Synchronization-Free Delay Tomography Based on Compressed Sensing

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***Abstract*—Delay tomography has so far burdened source and receiver measurement nodes in a network with two requirements: path establishment and clock synchronization between them. In this letter, we focus on the clock synchronization problem in delay tomography and propose a *synchronization-free delay tomography* scheme based on compressed sensing. The proposed scheme selects a path between source and receiver measurement nodes as the reference path, which results in a loss of equation in a conventional delay tomography problem. By utilizing compressed sensing, however, the proposed scheme becomes robust to the loss. Simulation experiments confirm that the proposed scheme works comparable to a conventional delay tomography scheme in a network with no clock synchronization between source and receiver measurement nodes.**

***Index Terms*—Delay tomography, compressed sensing, clock synchronization.**

I. Introduction

DELAY tomography means to estimate internal link delays in a network by means of measuring end-to-end path delays [1]. When an active measurement procedure is used, source measurement nodes transmit probe packets to receiver measurement nodes, and the end-to-end path delays are computed from the differences between the transmission and reception times of the probe packets.

So far, delay tomography has burdened source and receiver measurement nodes with two requirements: path establishment and clock synchronization between them. In this letter, we focus on the clock synchronization problem and propose a *synchronization-free delay tomography* scheme. Although there have been several works for the path establishment problem [2], to the best of the authors' knowledge, the clock synchronization problem has not been studied so far.

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The proposed scheme utilizes compressed sensing, which is a promising technique because it can reduce the number of paths between source and receiver measurement nodes [3], and identifies bottleneck links without any clock synchronization mechanism between them. In the proposed scheme, we construct a *differential routing matrix* by setting a path between source and receiver measurement nodes as the reference path, so it results in a loss of equation for the delay tomography problem. Since compressed sensing is robust to this problem, however, the proposed scheme works comparable to a conventional delay tomography scheme in networks with no clock synchronization between source and receiver measurement nodes.

The proposed scheme has a significant benefit in various network environments especially in wireless networks such as wireless sensor networks, in which electronic components of nodes are sometimes too untrustable to meet the requirement of clock synchronization in terms of accuracy and complexity.

II. Preliminary for Compressed Sensing

First, we define the ℓ_p norm $(p \geq 1)$ of a vector $\mathbf{x} = [x_1\ x_2 \ldots x_N]^\top \in \mathcal{R}^N$ as

$$\|\mathbf{x}\|_p = \left(\sum_{i=1}^{N} |x_i|^p\right)^{\frac{1}{p}} \tag{1}$$

where $\top$ denotes the transpose operator.

Now, we assume that through a matrix $\mathbf{A} \in \mathcal{R}^{M \times N} (M < N)$, a vector $\mathbf{y} = [y_1\ y_2 \ldots y_M]^\top \in \mathcal{R}^M$ is obtained for a vector $\mathbf{x}$ as $\mathbf{y} = \mathbf{A}\mathbf{x}$. When utilizing compressed sensing, whether or not one can recover a sparse vector $\mathbf{x}$ from $\mathbf{y}$ depends on the mathematical property of $\mathbf{A}$. Here, we define the mutual coherence $\mu(\mathbf{A})$, which can provide guarantees of the recovery of the sparse vector, as

$$\mu(\mathbf{A}) = \max_{1 \leq j, j' \leq N, j \neq j'} \frac{\left|\mathbf{a}_j^\top \mathbf{a}_{j'}\right|}{\|\mathbf{a}_j\|_2 \|\mathbf{a}_{j'}\|_2} \tag{2}$$

where $\mathbf{a}_j$ and $\mathbf{a}_{j'}$ are the j-th and j'-th column vectors of $\mathbf{A}$, respectively. If

$$k < \frac{1}{2}\left(1 + \frac{1}{\mu(\mathbf{A})}\right) \tag{3}$$

then there exists at most one vector $\mathbf{x}$ which has at most k nonzero components [4].

III. Conventional Delay Tomography

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ denote an undirected network, where $\mathcal{V}$ and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ denote sets of nodes and links, respectively. In addition, let $s \in \mathcal{V}$ and $r \in \mathcal{V}$ denote source and receiver measurement nodes, respectively. Here, we define $\mathcal{W} = \{path_{s,r}^{(l)};\ l = 1, 2, \ldots, |\mathcal{W}|\}$ as a subset of all paths from s to r, where $path_{s,r}^{(l)} = \{(s, v_{s,r}^{(l,1)}), (v_{s,r}^{(l,1)}, v_{s,r}^{(l,2)}), \ldots, (v_{s,r}^{(l,|path_{s,r}^{(l)}|-1)}, r)\} \subset \mathcal{E}$ represents the l-th path in $\mathcal{W}$ and $v_{s,r}^{(l,m)} \in \mathcal{V} \backslash \{s, r\}(m = 1, \ldots, |path_{s,r}^{(l)}| - 1)$ are intermediate nodes on the path. Furthermore, we reformulate $\mathcal{W}$ and $\mathcal{E}$ as $\mathcal{W} = \{w_1, w_2, \ldots, w_I\}$ and $\mathcal{E} = \{e_1, e_2, \ldots, e_J\}$, respectively, where $I = |\mathcal{W}|$ and $J = |\mathcal{E}|$ denote the numbers of paths and links, respectively, and define d_{e_j} as the link delay over e_j $(j = 1, 2, \ldots, J)$. Finally, we define a binary matrix $\mathbf{A} \in \{0, 1\}^{I \times J}$ as the *routing matrix* of $\mathcal{W}$ (each row of the matrix is a path), i.e., its (i,j)-th component is set to $a_{ij} = 1$ if $e_j \in w_i$, and $a_{ij} = 0$ otherwise.

Conventional delay tomography has been discussed on ideal networks which have no clock synchronization error between s and r. In this case, a packet transmitted from s on a path $w_i (i = 1, 2, \ldots, I)$ is successfully received at r with a path delay $D_{w_i} = \sum_{e_j \in w_i} d_{e_j}$, so defining *measurement vector* $\mathbf{y} = [y_1\ y_2 \ldots y_I]^\top$ and *link delay vector* $\mathbf{x} = [x_1\ x_2 \ldots x_J]^\top$ as

$$y_i = D_{w_i} = \sum_{e_j \in w_i} d_{e_j}, \quad x_j = d_{e_j} \tag{4}$$

and by using $\mathbf{A}$, we naturally obtain the following matrix/vector equation:

$$\mathbf{y} = \mathbf{A}\mathbf{x}. \tag{5}$$

When we are interested in identification of a limited number of bottleneck links with larger delays, we can apply compressed sensing. Namely, by attributing the delays only to the bottleneck links, we can approximate the elements of $\mathbf{x}$ corresponding to smaller link delays to be zero, so we can assume that $\mathbf{x}$ is approximately a sparse vector.

To calculate the mutual coherence of $\mathbf{A}$, by picking up the j-th and j'-th column vectors from $\mathbf{A}$, we define the partial matrix as

$$\mathbf{A}_{jj'} = [\mathbf{a}_j \quad \mathbf{a}_{j'}]. \tag{6}$$

When we obtain $\tilde{\mathbf{A}}_{jj'}$ by swapping any two row vectors of $\mathbf{A}_{jj'}$, from (2), we can see $\mu(\tilde{\mathbf{A}}_{jj'}) = \mu(\mathbf{A}_{jj'})$. So by repeating the swap, $\mathbf{A}_{jj'}$ leads to

$$\widetilde{\mathbf{A}}_{jj'} = \begin{bmatrix} \mathbf{d}_{jj'} & \overline{\mathbf{d}}_{jj'} \\ \mathbf{s}_{jj'} & \mathbf{s}_{jj'} \end{bmatrix} \tag{7}$$

where $\mathbf{d}_{jj'}$ and $\mathbf{s}_{jj'}$ are row vectors with adequate dimensions, respectively, and $\overline{(\cdot)}$ denotes the bit-reverse operator. This means that, by changing the order of the elements of $\mathbf{a}_j$ and $\mathbf{a}_{j'}$, they can be rearranged into two column vectors which have different elements in the upper part whereas the same elements in the lower part.

If $1/3 \leq \mu(\mathbf{A}) < 1$, namely, $k = 1$, $\mathbf{A}$ is referred to as *1-identifiable* matrix. In the following, we assume that $\mathbf{A}$ is 1-identifiable. In this case, from (7), for $1 \leq j, j' \leq J, j \neq j'$

$$\dim(\mathbf{d}_{jj'}) \neq 0 \tag{8}$$

is held.

IV. Proposed Differential Delay Tomography

When the clock of s is not synchronized to that of r, a measured path delay is contaminated with a synchronization error. That is, defining the clock synchronization error vector as $\boldsymbol{\Delta} = [\delta_1\ \delta_2 \ldots \delta_I]^\top$, the real measurement vector $\mathbf{z} = [z_1 z_2 \ldots z_I]^\top$ should be written as

$$\begin{aligned} \mathbf{z} &= \mathbf{y} + \boldsymbol{\Delta} & (9) \\ &= \mathbf{A}\mathbf{x} & (10) \end{aligned}$$

where $\mathbf{y}$ is the true measurement vector. From (9) and (10), we can see that the conventional delay tomography scheme does not work at all unless $\boldsymbol{\Delta}$ is estimated.

Now, in order to get rid of the synchronization errors, let us define the n-th component z_n of $\mathbf{z}$ and the n-th row of $\mathbf{A}$ as the *reference component* and the *reference row*, respectively. By subtracting the reference component and the reference row from all the other components and all the other rows, respectively, we have a new *differential measurement vector* $\mathbf{z}^{(n)} \in \mathbf{R}^{I-1}$ and a new *differential routing matrix* $\mathbf{A}^{(n)} \in \{-1, 0, 1\}^{(I-1) \times J}$ as

$$\mathbf{z}^{(n)} = [z_1 - z_n\ z_2 - z_n \ldots z_I - z_n]^\top \tag{11}$$

$$\mathbf{A}^{(n)} = \begin{bmatrix} a_{11} - a_{n1} & a_{12} - a_{n2} & \ldots & a_{1J} - a_{nJ} \\ a_{21} - a_{n1} & a_{22} - a_{n2} & \ldots & a_{2J} - a_{nJ} \\ \vdots & \vdots & \ddots & \vdots \\ a_{i1} - a_{n1} & a_{i2} - a_{n2} & \ldots & a_{iJ} - a_{nJ} \\ \vdots & \vdots & \ddots & \vdots \\ a_{I1} - a_{n1} & a_{I2} - a_{n2} & \ldots & a_{IJ} - a_{nJ} \end{bmatrix}. \tag{12}$$

As a result, we have a new matrix/vector equation as

$$\mathbf{z}^{(n)} = \mathbf{A}^{(n)}\mathbf{x}. \tag{13}$$

Note that the link delay vector $\mathbf{x}$ does not change through the subtraction.

From (9), $\mathbf{z}^{(n)}$ is decomposed as

$$\mathbf{z}^{(n)} = \mathbf{y}^{(n)} + \boldsymbol{\Delta}^{(n)} \tag{14}$$

where

$$\begin{aligned} \mathbf{y}^{(n)} &= [y_1 - y_n\ y_2 - y_n \ldots y_I - y_n]^\top, & (15) \\ \boldsymbol{\Delta}^{(n)} &= [\delta_1 - \delta_n\ \delta_2 - \delta_n \ldots \delta_I - \delta_n]^\top, & (16) \end{aligned}$$

so $\mathbf{z}^{(n)}$ still contains some synchronization errors. However, if the following condition is held:

$$|y_i - y_n| \gg |\delta_i - \delta_n| \quad (i = 1, 2, \ldots, I,\ i \neq n), \tag{17}$$

we have

$$\mathbf{z}^{(n)} \approx \mathbf{y}^{(n)}, \tag{18}$$

so the synchronization errors disappear from (13). Note $\delta_i - \delta_n$ means the clock skew which is caused by the clock frequency deviation between s and r.

In the proposed scheme, the trade-off for the asynchronism is a loss of equation. However, compressed sensing is a method to obtain a unique solution from an underdetermined linear system, so it is still simply applicable for the proposed scheme.

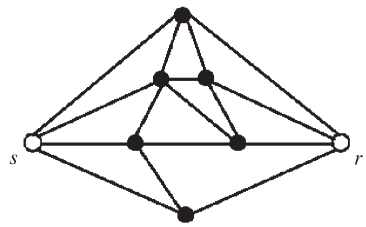

Fig. 1. Network Topology with 8 nodes and 16 links.

To discuss recoverability of a sparse vector $\mathbf{x}$ by means of the mutual coherence of $\mathbf{A}^{(n)}$, let us remind $\mathbf{A}$ is assumed to be 1-identifiable. Due to the limitation of the topology, if $\mathbf{A}$ has a rearranged partial matrix $\widetilde{\mathbf{A}}_{jj'}$ whose $\dim(\mathbf{d}_{jj'})$ equals I, then $\mu(\mathbf{A}^{(n)})$ always equals 1, that is, k of $\mathbf{A}^{(n)}$ always equals 0. Otherwise, $\mu(\mathbf{A}^{(n)})$ is still less than 1, regardless of the reference row, that is, there is no loss of capability in terms of the mutual coherence property given by (3).

Proof: See the Appendix A. ■

V. Performance Evaluations

A. Simulation Environment

Fig. 1 shows the network topology with 8 nodes and 16 links for the performance evaluation, where the clock of s is not synchronized to that of r. It is meaningful to compare the performance of the proposed scheme with that of the conventional scheme, so we assume that the clock of s is synchronized to that of r in the performance evaluation of the conventional scheme. We construct routing matrices in reference to the method in [2], and measure the path delays between s and r using an active measurement procedure. The probe packet transmission interval is defined as T_{prob}, and we set $T_{\text{prob}} = 1.0$ sec. In addition, we assume that the delay of a bottleneck link is constant with x^B whereas that of a normal link denoted by x^N is independent and identically distributed (i.i.d.) with average α_{x^N} and standard deviation σ_{x^N}. In this letter, we assume that all the nodes are wirelessly connected so x^N is Gaussian-distributed [5] with $\alpha_{x^N} = 15$ msec and $\sigma_{x^N} = 3$ msec [6], [7]. Furthermore, as the cause of the clock skew, we assume that each node has an i.i.d. clock frequency deviation of $\pm\Delta_{\text{dev}}$, and we set $\Delta_{\text{dev}} = 40$ ppm [8]. Finally, as an implementation of compressed sensing, we employ an $\ell_1 - \ell_2$ optimization [9].

In the following, we evaluate the delay tomography schemes in terms of two performance measures: *bottleneck link detection ratio* and *k-identifiability ratio*. The bottleneck link detection ratio is defined as the number of correctly detected bottleneck links divided by the total number of given bottleneck links, whereas the k-identifiability ratio is defined as $(R = N^{(k)}/{}_J C_k)$, where k and $N^{(k)}$ denote the number of bottleneck links and the number of the bottleneck link sets which can be identified from the routing matrix and the measurement vector, respectively.

B. Simulation Results

Table I shows three tested routing matrices. Their 1-identifiability ratios are guaranteed to be 1.0 according to their mutual coherences, but even when there is one bottleneck link in the network, if the link delay vector $\mathbf{x}$ cannot be regarded approximately as a sparse vector, the delay tomography schemes cannot correctly detect it. Therefore, first of all, we evaluate the sensitivity of the delay tomography schemes on the bottleneck delay. Fig. 2 shows the bottleneck link detection ratio versus the bottleneck delay for $k = 1$, where the number in the parenthesis attached to each routing matrix denotes its row size I. We confirmed that the performance of the proposed scheme is insensitive to the reference path selection, so the performance when selecting the first path as the reference $(n = 1)$ is shown in the figure. For both the proposed and conventional schemes, the bottleneck link detection ratio can be guaranteed to be 1.0 when x^B is more than 0.5 sec. Therefore, we set x^B to 0.5 sec, which corresponds to about 33.3 times as large as α_{x^N}.

Fig. 3 shows the k-identifiability ratio versus the number of bottleneck links. Note that (17) is held for the three differential routing matrices (see the Appendix B). From Fig. 3, we can see that the 1-identifiability ratio is guaranteed to be 1.0 for the proposed scheme using all the three tested matrices. Since the tested matrices have different row sizes which correspond to the numbers of path delay measurements, we can see that, as the number of path delay measurements increases, the performance is more improved for both the proposed and conventional schemes. For each of the numbers of path delay measurements, the proposed scheme performs comparable to the conventional scheme.

TABLE I
Tested Routing Matrices

Matrix	Size	Mutual Coherence
$\mathbf{P}$	8×16	0.816
$\mathbf{Q}$	10×16	0.816
$\mathbf{R}$	12×16	0.667

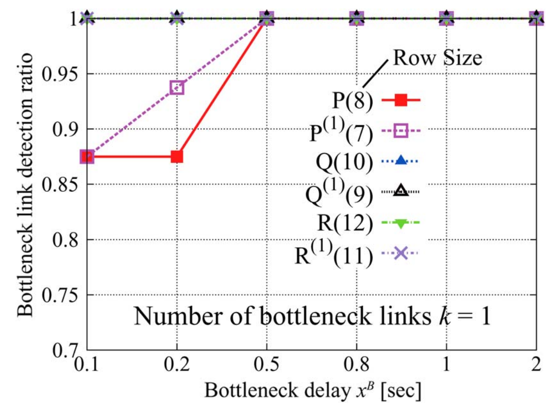

Fig. 2. The 1-identifiability ratio vs. the bottleneck delay.

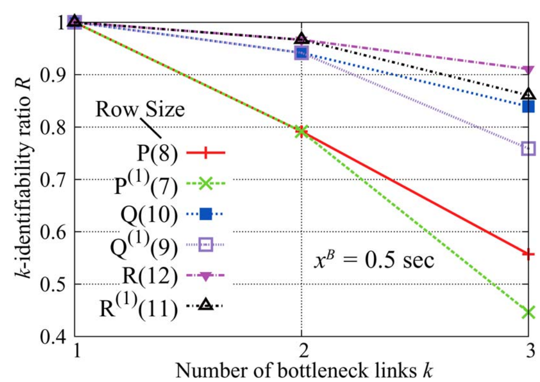

Fig. 3. The k-identifiability ratio R vs. the number of bottleneck links k.

Consequently, in the network, because of the clock asynchronism between source and receiver measurement nodes, we have to select a path as the reference in the proposed scheme, which leads to a loss of information. However, the information of the reference can be deleted not completely but partially, so the proposed scheme can effectively identify the bottleneck links.

VI. Conclusion

In this letter, we proposed a differential delay tomography scheme which enables us to infer link delays without clock synchronization between source and receiver measurement nodes in a network. We theoretically proved that the differential routing matrix preserves the mathematical property of the 1-identifiability of its original routing matrix and evaluated the performance of the proposed scheme by the simulation experiments. Furthermore, we confirmed that the clock offset can be canceled by the proposed scheme even when the clock frequency deviation exists between the source and receiver measurement nodes.

Some technical issues remain in the proposed scheme. Particularly, we have to reveal the connection between the capability of the proposed scheme and the reference selection, and propose an efficient reference selection method.

Appendix A

We define the n-th row of $\mathbf{d}_{jj'}$ and $\mathbf{s}_{jj'}$ as $d_{jj'}^{(n)} \in \{0,1\}$ and $s_{jj'}^{(n)} \in \{0,1\}$, respectively, and the matrices obtained by deleting the n-th row from $\mathbf{d}_{jj'}$ and $\mathbf{s}_{jj'}$ as $\mathbf{d}_{jj'}^{(-n)}$ and $\mathbf{s}_{jj'}^{(-n)}$, respectively. Incidentally, $\mathbf{A}^{(n)}$ represents the differential matrix of $\mathbf{A}$.

1) If $\dim(\mathbf{d}_{jj'}) = I$, that is, $\widetilde{\mathbf{A}}_{jj'} = [\mathbf{d}_{jj'} \ \overline{\mathbf{d}}_{jj'}]$:

$$\widetilde{\mathbf{A}}_{jj'}^{(n)} = \left[\mathbf{d}_{jj'}^{(-n)} - d_{jj'}^{(n)} \cdot \mathbf{1} \ \overline{\mathbf{d}}_{jj'}^{(-n)} - \overline{d}_{jj'}^{(n)} \cdot \mathbf{1}\right] = \begin{cases} \left[-\overline{\mathbf{d}}_{jj'}^{(-n)} \ \overline{\mathbf{d}}_{jj'}^{(-n)}\right] & \left(d_{jj'}^{(n)} = 1\right), \\ \left[\mathbf{d}_{jj'}^{(-n)} \ \ -\mathbf{d}_{jj'}^{(-n)}\right] & \left(d_{jj'}^{(n)} = 0\right). \end{cases}$$

Consequently, the two absolute column vectors of $\widetilde{\mathbf{A}}_{jj'}^{(n)}$ have no different element, so $\mu(\mathbf{A}^{(n)}) = 1$.

2) Otherwise:

a) If the n-th row is selected from $\mathbf{d}_{jj'}$ then

$$\widetilde{\mathbf{A}}_{jj'}^{(n)} = \begin{bmatrix} \mathbf{d}_{jj'}^{(-n)} - d_{jj'}^{(n)} \cdot \mathbf{1} & \overline{\mathbf{d}}_{jj'}^{(-n)} - \overline{d}_{jj'}^{(n)} \cdot \mathbf{1} \\ \mathbf{s}_{jj'} - d_{jj'}^{(n)} \cdot \mathbf{1} & \mathbf{s}_{jj'} - \overline{d}_{jj'}^{(n)} \cdot \mathbf{1} \end{bmatrix} = \begin{cases} \begin{bmatrix} -\overline{\mathbf{d}}_{jj'}^{(-n)} & \overline{\mathbf{d}}_{jj'}^{(-n)} \\ -\overline{\mathbf{s}}_{jj'} & \mathbf{s}_{jj'} \end{bmatrix} & \left(d_{jj'}^{(n)} = 1\right), \\ \begin{bmatrix} \mathbf{d}_{jj'}^{(-n)} & -\mathbf{d}_{jj'}^{(-n)} \\ \mathbf{s}_{jj'} & -\overline{\mathbf{s}}_{jj'} \end{bmatrix} & \left(d_{jj'}^{(n)} = 0\right). \end{cases}$$

b) If the n-th row is selected from $\mathbf{s}_{jj'}$ then

$$\widetilde{\mathbf{A}}_{jj'}^{(n)} = \begin{bmatrix} \mathbf{d}_{jj'} - s_{jj'}^{(n)} \cdot \mathbf{1} & \overline{\mathbf{d}}_{jj'} - s_{jj'}^{(n)} \cdot \mathbf{1} \\ \mathbf{s}_{jj'}^{(-n)} - s_{jj'}^{(n)} \cdot \mathbf{1} & \mathbf{s}_{jj'}^{(-n)} - s_{jj'}^{(n)} \cdot \mathbf{1} \end{bmatrix} = \begin{cases} \begin{bmatrix} -\overline{\mathbf{d}}_{jj'} & -\mathbf{d}_{jj'} \\ -\overline{\mathbf{s}}_{jj'}^{(-n)} & -\overline{\mathbf{s}}_{jj'}^{(-n)} \end{bmatrix} & \left(s_{jj'}^{(n)} = 1\right), \\ \begin{bmatrix} \mathbf{d}_{jj'} & \overline{\mathbf{d}}_{jj'} \\ \mathbf{s}_{jj'}^{(-n)} & \mathbf{s}_{jj'}^{(-n)} \end{bmatrix} & \left(s_{jj'}^{(n)} = 0\right). \end{cases}$$

From a) and b), consequently, the two absolute columns of $\widetilde{\mathbf{A}}_{jj'}^{(n)}$ still have the different element(s), namely, any two absolute column vectors of $\mathbf{A}^{(n)}$ are still not the same vectors, thus, $\mu(\mathbf{A}^{(n)}) < 1$.

Appendix B

Defining the clock frequency deviation between s and r and the arrival time of probe packet at r through the i-th path $(i = 1, 2, \ldots, I)$ as Δ_{sr} and T_i, respectively, the clock skew experienced in $z_i - z_n$ $(i = 1, 2, \ldots, I, \ i \neq n)$ is written as [5]

$$\delta_i - \delta_n = \Delta_{sr}(T_i - T_n).$$

Assuming that the clock frequency deviates uniformly in $[-\Delta_{\mathrm{dev}}, \ +\Delta_{\mathrm{dev}}]$, Δ_{sr} follows the triangular distribution $(-2\Delta_{\mathrm{dev}}, 0, +2\Delta_{\mathrm{dev}})$ [10]. Furthermore, assuming that probe packets can arrive anytime in one delay tomography session interval denoted by $T_{\mathrm{ses}}(= IT_{\mathrm{probe}})$, $T_i - T_n$ also follows the triangular distribution $(-T_{\mathrm{ses}}, 0, +T_{\mathrm{ses}})$. Therefore, $\delta_i - \delta_n$ follows the distribution of the product of the two triangular random variables, and for $\mathbf{R}$ with the largest $I(= 12)$, their average and standard deviation are calculated [10] as 0 msec and $1/6 \times 2\Delta_{\mathrm{dev}} T_{\mathrm{ses}} = 0.16$ msec, respectively.

On the other hand, from Fig. 1, $|y_i - y_n|$ can be the smallest when the i-th and n-th paths contain 2 and 2 normal links, respectively. In this case, $y_i - y_n$ follows the Gaussian distribution with average of 0 msec and standard deviation of $2\sigma_{x^N} = 6.0$ msec.

Consequently, comparing the averages and standard deviations between $y_i - y_n$ and $\delta_i - \delta_n$, respectively, *in the sense that the distribution of* $z_i - z_n = (y_i - y_n) + (\delta_i - \delta_n)$ *can be well approximated by that of* $y_i - y_n$, (17) is held.

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