Analysis on the Fluctuation Magnitude in Probe Interval for Active Measurement

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***Abstract*—Active measurement, which can provide end-to-end measurements of network performance, is critical since the Internet is managed by multiple organizations. Recently, on the active measurement of delay and loss, Baccelli et al reported that many probing policies can be used to provide appropriate estimation in addition to the traditional policy based on PASTA property if the volume of probe stream is negligible compared to the traffic stream. Probing schemes with fixed probe packet intervals suffer from the phase-lock phenomenon due to synchronization against the network performance; they do, however, provide superior accuracy. A remaining issue is how to decide the optimal probing policy while taking the phase-lock phenomenon into consideration. In this paper, we propose the probing policy that randomly fluctuates the probe packet interval to avoid the phase-lock phenomenon. We start by clarifying the relationships among the fluctuation magnitude, the properties of the target network, and estimation accuracy, and we discuss the optimal probing policy with regard to the properties of the target network.**

I. Introduction

The Internet has spread widely over the last several decades, and it has come to play a key role as an infrastructure supporting society and the economy. On the other hand, Quality of Service (QoS) which is represented by delay and loss in addition to connectivity, is appropriately regarded as a key technology goal. Controlling network quality demands an understanding of the internal state of the network, the key metric of which is congestion. However, the Internet is composed of multiple networks that are managed by different Internet Service Providers (ISP) or other organizations, and it is inherently difficult to gather the information needed from the different managers of the networks. Therefore, we need a way of measuring, from outside the Internet, the QoS of end-to-end connections.

Active measurement is an end-to-end measurement technique that can estimate the QoS of a network path. Active measurement derives the QoS from information obtained by injecting probe packets into the target network path [1], [2], [3]. From the the delay and loss experienced by probe packets, active measurement can estimate one-way delay, round-trip time (RTT), and loss rate on the network path. It can also estimate link capacity [4] and available bandwidth [5] from changes in the delay or interval of the probe packets.

Active measurement is easy for the end user to perform but it is not trouble free. The injection of probe packets impact the path's performance. We cannot estimate the true performance since that state occurs only without the probe stream. If we inject a lot of probe packets to increase sampling rate, estimation accuracy suffers due to the extra traffic. The problem of extra traffic is discussed in [6]. Therefore, it is important to achieve high accuracy with a limited number of probe packets.

Work by Baccelli et al [7], [8] gaves an important suggestion with respect to the accuracy of active measurement in assessing delay and loss. The current favorite is to use the PASTA (Poisson Arrivals See Time Averages) property by issuing probe packets so that their arrival intervals follow an exponential distribution, i.e. packet injection is a Poisson process [9]. Probing policy which is fixing the probe packet arrival intervals (periodic-probing) is also used because it is easier to manipulate. PASTA-based and periodic-probing was compared in [10]. However, recent work indicates that there may be many other distributions that can provide higher measurement accuracy if a non-intrusive context (the effect of probe packets is insignificant) can be assumed [7]. This property is named NIMASTA (Non-Intrusive Mixing Arrivals See Time Averages).

NIMASTA contains the following three assumptions.

1) The stochastic process that expresses the network state we are interested in (e.g. virtual delay and loss/no-loss indication) is stationary and ergodic. This process is called the ground truth process.
2) The point process of probe packets arrival $\{T_i\}$ $(i = 1, 2, \ldots, m)$ is stationary and *mixing*. Mixing is the requirement that guarantees joint ergodicity between the probe and ground truth processes (see [7] for details).
3) The last assumption is the non-intrusive context, i.e. we can ignore the impact of probe packets. Namely, the ratio of the probe stream to all streams is very small.

Under the above assumptions, Baccelli et al proved that the following equation holds,

$$\lim_{m\to\infty} \frac{1}{m} \sum_{i=1}^{m} f(X(T_i)) = \mathrm{E}[f(X(0))] \qquad \text{a.s.}, \tag{1}$$

where $X(t)$ and f are ground truth process and an arbitrary positive function, respectively. If we can obtain $X(T_i)$ (e.g. the delay of the probe packet or loss/no-loss of probe packet) from a probe packet injected at time T_i, (1) means that we can estimate $\mathrm{E}[f(X(0))]$ by injection of m probe packets.

The only requirements for the point process of probe packet arrival $\{T_i\}$ $(i = 1, 2, \dots)$ are that it be stationary and mixing. Therefore, there are a lot of point processes of probe packet arrival that can achieve appropriate estimation of ensemble mean $\mathrm{E}[f(X(0))]$ besides PASTA-based probing. Such mixing point process include those whose intervals follow the Gamma and the uniform distribution. Note that periodic-probing with fixed interval is not a mixing process, and does not satisfy (1).

Moreover, Baccelli et al investigated how to select an optimal probing process [8]. We can select an optimal probing process under a specific assumption by using the inter-probe time given by the parameterized Gamma distribution.

If we estimate the mean of $X(0)$ by using active measurement, we can use estimator $\hat{P} = \sum_{i=1}^{m} X(T_i)/m$. Thus the variance of $\hat{P}$ is

$$\mathrm{Var}[\hat{P}] = \frac{1}{m^2}\sum_{i=1}^{m}\sum_{j=1}^{m}\int_{-\infty}^{\infty} r(\tau) f_{i-j}(\tau)\mathrm{d}\tau\,, \qquad (2)$$

where f_{i-j} is probability density function (pdf) of $T_i - T_j$, $r(\tau) = \mathrm{Cov}\left(X(t), X(t+\tau)\right)$ is the autocovariance function (ACF) of the ground truth process $X(t)$ (we can express $r(\tau)$ by τ alone, because $X(t)$ is stationary).

If $r(\tau)$ is convex on interval $[0, \infty)$ and the average of the inter-probe time is μ, the following equation can be proven by using Jensen's inequality [11].

$$\int_{-\infty}^{\infty} r(\tau) f_k(\tau)\mathrm{d}\tau \geq \int_{-\infty}^{\infty} r(\tau)\delta(\tau - k\mu)\mathrm{d}\tau\,, \qquad (3)$$

where $\delta(\cdot)$ denotes Dirac δ function. (3) means that no other probing process with an average interval μ has a variance that is lower than that of periodic-probing. Estimator variance is associated with accuracy, lower is better, so periodic-probing is the best probing process if we focus only on asymptotic variance.

On the other hand, periodic-probing does not satisfy the assumptions of (1) due to non-mixing, so periodic-probing is not necessarily the best. This is because the phase-lock phenomenon may occur and the estimator may converge on a false value when a frequency component of the ground truth process is synchronized with the cycle of the probing process. Namely, periodic-probing has a bias. Therefore, we cannot categorize periodic-probing as the optimal probing process.

To tune the tradeoff between traditional probing process (which obeys Poisson arrival) and periodic-probing (which has a bias but has the advantage in terms of asymptotic variance), Baccelli et al proposed a probing process that gives an inter-probe time that obeys the parameterized Gamma distribution.

Gamma-probing provides multiple selections lying between traditional PASTA-based probing and periodic-probing through parameter, and it is a great advance in network measurement techniques. However, since Baccelli et al did not indicate how to decide upon the optimal parameter, it has remained a problem with no solution. Therefore, we cannot decide upon probing policy which we should use.

In this paper, we introduce a method to specify the optimal probing policy so as to maximize the accuracy of active measurements with regard to delay and loss (we don't use the approach with Gamma-probing). To avoid the phase-lock phenomenon, we add a random fluctuation to probe packet intervals. We clarify the relationships among the fluctuation magnitude imposed on probe packet interval, the property of the target network and accuracy of estimation, and we discuss the optimal probing policy corresponding to the property of the target network.

The rest of the paper is organized as follows. In Section II, we verify the cause of the phase-lock phenomenon and describe a method to assess it. Section III proposes our probing policy with random fluctuations. In Section IV, we analyze the relationships between the fluctuation magnitude and the accuracy of active measurement, and confirm the validity of the proposal by simulations. We conclude the paper in section V.

II. Evaluation of Phase-lock Phenomenon

Reference [8] did not mention the factor of phase-lock phenomenon in detail because it was not main topic of [8]. This study examines the phase-lock phenomenon in detail because our goal is to specify the optimal probing process. Accordingly, this section investigates the cause and effects of the phase-lock phenomenon and introduces an evaluation function that can assess this phenomenon appropriately.

The convex ACF $r(\tau)$ of the ground truth process $X(t)$ means $X(t)$ has no special periodicity. The phase-lock phenomenon occurs when the cycle of the ground truth process synchronizes to that of the probing process. Hence, phase-lock phenomenon will not occur if the ACF is strictly convex. This fact was proven by (3). However, the experiments of Baccelli et al demonstrated that there are multiple instances in which the accuracy of the estimator of periodic-probing was worse than that of other probing processes (periodic-probing is not always bad).

We consider that phase-lock phenomenon is caused by *accidental periodicity* which is realized by finite measurement periods. Even if there is no special periodicity of the ground truth process (in terms of long-time average), accidental periodicity can be generated. Thus, there is a possibility that a specific frequency component will be present by chance if the measurement period is limited. Even if ACF $r(\tau) = \mathrm{E}[X(t)X(t+\tau)] - \{\mathrm{E}[X(t)]\}^2$ in terms of ensemble mean (namely long-time average) is strictly convex, ACF $\int_0^{l-\tau} X(t)X(t+\tau)\mathrm{d}t - \int_0^{l-\tau} X(t)\mathrm{d}t \int_0^{l-\tau} X(t+\tau)\mathrm{d}t$ is, in terms of a time average on finite period $(0, l]$, not necessarily convex. In Fig. 1, we display the ACF of a simple ON-OFF process. The line of time average in Fig. 1 represents the time average for a single sample path generated by simulation.

Therefore, the accuracy will become bad if the ground truth process contains a lot of frequency component that can synchronize with the probing process. Conversely, if it contains few such frequency component so much, the accuracy will be quite good. The accuracy of periodic-probing may become extremely bad for the specific sample path though it is better on average.

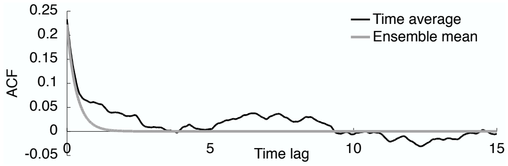

Fig. 1. ACF in terms of time average and ensemble mean

Accordingly, to assess the phase-lock phenomenon appropriately we must consider the accuracy of the target sample path. We assume the metric that we want to measure is the time average $\overline{X} = \int_0^l X(t)/l \, \mathrm{d}t$ on the measurement period $(0, l]$ and $\mathrm{E}[\hat{P}] = \overline{X}$ (namely bias-free) holds. By using a conditional variance, we can express the accuracy of the target sample path as follows.

$$\mathrm{Var}[\hat{P}|X(t)] \,. \tag{4}$$

According to the property of conditional variance, we get the following equation.

$$\mathrm{E}[\mathrm{Var}[\hat{P}|X(t)]] = \mathrm{Var}[\hat{P}] - \mathrm{Var}[\overline{X}] \,. \tag{5}$$

Baccelli et al proved that periodic-probing gives $\mathrm{Var}[\hat{P}]$, the minimal value (we introduced this in the Introduction). (5) means that the expectation of (4) is minimized with periodic-probing. Therefore, by using (5), we can assess the average accuracy (in which periodic-probing has a superiority). On the other hand, the effect of phase-lock phenomenon is that (4) is varied with the intensity of the frequency component (which is synchronized with the probing process). The effect of phase-lock phenomenon can be assessed by the following:

$$\mathrm{Var}[\mathrm{Var}[\hat{P}|X(t)]] \,. \tag{6}$$

This paper looks for the probing process which can avoid extremely bad accuracy due to phase-lock phenomenon (extremely bad accuracy occurs when the ground truth process contains high intensity of the frequency component which is synchronized with the probing process.). In addition, the probing process must be accurate in terms of average accuracy. To satisfy the two above requirements (which are assessed by (5) and (6), respectively), we introduce the following evaluation function:

$$\mathrm{E}[\mathrm{Var}[\hat{P}|X(t)]] + \sqrt{\mathrm{Var}[\mathrm{Var}[\hat{P}|X(t)]]} \,. \tag{7}$$

We define the probing process that minimizes evaluation function (7) as the optimal probing process, and we investigate the optimal probing process as determined by the property of the target network.

(7) is one of the simplest evaluation functions which can assess the phase-lock phenomenon. We can consider other various patterns of evaluation function. For example, we can use the square root of (6) multiplied by a constant instead of the second term of (7). Our evaluation function is simple and intuitive one though we cannot show that it is the best one.

III. Proposed Probing Method with Fluctuations

In this section, we will propose the probing process with fluctuations to specify the probing process that minimizes the evaluation function (7). To avoid the phase-lock phenomenon, fluctuated probe intervals is effective. In other words, we force the probing process to exhibit multiple phases (i.e. variable intervals). Actually, the Gamma-probing proposed by Baccelli et al in [8] is one approach to adding fluctuations. In this study, we propose a new method that takes three parameters into consideration: measurement period, number of probe packets, and the fluctuation magnitude. It is expected that measurement period has a critical affect on the intensity of accidental periodicity.

We add fluctuations that obey a normal distribution to the timing of probe packet arrivals, while specifying the measurement period. We assume that interval $(0, l]$ is the measurement period and m is the number of probe packets sent in the measurement period. Our probing method gives the point process of probe packet arrival $\{T_i\}\,(i = 1, 2, \ldots, m)$ as follows.

$$T_i = (S + G_i) - l \left\lfloor \frac{S + G_i}{l} \right\rfloor \,, \tag{8}$$

where S and G_i denote the random variables that follow uniform distribution $U(0, l/m)$ and normal distribution $N((i-1)l/m, \sigma^2)$, respectively, $\lfloor \cdot \rfloor$ denotes a floor function and σ denotes the magnitude parameter of the fluctuations. To prevent $\{T_i\}$ from taking a value on the outside of the measurement period $(0, l]$, we added the second term. $\{T_i\}$ given by (8) occurs at an equal probability at an arbitrary point in the measurement period $(0, l]$. Therefore, estimator $\hat{P} = \sum_{i=1}^{m} X(T_i)/m$ is an unbiased estimator for $\overline{X}$.

Our probing process provides multiple selections lying between traditional PASTA-based probing and periodic-probing as well as Gamma-probing. If $\sigma = 0$ (namely there is no fluctuation), $\{T_i\}$ corresponds to periodic-probing. On the other hand, if $\sigma \to \infty$, T_i follows a uniform distribution $U(0, l)$. Therefore, intervals between T_i follow an exponential distribution for sufficiently large l.

IV. Fluctuation Magnitude and Accuracy

In this section, we clarify the relationship between fluctuation magnitude, which is given by the probe timing, $r(\tau)$ (a property of the target network) and the evaluation function (7). We then detail a method that specifies the optimal fluctuation magnitude, theoretically. As we mentioned in Section II, periodic-probing is the optimal probing process if we consider only average accuracy. It is important to specify the minimal fluctuation magnitude that can avoid the phase-lock phenomenon, because large fluctuations cause the probing process to deviate too far from periodic-probing.

Our evaluation function (7) is composed of the average and variance of (4). Therefore, we first address the stochastic behavior of (4).

We define $\tilde{X}(t) = X(t - l\lfloor t/l \rfloor)$ which depends on stochastic process $X(t)$ in measurement period $(0, l]$, and

$\tilde{T}_i = S + G_i$ which is composed of a simple uniform and normal random variable. Since $X(t)$ observed at the timing of $\{T_i\}$ is equal to $\tilde{X}(t)$ observed at the timing of $\{\tilde{T}_i\}$, $\hat{P} = \sum_{i=1}^{m} \tilde{X}(\tilde{T}_i)/m$ holds. Therefore, by using ACF $\tilde{R}(\tau) = \int_0^l \tilde{X}(t)\tilde{X}(t+\tau)/l\,\mathrm{d}t - \{\int_0^l \tilde{X}(t)/l\,\mathrm{d}t\}^2$ in terms of time average, (4) can be expressed as follows (as well as (2)).

$$\mathrm{Var}[\hat{P}|X(t)] = \frac{1}{m^2}\sum_{i=1}^{m}\sum_{j=1}^{m}\int_{-\infty}^{\infty}\tilde{R}(\tau)f_{i-j}(\tau)\mathrm{d}\tau\,.$$

Note that $\tilde{R}(\tau)$ is a stochastic process that depends on $X(t)$.

Furthermore, when we expand $\tilde{R}(\tau)$ in Fourier series, we find that

$$\begin{aligned}&\mathrm{Var}[\hat{P}|X(t)]\\&=\frac{1}{m^2}\sum_{n=1}^{\infty}K_n\left\{\sum_{i=1}^{m}\sum_{j=1}^{m}\int_{-\infty}^{\infty}\cos\left(\frac{2\pi n}{l}\tau\right)f_{i-j}(\tau)\mathrm{d}\tau\right\}\,,\end{aligned} \tag{9}$$

$$K_n = \frac{2}{l}\int_0^l \cos\left(\frac{2\pi n}{l}\tau\right)\tilde{R}(\tau)\mathrm{d}\tau\,.$$

Note that $\{K_n\}\,(n = 1, 2, \cdots)$ are random variables that depend on $X(t)$; they represent the intensity of each frequency component of $X(t)$.

Moreover, the summation in (9) reduces as follows.

$$\begin{aligned}&\sum_{i=1}^{m}\sum_{j=1}^{m}\int_{-\infty}^{\infty}\cos\left(\frac{2\pi n}{l}\tau\right)f_{i-j}(\tau)\mathrm{d}\tau\\
&= m\sum_{i=0}^{m-1}\int_{-\infty}^{\infty}\cos\left(\frac{2\pi n}{l}\tau\right)f_i(\tau)\mathrm{d}\tau\\
&= m + m\int_{-\infty}^{\infty}\cos\left(\frac{2\pi n}{l}\tau\right)\sum_{i=1}^{m-1}\frac{1}{2\sigma\sqrt{\pi}}\mathrm{e}^{-\frac{(\tau-il/m)^2}{4\sigma^2}}\mathrm{d}\tau\\
&= \begin{cases} m + (m^2-m)\int_0^\infty \cos\left(\frac{2\pi n}{l}\tau\right)\frac{1}{\sigma\sqrt{\pi}}\mathrm{e}^{-\frac{\tau^2}{4\sigma^2}}\mathrm{d}\tau, & \\ \qquad n = mj\,(j=1,2,\dots) & \\ m - m\int_0^\infty \cos\left(\frac{2\pi n}{l}\tau\right)\frac{1}{\sigma\sqrt{\pi}}\mathrm{e}^{-\frac{\tau^2}{4\sigma^2}}\mathrm{d}\tau, & \text{otherwise}\end{cases}\\
&= \begin{cases} m + (m^2-m)\mathrm{e}^{-\left(\frac{2\pi n}{l}\right)^2\sigma^2}, & n = mj\,(j=1,2,\dots)\\ m - m\,\mathrm{e}^{-\left(\frac{2\pi n}{l}\right)^2\sigma^2}, & \text{otherwise}\end{cases}\,.\end{aligned} \tag{10}$$

Substituting (10) for (9), the accuracy achieved for target sample path $\mathrm{Var}[\hat{P}|X(t)]$ is expressed as follows using the intensity of each frequency component $\{K_n\}\,(n = 1, 2, \cdots)$.

$$\mathrm{Var}[\hat{P}|X(t)] = \sum_{i=1}^{\infty} w_i K_i\,, \tag{11}$$

$$w_i = \begin{cases}\frac{1+(m-1)\mathrm{e}^{-\left(\frac{2\pi i}{l}\right)^2\sigma^2}}{m}, & i = mj\,(j=1,2,\dots)\\ \frac{1-\mathrm{e}^{-\left(\frac{2\pi i}{l}\right)^2\sigma^2}}{m}, & \text{otherwise}\end{cases}\,.$$

Note that $K_0 = 0$ because $\int_0^l \tilde{R}(\tau)\mathrm{d}\tau = 0$.

To clarify the average and standard deviation of $\mathrm{Var}[\hat{P}|X(t)]$, we must investigate the stochastic behavior of $\{K_n\}\,(n = 1, 2, \cdots)$. Since Fourier transformation of autocorrelation function $\tilde{R}(t) + \overline{X}^2$ of the stochastic process yields a power spectrum (Wiener-Khintchin theorem [12]), $\mathcal{F}[\tilde{R}(t) + \overline{X}^2] = |\mathcal{F}[\tilde{X}(t)]|^2$ holds. Consequently, K_n relates to Fourier coefficient $C_n = 2\int_0^l \cos(2\pi nt/l)X(t)\mathrm{d}t/l$ and $S_n = 2\int_0^l \sin(2\pi nt/l)X(t)\mathrm{d}t/l$ as follows.

$$K_n = \frac{{C_n}^2 + {S_n}^2}{2}\,. \tag{12}$$

Note that C_n and S_n are random variables because they depend on $X(t)$.

Hence, by using (11) and (12), (5) which composes our evaluation function is given as follows.

$$\mathrm{E}\left[\mathrm{Var}[\hat{P}|X(t)]\right] = \frac{1}{2}\sum_{i=1}^{\infty} w_i\left\{\mathrm{Var}[C_i] + \mathrm{Var}[S_i]\right\}\,. \tag{13}$$

Similarly, if we assume C_i and S_i follow normal distributions, the other component (6) composing the evaluation function is as follows.

$$\begin{aligned}\mathrm{Var}\left[\mathrm{Var}[\hat{P}|X(t)]\right] &= \frac{1}{4}\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} w_i w_j\Big\{\mathrm{Cov}\left({C_i}^2, {C_j}^2\right)\\ &\quad + 2\mathrm{Cov}\left({C_i}^2, {S_j}^2\right) + \mathrm{Cov}\left({S_i}^2, {S_j}^2\right)\Big\}\\
&= \frac{1}{4}\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} w_i w_j\Big\{2\left\{\mathrm{Cov}\left(C_i, C_j\right)\right\}^2\\ &\quad + 4\left\{\mathrm{Cov}\left(C_i, S_j\right)\right\}^2 + 2\left\{\mathrm{Cov}\left(S_i, S_j\right)\right\}^2\Big\}\,,\end{aligned} \tag{14}$$

where the second equality follows from property of the moment of bivariate normal distributions.

Finally, if we can relate $\mathrm{Cov}(C_i, S_j)$, $\mathrm{Cov}(C_i, C_j)$ and $\mathrm{Cov}(S_i, S_j)$ to $r(\tau)$, we can evaluate any probing method by our evaluation function. Calculating $\mathrm{Cov}(C_i, S_j)$, we have

$$\begin{aligned}&\mathrm{Cov}(C_i, S_j)\\
&= \frac{4}{l^2}\int_0^l\int_0^l \cos\left(\frac{2\pi i}{l}t\right)\sin\left(\frac{2\pi j}{l}s\right)r(s-t)\mathrm{d}s\mathrm{d}t\\
&= \frac{4}{l^2}\int_0^l\int_{-t}^{l-t} \cos\left(\frac{2\pi i}{l}t\right)\sin\left(\frac{2\pi j}{l}(\tau-t)\right)r(\tau)\mathrm{d}\tau\mathrm{d}t\\
&= \frac{2}{l^2}\int_0^l\int_{-l}^{-\tau}\left\{\sin\left(\frac{2\pi(j+i)}{l}t + \frac{2\pi j}{l}\tau\right)\right.\\
&\qquad \left. + \sin\left(\frac{2\pi(j-i)}{l}t + \frac{2\pi j}{l}\tau\right)\right\}\mathrm{d}t\,r(\tau)\mathrm{d}\tau\\
&\qquad + \frac{2}{l^2}\int_{-l}^{0}\int_{-\tau}^{l}\left\{\sin\left(\frac{2\pi(j+i)}{l}t + \frac{2\pi j}{l}\tau\right)\right.\\
&\qquad \left. + \sin\left(\frac{2\pi(j-i)}{l}t + \frac{2\pi j}{l}\tau\right)\right\}\mathrm{d}t\,r(\tau)\mathrm{d}\tau\,.\end{aligned}$$

Moreover, by creating two cases $i = j$ and $i \neq j$ and integrating the result, we get $\mathrm{Cov}(C_i, S_j) = 0$. Similarly, we can calculate $\mathrm{Cov}(C_i, C_j)$ and $\mathrm{Cov}(S_i, S_j)$ as follows.

$$\mathrm{Cov}(C_i, C_j) = \begin{cases} -\frac{2}{il\pi}r_{S,i} + \frac{4}{l}r_{C,i}, & (i = j)\\ \frac{4}{l\pi(j^2-i^2)}\left(ir_{S,i} - jr_{S,j}\right), & (i \neq j)\end{cases}\,,$$

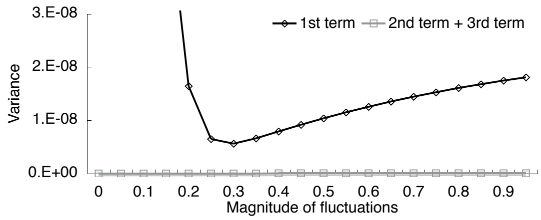

Fig. 2. Comparison of first and second terms on the right-hand side of (15)?D

$$\mathrm{Cov}(S_i, S_j) = \begin{cases} \frac{2}{il\pi} r_{S,i} + \frac{4}{l} r_{C,i}, & (i = j) \\ \frac{4}{l\pi(j^2-i^2)} \left(j r_{S,i} - i r_{S,j} \right), & (i \neq j) \end{cases},$$

$$r_{S,i} = \int_0^l \sin\left(\frac{2\pi i}{l}\tau\right) r(\tau)\mathrm{d}\tau\,,$$

$$r_{C,i} = \int_0^l \left(1 - \frac{\tau}{l}\right) \cos\left(\frac{2\pi i}{l}\tau\right) r(\tau)\mathrm{d}\tau\,.$$

Substituting them into (13), we have

$$\mathrm{E}\left[\mathrm{Var}[\hat{P}|X(t)]\right] = \sum_{i=1}^{\infty} \frac{4}{l} w_i r_{C,i}\,,$$

$$\begin{aligned} &\mathrm{Var}\left[\mathrm{Var}[\hat{P}|X(t)]\right] \\ &= \sum_{i=1}^{\infty} \left\{\frac{4}{l} w_i r_{C,i}\right\}^2 + \sum_{i=1}^{\infty} \left\{\frac{2}{li\pi} w_i r_{S,i}\right\}^2 \\ &\quad + \sum_{i \neq j} 16 w_i w_j \frac{(i^2+j^2)\left\{{r_{S,i}}^2 + {r_{S,j}}^2\right\} - 4ij r_{S,i} r_{S,j}}{l^2\pi^2(i^2-j^2)^2}\,. \end{aligned} \tag{15}$$

However, the second and third terms on the right-hand side of (15) are trivial compared with the first term if we give actual parameters to $r(\tau)$, l and m. Hence, (15) is approximated as

$$\mathrm{Var}\left[\mathrm{Var}[\hat{P}|X(t)]\right] \simeq \sum_{i=1}^{\infty} \left\{\frac{4}{l} w_i r_{C,i}\right\}^2\,. \tag{16}$$

Now, if $X(t)$ denotes a simple ON-OFF process whose ON and OFF periods obey exponential distribution with parameters λ_1 and λ_2, $r(\tau) = (\lambda_1\lambda_2)/(\lambda_1+\lambda_2)^2 \cdot \mathrm{e}^{-(\lambda_1+\lambda_2)\tau}$ is easily verified. In Fig. 2, we show the first term and other terms on the right-hand side of (15) for the case of $(\lambda_1, \lambda_2, l, m) = (2, 1, 100, 100)$. Obviously, we can judge that the approximation is appropriate according to Fig. 2.

Thus evaluation function $e(\sigma)$ is given by the following:

$$e(\sigma) = \sum_{i=1}^{\infty} \frac{4}{l} w_i r_{C,i} + \sqrt{\sum_{i=1}^{\infty} \left\{\frac{4}{l} w_i r_{C,i}\right\}^2}\,. \tag{17}$$

We can plot evaluation function $e(\sigma)$ if we specify the following: measurement period l, the number of probe packets m, the ACF $r(\tau)$ of $X(t)$, and the fluctuation magnitude σ. This means that we can obtain the optimal probing process as per the properties of the target network.

Finally, we conduct simulations to assess the validity of (17). We executed Monte-Carlo simulations that assumed above mentioned ON-OFF processes, and we calculated (7) directly. In Fig. 3, we display comparisons between the simulation-based value and (17). We can confirm that (17) is appropriate, and we can judge that the optimal σ are 0.2.

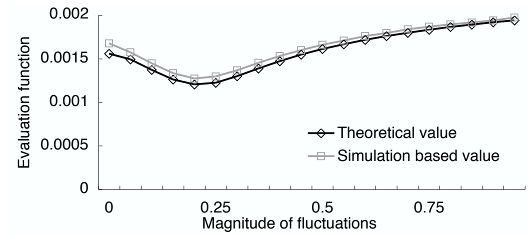

Fig. 3. Comparison of the simulation-based evaluation function and (17).

V. Conclusion

In this paper, we proposed a new probing policy that can avoid the phase-lock phenomenon by randomly perturbing the timing of probe packets. Moreover, we defined an evaluation function which can well assess the phase-lock phenomenon, and we provided a method that can specify the optimal fluctuation magnitude. Consequently, given the properties of the target network, we can identify the optimal probing policy.

We confirmed the validity of our method through simulations. In the future, we will validate the method in detail including verification on an actual network.

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