

Software Requirements Specification for Projectile

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1 Reference Material

This section records information for easy reference.

1.1 Table of Units

The unit system used throughout is SI (Système International d'Unités). In addition to the basic units, several derived units are also used. For each unit, the [Table of Units](#) lists the symbol, a description, and the SI name.

Table 1: Table of Units

| Symbol | Description | SI Name |
|--------|-------------|---------|
| m | length | metre |
| rad | angle | radian |
| s | time | second |

1.2 Table of Symbols

The symbols used in this document are summarized in the [Table of Symbols](#) along with their units. Throughout the document, symbols in bold will represent vectors, and scalars otherwise. The symbols are listed in alphabetical order. For vector quantities, the units shown are for each component of the vector.

Table 2: Table of Symbols

| Symbol | Description | Units |
|---------------------|---|-------------------------------|
| a | Scalar acceleration | $\frac{\text{m}}{\text{s}^2}$ |
| a^c | Constant acceleration | $\frac{\text{m}}{\text{s}^2}$ |
| a_x | x -component of acceleration | $\frac{\text{m}}{\text{s}^2}$ |
| a_x^c | x -component of constant acceleration | $\frac{\text{m}}{\text{s}^2}$ |
| a_y | y -component of acceleration | $\frac{\text{m}}{\text{s}^2}$ |
| a_y^c | y -component of constant acceleration | $\frac{\text{m}}{\text{s}^2}$ |
| $\mathbf{a}(t)$ | Acceleration | $\frac{\text{m}}{\text{s}^2}$ |
| \mathbf{a}^c | Constant acceleration vector | $\frac{\text{m}}{\text{s}^2}$ |
| d_{offset} | Distance between the target position and the landing position | m |
| g | Magnitude of gravitational acceleration | $\frac{\text{m}}{\text{s}^2}$ |
| p | Scalar position | m |

Continued on next page

Table 2: Table of Symbols (Continued)

| Symbol | Description | Units |
|---------------------|---|-----------------------------|
| $p(t)$ | 1D position | m |
| p^i | Initial position | m |
| p_{land} | Landing position | m |
| p_{target} | Target position | m |
| p_x | x -component of position | m |
| p_x^i | x -component of initial position | m |
| p_y | y -component of position | m |
| p_y^i | y -component of initial position | m |
| $\mathbf{p}(t)$ | Position | m |
| s | Output message as a string | — |
| t | Time | s |
| t_{flight} | Flight duration | s |
| v | Speed | $\frac{\text{m}}{\text{s}}$ |
| $v(t)$ | 1D speed | $\frac{\text{m}}{\text{s}}$ |
| v^i | Initial speed | $\frac{\text{m}}{\text{s}}$ |
| v_{launch} | Launch speed | $\frac{\text{m}}{\text{s}}$ |
| v_x | x -component of velocity | $\frac{\text{m}}{\text{s}}$ |
| v_x^i | x -component of initial velocity | $\frac{\text{m}}{\text{s}}$ |
| v_y | y -component of velocity | $\frac{\text{m}}{\text{s}}$ |
| v_y^i | y -component of initial velocity | $\frac{\text{m}}{\text{s}}$ |
| $\mathbf{v}(t)$ | Velocity | $\frac{\text{m}}{\text{s}}$ |
| \mathbf{v}^i | Initial velocity | $\frac{\text{m}}{\text{s}}$ |
| ε | Hit tolerance | — |
| θ | Launch angle | rad |
| π | Ratio of circumference to diameter for any circle | — |

1.3 Abbreviations and Acronyms

Table 3: Abbreviations and Acronyms

| Abbreviation | Full Form |
|--------------|-------------------------------------|
| 1D | One-Dimensional |
| 2D | Two-Dimensional |
| A | Assumption |
| DD | Data Definition |
| GD | General Definition |
| GS | Goal Statement |
| IM | Instance Model |
| PS | Physical System Description |
| R | Requirement |
| RefBy | Referenced by |
| Refname | Reference Name |
| SRS | Software Requirements Specification |
| TM | Theoretical Model |
| Uncert. | Typical Uncertainty |

2 Introduction

Projectile motion is a common problem in physics. Therefore, it is useful to have a program to solve and model these types of problems. Common examples of projectile motion include ballistics problems (missiles, bullets, etc.) and the flight of balls in various sports (baseball, golf, football, etc.). The program documented here is called Projectile.

The following section provides an overview of the Software Requirements Specification (SRS) for Projectile. This section explains the purpose of this document, the scope of the requirements, the characteristics of the intended reader, and the organization of the document.

2.1 Purpose of Document

The primary purpose of this document is to record the requirements of Projectile. Goals, assumptions, theoretical models, definitions, and other model derivation information are specified, allowing the reader to fully understand and verify the purpose and scientific basis of Projectile. With the exception of **system constraints**, this SRS will remain abstract, describing what problem is being solved, but not how to solve it.

This document will be used as a starting point for subsequent development phases, including writing the design specification and the software verification and validation plan. The design document will show how the requirements are to be realized, including deci-

sions on the numerical algorithms and programming environment. The verification and validation plan will show the steps that will be used to increase confidence in the software documentation and the implementation. Although the SRS fits in a series of documents that follow the so-called waterfall model, the actual development process is not constrained in any way. Even when the waterfall model is not followed, as Parnas and Clements point out [parnasClements1986], the most logical way to present the documentation is still to “fake” a rational design process.

2.2 Scope of Requirements

The scope of the requirements includes the analysis of a two-dimensional (2D) projectile motion problem with constant acceleration.

[We are only interested in the position of the projectile, not its orientation A:noOrientation. —SS]

[We assume that forces are not relevant for the model so that we only need kinematic equations A:kinematicOnly. —SS]

2.3 Characteristics of Intended Reader

Reviewers of this documentation should have an understanding of undergraduate level 1 physics and undergraduate level 1 calculus. The users of Projectile can have a lower level of expertise, as explained in Sec:User Characteristics.

2.4 Organization of Document

The organization of this document follows the template for an SRS for scientific computing software proposed by [koothoor2013], [smithLai2005], [smithEtAl2007], and [smithKoothoor2016]. The presentation follows the standard pattern of presenting goals, theories, definitions, and assumptions. For readers that would like a more bottom up approach, they can start reading the instance models and trace back to find any additional information they require.

The goal statements are refined to the theoretical models and the theoretical models to the instance models.

3 General System Description

This section provides general information about the system. It identifies the interfaces between the system and its environment, describes the user characteristics, and lists the system constraints.



Figure 1: System Context

3.1 System Context

Fig:sysCtxDiag shows the system context. A circle represents an entity external to the software, the user in this case. A rectangle represents the software system itself (Projectile). Arrows are used to show the data flow between the system and its environment.

The interaction between the product and the user is through an application programming interface. The responsibilities of the user and the system are as follows:

- User Responsibilities
 - Provide initial conditions of the physical state of the motion and the input data related to the Projectile, ensuring no errors in the data entry.
 - Ensure that consistent units are used for input variables.
 - Ensure required **software assumptions** are appropriate for any particular problem input to the software.
- Projectile Responsibilities
 - Detect data type mismatch, such as a string of characters input instead of a floating point number.
 - Determine if the inputs satisfy the required physical and software constraints.
 - Calculate the required outputs.

3.2 User Characteristics

The end user of Projectile should have an understanding of high school physics and high school calculus.

3.3 System Constraints

There are no system constraints.

4 Specific System Description

This section first presents the problem description, which gives a high-level view of the problem to be solved. This is followed by the solution characteristics specification, which presents the assumptions, theories, and definitions that are used.

4.1 Problem Description

A system is needed to predict whether a launched projectile hits its target.

4.1.1 Terminology and Definitions

This subsection provides a list of terms that are used in the subsequent sections and their meaning, with the purpose of reducing ambiguity and making it easier to correctly understand the requirements.

- Launcher: Where the projectile is launched from and the device that does the launching.
- Projectile: The object to be launched at the target.
- Target: Where the projectile should be launched to.
- Gravity: The force that attracts one physical body with mass to another.
- Cartesian coordinate system: A coordinate system that specifies each point uniquely in a plane by a set of numerical coordinates, which are the signed distances to the point from two fixed perpendicular oriented lines, measured in the same unit of length (from [2]).
- Rectilinear: Occurring [\[fixed typo in “Ocurring” —SS\]](#) in one dimension.

4.1.2 Physical System Description

The physical system of Projectile, as shown in [Fig:Launch](#), includes the following elements:

PS1: The launcher.

PS2: The projectile (with initial velocity \mathbf{v}^i and launch angle θ).

PS3: The target.

4.1.3 Goal Statements

Given the initial velocity vector of the projectile and the geometric layout of the launcher and target, the goal statement is:

targetHit: Determine if the projectile hits the target.

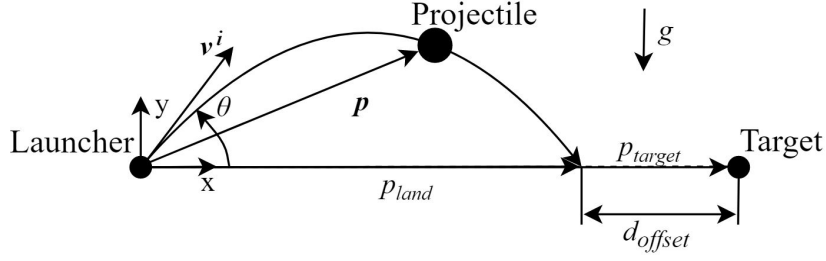


Figure 2: The physical system

4.2 Solution Characteristics Specification

The instance models that govern Projectile are presented in the [Instance Model Section](#). The information to understand the meaning of the instance models and their derivation is also presented, so that the instance models can be verified.

4.2.1 [Scope Assumptions —SS]

This section simplifies the original problem and helps in developing the theoretical models by filling in the missing information for the physical system. The assumptions refine the scope by providing more detail.

noOrientation: [The orientation of the projectile is ignored. We only care about its translation not its rotation. (RefBy: [TM:acceleration](#) and [TM:velocity](#).) —SS]

kinematicOnly: [The motion of the projectile is modelled with only kinematic equations. Forces are not considered. —SS]

4.2.2 [Background Theory Assumptions —SS]

cartSyst: A Cartesian coordinate system is used (from [A:neglectCurv](#)). (RefBy: [GD:velVec](#) and [GD:posVec](#).)

4.2.3 [Helper Theory Assumptions (GD:rectVel, GD:rectPos) —SS]

cartSyst: The motion of the particle is one dimensional. (RefBy: [GD:velVec](#) and [GD:posVec](#).)

constAccel: The acceleration is constant (from [A:accelXZero](#), [A:accelYGravity](#), [A:neglectDrag](#), and [A:freeFlight](#)). (RefBy: [GD:velVec](#) and [GD:posVec](#).)

4.2.4 [Helper Theory Assumptions (GD:velVec, GD:posVec) —SS]

twoDMotion: The projectile motion is two-dimensional (2D). (RefBy: GD:velVec and GD:posVec.)

constAccelX: [The acceleration is constant in the x direction. (RefBy: GD:velVec and GD:posVec.) —SS]

constAccelY: [The acceleration is constant in the y direction. (RefBy: GD:velVec and GD:posVec.) —SS]

4.2.5 [Theory Assumptions —SS]

twoDMotion: The projectile motion is two-dimensional (2D). (RefBy: GD:velVec and GD:posVec.)

yAxisGravity: The direction of the y -axis is directed opposite to gravity. (RefBy: IM:calOfLandingDist, IM:calOfLandingTime, and A:accelyGravity.)

launchOrigin: The launcher is coincident with the origin. (RefBy: IM:calOfLandingDist and IM:calOfLandingTime.)

targetXAxis: The target lies on the x -axis (from A:neglectCurv). (RefBy: IM:calOfLandingTime.)

posXDirection: The positive x -direction is from the launcher to the target. (RefBy: IM:offsetIM, IM:messageIM, IM:calOfLandingDist, and IM:calOfLandingTime.)

accelXZero: The acceleration in the x -direction is zero. (RefBy: IM:calOfLandingDist and A:constAccel.)

accelyGravity: The acceleration in the y -direction is the acceleration due to gravity (from A:yAxisGravity). (RefBy: IM:calOfLandingTime and A:constAccel.)

neglectDrag: Air drag is neglected. (RefBy: A:constAccel.)

pointMass: The size and shape of the projectile are negligible, so that it can be modelled as a point mass. (RefBy: GD:rectVel and GD:rectPos.)

freeFlight: The flight is free; there are no collisions during the trajectory of the projectile. (RefBy: A:constAccel.)

neglectCurv: The distance is small enough that the curvature of the celestial body can be neglected. (RefBy: A:targetXAxis and A:cartSyst.)

timeStartZero: Time starts at zero. (RefBy: GD:velVec, GD:rectVel, GD:rectPos, GD:posVec, and IM:calOfLandingTime.)

gravAccelValue: The acceleration due to gravity is assumed to have the value provided in the section for Values of Auxiliary Constants. (RefBy: IM:calOfLandingDist and IM:calOfLandingTime.)

4.2.6 [Final Theory Assumptions —SS]

- twoDMotion: The projectile motion is two-dimensional (2D). (RefBy: **GD:velVec** and **GD:posVec**.)
- yAxisGravity: The direction of the y -axis is directed opposite to gravity. (RefBy: **IM:calOfLandingDist**, **IM:calOfLandingTime**, and **A:accelYGravity**.)
- launchOrigin: The launcher is coincident with the origin. (RefBy: **IM:calOfLandingDist** and **IM:calOfLandingTime**.)
- targetXAxis: The target lies on the x -axis (from **A:neglectCurv**). (RefBy: **IM:calOfLandingTime**.)
- posXDirection: The positive x -direction is from the launcher to the target. (RefBy: **IM:offsetIM**, **IM:messageIM**, **IM:calOfLandingDist**, and **IM:calOfLandingTime**.)
- accelXZero: The acceleration in the x -direction is zero. (RefBy: **IM:calOfLandingDist** and **A:constAccel**.)
- accelYGravity: The acceleration in the y -direction is the acceleration due to gravity (from **A:yAxisGravity**). (RefBy: **IM:calOfLandingTime** and **A:constAccel**.)
- neglectDrag: Air drag is neglected. (RefBy: **A:constAccel**.)
- pointMass: The size and shape of the projectile are negligible, so that it can be modelled as a point mass. (RefBy: **GD:rectVel** and **GD:rectPos**.)
- freeFlight: The flight is free; there are no collisions during the trajectory of the projectile. (RefBy: **A:constAccel**.)
- neglectCurv: The distance is small enough that the curvature of the celestial body can be neglected. (RefBy: **A:targetXAxis** and **A:cartSyst**.)
- timeStartZero: Time starts at zero. (RefBy: **GD:velVec**, **GD:rectVel**, **GD:rectPos**, **GD:posVec**, and **IM:calOfLandingTime**.)
- gravAccelValue: The acceleration due to gravity is assumed to have the value provided in the section for **Values of Auxiliary Constants**. (RefBy: **IM:calOfLandingDist** and **IM:calOfLandingTime**.)

4.2.7 [Context Theories —SS]

[Some theories do not have to be explicitly invoked. They are part of the context for the other theories, without having to be explicitly stated or defined. The context theories for this problem are as follows:

- real arithmetic
- vectors

- Cartesian coordinate system
- differentiation
- integration

—SS]

4.2.8 [Background Theories (BT) —SS]

This section focuses on the general equations and laws that Projectile is based on. [Maybe relabel all of the background theories with the prefix BT, instead of TM? —SS]

| Refname | TM:acceleration |
|--------------------------------|---|
| Label | Acceleration |
| [Input —SS] | $[\mathbf{v}(t) : \mathbb{R} \rightarrow \mathbb{R}^3 \text{ —SS}]$ |
| [Output —SS] | $[\mathbf{a}(t) : \mathbb{R} \rightarrow \mathbb{R}^3 \text{ —SS}]$ |
| [Input Constraints —SS] | [None —SS] |
| [Output Constraints —SS] | [None —SS] |
| Equation | $\mathbf{a}(t) = \frac{d\mathbf{v}(t)}{dt}$ |
| Description | $\mathbf{a}(t)$ is the acceleration ($\frac{\text{m}}{\text{s}^2}$) t is the time (s) $\mathbf{v}(t)$ is the velocity ($\frac{\text{m}}{\text{s}}$) |
| Notes | The velocity and acceleration of the body are expressed using a Cartesian coordinate system (A:cartSyst). That is, the coordinate system is rectangular (orthonormal). |
| Source | [1] |
| RefBy | GD:rectVel |

Preconditions for TM:acceleration

- **A:cartSyst**

| Refname | TM:velocity |
|--------------------------------|--|
| Label | Velocity |
| [Input —SS] | $[\mathbf{p}(t) : \mathbb{R} \rightarrow \mathbb{R}^3 \text{ —SS}]$ |
| [Output —SS] | $[\mathbf{v}(t) : \mathbb{R} \rightarrow \mathbb{R}^3 \text{ —SS}]$ |
| [Input Constraints —SS] | [None —SS] |
| [Output Constraints —SS] | [None —SS] |
| Equation | $\mathbf{v}(t) = \frac{d\mathbf{p}(t)}{dt}$ |
| Description | <p>$\mathbf{v}(t)$ is the velocity ($\frac{\text{m}}{\text{s}}$)</p> <p>$t$ is the time (s)</p> <p>$\mathbf{p}(t)$ is the position (m)</p> |
| Notes | The position and velocity of the body are expressed using a Cartesian coordinate system (A:cartSyst). That is, the coordinate system is rectangular (orthonormal). |
| Source | [3] |
| RefBy | GD:rectPos |

Preconditions for TM:velocity

- **A:cartSyst**

4.2.9 [Helper Theories (GD:rectVel and GD:rectPos) —SS]

This section collects the laws and equations that will be used to build the instance models.
[\[We should remove the prefix GD. Maybe we should replace it with the prefix TM? —SS\]](#)

| Refname | GD:rectVel |
|--------------------------------|---|
| Label | Rectilinear (1D) velocity as a function of time for constant acceleration |
| Units | $\frac{\text{m}}{\text{s}}$ [Do we need units as a separate field? user option? —SS] |
| [Input —SS] | $[v^i : \mathbb{R}, a^c : \mathbb{R}, t : \mathbb{R} \text{ —SS}]$ |
| [Output —SS] | $[v(t) : \mathbb{R} \rightarrow \mathbb{R} \text{ —SS}]$ |
| [Input Constraints —SS] | $[t \geq 0 \text{ —SS}]$ [A:timeStartZero —SS] |
| [Output Constraints —SS] | $[\text{None} \text{ —SS}]$ |
| Equation | $v(t) = v^i + a^c t$ |
| Description | $v(t)$ is the 1D speed ($\frac{\text{m}}{\text{s}}$) v^i is the initial speed ($\frac{\text{m}}{\text{s}}$) a^c is the constant acceleration ($\frac{\text{m}}{\text{s}^2}$) t is the time (s) |
| [Notes —SS] | [See detailed derivation below —SS] |
| Source | [4, (pg. 8)] |
| RefBy | GD:velVec and GD:rectPos |

Detailed derivation of rectilinear velocity: [\[We start from the theory of acceleration TM:acceleration for a body in 3D Cartesian space: —SS\]](#)

$$\mathbf{a}(t) = \frac{d\mathbf{v}(t)}{dt}$$

[At this point we assume the body only travels in a straight line along one dimension of the coordinate system (**A:oneD**): —SS]

$$a(t) = \frac{dv(t)}{dt}$$

[We now assume that the acceleration is constant (does not vary with time) (**A:constAccel**) represented by a^c . The initial velocity (at $t = 0$, from **A:timeStartZero**) is represented by v^i . We now have: —SS]

$$a^c = \frac{dv}{dt}$$

Rearranging and integrating, we have:

$$\int_{v^i}^v 1 dv = \int_0^t a^c dt$$

Performing the integration, we have the required equation:

$$v(t) = v^i + a^c t$$

[**Preconditions for GD:rectVel** —SS]

- **A:cartSyst** (inherited from **TM:acceleration**)
- **A:oneD**
- **A:constAccel**
- **A:timeStartZero**

| | |
|--------------------------------|---|
| Refname | GD:rectPos |
| Label | Rectilinear (1D) position as a function of time for constant acceleration |
| Units | m |
| [Input —SS] | $[p^i : \mathbb{R}, v^i : \mathbb{R}, a^c : \mathbb{R}, t : \mathbb{R} \text{ —SS}]$ |
| [Output —SS] | $[p(t) : \mathbb{R} \rightarrow \mathbb{R} \text{ —SS}]$ |
| [Input Constraints —SS] | $[t \geq 0 \text{ —SS}]$ [A:timeStartZero —SS] |
| [Output Constraints —SS] | $[None \text{ —SS}]$ |
| Equation | $p(t) = p^i + v^i t + \frac{a^c t^2}{2}$ |
| Description | <p> $p(t)$ is the 1D position (m) p^i is the initial position (m) v^i is the initial speed ($\frac{\text{m}}{\text{s}}$) t is the time (s) a^c is the constant acceleration ($\frac{\text{m}}{\text{s}^2}$) </p> |
| [Notes —SS] | [See detailed derivation below —SS] |
| Source | [4, (pg. 8)] |
| RefBy | GD:posVec |

Detailed derivation of rectilinear position: [We start from the kinematic equation for velocity TM:velocity for a body in 3D Cartesian space: —SS]

$$\mathbf{v}(t) = \frac{d\mathbf{p}(t)}{dt}$$

[At this point we assume the body only travels in a straight line along one dimension of the coordinate system (**A:oneD**): —SS]

$$v(t) = \frac{dp(t)}{dt}$$

[The initial position (at $t = 0$, from **A:timeStartZero**) is represented by p^i . Rearranging the above equation and integrating we have: —SS]

$$\int_{p^i}^{p(t)} 1 dp = \int_0^t v(t) dt$$

[This equation has been changed from the previous version to show $p(t)$ and $v(t)$ —SS]

[We now assume that the acceleration is constant (does not vary with time) (**A:constAccel**) represented by a^c . The initial velocity (at $t = 0$, from **A:timeStartZero**) is represented by v^i . Since we satisfy the preconditions for **GD:rectVel** we can replace $v(t)$ to get: —SS]

$$\int_{p^i}^{p(t)} 1 dp = \int_0^t v^i + a^c t dt$$

[The above equation has been changed from the previous version to show $p(t)$. —SS] Performing the integration, we have the required equation:

$$p(t) = p^i + v^i t + \frac{a^c t^2}{2}$$

Preconditions for **GD:rectPos**

- **A:cartSyst** (inherited from **TM:velocity**)
- **A:oneD**
- **A:constAccel**
- **A:timeStartZero**

4.2.10 [Helper Theories (GD:velVec and GD:posVec) —SS]

| Refname | GD:velVec |
|-------------|--|
| Label | Velocity vector as a function of time for 2D motion under constant acceleration |
| Units | $\frac{\text{m}}{\text{s}}$ |
| Equation | $\mathbf{v}(t) = \begin{bmatrix} v_x^i + a_x^c t \\ v_y^i + a_y^c t \end{bmatrix}$ |
| Description | <p> $\mathbf{v}(t)$ is the velocity ($\frac{\text{m}}{\text{s}}$) v_x^i is the x-component of initial velocity ($\frac{\text{m}}{\text{s}}$) a_x^c is the x-component of constant acceleration ($\frac{\text{m}}{\text{s}^2}$) t is the time (s) v_y^i is the y-component of initial velocity ($\frac{\text{m}}{\text{s}}$) a_y^c is the y-component of constant acceleration ($\frac{\text{m}}{\text{s}^2}$) </p> |
| Source | — |
| RefBy | |

Detailed derivation of velocity vector: For a two-dimensional Cartesian coordinate system ([A:twoDMotion](#) and [A:cartSyst](#)), we can represent the velocity vector as $\mathbf{v}(t) = \begin{bmatrix} v_x \\ v_y \end{bmatrix}$ and the acceleration vector as $\mathbf{a}(t) = \begin{bmatrix} a_x \\ a_y \end{bmatrix}$. The acceleration is assumed to be constant in both the x direction a_x^c ([A:constAccelX](#)) and y direction a_y^c ([A:constAccelY](#)). The constant acceleration vector is represented as $\mathbf{a}^c = \begin{bmatrix} a_x^c \\ a_y^c \end{bmatrix}$. The initial velocity (at $t = 0$, from [A:timeStartZero](#)) is represented by $\mathbf{v}^i = \begin{bmatrix} v_x^i \\ v_y^i \end{bmatrix}$. [For each coordinate direction we satisfy the preconditions for [GD:rectVel](#). This means we can use the one dimensional equation for each of the two coordinate directions to yield the required equation: —SS]

$$\mathbf{v}(t) = \begin{bmatrix} v_x^i + a_x^c t \\ v_y^i + a_y^c t \end{bmatrix}$$

| | |
|-------------|---|
| Refname | GD:posVec |
| Label | Position vector as a function of time for 2D motion under constant acceleration |
| Units | m |
| Equation | $\mathbf{p}(t) = \begin{bmatrix} p_x^i + v_x^i t + \frac{a_x^c t^2}{2} \\ p_y^i + v_y^i t + \frac{a_y^c t^2}{2} \end{bmatrix}$ |
| Description | <p>$\mathbf{p}(t)$ is the position (m)</p> <p>p_x^i is the x-component of initial position (m)</p> <p>v_x^i is the x-component of initial velocity ($\frac{\text{m}}{\text{s}}$)</p> <p>$t$ is the time (s)</p> <p>a_x^c is the x-component of constant acceleration ($\frac{\text{m}}{\text{s}^2}$)</p> <p>$p_y^i$ is the y-component of initial position (m)</p> <p>v_y^i is the y-component of initial velocity ($\frac{\text{m}}{\text{s}}$)</p> <p>$a_y^c$ is the y-component of constant acceleration ($\frac{\text{m}}{\text{s}^2}$)</p> |
| Source | — |
| RefBy | IM:calOfLandingDist and IM:calOfLandingTime |

Detailed derivation of position vector: For a two-dimensional Cartesian coordinate system ([A:twoDMotion](#) and [A:cartSyst](#)), we can represent the position vector as $\mathbf{p}(t) = \begin{bmatrix} p_x \\ p_y \end{bmatrix}$, the velocity vector as $\mathbf{v}(t) = \begin{bmatrix} v_x \\ v_y \end{bmatrix}$, and the acceleration vector as $\mathbf{a}(t) = \begin{bmatrix} a_x \\ a_y \end{bmatrix}$. The acceleration is assumed to be constant ([A:constAccel](#)) and the constant acceleration vector is represented as $\mathbf{a}^c = \begin{bmatrix} a_x^c \\ a_y^c \end{bmatrix}$. The initial velocity (at $t = 0$, from [A:timeStartZero](#)) is represented by $\mathbf{v}^i = \begin{bmatrix} v_x^i \\ v_y^i \end{bmatrix}$. Since we have a Cartesian coordinate system, [GD:rectPos](#) can be applied to each coordinate of the position vector to yield the required equation:

$$\mathbf{p}(t) = \begin{bmatrix} p_x^i + v_x^i t + \frac{a_x^c t^2}{2} \\ p_y^i + v_y^i t + \frac{a_y^c t^2}{2} \end{bmatrix}$$

4.2.11 Data Definitions

This section collects and defines all the data needed to build the instance models.

| | |
|-------------|---|
| Refname | DD:vecMag |
| Label | Speed |
| Symbol | v |
| Units | $\frac{\text{m}}{\text{s}}$ |
| Equation | $v = \ \mathbf{v}(t)\ $ |
| Description | v is the speed ($\frac{\text{m}}{\text{s}}$) $\mathbf{v}(t)$ is the velocity ($\frac{\text{m}}{\text{s}}$) |
| Notes | For a given velocity vector $\mathbf{v}(t)$, the magnitude of the vector ($\ \mathbf{v}(t)\ $) is the scalar called speed. |
| Source | – |
| RefBy | DD:speedIY and DD:speedIX |

| | |
|-------------|---|
| Refname | DD:speedIX |
| Label | x -component of initial velocity |
| Symbol | v_x^i |
| Units | $\frac{\text{m}}{\text{s}}$ |
| Equation | $v_x^i = v^i \cos(\theta)$ |
| Description | v_x^i is the x -component of initial velocity ($\frac{\text{m}}{\text{s}}$) v^i is the initial speed ($\frac{\text{m}}{\text{s}}$) θ is the launch angle (rad) |
| Notes | v^i is from DD:vecMag. θ is shown in Fig:Launch. |
| Source | — |
| RefBy | IM:calOfLandingDist |

| | |
|-------------|---|
| Refname | DD:speedIY |
| Label | y -component of initial velocity |
| Symbol | v_y^i |
| Units | $\frac{\text{m}}{\text{s}}$ |
| Equation | $v_y^i = v^i \sin(\theta)$ |
| Description | v_y^i is the y -component of initial velocity ($\frac{\text{m}}{\text{s}}$) v^i is the initial speed ($\frac{\text{m}}{\text{s}}$) θ is the launch angle (rad) |
| Notes | v^i is from DD:vecMag. θ is shown in Fig:Launch. |
| Source | — |
| RefBy | IM:calOfLandingTime |

4.2.12 Instance Models

This section transforms the problem defined in the [problem description](#) into one which is expressed in mathematical terms. It uses concrete symbols defined in the [data definitions](#) to replace the abstract symbols in the models identified in [theoretical models](#) and [general definitions](#).

| | | | |
|--------------------|---|--|--|
| Refname | IM:calOfLandingTime | | |
| Label | Calculation of landing time | | |
| Input | $v_{\text{launch}}, \theta$ | | |
| Output | t_{flight} | | |
| Input Constraints | $v_{\text{launch}} > 0$ $0 < \theta < \frac{\pi}{2}$ | | |
| Output Constraints | $t_{\text{flight}} > 0$ | | |
| Equation | $t_{\text{flight}} = \frac{2v_{\text{launch}} \sin(\theta)}{g}$ | | |
| Description | t_{flight} is the flight duration (s) v_{launch} is the launch speed ($\frac{\text{m}}{\text{s}}$) θ is the launch angle (rad) g is the magnitude of gravitational acceleration ($\frac{\text{m}}{\text{s}^2}$) | | |
| Notes | The constraint $0 < \theta < \frac{\pi}{2}$ is from A:posXDirection and A:yAxisGravity , and is shown in Fig:Launch . g is defined in A:gravAccelValue . The constraint $t_{\text{flight}} > 0$ is from A:timeStartZero . | | |
| Source | – | | |
| RefBy | IM:calOfLandingDist , FR:Output-Values , and FR:Calculate-Values | | |

Detailed derivation of flight duration: We know that $p_y^i = 0$ ([A:launchOrigin](#)) and $a_y^c = -g$ ([A:accelYGravity](#)). Substituting these values into the y-direction of [GD:posVec](#) gives us:

$$p_y = v_y^i t - \frac{gt^2}{2}$$

To find the time that the projectile lands, we want to find the t value (t_{flight}) where $p_y = 0$ (since the target is on the x -axis from **A:targetXAxis**). From the equation above we get:

$$v_y^i t_{\text{flight}} - \frac{gt_{\text{flight}}^2}{2} = 0$$

Dividing by t_{flight} (with the constraint $t_{\text{flight}} > 0$) gives us:

$$v_y^i - \frac{gt_{\text{flight}}}{2} = 0$$

Solving for t_{flight} gives us:

$$t_{\text{flight}} = \frac{2v_y^i}{g}$$

From **DD:speedIY** (with $v^i = v_{\text{launch}}$) we can replace v_y^i :

$$t_{\text{flight}} = \frac{2v_{\text{launch}} \sin(\theta)}{g}$$

| | | | |
|--------------------|--|--|--|
| Refname | IM:calOfLandingDist | | |
| Label | Calculation of landing position | | |
| Input | $v_{\text{launch}}, \theta$ | | |
| Output | p_{land} | | |
| Input Constraints | $v_{\text{launch}} > 0$ $0 < \theta < \frac{\pi}{2}$ | | |
| Output Constraints | $p_{\text{land}} > 0$ | | |
| Equation | $p_{\text{land}} = \frac{2v_{\text{launch}}^2 \sin(\theta) \cos(\theta)}{g}$ | | |
| Description | <p>p_{land} is the landing position (m) v_{launch} is the launch speed ($\frac{\text{m}}{\text{s}}$) θ is the launch angle (rad) g is the magnitude of gravitational acceleration ($\frac{\text{m}}{\text{s}^2}$)</p> | | |
| Notes | <p>The constraint $0 < \theta < \frac{\pi}{2}$ is from A:posXDirection and A:yAxisGravity, and is shown in Fig:Launch. g is defined in A:gravAccelValue. The constraint $p_{\text{land}} > 0$ is from A:posXDirection.</p> | | |
| Source | – | | |
| RefBy | IM:offsetIM and FR:Calculate-Values | | |

Detailed derivation of landing position: We know that $p_x^i = 0$ ([A:launchOrigin](#)) and $a_x^c = 0$ ([A:accelXZero](#)). Substituting these values into the x-direction of [GD:posVec](#) gives us:

$$p_x = v_x^i t$$

To find the landing position, we want to find the p_x value (p_{land}) at flight duration (from **IM:calOfLandingTime**):

$$p_{\text{land}} = \frac{v_x^i \cdot 2v_{\text{launch}} \sin(\theta)}{g}$$

From **DD:speedIX** (with $v^i = v_{\text{launch}}$) we can replace v_x^i :

$$p_{\text{land}} = \frac{v_{\text{launch}} \cos(\theta) \cdot 2v_{\text{launch}} \sin(\theta)}{g}$$

Rearranging this gives us the required equation:

$$p_{\text{land}} = \frac{2v_{\text{launch}}^2 \sin(\theta) \cos(\theta)}{g}$$

| | | | |
|-----------------------|---|--|--|
| Refname | IM:offsetIM | | |
| Label | Offset | | |
| Input | $p_{\text{land}}, p_{\text{target}}$ | | |
| Output | d_{offset} | | |
| Input Constraints | $p_{\text{land}} > 0$ $p_{\text{target}} > 0$ | | |
| Output Constraints | | | |
| Equation | $d_{\text{offset}} = p_{\text{land}} - p_{\text{target}}$ | | |
| Description | d_{offset} is the distance between the target position and the landing position (m) p_{land} is the landing position (m) p_{target} is the target position (m) | | |
| Notes | p_{land} is from IM:calOfLandingDist . The constraints $p_{\text{land}} > 0$ and $p_{\text{target}} > 0$ are from A:posXDirection . | | |
| Source | — | | |
| RefBy | IM:messageIM , FR:Output-Values , and FR:Calculate-Values | | |

| | |
|--------------------|--|
| Refname | IM:messageIM |
| Label | Output message |
| Input | $d_{\text{offset}}, p_{\text{target}}$ |
| Output | s |
| Input Constraints | $d_{\text{offset}} > -p_{\text{target}}$ $p_{\text{target}} > 0$ |
| Output Constraints | |
| Equation | $s = \begin{cases} \text{“The target was hit.”}, & \left \frac{d_{\text{offset}}}{p_{\text{target}}} \right < \varepsilon \\ \text{“The projectile fell short.”}, & d_{\text{offset}} < 0 \\ \text{“The projectile went long.”}, & d_{\text{offset}} > 0 \end{cases}$ |
| Description | <p>s is the output message as a string (Unitless)</p> <p>d_{offset} is the distance between the target position and the landing position (m)</p> <p>p_{target} is the target position (m)</p> <p>ε is the hit tolerance (Unitless)</p> |
| Notes | <p>d_{offset} is from IM:offsetIM.</p> <p>The constraint $p_{\text{target}} > 0$ is from A:posXDirection.</p> <p>The constraint $d_{\text{offset}} > -p_{\text{target}}$ is from the fact that $p_{\text{land}} > 0$, from A:posXDirection.</p> <p>ε is defined in Sec:Values of Auxiliary Constants.</p> |
| Source | — |
| RefBy | FR:Output-Values and FR:Calculate-Values |

4.2.13 Data Constraints

The **Data Constraints Table** shows the data constraints on the input variables. The column for physical constraints gives the physical limitations on the range of values that can be taken by the variable. The uncertainty column provides an estimate of the confidence with which the physical quantities can be measured. This information would be part of the input if one were performing an uncertainty quantification exercise. The constraints are conservative to give the user of the model the flexibility to experiment with unusual situations. The column of typical values is intended to provide a feel for a common scenario.

Table 4: Input Data Constraints

| Var | Physical Constraints | Typical Value | Uncert. |
|---------------------|------------------------------|---------------------------------|---------|
| p_{target} | $p_{\text{target}} > 0$ | 1000 m | 10% |
| v_{launch} | $v_{\text{launch}} > 0$ | $100 \frac{\text{m}}{\text{s}}$ | 10% |
| θ | $0 < \theta < \frac{\pi}{2}$ | $\frac{\pi}{4}$ rad | 10% |

4.2.14 Properties of a Correct Solution

The **Data Constraints Table** shows the data constraints on the output variables. The column for physical constraints gives the physical limitations on the range of values that can be taken by the variable.

Table 5: Output Data Constraints

| Var | Physical Constraints |
|---------------------|--|
| p_{land} | $p_{\text{land}} > 0$ |
| d_{offset} | $d_{\text{offset}} > -p_{\text{target}}$ |
| t_{flight} | $t_{\text{flight}} > 0$ |

5 Requirements

This section provides the functional requirements, the tasks and behaviours that the software is expected to complete, and the non-functional requirements, the qualities that the software is expected to exhibit.

5.1 Functional Requirements

This section provides the functional requirements, the tasks and behaviours that the software is expected to complete.

Input-Values: Input the values from **Tab:ReqInputs**.

- Verify-Input-Values: Check the entered input values to ensure that they do not exceed the **data constraints**. If any of the input values are out of bounds, an error message is displayed and the calculations stop.
- Calculate-Values: Calculate the following values: t_{flight} (from **IM:calOfLandingTime**), p_{land} (from **IM:calOfLandingDist**), d_{offset} (from **IM:offsetIM**), and s (from **IM:messageIM**).
- Output-Values: Output t_{flight} (from **IM:calOfLandingTime**), s (from **IM:messageIM**), and d_{offset} (from **IM:offsetIM**).

Table 6: Required Inputs following
FR:Input-Values

| Symbol | Description | Units |
|---------------------|-----------------|-----------------------------|
| p_{target} | Target position | m |
| v_{launch} | Launch speed | $\frac{\text{m}}{\text{s}}$ |
| θ | Launch angle | rad |

5.2 Non-Functional Requirements

This section provides the non-functional requirements, the qualities that the software is expected to exhibit.

Correct: The outputs of the code have the properties described in **Properties of a Correct Solution**.

Verifiable: The code is tested with complete verification and validation plan.

Understandable: The code is modularized with complete module guide and module interface specification.

Reusable: The code is modularized.

Maintainable: The traceability between requirements, assumptions, theoretical models, general definitions, data definitions, instance models, likely changes, unlikely changes, and modules is completely recorded in traceability matrices in the SRS and module guide.

Portable: The code is able to be run in different environments.

5.3 [Rationale —SS]

[Capture the rationale for the scope assumptions and final theory assumptions. The rationale could vary between problems. For instance, for projectile motion the rationale could be that it is being used for teaching purposes. If the theories are used to solve an actual science or

engineering problem, the rationale would need more justification. —SS] [Neglecting rotation could be justified by assuming a point mass? A:pointMass) —SS]. [Should requirements be added related to guaranteeing assumptions and constraints? (As is done after a hazard analysis.) Requirements could be added to check the input constraints, like $x > 0$. Requirements could be added to check neglecting curvature. Would need the radius of the planet, or are we assuming it's Earth? —SS]

6 Traceability Matrices and Graphs

The purpose of the traceability matrices is to provide easy references on what has to be additionally modified if a certain component is changed. Every time a component is changed, the items in the column of that component that are marked with an “X” should be modified as well. Tab:TraceMatAvsA shows the dependencies of the assumptions on each other. Tab:TraceMatAvsAll shows the dependencies of the data definitions, theoretical models, general definitions, instance models, requirements, likely changes, and unlikely changes on the assumptions. Tab:TraceMatRefvsRef shows the dependencies of the data definitions, theoretical models, general definitions, and instance models on each other. Tab:TraceMatAllvsR shows the dependencies of the requirements and goal statements on the data definitions, theoretical models, general definitions, and instance models.

| | A:twoDMotion | A:cartSyst | A:yAxisGravity | A:launchOrigin | A:targetX |
|-----------------|--------------|------------|----------------|----------------|-----------|
| A:twoDMotion | | | | | |
| A:cartSyst | | | | | |
| A:yAxisGravity | | | | | |
| A:launchOrigin | | | | | |
| A:targetXAxis | | | | | |
| A:posXDirection | | | | | |
| A:constAccel | | | | | |
| A:accelXZero | | | | | |
| A:accelYGravity | | | X | | |
| A:neglectDrag | | | | | |
| A:pointMass | | | | | |
| A:freeFlight | | | | | |
| A:neglectCurv | | | | | |
| A:timeStartZero | | | | | |

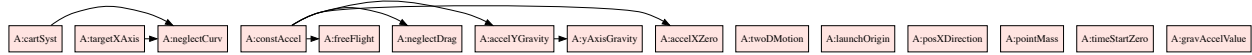
Table 7: T

| | A:twoDMotion | A:cartSyst | A:yAxisGravity | A:launchOrigin | A:target2 |
|------------------------|--------------|------------|----------------|----------------|-----------|
| A:gravAccelValue | | | | | |
| DD:vecMag | | | | | |
| DD:speedIX | | | | | |
| DD:speedIY | | | | | |
| TM:acceleration | | | | | |
| TM:velocity | | | | | |
| GD:rectVel | | | | | |
| GD:rectPos | | | | | |
| GD:velVec | X | X | | | |
| GD:posVec | X | X | | | |
| IM:calOfLandingTime | | | X | X | X |
| IM:calOfLandingDist | | | X | X | |
| IM:offsetIM | | | | | |
| IM:messageIM | | | | | |
| FR:Input-Values | | | | | |
| FR:Verify-Input-Values | | | | | |
| FR:Calculate-Values | | | | | |
| FR:Output-Values | | | | | |
| NFR:Correct | | | | | |
| NFR:Verifiable | | | | | |
| NFR:Understandable | | | | | |
| NFR:Reusable | | | | | |
| NFR:Maintainable | | | | | |
| NFR:Portable | | | | | |

Table 9: Traceability

| | DD:vecMag | DD:speedIX | DD:speedIY | TM:acceleration | TM:velocity |
|------------------------|-----------|------------|------------|-----------------|-------------|
| DD:vecMag | | | | | |
| DD:speedIX | X | | | | |
| DD:speedIY | X | | | | |
| TM:acceleration | | | | | |
| TM:velocity | | | | | |
| GD:rectVel | | | | X | |
| GD:rectPos | | | | | X |
| GD:velVec | | | | | |
| GD:posVec | | | | | |
| IM:calOfLandingTime | | | X | | |
| IM:calOfLandingDist | | X | | | |
| IM:offsetIM | | | | | |
| IM:messageIM | | | | | |
| | DD:vecMag | DD:speedIX | DD:speedIY | TM:acceleration | TM:velocity |
| GS:targetHit | | | | | |
| FR:Input-Values | | | | | |
| FR:Verify-Input-Values | | | | | |
| FR:Calculate-Values | | | | | |
| FR:Output-Values | | | | | |
| NFR:Correct | | | | | |
| NFR:Verifiable | | | | | |
| NFR:Understandable | | | | | |
| NFR:Reusable | | | | | |
| NFR:Maintainable | | | | | |
| NFR:Portable | | | | | |

The purpose of the traceability graphs is also to provide easy references on what has to be additionally modified if a certain component is changed. The arrows in the graphs represent dependencies. The component at the tail of an arrow is depended on by the com-



ponent at the head of that arrow. Therefore, if a component is changed, the components that it points to should also be changed. **Fig:TraceGraphAvsA** shows the dependencies of assumptions on each other. **Fig:TraceGraphAvsAll** shows the dependencies of data definitions, theoretical models, general definitions, instance models, requirements, likely changes, and unlikely changes on the assumptions. **Fig:TraceGraphRefsRef** shows the dependencies of data definitions, theoretical models, general definitions, and instance models on each other. **Fig:TraceGraphAllvsR** shows the dependencies of requirements and goal statements on the data definitions, theoretical models, general definitions, and instance models. **Fig:Trace-GraphAllvsAll** shows the dependencies of dependencies of assumptions, models, definitions, requirements, goals, and changes with each other.



Figure 6: TraceGraphAllvsR

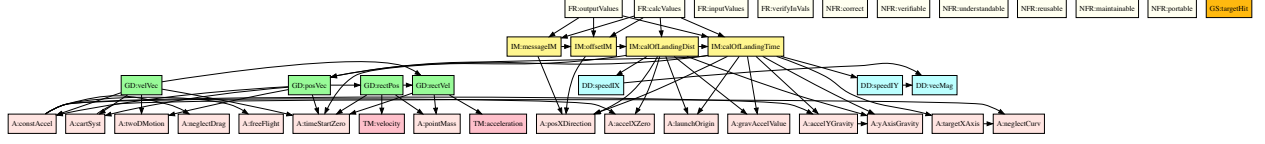


Figure 7: TraceGraphAllvsAll

7 Values of Auxiliary Constants

This section contains the standard values that are used for calculations in Projectile.

Table 11: Auxiliary Constants

| Symbol | Description | Value | Unit |
|------------|---|------------|-------------------------------|
| g | magnitude of gravitational acceleration | 9.8 | $\frac{\text{m}}{\text{s}^2}$ |
| ϵ | hit tolerance | 2.0% | — |
| π | ratio of circumference to diameter for any circle | 3.14159265 | — |

8 References

- [1] Wikipedia Contributors. *Acceleration*. <https://en.wikipedia.org/wiki/Acceleration>. June 2019.
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- [4] R. C. Hibbeler. *Engineering Mechanics: Dynamics*. Pearson Prentice Hall, 2004.