A Change-of-Measure Approach to Per-Flow Delay Measurement Combining Passive and Active Methods: Mathematical Formulation for CoMPACT Monitor

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***Abstract*— One problem with active measurement is that, while it is suitable for measuring time-average network performance, it is difficult to measure per-flow QoS, which is defined as the average over packets in the flow. To achieve such per-flow QoS measurement, the authors proposed a new technique, called the CoMPACT Monitor, which is based on the change-of-measure framework in probability/measure theory and transforms actively obtained information by using passively monitored data. This technique enables us to concurrently measure one-way delay information about individual users, applications and organizations in detail in a lightweight manner. This paper presents the mathematical formulation for the CoMPACT Monitor and verifies that it works well under some weak conditions. In addition, we investigate its characteristics regarding several implementation issues through simulation and actual network experiments. The results reveal that our technique provides highly qualified estimates involving only a limited amount of extra traffic from active probes.**

Index Terms— QoS measurement, delay measurement, active monitoring, passive monitoring, random sampling, change of measure

I. Introduction

Measuring performance and the quality of service (QoS) in the Internet is crucial for controlling, managing and provisioning the network. Measuring QoS indices such as the delay and loss for each of multiple flows (*e.g.*, users, applications or organizations) is also important since these are used as key parameters in Service Level Agreements (SLAs) between an ISP and its customers. Since the traffic conveyed through the Internet is generated by a wide variety of applications, which have different characteristics and different quality requirements, there are still problems in measuring the QoS for individual flows.

Many tools have been developed to measure network performance and/or QoS ([1], [2], [25]) and the results from these have also been reported ([11], [26], [27]). Conventional means of measuring network performance and QoS can be classified into two; that is, active and passive. These are briefly summarized below.

Active measurement measures network performance and/or QoS by injecting probe packets into a network path and monitoring them ([6], [7], [22], [24]). Various active methods have been proposed to measure different performance indices such as delay, loss and available bandwidth ([18]). In measuring one-way delay in a flow, we monitor the one-way delays of probe packets and generalize them as representing the one-way delays of flow packets. Problems with active measurement are as follows:

- The probe stream may cause a non-negligible amount of extra traffic on the network and this may affect its performance and the QoS of regular traffic (see Fig. 1). Thus, to obtain what we want to measure, an additional step of inversion is required to eliminate the influence of probe traffic. Furthermore, while it can measure the time-average performance of a network when the probe stream is a Poisson process (PASTA property; see [30], [10]), it is difficult to measure the per-flow QoS, which is defined as the average over packets (such as the delay and the loss for each flow).
- If we use light probe traffic with short probe packets to make extra traffic negligible, not only Poisson processes but also numerous probing processes (independent of regular traffic) are available to measure the time-average performance of the network (*e.g.*, see [10], [29]). However, it is still problematic to measure the QoS defined as the average over packets.

Let us make an additional remark here on averages over time and averages over packets. It is a well known fact in

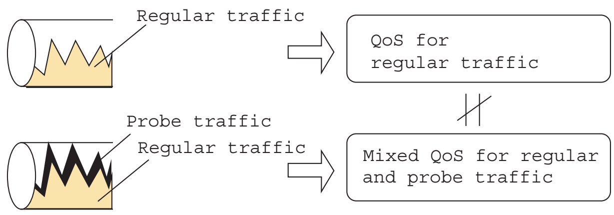

Fig. 1. Non-negligible amount of extra traffic caused by probe stream.

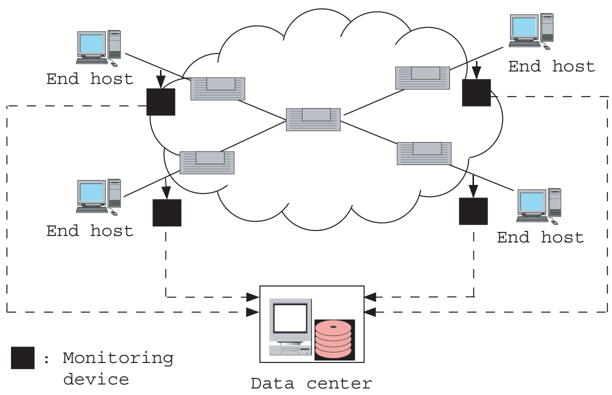

Fig. 2. Two-point passive monitoring.

the literature on queueing theory that the average over time is generally not equal to the average over packets, while they are occasionally confused within the context of active measurement. Since one-way delay in a flow is evaluated through the one-way delays experienced by flow packets, it should literally be defined as the average over flow packets. While active measurement, if it works, is suitable for measuring time averages, it is difficult to directly measure averages over packets. If the flow packets arrive according to a Poisson process, the average over the flow packets can be identical to the time average (PASTA) and active measurement may evaluate one-way delay in the flow. However, as actual Internet traffic has bursty properties and is no longer Poissonian ([23]), simple active measurement generalization is inappropriate.

Passive measurement is mainly used to monitor the volume of traffic but can be used to measure the per-flow QoS as well. Passive measurement can be categorized into two-point and one-point monitoring.

- Two-point monitoring requires two monitoring devices deployed at the ingress and egress points of the network path (see Fig. 2). The devices sequentially extract the packet data, and the QoS such as delay and loss can be evaluated by comparing the data from corresponding packets extracted at the two points. While this enables accurate measurement, we are faced with the following problems. Each packet should be identified by comparing the data in its header and/or content extracted by the two devices. Such data-matching processes are too heavy with huge volumes of traffic, as in large-scale networks. Furthermore, all the data should be collected to enable data to be matched (*e.g.*, at a data center, see Fig. 2). This process may require non-negligible bandwidth. While sub-sampling the packets reduces these problems, even here, data matching is still required for each flow when measuring multiple flows.
- One-point monitoring uses the TCP acknowledgment mechanism. When the TCP-sink receives a packet from the TCP-source, it sends back an acknowledgment packet, called an ack ([28]). Therefore, we can estimate the round-trip delay between the monitoring point and the sink by monitoring the packet-ack pair at a point in the network. Packet loss can also be similarly detected. However, one-point monitoring is restricted to measuring TCP flows. In addition, it needs to be noted that TCP acknowledgment implements cumulative acknowledgment and an ack is not immediately generated when a packet is received.

To ease these difficulties with active and passive methods, the authors proposed a new technique of measurement; change-of-measure-based passive/active monitoring (the CoMPACT Monitor) [4], [17]. The CoMPACT Monitor combines both active and passive methods in an easy-to-measure way and provides a solution to the generalization problem in active measurement. This is based on the change-of-measure framework in probability/measure theory and transforms actively obtained information by using passively monitored data. We can concurrently measure one-way delays for individual users, organizations and applications with this technique. It is also lightweight and scalable in the sense that the monitoring system does not become too complex even when the number of target flows that share the common path increases (see Sec. II). The authors' previous works [4], [17], however, were rather intuitive and lacked a theoretical basis; that is, they did not fully clarify why this technique worked so well. They also only investigated a simple implementation of the CoMPACT Monitor, which might be insufficient to achieve the targets the technique was aimed at. The current paper, which refines [5] in many respects, presents the mathematical formulation for the CoMPACT Monitor and corroborates that it works well under some weak conditions. Furthermore, we investigate its characteristics regarding a number of typical implementation issues via simulation and actual network experiments. Although we only discuss one-way delay measurement in this paper, the concept of the CoMPACT Monitor can also be applied to other QoS measurements and was recently applied to the measurement of packet loss ([16]).

Lindh ([19], [20]) proposed another technique that combined passive and active methods to detect packet loss, which used active probe packets in IP networks just like OAM cells in ATM networks. In his technique, a router sends a probe packet at every interval such that a predefined fixed number of target flow packets pass through it. A passive monitoring device is used to count the number of target flow packets passing through the router. Since the number of target flow packets between adjacent probes should be constant, we can detect packet loss in the target flow by counting them. Although this idea can also be generalized to other QoS measurements, it has the following drawbacks:

- It requires many probe packets to be inserted when there are numerous target flow packets. This means that the number of probe packets tends to increase as the network becomes congested. Thus, the probe traffic may actually affect regular traffic.
- When we intend to measure the individual characteristics of multiple flows, Lindh's idea requires respective probe streams to correspond to flows. Thus, it is not scalable with respect to the number of target flows.

The amount of extra traffic generated by active probes with the CoMPACT Monitor, on the other hand, can be independent of regular traffic and we can reduce the amount of extra traffic to be sufficiently light not to affect regular traffic. Furthermore, one-way delays in individual flows can be measured by using a common probe stream as long as the flows share the same network path.

The rest of the paper is organized as follows. We describe the fundamental concept behind the CoMPACT Monitor in Sec. II to enable the underlying idea to be intuitively understood. Then, we present the mathematical formulation for our technique in Sec. III, which is based on fluid approximation, sample-path argument and random sampling. Section IV discusses the practical implementation of the technique. Sections V and VI present some results obtained from simulation and actual network experiments to demonstrate the features of the CoMPACT Monitor, where we discuss our evaluations of marginal one-way delay distributions for individual flows. In addition, we investigate its characteristics in terms of several implementation issues. We conclude the paper in Sec. VII.

II. The Concept

This section discusses the problem of generalizing active measurement and states the concept underlying our new technique; that is, the CoMPACT Monitor. Throughout the paper, we use the term flow to denote the packet stream of a user, an application, an organization or a combination of them, which can be identified by its source/destination IP addresses, its higher-layer protocol, its port number or any other information in the header or payload of its packets. In the following, we assume that one flow goes through one network path.

Let us focus, for a while, on a flow through a path on the network being considered and refer to it as the target flow. Let $V(t)$ denote the virtual one-way delay in the network path at time t (≥ 0); that is, if a flow packet or a probe packet is introduced into the path at time T (≥ 0), the one-way delay of the packet is given by $V(T)$. The virtual one-way delay reveals the amount of congestion in the network path and is not only influenced by the target flow and the probe stream but also by other flows that share all or some of the path.

Let A_n, $n = 1, 2, \ldots$, satisfy $A_n \leq A_{n+1}$ and denote the time at which the nth target flow packet is introduced into the network and let $X_n = V(A_n)$, $n = 1, 2, \ldots$, denote the one-way delay of the nth target flow packet. We assume that X_n, $n = 1, 2, \ldots$, have a common marginal distribution function F (this assumption is relaxed in the next section). Of course, we do not know distribution F in advance and this is usually evaluated as follows: For any subset $C \subset [0, \infty)$, we have

$$\begin{aligned} \mathrm{P}_F(X_n \in C) &= \int 1_{\{x \in C\}} \, \mathrm{d}F(x) \\ &= \mathrm{E}_F \left[1_{\{X_n \in C\}} \right], \end{aligned}$$

where P_F and E_F respectively denote the probability measure and the expectation with respect to it endowed with distribution F, and 1_E denotes the indicator that takes a value of 1 if event E is true and 0 otherwise. For example, choosing $C = [0, c]$, $c \geq 0$, we have $\mathrm{P}_F(X_n \leq c) = F(c)$. If we can directly monitor sequence X_n, $n = 1, 2, \ldots$, its marginal distribution can be estimated by

$$\frac{1}{m} \sum_{n=1}^{m} 1_{\{X_n \in C\}} \quad \text{for sufficiently large } m. \tag{1}$$

Now, suppose that we cannot directly monitor X_n, $n = 1, 2, \ldots$, and cannot estimate the marginal distribution with (1). Let T_n, $n = 1, 2, \ldots$, satisfy $T_n < T_{n+1}$ and denote the time at which the nth probe packet is introduced into the same network path and let $Y_n = V(T_n)$, $n = 1, 2, \ldots$, denote the one-way delay of the nth probe packet. We assume that Y_n, $n = 1, 2, \ldots$, have a common marginal distribution function G (this is also relaxed in the next section). Here, the problem is that G is generally different from F. If the probe traffic is negligible (but jointly stationary with regular traffic), we can demonstrate that $\mathrm{P}_G(Y_n \in C) = \mathrm{P}(V(t) \in C)$, where P_G denotes the probability measure endowed with distribution G and P denotes the probability measure that $\{V(t)\}_{t \geq 0}$ follows (*e.g.*, see [10]). Also, if the probe traffic is not negligible but is inserted according to a Poisson process (independent of but jointly stationary with regular traffic), then PASTA [30] can state that $\mathrm{P}_G(V(T_n-) \in C) = \mathrm{P}(V(t) \in C)$, where $V(T_n-)$ denotes the virtual delay just before the nth probe packet arrives; that is, it represents the queueing delay of the nth probe packet. Here, it should be noted that, under the ergodicity assumption,

$$\mathrm{P}(V(t) \in C) = \lim_{T \to \infty} \frac{1}{T} \int_0^T 1_{\{V(t) \in C\}} \, \mathrm{d}t.$$

That is, active measurement (P_G), if it works, can be used to evaluate the time average (P), but generally cannot be used to evaluate the average over flow packets (P_F) as obtained by (1) directly. If $\{A_n\}_{n \geq 1}$ is a Poisson process, we can show $\mathrm{P}(V(t) \in C) = \mathrm{P}_F(V(A_n-) \in C)$ under a weak condition (PASTA), but the Poisson assumption of $\{A_n\}_{n \geq 1}$ is unexpected in Internet traffic.

To overcome this problem, we consider transforming P_F to P_G using the change-of-measure framework. Under one certain condition (F is absolutely continuous with respect to G), we have

$$\begin{aligned} \mathrm{P}_F(X_n \in C) &= \int 1_{\{y \in C\}} \frac{\mathrm{d}F(y)}{\mathrm{d}G(y)} \, \mathrm{d}G(y) \\ &= \mathrm{E}_G \left[1_{\{Y_n \in C\}} \frac{\mathrm{d}F(Y_n)}{\mathrm{d}G(Y_n)} \right], \end{aligned}$$

where E_G denotes the expectation with respect to P_G and $\mathrm{d}F/\mathrm{d}G$ is called the likelihood ratio of F with respect to G.

Since Y_n, $n = 1, 2, \ldots$, are monitored by active measurement, once we can evaluate $\mathrm{d}F(Y_n)/\mathrm{d}G(Y_n)$, $n = 1, 2, \ldots$, we can estimate marginal one-way delay distribution $\mathrm{P}_F(X_n \in C)$ by

$$\frac{1}{m}\sum_{n=1}^{m} 1_{\{Y_n \in C\}} \frac{\mathrm{d}F(Y_n)}{\mathrm{d}G(Y_n)} \quad \text{for sufficiently large } m. \tag{2}$$

Note that (2) forms a weighted mean of $1_{\{Y_n \in C\}}$ over $n = 1, \ldots, m$ with weight $\mathrm{d}F(Y_n)/\mathrm{d}G(Y_n)$ endowed with the likelihood ratio. As might be intuited and will be verified in the following sections, each weight is evaluated as proportional to how much traffic in the target flow is sent into the path at about the same time as a probe packet. That is, the values of the weights are approximately obtained by simply counting the packets in the target flow by passively monitoring them.

The fundamental concept underlying the CoMPACT Monitor is as follows. It is indeed difficult to directly monitor X_n, $n = 1, 2, \ldots$, and to measure one-way delay distribution $\mathrm{P}_F(X_n \in C)$ through (1). However, it is easy to approximately evaluate $\mathrm{d}F(Y_n)/\mathrm{d}G(Y_n)$, $n = 1, 2, \ldots$, from the passive monitoring data. Thus, we can measure distribution $\mathrm{P}_F(X_n \in C)$ through (2) by monitoring Y_n instead of X_n, $n = 1, 2, \ldots$. In addition, (2) means that if we have $\mathrm{d}F(Y_n)/\mathrm{d}G(Y_n)$, $n = 1, 2, \ldots$, for individuals of multiple flows (not only users but also user groups, applications, organizations and their combinations) on the same network path, we can concurrently evaluate the one-way delay distributions for the respective flows by using common actively monitored data Y_n, $n = 1, 2, \ldots$.

The expected advantages of the CoMPACT Monitor are as follows:

- Simple monitoring system: The setting for the active monitoring hosts is the same as for standard active measurement. The passive monitoring by the CoMPACT Monitor is just one-point packet counting; that is, the monitoring device counts the number of target flow packets within a certain small neighborhood for each probe packet. Here, no clock synchronization is required between the passive monitoring device and the active hosts since the packet counting process is only triggered when the arrival of a probe is detected at the passive monitoring device (see Sec. IV).
- Negligible extra traffic from active probes: The CoMPACT Monitor is a weighted sampling technique and the sampling is done by the probe packets. Although sampling techniques generally perform better with more samples, in active measurement, many probes may affect regular traffic. As found through the results of the experiments discussed in Secs. V and VI, the CoMPACT Monitor yields highly qualified estimates with little extra traffic from active probes, as with conventional active measurement, so that regular traffic remains largely unaffected (appropriate number of probes in a given interval is discussed in [13]).
- Concurrent measurements of multiple flows: By evaluating the individual likelihood-ratio weights for flows that share a common path, we can measure the one-way delays in respective flows by using common actively monitored data. This means that the CoMPACT Monitor is scalable in the sense that the monitoring system does not become too complicated even if the number of the target flows sharing the same path increases.
- Matching process for light amounts of data: Data matching is also required between actively monitored data and the corresponding likelihood-ratio weights obtained from passively monitored data. However, the probe traffic in the CoMPACT Monitor is light and so is the data matching process.
- Protocol independence: The CoMPACT Monitor can be applied to non-TCP protocols as well.

Note that the target flows that can be measured from common actively monitored data are restricted to those sharing the same physical and logical paths (*e.g.*, priority queueing at a router causes different logical paths on the same physical path). If we want to measure the flows on different paths, it is necessary to use different probe streams (this is, of course, the usual feature in conventional active measurement). However, even when measuring flows on different paths, the configuration for passive monitoring devices is not always complicated. If the different paths include a common link, we can reduce the number of passive monitoring devices by placing one on the common link (see Fig. 3).

If we randomly select packets from one of the multiple target flows on the same path and make them play the role of probe packets as previously described with the CoMPACT Monitor, we can evaluate the one-way delays in all the target flows in the same way without a probe stream (that is, a combination of standard two-point passive monitoring and packet counting to evaluate the likelihood-ratio weights). Although this seems useful, we adopt an active probe stream since regular traffic can not be controlled and also because it is much easier to match probe packets than flow packets.

III. Mathematical Formulation

This section presents the mathematical formulation for the CoMPACT Monitor to fully clarify why this technique works so well. In the next section, we will show, based on the result obtained here, how the sequence of the likelihood-ratio weights in (2) can approximately be evaluated by passive monitoring in practice.

A. General Formulation

We will discuss the formulation by making as general assumptions as possible since Internet traffic has a wide variety of characteristics. We assume that there are many more flow packets traveling through the network than probe packets in the framework for the CoMPACT Monitor. Thus, we consider a fluid approximation for regular traffic; that is, the flows are approximated as fluid while the probe arrivals are modeled as a point process. Throughout this subsection, we follow a standard sample-path argument within the theory of stochastic processes; that is, a sample path for the stochastic fluid process is fixed and is discussed as being deterministic.

Let $a(t)$ denote the input rate of the target flow fluid at time t (≥ 0), where $\{a(t)\}_{t\geq 0}$ is a nonnegative deterministic

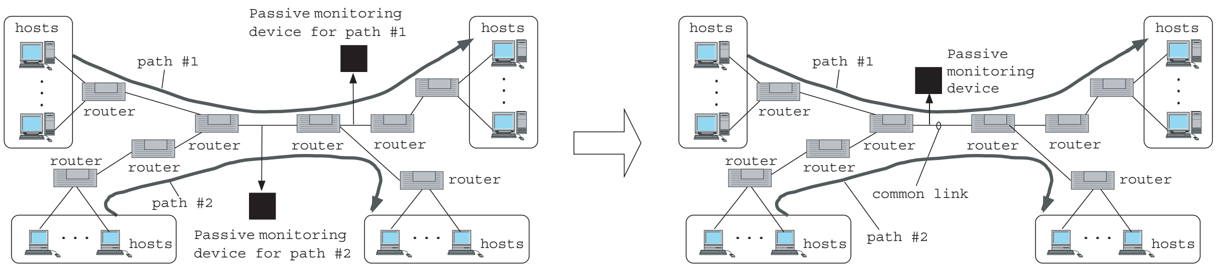

Fig. 3. Simplification of configuration for passive monitoring devices.

process assumed to be right-continuous with left limits and bounded on $t \geq 0$. The cumulative amount of target flow fluid transmitted during $(0, t]$ is then represented by $\int_0^t a(s)\,\mathrm{d}s$, where we further assume that $\liminf_{t\to\infty} t^{-1}\int_0^t a(s)\,\mathrm{d}s > 0$. As in the preceding section, let $V(t)$ denote the virtual one-way delay in the network path at time t (≥ 0). We assume that $\{V(t)\}_{t\geq 0}$ is also a nonnegative deterministic process and is right-continuous with left limits in $t \geq 0$. The deterministic process $\{(V(t), a(t))\}_{t\geq 0}$ can be considered a sample path extracted from the corresponding stochastic process, where we assume neither stationarity nor ergodicity. Note that, in such a probabilistic sense, $\{V(t)\}_{t\geq 0}$ can not only be influenced by $\{a(t)\}_{t\geq 0}$ but also by other flows that share all or some of the network path.

With the above setting, the empirical average one-way delay distribution for the target flow fluid transmitted over $(0, t]$, for any $t > 0$ such that $\int_0^t a(s)\,\mathrm{d}s > 0$, is given by

$$\pi_t(C) = \frac{\int_0^t 1_{\{V(s)\in C\}}\, a(s)\,\mathrm{d}s}{\int_0^t a(s)\,\mathrm{d}s}, \quad C \in \mathcal{B}(\mathbb{R}_+), \tag{3}$$

where $\mathcal{B}(\mathbb{R}_+)$ denotes the Borel σ-field on $\mathbb{R}_+$. The denominator on the right-hand side of (3) represents the total amount of target flow fluid transmitted during $(0, t]$ and the numerator represents the amount of target flow fluid during $(0, t]$ whose one-way delay is in the range of C. Therefore, (3) expresses the average over the target flow fluid transmitted during $(0, t]$. If both limits of

$$\lim_{t\to\infty} \frac{1}{t}\int_0^t 1_{\{V(s)\in C\}}\, a(s)\,\mathrm{d}s, \tag{4}$$

$$\lim_{t\to\infty} \frac{1}{t}\int_0^t a(s)\,\mathrm{d}s, \tag{5}$$

exist, the long-term average one-way delay distribution for the target flow fluid also exists and is given by

$$\pi(C) = \lim_{t\to\infty} \pi_t(C), \quad C \in \mathcal{B}(\mathbb{R}_+), \tag{6}$$

where $\lim_{t\to\infty} t^{-1}\int_0^t a(s)\,\mathrm{d}s > 0$ is ensured by the assumption.

Now, let us consider evaluating long-term average distribution $\pi(C)$, $C \in \mathcal{B}(\mathbb{R}_+)$, in (6) by monitoring the network path at random sampling epochs. These monitoring epochs correspond to the arrival times of probe packets. Let $\{N(t)\}_{t\geq 0}$ denote a simple counting process representing the monitoring epochs and let $\{T_n\}_{n\geq 1}$ be the corresponding point sequence; that is, $N(t)$ denotes the number of monitoring epochs during $(0, t]$ and $T_n = \inf\{t \geq 0 : N(t) \geq n\}$ for $n = 1, 2, \ldots$. The simplicity of counting processes is defined as $T_n < T_{n+1}$ for $n = 1, 2, \ldots$ almost surely with respect to a probability measure P (P-a.s. for short). We assume that $\{N(t)\}_{t\geq 0}$ has stationary and ergodic increments with respect to P and also has a positive and finite intensity $\lambda_N = \mathrm{E}[N(1)]$, where E denotes the expectation with respect to P. We can then verify the following:

Theorem 1 *If the following holds with constant $\alpha(C)$ for any fixed $C \in \mathcal{B}(\mathbb{R}_+)$,*

$$\lim_{m\to\infty} \frac{1}{m}\sum_{n=1}^{m} 1_{\{V(T_n)\in C\}}\, a(T_n) = \alpha(C), \quad \mathrm{P}\text{-a.s.}, \tag{7}$$

then the limits (4) and (5) also exist, and we have

$$\pi(C) = \frac{\alpha(C)}{\alpha(\mathbb{R}_+)} = \lim_{m\to\infty} \frac{\sum_{n=1}^{m} 1_{\{V(T_n)\in C\}}\, a(T_n)}{\sum_{n=1}^{m} a(T_n)}, \quad \mathrm{P}\text{-a.s.} \tag{8}$$

Theorem 1 demonstrates that the long-term average one-way delay distribution $\pi(C)$, $C \in \mathcal{B}(\mathbb{R}_+)$, for the target flow fluid is estimated through m times monitoring by

$$Z_m(C \mid N) = \frac{\sum_{n=1}^{m} 1_{\{V(T_n)\in C\}}\, a(T_n)}{\sum_{n=1}^{m} a(T_n)}, \tag{9}$$

for sufficiently large m. The estimator (9) is strongly consistent within the sense of (8) for each given $\{(V(t), a(t))\}_{t\geq 0}$ regardless of any realizations of sampling process $\{N(t)\}_{t\geq 0}$ as long as (7) holds. In (9), $V(T_n)$ represents one-way delay of the nth probe packet; that is, $V(T_n) = Y_n$, $n = 1, 2, \ldots$. Thus, multiplying both the numerator and the denominator in (9) by $1/m$, we can see that $a(T_n)/\bigl(m^{-1}\sum_{n=1}^{m} a(T_n)\bigr)$ corresponds to the likelihood-ratio weight $\mathrm{d}F(Y_n)/\mathrm{d}G(Y_n)$ in (2). In Sec. IV, we explain how this likelihood-ratio weight can approximately be evaluated by simply counting the packets in the target flow in practical situations where the traffic is not fluid.

Proof: We show that there exists the limit in (4) and this is equal to $\alpha(C)$ in (7) along similar lines to the proof of Theorem 3.1 in [15]. First, since $\{(V(t), a(t))\}_{t\geq 0}$ is deterministic, we have

$$\mathrm{E}\left[\frac{1}{t}\sum_{n=1}^{N(t)} 1_{\{V(T_n)\in C\}}\, a(T_n)\right]$$

$$= \mathrm{E}\biggl[\frac{1}{t}\int_0^t 1_{\{V(s)\in C\}}\, a(s)\,\mathrm{d}N(s)\biggr]$$
$$= \frac{1}{t}\int_0^t 1_{\{V(s)\in C\}}\, a(s)\,\lambda_N\,\mathrm{d}s, \tag{10}$$

where the first equality follows from the simplicity of $\{N(t)\}_{t\ge 0}$; that is, $N(t)$ only increases by one at $t = T_n$, $n = 1, 2, \ldots$, and in the second equality, we use $\mathrm{E}[N(t)] = \lambda_N\, t$ due to the stationarity of $\{N(t)\}_{t\ge 0}$. Here, for the inside of the expectation on the left-hand side of (10), we have under the condition in (7) and the ergodicity of $\{N(t)\}_{t\ge 0}$,

$$\lim_{t\to\infty}\frac{1}{t}\sum_{n=1}^{N(t)} 1_{\{V(T_n)\in C\}}\, a(T_n)$$
$$= \lim_{t\to\infty}\frac{N(t)}{t}\,\frac{1}{N(t)}\sum_{n=1}^{N(t)} 1_{\{V(T_n)\in C\}}\, a(T_n)$$
$$= \lambda_N\,\alpha(C), \quad \text{P-a.s.} \tag{11}$$

Furthermore, since $\{a(t)\}_{t\ge 0}$ is bounded,

$$\frac{1}{t}\sum_{n=1}^{N(t)} 1_{\{V(T_n)\in C\}}\, a(T_n) \le a^{\sup}\,\frac{N(t)}{t},$$

where $a^{\sup} = \sup_{t\ge 0}\{a(t)\}$, and we have $\mathrm{E}[N(t)/t] = \lambda_N < \infty$. Therefore, taking $t \to \infty$ in (10) and applying the dominated convergence theorem to exchange the limit and the expectation on the left-hand side, we obtain

$$\lim_{t\to\infty}\frac{\lambda_N}{t}\int_0^t 1_{\{V(s)\in C\}}\, a(s)\,\mathrm{d}s$$
$$= \mathrm{E}\biggl[\lim_{t\to\infty}\frac{1}{t}\sum_{n=1}^{N(t)} 1_{\{V(T_n)\in C\}}\, a(T_n)\biggr]$$
$$= \lambda_N\,\alpha(C),$$

where the last equality follows from (11). Hence, there exists the limit in (4) and this is equal to $\alpha(C)$. The existence of the limit in (5) and its equality with $\alpha(\mathbb{R}_+)$ can similarly be verified by replacing C with $\mathbb{R}_+$. Thus, we have the result in (8) from (3) and (6). ■

We can make a few remarks on Theorem 1 from the stochastic point of view:

Remark 1 $\{(V(t), a(t))\}_{t\ge 0}$ in the above discussion can be interpreted as a sample path extracted from the corresponding stochastic process, where we assume neither stationarity nor ergodicity. Therefore, even if C is fixed, limit $\alpha(C)$ in (7) can take different values according to different sample paths. The condition that limit $\alpha(C)$ is a constant (up to the sample path) for a given C, on the other hand, means that once a sample path for $\{(V(t), a(t))\}_{t\ge 0}$ is given and C is fixed, this limit remains the same for any realizations of the counting process $\{N(t)\}_{t\ge 0}$. In other words, while the condition allows sufficient generality for $\{(V(t), a(t))\}_{t\ge 0}$, it requires sufficient randomness for $\{N(t)\}_{t\ge 0}$ (that is, (7) is the condition on $\{N(t)\}_{t\ge 0}$ rather than that on $\{(V(t), a(t))\}_{t\ge 0}$).

We can also slightly weaken the ergodicity assumption for $\{N(t)\}_{t\ge 0}$ such that it is asymptotically ergodic. Thus, we can choose $\{N(t)\}_{t\ge 0}$ from a broader class of standard non-delayed renewal processes with spread-out interarrival distributions (note that a stationary renewal process is delayed with the delay distribution given as a stationary excess distribution, *e.g.*, see [8, Chaps. V and VII] for fundamental renewal theory and spread-out distributions).

Remark 2 If we assume that $\{(V(t), a(t))\}_{t\ge 0}$ is stochastic and jointly ergodic with sampling process $\{N(t)\}_{t\ge 0}$, we can then show that the condition (7) in Theorem 1 holds under the mutual independence of $\{(V(t), a(t))\}_{t\ge 0}$ and $\{N(t)\}_{t\ge 0}$. Here, the result in (8) reduces to

$$\mathrm{P}_A(V(0)\in C) = \frac{\mathrm{E}_N^0[1_{\{V(0)\in C\}}\, a(0)]}{\mathrm{E}_N^0[a(0)]}, \tag{12}$$

where P_A denotes the Palm probability for random measure $a(t)\,\mathrm{d}t$ and E_N^0 denotes the expectation with respect to Palm probability P_N^0 for point process $\{N(t)\}_{t\ge 0}$ (*e.g.*, see [9, Chap. 1] and [14, Chap. 12] for the theory of Palm probabilities). In (12), we can see that $a(0)/\mathrm{E}_N^0[a(0)]$ plays the role of likelihood ratio $\mathrm{dP}_A/\mathrm{dP}_N^0$ in the change-of-measure framework.

Remark 3 We considered $\{(V(t), a(t))\}_{t\ge 0}$ to be stochastic in the above two remarks but still assumed that it was independent of sampling process $\{N(t)\}_{t\ge 0}$. Now, let us consider a case where $(V(t), a(t))$ depends on $\{N(s)\}_{s<t}$ but is independent of $\{N(u) - N(t)\}_{u\ge t}$ for $t > 0$. Here, if $\{N(t)\}_{t\ge 0}$ is a Poisson process and is jointly ergodic with $\{(V(t), a(t))\}_{t\ge 0}$, we can verify the following by using Poisson calculus [12, Chap. 9] (also see PASTA property [30]):

$$\mathrm{P}_A(V(0)\in C) = \frac{\mathrm{E}_N^0[1_{\{V(0-)\in C\}}\, a(0-)]}{\mathrm{E}_N^0[a(0-)]},$$

where $V(0-) = \lim_{t\uparrow 0} V(t)$ and $a(0-) = \lim_{t\uparrow 0} a(t)$. The corresponding estimator is given by

$$Z_m^-(C \mid N) = \frac{\sum_{n=1}^m 1_{\{V(T_n-)\in C\}}\, a(T_n-)}{\sum_{n=1}^m a(T_n-)}. \tag{13}$$

B. Variance Analysis in Special Case

Here, we discuss our analysis of the variance of the estimator (9) under a special simplifying assumption. Although the assumption may be too restricted, we believe that variance analysis even under such an assumption can provide some insights into using our technique in the real world.

Let us consider the setting in Remark 2; that is, we assume that $\{(V(t), a(t))\}_{t\ge 0}$ is stochastic and also jointly ergodic with but independent of sampling process $\{N(t)\}_{t\ge 0}$. Moreover, we assume that the input rate process $\{a(t)\}_{t\ge 0}$ of the target flow is ON-OFF; that is, $a(t) = a\,1_{\{a(t)>0\}}$, $t \ge 0$, with a constant $a > 0$. With this setting, we have the following:

Proposition 1 *Assume that, for each $n = 1, 2, \ldots$, $V(T_n)$ is conditionally independent of $\{(V(T_l), a(T_l))\}_{l\ne n}$ given $a(T_n)$. Then, the variance of $Z_m(C \mid N)$, $C \in \mathcal{B}(\mathbb{R}_+)$, in (9) reduces to*

$$\mathrm{Var}_N^0[Z_m(C \mid N)]$$

$$= \pi(C)\,(1-\pi(C))\,\mathrm{E}_N^0\left[\frac{1}{\sum_{n=1}^m 1_{\{a(T_n)>0\}}}\right], \tag{14}$$

where Var_N^0 *denotes the variance with respect to Palm probability* P_N^0.

Indeed, the condition for the proposition may be too strong and nonrealistic. However, we believe that (14) can provide good insights into the practical use of our technique; that is, the quality of estimates mainly depends on the number of sampling points T_n with $a(T_n) > 0$, which suggests that lighter traffic flows require more sampling points.

Proof: Since $a(t) = a\,1_{\{a(t)>0\}}$, $t \geq 0$, estimator $Z_m(C \mid N)$ in (9) reduces to

$$Z_m(C \mid N) = \frac{\sum_{n=1}^m 1_{\{V(T_n)\in C, a(T_n)>0\}}}{\sum_{n=1}^m 1_{\{a(T_n)>0\}}}.$$

Here, define sub-σ-field $\mathcal{A}_m$, $m = 1, 2, \ldots$, generated by $\{a(T_1), \ldots, a(T_m)\}$. To evaluate expectation $\mathrm{E}_N^0[Z_m(C \mid N)]$, we first consider the conditional expectation given $\mathcal{A}_m$. Under the conditional independence in the proposition, we have

$$\begin{aligned}
&\mathrm{E}_N^0[Z_m(C \mid N) \mid \mathcal{A}_m] \\
&= \frac{\sum_{n=1}^m \mathrm{P}_N^0\big(V(T_n) \in C \mid a(T_n) > 0\big)\, 1_{\{a(T_n)>0\}}}{\sum_{n=1}^m 1_{\{a(T_n)>0\}}} \\
&= \mathrm{P}_N^0\big(V(0) \in C \mid a(0) > 0\big),
\end{aligned}$$

where the second equality follows from the joint stationarity of $\{(V(t), a(t))\}_{t\geq 0}$ and $\{N(t)\}_{t\geq 0}$. Therefore, we have $\mathrm{E}_N^0[Z_m(C \mid N)] = \mathrm{P}_N^0\big(V(0) \in C \mid a(0) > 0\big)$.

On the other hand, using (3)–(6), we have under the ergodicity assumption,

$$\pi(C) = \frac{\mathrm{E}[1_{\{V(0)\in C\}}\, a(0)]}{\mathrm{E}[a(0)]} = \mathrm{P}_A(V(0) \in C),$$

where the second equality follows from the property of the Palm probability with respect to a random measure with its density. Here, since $a(0) = a\,1_{\{a(0)>0\}}$, from (12), we have

$$\pi(C) = \mathrm{P}_N^0\big(V(0) \in C \mid a(0) > 0\big);$$

that is, in this very restricted case, $Z_m(C \mid N)$ in (9) gives an unbiased estimator for one-way delay distribution $\pi(C)$.

Under the conditional independence in the proposition, the conditional second moment of $Z_m(C \mid N)$ given $\mathcal{A}_m$ is derived in a similar way as

$$\begin{aligned}
\mathrm{E}_N^0[Z_m(C \mid N)^2 \mid \mathcal{A}_m] &= \frac{\pi(C)}{\sum_{n=1}^m 1_{\{a(T_n)>0\}}} \\
&\quad + \pi(C)^2\left(1 - \frac{1}{\sum_{n=1}^m 1_{\{a(T_n)>0\}}}\right).
\end{aligned}$$

Hence, taking the expectation and subtracting $\mathrm{E}_N^0[Z_m(C \mid N)]^2 = \pi(C)^2$, we have the variance (14). ■

IV. Implementation

Since the traffic is not fluid in practice and the point monitored by the passive device is not the entrance to the network path (see Fig. 3), we must carefully consider the practical implementation of estimator $Z_m(C \mid N)$ in (9) as well as $Z_m^-(C \mid N)$ in (13).

Note that the monitoring process at the passive device is triggered when a probe arrives at the device (not when the probe is introduced into the network). Suppose that the probe, which is introduced into the path at T_n, $n = 1, 2, \ldots$, reaches the monitoring point of the passive device at T'_n (see Fig. 4). The passive device then counts the number of target flow packets in some small neighborhood of T'_n. Let $\tilde{a}^+(T_n, \delta)$ and $\tilde{a}^-(T_n, \delta)$ respectively denote the numbers of target flow packets going through the monitoring point during $[T'_n, \min\{T'_n + \delta, T'_{n+1}\})$ and $[\max\{T'_n - \delta, T'_{n-1}\}, T'_n)$, where δ is a predefined small positive number (we conventionally set $T'_0 = 0$). If the influence of probe traffic is negligible, we can assume that $\{(V(t), a(t))\}_{t\geq 0}$ and sampling process $\{N(t)\}_{t\geq 0}$ are mutually independent. In this case, estimator $Z_m(C \mid N)$ in (9) can be implemented by any of the following:

$$\tilde{Z}_m^+(C \mid N) = \frac{\sum_{n=1}^m 1_{\{Y_n\in C\}}\, \tilde{a}^+(T_n, \delta)}{\sum_{n=1}^m \tilde{a}^+(T_n, \delta)}, \tag{15}$$

$$\tilde{Z}_m^-(C \mid N) = \frac{\sum_{n=1}^m 1_{\{Y_n\in C\}}\, \tilde{a}^-(T_n, \delta)}{\sum_{n=1}^m \tilde{a}^-(T_n, \delta)}; \tag{16}$$

that is, $a(T_n)$, $n = 1, \ldots, m$, in (9) can be approximated by any of $\tilde{a}^+(T_n, \delta)$ and $\tilde{a}^-(T_n, \delta)$, where Y_n in (15) and (16) represents one-way delay of the nth probe packet. When the influence of probe traffic is not negligible, on the other hand, by letting the probe stream be a Poisson process (Remark 3 in the preceding section), estimator $Z_m^-(C \mid N)$ in (13) can be implemented with (16); that is, $a(T_n-)$ can be approximated by $\tilde{a}^-(T_n, \delta)$. Here, we still regard $V(T_n-)$ as Y_n; that is, the propagation delay and the transmission delay of the probe packets are ignored since they are generally smaller than those of the flow packets (which have already been ignored by the fluid approximation in (13)). Note that the target flow packets going through the monitoring point in the δ-neighborhood of T'_n do not exactly correspond to those entering the path in the δ-neighborhood of T_n. However, since δ is small, they certainly correspond to the target flow packets entering the path in some small neighborhood of T_n (depicted by δ_n^+ and δ_n^- in Fig. 4).

Now, let us consider achieving a passive monitoring device to respectively evaluate $\tilde{a}^+(T_n, \delta)$ and $\tilde{a}^-(T_n, \delta)$ in (15) and (16). Recent developments in hardware technology have made on-line high-speed packet filtering possible (*e.g.*, see [21]) and enabled realtime counting. In this case, the passive device can record the number of target flow packets observed during $[T'_n, \min\{T'_n + \delta, T'_{n+1}\})$ or $[\max\{T'_n - \delta, T'_{n-1}\}, T'_n)$ on-line. In achieving $\tilde{a}^+(T_n, \delta)$, when a probe arrives at the passive monitoring device at T'_n, $n = 1, \ldots, m$, the device starts counting packets for the target flow during $[T'_n, \min\{T'_n + \delta, T'_{n+1}\})$. In achieving $\tilde{a}^-(T_n, \delta)$, on the other hand, the passive monitoring device always holds the number of target flow packets during $[\max\{t - \delta, T'_-(t)\}, t)$ for any time t,

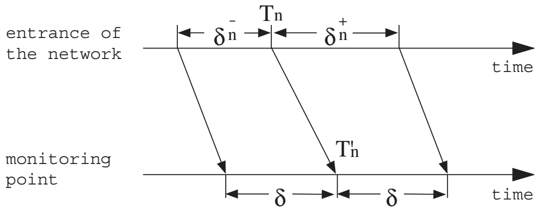

Fig. 4. Difference in implementations.

and remains ready for the arrival of a probe, where $T'_-(t) = \max\{T'_n < t\}$. Once the probe arrives at T'_n, the device can immediately read out the number of target flow packets during $[\max\{T'_n - \delta, T'_{n-1}\}, T'_n)$. Therefore, in both implementations of (15) and (16), the passive monitoring device only maintains one packet counter for each target flow.

Even if high-speed packet filtering is not available and the realtime counting is difficult for high-speed links, both (15) and (16) can be implemented. The passive monitoring device monitors all flow packets and records parts of them that include sufficient information for filtering packets. Then, an additional procedure extracts the flow packet data observed during $[T'_n, \min\{T'_n + \delta, T'_{n+1}\})$ or $[\min\{T'_n - \delta, T'_{n-1}\}, T'_n)$ from the recorded data, and counts these for each target flow. However, in this scenario, the volume of data sent to the data center is extremely large and may neutralize one of the benefits of the CoMPACT Monitor. Therefore, realtime counting is preferable for efficient implementation (note that realtime counting is also essential to Lindh's technique [19], [20] discussed in the introduction).

We have not yet found an appropriate value for δ in these implementations. It must neither be too large nor too small. It will probably depend on such factors as the observation period, the number and variety of applications, the packet sizes of the flows (ignored by fluid approximation), link capacities and the number of hops from the entrance of the network to the passive monitoring point. In the previous works [4], [17], another implementation of counting packets during $[T'_n, T'_{n+1})$ or $[T'_{n-1}, T'_n)$ was adopted, which corresponded to the case of $\delta \to \infty$. The experimental results obtained by comparing these implementations can be found in Sec. VI.

V. Simulation Experiments

This section discusses our investigations into the characteristics of the CoMPACT Monitor obtained through simulation experiments.

A. Network and Traffic Models

The network model with 20 pairs of source and destination end hosts shown in Fig. 5 was used for all experiments discussed in this section except for the one in Sec. V-F. Each link capacity between the end host and the edge router is 1.5 Mbps, that between the edge router and the core router is 8 Mbps, and that between the core routers is 10 Mbps. Each end host on the left of Fig. 5 is a source and transfers packets by TCP/IP to the corresponding destination end host on the right. Here, we refer to the packet stream between a pair of source and destination end hosts as a flow. The flows are given as ON-OFF processes and categorized into four different types of application traffic listed in Table I; that is, every five flows are assigned to one type of traffic. The size of the flow packets is identical at 1500 bytes.

In addition, two pairs of hosts for sending and receiving probe packets to support the active probe traffic are connected in the same manner as the end hosts, and are depicted by A's and B's in Fig. 5. The probe packets are 64 bytes long and are introduced into the network according to a Poisson process. Note that the probe stream is not necessarily Poissonian (except for the case of Remark 3 in Sec. III), but is necessary to avoid synchronization with any periodical process in the network. The passive monitoring device is located at the common link between the core routers to count the packets in the flows to evaluate their likelihood-ratio weights.

Each experiment in the following subsections was conducted using ns2 ([3]), where the run length was 3600 s, except for cases where it is otherwise specified, and the marginal delay distributions were estimated. Here, the delay of a packet denotes the time taken from the packet to arrive at the edge router on the left to its arrival at the edge router on the right. A common sample path for a set of 20 flows was used throughout the experiments except for the one in Sec. V-F. 30 independent replicas of the probe streams were inserted into such a common sample path for regular traffic and the estimates with the CoMPACT Monitor were computed with 95% confidence intervals. The point and interval estimates for the marginal delay distribution of the probe stream itself were also evaluated for comparison.

B. Delay Distributions in Individual Flows

This subsection explains that the CoMPACT Monitor can measure the marginal delay distributions for individual flows on the same network path by using a common sequence of the delays of probes. Here, the implementation in (16) was adopted and the value of δ was set at 40 ms to evaluate $\tilde{a}^-(T_n, \delta)$, $n = 1, 2, \ldots$; that is, the flow packets were counted for 40 ms when each probe arrived. The mean interval for probes introduced into the network was 500 ms. As a result, the influence on regular traffic was considered to be negligible because the extra traffic caused by the probe stream was only about 0.013% of the link capacity of 8 Mbps and 0.020% of the link capacity of 10 Mbps.

The estimates for the marginal delay distributions (complementary cumulative distribution functions: CDFs) of the #1, #6, #11, and #16 flows are in Figs. 6–9, representing the four different types of traffic in the 20 flows in Table I. The empirical marginal delay distributions for the flows computed directly using (1) from the given sample path for regular traffic are also displayed in these figures, together with the point and interval estimates derived from the CoMPACT Monitor and those for the probe streams. The estimates for the probe stream in Figs. 6 and 7 are common as are those in Figs. 8 and 9.

We can see that the CoMPACT Monitor can accurately evaluate the delay distributions for the flows (packet-average)

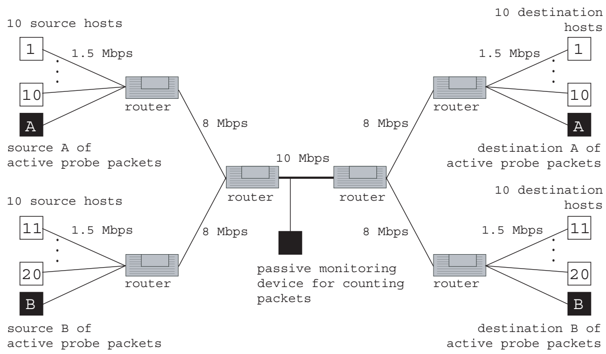

Fig. 5. Network model.

TABLE I
TRAFFIC MODEL.

Traffic type	Flow ID	Transport layer		Application layer				
		Protocol	Packet size	Mean ON period	Mean OFF period	distribution of ON/OFF length	Shape parameter	Rate at ON period
Type 1	##1–5	TCP	1.5 KB	10 s	5 s	exp	—	1 Mbps
Type 2	##6–10	TCP	1.5 KB	5 s	10 s	exp	—	1 Mbps
Type 3	##11–15	TCP	1.5 KB	5 s	15 s	Pareto	1.5	1.5 Mbps
Type 4	##16–20	TCP	1.5 KB	2 s	8 s	Pareto	1.5	1.5 Mbps

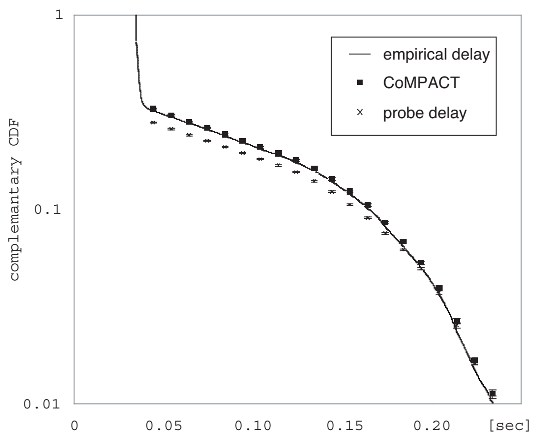

Fig. 6. Delay distribution estimates for flow #1 (Type 1).

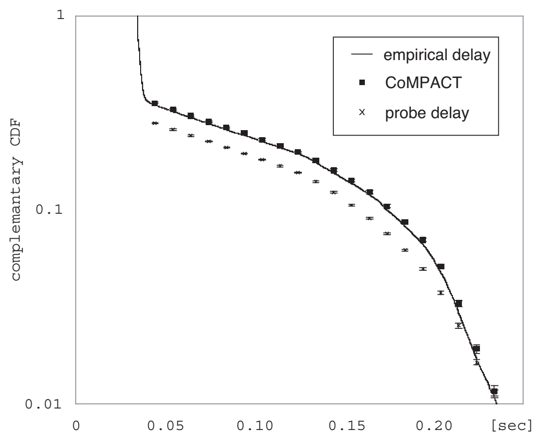

Fig. 7. Delay distribution estimates for flow #6 (Type 2).

according to their characteristics, while simple active measurement (time-average) cannot. In addition, we can see that the delay in the probe stream is smaller than that in each flow in distribution. This implies that $\{Y_n > c\}$ often occurs at the same time as $\tilde{a}^-(T_n, \delta) \geq m^{-1} \sum_{n=1}^{m} \tilde{a}^-(T_n, \delta)$ in (16); that is, the delays of probes tend to be long when there is a relatively large amount of traffic for the target flow.

C. Comparison of Implementations (15) and (16)

This subsection compares the two implementations of (15) and (16) in Sec. IV. The difference between these implementations is that between $\tilde{a}^+(T_n, \delta)$ and $\tilde{a}^-(T_n, \delta)$; that is, the period in which packets are counted occurs either just after or just before a probe arrives at the passive device (see Fig. 4). Flow #16 was chosen from the flows in Table I for the comparative experiments. The type of traffic that includes flow #16 (Type 4) has the lowest average rate and the most burstiness. Figure 10 has the estimates for the marginal delay distribution obtained by the two implementations with a mean probe interval of 500 ms and a δ of 40 ms. There are no significant differences between the two implementations, at least from this result.

D. Number of Active Probes

This subsection investigates the impact of the number of probes introduced into the network. Let us first consider the mean probe intervals. Flow #16 was again chosen as in the preceding experiment. Figures 11 and 12 have the estimates for marginal delay distributions using implementation (16) with mean probe intervals of 1000 ms and 250 ms, where $\delta = 40$ ms

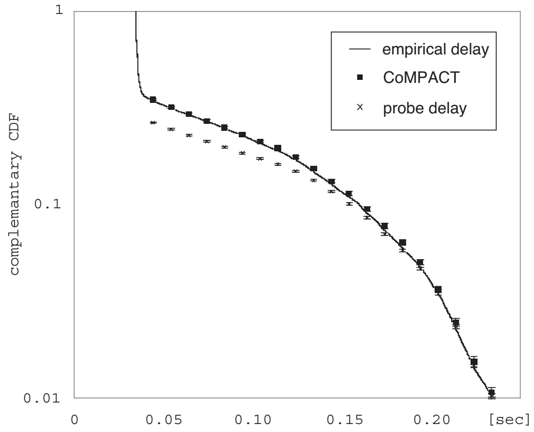

Fig. 8. Delay distribution estimates for flow #11 (Type 3).

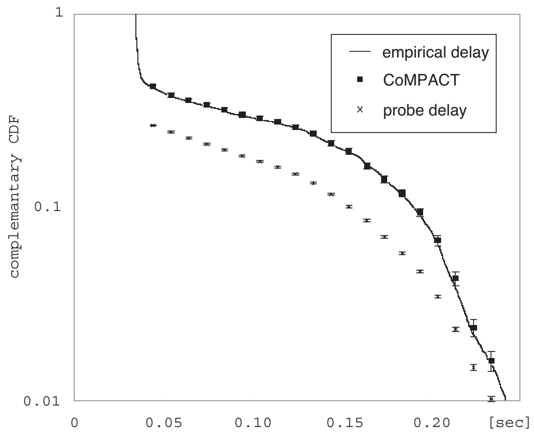

Fig. 9. Delay distribution estimates for flow #16 (Type 4).

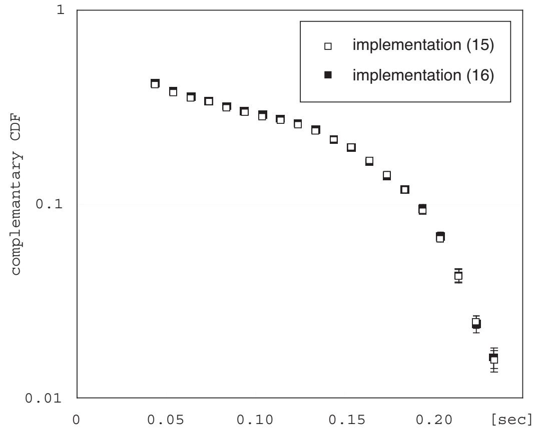

Fig. 10. Delay distribution estimates obtained by implementations (15) and (16).

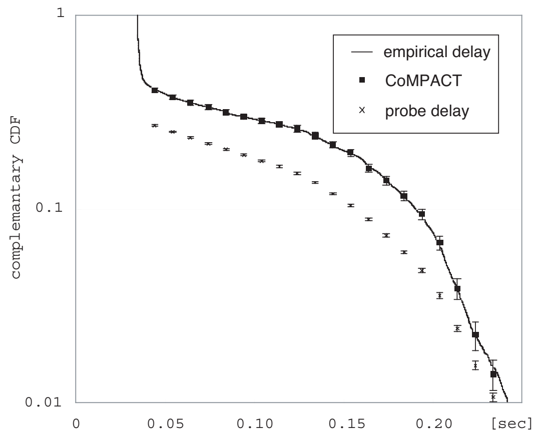

Fig. 11. Delay distribution estimates with mean probe interval of 1000 ms.

as the preceding experiment. The empirical delay distribution computed with (1) from the given sample path of regular traffic is also plotted in each figure. In each case, the extra traffic caused by the probes was negligible; that is, about 0.007% of the link capacity of 8 Mbps and 0.010% of the link capacity of 10 Mbps for the mean probe interval of 1000 ms. This is about 0.026% of the link capacity of 8 Mbps and 0.040% of the link capacity of 10 Mbps for the mean probe interval of 250 ms. Comparing Figs. 11, 12, and Fig. 9 for the mean probe interval of 500 ms, we can see that the shorter mean probe intervals yield more confident estimates (particularly looking at the distributional tails). However, we can surmise from the framework of the CoMPACT Monitor that the reason for this is not the mean interval itself but the number of sampling points by the probes.

To confirm this conjecture, let us next investigate the sample coefficients of variation (square root of sample variance divided by sample mean) in the estimates with respect to the run length. The implementation in (16) was again chosen with a mean probe interval of 500 ms and a δ of 40 ms. The sample coefficients of variation were computed from the 30 independent replicas of probe streams introduced into the common sample path of regular traffic. Here, flows #1 (Type 1) and #16 (Type 4) were selected since they had the largest and smallest average rates for the four types of traffic. It was found that flow #1 had about 4670 sampling points T_n with $\tilde{a}^-(T_n, \delta) > 0$ and #16 had about 560 over the 30 observations of independent probe traffic in common regular traffic for 3600 s. Based on this observation, two additional cases were examined for comparison. In the first, the sample coefficients of variation were computed with a short sample path which was part of the original such that flow #1 had about 560 sampling points T_n with $\tilde{a}^-(T_n, \delta) > 0$. In the second, a longer sample path was constructed such that flow #16 had about 4670 sampling points T_n with $\tilde{a}^-(T_n, \delta) > 0$, and the sample coefficients of variation were computed similarly. Both cases are plotted in Fig. 13, where the horizontal axis represents the point estimates for the marginal complementary delay distributions and the vertical axis represents the sample coefficients of variation. We can see that when the number of sampling points T_n with $\tilde{a}^-(T_n, \delta) > 0$ is almost the same, the coefficients of variation exhibit similar behavior regardless of the types of traffic. As a result, variations in estimates obtained with the CoMPACT Monitor depend on the number of sampling points T_n with $\tilde{a}^-(T_n, \delta) > 0$ as well as the delay-probability values, which indeed corresponds to the result of Proposition 1 in Sec. III.

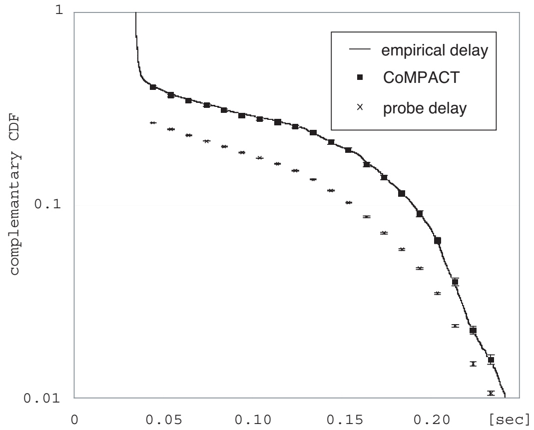

Fig. 12. Delay distribution estimates with mean probe interval of 250 ms.

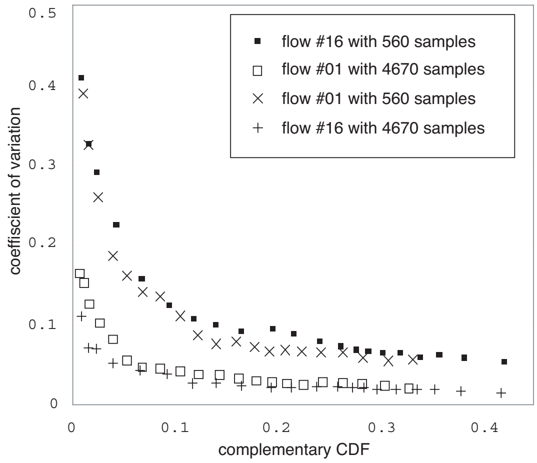

Fig. 13. Sample coefficients of variation.

E. Coefficients of Variation

Here, we compare the sample coefficients of variation obtained in the preceding subsection with the coefficients of variation derived from (14) to confirm the validity of variance in the estimates derived under a restricted assumption in Proposition 1. Under the assumption in Proposition 1, the coefficient of variation in the estimator (9) reduces to

$$\frac{\sqrt{\mathrm{Var}_N^0[Z_m(C|N)]}}{\mathrm{E}_N^0[Z_m(C|N)]} = \sqrt{\frac{1-\pi(C)}{\pi(C)} \mathrm{E}_N^0\left[\frac{1}{\sum_{n=1}^m 1_{\{a(T_n)>0\}}}\right]}. \tag{17}$$

To evaluate (17), $\pi(C)$ was replaced by the empirical marginal delay distributions for the flows computed with (1) from the given sample path of regular traffic, and $\mathrm{E}_N^0[1/\sum_{n=1}^m 1_{\{a(T_n)>0\}}]$ was replaced with the sample mean of $1/\sum_{n=1}^m 1_{\{\tilde{a}^-(T_n,\delta)>0\}}$ taken from the 30 independent replicas of probe streams introduced into common regular traffic.

Figure 14 has these estimates for the coefficients of variation for flows #1 and #16 over the 3600-s run. The horizontal and vertical axes are the same as those in Fig. 13 and the plots with sign "■" and those with "□" are also the same in both figures. This result indicates that variations in the estimates obtained with the CoMPACT Monitor can be accurately predicted by (14), despite being derived under the restricted assumption.

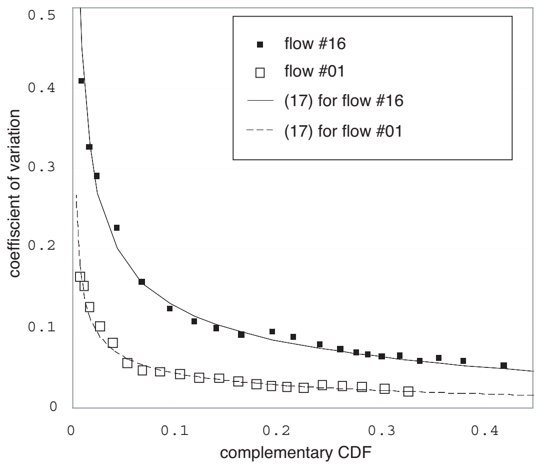

Fig. 14. Estimates for coefficients of variation.

F. Number of Flows

This subsection describes our investigations into the impact of the number of flows, particularly when there are many of these. An enhanced network model with 200 pairs of source and destination end hosts was used. Although the network configuration for the enhanced model is almost the same as that in Fig. 5, there are 50 flows for each type (five in the original model). The traffic model for each type of flow is the same as that described in Table I. Type 1 includes flows ##1–50, Type 2 includes ##51–100, Type 3 includes ##101–150 and Type 4 includes ##151–200. Each link capacity between the edge router and the core router is increased to 60 Mbps and that between the core routers is increased to 75 Mbps due to the greater number of flows (these are 8 Mbps and 10 Mbps in the original model). The length of the run was 1000 s in the experiment. The other conditions were the same as those described in Secs. V-A and V-B.

The estimates for the marginal delay distributions (complementary CDFs) of the three #151, #152, and #153 flows in Type 4 are in Figs. 15–17. The empirical marginal delay distributions for the flows computed using (1) from the given sample path of regular traffic are plotted in these figures, together with the point and interval estimates derived from the CoMPACT Monitor and those for the probe stream. The estimates for the probe stream are common in Figs. 15–17.

We can see that the CoMPACT Monitor can accurately evaluate the delay distributions for individual flows even if there are many of these. It is worth noting that the delay distributions for these flows are different even though their traffic characteristics are the same (Type 4). This implies that the simulation run has not reached the equilibrium state and that the CoMPACT Monitor works well for non-stationary traffic. In addition, contrary to the original 20-flow model, we can see cases where the delay in the probe stream is greater than that in the flow in distribution (see Fig. 16). This may be because one flow has a relatively small impact on the total

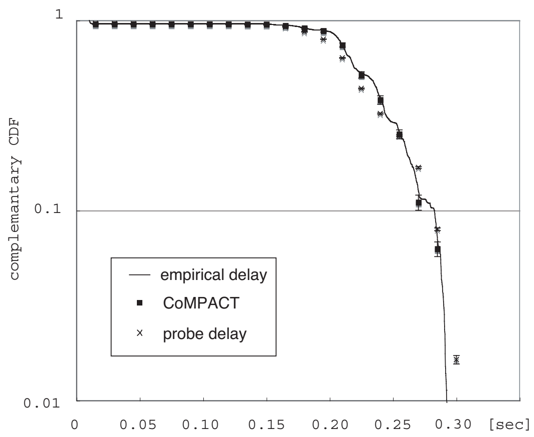

Fig. 15. Delay distribution estimates for flow #151.

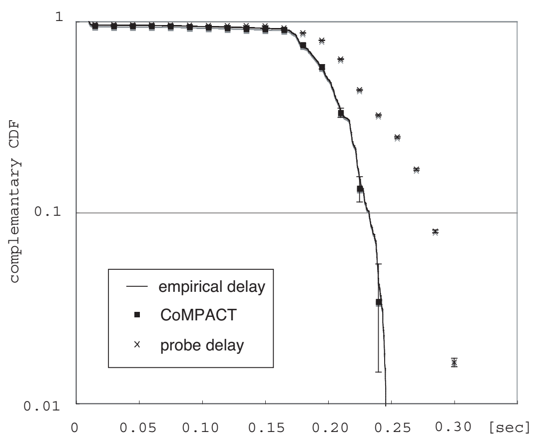

Fig. 16. Delay distribution estimates for flow #152.

amount of traffic when there are many flows, so probe delays can be prolonged when there is a large amount of total traffic, even when there is a small amount of traffic for the target flow. The experimental results demonstrate that the CoMPACT Monitor works well in such situations.

VI. Actual Network Experiments

Let us further characterize our technique through some experiments using a simple actual network. Here, we investigate the impact of neighborhood size δ; that is, the length of time for counting packets. The path from the PC of an asymmetric digital subscriber line (ADSL) customer to another PC in a company's LAN was used in the experiments, where both PCs were GPS-synchronized. The LAN for the ADSL customer and the company's LAN were connected via two ISPs and the path between the LANs consisted of 15 hops. 64 byte UDP packets were sent from the ADSL customer to the PC in the company's LAN according to a Poisson process with a mean interval of 20 ms and their one-way end-to-end delays were monitored for 900 s from about 10:00am on a Friday 2003. The maximum delay was 207 ms, the mean was 60 ms and the minimum was 20 ms.

The target flow traffic and the active probe traffic were artificially constructed as follows from these packet data for

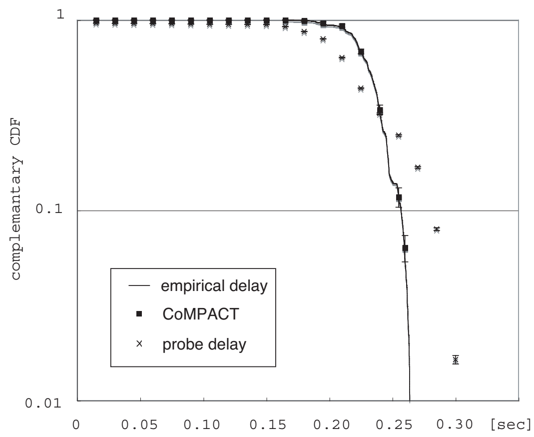

Fig. 17. Delay distribution estimates for flow #153.

900 s. When the delay of a packet was greater than 50 ms, it was regarded as a target flow packet with probability 0.6, otherwise, it was regarded as a target flow packet with probability 0.2. The end-to-end delay distribution was evaluated using this sequence of target flow packets. The probe packets, on the other hand, were randomly selected such as those with a mean interval of 500 ms (regardless of the overlap with target flow packets). Such probe-packet constructions were independently and identically repeated 30 times, and estimates obtained by implementation (16) were computed with 95% confidence intervals. The point and interval estimates for the end-to-end delay distribution of the probe stream itself were also evaluated for comparison.

The experimental results are in Figs. 18–20. The first two figures show the results for the CoMPACT Monitor with $\delta = 40$ ms and 100 ms. The last figure has the result for another implementation with packets counted during $[T'_{n-1}, T'_n)$, which corresponds to $\delta \to \infty$ considered in the previous works [4], [17]. The empirical end-to-end delay distribution for the target flow computed using (1) and the estimates for the end-to-end delay distribution of the probe stream are common in the three figures. Similar to the simulation experiments in Sec. V, we can see that simple active measurement cannot accurately estimate the delay distribution for the target flow although the CoMPACT Monitor can.

Next, we compare the relative errors,

$$\frac{\tilde{Z}^{-}((c,\infty) \mid N) - \tilde{\pi}(c,\infty)}{\tilde{\pi}(c,\infty)},$$

to study the impact of the neighborhood size δ, where $\tilde{\pi}(c,\infty)$ denotes the empirical delay distribution computed by (1) with $C = (c,\infty)$, and $\tilde{Z}^{-}((c,\infty) \mid N)$ denotes the estimate for the corresponding distribution obtained with the CoMPACT Monitor. Such estimates for the relative errors were computed with 95% confidence intervals taken from the 30 independent constructions of probe streams.

The horizontal and vertical axes in Fig. 21 represent the values for c and the relative errors. The absolute values of the relative errors are larger around 50 ms in all implementations, and this may be because the statistics for our artificial target flow traffic differed depending on whether the packet delay

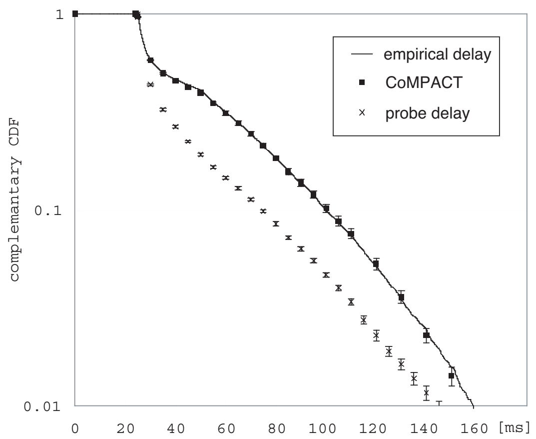

Fig. 18. End-to-end delay distribution estimates with $\delta = 40$ ms.

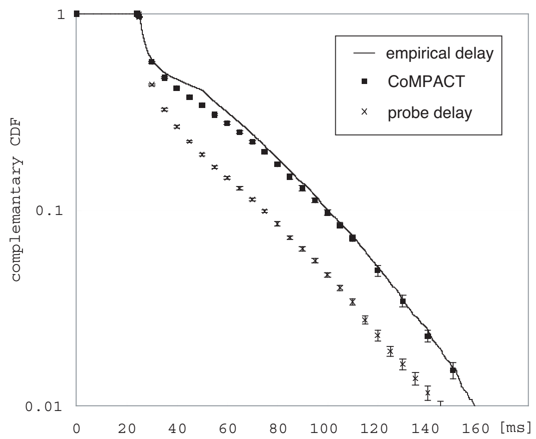

Fig. 20. End-to-end delay distribution estimates with $\delta \to \infty$.

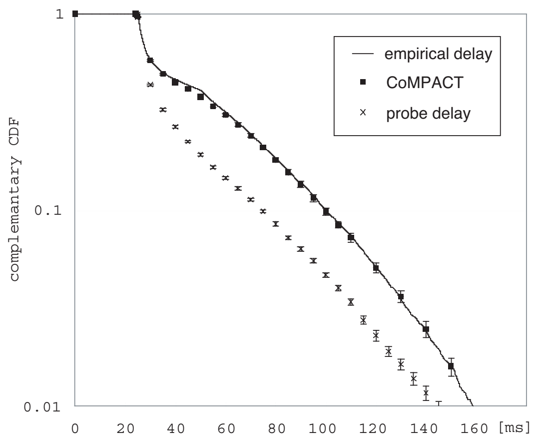

Fig. 19. End-to-end delay distribution estimates with $\delta = 100$ ms.

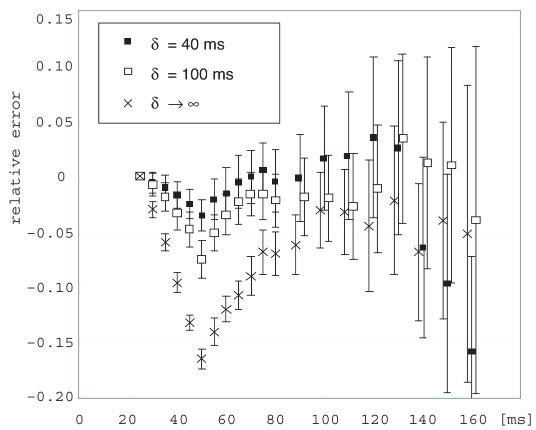

Fig. 21. Relative errors in estimates with respect to neighborhood sizes.

was greater or less than 50 ms. We can see that the smaller δ gives more accurate estimates when c is small. When c is large, on the other hand, the estimates with smaller δ have larger absolute values for relative errors. This may be because the smaller δ and larger c yield a smaller number of sampling points T_n satisfying both $\tilde{a}^{-}(T_n, \delta) > 0$ and $Y_n > c$. Furthermore, we need to note that, in all the three implementations, the variations in the estimates increase according to the value of c regardless of δ. This corresponds to that the standard deviation for the relative error of estimator (9) is also given by the right-hand side of (17) under the assumption in Proposition 1, where $(1 - \pi(c, \infty))/\pi(c, \infty)$ is nondecreasing in c (≥ 0).

VII. Conclusions

In this paper, we have presented the mathematical formulation for the CoMPACT Monitor under the quite general assumption of regular traffic. The characteristics and efficiency of our technique involving various implementation issues have also been investigated through simulation and actual network experiments, where we have found that the number of sampling points is crucial to ensure the quality of the estimates (although the required number is, of course, much smaller than that affecting regular traffic).

Although we have only discussed one-way delay distributions with respect to packets, the framework of the CoMPACT Monitor also applies to one-way delay distributions with respect to bytes if the passive monitoring device counts bytes in packets (instead of counting the number of packets). Furthermore, other than one-way delay, there are significant QoS parameters such as loss, round-trip delay, throughput and Web server workload experienced by a flow. The concept of the CoMPACT Monitor can be applied to measuring the packet-loss rate and CPU utilization by Web servers, and one application to measuring packet loss has recently been reported [16]. Further extension to measuring other QoS parameters is expected and this has been left for future study.

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