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Stochastic Energy Systems with Optimal Parameters

Research Supervisor: **Dr. Douglas Down**,
McMaster University

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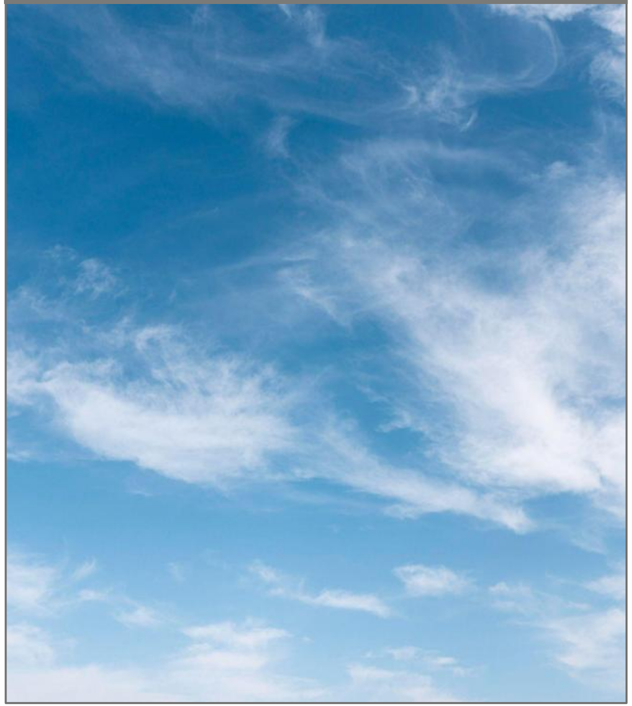

Relevant Research Projects

This research extends the previous project "Stochastic Energy Systems with Mechanism Choices", with 2 more optimal analysis.

Project link: https://github.com/daijingz/Research_Projects/tree/main/Energy_Systems_Total_Cost_Optimization

Main Contributions: Discover relationships between values of the parameter k2 and the total cost of the energy system. Also, prove that non-polarized k2 values are able to exist in the optimal parameter group.

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02 Introductions

Background information of the stochastic energy system, and its resource costs

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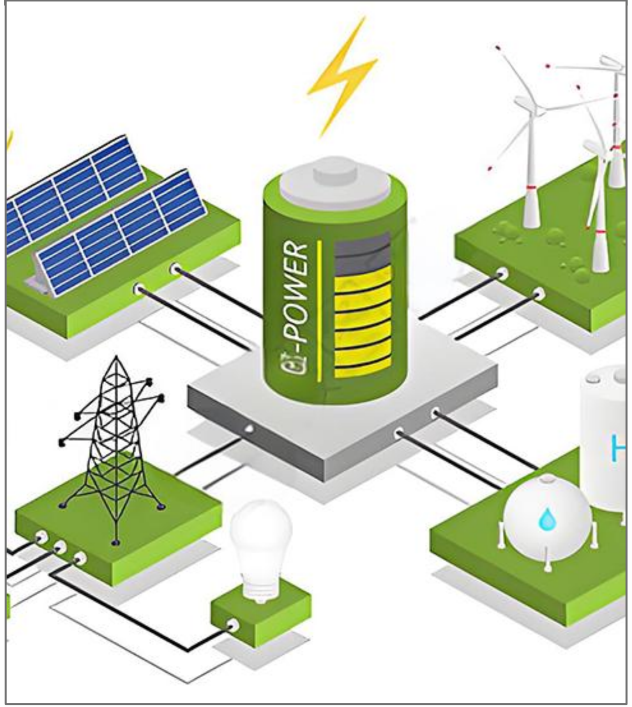

Stochastic Energy Systems of Consumer Communities

Energy systems of consumer communities
Common characteristics:

- With large-scale users, whose requests are identical or very similar in processing.
- Server's processing rate are constant (Keep the same in all of time)
- Systems serve requests in a first-in-first-out order(First-arrived orders serve first)

Examples

- Energy systems in the community (serving 50 – 100 homes)

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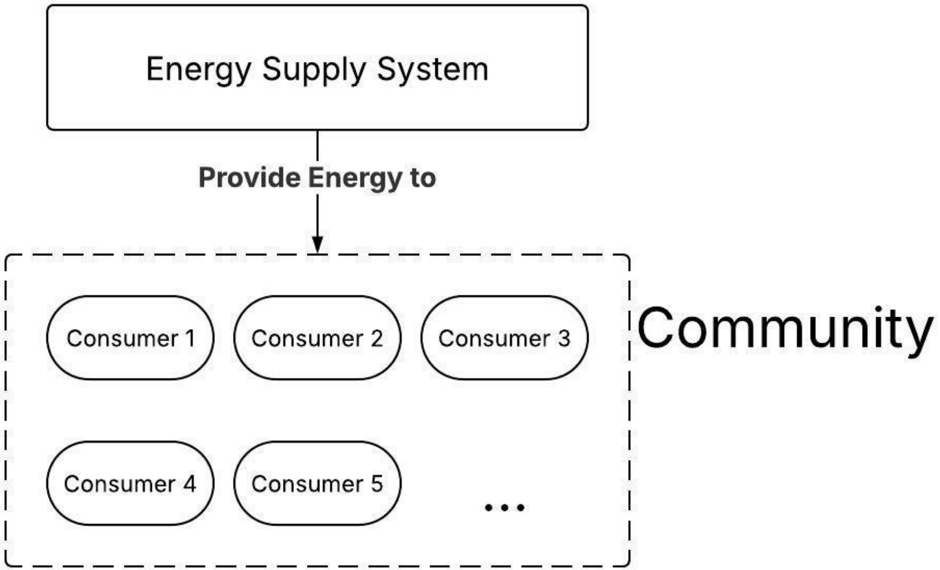
Fig 1 - Sample of Energy Systems



- Each consumer represents a house or a room requires energy supply.
- The energy supply system offers energy to energy consumers. e.g., a power plant operating 24 hours

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Costs of Such Stochastic Energy Systems

There are two parts of costs mainly:

- **Energy Consumptions** – Energy consumed by energy systems' multi-level processing.
  - Using higher processing rates will have a higher power consumption.
- **The Number of Waiting Jobs** – representing the crowdedness of such energy systems.
  - When more jobs are waiting to be processed, such costs will become heavier. This will be determined by **the expected number of jobs**.

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Such Systems' Cost Optimizations

Total_Cost = Power_Consumption_Cost + Waiting_Cost

Engineers' ultimate goal: **To minimize Total_Cost!**

1. Both power consumption costs and waiting job costs need to be minimized to trigger the minimal sum.
2. **Tradeoffs** between different choices: Increasing energy consumption rates may decrease waiting job cost; however, this will lead to an increase on power consumption costs.

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03 Problem Environments

The concrete problem this project aims to solve, with parameter information

Problem 1 (Already Solved)

Already solved, with solution here: https://github.com/daijingz/Research_Projects/tree/main/Energy_Systems_Total_Cost_Optimization

In this project, we determined that polarized k_2 values **(max and min k_2)** may not always targeting at the minimum total cost, and on some cases, we discovered formulas accurately determine the k_2 value corresponding to optimal (min) total costs.

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Problem 2

1. The energy system is a **continuous time Markov chain (CTMC)**, where job arrivals follow a Poisson distribution, whose parameter is a positive constant.
2. The energy system will not service the arrived jobs (that is, when the server is OFF) until the waiting job amount reaches **k1**, where k1 >= 1. Then the system will turn on with **γ** (the turn-on rate).
3. The system always use the normal service rate, whatever the number of waiting job is in the system.

Problem 2

4. When there is no waiting jobs and **will_turn_off = True** (the system is allowed to turn off), the system will turn off, otherwise it will keep turning on.
5. **α** is the idle rate of the energy system, a positive finite number (**α cannot be infinity**).
6. **γ** is the turn-on rate of the energy system, a positive finite number (**γ cannot be infinity**).
7. In order to make the whole system under stable and expected performance, **$\lambda < \mu$**. The normal service rate is the finite variable, cannot equal infinity.

Problem 2

8. 1 <= k1, as the energy system cannot turn on when there is no jobs waiting. (Or a waste of energy)
9. When **will_turn_off = False**, the energy system will never go back to OFF states (to become idle), so we can ignore all states without service processing. This results in a typical M/M/1 queue.
10. When **will_turn_off = True**, the energy system will go back to idle when there is no waiting job. This has more complicated structures, and it does not belong to any special CTMC cases (e.g., M/M/1).

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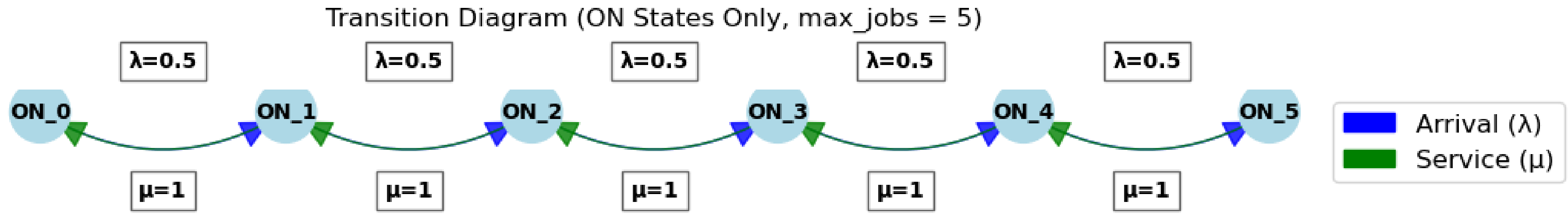

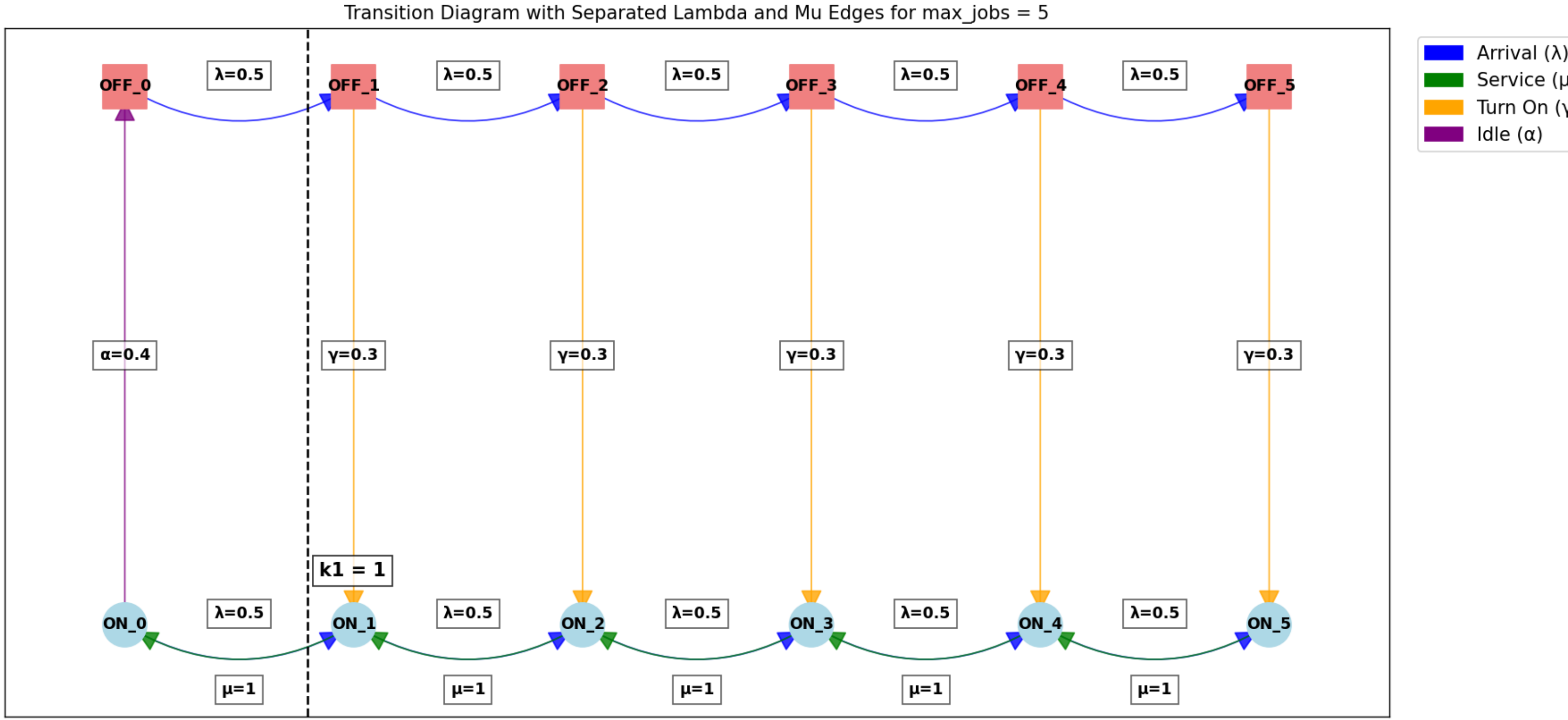
Transition Diagram with Separated Lambda and Mu Edges for max_jobs = 5
OFF_0
OFF_1
OFF_2
OFF_3
OFF_4
OFF_5
ON_0
ON_1
ON_2
ON_3
ON_4
ON_5
λ=0.5
α=0.4
γ=0.3
μ=1
k1 = 1
Arrival (λ)
Service (μ
Turn On (γ
Idle (α)

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Problem 2 Variant - Power-based E[E] Rate

Problem 2 (Variant) P2-V is an extension of Problem 2. In **Problem 2 (Variant)**, all other constraints are the same, but the power consumption of normal states is

μ^θ, where **θ** is the parameter of power consumption.

- **θ >= 1**
- **θ** can be any **real numbers** in that range (**θ** does not need to be an integer)

Problem 3

1. The energy system is a **continuous time Markov chain (CTMC)**, where job arrivals follow a Poisson distribution, whose parameter is a constant.
2. The energy system will not service the arrived jobs (that is, when the server is OFF) until the waiting job amount reaches k1, where **k1 >= 1**. Then the system will turn on with **γ** (the turn-on rate).
3. When the waiting job amount **< k2**, the system uses the normal processing rate (**μ**), otherwise when it **>= k2**, the system will use the enhanced process rate (**μ̲**).

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Problem 3

4. When there is no waiting jobs and **will_turn_off = True** (the system is allowed to turn off), the system will turn off, otherwise it will keep turning on.
5. **α** is the idle rate of the energy system, a positive finite number (**α cannot be infinity**).
6. **γ** is the turn-on rate of the energy system, a positive finite number (**γ cannot be infinity**).
7. In order to make the whole system under stable and expected performance, $\boldsymbol{\lambda < \mu < \underline{\mu}}$. The normal service rate is the finite variable, cannot equal infinity.

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Problem 3

8. 1 <= k1, as the energy system cannot turn on when there is no jobs waiting. (Or a waste of energy)
9. k1 <= k2, as the enhanced service rate come after the normal service rate.
10. When **will_turn_off = False**, the energy system will never go back to OFF states (to become idle), so we can ignore all states without service processing. This results in a variant M/M/1 queue. (Same architecture with M/M/1, but with two different processing rates)

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Problem 3

11. When **will_turn_off = True**, the energy system will go back to idle when there is no waiting job. This has more complicated structures, and it does not belong to any special CTMC cases (e.g., M/M/1).
12. Two cases (**will_turn_off = False/True**) has distinct transition diagrams and parameters. (Remember, when will_turn_off = False, k1, α, and γ will become meaningless)

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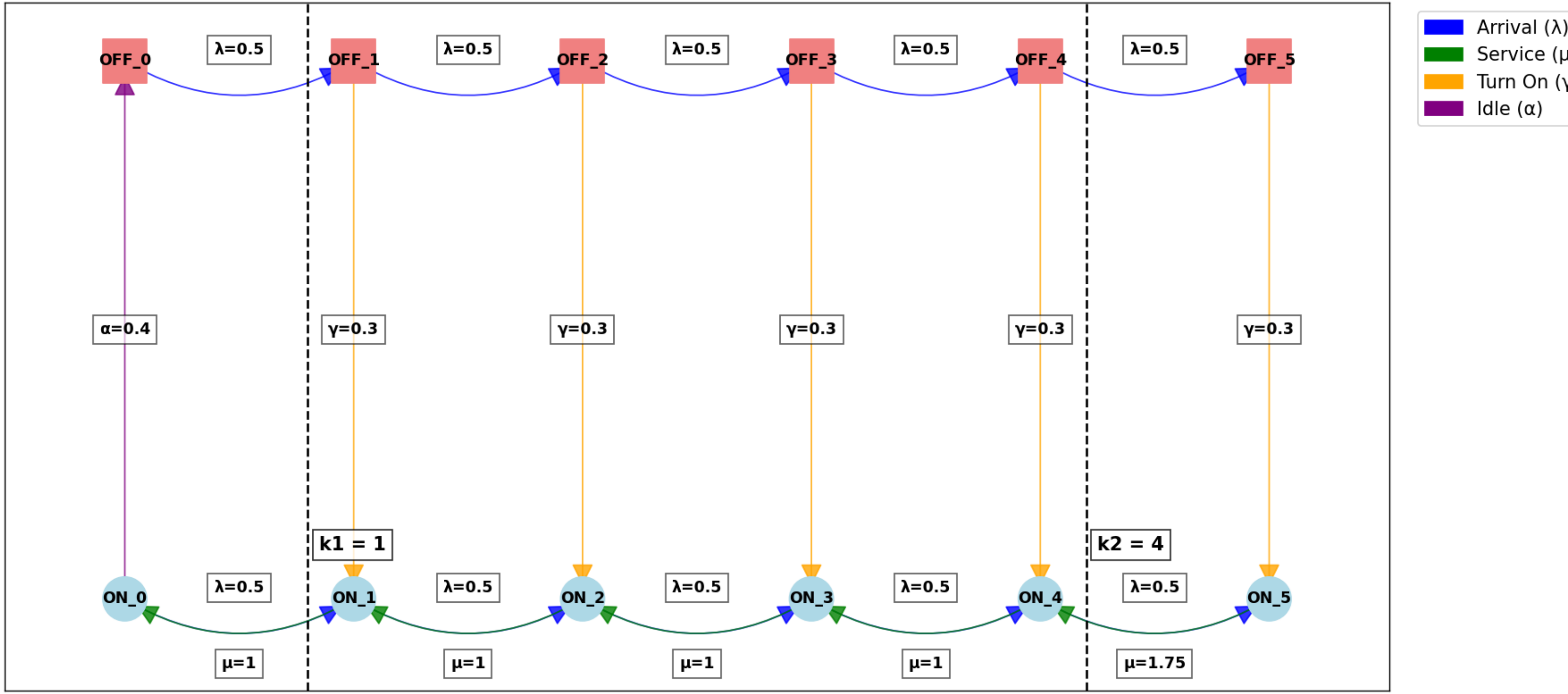
Transition Diagram with Separated Lambda and Mu Edges for max_jobs = 5



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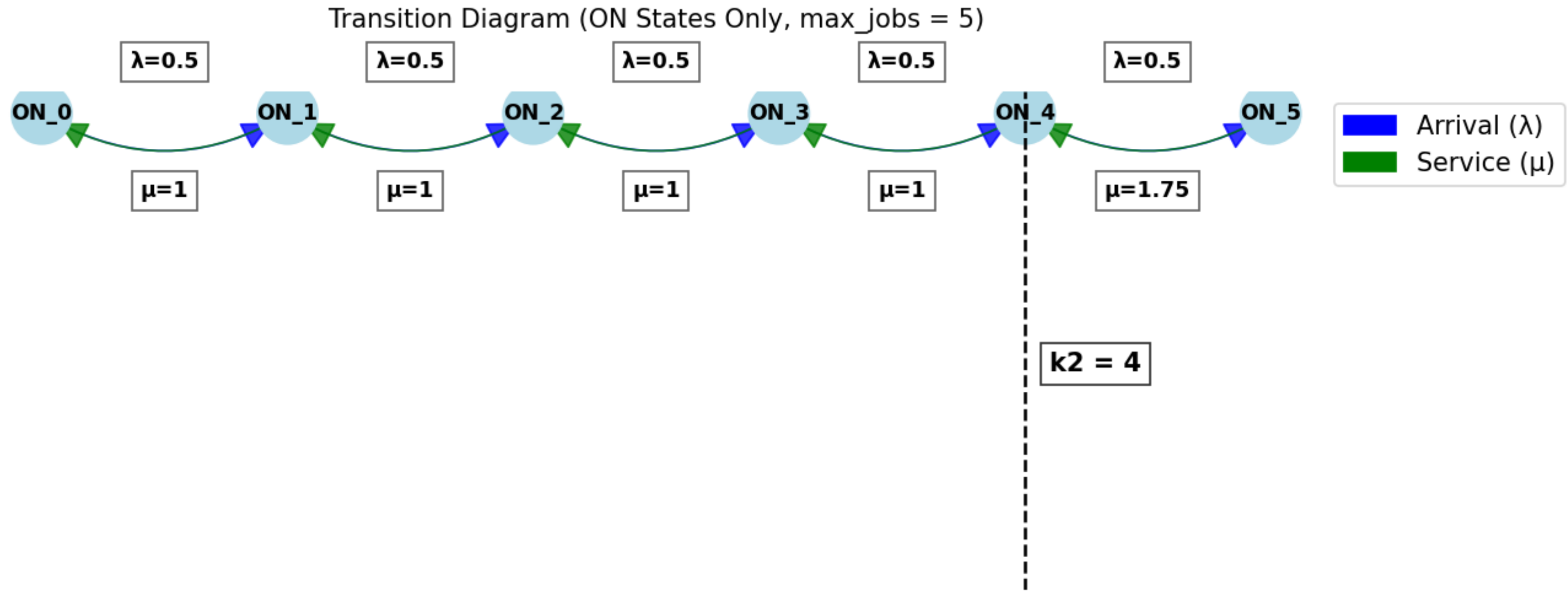

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04 Objectives

The final destination of this problem's solutions (Specified by each problem)

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Problem 2

In such provided energy systems, does the optimal total_cost always occur with polarized **μ** values?
1. Maximum **μ**: **μ infinitely approaches the value of λ**
2. Minimum **μ**: **μ = ∞**

Or, non-polarized μ (neither min nor max) values can also occur in the optimal parameter group with:
1. will_turn_off = False
2. will_turn_off = True

Problem 3

In such provided energy systems, does the optimal total_cost always occur with polarized **μ** values?
1. Maximum **μ**: **μ infinitely approaches the value of μ**
2. Minimum **μ**: **μ = ∞**

Or, non-polarized μ (neither min nor max) values can also occur in the optimal parameter group with:
1. will_turn_off = False
2. will_turn_off = True

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Problem 3 Variant – Abstract E[E] Rate

Problem 3 (Variant) – P3-V is an extension of Problem 3. In **P3-V**, all other constraints are the same, but the power consumption of enhanced rates is

μ^**θ**, where **θ** is the parameter of power consumption.

- **θ >= 1, not just θ = 2**
- **θ** can be any **real numbers** in that range (**θ** does not need to be an integer)

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05 Methodologies

Methods used to solve each specified problem

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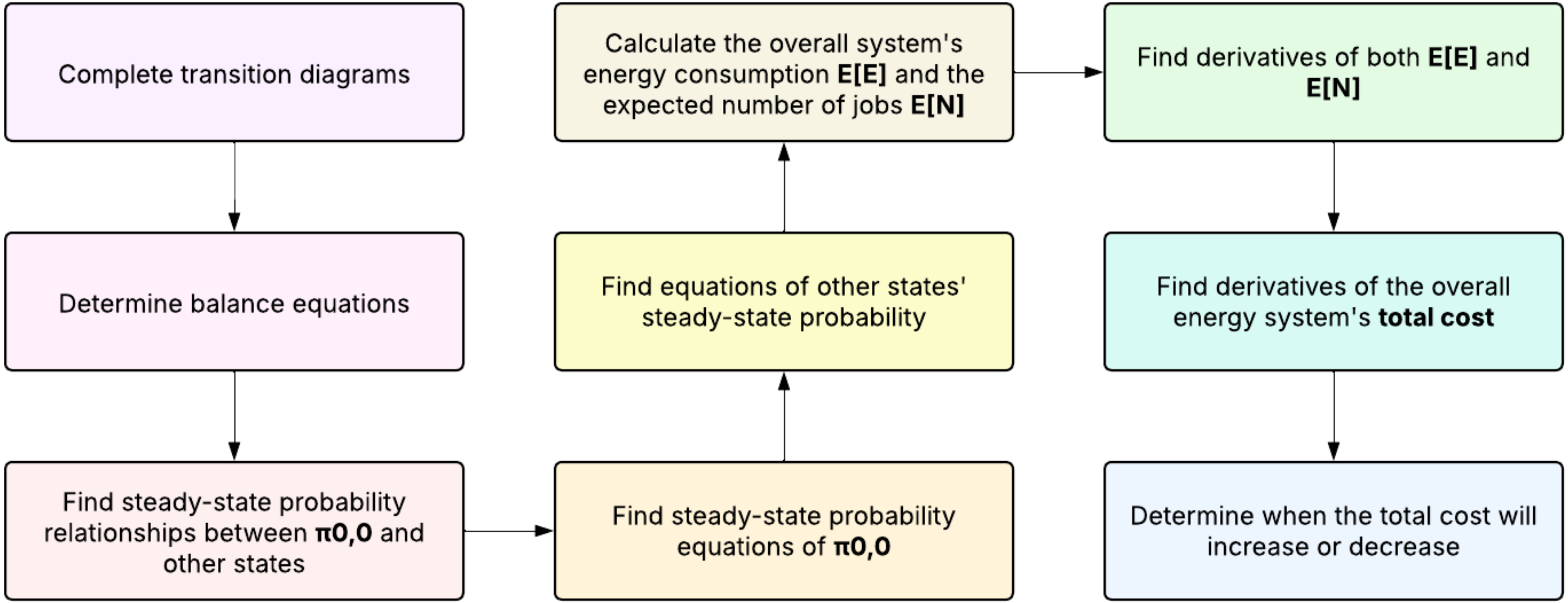

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Problem 2 & 3 Key - Using CTMC

1. Both cases can be modelled as CTMC (continuous-time Markov chain), so the method of solving CTMC balanced equations can be used.
2. After getting equations of all states' steady-state probability, we can determine the overall system's equation of energy consumption and # of Jobs, which determines the total cost.
3. By discovering the μ and $\underline{\mu}$ derivative of such system's total cost, we can find local max/min positions (with optimal μ and $\underline{\mu}$).

P2 Step 1 - Complete Transition Diagrams

1. Listed all states on both conditions.

π0, x represent OFF states. (e.g., **π0, 4** represents when the system is OFF, with 4 waiting jobs)

π1, x represent ON states. (e.g., **π1, 2** represents when the system is ON, with 2 waiting jobs)

2. Determine all states' transmission rates and targets **(λ, μ, γ, α)**

3. Draw transition diagrams on both conditions.

P2 Step 2 & 3 - Solving Balance Equations

1. For each state, find all **IN** and **OUT** rates, and summarize them to an equation between two nearby states' **steady-state probability**.

2. Solve these equations, to get relationships between different states' **steady-state probability**.

3. By substitutions, find **all other states' steady-state probability** equations with **π0, 0**.

P2 Step 4 - Determine Formula of $\pi_{0,0}$

Now all other states can be expressed by (…) * $\pi_{0,0}$.

By adding them together, we have:
(Net Term) * $\pi_{0,0} = 1$.

So $\pi_{0,0}$ will equal **1/(Net Term)**.

Reminder: Whatever the system's parameters change, $\pi_{0,0}$ will always be positive.

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P2 Step 5 - Determine Formula of states

From **Step 3**, we already have the relationship equation of different formulas between each other state and π0, 0.

e.g., (…) * π0, 2 = (…) * π0, 0.

By substituting **π0, 0** and transforming the formula, we can get equations of **all other states**. E.g., π0, 2 = (…)/(…) <- here π0, 0, are already substituted.

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P2 Step 6 - Determine E[E] and E[N]

- **Power Consumption E[E]** = Sum of (**each state's steady-state probability * power consumption coefficient**)

Power consumption coefficient:

- **0** for OFF states when the system is not turning on, **1** for OFF turning-on states. **σ** for ON0, **θ** * **μ** for ON states with the regular service rate.
- The idle power consumption coefficient **σ** is greater than **0** (Can be very large).
- The power consumption **θ** is greater than **0**.

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P2 Step 6 - Determine E[E] and E[N]

- **Expected Number of Jobs (E[N]) =** Sum of (**each state's Steady-State probability * # of waiting jobs**)
- ON0 means 0 waiting jobs, ON1 means 1, and so on…

e.g., Pr(ON0) = 0.5, Pr(ON1) = 0.3, Pr(ON2) = 0.2.

Expected Number of Jobs (E[N]) = 0.5 * 0 + 0.3 * 1 + 0.2 * 2 = 0.7

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P2 Step 6 - Determine E[E] and E[N]

- **Total_Cost = Power Consumption E[E] * β + Expected Number of Jobs E[N].**
- **β** is a constant weight parameter, greater than 0.

e.g., E[E] = 100, β = 0.5, E[N] = 70.

Total_Cost = 100 * 0.5 + 70 = 120

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P2 Step 7 - Find E[E] & E[N] Derivatives

Here we need to find the **μ** derivative of **E[E]** & **E[N]**. (Considering **μ** is the variable, while others are all abstract constants)

Common characteristics of **E[E]** and **E[N]**:

- Because both are derived from **π0**, **0**, E[E] and E[N] have the same denominator.
- A lot of E[E] and E[N] terms are very similar, so they are able to be re-combined and cancelled.

(This aims to simplify the total_cost function later)

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P2 Step 7 - Find E[E] & E[N] Derivatives

For both **E[E]** and **E[N]**, first use the quotient rule, and then simplify both sides' numerator terms.

$$h(x) = \frac{f(x)}{g(x)} \qquad h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

(No need to simplify the denominator terms. Since by the quotient rule, both **E[E]** and **E[N]** derivative's denominator is **D^2**, where **D** is the denominator of **E[E]**)

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P2 Step 8 - total_cost & its Derivatives

From previous slides, we already know **total_cost = E[E] * β + E[N]**, where **β** is the positive weight coefficient (always **> 0**).

And from previous steps, we already have the equations of both **E[E]** and **E[N]**.

Considering **β** as an abstract constant, substitute **E[E]** and **E[N]**. Since both have the same denominator, we just combine their numerators with **β**.

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P2 Step 8 - total_cost & its Derivatives

To find the **μ** derivative of **total_cost**, we know:

$$TotalCost = \beta * E[E] + E[N]$$

And them we have:

$$\frac{d}{d\mu}(\text{TotalCost}) = \frac{d}{d\mu}(\beta * E[E] + E[N])$$

Which is:

$$\frac{d}{d\mu}(\text{TotalCost}) = \beta * \frac{d}{d\mu}(E[E]) + \frac{d}{d\mu}(E[N])$$

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P2 Step 8 - total_cost & its Derivatives

On Step 7, We already know the equations of

$$\frac{d}{d\mu}(E[E]) \quad \text{AND} \quad \frac{d}{d\mu}(E[N])$$

Just substitute both parts from the total_cost equation, Since both parts have the same denominator again, we can cancel and re-combine a lot of both parts' numerator terms, contributes to a simplified equation.

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P2 Step 9 - total_cost's μ Derivatives

Now we already have the **μ** derivatives of total_cost, so we are able to find when it will be **positive** (total cost is increasing), and when it will be **negative** (total cost is decreasing).

Inside, some sub-parts are continuously positive or negative (Whatever other parameters have any value changes, their signs did not change.)

We can also determine the energy system's optimal **μ** values on both conditions.

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Problem 2 Formal Mathematical Proofs

Both conditions have considered three cases independently:

1. β is a **very small** value, meaning only **E[N]** is important for determining the total cost's values and tendencies.
2. β is a **very large** value, meaning only **E[E]** is important for determining the total cost's values and tendencies.
3. β is a **balanced value**, meaning both **E[E]** and **E[N]** are important for determining the total cost's values and tendencies.

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e.g., $(\ldots) * \pi_{0,2} = (\ldots) * \pi_{0,0}$.

By substituting **$\pi_{0,0}$** and transforming the formula, we can get equations of **all other states**. E.g., $\pi_{0,2} = (\ldots)/(\ldots)$ <- here $\pi_{0,0}$, are already substituted.

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(This aims to simplify the total_cost function later)

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To find the **µ** derivative of **total_cost**, we know:

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And them we have:

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Which is:

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Just substitute both parts from the total_cost equation, Since both parts have the same denominator again, we can cancel and re-combine a lot of both parts' numerator terms, contributes to a simplified equation.

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Both conditions have considered three cases independently:

1. **β** is a **very small** value, meaning only **E[N]** is important for determining the total cost's values and tendencies.
2. **β** is a **very large** value, meaning only **E[E]** is important for determining the total cost's values and tendencies.
3. **β** is a **balanced value**, meaning both **E[E]** and **E[N]** are important for determining the total cost's values and tendencies.

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06 Analysis

Experimental results and their meanings

Problem 2 Case A - will_turn_off = False

$$E[E] = \left(1 - \frac{\lambda}{\mu}\right) * \sigma + \frac{\lambda}{\mu} * \mu * \theta = \left(1 - \frac{\lambda}{\mu}\right) * \sigma + \lambda * \theta = \sigma + \lambda * \theta - \frac{\lambda}{\mu} * \sigma$$

$$E[N] = \frac{\rho}{1 - \rho} = \frac{\lambda}{\mu - \lambda}$$

$$TotalCost = E[E] * \beta + E[N]$$

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Problem 2 Case A - will_turn_off = False

$$\frac{d}{d\mu}\left(E\left[E\right]\right) = \frac{\lambda}{\mu^2} * \sigma$$

$$\frac{d}{d\mu}\left(TotalCost\right) = \frac{d}{d\mu}\left(E[E] * \beta + E[N]\right)$$

$$\frac{d}{d\mu}\left(E\left[N\right]\right) = -\frac{\lambda}{\left(\mu - \lambda\right)^2}$$

$$\frac{d}{d\mu}\left(TotalCost\right) = \frac{\lambda}{\mu^2} * \sigma * \beta - \frac{\lambda}{\left(\mu - \lambda\right)^2}$$

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Problem 2 Case A - will_turn_off = False

Solutions: Yes, non-polarized μ values can exist in the optimal parameter group in the **sub-condition 3.**

Sub-condition 1 - **β** is a **very small** value. μ = λ

Sub-condition 2 - **β** is a **very large** value. μ = ∞

Sub-condition 3 - **β** is a **balanced** value.

- Case 1: **βσ > 1**, μ is minimized (μ approaches the value of λ), or $\mu = \frac{\sqrt{\beta\sigma * \lambda}}{\sqrt{\beta\sigma - 1}}$
- Case 2: **βσ < 1**, μ is maximized (μ = ∞)
- Case 3: **βσ = 1**, μ is maximized (μ = ∞)

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Problem 2-V Case A - will_turn_off = False

$$E[E]=\left(1-\frac{\lambda}{\mu}\right)*\sigma+\frac{\lambda}{\mu}*\mu^{\theta}=\left(1-\frac{\lambda}{\mu}\right)*\sigma+\lambda\mu^{\theta-1}=\sigma+\lambda\mu^{\theta-1}-\frac{\lambda}{\mu}*\sigma$$

$$E[N]=\frac{\rho}{1-\rho}=\frac{\lambda}{\mu-\lambda}$$

$$TotalCost = E[E]*\beta+E[N]$$

Problem 2-V Case A - will_turn_off = False

$$\frac{d}{d\mu}E\left[E\right] = \lambda(\theta-1)\mu^{\theta-2} + \frac{\lambda}{\mu^2}\sigma \qquad \frac{d}{d\mu}\left(TotalCost\right) = \frac{d}{d\mu}\left(E[E] * \beta + E[N]\right)$$

$$\frac{d}{d\mu}\left(E\left[N\right]\right) = -\frac{\lambda}{\left(\mu-\lambda\right)^2} \qquad \frac{d}{d\mu}\left(TotalCost\right) = \left(\lambda(\theta-1)\mu^{\theta-2} + \frac{\lambda}{\mu^2}\sigma\right) * \beta - \frac{\lambda}{\left(\mu-\lambda\right)^2}$$

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Problem 2-V **Case A** - will_turn_off = False

Solutions: Yes, non-polarized μ values can exist in the optimal parameter group in the **sub-condition 3.**
Sub-condition 1 - β is a **very small** value. $\mu = \lambda$
Sub-condition 2 - β is a **very large** value. $\mu = \infty$

Sub-condition 3 - β is a **balanced** value.

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Problem 2 Case B - will_turn_off = True

$$E[E] = \frac{\text{Numerator1}}{\text{Denominator}} \qquad \text{Numerator1} = \frac{\lambda}{\gamma} + \frac{\lambda}{\alpha}\sigma + \frac{\lambda}{\mu - \lambda} * \mu * \left(k_1 + \frac{\lambda}{\alpha} + \frac{\lambda}{\gamma}\right) * \theta$$

$$\text{Denominator} = k_1 + \frac{\lambda}{\gamma} + \frac{\lambda}{\alpha} + \frac{\lambda}{\mu - \lambda} * \left(k_1 + \frac{\lambda}{\alpha} + \frac{\lambda}{\gamma}\right)$$

$$E[N] = \frac{\text{Numerator2}}{\text{Denominator}}$$

Denominator is the same,

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Problem 2 Case B - will_turn_off = True

$$
\begin{aligned}
\text{Numerator2} = {} & \frac{k_1 * (k_1 - 1)}{2} + \frac{\lambda}{\gamma} * \left(k_1 + \frac{\lambda}{\gamma}\right) \\
& + \frac{\lambda}{\mu - \lambda} * \frac{k_1 * (k_1 + 1)}{2} \\
& + \frac{\mu^2}{(\mu - \lambda)^2} * \left(\frac{\lambda}{\mu}\left(\frac{\lambda}{\alpha} - \frac{\lambda}{\mu - \lambda}\right) + \left(\frac{\lambda}{\mu}\right)^3 * \left(k_1\left(1 - \left(\frac{\lambda}{\mu}\right)^{k_1}\right) + \frac{\mu}{\mu - \lambda}\right)\right) \\
& + \frac{\mu^2}{(\mu - \lambda)^2} * \left(\left(\frac{\lambda}{\mu}\right)^{k_1+1} * \left(1 - \frac{\lambda}{\mu}\right) * \left(\frac{(k_1+1)\,\mu}{\mu - \lambda} * \left(1 + \frac{\lambda}{\mu}\right) + \frac{\lambda}{\alpha}\right)\right) \\
& + \frac{\lambda}{\lambda + \gamma - \mu} * \left(\frac{\frac{\lambda}{\mu}\left(1 + k_1\left(1 - \frac{\lambda}{\mu}\right)\right)}{\left(1 - \frac{\lambda}{\mu}\right)^2} - \frac{\frac{\lambda}{\lambda+\gamma}\left(1 + k_1\left(1 - \frac{\lambda}{\lambda+\gamma}\right)\right)}{\left(1 - \frac{\lambda}{\lambda+\gamma}\right)^2}\right)
\end{aligned}
$$

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Problem 2 Case B - will_turn_off = True

$$\frac{d}{d\mu}(E[E]) = \frac{\frac{\lambda\left(k_1+\frac{\lambda}{\alpha}+\frac{\lambda}{\gamma}\right)}{(\mu-\lambda)^2}\left(\frac{\lambda}{\gamma}+\frac{\lambda}{\alpha}\sigma+\frac{\lambda}{\mu-\lambda}\mu\left(k_1+\frac{\lambda}{\alpha}+\frac{\lambda}{\gamma}\right)\theta\right)}{\left(\left(1+\frac{\lambda}{\mu-\lambda}\right)\left(k_1+\frac{\lambda}{\alpha}+\frac{\lambda}{\gamma}\right)\right)^2}$$

$$\frac{d}{d\mu}(E[N]) = \frac{(S_2'+S_3'+S_4'+S_5')\cdot \text{Denominator} - \text{Numerator2}\cdot \text{Denominator}'}{(\text{Denominator})^2}$$

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Problem 2 Case B - will_turn_off = True

$$S_2' = -\frac{\lambda}{(\mu-\lambda)^2} \cdot \frac{k_1(k_1+1)}{2}$$

$$S_3' = -\left(\frac{2\lambda^3}{\alpha} + \frac{3\lambda^3}{\mu}\right) \cdot \frac{1}{(\mu-\lambda)^3} - \frac{2\mu\lambda}{(\mu-\lambda)^3} \cdot \left(\frac{\lambda}{\mu}\right)^3 \cdot k_1 \left(1 - \left(\frac{\lambda}{\mu}\right)^{k_1}\right)$$

$$+ \frac{2\lambda^3 + \lambda^2(2\mu-\lambda) - 3\mu^2\lambda\left(\frac{\lambda}{\mu}\right)^3}{(\mu-\lambda)^4}$$

$$- \frac{1}{(\mu-\lambda)^2} \cdot \left(\frac{\lambda^2}{\alpha} + \frac{3\lambda^3}{\mu^2} \cdot k_1 \left(1 - \left(\frac{\lambda}{\mu}\right)^{k_1}\right)\right) + \frac{k_1^2\lambda^{k_1+3}}{(\mu-\lambda)^2\mu^{k_1+2}}$$

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Problem 2 Case B - will_turn_off = True

$$S'_4 = -\frac{2\mu\lambda}{(\mu-\lambda)^3} \cdot \left(\frac{\lambda}{\mu}\right)^{k_1} \left(1-\frac{\lambda}{\mu}\right) \cdot \left[\frac{k_1\mu}{\mu-\lambda}\left(1+\frac{\lambda}{\mu}\right)+\frac{\lambda}{\alpha}\right]$$
$$+\frac{\mu^2}{(\mu-\lambda)^2} \cdot \left[-\frac{k_1\lambda^{k_1}}{\mu^{k_1+1}}\left(1-\frac{\lambda}{\mu}\right)+\frac{\lambda^{k_1+1}}{\mu^{k_1+2}}\right] \cdot \left[\frac{k_1\mu}{\mu-\lambda}\left(1+\frac{\lambda}{\mu}\right)+\frac{\lambda}{\alpha}\right]$$
$$-\frac{\mu^2}{(\mu-\lambda)^2} \cdot \left(\frac{\lambda}{\mu}\right)^{k_1} \left(1-\frac{\lambda}{\mu}\right) \cdot \left[\frac{k_1\lambda}{(\mu-\lambda)^2}\left(1+\frac{\lambda}{\mu}\right)+\frac{k_1\lambda}{\mu(\mu-\lambda)}\right]$$

$$S'_5 = \frac{\lambda}{(\lambda+\gamma-\mu)^2} \cdot \frac{\lambda\mu(1+k_1)-k_1\lambda^2}{(\mu-\lambda)^2}$$
$$+\frac{\lambda}{\lambda+\gamma-\mu} \cdot \frac{-\lambda\mu(1+k_1)+\lambda^2(k_1-1)}{(\mu-\lambda)^3}$$

Problem 2 Case B - will_turn_off = True

$$\text{Denominator}' = -\frac{\lambda}{(\mu-\lambda)^2} \cdot \left(k_1 + \frac{\lambda}{\alpha} + \frac{\lambda}{\gamma}\right)$$

$$\text{Denominator} = k_1 + \frac{\lambda}{\gamma} + \frac{\lambda}{\alpha} + \frac{\lambda}{\mu-\lambda} \cdot \left(k_1 + \frac{\lambda}{\alpha} + \frac{\lambda}{\gamma}\right)$$

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Problem 2 Case B - will_turn_off = True

Solutions: Yes, non-polarized μ values can exist in the optimal parameter group in **2 of 3 sub-conditions.**

Sub-condition 1 - β is a **very small** value.
Sub-condition 2 - β is a **very large** value. In this case, the derivative is continuously positive, which disables non-polarized **μ** values to exist as optimal.
Sub-condition 3 - β is a **balanced** value.

Problem 2-V Case B - will_turn_off = True

$$E[E] = \frac{\text{Numerator1}}{\text{Denominator}}$$

$$\text{Numerator1} = \frac{\lambda}{\gamma} + \frac{\lambda}{\alpha}\sigma + \frac{\lambda}{\mu - \lambda} * \mu^{\theta} * \left(k_1 + \frac{\lambda}{\alpha} + \frac{\lambda}{\gamma}\right)$$

$$\text{Denominator} = k_1 + \frac{\lambda}{\gamma} + \frac{\lambda}{\alpha} + \frac{\lambda}{\mu - \lambda} * \left(k_1 + \frac{\lambda}{\alpha} + \frac{\lambda}{\gamma}\right) = \left(1 + \frac{\lambda}{\mu - \lambda}\right) * \left(k_1 + \frac{\lambda}{\alpha} + \frac{\lambda}{\gamma}\right)$$

Problem 2-V Case B - will_turn_off = True

$$E[N] = \frac{\text{Numerator2}}{\text{Denominator}} \qquad \text{Denominator} = k_1 + \frac{\lambda}{\gamma} + \frac{\lambda}{\alpha} + \frac{\lambda}{\mu-\lambda} * \left(k_1 + \frac{\lambda}{\alpha} + \frac{\lambda}{\gamma}\right) = \left(1 + \frac{\lambda}{\mu-\lambda}\right) * \left(k_1 + \frac{\lambda}{\alpha} + \frac{\lambda}{\gamma}\right)$$

$$\begin{aligned}
\text{Numerator2} = {} & \frac{k_1 * (k_1 - 1)}{2} + \frac{\lambda}{\gamma} * \left(k_1 + \frac{\lambda}{\gamma}\right) \\
& + \frac{\lambda}{\mu-\lambda} * \frac{k_1 * (k_1 + 1)}{2} \\
& + \frac{\mu^2}{(\mu-\lambda)^2} * \left(\frac{\lambda}{\mu}\left(\frac{\lambda}{\alpha} - \frac{\lambda}{\mu-\lambda}\right) + \left(\frac{\lambda}{\mu}\right)^3 * \left(k_1\left(1 - \left(\frac{\lambda}{\mu}\right)^{k_1}\right) + \frac{\mu}{\mu-\lambda}\right)\right) \\
& + \frac{\mu^2}{(\mu-\lambda)^2} * \left(\left(\frac{\lambda}{\mu}\right)^{k_1+1} * \left(1 - \frac{\lambda}{\mu}\right) * \left(\frac{(k_1+1)\,\mu}{\mu-\lambda} * \left(1 + \frac{\lambda}{\mu}\right) + \frac{\lambda}{\alpha}\right)\right) \\
& + \frac{\lambda}{\lambda+\gamma-\mu} * \left(\frac{\frac{\lambda}{\mu}\left(1 + k_1\left(1 - \frac{\lambda}{\mu}\right)\right)}{\left(1 - \frac{\lambda}{\mu}\right)^2} - \frac{\frac{\lambda}{\lambda+\gamma}\left(1 + k_1\left(1 - \frac{\lambda}{\lambda+\gamma}\right)\right)}{\left(1 - \frac{\lambda}{\lambda+\gamma}\right)^2}\right)
\end{aligned}$$

Problem 2-V Case B - will_turn_off = True

$$TotalCost = E[E] * \beta + E[N]$$

$$\frac{d}{d\mu}(E[E]) = \frac{\left(k_1 + \frac{\lambda}{\alpha} + \frac{\lambda}{\gamma}\right) \cdot \frac{\mu^\theta(\lambda \cdot \theta \cdot (\mu - \lambda) - \mu) - \lambda\left(\frac{\lambda}{\gamma} + \frac{\lambda}{\alpha}\sigma + \frac{\lambda}{\mu - \lambda} \cdot \mu^\theta\right)}{(\mu - \lambda)^3}}{\left(\left(1 + \frac{\lambda}{\mu - \lambda}\right)\left(k_1 + \frac{\lambda}{\alpha} + \frac{\lambda}{\gamma}\right)\right)^2}$$

$$\frac{d}{d\mu}(E[N]) = \frac{(S_2' + S_3' + S_4' + S_5') \cdot \text{Denominator} - \text{Numerator2} \cdot \text{Denominator}'}{(\text{Denominator})^2}$$

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Problem 2-V Case B - will_turn_off = True

$$S_2' = -\frac{\lambda}{(\mu-\lambda)^2} \cdot \frac{k_1(k_1+1)}{2}$$

$$\begin{aligned} S_3' = &-\left(\frac{2\lambda^3}{\alpha} + \frac{3\lambda^3}{\mu}\right) \cdot \frac{1}{(\mu-\lambda)^3} - \frac{2\mu\lambda}{(\mu-\lambda)^3} \cdot \left(\frac{\lambda}{\mu}\right)^3 \cdot k_1 \left(1 - \left(\frac{\lambda}{\mu}\right)^{k_1}\right) \\ &+ \frac{2\lambda^3 + \lambda^2(2\mu-\lambda) - 3\mu^2\lambda\left(\frac{\lambda}{\mu}\right)^3}{(\mu-\lambda)^4} \\ &- \frac{1}{(\mu-\lambda)^2} \cdot \left(\frac{\lambda^2}{\alpha} + \frac{3\lambda^3}{\mu^2} \cdot k_1 \left(1 - \left(\frac{\lambda}{\mu}\right)^{k_1}\right)\right) + \frac{k_1^2 \lambda^{k_1+3}}{(\mu-\lambda)^2 \mu^{k_1+2}} \end{aligned}$$

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Problem 2-V Case B - will_turn_off = True

$$
\begin{aligned}
S_4' = &-\frac{2\mu\lambda}{(\mu-\lambda)^3}\cdot\left(\frac{\lambda}{\mu}\right)^{k_1}\left(1-\frac{\lambda}{\mu}\right)\cdot\left[\frac{k_1\mu}{\mu-\lambda}\left(1+\frac{\lambda}{\mu}\right)+\frac{\lambda}{\alpha}\right]\\
&+\frac{\mu^2}{(\mu-\lambda)^2}\cdot\left[-\frac{k_1\lambda^{k_1}}{\mu^{k_1+1}}\left(1-\frac{\lambda}{\mu}\right)+\frac{\lambda^{k_1+1}}{\mu^{k_1+2}}\right]\cdot\left[\frac{k_1\mu}{\mu-\lambda}\left(1+\frac{\lambda}{\mu}\right)+\frac{\lambda}{\alpha}\right]\\
&-\frac{\mu^2}{(\mu-\lambda)^2}\cdot\left(\frac{\lambda}{\mu}\right)^{k_1}\left(1-\frac{\lambda}{\mu}\right)\cdot\left[\frac{k_1\lambda}{(\mu-\lambda)^2}\left(1+\frac{\lambda}{\mu}\right)+\frac{k_1\lambda}{\mu(\mu-\lambda)}\right]
\end{aligned}
$$

$$
\begin{aligned}
S_5' = &\frac{\lambda}{(\lambda+\gamma-\mu)^2}\cdot\frac{\lambda\mu(1+k_1)-k_1\lambda^2}{(\mu-\lambda)^2}\\
&+\frac{\lambda}{\lambda+\gamma-\mu}\cdot\frac{-\lambda\mu(1+k_1)+\lambda^2(k_1-1)}{(\mu-\lambda)^3}
\end{aligned}
$$

$$
\text{Denominator} = k_1+\frac{\lambda}{\gamma}+\frac{\lambda}{\alpha}+\frac{\lambda}{\mu-\lambda}\cdot\left(k_1+\frac{\lambda}{\alpha}+\frac{\lambda}{\gamma}\right)
$$

$$
\text{Denominator}' = -\frac{\lambda}{(\mu-\lambda)^2}\cdot\left(k_1+\frac{\lambda}{\alpha}+\frac{\lambda}{\gamma}\right)
$$

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Problem 2 Case B - will_turn_off = True

Solutions: Yes, non-polarized μ values can exist in the optimal parameter group in **all 3 sub-conditions.**

Sub-condition 1 - β is a **very small** value.
Sub-condition 2 - β is a **very large** value.
Sub-condition 3 - β is a **balanced** value.

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Problem 3 Case A - will_turn_off = False

$$E\left[E\right] = \frac{\sigma \cdot \left(1 + \frac{\lambda}{\mu}\right)\left(1 - \frac{\lambda}{\mu}\right) + \left(\frac{\lambda}{\mu}\right)^{k_2} \cdot \left(\left(1 - \frac{\lambda}{\mu}\right) \cdot \bar{\mu}^2 - \left(1 - \frac{\lambda}{\bar{\mu}}\right)\right)}{\left(1 - \left(\frac{\lambda}{\mu}\right)^{k_2}\right)\left(1 - \frac{\lambda}{\bar{\mu}}\right) + \left(\frac{\lambda}{\mu}\right)^{k_2} \cdot \left(1 - \frac{\lambda}{\mu}\right)}$$

$$E\left[N\right] = \frac{PartA + PartB}{PartD}$$

$$PartA = k_2 \cdot \left(\frac{\lambda}{\mu}\right)^{k_2} \cdot \left(\frac{1}{\left(1 - \frac{\lambda}{\bar{\mu}}\right)} - \frac{1}{\left(1 - \frac{\lambda}{\mu}\right)}\right) \cdot \left(1 - \frac{\lambda}{\bar{\mu}}\right) \cdot \left(1 - \frac{\lambda}{\mu}\right)$$

$$PartB = \left(\frac{\frac{\lambda}{\mu}}{\left(1 - \frac{\lambda}{\mu}\right)^2} \cdot \left(1 - \left(\frac{\lambda}{\mu}\right)^{k_2}\right) + \frac{\frac{\lambda}{\bar{\mu}}}{\left(1 - \frac{\lambda}{\bar{\mu}}\right)^2} \cdot \left(\frac{\lambda}{\mu}\right)^{k_2}\right) \cdot \left(1 - \frac{\lambda}{\bar{\mu}}\right) \cdot \left(1 - \frac{\lambda}{\mu}\right)$$

$$PartD = \left(1 - \left(\frac{\lambda}{\mu}\right)^{k_2}\right)\left(1 - \frac{\lambda}{\bar{\mu}}\right) + \left(\frac{\lambda}{\mu}\right)^{k_2} \cdot \left(1 - \frac{\lambda}{\mu}\right)$$

Problem 3 Case A - will_turn_off = False

$$\frac{d}{d\bar{\mu}}E[E] = \frac{PartA \cdot \left(\frac{\lambda}{\mu}\right)^{k_2} \cdot \left[2\bar{\mu}\left(1-\frac{\lambda}{\mu}\right)+\frac{\lambda}{\bar{\mu}^2}\right] - PartB \cdot \left(1-\left(\frac{\lambda}{\mu}\right)^{k_2}\right) \cdot \frac{\lambda}{\bar{\mu}^2}}{\left[\left(1-\left(\frac{\lambda}{\mu}\right)^{k_2}\right)\left(1-\frac{\lambda}{\bar{\mu}}\right)+\left(\frac{\lambda}{\mu}\right)^{k_2}\left(1-\frac{\lambda}{\mu}\right)\right]^2}$$

$$PartA = \left[\left(1-\left(\frac{\lambda}{\mu}\right)^{k_2}\right)\left(1-\frac{\lambda}{\bar{\mu}}\right)+\left(\frac{\lambda}{\mu}\right)^{k_2}\left(1-\frac{\lambda}{\mu}\right)\right]$$

$$PartB = \left[\sigma\left(1+\frac{\lambda}{\mu}\right)\left(1-\frac{\lambda}{\mu}\right)+\left(\frac{\lambda}{\mu}\right)^{k_2}\left(\left(1-\frac{\lambda}{\mu}\right)\bar{\mu}^2-\left(1-\frac{\lambda}{\bar{\mu}}\right)\right)\right]$$

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Problem 3 Case A - will_turn_off = False

$$\frac{d}{d\bar{\mu}}E[E] = \frac{\left(1-\frac{\lambda}{\mu}\right)\left[PartC\cdot\left(\frac{\lambda}{\mu}\right)^{k_2}\cdot\left(2\bar{\mu}\left(1-\frac{\lambda}{\mu}\right)+\frac{\lambda}{\bar{\mu}^2}\right)-PartD\cdot\left(1-\left(\frac{\lambda}{\mu}\right)^{k_2}\right)\cdot\frac{\lambda}{\bar{\mu}^2}\right]}{[Denominator]^2}$$

$$PartC = \left[\left(1-\left(\frac{\lambda}{\mu}\right)^{k_2}\right)\left(1-\frac{\lambda}{\bar{\mu}}\right)+\left(\frac{\lambda}{\mu}\right)^{k_2}\right]$$

$$PartD = \left[\sigma\left(1+\frac{\lambda}{\mu}\right)\left(1-\frac{\lambda}{\mu}\right)+\left(\frac{\lambda}{\mu}\right)^{k_2}\left(\left(1-\frac{\lambda}{\mu}\right)\bar{\mu}^2-\left(1-\frac{\lambda}{\bar{\mu}}\right)\right)\right]$$

Problem 3 Case A - will_turn_off = False

$$\frac{d}{d\bar{\mu}}E[N] = \frac{PartE - PartF}{[Denominator]^2}$$

$$PartE = \left[\left(1-\left(\frac{\lambda}{\mu}\right)^{k_2}\right)\left(1-\frac{\lambda}{\bar{\mu}}\right) + \left(\frac{\lambda}{\mu}\right)^{k_2}\left(1-\frac{\lambda}{\mu}\right)\right]\cdot\frac{\lambda}{\bar{\mu}^2}$$
$$\left[-k_2\left(\frac{\lambda}{\mu}\right)^{k_2}\cdot\frac{\left(1-\frac{\lambda}{\mu}\right)}{\left(1-\frac{\lambda}{\bar{\mu}}\right)} + \frac{(\mu-\bar{\mu})}{\left(1-\frac{\lambda}{\mu}\right)^2}\left(1-\frac{\lambda}{\mu}\right)\right]$$

$$PartF = \left[k_2\left(\frac{\lambda}{\mu}\right)^{k_2}\left[\left(\frac{1}{1-\frac{\lambda}{\bar{\mu}}} - \frac{1}{1-\frac{\lambda}{\mu}}\right)\left(1-\frac{\lambda}{\bar{\mu}}\right)\left(1-\frac{\lambda}{\mu}\right)\right]\right]\cdot\frac{\lambda}{\bar{\mu}^2}\left(1-\left(\frac{\lambda}{\mu}\right)^{k_2}\right)$$

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Problem 3 **Case A** - will_turn_off = False

Solutions: Yes, non-polarized μ values can exist in the optimal parameter group in **all 3 sub-condition.**

Sub-condition 1 - **β** is a **very small** value.
Sub-condition 2 - **β** is a **very large** value.
Sub-condition 3 - **β** is a **balanced** value.

Problem 3-V Case A - will_turn_off = False

$$E\left[E\right] = \frac{\sigma \cdot \left(1 + \frac{\lambda}{\mu}\right)\left(1 - \frac{\lambda}{\mu}\right) + \left(\frac{\lambda}{\mu}\right)^{k_2} \cdot \left(\left(1 - \frac{\lambda}{\mu}\right) \cdot \bar{\mu}^{\theta} - \left(1 - \frac{\lambda}{\bar{\mu}}\right)\right)}{\left(1 - \left(\frac{\lambda}{\mu}\right)^{k_2}\right)\left(1 - \frac{\lambda}{\bar{\mu}}\right) + \left(\frac{\lambda}{\mu}\right)^{k_2} \cdot \left(1 - \frac{\lambda}{\mu}\right)} \qquad E\left[N\right] = \frac{PartA + PartB}{PartD}$$

$$PartA = k_2 \cdot \left(\frac{\lambda}{\mu}\right)^{k_2} \cdot \left(\frac{1}{\left(1 - \frac{\lambda}{\bar{\mu}}\right)} - \frac{1}{\left(1 - \frac{\lambda}{\mu}\right)}\right) \cdot \left(1 - \frac{\lambda}{\bar{\mu}}\right) \cdot \left(1 - \frac{\lambda}{\mu}\right)$$

$$PartB = \left(\frac{\frac{\lambda}{\mu}}{\left(1 - \frac{\lambda}{\mu}\right)^2} \cdot \left(1 - \left(\frac{\lambda}{\mu}\right)^{k_2}\right) + \frac{\frac{\lambda}{\bar{\mu}}}{\left(1 - \frac{\lambda}{\bar{\mu}}\right)^2} \cdot \left(\frac{\lambda}{\mu}\right)^{k_2}\right) \cdot \left(1 - \frac{\lambda}{\bar{\mu}}\right) \cdot \left(1 - \frac{\lambda}{\mu}\right)$$

$$PartD = \left(1 - \left(\frac{\lambda}{\mu}\right)^{k_2}\right)\left(1 - \frac{\lambda}{\bar{\mu}}\right) + \left(\frac{\lambda}{\mu}\right)^{k_2} \cdot \left(1 - \frac{\lambda}{\mu}\right)$$

Problem 3-V Case A - will_turn_off = False

$$\frac{d}{d\bar{\mu}}\left(E[E]\right)=\frac{\left(1-\left(\frac{\lambda}{\mu}\right)^{k_2}\right)\left(1-\frac{\lambda}{\bar{\mu}}\right)+\left(\frac{\lambda}{\mu}\right)^{k_2}\left(1-\frac{\lambda}{\mu}\right)\cdot\left[\left(\frac{\lambda}{\mu}\right)^{k_2}\left(\left(1-\frac{\lambda}{\mu}\right)\theta\bar{\mu}^{\theta-1}+\frac{\lambda}{\bar{\mu}^2}\right)-\left(1-\left(\frac{\lambda}{\mu}\right)^{k_2}\right)\cdot\frac{\lambda}{\bar{\mu}^2}\right]}{\left[\left(1-\left(\frac{\lambda}{\mu}\right)^{k_2}\right)\left(1-\frac{\lambda}{\bar{\mu}}\right)+\left(\frac{\lambda}{\mu}\right)^{k_2}\right]}$$

$$\frac{d}{d\bar{\mu}}E[N]=\frac{PartE-PartF}{[Denominator]^2}$$

$$PartF=\left[k_2\left(\frac{\lambda}{\mu}\right)^{k_2}\left[\left(\frac{1}{1-\frac{\lambda}{\bar{\mu}}}-\frac{1}{1-\frac{\lambda}{\mu}}\right)\left(1-\frac{\lambda}{\bar{\mu}}\right)\left(1-\frac{\lambda}{\mu}\right)\right]\right]\cdot\frac{\lambda}{\bar{\mu}^2}\left(1-\left(\frac{\lambda}{\mu}\right)^{k_2}\right)$$

$$PartE=\left[\left(1-\left(\frac{\lambda}{\mu}\right)^{k_2}\right)\left(1-\frac{\lambda}{\bar{\mu}}\right)\right.$$

$$\left.+\left(\frac{\lambda}{\mu}\right)^{k_2}\left(1-\frac{\lambda}{\mu}\right)\right]\cdot\frac{\lambda}{\bar{\mu}^2}\quad\left[-k_2\left(\frac{\lambda}{\mu}\right)^{k_2}\cdot\frac{\left(1-\frac{\lambda}{\bar{\mu}}\right)}{\left(1-\frac{\lambda}{\bar{\mu}}\right)}+\frac{\left(\mu-\bar{\mu}\right)}{\left(1-\frac{\lambda}{\mu}\right)^2}\left(1-\frac{\lambda}{\mu}\right)\right]$$

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Problem 3-V **Case A** - will_turn_off = False

Solutions: Yes, non-polarized μ values can exist in the optimal parameter group in **all 3 sub-condition.**

Sub-condition 1 - **β** is a **very small** value.
Sub-condition 2 - **β** is a **very large** value.
Sub-condition 3 - **β** is a **balanced** value.

Problem 3 Case B - will_turn_off = True

$$E[E] = \frac{\text{Numerator1}}{\text{Denominator}} \qquad E[N] = \frac{\text{Numerator2}}{\text{Denominator}}$$

$$\begin{aligned}\text{Numerator1} = {} & \frac{\lambda}{\gamma} - 0.5 \times \frac{\lambda}{\alpha} + \left(\left(\frac{\lambda}{\alpha} - \frac{\rho}{1-\rho} \right) \times \frac{1-\rho^{k_2}}{1-\rho} - \frac{\lambda}{\alpha} + k_2 \times \frac{\rho}{1-\rho} \right) \\ & + \frac{\lambda}{\bar{\mu} - \lambda} \times \left(\frac{\lambda}{\alpha} \times \rho^{k_2} + \frac{\rho^{k_2-k_1+1}}{1-\rho} \times (1-\rho^{k_1}) \right) \\ & + \frac{\lambda}{\bar{\mu} - \lambda} \times \left(\frac{\frac{\lambda}{\lambda+\gamma} \times \rho^{k_2-k_1}}{1 - \frac{\lambda}{\lambda+\gamma} \times \rho} \times \left(1 - \frac{\lambda}{\lambda+\gamma} \times \rho^{k_2-k_1} \right) \right) \\ & + \frac{\lambda+\gamma}{\lambda+\gamma-1} * \left(\frac{\lambda}{\lambda+\gamma} \right)^{k_2-k_1+1} * \bar{\mu}^2\end{aligned}$$

Problem 3 Case B - will_turn_off = True

$$
\begin{aligned}
\text{Numerator2} = \; & \frac{k_1 * (k_1 - 1)}{2} + \left(\frac{\lambda * (\lambda + \gamma)}{\gamma} + (k_1 - 1) * \frac{\lambda}{\gamma} \right) \\
& + \frac{\rho * \left(1 - \rho^{k_1} * (1 + k_1 (1 - \rho))\right)}{(1 - \rho)^2} + \frac{\rho}{1 - \rho} * \frac{k_1 * (k_1 + 1)}{2} \\
& + \frac{(k_2 - k_1)(k_2 + k_1 + 1)}{2} * \left(\frac{\lambda}{\alpha} * \rho^{k_2} + \rho^{k_2 - k_1 + 1} * \frac{1 - \rho^{k_1}}{1 - \rho} \right) \\
& + \frac{(k_2 - k_1)(k_2 + k_1 + 1)}{2} * \left(\frac{\lambda}{\lambda + \gamma} * \rho^{k_2 - k_1} * \frac{1 - \frac{\lambda}{\lambda + \gamma} * \rho^{k_2 - k_1}}{1 - \frac{\lambda}{\lambda + \gamma} * \rho} \right) \\
& + \frac{\lambda}{\bar{\mu} - \lambda} * \left(k_2 + \frac{1}{\bar{\mu} - \lambda} \right) * \left(\frac{\lambda}{\alpha} * \rho^{k_2} + \frac{\rho^{k_2 - k_1 + 1} * (1 - \rho^{k_1})}{1 - \rho} \right) \\
& + \frac{\lambda}{\bar{\mu} - \lambda} * \left(k_2 + \frac{1}{\bar{\mu} - \lambda} \right) * \left(\frac{\frac{\lambda}{\lambda + \gamma} * \rho^{k_2 - k_1}}{1 - \frac{\lambda}{\lambda + \gamma} * \rho} * \left(1 - \frac{\lambda}{\lambda + \gamma} * \rho^{k_2 - k_1} \right) \right)
\end{aligned}
$$

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Problem 3 Case B - will_turn_off = True

$$
\begin{aligned}
\text{Denominator} = k_1 &+ \frac{\lambda}{\gamma} + \left(k_2 - \frac{1-\rho^{k_2}}{1-\rho}\right) \times \frac{\lambda}{\mu - \lambda} \\
&+ \frac{\lambda}{\bar{\mu} - \lambda} \times \left(\frac{\lambda}{\alpha} \times \rho^{k_2} + \frac{\frac{\lambda}{\lambda+\gamma} \times \rho^{k_2-k_1}}{1 - \frac{\lambda}{\lambda+\gamma} \times \rho} \times \left(1 - \frac{\lambda}{\lambda+\gamma} \times \rho^{k_2-k_1}\right)\right) \\
&+ \frac{\lambda}{\bar{\mu} - \lambda} \times \frac{\rho^{k_2-k_1+1} \times (1-\rho^{k_1})}{1-\rho}
\end{aligned}
$$

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Problem 3 Case B - will_turn_off = True

$$\frac{d}{d\bar{\mu}}\left(E[E]\right) = \frac{d}{d\bar{\mu}}\left(\frac{f(\bar{\mu})}{g(\bar{\mu})}\right) = \frac{g(\bar{\mu})\cdot\frac{d}{d\bar{\mu}}f(\bar{\mu}) - f(\bar{\mu})\cdot\frac{d}{d\bar{\mu}}g(\bar{\mu})}{g(\bar{\mu})^2}$$

$$\begin{aligned}
f(\bar{\mu}) = {} & \frac{\lambda}{\bar{\mu}-\lambda} \\
& + \frac{\lambda}{\bar{\mu}-\lambda}\left(\frac{\lambda}{\alpha}\rho^{k_2} + \frac{\rho^{k_2-k_1+1}}{1-\rho}(1-\rho^{k_1})\right) \\
& + \frac{\lambda}{\bar{\mu}-\lambda}\left(\frac{\frac{\lambda}{\lambda+\gamma}\rho^{k_2-k_1}}{1-\frac{\lambda}{\lambda+\gamma}\rho}\left(1-\frac{\lambda}{\lambda+\gamma}\rho^{k_2-k_1}\right)\right) \\
& + \frac{\lambda+\gamma}{\lambda+\gamma-1}\left(\frac{\lambda}{\lambda+\gamma}\right)^{k_2-k_1+1} * \bar{\mu}^2
\end{aligned}$$

$$\begin{aligned}
f'(\bar{\mu}) = {} & -\frac{\lambda}{(\bar{\mu}-\lambda)^2} \\
& - \frac{\lambda}{(\bar{\mu}-\lambda)^2}\left(\frac{\lambda}{\alpha}\rho^{k_2} + \frac{\rho^{k_2-k_1+1}}{1-\rho}(1-\rho^{k_1})\right) \\
& - \frac{\lambda}{(\bar{\mu}-\lambda)^2}\left(\frac{\frac{\lambda}{\lambda+\gamma}\rho^{k_2-k_1}}{1-\frac{\lambda}{\lambda+\gamma}\rho}\left(1-\frac{\lambda}{\lambda+\gamma}\rho^{k_2-k_1}\right)\right) \\
& + 2\times\frac{\lambda+\gamma}{\lambda+\gamma-1}\times\left(\frac{\lambda}{\lambda+\gamma}\right)^{k_2-k_1+1} * \bar{\mu}
\end{aligned}$$

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Problem 3 Case B - will_turn_off = True

$$
\begin{aligned}
g(\bar{\mu}) = Denominator = k_1 + \frac{\lambda}{\gamma} \\
&+ \left(k_2 - \frac{1-\rho^{k_2}}{1-\rho}\right) \times \frac{\lambda}{\mu - \lambda} \\
&+ \frac{\lambda}{\bar{\mu}-\lambda}\left(\frac{\lambda}{\alpha}\rho^{k_2} + \frac{\frac{\lambda}{\lambda+\gamma}\rho^{k_2-k_1}}{1-\frac{\lambda}{\lambda+\gamma}\rho}\left(1 - \frac{\lambda}{\lambda+\gamma}\rho^{k_2-k_1}\right)\right) \\
&+ \frac{\lambda}{\bar{\mu}-\lambda}\frac{\rho^{k_2-k_1+1}\left(1-\rho^{k_1}\right)}{1-\rho}
\end{aligned}
$$

$$
\begin{aligned}
g'(\bar{\mu}) = Denominator' = -\frac{\lambda}{(\bar{\mu}-\lambda)^2}\left(\frac{\lambda}{\alpha}\rho^{k_2} + \frac{\frac{\lambda}{\lambda+\gamma}\rho^{k_2-k_1}}{1-\frac{\lambda}{\lambda+\gamma}\rho}\left(1 - \frac{\lambda}{\lambda+\gamma}\rho^{k_2-k_1}\right)\right) \\
&- \frac{\lambda}{(\bar{\mu}-\lambda)^2}\frac{\rho^{k_2-k_1+1}\left(1-\rho^{k_1}\right)}{1-\rho}
\end{aligned}
$$

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Problem 3 Case B - will_turn_off = True

$$\frac{d}{d\bar{\mu}}E[N] = \frac{\text{Denominator} \cdot \frac{d}{d\bar{\mu}}\text{Numerator2} - \text{Numerator2} \cdot \frac{d}{d\bar{\mu}}\text{Denominator}}{\text{Denominator}^2}$$

$$\begin{aligned}\frac{d}{d\bar{\mu}}(\text{Numerator2}) = &-\frac{\lambda}{(\bar{\mu}-\lambda)^2} \cdot \left(k_2 + \frac{1}{\bar{\mu}-\lambda}\right) \cdot \left(\frac{\lambda}{\alpha} \cdot \rho^{k_2} + \frac{\rho^{k_2-k_1+1}\left(1-\rho^{k_1}\right)}{1-\rho}\right) \\ &- \frac{\lambda}{(\bar{\mu}-\lambda)^2} \cdot \left(k_2 + \frac{1}{\bar{\mu}-\lambda}\right) \cdot \left(\frac{\frac{\lambda}{\lambda+\gamma} \cdot \rho^{k_2-k_1}}{1-\frac{\lambda}{\lambda+\gamma} \cdot \rho} \cdot \left(1 - \frac{\lambda}{\lambda+\gamma} \cdot \rho^{k_2-k_1}\right)\right)\end{aligned}$$

$$\begin{aligned}Denominator' = &-\frac{\lambda}{(\bar{\mu}-\lambda)^2}\left(\frac{\lambda}{\alpha}\rho^{k_2} + \frac{\frac{\lambda}{\lambda+\gamma}\rho^{k_2-k_1}}{1-\frac{\lambda}{\lambda+\gamma}\rho}\left(1-\frac{\lambda}{\lambda+\gamma}\rho^{k_2-k_1}\right)\right) \\ &- \frac{\lambda}{(\bar{\mu}-\lambda)^2}\frac{\rho^{k_2-k_1+1}\left(1-\rho^{k_1}\right)}{1-\rho}\end{aligned}$$

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Problem 3 Case B - will_turn_off = True

Solutions: Yes, non-polarized μ values can exist in the optimal parameter group in **all 3 sub-condition.**

Sub-condition 1 - β is a **very small** value.
Sub-condition 2 - β is a **very large** value.
Sub-condition 3 - β is a **balanced** value.

Problem 3-V Case B - will_turn_off = True

$$E[E] = \frac{\text{Numerator1}}{\text{Denominator}}$$

$$\begin{aligned}\text{Numerator1} = & \frac{\lambda}{\gamma} - \sigma \times \frac{\lambda}{\alpha} + \left(\left(\frac{\lambda}{\alpha} - \frac{\rho}{1-\rho} \right) \times \frac{1-\rho^{k_2}}{1-\rho} - \frac{\lambda}{\alpha} + k_2 \times \frac{\rho}{1-\rho} \right) \\ & + \frac{\lambda}{\bar{\mu} - \lambda} \times \left(\frac{\lambda}{\alpha} \times \rho^{k_2} + \frac{\rho^{k_2-k_1+1}}{1-\rho} \times (1 - \rho^{k_1}) \right) \\ & + \frac{\lambda}{\bar{\mu} - \lambda} \times \left(\frac{\frac{\lambda}{\lambda+\gamma} \times \rho^{k_2-k_1}}{1 - \frac{\lambda}{\lambda+\gamma} \times \rho} \times \left(1 - \frac{\lambda}{\lambda+\gamma} \times \rho^{k_2-k_1} \right) \right) \\ & + \frac{\lambda+\gamma}{\lambda+\gamma-1} * \left(\frac{\lambda}{\lambda+\gamma} \right)^{k_2-k_1+1} * \bar{\mu}^{\theta}\end{aligned}$$

Problem 3-V Case B - will_turn_off = True

$$
\begin{aligned}
\text{Denominator} = k_1 + \frac{\lambda}{\gamma} + \left(k_2 - \frac{1-\rho^{k_2}}{1-\rho}\right) \times \frac{\lambda}{\mu - \lambda} \\
+ \frac{\lambda}{\bar{\mu} - \lambda} \times \left(\frac{\lambda}{\alpha} \times \rho^{k_2} + \frac{\frac{\lambda}{\lambda+\gamma} \times \rho^{k_2-k_1}}{1 - \frac{\lambda}{\lambda+\gamma} \times \rho} \times \left(1 - \frac{\lambda}{\lambda+\gamma} \times \rho^{k_2-k_1}\right)\right) \\
+ \frac{\lambda}{\bar{\mu} - \lambda} \times \frac{\rho^{k_2-k_1+1} \times \left(1 - \rho^{k_1}\right)}{1-\rho}
\end{aligned}
$$

$$
E[N] = \frac{\text{Numerator2}}{\text{Denominator}}
$$

Problem 3-V Case B - will_turn_off = True

$$
\begin{aligned}
\text{Numerator2} = &\frac{k_1 * (k_1 - 1)}{2} + \left(\frac{\lambda * (\lambda + \gamma)}{\gamma} + (k_1 - 1) * \frac{\lambda}{\gamma}\right) \\
&+ \frac{\rho * \left(1 - \rho^{k_1} * (1 + k_1 (1 - \rho))\right)}{(1 - \rho)^2} + \frac{\rho}{1 - \rho} * \frac{k_1 * (k_1 + 1)}{2} \\
&+ \frac{(k_2 - k_1)(k_2 + k_1 + 1)}{2} * \left(\frac{\lambda}{\alpha} * \rho^{k_2} + \rho^{k_2 - k_1 + 1} * \frac{1 - \rho^{k_1}}{1 - \rho}\right) \\
&+ \frac{(k_2 - k_1)(k_2 + k_1 + 1)}{2} * \left(\frac{\lambda}{\lambda + \gamma} * \rho^{k_2 - k_1} * \frac{1 - \frac{\lambda}{\lambda+\gamma} * \rho^{k_2 - k_1}}{1 - \frac{\lambda}{\lambda+\gamma} * \rho}\right) \\
&+ \frac{\lambda}{\bar{\mu} - \lambda} * \left(k_2 + \frac{1}{\bar{\mu} - \lambda}\right) * \left(\frac{\lambda}{\alpha} * \rho^{k_2} + \frac{\rho^{k_2 - k_1 + 1} * (1 - \rho^{k_1})}{1 - \rho}\right) \\
&+ \frac{\lambda}{\bar{\mu} - \lambda} * \left(k_2 + \frac{1}{\bar{\mu} - \lambda}\right) * \left(\frac{\frac{\lambda}{\lambda+\gamma} * \rho^{k_2 - k_1}}{1 - \frac{\lambda}{\lambda+\gamma} * \rho} * \left(1 - \frac{\lambda}{\lambda + \gamma} * \rho^{k_2 - k_1}\right)\right)
\end{aligned}
$$

$$\rho = \frac{\lambda}{\mu}$$

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Problem 3-V Case B - will_turn_off = True

$$\frac{d}{d\bar{\mu}}(E[E]) = \frac{d}{d\bar{\mu}}\left(\frac{f(\bar{\mu})}{g(\bar{\mu})}\right) = \frac{g(\bar{\mu})\cdot\frac{d}{d\bar{\mu}}f(\bar{\mu}) - f(\bar{\mu})\cdot\frac{d}{d\bar{\mu}}g(\bar{\mu})}{g(\bar{\mu})^2}$$

$$\begin{aligned} f(\bar{\mu}) = {} & \frac{\lambda}{\bar{\mu}-\lambda} \\ & + \frac{\lambda}{\bar{\mu}-\lambda}\left(\frac{\lambda}{\alpha}\rho^{k_2} + \frac{\rho^{k_2-k_1+1}}{1-\rho}(1-\rho^{k_1})\right) \\ & + \frac{\lambda}{\bar{\mu}-\lambda}\left(\frac{\frac{\lambda}{\lambda+\gamma}\rho^{k_2-k_1}}{1-\frac{\lambda}{\lambda+\gamma}\rho}\left(1-\frac{\lambda}{\lambda+\gamma}\rho^{k_2-k_1}\right)\right) \\ & + \frac{\lambda+\gamma}{\lambda+\gamma-1}\left(\frac{\lambda}{\lambda+\gamma}\right)^{k_2-k_1+1} * \bar{\mu}^{\theta} \end{aligned}$$

$$\begin{aligned} f'(\bar{\mu}) = {} & -\frac{\lambda}{(\bar{\mu}-\lambda)^2} \\ & - \frac{\lambda}{(\bar{\mu}-\lambda)^2}\left(\frac{\lambda}{\alpha}\rho^{k_2} + \frac{\rho^{k_2-k_1+1}}{1-\rho}(1-\rho^{k_1})\right) \\ & - \frac{\lambda}{(\bar{\mu}-\lambda)^2}\left(\frac{\frac{\lambda}{\lambda+\gamma}\rho^{k_2-k_1}}{1-\frac{\lambda}{\lambda+\gamma}\rho}\left(1-\frac{\lambda}{\lambda+\gamma}\rho^{k_2-k_1}\right)\right) \\ & + \theta \times \frac{\lambda+\gamma}{\lambda+\gamma-1} \times \left(\frac{\lambda}{\lambda+\gamma}\right)^{k_2-k_1+1} * (\bar{\mu})^{\theta-1} \end{aligned}$$

Problem 3-V Case B - will_turn_off = True

$$
\begin{aligned}
g(\bar{\mu}) = Denominator = k_1 &+ \frac{\lambda}{\gamma} \\
&+ \left(k_2 - \frac{1-\rho^{k_2}}{1-\rho}\right) \times \frac{\lambda}{\mu - \lambda} \\
&+ \frac{\lambda}{\bar{\mu}-\lambda}\left(\frac{\lambda}{\alpha}\rho^{k_2} + \frac{\frac{\lambda}{\lambda+\gamma}\rho^{k_2-k_1}}{1-\frac{\lambda}{\lambda+\gamma}\rho}\left(1 - \frac{\lambda}{\lambda+\gamma}\rho^{k_2-k_1}\right)\right) \\
&+ \frac{\lambda}{\bar{\mu}-\lambda}\frac{\rho^{k_2-k_1+1}\left(1-\rho^{k_1}\right)}{1-\rho}
\end{aligned}
$$

$$
\begin{aligned}
g'(\bar{\mu}) = Denominator' = &-\frac{\lambda}{(\bar{\mu}-\lambda)^2}\left(\frac{\lambda}{\alpha}\rho^{k_2} + \frac{\frac{\lambda}{\lambda+\gamma}\rho^{k_2-k_1}}{1-\frac{\lambda}{\lambda+\gamma}\rho}\left(1 - \frac{\lambda}{\lambda+\gamma}\rho^{k_2-k_1}\right)\right) \\
&- \frac{\lambda}{(\bar{\mu}-\lambda)^2}\frac{\rho^{k_2-k_1+1}\left(1-\rho^{k_1}\right)}{1-\rho}
\end{aligned}
$$

Problem 3-V Case B - will_turn_off = True

$$\frac{d}{d\bar{\mu}}E[N] = \frac{\text{Denominator} \cdot \frac{d}{d\bar{\mu}}\text{Numerator2} - \text{Numerator2} \cdot \frac{d}{d\bar{\mu}}\text{Denominator}}{\text{Denominator}^2}$$

$$\begin{aligned}\frac{d}{d\bar{\mu}}\left(\text{Numerator2}\right) = &-\frac{\lambda}{(\bar{\mu}-\lambda)^2} \cdot \left(k_2 + \frac{1}{\bar{\mu}-\lambda}\right) \cdot \left(\frac{\lambda}{\alpha} \cdot \rho^{k_2} + \frac{\rho^{k_2-k_1+1}\left(1-\rho^{k_1}\right)}{1-\rho}\right) \\ &-\frac{\lambda}{(\bar{\mu}-\lambda)^2} \cdot \left(k_2 + \frac{1}{\bar{\mu}-\lambda}\right) \cdot \left(\frac{\frac{\lambda}{\lambda+\gamma} \cdot \rho^{k_2-k_1}}{1-\frac{\lambda}{\lambda+\gamma} \cdot \rho} \cdot \left(1 - \frac{\lambda}{\lambda+\gamma} \cdot \rho^{k_2-k_1}\right)\right)\end{aligned}$$

$$\begin{aligned}Denominator' = &-\frac{\lambda}{(\bar{\mu}-\lambda)^2}\left(\frac{\lambda}{\alpha}\rho^{k_2} + \frac{\frac{\lambda}{\lambda+\gamma}\rho^{k_2-k_1}}{1-\frac{\lambda}{\lambda+\gamma}\rho}\left(1-\frac{\lambda}{\lambda+\gamma}\rho^{k_2-k_1}\right)\right) \\ &-\frac{\lambda}{(\bar{\mu}-\lambda)^2}\frac{\rho^{k_2-k_1+1}\left(1-\rho^{k_1}\right)}{1-\rho}\end{aligned}$$

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Problem 3-V Case B - will_turn_off = True

Solutions: Yes, non-polarized μ values can exist in the optimal parameter group in **all 3 sub-condition.**

Sub-condition 1 - **β** is a **very small** value.
Sub-condition 2 - **β** is a **very large** value.
Sub-condition 3 - **β** is a **balanced** value.

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07 Conclusions

The final summarized results

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Problem 2

Does it allow non-polarized **μ/μ** to exist as optimal?	Problem 2 **Case A** will_turn_off = **False**	Problem 2 **Case B** will_turn_off = **True**	Problem 2-V **Case A** will_turn_off = **False**	Problem 2-V **Case B** will_turn_off = **True**
Sub-condition 1 - β is a **very small** value	✗	✓	✗	✓
Sub-condition 2 - β is a **very large** value	✗	✗	✗	✓
Sub-condition 3 - β is a **balanced** value	✓	✓	✓	✓

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Problem 3

Does it allow non-polarized **μ/μ** to exist as optimal?	Problem 3 **Case A** will_turn_off = **False**	Problem 3 **Case B** will_turn_off = **True**	Problem 3-V **Case A** will_turn_off = **False**	Problem 3-V **Case B** will_turn_off = **True**
Sub-condition 1 - β is a **very small** value	✓	✓	✓	✓
Sub-condition 2 - β is a **very large** value	✓	✓	✓	✓
Sub-condition 3 - β is a **balanced** value	✓	✓	✓	✓

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Interesting Discoveries

Both problems' **non-polarized** $\boldsymbol{\mu}/\underline{\boldsymbol{\mu}}$ **cases** have 2 common characteristics:

1. With a relatively much larger $\boldsymbol{\sigma}$ value. The normal states' ratio of energy consumption and processing rate is 1, while the value of $\boldsymbol{\sigma}$ should be much larger than 1.

2. With a high utilization $\boldsymbol{\rho}$. That means, non-polarized $\boldsymbol{\mu}$ values usually occur with a high $\boldsymbol{\lambda}$ value, closed to the value of $\boldsymbol{\mu}$. (e.g., $\lambda = 0.95$ and $\mu = 1.0$)

Section 08 - Asking Questions!

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End of this Final Presentation.

Thank you for your visiting.

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