} \tag{23}$$

which implies that the left-eigenvectors of $\mathbf{D}_{ik}^i$ associated to the non-zero eigenvalues lie in a subspace orthogonal to $\mathcal{V}(\mathbf{Z}_{ik}^j)$. Let $\mathbf{T}_i^1 = \mathbf{\Upsilon}_{\mathbf{T}^1}\mathbf{\Sigma}_{\mathbf{T}^1}\mathbf{\Xi}_{\mathbf{T}^1}^H$ and $\tilde{\mathbf{H}}_{ik}^i = \mathbf{\Upsilon}_{\tilde{\mathbf{H}}}\mathbf{\Sigma}_{\tilde{\mathbf{H}}}\mathbf{\Xi}_{\tilde{\mathbf{H}}}^H$ be the SVD of the matrix elements comprising $\mathbf{D}_{ik}^i$, and substituting them into (22) yields [27], [see Appendix A]:

$$\begin{aligned}\mathbf{D}_{ik}^{i} &= [\mathbf{\Upsilon}_{\mathbf{T}^1}\mathbf{\Sigma}_{\mathbf{T}^1}\mathbf{\Xi}_{\mathbf{T}^1}^H \otimes (\mathbf{\Upsilon}_{\tilde{\mathbf{H}}}\mathbf{\Sigma}_{\tilde{\mathbf{H}}}\mathbf{\Xi}_{\tilde{\mathbf{H}}}^H)\mathbf{T}_i^2], \\ &= [\mathbf{\Upsilon}_{\mathbf{T}^1} \otimes \mathbf{\Upsilon}_{\tilde{\mathbf{H}}}][\mathbf{\Sigma}_{\mathbf{T}^1} \otimes \mathbf{\Sigma}_{\tilde{\mathbf{H}}}] \\ &\quad \times [\mathbf{\Xi}_{\mathbf{T}^1} \otimes \mathbf{\Xi}_{\tilde{\mathbf{H}}}]^H[\mathbf{I}_\beta \otimes \mathbf{T}_i^2].\end{aligned} \tag{24}$$

The matrix $[\mathbf{I}_\beta \otimes \mathbf{T}_i^2] \in \mathbb{C}^{\beta M \times \beta M}$ performs a rotation of $[\mathbf{T}_i^1 \otimes \tilde{\mathbf{H}}_{ik}^i]$ over the spatial dimension. According to [28, Section 6.1], $\mathcal{V}(\tilde{\mathbf{H}}_{ik}^i\mathbf{T}_i^2) \subset \mathcal{V}(\tilde{\mathbf{H}}_{ik}^i)$, hence to preserve the dimension of the subspace spanned by the channel $\tilde{\mathbf{H}}_{ik}^i$, without lost of generality, set $\mathbf{T}_i^2 = \mathbf{I}_M$ in (24). Notice that $\text{rank}(\mathbf{T}_i^1) = \beta$ and $\text{rank}(\tilde{\mathbf{H}}_{ik}^i) = N$, and from the Kronecker product $[\mathbf{\Sigma}_{\mathbf{T}^1} \otimes \mathbf{\Sigma}_{\tilde{\mathbf{H}}}]$ in (24) it can be observed that $\mathbf{D}_{ik}^i$ has βN non-zero eigenvalues, [cf. Appendix A]. By letting $\mathbf{W}_{ik}$ be the first d dominant left-eigenvectors of $\mathbf{D}_{ik}^i$, the receive filter will perform a projection (spatio-temporal) of the dominant eigenvectors of $\tilde{\mathbf{H}}_{ik}^i$ onto a number of basis of $\mathcal{N}_L(\mathbf{Z}_{ik})$, which are constructed from $\mathbf{T}_i^1$. □

From the previous results several design guidelines regarding the operational values of the parameters can be provided and some observations are in order:

1) The theorem implies that no cooperation among transmitters is required, i.e., dedicated training in the sense of [24] or learning the cross channels (or their null space) as in [8], [9], [29] can be avoided. Since $\mathbf{W}_{ik}$ lies in $\mathcal{V}(\mathbf{H}_{ik}^i\mathbf{\Phi}_i)$, knowledge of the full subspace $\mathcal{N}_L(\mathbf{Z}_{ik})$ is not needed.
2) The design of matrix $\mathbf{\Phi}_i$ as suggested above guarantees the suppression of the ICI and makes $\mathbf{W}_{ik}$ a singular value decomposition (SVD)-based receive filter [30]. In other words, $\mathbf{W}_{ik}$ is a matched receive filter that can be easily constructed since $\mathbf{\Phi}_i$ is a fixed matrix and $\tilde{\mathbf{H}}_{ik}^i$ is learned at the beginning of the training phase.
3) The analysis above shows that the spatial processing matrix, $\mathbf{T}_i^2$, should be designed so that the dimension of the subspace spanned by the direct channel is not affected, i.e., $\mathcal{V}(\tilde{\mathbf{H}}_{ik}^i\mathbf{T}_i^2) = \mathcal{V}(\tilde{\mathbf{H}}_{ik}^i)$. This is achieved if $\mathbf{T}_i^2 \in \mathbb{C}^{M\times M}$ has linearly independent columns, e.g., it is a unitary matrix. This contrasts with the design proposed in [9], where the spatial processing matrix is drawn from a continuous distribution.
4) To illustrate the ICI suppression, consider a toy example with two transmitters i and j, e.g. Fig. 1, so that $\mathbf{T}_i^1 = [1,1]^T$, $\mathbf{T}_j^1 = [1,-1]^T$, and $\mathbf{T}_i^2 = \mathbf{T}_j^2 = \mathbf{I}_M$. On the one hand, we can see that the spatial directions of the direct channel of the (i,k) user $\mathbf{D}_{ik}^i$ in (22) and its cross channel $\mathbf{Z}_{ik}^i$ in (11) are orthogonal according to (23). On the other hand, antenna selection can be performed as in [13], [14], if the spatial processing matrix $\mathbf{T}_i^2$ has some of its diagonal elements equal to zero.

B. Inner Precoder Optimization

As previously outlined in Section III-A, the effective channel of the (i,k)-th user, $\bar{\mathbf{H}}_{ik}^i$, is sent to the i-th transmitter and the matrix $\bar{\mathbf{H}}_{ik} \in \mathbb{C}^{[d(K-1)]\times\beta M}$ is constructed, cf. (13). The inner precoders must fulfill the zero-interference constraint (9), i.e., $\bar{\mathbf{H}}_{ik}^i\mathbf{V}_{il} = \mathbf{0}$, $\forall k \neq l$. Since the ICI has been mitigated at the receivers, we can formulate the capacity maximization problem subject to zero IUI, $\forall d \geq 1$ as a block diagonalization optimization at the i-th transmitter [17]:

$$\begin{aligned}\text{maximize} \quad & \sum_{k=1}^{K}\log_2\left|\mathbf{I}_d + \bar{\mathbf{H}}_{ik}^i\mathbf{V}_{ik}\mathbf{Q}_{ik}\mathbf{V}_{ik}^H(\bar{\mathbf{H}}_{ik}^i)^H\right| \\ \text{subject to} \quad & \mathbf{H}_{ik}^i\mathbf{V}_{il} = \mathbf{0}, \qquad \forall k \neq l \\ & \sum_{k=1}^{K} Tr(\mathbf{Q}_{ik}) \leq P\end{aligned} \tag{25}$$

where $\mathbf{Q}_{ik} = \mathbb{E}[\mathbf{s}_{ik}\mathbf{s}_{ik}^H]$ is the input covariance matrix of user (i,k). The solution of (25) can be obtained by the water-filling algorithm (to construct the elements in the diagonal of $\mathbf{Q}_{ik}$), over the eigenvalues of $\bar{\mathbf{H}}_{ik}^i\mathbf{V}_{ik}\mathbf{V}_{ik}^H(\bar{\mathbf{H}}_{ik}^i)^H$.

In this section we are concern about the construction of the precoders $\mathbf{V}_{ik}$, which can be found using SVD [17]. To guarantee that for all users the constraint (9) is satisfied, the inner precoder $\mathbf{V}_{ik} \in \mathbb{C}^{\beta M\times d}$ must be constructed within the IUI-free subspace $\mathcal{N}_R(\bar{\mathbf{H}}_{ik})$, whose dimension is

$$\dim(\mathcal{N}_R(\bar{\mathbf{H}}_{ik})) = \beta M - d(K-1), \tag{26}$$

where the condition (19) must hold. If the number of scheduled users K is less than the upper bound in (21), then (26) is strictly larger than d, and each transmitter can optimize $\mathbf{V}_{ik}$ by properly combining the basis of $\mathcal{N}_R(\bar{\mathbf{H}}_{ik})$.

Let $\mathbf{N}_{\bar{\mathbf{H}}_{ik}}$ be the matrix whose columns are the orthonormal basis of $\mathcal{N}_R(\bar{\mathbf{H}}_{ik})$, and compute the SVD of the product $(\bar{\mathbf{H}}^i_{ik}\mathbf{N}_{\bar{\mathbf{H}}_{ik}}) = \bar{\mathbf{U}}_{ik}\bar{\mathbf{\Sigma}}_{ik}\bar{\mathbf{O}}^H_{ik}$. The inner precoder of the (i,k)-th user is defined as

$$\mathbf{V}_{ik} = \mathbf{N}_{\bar{\mathbf{H}}_{ik}}[\bar{\mathbf{O}}_{ik}]_{1:d}, \tag{27}$$

where $[\mathbf{X}]_{1:q}$ denotes a reduced matrix consisting of the first $\{1,\ldots,q\}$ leftmost columns of the operated matrix $\mathbf{X}$. The works [8], [9] proposed iterative algorithms to compute and refine the IUI precoders based on zero forcing. However, the feedback overhead and computational load may become too large as M and T_c increase.

V. Coordinated IAS

For the sake of comparison, in this section we present the ICI coordination technique necessary when the outer precoders, $\mathbf{\Phi}_i$, lack of structure and have random entries,[1] cf. [9]. For such a design, a training phase is required where pilot signals are sent from all transmitters to all users, such that the (i,k)-th user can learn the direct $(\mathbf{H}^i_{ik}\mathbf{\Phi}_i)$ and cross $(\mathbf{H}^j_{ik}\mathbf{\Phi}_j)$ channels, $\forall j \neq i$.

A. Receive Filter Design

After the training phase, the (i,k)-th user has learned $\mathbf{Z}^j_{ik}$, $\forall j \neq i$, and can construct the receive filter $\mathbf{W}_{ik} \in \mathcal{N}_L(\mathbf{Z}_{ik})$. From the parametrization in Section III-B one can see that increasing the parameters T_c and N results in a larger $\dim(\mathcal{N}_L(\mathbf{Z}_{ik}))$, cf. (18), which implies that we attain more dimensions than required to design the receive filter. To maximize the magnitude of $\bar{\mathbf{H}}^i_{ik} = \mathbf{W}^H_{ik}\mathbf{H}^i_{ik}\mathbf{\Phi}_i$, the receive filter $\mathbf{W}_{ik}$ must be constructed taking into account the direct channel, so that we can achieve coherent combining gain from the desired signal [10]. Let $\mathbf{N}_{\mathbf{Z}_{ik}}$ be the matrix whose columns are the orthonormal basis of $\mathcal{N}_L(\mathbf{Z}_{ik})$, and compute the SVD of the product $(\mathbf{N}_{\mathbf{Z}_{ik}}\mathbf{H}^i_{ik}\mathbf{\Phi}_i) = \mathbf{U}_{ik}\mathbf{\Sigma}_{ik}\mathbf{O}^H_{ik}$. The receive filter aligned to the direct channel is given by

$$\mathbf{W}_{ik} = \mathbf{N}_{\mathbf{Z}_{ik}}[\mathbf{U}_{ik}]_{1:d}. \tag{28}$$

[1]The design presented in [8] lies within this category, where the transmission scheme is defined for $B=2$, $T_c=1$, and random outer precoders.

The design of $\mathbf{W}_{ik}$ can be enhanced as in [10], [12], i.e., if the (i,k)-th user has knowledge of the interference plus noise covariance matrix, such information can be used to maximize the SINR by implementing an MMSE-like receiver.

B. Discussion

For the design presented in Section III the construction of the precoders satisfy the orthogonality constraint $(\mathbf{\Phi}_i\mathbf{V}_{ik})^H(\mathbf{\Phi}_i\mathbf{V}_{ik}) = \mathbf{I}_d$, i.e., the symbols in $\mathbf{s}_{ik}$ can be processed independently at the (i,k)-th receiver. However, if the matrices $\mathbf{T}^1_i$ and $\mathbf{T}^2_i$ are random, the overall design loses flexibility and the number of valid configurations is reduced. One of the main drawbacks of the random matrix design in [9] is that each user can receive at most one data stream. When $\mathbf{\Phi}_i$ is constructed from random matrices its columns will be correlated, which results in $\mathbf{F}^H_{ik}\mathbf{F}_{ik} \neq \mathbf{I}_d$ corrupting the symbol vector $\mathbf{s}_{ik}$. The orthogonality constraint can be satisfied if both $\mathbf{T}^1_i$ and $\mathbf{T}^2_i$ are unitary random matrices, but if orthogonality between $\mathbf{T}^1_i$ and $\mathbf{T}^1_j$, $\forall i \neq j$, is not provided, then the constructions of the receive filter $\mathbf{W}_{ik}$ relies on the knowledge of $\mathcal{N}_L(\mathbf{Z}_{ik})$.

VI. Performance Evaluation

Through numerical simulations, we evaluate the performance of the proposed IAS for downlink transmission. We consider a deployment with the same number of users per transmitter and i.i.d. flat Rayleigh fading propagation channels. We evaluate the performance as a function of the SNR, β, M over 1e6 channel realizations. Equal power allocation is used across the data streams, i.e., $\mathbf{Q}_{ik} = \rho\mathbf{I}_d$, $\forall i,k$, where $\rho = \frac{P}{Kd}$. This simulation setup is suitable for scenarios with B transmitters, where all users have the same long-term channel power gain. This is a common assumption in the literature, e.g., users deployed at the center of a multi-cell cluster or facing sectors of three base stations, where most gains are expected from cooperative or coordinated transmission [12], [20].

Our aim is to assess the proposed parametrization, investigating the role of T_c, B, M, and N on the overall performance and defining guidelines for network design. Assessing the system-level performance considering either a particular cellular or heterogeneous network, and users with different power profiles is beyond the scope of this paper and will be the subject of future work.

In the figures we refer to our proposed design and parametrization as IAS, where $\mathbf{T}^2_i = \mathbf{I}_M$, and the DFT matrix $\mathbf{T}$ is used to construct the matrices $\mathbf{T}^1_i$, $\forall i$. The

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TWC.2017.2670540, IEEE Transactions on Wireless Communications

outer precoders are designed off-line and the implementation of IAS does not require on-line exchange of CSI or user data between transmitters. The curves labeled as M-D refer to the scheme where the random matrices $\mathbf{T}_i^1$, $\mathbf{T}_i^2$ are drawn from a continuous distribution, as proposed by Medra and Davidson in [9]. Notice that for the M-D scheme the receive filters must be computed as described in Section V-A, i.e., the training phase is required by the users to learn their cross channels and compute $\mathbf{W}_{ik} \in \mathcal{N}_L(\mathbf{Z}_{ik})$.

For comparison, we also consider time-division multiplexing (TDM) at the transmitter level. In this scheme only one transmitter sends data streams, while the other $B-1$ transmitters remain silent. We have used the precoding scheme in [17] for the TDM multiuser MIMO system. TDM is a common technique for ICI coordination used in cellular and heterogeneous networks. In standardized systems one transmitter sends data to its users whereas adjacent transmitters send *almost black subframes* preventing ICI at a certain time-frequency resource, which only requires the exchange of control signals among transmitters [31].

A. Capacity as a function of the SNR

To assess the performance of proposed design and parametrization we analyze different $N \times M$ antenna configurations with fixed parameters K and d. The proposed design of $\mathbf{\Phi}_i$ provides considerable spectral efficiency gains w.r.t. the M-D approach. For a target SNR of 20 dB the spectral efficiency of IAS over M-D is about 155% and 163% for the settings in Fig. 4 and Fig. 5, respectively. The results show that the specific choices of $\mathbf{T}_i^1$ and $\mathbf{T}_i^2$ have an impact in the achievable capacity and stress the fact the outer precoders must be jointly optimized (off-line) for all transmitters, in contrast to the approaches in [8], [9]. The proposed IAS outperforms TDM for both configurations in Fig. 4 and Fig. 5. IAS can be applied when all transmitters have acquired their local CSI simultaneously, instead of coordinating their idle stages as in TDM. It is worth noticing that for IAS and M-D schemes the ICI and IUI are zero forced, and the overall performance depends on the correlation between $\mathbf{H}_{ik}$ and $\bar{\mathbf{H}}_{ik}$, as well as the characteristics of the cross channels $\mathbf{Z}_{ik}$ for the M-D scheme. Nonetheless, IAS is more robust than M-D since it does not depend on the null space consistency time [29], i.e., the value of T_c for which $\mathcal{N}_L(\mathbf{Z}_{ik})$ remains constant.

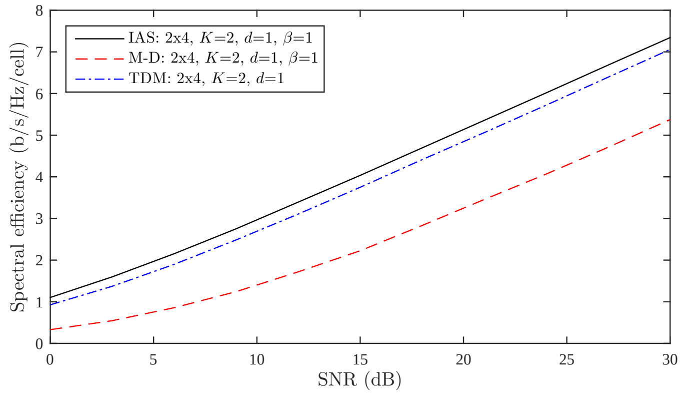

Fig. 4. Spectral efficiency as a function of the SNR for a MIMO setting with $T_c = 3$, $B = 3$, $M = 4$, $N = 2$, $d = 1$ and $K = 2$.

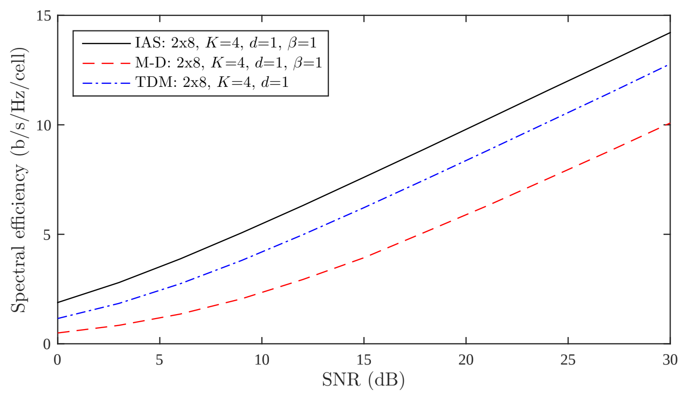

Fig. 5. Spectral efficiency as a function of the SNR for a MIMO setting with $T_c = 3$, $B = 3$, $M = 8$, $N = 2$, $d = 1$ and $K = 4$.

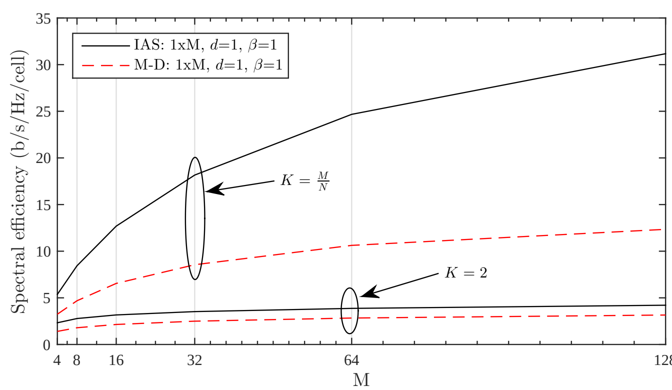

Fig. 6. Spectral efficiency per cell as a function of M in a large-scale MISO scenario for an SNR of 20 dB with $T_c = 3$, $B = 3$, $N = 1$, and $d = 1$.

B. Large-Scale MIMO Scenarios

Large-scale MIMO refers to setting with unconventionally many active antenna elements at the transmitter that can serve a large number of user terminals. Fig. 6 presents a MISO $1 \times M$ configuration and different number of users at the transmitters. Two configurations are presented, $K = 2$ and a fully loaded network

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TWC.2017.2670540, IEEE Transactions on Wireless Communications

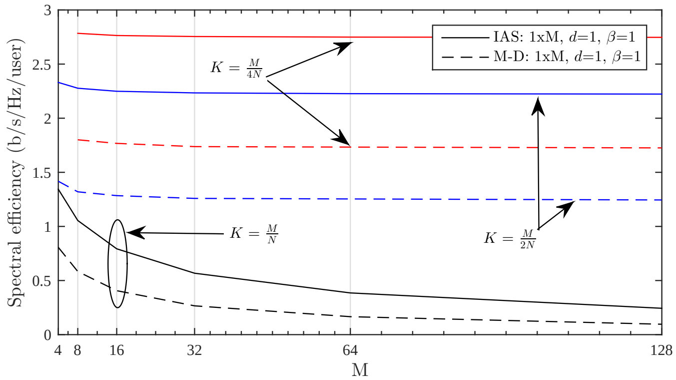

Fig. 7. Spectral efficiency per user as a function of M in a large-scale MISO scenario for an SNR of 20 dB with $T_c = 3$, $B = 3$, $N = 1$, and $d = 1$.

$Kd = \beta M$, cf. (21). For the latter configuration all temporal and spatial dimensions are occupied, i.e., no additional resources to enhance the signal properties are available. The M-D scheme fails to exploit the diversity provided by large values of M, resulting in a spectral efficiency gap of 14 (b/s/Hz/cell) w.r.t. IAS for $K = M/N$, $M = 64$ and $N = 1$.

An important design guideline for large-scale MIMO in [32] establishes that depending on the type of inner precoder, there exists an optimal ratio $\frac{M}{Kd}$, where the spectral efficiency is maximized. Moreover, authors in [5], [33] stress the fact that despite of large values of M, the MIMO channels attain few dominant eigen-directions, which should be used for transmission. Fig. 7 presents the spectral efficiency per user, where the load per transmitter (the number of scheduled users) is defined as $K = \frac{M}{c_1 N}$, where $c_1 \in \mathbb{Z}_+$ is an arbitrary design parameter. Observe that for configurations with $c_1 > 1$ the capacity per user does not degrade as long as the load is $Kd < M$. The performance gain increases because independent and identically distributed large-scale MIMO channels get spatially uncorrelated as M grows, i.e., power transfer across precoded MIMO channels is more efficient [2], [32].

For the partially loaded settings in Fig. 7 the subspace used to construct the inner precoders, cf. (26), is such that $\mathcal{N}_R(\bar{\mathbf{H}}_{ik})$ provides additional dimensions to improve the signal subspace of each scheduled user; cf. (19) and (20). As a result, the zero forcing operation necessary to suppress IUI does not degrade severely the channel magnitude after precoding such that $\|\bar{\mathbf{H}}_{ik}^i \mathbf{V}_{ik}\|_F$ approaches to $\|\bar{\mathbf{H}}_{ik}^i\|_F$. Notice that such gain is achieved for $\beta = 1$, cf. (26), i.e., IAS requires the minimum value of T_c to cancel ICI, and to profit from a large-scale MIMO setting for a deployment with B transmitters.

In partially loaded scenarios, $Kd \ll \beta M$, one may consider different user classes so that other services, such as broadcasting [34], can profit from the additional spatio-temporal dimensions. Notice that user scheduling can improve the performance per stream by selecting the most spatially compatible users (see [4], [35] and references therein). Scheduling can improve the characteristics of $\mathcal{N}_R(\bar{\mathbf{H}}_{ik})$ w.r.t. $\bar{\mathbf{H}}_{ik}^i$, and consequently maximize $\|\bar{\mathbf{H}}_{ik}^i \mathbf{V}_{ik}\|_F$. Designing such scheduling protocols, assessing their joint performance with the proposed scheme, and determining the best operative ranges of K given M are interesting open problems.

C. Other Scenarios

The IAS design might have applications in systems where a set of B transmitters can potentially interfere with each other. In [8], Suh *et al.* presented some applications for an IA scheme for two-transmitter ($B = 2$ and $d = 1$) deployments. The main idea was to design a cellular network so that the dominant ICI was fully canceled. The work in [9] extended the IA scheme to scenarios where $B = 3$, and the spatio-temporal precoding can be used to suppress interference in hexagonal cell arrangements (in a frequency reuse fashion) and unbounded linear-cellular configurations with two dominant interference sources. The IAS extends the ICI suppression for B transmitters and is flexible enough to design heterogeneous networks with several coexisting tiers.

It has been recently reported the combination of frequency reuse, cell sectorization, and coplanar antennae arrays at the transmitters that can achieve high-selectivity angular bemaforming [2]. Among the state-of-the-art antenna array configurations one can find a two-dimensional (2D) grid of many antennas, which is known as full dimension (FD) MIMO [36]. A FD-MIMO channel, $\tilde{\mathbf{H}}_{ik}^{3D}$, can be constructed from the azimuth ($\tilde{\mathbf{H}}_{ik}^{h}$) and elevation ($\tilde{\mathbf{H}}_{ik}^{v}$) domain channels, according to recent 3GPP standard enhancement schemes, such that $\tilde{\mathbf{H}}_{ik}^{3D} = \tilde{\mathbf{H}}_{ik}^{h} \otimes \tilde{\mathbf{H}}_{ik}^{v}$, cf. [36]. This structure not only allows a large number of joint or independent optimization techniques over both spatial domains (azimuth and elevation), but can also be combined with the temporal extension in (4) to define 3-level decomposable structures with several degrees of freedom over multiple optimization domains. Based on the FD-MIMO concept, recent works have developed three-dimensional beamforming schemes that increase the capacity and spatial multiplexing gains in single-cell deployments with single-antenna user terminals, e.g., [37] and references therein. The joint design of reuse precoding and FD-MIMO structures is an interesting open problem.

VII. Conclusions

In this paper we established a framework to design precoding reuse schemes based on interference alignment for downlink communications. We derived necessary conditions to mitigate ICI and IUI either in a coordinated or uncoordinated fashion. We defined the role and operational ranges of the parameters according to the available CSI at the receivers and level of coordination between transmitters. Numerical results show that IAS can be used in several MIMO settings and network deployments, avoiding centralized processing or exchange of CSI between transmitters. We aimed to provide several network-level and signal processing design guidelines to exploit the available spatial and temporal resources. The paper leads to several interesting open problems and the proposed IAS scheme can be used to mitigate interference for multiple antenna configurations.

Appendix A
Properties of Kronecker Product

For sake of completeness, we present a set of formulas used to derive the results in Section III and Section IV. Given a matrix $\mathbf{A} \in \mathbb{C}^{m \times n}$, according to the fundamental theorem of algebra

$$\dim(\mathcal{N}_R(\mathbf{A})) = n - \text{rank}(\mathbf{A}),$$
$$\dim(\mathcal{N}_L(\mathbf{A})) = m - \text{rank}(\mathbf{A}).$$

These formulas can be applied to Kronecker products considering the following properties [27]:

$$(\mathbf{A} \otimes \mathbf{B})^H = \mathbf{A}^H \otimes \mathbf{B}^H$$
$$(\mathbf{A} \otimes \mathbf{B})(\mathbf{C} \otimes \mathbf{D}) = \mathbf{AC} \otimes \mathbf{BD}$$
$$\text{rank}(\mathbf{A} \otimes \mathbf{B}) = \text{rank}(\mathbf{A})\text{rank}(\mathbf{B})$$
$$\text{eig}(\mathbf{A} \otimes \mathbf{B}) = \text{eig}(\mathbf{A}) \otimes \text{eig}(\mathbf{B})$$

where $\text{eig}(\mathbf{A})$ denotes the eigenvalues of matrix $\mathbf{A}$.

References

[1] J. Andrews, W. Choi, and R. Heath, "Overcoming interference in spatial multiplexing MIMO cellular networks," *IEEE Wireless Commun.*, vol. 14, no. 6, pp. 95–104, Dec. 2007.

[2] K. Zheng, L. Zhao, J. Mei, B. Shao, W. Xiang, and L. Hanzo, "Survey of Large-Scale MIMO Systems," *IEEE Commun. Surveys & Tutorials*, vol. 17, no. 3, pp. 1738–1760, 3Q 2015.

[3] D. Gesbert, S. Hanly, H. Huang, S. Shamai Shitz, O. Simeone, and W. Yu, "Multi-Cell MIMO Cooperative Networks: A New Look at Interference," *IEEE Journal on Selected Areas in Commun.*, vol. 28, no. 9, pp. 1380–1408, Dec. 2010.

[4] E. Castañeda, A. Silva, A. Gameiro, and M. Kountouris, "An Overview on Resource Allocation Techniques for Multi-User MIMO Systems," *IEEE Commun. Surveys & Tutorials*, 2016.

[5] A. Liu and V. Lau, "Hierarchical Interference Mitigation for Massive MIMO Cellular Networks," *IEEE Transactions on Signal Processing*, vol. 62, no. 18, pp. 4786–4797, Sept 2014.

[6] J. Chen and V. Lau, "Two-Tier Precoding for FDD Multi-Cell Massive MIMO Time-Varying Interference Networks," *IEEE Journal on Selected Areas in Communications*, vol. 32, no. 6, pp. 1230–1238, June 2014.

[7] S. A. Jafar, *Interference Alignment - A New Look at Signal Dimensions in a Communication Network*. Foundations and Trends(r) in Communications and Information Theory, 2011.

[8] C. Suh, M. Ho, and D. Tse, "Downlink Interference Alignment," *IEEE Trans. on Commun.*, vol. 59, no. 9, pp. 2616–2626, 2011.

[9] A. Medra and T. N. Davidson, "Spatial Reuse Precoding for Scalable Downlink Networks," *IEEE Transactions on Signal Processing*, vol. 63, no. 22, pp. 5976–5989, Oct. 2015.

[10] K. Gomadam, V. Cadambe, and S. Jafar, "A Distributed Numerical Approach to Interference Alignment and Applications to Wireless Interference Networks," *IEEE Transactions on Information Theory*, vol. 57, no. 6, pp. 3309–3322, June 2011.

[11] M. Razaviyayn, G. Lyubeznik, and Z.-Q. Luo, "On the Degrees of Freedom Achievable Through Interference Alignment in a MIMO Interference Channel," *IEEE Transactions on Signal Processing*, vol. 60, no. 2, pp. 812–821, Feb 2012.

[12] P. Ferrand and J.-M. Gorce, "Downlink Cellular Interference Alignment," INRIA, Research Report RR-8543, May 2014. [Online]. Available: https://hal.inria.fr/hal-00996728

[13] X. Rao and V. K. N. Lau, "Interference alignment with partial csi feedback in mimo cellular networks," *IEEE Transactions on Signal Processing*, vol. 62, no. 8, pp. 2100–2110, April 2014.

[14] P. d. Kerret and D. Gesbert, "Interference Alignment with Incomplete CSIT Sharing," *IEEE Transactions on Wireless Communications*, vol. 13, no. 5, pp. 2563–2573, May 2014.

[15] X. Rao, L. Ruan, and V. K. N. Lau, "CSI Feedback Reduction for MIMO Interference Alignment," *IEEE Transactions on Signal Processing*, vol. 61, no. 18, pp. 4428–4437, Sept 2013.

[16] O. E. Ayach and R. W. Heath, "Interference alignment with analog channel state feedback," *IEEE Transactions on Wireless Communications*, vol. 11, no. 2, pp. 626–636, February 2012.

[17] Q. Spencer, A. Swindlehurst, and M. Haardt, "Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels," *IEEE Trans. on Signal Processing*, vol. 52, no. 2, pp. 461–471, Feb 2004.

[18] D. Nguyen and T. Le-Ngoc, *Wireless Coordinated Multicell Systems: Architectures and Precoding Designs*, ser. SpringerBriefs in computer science. Springer, 2014.

[19] M. Hong and Z.-Q. Luo, "Academic Press Library in Signal Processing: Communications and Radar Signal Processing," S. Theodoridis and R. Chellappa, Eds. Elsevier Science, 2013, vol. 2, ch. 8 - Signal Processing and Optimal Resource Allocation for the Interference Channel.

[20] L.-C. Wang and C.-J. Yeh, "3-Cell Network MIMO Architectures with Sectorization and Fractional Frequency Reuse," *IEEE J. on S. Areas in Commun.*, vol. 29, no. 6, pp. 1185–1199, 2011.

[21] E. Biglieri, J. Proakis, and S. Shamai, "Fading Channels: Information-Theoretic and Communications Aspects," *IEEE Trans. on Inf. Theory*, vol. 44, no. 6, pp. 2619–2692, Oct 1998.

[22] C. S. C. Suh and D. Tse, "Interference Alignment for Cellular Networks," *Allerton Conf. on Commun., Control, and Computing*, 2008.

[23] M. Razaviyayn, M. Sanjabi, and Z.-Q. Luo, "Linear Transceiver Design for Interference Alignment: Complexity and Computation," *IEEE Trans. on Information Theory*, vol. 58, no. 5, pp. 2896–2910, May 2012.

[24] G. Caire, N. Jindal, M. Kobayashi, and N. Ravindran, "Multiuser MIMO Achievable Rates With Downlink Training and

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TWC.2017.2670540, IEEE Transactions on Wireless Communications

Channel State Feedback," *IEEE Trans. on Information Theory*, vol. 56, no. 6, pp. 2845–2866, 2010.

[25] L. Korowajczuk, *LTE, WiMAX and WLAN Network Design, Optimization and Performance Analysis*. Wiley, 2011.

[26] O. Elijah, C. Leow, T. Rahman, S. Nunoo, and S. Iliya, "A Comprehensive Survey of Pilot Contamination in Massive MIMO - 5G System," *IEEE Communications Surveys & Tutorials*, 2015.

[27] W. Steeb and Y. Hardy, *Matrix Calculus and Kronecker Product: A Practical Approach to Linear and Multilinear Algebra*. World Scientific, 2011.

[28] I. C. F. Ipsen, *Numerical Matrix Analysis*. Siam, 2009.

[29] A. Manolakos, Y. Noam, and A. Goldsmith, "Null Space Learning in Cooperative MIMO Cellular Networks Using Interference Feedback," *IEEE Trans. on Wireless Commun.*, vol. 14, no. 7, pp. 3961–3977, 2015.

[30] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge University Press, 2005.

[31] E. Dahlman, S. Parkvall, and J. Skold, *4G: LTE/LTE-Advanced for Mobile Broadband*, 2nd ed. Elsevier Science, 2014.

[32] E. Bjornson, E. Larsson, and T. Marzetta, "Massive MIMO: ten myths and one critical question," *IEEE Communications Magazine*, vol. 54, no. 2, pp. 114–123, February 2016.

[33] G. Lee and Y. Sung, "Asymptotically optimal simple user scheduling for massive MIMO downlink with two-stage beamforming," in *IEEE International Workshop on Signal Processing Advances in Wireless Communications*, June 2014, pp. 60–64.

[34] E. G. Larsson and H. V. Poor, "Joint Beamforming and Broadcasting in Massive MIMO," *IEEE Transactions on Wireless Communications*, vol. 15, no. 4, pp. 3058–3070, April 2016.

[35] H. Liu, H. Gao, S. Yang, and T. Lv, "Low-Complexity Downlink User Selection for Massive MIMO Systems," *IEEE Systems Journal*, 2015.

[36] Y.-H. Nam, M. S. Rahman, Y. Li, G. Xu, E. Onggosanusi, J. Zhang, and J.-Y. Seol, "Full Dimension MIMO for LTE-Advanced and 5G," in *Information Theory and Applications Workshop*, UCSD, 2015.

[37] X. Li, S. Jin, X. Gao, and R. Heath, "3D Beamforming for Large-scale FD-MIMO Systems Exploiting Statistical Channel State Information," *IEEE Trans. on Vehicular Technology*, 2016.

Eduardo Castañeda obtained the B.S. (Hons.) degree in telecommunications engineering form the National University of Mexico (UNAM) in 2009, and received the Ph.D. degree in electrical engineering from the University of Aveiro, Portugal, in 2015. He is currently a Postdoctoral Researcher with the Instituto de Telecomunicações (IT) and part of the CMU-Portugal Program. He has been involved in several national and European projects, FUTON, ADIN, and COPWIN. His research interests lie in radio resource management, signal processing for digital communications, cooperative networks and software define radio.

Daniel Castanheira graduated in Electronics and Telecommunications in 2007 and completed a PhD degree in Telecommunications in 2012, from the Universidade de Aveiro, Portugal. He is currently a post-doctoral researcher in the mobile networks (MOBNET) group at Instituto de Telecomunicações, Portugal, where he has been involved in several national and european projects, namely COPWIN, HETCOP, PURE-5GNET and SWING2, within the Portuguese National Scientific Foundation, and CODIV, FUTON and QOSMOS within the European Seventh Framework Programme (FP7). His research interests lie in signal processing techniques for digital communications, with emphasis for physical layer issues including channel coding, precoding/equalization and interference cancellation. In 2011 he was with Departamento de Eletrónica, Telecomunicações e Informática at Universidade de Aveiro, as an assistant professor.

Adão Silva received the M.Sc. and Ph.D. degrees in electronics and telecommunications from the University of Aveiro, Aveiro, Portugal, in 2002 and 2007, respectively. He is currently an Assistant Professor with the Department of Electronics, Telecommunications and Informatics, University of Aveiro, and a Researcher with the Instituto de Telecomunicações. He has been participating in several European research projects in the field of wireless networks. He has led several research projects, in the broadband wireless communications area, at the national level. His interests include multiuser MIMO, multicarrier-based systems, cooperative networks, precoding, and multiuser detection. Dr. Silva acted as a member of the TPC of several international conferences.

Atílio Gameiro received the Licenciatura and the Ph.D. degrees from the University of Aveiro, Aveiro, Portugal, in 1985 and 1993, respectively. He is currently an Associate Professor with the Department of Electronics and Telecommunications, University of Aveiro, and a Researcher with the Instituto de Telecomunicações (IT), Pólo de Aveiro, where he is head of group. His industrial experience includes a period of one year at BT Labs and one year at NKT Elektronik. His main interests lie in signal processing techniques for digital communications and communication protocols, and within this research line, he has done work for optical and mobile communications, either at the theoretical and experimental level, and has published over 200 technical papers in international journals and conferences. His current research activities involve spacetimefrequency algorithms for the broadband wireless systems and cross-layer design. He has been involved and has led IT and University of Aveiro participation on more than 20 national and European projects.