Discrimination by m_b-M_S

Akio Katsumata
(Meteorological Research Institute, Japan Meteorological Agency)

Contents

1 Magnitude **1**
1.1 Local Magnitude (M_L) . . . 1
1.2 Body-wave Magnitude (m_b) . . . 1
1.3 Surface-wave Magnitude (M_S) . . . 1
1.4 Moment Magnitude (M_w) . . . 3
1.5 Relationship of Various Magnitude Scales . . . 4

2 Response of Seismographs **5**
2.1 SP . . . 5
2.2 LP . . . 5

3 Scaling Relationship **7**

4 Discrimination by by m_b-M_S **9**

5 Practice of Magnitude determination **10**
5.1 Amplitude and Period Measurement . . . 10
5.2 SAC2000 commands . . . 10
5.2.1 Magnitude calculation . . . 10
5.2.2 Coordinate rotation . . . 11
5.2.3 Two-dimensional Plot . . . 11
5.2.4 Phase Picking . . . 11
5.3 Sample Files . . . 13

6 References **16**

A Appendix **17**
A.1 Seismic moment . . . 17
A.1.1 Seismic moment . . . 17
A.1.2 Seismic moment and double-couple . . . 18
A.1.3 Moment tensor . . . 19
A.1.4 Seismic moment and seismic waves . . . 21

1 Magnitude

There are many definition of magnitude owing to many types of instruments. Definitions of magnitudes are described in this section.

1.1 Local Magnitude (M_L)

Richter (1935) defined earthquake magnitude at the first for local shocks in Southern California as

$$M_L = log_{10} A,$$

where A is the maximum trace amplitude (μm) recorded by the Wood-Anderson seismometer ($T_0 = 0.8$s, $h = 0.8$, $V_0 = 2800$) at a epicentral distance of 100 km. This kind of magnitude is called local magnitude. Richter (1935) gave empirical correction value for epicentral distance $\Delta = 25$–600 km.

The mass media usually call earthquake magnitude as Richter scale, but magnitude kind is not specified by the word of "the Richter scale".

1.2 Body-wave Magnitude (m_b)

Gutenberg extended the magnitude scale to teleseismic data and deeper shocks. Gutenberg Gutenberg (1945a,b) gave a formula as

$$m_B = \log(A/T)_{max} + q(\Delta, h),$$

where A is a ground amplitude in micro-meter (μm), T is a period in seconds, and $q(\Delta, h)$ is calibrating function to correct amplitude decay for a epicentral distance (Δ) and focal depth (h). $q(\Delta, h)$ were gave for different types of waves(PZ, PPZ, SH). Fig. 1 shows $q(\Delta, h)$ for P-wave by Gutenberg and Richter (1956).

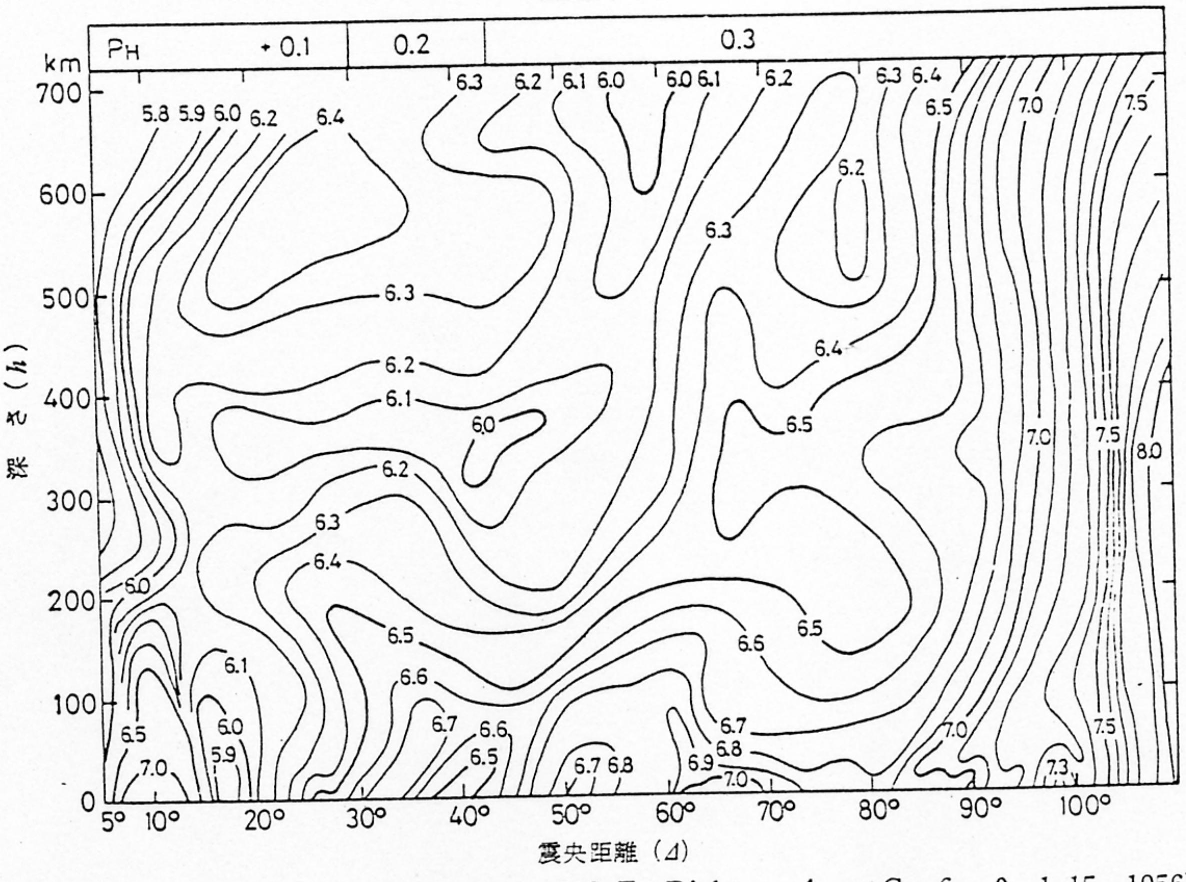
Body-wave magnitudes currently reported by the ISC and the NEIC (USGS) (m_b) are not identical with the body-wave magnitude m_B by Gutenberg (1945a, 1945b), because the amplitude from only short-period instruments are used for m_b. Gutenberg used data from instruments of longer natural periods. Maximum ground amplitude obtained with WWSS SP component is used for m_b. The same function as Fig. 1 is used for m_b determination. The maximum should be read within 20 seconds from the onset, and $T \leq 3$ sec in the ISC.

1.3 Surface-wave Magnitude (M_S)

Surface magnitude is determined with the formula (Vaněk *et al.*(1962) of

$$M_S = \log(A/T) + 1.66 \log \Delta + 3.3,$$

where A is maximum ground amplitude recorded by WWSS LP vertical component in μm, T is corresponding period (seconds), and Δ is epicentral distance in degree. T and



$q(\Delta, h)$ の値 (Gutenberg, B. and C. F. Richter: *Ann. Geofis*, 9, 1–15, 1956)

Figure 1: $q(\Delta, h)$ for m_b.

Δ should be $10 \leq T \leq 60$ (sec) ($18 \leq T \leq 22$ recommended), $20 \leq \Delta \leq 160$ (deg) (ISC), and calculated only for shallow events ($depth \leq 60$km).

The original formula by Gutenberg(1945c) is a little different from the above formula.

$$M_S = \log A + 1.656 \log \Delta + 1.818.$$

1.4 Moment Magnitude (M_w)

Seismic moment is considered to be the best way to indicate of sizes of earthquakes. Seismic moment M_0 is calculated as

$$M_0 = \mu DS,$$

where μ is rigidity of underground rock, D is dislocation on a fault, nad S is area of the fault.

There is an empirical relationship between seismic moment M_0(Nm) and surface-wave magnitude M_S (Kanamori, 1977).

$$\log_{10} M_0 = 1.5 M_S + 9.1 \tag{1}$$

Based on this relationship, Kanamori (1977) defined a moment magnitude (energy magnitude) M_w.

$$M_w = (\log_{10} M_0 - 9.1)/1.5 \tag{2}$$

Since this kind of magnitude has a definite physical meaning, the moment magnitude is often referred to. Conventional magnitudes show "saturation" due to limitation of observable period of seismic waves. The moment magnitude does have "limitation" of magnitude because it it not related to instrumental responses.

1.5 Relationship of Various Magnitude Scales

Fig. 2 shows relationships among various magnitude scales by Utsu (1982).

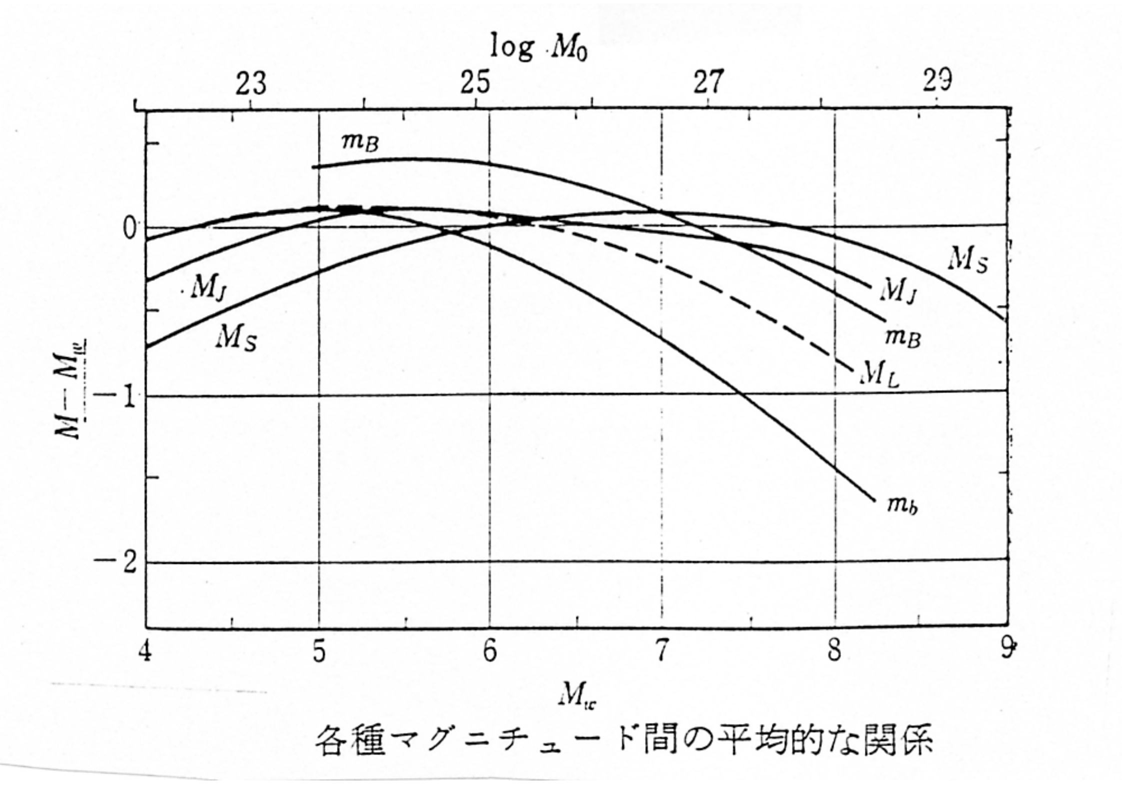


Figure 2: Relationship of various magnitude scales for shallow earthquakes (Utsu, 1982).

2 Response of Seismographs

2.1 SP

Fig. 3 shows response of LP component.

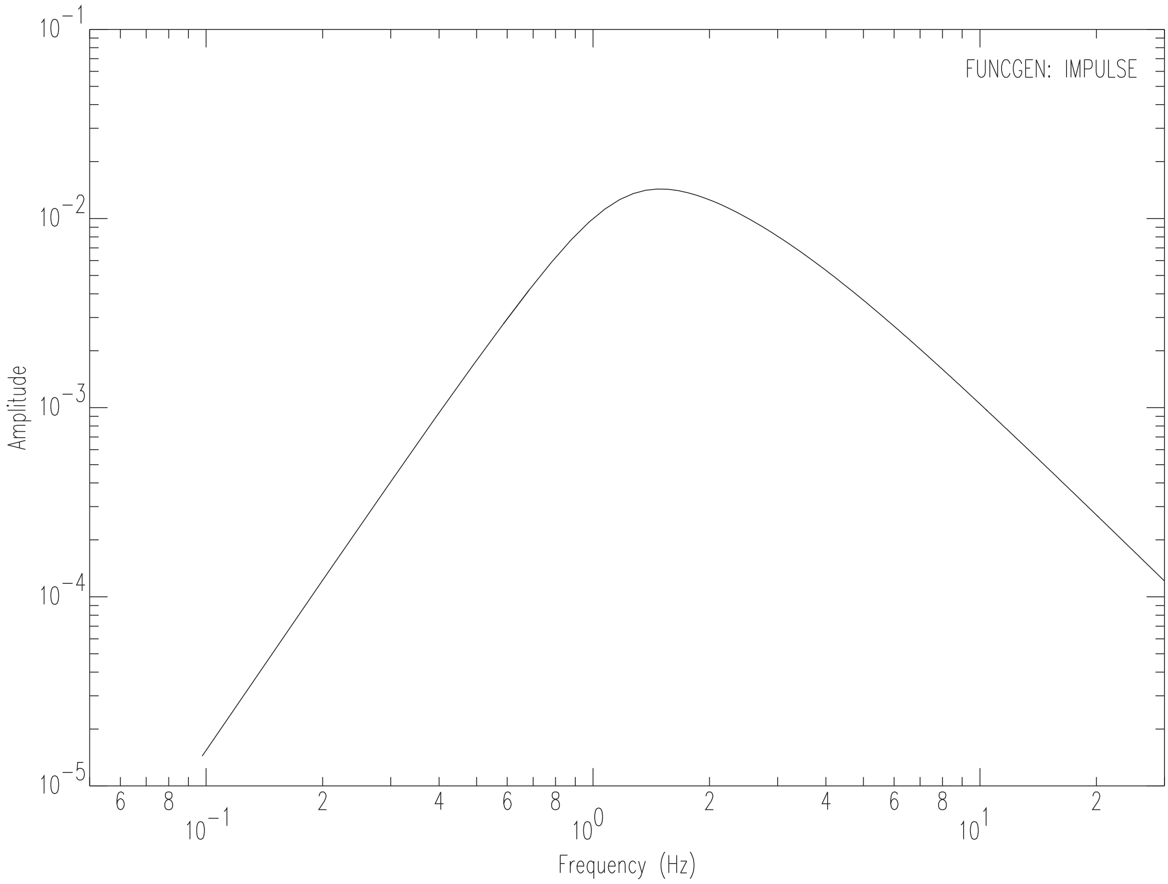


Figure 3: Response of SP component.

2.2 LP

The long-period seismographs of the WWSSN consisted of a long-period electrodynamic seismometer normally tuned to a free period of 15 sec, and a long-period mirror-galvanometer with a free period around 90 sec. Fig. 4 shows response of LP component.

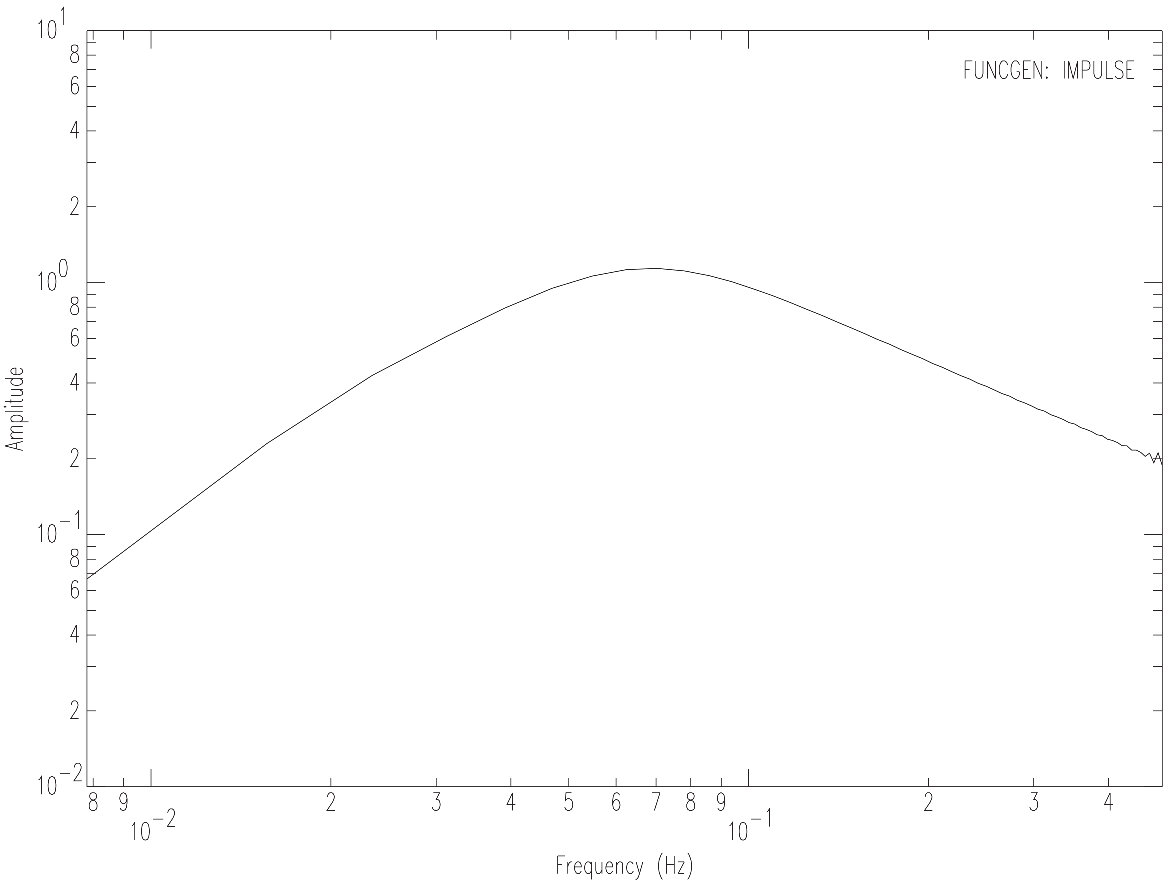


Figure 4: Response of LP component.

3 Scaling Relationship

If stress of σ is released on a fault of length L, the dislocation will be $D \simeq L\sigma/\mu$. The fault width is assumed to be proportional to the fault length. The seismic moment of the case is

$$\begin{aligned} M_0 &= \mu D S \\ &\simeq \mu(L\sigma/\mu)(LW) \\ &\propto \sigma L^3. \end{aligned}$$

If the stress drop σ is unchanged, the seismic moment would be proportional to cube of fault length L^3.

If the rupture velocity V_R is almost constant, the duration of the fault rupture T_r would be proportional to $M_0^{1/3}$.

$$\begin{aligned} T_r &\simeq \frac{L}{V_R} \\ &\propto M_0^{1/3} \end{aligned}$$

The spectrum of seismic wave is expressed as Fig. 5 (Aki, 1967). For the surface-wave magnitude, seismic wave of period 20sec is evaluated (arrow in Fig. 5).

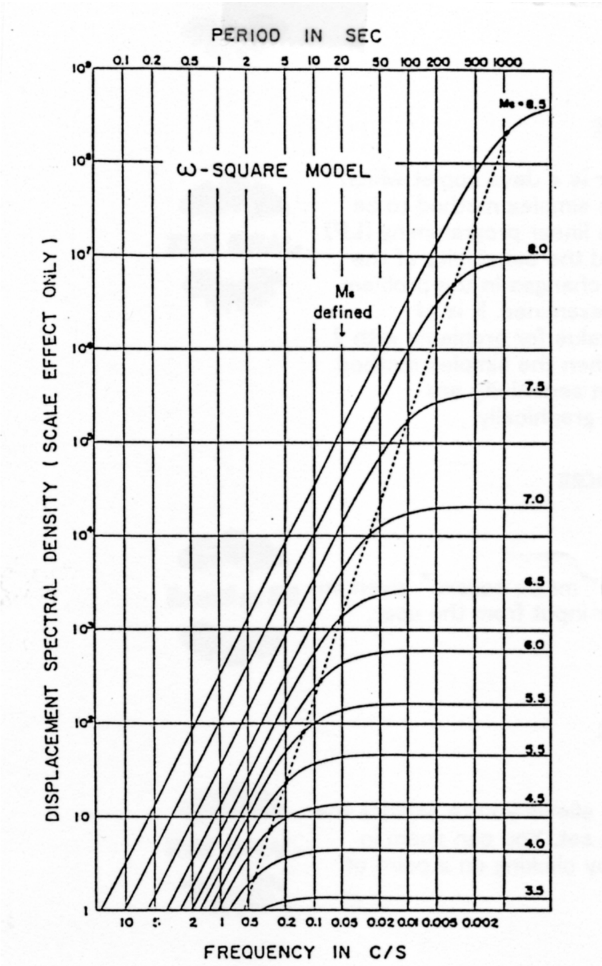


Fig. 3. Dependence of amplitude spectral density of earthquake magnitude M_s for the ω-square model.

Figure 5: Scaling of earthquakes

4 Discrimination by by m_b-M_S

We follow discussion on difference of $m_b - Ms$ between earthquake and explosion by Stevens and Steven (1985) in this section.

M_s and m_b are considered to be related to amplitude at 0.05 Hz (f_1) and 1 Hz ($_f2$), respectively. It is assumed that the M_s and m_b of an earthquake are expressed as

$$\begin{aligned} M_s^q(f_1) &= \log|M_q(f_1)\chi_q(f_1)| + C_s \qquad (3) \\ m_b^q(f_2) &= \log|M_q(f_2)F_p/(\rho_q\alpha_q)^3)^{1/2}| + C_b \qquad (4) \end{aligned}$$

where q indicates earthquake. $M(f)$ is spectrum of seismic wave; rho_q and α_q are the density and P wave speed at earthquake source; F_p is hte earthquake P wave radiation pattern; χ_q is the surface wave excitation function which depends on source depth; C_b and C_s are constants.

Likewise M_s and m_b of an explosion are expressed as

$$\begin{aligned} M_s^x(f1) &= \log|M_x(f1)\chi_x(f_1)| + C_s \qquad (5) \\ m_b^x(f2) &= \log|M_x(f2)F_p/(\rho_x\alpha_x)^3)^{1/2}| + C_b, \qquad (6) \end{aligned}$$

where x indicates explosion.

Assume that we compare $m_b - M_s$ of earthquake (q) and explosion (x).

$$(m_b^x - M_s^x) - (m_b^q - M_s^q) = \log\left[\left|\frac{M_x(f_2)M_q(f_1)}{M_x(f_1)M_q(f_2)}\right|\left(\frac{\rho_q\alpha_q^3}{\rho_x\alpha_x^3}\right)^{1/2}\left|\frac{\chi_q(f_1)}{F_p\chi_x(f_1)}\right|\right] \qquad (7)$$

The first factor gives the effect of source spectral differences. The second factor is the effect of source region elastic properties. The third factor is the difference in focal mechanism and excitation of surface wave.

Since source duration of explosion is very short, spectral amplitude at 1 Hz and 0.05 Hz are not different very much, $M_x(f_1) \sim M_x(f_2)$. On the other hand, rupture of natural earthquakes propagate with limited speed (less than S wave speed). This makes difference between $M_x(f)$ and $M_q(f)$ for events of the same size. But, if the source duration is shorter than the period of f_2, the difference between $M_q(f_1)$ and $M_q(f_2)$ is not large.

In general, explosions are at much shallower depths than most earthquakes. So the factor of $\log\left(\rho_q\alpha_q^3/\rho_x\alpha_x^3\right)^{1/2}$ tends to be positive value. Explosion in tuff (for example $\alpha_x = 2400$ m/s, $\rho_x = 1915$ kg/m^3) and an earthquake in crystalline crustal rock ($\alpha_q = 6000$ km/s, $\rho_q = 2700$km/m^3) makes a contribution of 0.7 magnitude unit.

F_p and χ_q are associated with specific source orientations. Averaged effect of $\log|\chi_q(f_1)/F_p\chi_x(f_1)|$ is estimated at about 0.35 by simulation. The focal mechanism contribution is expected to enhance earthquake/explosion separation.

$$\begin{aligned} < \log(\chi_q/\chi_x) > &\simeq -0.1 \\ < \log(1/F_p) > &\simeq 0.45 \end{aligned}$$

5 Practice of Magnitude determination

1. Start Linux.
2. Login earth.
3. cd sac_data

5.1 Amplitude and Period Measurement

Amplitude A for magnitude determination is taken as half of total amplitude (Fig. 6). The period T is measured as time interval between two adjacent peaks or troughs.

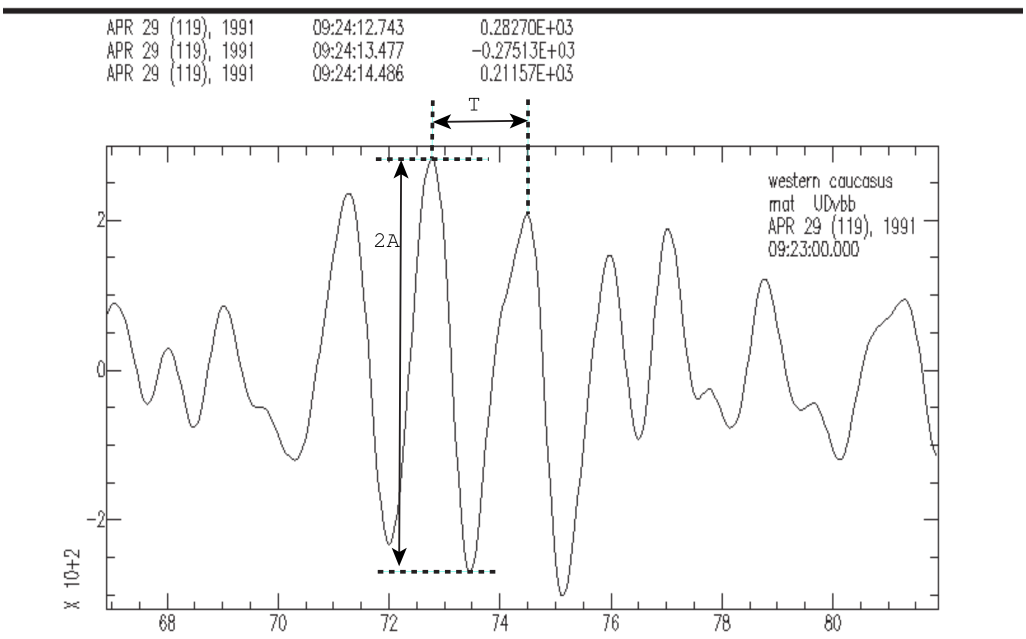


Figure 6: Amplitude and period measurement.

5.2 SAC2000 commands

5.2.1 Magnitude calculation

```
 qdp off           quick and dirty plot off
 r matlymddhh.z    read UD component. LP is for Ms.
(r matbymddhh.z                         VBB is for mb.)
 p or p1           plot waveform.
 rmean             remove bias.
 rtrend            remove linear trend
```

```
tran from polezero subtype calb.irislp to wwlpbn
                  instrumental response correction and simulation.
                  If you want instrumental correction only,
                  trans from polezero subtype calb.irislp to none
tran from polezero subtype calb.irisvb to wwspbn
                  instrumental response correction and simulation.
                  If you want instrumental correction only,
                  trans from polezero subtype calb.irisvb to none

                 'trans' can be approximated by the following procedure
                  mul 0.25        for LP = 4*e9 counts/(meter/s)
                 (mul 1.0         for VBB = 1.0*e9)
                  int             numerical integration by tranpezoidal rule.
                  bp bu co 0.04 0.1 butterworth bandpass filter for LP
                           1.0  2.0                            for VBB

ppk               read peak to peak amplitude and time difference
                  amplitude in nano-meter(1.0e-9 meter),
                  time      in UTC.
```

5.2.2 Coordinate rotation

```
r matlymddhh.n matlymddhh.e   read horizontal components
                              (NS first, then EW)
rotate to gcp                 rotate horizontal components to the
                              coordinate referring to the great circle.
                              (Radial -- Transverse)
write matlymddhh.R matlymddhh.T create new files.
write matbymddhh.R matbymddhh.T
```

5.2.3 Two-dimensional Plot

```
r matlymddhh.(e/n/z)   read the channel that should be on y-axis.
     b       (R/T/z)
r more matlymddhh.(e/n/z)  read the channel that should be on x-axis.
          b       (R/T/z)
p1                         Plot two channels in one window.
xlim (t1) (t2)             Assign period to be plotted (from t1 to t2).
xlim off                   Cancel xlim.
ppm                        Two dimensional particle motion plot.
```

5.2.4 Phase Picking

```
r matlymddhh.*             read three components at one time.
     b
```

ylim all	Unity normalization factor to enable amplitude comparison between the channels.
ylim off	Cancel ylim.
picks on	Display all picked marks in the header.
picks off	Cancel picks.
ppk	plot waveform, with cursor available. Be sure that the waveform window is focused to use a cursor.
x,(any key)	zoom in (from 'x' to '(any key)').
o	Zoom out.
l	display cursor position, time is in UTC, amplitude is in the ordinate unit.
p	P phase picking --> stored in 'A'.
s	S phase picking --> stored in 'T0'.
Tn (n=1...9)	arbitrary phase picking
q	quit ppk
ch ktn '(phase name)'	give phase name, ex) PKPab, pP, etc.

(ktn's n corresponds to Tn's n.)

lh p	display picking header list
write over	Store picking results in the file by overwrite.

5.3 Sample Files

For m_b.

```
file=matb142909.z
   ev_lat=42.5000
   ev_lon=43.7000
   ev_dep=17.00(km)
   dist=7711.90(km),69.4072(deg)
file=matb252105.z
   ev_lat=41.6000
   ev_lon=88.8000
   ev_dep=0.00(km)
   dist=4255.28(km),38.2976(deg)
file=matb380808.z
   ev_lat=13.3000
   ev_lon=144.0000
   ev_dep=50.00(km)
   dist=2634.65(km),23.7119(deg)
file=matb540722.z
   ev_lat=-15.1000
   ev_lon=186.4000
   ev_dep=33.00(km)
   dist=7630.31(km),68.6729(deg)
```

For M_s.

```
file=matl142909.z
   ev_lat=42.5000
   ev_lon=43.7000
   ev_dep=17.00(km)
   dist=7711.90(km),69.4072(deg)
file=matl160711.z
   ev_lat=-7.2000
   ev_lon=122.5000
   ev_dep=536.00(km)
   dist=5107.75(km),45.9698(deg)
file=matl251507.z
   ev_lat=-6.1000
   ev_lon=147.6000
   ev_dep=58.00(km)
   dist=4816.54(km),43.3489(deg)
file=matl252105.z
   ev_lat=41.6000
   ev_lon=88.8000
   ev_dep=0.00(km)
```

```
   dist=4255.28(km),38.2976(deg)
file=matl262812.z
   ev_lat=34.2000
   ev_lon=-116.4000
   ev_dep=1.00(km)
   dist=9018.11(km),81.1631(deg)
file=matl290200.z
   ev_lat=11.7000
   ev_lon=-87.3000
   ev_dep=45.00(km)
   dist=12863.51(km),115.7717(deg)
file=matl351621.z
   ev_lat=-15.3000
   ev_lon=-173.3000
   ev_dep=21.00(km)
   dist=7668.03(km),69.0123(deg)
file=matl380808.z
   ev_lat=13.3000
   ev_lon=144.0000
   ev_dep=50.00(km)
   dist=2634.65(km),23.7119(deg)
file=matl391019.z
   ev_lat=14.7000
   ev_lon=-92.6000
   ev_dep=34.00(km)
   dist=12238.33(km),110.1451(deg)
file=matl430923.z
   ev_lat=-18.0000
   ev_lon=-178.4000
   ev_dep=562.00(km)
   dist=7563.06(km),68.0676(deg)
file=matl460900.z
   ev_lat=-13.8000
   ev_lon=-67.6000
   ev_dep=631.00(km)
   dist=16423.52(km),147.8118(deg)
file=matl540722.z
   ev_lat=-15.1000
   ev_lon=186.4000
   ev_dep=33.00(km)
   dist=7630.31(km),68.6729(deg)
file=matl570320.z
   ev_lat=-29.2000
   ev_lon=182.3000
```

```
   ev_dep=33.00(km)
   dist=8611.18(km),77.5007(deg)
file=matl591414.z
   ev_lat=16.7000
   ev_lon=261.5000
   ev_dep=33.00(km)
   dist=11646.76(km),104.8209(deg)
```

6 References

Aki, K. (1967), Scaling law of seismic spectrum, *J. Geophys. Res.*, **72**, 1217–1231.

Aki, K. and P. G. Richards(1980), Quantitative Seismology: Theory and Methods, W. H. Freeman and Company, San Francisco,USA.

Kanamori H. (1977), The energy release in great earthquakes, *J. Geophys. Res.*, 82, 2981–2987.

Stevnes, J. L. and M. D. Steven (1985), The physical basis of $m_b : M_s$ and variable frequency magnitude methods for earthquake/explosion discrimination, *J. Geophys. Res.*, 90, 3009–3020.

Richter, C. F.(1935), An Instrumental Earthquake Magnitude Scale, Bull. Seism. Soc. Am., 25, 1-32.

Gutenberg, B. and C. F. Richter(1949), Seismicity of the Earth, Princeton University Press, Princeton, N.J..

Vaněk, j., A. Zá topek, V. Ká rní k, N. V. Kondroskaya, Y. V. Riznichenko, E. F. Savarensky, S. L. Solov'er, and N.V. Shebalin (1962), Standardization of magnitude scales, Izu, Acad. Sci. USSR, Geophys. Ser., 108-111.

Gutenberg, B.(1945a), Amplitudes of P, PP and magnitudes of shallow earthquakes, Bull. Seism. Soc. Am., 35, 57-69.

Gutenberg, B.(1945b), Magnitude determination for deep-focus earthquakes, Bull. Seism. Soc. Am., 35, 117-130.

Gutenberg, B.(1945c), Amplitudes of surface waves and magnitude of shallow earthquakes, Bull. Seism. Soc. Ame., 35, 3-12.

Gutenberg, B. and C. F. Richter (1956), Magnitude and energy of earthquakes, *Ann. Geofis.*, 9, 1-15.

Utsu, T. (1982), Relationships between earthquake magnitude scales, *Bull. Earthq. Res. Inst.*, 57, 465-497. (In Japanese with English abstract)

A Appendix

A.1 Seismic moment

A.1.1 Seismic moment

Earthquake magnitude was defined to quantify the size of an earthquake, which was an empirical quantity and was not linked to a physical quantity originally. Currently an earthquake is recognized as a slip on a fault plane, and determination of seismic moment is thought to be the best way to quantify the size of an earthquake. The seismic moment (M_0) is expressed as

$$M_0 = \mu DS \tag{8}$$

where μ is rigidity, D is average dislocation on the fault, and S is area of the fault. The seismic moment is usually estimated by comparing observed seismic records with synthetic ones.

The unit of seismic moment is Nm (Newton meter) or dyne-cm (dyne centimeter), which is the same as the unit to measure the force to rotate something (moment of force).

Based on a pillar model (Fig. 1), relationship between dislocation and seismic moment is considered. We assume that the force F caused a dislocation of D on a pillar of length L, cross-section area S, and rigidity μ. The relationship between the stress on the edge of the pillar τ and the dislocation is

$$\tau = \mu \frac{D}{L}$$

where the stress is force per unit area.

$$\begin{aligned} \text{stress} &= \frac{\text{force}}{\text{area}} \\ &= \frac{F}{S} \end{aligned}$$

$\frac{D}{L}$ is strain.

$$\text{strain} = \frac{\text{displacement}}{\text{length}}$$

μ is rigidity, which is ratio of stress to strain.

Then we calculate a value of (stress)×(volume).

$$\begin{aligned} (\text{stress}) \times (\text{volume}) &= (\mu \frac{D}{L}) \times (SL) \\ &= \mu DS, \end{aligned}$$

which corresponds to the seismic moment.

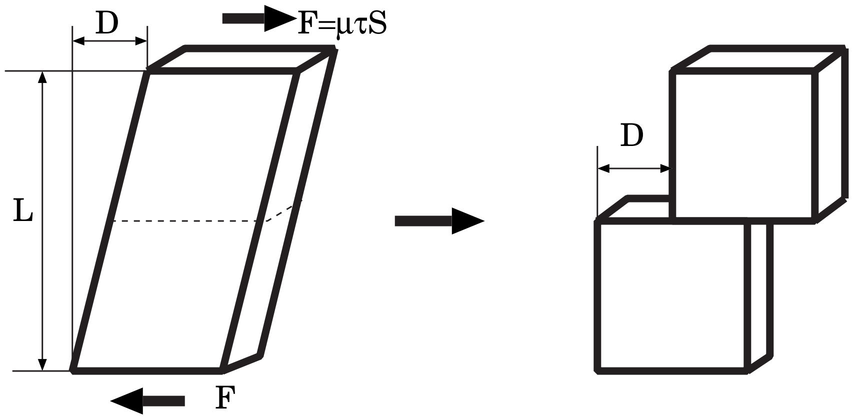


Figure 7: dislocation and the seismic moment

A.1.2 Seismic moment and double-couple

In the previous section, we ignore the "reaction" to the force "F". If the force "F" is applied on the pillar, the pillar will to begin to rotate. The rock around the fault under the ground should not rotate. Some force should work to stop the rotation.

In the case of Fig.2, the force couple 2 stops the rotation.

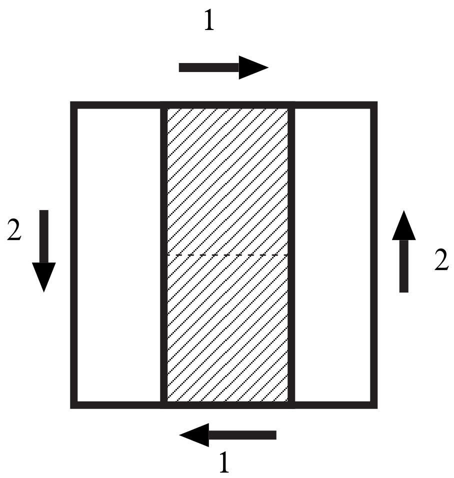


Figure 8: Double couple

Combination of two force couples make force couples of compression and dilatation. (Fig.3) When an earthquake occurs, there is a stress change. But no net moment of force works.

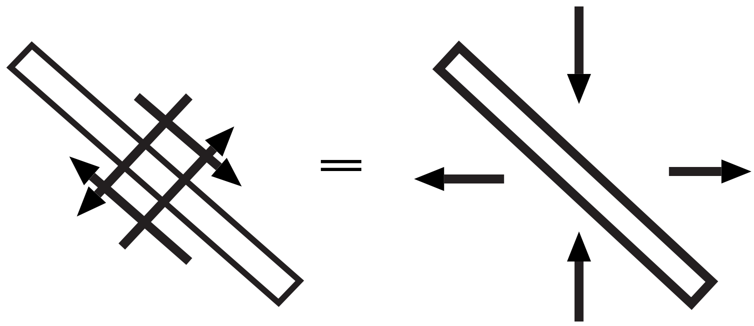


Figure 9: Equivalent double couple.

A.1.3 Moment tensor

Since the seismic moment is related to force direction and direction of the "arm", there are $3 \times 3 = 9$ components. Fig. 4 shows the component of seismic moment. A component of m_{ij} $(i \neq j)$ is related to shear force. A component of m_{ij} $(i = j)$ is related volumetric change (ex. explosion).

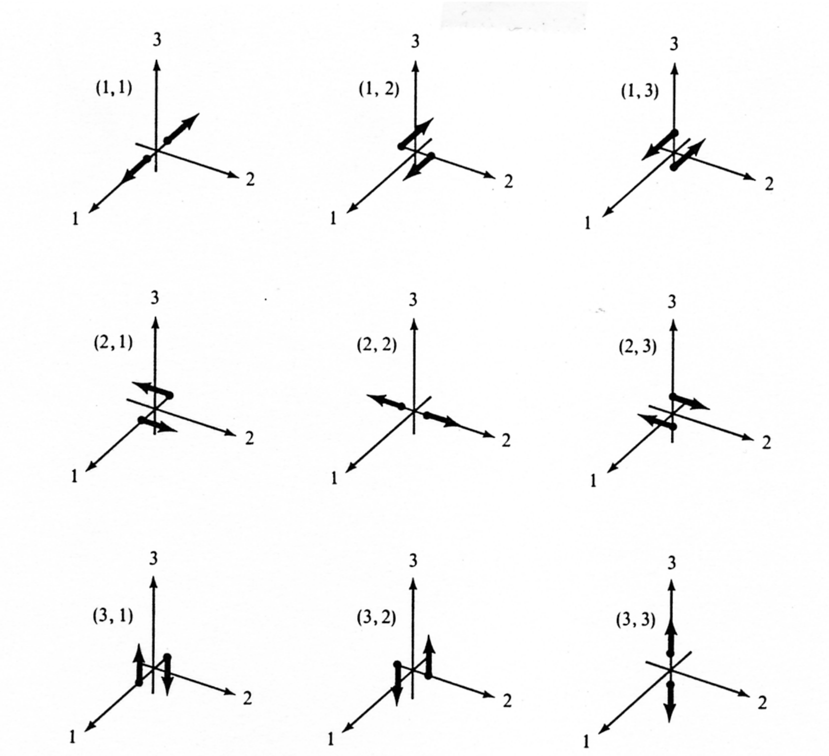


FIGURE **3.7**

The nine possible couples that are required to obtain equivalent forces for a generally oriented displacement discontinuity in anisotropic media.

Figure 10: Component of seismic moment. (Aki and Richards, 1980)

A.1.4 Seismic moment and seismic waves

Displacement (U_n) due to seismic moment (M_{pq}) is expressed as (Aki and Richards, 1980)

$$
\begin{aligned}
U_n &= M_{pq} * G_{np,q} \\
& \mu(\bar{u_p}\nu_q + \bar{u_q}\nu_p)A * G_{np,q} \\
&= \frac{30\gamma_n\gamma_p\gamma_q\nu_q - 6\nu_n\gamma_p - 6\delta_{np}\gamma_q\nu_q}{4\pi\rho r^4}\mu A \int_{r/\alpha}^{r/\beta} \tau\bar{u_p}(t-\tau)d\tau \\
& + \frac{12\gamma_n\gamma_p\gamma_q\nu_q - 2\nu_n\gamma_p - 2\delta_{np}\gamma_q\gamma_q}{4\pi\rho\alpha^2 r^2}\mu A\bar{u_p}(t-\frac{r}{\alpha}) \\
& - \frac{12\gamma_n\gamma_p\gamma_q\nu_q - 3\nu_n\gamma_p - 3\delta_{np}\gamma_q\gamma_q}{4\pi\rho\beta^2 r^2}\mu A\bar{u_p}(t-\frac{r}{\beta}) \\
& + \frac{2\gamma_n\gamma_p\gamma_q\nu_q}{4\pi\rho\alpha^3 r}\mu A\dot{\bar{u_p}}(t-\frac{r}{\alpha}) \\
& - \frac{2\gamma_n\gamma_P\gamma_q\nu_q - \nu_n\gamma_p - \delta_{np}\gamma_q\nu_q}{4\pi\rho\beta^3 r}\mu A\dot{\bar{u_p}}(t-\frac{r}{\beta})
\end{aligned}
$$

where $\bar{u_i}$ is dislocation on the fault; ν_i, normal vector to the fault plane; A, area of the fault; γ_i, directional cosine to the observation point. The last two terms attenuate as $\frac{1}{r}$, and are referred to as far-field terms. The other terms attenuate as $\frac{1}{r^2}$ or $\frac{1}{r^4}$, and are referred to near-field terms. The far field terms are proportional to a derivative of μAu, which is rate of seismic moment change.