Stochastic Energy Systems with Mechanism Choices

Jingze Dai, McMaster University
Research Supervisor: **Dr.Douglas Down**

Table of contents

01

Introductions

Background information of the energy system

02

Problem Environment

The concrete problem this project aims to solve

03

Objectives

The final destination of this problem's solutions

04

Methodology

Methods used to solve this problem

05

Analysis

Experimental results and their meanings

06

Conclusions

The final summarized results

01

Introductions

Energy System Fundamentals

Background

Energy systems of consumer communities

An energy system used for supplying energy to a group of energy consumer entities, all energy consumptions are in a First-In-First-Out order.

Examples of Such energy systems

e.g., power plant of an apartment, where each room requires energy supply equally.

Two parts of energy systems' cost origins

1. Energy consumption spending **(More energy consumption, more cost)**
2. Energy supply congestion spending **(More waiting supplies, most cost)**

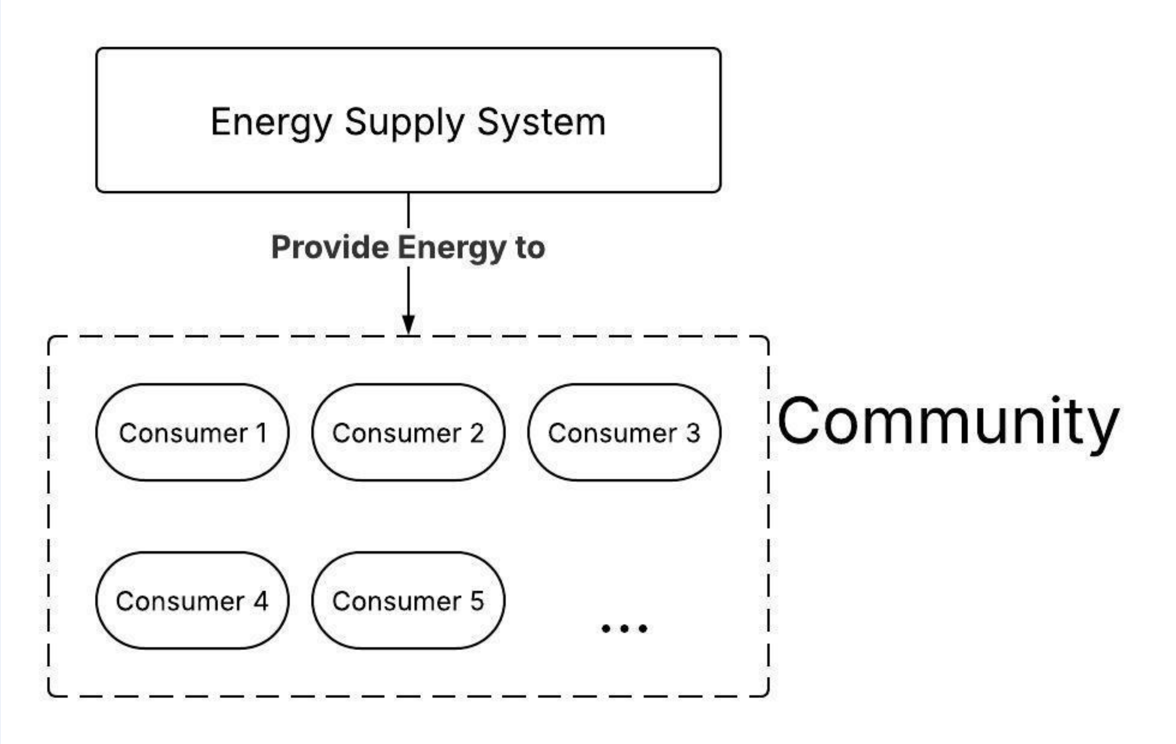


Fig 1 – Sample of Energy Systems

Engineer’s Paralleled Goals

Goal 1: To minimize energy consumption of the whole system

Goal 2: To minimize waiting in this system’s energy serving

02

Problem Environment

Energy System Concrete Problems' Settings

On usual cases:

1. The energy system is a **continuous time Markov chain (CTMC)**, where job arrivals are under Poisson distributions, whose parameter is a constant.
2. The energy system will not service the arrived jobs until the waiting job amount reaches k1, where k1 >= 1. Then the system will turn on with gamma (the turn-on rate).
3. When the waiting job amount < k2, the system uses the normal process rate (mu), otherwise when it >= k2, the system will use the enhanced process rate (mu_bar).

On usual cases:

4. In order to make the whole system under stable and expected performance, lambda < mu < mu_bar.
5. Alpha is the idle rate of the energy system, available on 1 of 2 conditions. (**Mentioned on slide 11**)
6. Remember gamma (the turning-on rate) and alpha (the idle rate) also exists, both are greater than 0.
7. will_turn_off represents the ability to turn off. If it is true, then alpha > 0, or it is 0. -> (the system cannot turn off)

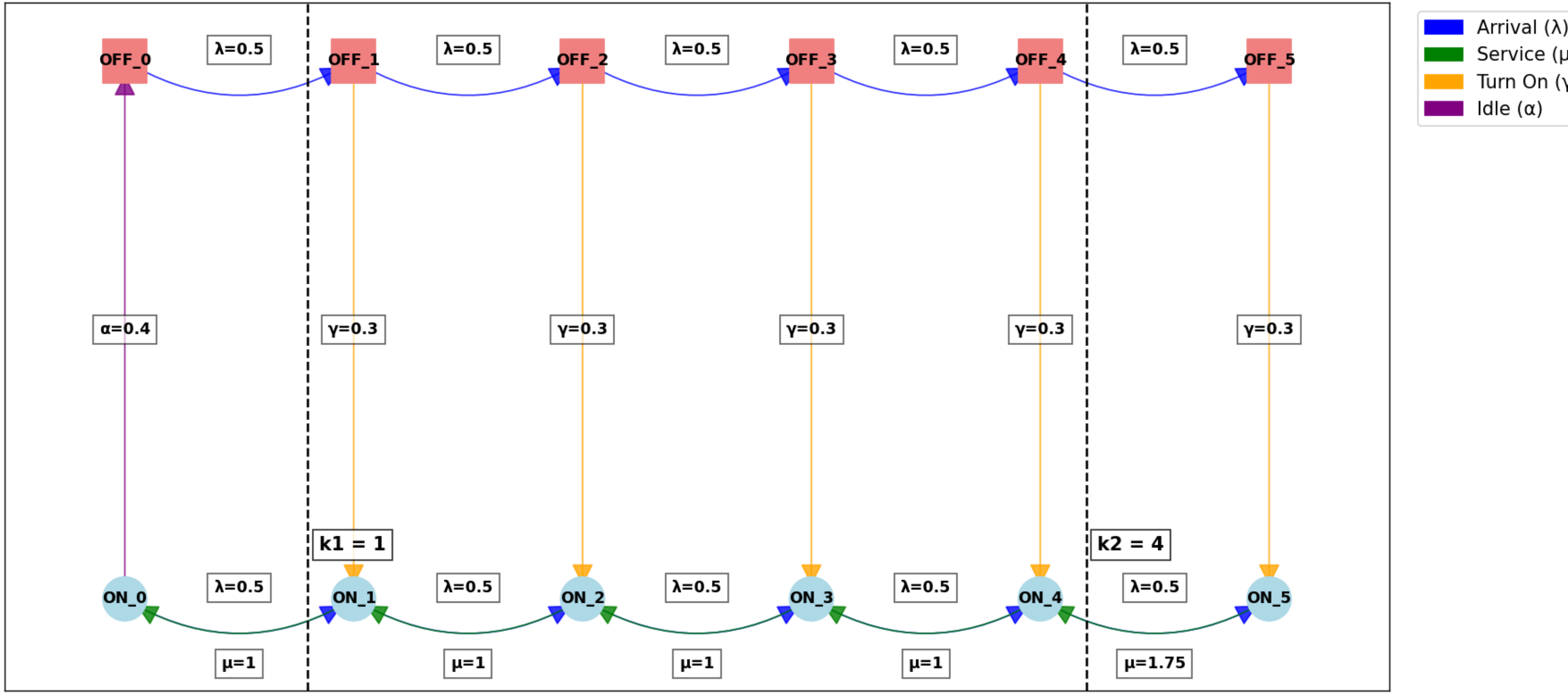
On usual cases:

8. 1 <= k1 <= k2, as the enhanced service rate only occurs after normal service rate (represents overload). Note that both k1 and k2 can reach indefinity.
9. **Important**: parameters k1, k2, and will_turn_off are variables (can be adjusted by system controls), while others are all constant (cannot be changed).

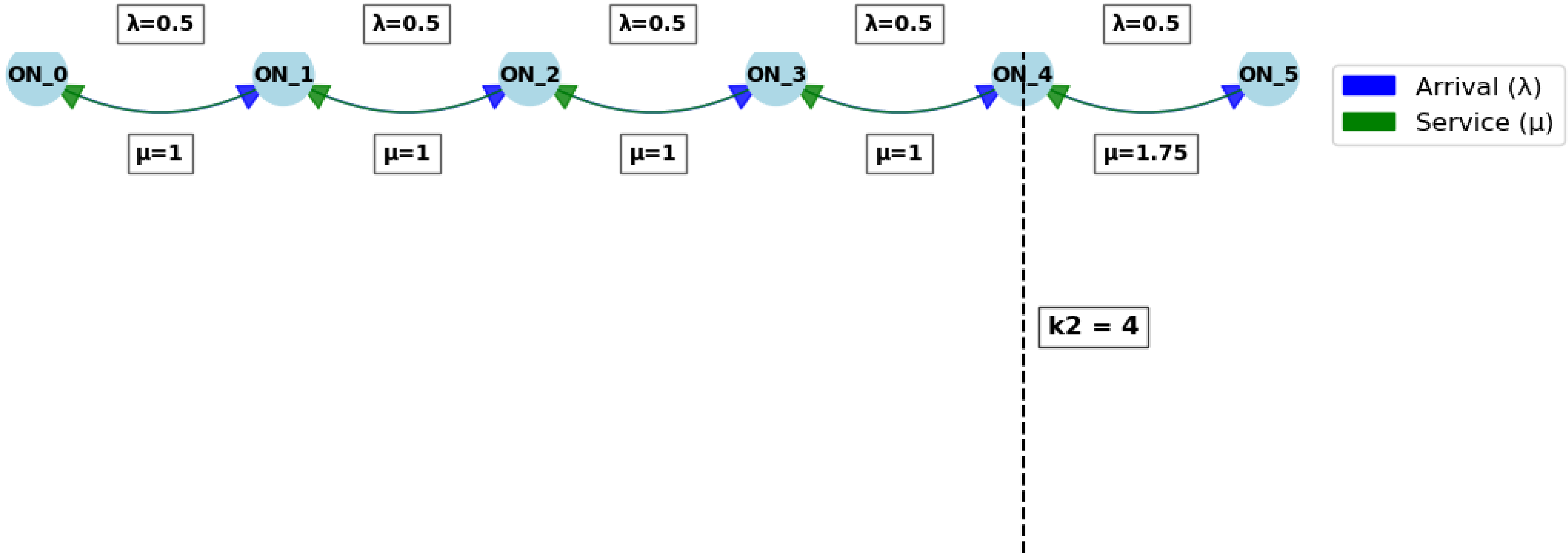
will_turn_off determines distinctions

1. When will_turn_off = False, the energy system will never go back to OFF states (to become idle), so we can ignore all states without service processing. This results in a **variant M/M/1 queue** (**Two service rates**).
2. When will_turn_off = True, the energy system will go back to idle when there is no waiting job. This has more complicated structures, and it **does not belong to any special CTMC cases** (e.g., M/M/1).

Transition Diagram with Separated Lambda and Mu Edges for max_jobs = 5



Transition Diagram (ON States Only, max_jobs = 5)



Power Consumption and Total Cost

1. Power consumption = Sum of (each state's steady-state probability * power consumption coefficient)
2. Power consumption coefficient:
0 for OFF states when the system is not turning on, 1 for OFF turning-on states. sigma for ON0, 1 for ON states with the regular service rate, and (mu_bar)^2 for ON states with the enhanced service rate.
3. The idle power consumption coefficient sigma is greater than 0 and less than 1.

Total Cost

1. Expected Number of Jobs (Mean) = Sum of (each state's steady-state probability * the waiting jobs)
2. ON0 means 0 waiting jobs, ON1 means 1, and so on...

1. **Total_Cost** = Power Consumption * beta + Expected Number of Jobs.
2. beta is a constant weight parameter, greater than 0.

03

Objectives

Problems' solution objectives

Solved **Question 1**

Our goal is to minimize the Total Cost of the whole energy system. Previously there are two choices:

1. Maximize k2 value, which aims to minimize the energy consumption caused by enhanced service rate usage.
2. Setting will_turn_off = True.
3. Setting will_turn_off = False.

Question: Can 1 and 2, or 1 and 3, co-exist in the optimal parameter group?

Solved **Question 2**

Question 2 is a **variant** problem of Question 1:
1. Parameter k2 is a non-polarized value, which means <u>it is neither maximized nor minimized</u>.
2. Setting will_turn_off = True.
3. Setting will_turn_off = False.

Question: <u>Can 1 and 2, or 1 and 3, co-exist in the optimal parameter group?</u>

04

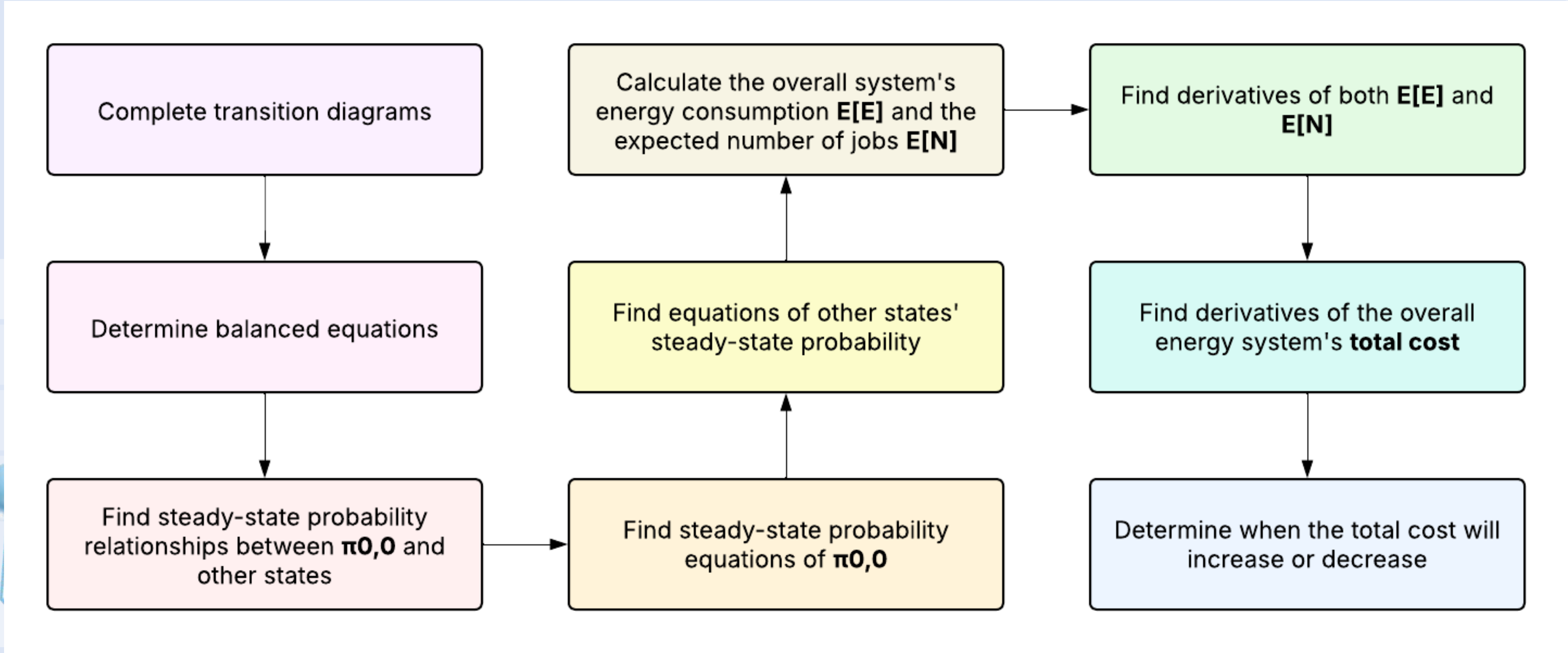
Methodologies

Methods used to solve these problems

Key – Solving CTMC

1. Both cases are under CTMC (continuous-time Markov chain), so the method of solving CTMC balanced equations will be efficient.
2. After getting equations of all states' steady-state probability, we can determine the overall system's equation of energy consumption and # of Jobs, which points out the equation of total cost.
3. By discovering the k2 derivative of such system's total cost, we can find local max/min positions (which k2 is).

Mandatory Processes



Step 1 – Complete Transition Diagrams

1. Listed all states on both conditions.
π0, x represent OFF states.
π1, x represent ON states.

2. Determine all states' transmission rates and targets (lambda, mu, mu bar, gamma, alpha)

Step 2 & 3 – Solving balanced equations

1. For each state, find all IN and OUT rates, and summarize them to an equation between two nearby states' steady-state probability.

2. Solve these equations, to get relationships between different states' steady-state probability.

3. By substitutions, find all other states' steady-state probability equations with **π0, 0**.

Step 4 – Determine the formula of **π0, 0**

Now all other states can be expressed by (...) * **π0, 0**.
By adding them together, we have:
(Net Term) * **π0, 0** = 1.

So **π0, 0** will equal **1/(Net Term)**.

Reminder: Whatever the system's parameters **π0, 0** change, will always be positive.

Step 5 – Determine the formula of **states**

From Step 3, we already have the relationship equation of different formulas between each other state and **π0, 0**. e.g., (...) * **π0, 2** = (...) * **π0, 0**.

By substituting **π0, 0** and transforming the formula, we can get equations of all other states. E.g., **π0, 2 = (...)/(...) <- here π0, 0, are already substituted.**

Step 6 – Determine E[E] and E[N]

To calculate the expected jobs E[N]:
E[N] = sum of {each state's steady-state probability * its corresponding number of jobs}

e.g., **π1, 0** has the steady-state probability of 0.3, and its expected number of jobs is 0, so therefore its expected number of jobs is 0.3 * 0 = 0.

Step 7 – Find derivatives

Some interesting things exist on both E[E] and E[N]:

1. They have the same denominator, because both of them are calculated from **π0, 0**
2. On both numerator side, a lot of terms look similar, since there are a lot of common terms inside.

Step 7 – Find k2 derivatives

First use the quotient rule for both body, and then simplify both sides' numerator terms.

(Not need to simplify the denominator terms. Since by the quotient rule, both **E[E]** and **E[N]** derivative's denominator is **D^2**, where **D** is the denominator of **E[E]**)

Step 8 – Find total cost and its derivatives

From slide 15, we already know **total cost = E[E] * beta + E[N]**, where beta is the weight coefficient **(always > 0)**.

Substitute the total cost functions with actual equations of **E[E]** and **E[N]**. On the numerator side, a lot of terms can be combined or cancelled.

Step 9 – Find total cost's k2 derivatives

From slide 15, we already know **total cost = E[E] * beta + E[N]**, where beta is the weight coefficient **(always > 0)**.

Considering beta as an abstract constant, then we have the total cost's k2 derivative here (using **constant rule**):

$$\frac{d}{dk2}(total\ cost) = \frac{d}{dk2}(E[E]) \cdot \beta + \frac{d}{dk2}(E[N])$$

Step 9 – Find total cost’s k2 derivatives

With such k2 derivative, now we are able to find when it will be **positive** (total cost is increasing), and when it will be **negative** (total cost is decreasing).

We can also determine the energy system’s optimal k2 values on both conditions.

Formal Proof (on both problems' solving)

Both conditions have considered three cases independently:
1. β is a very small value, meaning only E[N] is important for determining the total cost's values and tendencies.
2. β is a very large value, meaning only E[E] is important for determining the total cost's values and tendencies.
3. β is a balanced value, meaning both E[E] and E[N] are important for determining the total cost's values and tendencies.

05

Analysis

Experimental results

Case A – will_turn_off = False

$$E\left[E\right] = \frac{\sigma \cdot \left(1 + \frac{\lambda}{\mu}\right)\left(1 - \frac{\lambda}{\mu}\right) + \left(\frac{\lambda}{\mu}\right)^{k_2} \cdot \left(\left(1 - \frac{\lambda}{\mu}\right) \cdot \bar{\mu}^2 - \left(1 - \frac{\lambda}{\bar{\mu}}\right)\right)}{\left(1 - \left(\frac{\lambda}{\mu}\right)^{k_2}\right)\left(1 - \frac{\lambda}{\bar{\mu}}\right) + \left(\frac{\lambda}{\mu}\right)^{k_2} \cdot \left(1 - \frac{\lambda}{\mu}\right)}$$

$$E\left[N\right] = \frac{PartA + PartB}{PartD}$$

$$PartA = k_2 \cdot \left(\frac{\lambda}{\mu}\right)^{k_2} \cdot \left(\frac{1}{\left(1 - \frac{\lambda}{\bar{\mu}}\right)} - \frac{1}{\left(1 - \frac{\lambda}{\mu}\right)}\right) \cdot \left(1 - \frac{\lambda}{\bar{\mu}}\right) \cdot \left(1 - \frac{\lambda}{\mu}\right)$$

$$PartB = \left(\frac{\frac{\lambda}{\mu}}{\left(1 - \frac{\lambda}{\mu}\right)^2} \cdot \left(1 - \left(\frac{\lambda}{\mu}\right)^{k_2}\right) + \frac{\frac{\lambda}{\bar{\mu}}}{\left(1 - \frac{\lambda}{\bar{\mu}}\right)^2} \cdot \left(\frac{\lambda}{\mu}\right)^{k_2}\right) \cdot \left(1 - \frac{\lambda}{\bar{\mu}}\right) \cdot \left(1 - \frac{\lambda}{\mu}\right)$$

$$PartD = \left(1 - \left(\frac{\lambda}{\mu}\right)^{k_2}\right)\left(1 - \frac{\lambda}{\bar{\mu}}\right) + \left(\frac{\lambda}{\mu}\right)^{k_2} \cdot \left(1 - \frac{\lambda}{\mu}\right)$$

Case A – will_turn_off = False

$$\frac{d}{dk_2}(TotalCost) = \frac{Numerator}{NewDenominator^2}$$

$$Numerator = \left(\frac{\lambda}{\mu}\right)^{k_2} \cdot \ln\left(\frac{\lambda}{\mu}\right) \cdot \left(\frac{\lambda}{\bar{\mu}} - \frac{\lambda}{\mu}\right) \cdot \left(\left(k_2 - \sigma \cdot \left(1 + \frac{\lambda}{\mu}\right) \cdot \left(1 - \frac{\lambda}{\mu}\right)\right) \cdot \beta + \left(1 - \frac{\lambda}{\bar{\mu}}\right) - \left(1 - \frac{\lambda}{\mu}\right) \cdot \left(\bar{\mu}^2 + \frac{\frac{\lambda}{\bar{\mu}}}{1 - \frac{\lambda}{\bar{\mu}}}\right)\right)$$

$$NewDenominator = \left(1 - \frac{\lambda}{\mu} + \left(\frac{\lambda}{\mu}\right)^{k_2} \cdot \left(\frac{\lambda}{\bar{\mu}} - \frac{\lambda}{\mu}\right)\right)^2$$

Case A – will_turn_off = False

Problem 1 solution: Yes, 1 and 3 can co-exist in the optimal parameter group.

Observations:

1. Maximum k2 minimizes **E[E]** but maximizes **E[N]**, and will_turn_off = False increase **E[E]**
2. However, a small k1 value can lower **E[N]** and offset the effects of the maximum k2.

Case A – will_turn_off = False

Evidence:
lambda = 0.2, mu = 1, mu_bar = 10000, gamma = 0.5, alpha = 0.5, sigma = 0.5, beta = 1.

```
Best Parameters: k1=1, k2=500, will_turn_off=False
Minimum Total Cost: 0.8500000000086299
Elapsed Time: 154.80 seconds
```

Case A – will_turn_off = False

Problem 2 solution: Yes, 1 and 3 can co-exist in the optimal parameter group.

Observations:

1. On slide 32 case 1 & 2, the k2 derivative of total cost is continuously positive or negative, which disallow non-polarized k2 value to exist in the optimal parameter group.
2. However, on case 3, the k2 derivative of total cost is first negative then positive, allowing non-polarized k2 vals.

Case A – will_turn_off = False

Evidence:

lambda = 0.2, mu = 1, mu_bar = 100, gamma = 0.5, alpha = 0.5, sigma = 0.5, beta = 1.

```
Best Parameters: k1=1, k2=80, will_turn_off=False
Minimum Total Cost: 0.85000000000026108
Elapsed Time: 149.54 seconds
```

Case A – Methods to find optimal k2 values

There are several terms in the k2 derivative of total cost, but only one term possible to change sign:

$$\left(\left(k_2 - \sigma\cdot\left(1+\frac{\lambda}{\mu}\right)\cdot\left(1-\frac{\lambda}{\mu}\right)\right)\cdot\beta+\left(1-\frac{\lambda}{\bar{\mu}}\right)-\left(1-\frac{\lambda}{\mu}\right)\cdot\left(\bar{\mu}^2+\frac{\frac{\lambda}{\bar{\mu}}}{1-\frac{\lambda}{\bar{\mu}}}\right)\right)$$

So, we need to solve:

$$\left(\left(k_2 - \sigma\cdot\left(1+\frac{\lambda}{\mu}\right)\cdot\left(1-\frac{\lambda}{\mu}\right)\right)\cdot\beta+\left(1-\frac{\lambda}{\bar{\mu}}\right)-\left(1-\frac{\lambda}{\mu}\right)\cdot\left(\bar{\mu}^2+\frac{\frac{\lambda}{\bar{\mu}}}{1-\frac{\lambda}{\bar{\mu}}}\right)\right) < 0$$

Case A – Methods to find optimal k2 values

Example

Given $\lambda = 0.2$, $\mu = 1$, $\gamma = 0.8$, $\alpha = 1.5$, $\bar{\mu} = 2$, $\beta = 1$, $\sigma = 0.5$, $k_1 = 1$, and k_2 from 1 to ∞. (k_2 is the only variable.)

By substituting all these into this formula:

$$\left(\left(k_2 - \sigma \cdot \left(1 + \frac{\lambda}{\mu}\right) \cdot \left(1 - \frac{\lambda}{\mu}\right)\right) \cdot \beta + \left(1 - \frac{\lambda}{\bar{\mu}}\right) - \left(1 - \frac{\lambda}{\mu}\right) \cdot \left(\bar{\mu}^2 + \frac{\frac{\lambda}{\bar{\mu}}}{1 - \frac{\lambda}{\bar{\mu}}}\right)\right) < 0$$

We obtained $k_2 - 2.8689 < 0$, which is $k_2 < 2.8689$. Since k_2 can only be an integer, the greatest k_2 value fulfils this is $k_2 = 2$, which is regionally optimal.

Case B – will_turn_off = True

$$E[E] = \frac{\text{Numerator1}}{\text{Denominator}}$$

$$\begin{aligned}
\text{Numerator1} = {} & \frac{\lambda}{\gamma} - \sigma \times \frac{\lambda}{\alpha} + \left(\left(\frac{\lambda}{\alpha} - \frac{\rho}{1-\rho}\right) \times \frac{1-\rho^{k_2}}{1-\rho} - \frac{\lambda}{\alpha} + k_2 \times \frac{\rho}{1-\rho}\right) \\
& + \frac{\lambda}{\bar{\mu}-\lambda} \times \left(\frac{\lambda}{\alpha} \times \rho^{k_2} + \frac{\rho^{k_2-k_1+1}}{1-\rho} \times \left(1-\rho^{k_1}\right)\right) \\
& + \frac{\lambda}{\bar{\mu}-\lambda} \times \left(\frac{\frac{\lambda}{\lambda+\gamma} \times \rho^{k_2-k_1}}{1-\frac{\lambda}{\lambda+\gamma} \times \rho} \times \left(1 - \frac{\lambda}{\lambda+\gamma} \times \rho^{k_2-k_1}\right)\right) \\
& + \frac{\lambda+\gamma}{\lambda+\gamma-1} * \left(\frac{\lambda}{\lambda+\gamma}\right)^{k_2-k_1+1} * \bar{\mu}^2
\end{aligned}$$

$$\begin{aligned}
\text{Denominator} = {} & k_1 + \frac{\lambda}{\gamma} + \left(k_2 - \frac{1-\rho^{k_2}}{1-\rho}\right) \times \frac{\lambda}{\mu-\lambda} \\
& + \frac{\lambda}{\bar{\mu}-\lambda} \times \left(\frac{\lambda}{\alpha} \times \rho^{k_2} + \frac{\frac{\lambda}{\lambda+\gamma} \times \rho^{k_2-k_1}}{1-\frac{\lambda}{\lambda+\gamma} \times \rho} \times \left(1 - \frac{\lambda}{\lambda+\gamma} \times \rho^{k_2-k_1}\right)\right) \\
& + \frac{\lambda}{\bar{\mu}-\lambda} \times \frac{\rho^{k_2-k_1+1} \times \left(1-\rho^{k_1}\right)}{1-\rho}
\end{aligned}$$

Case B – will_turn_off = True

$$E[N] = \frac{\text{Numerator2}}{\text{Denominator}}$$

$$\begin{aligned}
\text{Numerator2} = {} & \frac{k_1 * (k_1 - 1)}{2} + \left(\frac{\lambda * (\lambda + \gamma)}{\gamma} + (k_1 - 1) * \frac{\lambda}{\gamma}\right) \\
& + \frac{\rho * \left(1 - \rho^{k_1} * (1 + k_1 (1 - \rho))\right)}{(1 - \rho)^2} + \frac{\rho}{1 - \rho} * \frac{k_1 * (k_1 + 1)}{2} \\
& + \frac{(k_2 - k_1)(k_2 + k_1 + 1)}{2} * \left(\frac{\lambda}{\alpha} * \rho^{k_2} + \rho^{k_2 - k_1 + 1} * \frac{1 - \rho^{k_1}}{1 - \rho}\right) \\
& + \frac{(k_2 - k_1)(k_2 + k_1 + 1)}{2} * \left(\frac{\lambda}{\lambda + \gamma} * \rho^{k_2 - k_1} * \frac{1 - \frac{\lambda}{\lambda+\gamma} * \rho^{k_2 - k_1}}{1 - \frac{\lambda}{\lambda+\gamma} * \rho}\right) \\
& + \frac{\lambda}{\bar{\mu} - \lambda} * \left(k_2 + \frac{1}{\bar{\mu} - \lambda}\right) * \left(\frac{\lambda}{\alpha} * \rho^{k_2} + \frac{\rho^{k_2 - k_1 + 1} * (1 - \rho^{k_1})}{1 - \rho}\right) \\
& + \frac{\lambda}{\bar{\mu} - \lambda} * \left(k_2 + \frac{1}{\bar{\mu} - \lambda}\right) * \left(\frac{\frac{\lambda}{\lambda+\gamma} * \rho^{k_2 - k_1}}{1 - \frac{\lambda}{\lambda+\gamma} * \rho} * \left(1 - \frac{\lambda}{\lambda + \gamma} * \rho^{k_2 - k_1}\right)\right)
\end{aligned}$$

The denominator of E[N] is the same as E[E].

Case B – will_turn_off = True

Formulas of the k2 derivative of total cost is available at:
https://www.overleaf.com/read/rpbgzbfpsktm#65aff2

Case B – will_turn_off = True

Problem 1 solution: Yes, 1 and 2 can co-exist in the optimal parameter group.

Observations:
1. Selecting higher idle rates and turn-on rates trigger preferences on will_turn_off = True
2. Parameter mu and mu bar are significant on determining the total cost actual values, but they are trivial in making preferences on will_turn_off options.

Case B – will_turn_off = True

Evidence:
lambda = 0.5, mu = 1, mu_bar = 17.5, gamma = 1.5, alpha = 1.5, sigma = 0.5, beta = 500.

```
Best Parameters: k1=1, k2=500, will_turn_off=True
Minimum Total Cost: 326.2666666667114
Elapsed Time: 152.24 seconds
```

Case B – will_turn_off = True

Problem 2 solution: Yes, 1 and 2 can co-exist in the optimal parameter group.

Observations:

1. On slide 32 all three cases, there exists cases allowing the non-polarized k2 value to exist as optimal.
2. To have such solutions, parameter lambda should have a small value (**lambda <= 10 % of mu**)

Case B – will_turn_off = True

Case 1 Evidence:

Counter-example: $\lambda = 0.05$, $\mu = 1$, $\bar{\mu} = 1.1$, $\gamma = 0.8$, $\alpha = 0.1$, $\sigma = 0.5$, and $\beta = 500$.

Actual optimal parameter: Best Parameters: $k_1 = 1$, $k_2 = 493$, will_turn_off=True Minimum Total Cost: 120.0951315789538 5.

Case B – will_turn_off = True

Case 2 Evidence:

Counter-example: $\lambda = 0.02$, $\mu = 1$, $\bar{\mu} = 1000000$, $\gamma = 3$, $\alpha = 4$, $\sigma = 0.5$, and $\beta = 0.08$.

Actual optimal parameter: Best Parameters: $k_1 = 1$, $k_2 = 10$, will_turn_off=True Minimum Total Cost: 0.02935225936.

Case B – will_turn_off = True

Case 3 Evidence:

Counter-example: $\lambda = 0.2$, $\mu = 1$, $\bar{\mu} = 10$, $\gamma = 3$, $\alpha = 4$, $\sigma = 0.5$, and $\beta = 1$.

Actual optimal parameter: Best Parameters: $k_1 = 1$, $k_2 = 8$, will_turn_off=True
Minimum Total Cost: 0.5793526693.

06

Conclusions

Final Conclusions

Conclusions

1. Non-polarized k2 values exist on all conditions' most cases. This means **selecting polarized k2 values may not always contributes to optimal total cost**.
2. Interesting discoveries: although these non-polarized k2 values exist, the k2 values greater than them don't have a much larger total cost (**their differences are trivial**)


End of
Presentation