

# Proposal and Evaluation of Method to Estimate Packet Loss-Rate Using Correlation of Packet Delay and Loss

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**SUMMARY** We previously proposed a change-of-measure based performance measurement method which combines active and passive measurement to estimate performance experienced by user packets and applied this to estimate packet delay. In this paper, we apply it to estimating loss rate. Since packets are rarely lost in current networks, rate measurement usually requires a huge number of probe packets, which imposes a non-negligible load on networks. We propose a loss-rate estimation method which requires significantly fewer number of probe packets. In our proposed method, the correlation between delay and loss is measured in advance, and at the time of measurement, the time-averaged loss rate is estimated by using the delay of probe packets and the correlation. We also applied our change-of-measure framework to estimating the loss rate in user packets by using this time-averaged loss rate. We prove that the mean square error in our method is lower than that simple loss measurement, which is estimated by dividing the number of lost packets by the total number of sent packets. We evaluated our method through simulations and actual measurements and found that it can estimate below  $10^{-3}$  packet loss rate with only 900 probe packets.

**key words:** packet loss rate, transmission delay, change of measure, QoS

## 1. Introduction

We previously proposed a lightweight and scalable method of measuring performance, *CoMPACT Monitor*, that combines active and passive measurement to estimate performance experienced by users. We also used it to estimate the delay in user packets [1], [2]. In this paper, we report the use of it to estimate the end-to-end loss rate for user packets.

Packet losses significantly degrade the QoS of UDP applications such as streaming or Voice over IP [3], [4]. TCP throughput also depends on a packet loss rate in a large bandwidth-delay product environment [5]. Therefore, it is necessary to measure the packet loss rate to manage the service level of these applications. In terms of managing service level, we need to know not only the stationary packet loss rate as an indicator of network performance, but also the time-varying loss rates for relatively short time periods (*e.g.*, duration of streaming videos), as service-level statistics.

In general, methods of measuring network performance can be divided into two types: passive and active. Passive methods capture user packets and deter-

mine network performance using their data. For example, we can detect the loss in packets by comparing two sets of time-series data captured with monitoring devices deployed at ingress and egress points on the network. By measuring the performance of the network passively, we can evaluate the performance experienced by users packets. However, these methods require the identification of each packet by header and/or content, which is difficult with the huge traffic volumes in high-speed networks. Furthermore, the passive devices may fail to capture all packets [6], which leads to errors in estimating the loss rate. A management information base (MIB) such as "IfOutDiscards" can be used to monitor the packet loss at the router, but cannot be used to obtain end-to-end loss rate.

Active methods, on the other hand, can simply be used to measure the performance of a network by sending probe packets and monitoring the delay or loss in these. However, there are two drawbacks with current active loss measurement.

First, when loss rate is low, active methods send a huge number of probe packets to detect packet losses that rarely occur. This imposes a non-negligible load on the network, especially when we measure the loss rate for a short period of time. While there are many tools for actively measuring packet loss rate [7], they only make use of information on lost probe packets and do not use other available packet information. With those methods, the required number of probe packets to measure the loss rate is roughly an order more than the reciprocal of the rate when losses occur independently. When there is a correlation between loss events, which has been reported in many Internet loss measurements [8]–[13], we do not need as many probe packets. Yet, the required number of probe packets is still large and those probe packets may perturb the networks [16]. Matsumoto et al. proposed a method of estimating packet loss rate through packet trains [14]. They used the increase in loss rate for successive packets in the train to estimate the loss rate of first packet, which is the time-average loss rate. Although their accuracy of estimating loss rate is expected to be improved beyond conventional methods, it also only uses information on lost probe packets, so there is room to improve accuracy by fully using all available information.

Second, the loss rate measured by probe packets

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may not be the same as that experienced by user packets [16]. If we assume that active monitoring measures the time average of network performance and that user traffic is Poissonian, then the performance experienced by users and actively measured performance will be the same. This is a well-known property called PASTA (which stands for “Poisson Arrivals See Time Average”) [17]. It is known, however, that current Internet traffic exhibits burstiness and is not Poissonian, in general [18]. In that case, more user packets are transmitted during congested periods, which means that more user packets experience a high loss rate. Thus, the loss rate experienced by users may actually be higher than that measured by those active monitoring. We previously proposed a method that can estimate the performance experienced by users by combining active and passive monitoring [1], [2]. We also applied this method to packet-delay estimation. However, if we simply apply it to estimating loss rate without considering the first drawback, the estimated loss rate may deviate from the actual loss rate when the number of probe packets is limited.

In this paper, we propose a method of estimating packet loss-rate using delay information in probe packets to overcome the first drawback. We use an intuitive expectation that when the delay of a probe packet is large, then the loss probability of user packets sent near the probe packet is high. We propose an estimator for the time-average loss rate that uses the correlation between loss and delay. As for using the delay measurement to estimate the loss rate, there is also a research that uses large deviation theory and estimates the buffer overflow probability (loss probability) by measuring the buffer occupancy (delay) [15]. However this scheme use the buffer size in the estimation, thus the size should be known, which is hard in current networks. Furthermore, the method can only be applied single-hop case. Our method does not require any network specific information, rather measures the information itself. Thus it can be applied to estimate the loss rate for multiple-hop path. We also combine the concept of *CoMPACT Monitor* and this estimator, and propose an estimator for the loss rate experienced by users. We prove that the mean square error (MSE) of our method is smaller than those of simple loss measurement. The method was also evaluated through simulations and actual measurements and those showed that it can estimate below  $10^{-3}$  packet loss rate with only 900 probe packets.

The rest of this paper is organized as follows. Section 2 presents the proposed loss measurement method. Simulations and actual measurement experiments are described in Sect.3. Finally, we give summary in Sect.4.

## 2. Proposed Measurement Method

Our method of estimating the user-packets loss rate involves the following two steps:

**Step 1:** Estimating the time-average loss rate using actively measured delay and loss.

**Step 2:** Converting the time-average loss rate to the loss rate experienced by user packets by using passively measured traffic intensity.

These steps are explained in more details in the next two subsections.

### 2.1 Time-Averaged Loss Rate

#### 2.1.1 Loss Rate Estimation Using Delay Information

The main difficulty in this step is measuring rare events with fewer samples (probe packets). We overcome this difficulty by using delay information about probe packets that are not lost.

The objective of this step, time-average loss rate during  $(t_1, t_2]$   $LR(t_1, t_2)$ , is defined as

$$LR(t_1, t_2) := \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \mathbf{1}_{\{V(t)=D_l\}} dt, \quad (1)$$

where  $\mathbf{1}_{\{\cdot\}}$  is the indicator function,  $V(t)$  is the virtual delay of the packet sent at  $t$ ,  $D_l$  is a value set larger than the maximum delay, and  $V(t) = D_l$  indicates the packet is lost. Here, we use the word “virtual” because there may not be any actual user packets at time  $t$  [17].

A simple estimator of  $LR(t_1, t_2)$  with  $n$  probe packets sent during  $(t_1, t_2]$ ,  $SELR(n)$ , is

$$SELR(n) := \frac{1}{n} \sum_{i=1}^n \mathbf{1}_{\{Y_i=D_l\}}, \quad (2)$$

where  $Y_i$  is the delay for  $i$ -th probe packet and  $Y_i = D_l$  indicates the packet is lost. We also define  $t_i$  as the transmission time for the  $i$ -th probe packet, so  $V(t_i) = Y_i$ . Actually, if probe packets are sent independently of  $V(t)$ , *i.e.* in a Poisson manner, (2) is a consistent estimator of (1). However, when seeing an individual estimation for a measurement of a short time period, it may be significantly different from  $LR(t_1, t_2)$  when the loss rate is low and the number of probe packets is limited. Now, we can intuitively expect that even if a probe packet is not lost, when the delay for the probe is large, then the loss rate around the time of the probe packet is high, as reported in [10]. Therefore, we propose an estimation method that uses the correlation between the delay of a packet and the loss of the neighboring packets.

First, let us define the conditional loss probability  $l_c(t, \tau, x)$  as follows:

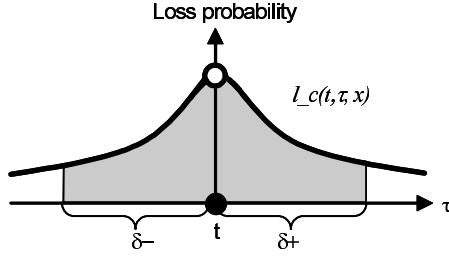


Fig. 1 Conditional loss probability  $l_c(t, \tau, x)$ .

$$l_c(t, \tau, x) := \Pr[V(t + \tau) = D_l | V(t) = x]. \quad (3)$$

For large delay  $x$ , the conditional loss probability of a packet sent near  $t$  is expected to be high and to decrease as  $|\tau|$  increases (Fig. 1). Here, we assume that  $l_c(t, \tau, x)$  is stationary (independent of  $t$ ), and denote this as  $l_c(\tau, x)$ .

Then, given the delay of the  $i$ -th probe packet,  $Y_i$ , we obtain the unconditional loss probability for time  $t_i + \tau$  as  $l_c(\tau, Y_i)$ . Therefore, the loss rate in the neighborhood of  $t_i$ ,  $(t_i - \delta_-, t_i + \delta_+)$  ( $\delta_-, \delta_+ \geq 0$ ), is estimated by

$$\frac{1}{\delta_+ + \delta_-} \int_{-\delta_-}^{\delta_+} l_c(\tau, Y_i) d\tau. \quad (4)$$

While  $\mathbf{1}_{\{Y_i = D_l\}}$  in (2) only takes 0 or 1, (4) takes value from 0 to 1 even if  $Y_i = D_l$ .

The expectation of (4) in terms of  $Y_i$  agrees with  $E[LR(t_i - \delta_-, t_i + \delta_+)]$  because

$$\begin{aligned} E \left[ \int_{-\delta_-}^{\delta_+} l_c(\tau, Y_i) d\tau \right] &= \int_{-\delta_-}^{\delta_+} E[l_c(\tau, Y_i)] d\tau \\ &= \int_{-\delta_-}^{\delta_+} \int_0^{D_l} l_c(\tau, x) dF_i(x) d\tau \\ &= \int_{-\delta_-}^{\delta_+} \Pr[V(t_i + \tau) = D_l] d\tau \\ &= \int_{-\delta_-}^{\delta_+} E[\mathbf{1}_{\{V(t_i + \tau) = D_l\}}] d\tau \\ &= E \left[ \int_{t_i - \delta_-}^{t_i + \delta_+} \mathbf{1}_{\{V(\tau) = D_l\}} d\tau \right], \quad (5) \end{aligned}$$

where  $F_i(x) = \Pr[V(t_i) \leq x]$ . Therefore, using conditional probability  $l_c(\tau, x)$  and  $n$  measurements of probe packets  $\{Y_1, Y_2, \dots, Y_n\}$  sent in  $(t_1, t_2]$ , we obtain another estimator for  $LR(t_1, t_2)$  by changing  $\mathbf{1}_{\{Y(i) = D_l\}}$  in  $SELR$  to (4) as

$$ELR(n, \delta_+, \delta_-) := \frac{1}{n} \sum_{i=1}^n \frac{1}{\delta_+ + \delta_-} \int_{-\delta_-}^{\delta_+} l_c(\tau, Y_i) d\tau. \quad (6)$$

Compared to (2) which only uses the loss rate in timing the probe packet transmission, the estimator (6) is expected to be more accurate because it uses the

loss rate of the neighborhood of the probe packets. By using the delay in probe packets and the correlation between the delay and loss probability. Actually, we can show that the mean square error in the proposed method is smaller than that in simple loss rate estimation (2). Let  $R(x)$  be  $R(x) := \int_{-\delta_-}^{\delta_+} l_c(t, x) dt / (\delta_+ + \delta_-)$  and  $f_1(D_l)$  be the stationary loss probability, i.e.,  $f_1(D_l) := E[\mathbf{1}_{\{V(0) = D_l\}}] = 1 - F_1(D_l -)$ . Then, the difference between the mean square error (MSE) of the proposed method and MSE of the simple estimation for one probe packet case is given as

$$\begin{aligned} &E[(SELR(1) - LR(1, -\delta_-, \delta_+))^2] \\ &- E[(ELR(1) - LR(-\delta_-, \delta_+))^2] \\ &= f_1(D_l) (1 - R(D_l))^2 + \int_0^{D_l -} R(x)^2 dF_1(x) \geq 0. \quad (7) \end{aligned}$$

The proof is in the Appendix. Errors in multiple packets can roughly be obtained by dividing the MSE by the number of probe packets if we can assume that error between probe packets is independent. Equation (7) indicates that the error in our estimator is smaller than or equal to that of a simple loss estimator. The first term equals zero if the mean packet loss probability near the lost probe packet is one. The second term is zero if the mean loss probability near a probe packet is zero even when the delay of the probe packet is large. Although packet loss on the Internet exhibits a high degree of burstiness [11], a situation where both terms equal zero can hardly be expected, especially for a relatively large  $\delta$ .

### 2.1.2 Conditional Loss Probability

Our estimator (6) requires conditional loss probability  $l_c(\tau, x)$ . This can be obtained through either preprocessing or in an online way. In preprocessing, probe packets are sent with short intervals, and we can take the loss and delay pairs of the interval. The interval should be sufficiently short that the integral in (6) can be approximated by the sum of the intervals. Then,  $l_c(\tau, x)$  can be calculated using the loss and delay pairs. Of course, the accuracy of the estimate depends on the number of pairs used to calculate conditional probability. We will discuss this in Sect. 3.2.2. We expect this probability remain almost unchanged in a network path during a period of time that can be considered as stationary (such as a busy period). Thus, once the  $l_c(\tau, x)$  of a period of time has been obtained, we can use this for other measurements done in the same period of time. However, to cope with gradual changes in network conditions, introducing an online update of  $l_c(\tau, x)$  may be better. In that case the delay and loss pairs for different intervals can be obtained by sending probe packets with different intervals (e.g. Poisson process), while simultaneously measuring the loss rate. Then,  $l_c(\tau, x)$  is updated with these delay and loss pairs,

for example, by taking a moving average.

## 2.2 User-Packet Loss Rate

This step uses our previously proposed method to estimate the delay in user packets [1], [2]. In this subsection, we present a brief review of it and its combination with the results obtained in Step 1. The mathematical foundation of the method can be found in [1], [2].

Our proposed change-of-measure based network performance measurement method, *CoMPACT Monitor*, was aimed at estimating the network performance experienced by users. To do this, it actively measures the performance through probe packets, and passively measures the number of user packets sent near the probe packets. Then, by changing the measure for the performance of probe packets to that of user packets, we can obtain an estimator of network performance experienced by users.

The objective of this step is to estimate the loss rate experienced by user packets sent in  $(t_1, t_2]$ ,  $ULR(t_1, t_2)$ :

$$ULR(t_1, t_2) = \frac{\int_{t_1}^{t_2} \mathbf{1}_{\{V(t)=D_l\}} dA(t)}{\int_{t_1}^{t_2} dA(t)}, \quad (8)$$

where  $A(t)$  is the arrival process for user packets. Here, the denominator represents the number of user packets sent in  $(t_1, t_2]$  and the numerator represents the number of lost user packets.

Modeling  $A(t)$  as a fluid, we demonstrated that an empirical distribution of the delay for any user fluid  $A(t)$  could be obtained using actively and passively measured values [1]. Let  $a(i)$  be user traffic intensity at  $t_i$ , i.e.,  $dA(t)/dt$  at the  $i$ -th active measurement timing. The  $a(i)$  is obtained through passive measurement by counting the number of user packets. Assuming that measurement timing is stationary, it can be proved for any  $D \in \mathbf{R}_+$  that

$$\begin{aligned} & \lim_{t_2 \rightarrow \infty} \frac{\int_{t_1}^{t_2} \mathbf{1}_{\{V(t)>D\}} dA(t)}{\int_{t_1}^{t_2} dA(t)} \\ &= \lim_{n \rightarrow \infty} \frac{\sum_{i=1}^n \mathbf{1}_{\{Y_i>D\}} a(i)}{\sum_{i=0}^n a(i)} \text{ a.s.} \end{aligned} \quad (9)$$

Therefore, by setting  $D$  to  $D_l$ ,  $a(i)$  as the number of user packets sent in  $(t_i - \delta_-, t_i + \delta_+]$ , and  $u$  as the  $\sum_{i=1}^n a(i)$ , we obtain an estimator of (8) by simply applying the change-of-measure method as

$$SEULR(n) := \frac{1}{u} \sum_{i=1}^n \mathbf{1}_{\{Y_i=D_l\}} a(i). \quad (10)$$

Compared with (2), the estimator (10) weights the event  $Y(i) = D_l$  by  $a(i)$  to convert the time-average loss rate to the user-experienced loss rate. *SELR* has been proved to agree with *ULR* if the measurement

lasts long enough. However, because it assumes that user packets sent near the lost probe packet will all be lost, otherwise no user packets are lost, it shares the same problem as (2).

Therefore, by combining the results of Step 1 and the change-of-measure based method, we propose an estimator to determine the loss rate for user packets  $EULR(n, \delta_+, \delta_-)$  as

$$\begin{aligned} & EULR(n, \delta_+, \delta_-) \\ &:= \frac{1}{u} \sum_{i=1}^n \frac{a(i)}{\delta_+ + \delta_-} \int_{-\delta_-}^{\delta_+} l_c(\tau, Y_i) d\tau. \end{aligned} \quad (11)$$

This estimator is obtained by changing  $\mathbf{1}_{\{Y_i=D_l\}}$  in (10) to  $\frac{1}{\delta_+ + \delta_-} \int_{-\delta_-}^{\delta_+} l_c(\tau, Y_i) d\tau$  the same as the relationship between (2) and (6).

## 3. Evaluation

### 3.1 Simulation Results

We evaluated our method through *ns-2* simulator [19] for the network in Fig. 2. User sources generate exponential On-Off traffic, whose means are 1 s and 14 s, respectively. During the On period, 1,000-byte packets were sent with an exponentially distributed interval with a mean of 10.6 ms. Thus, each source sent packets at 50 kbps on average. We varied the number of sources as 100, 140, and 180 so that the utilization rate of the intermediate link was 0.5, 0.7, and 0.9, respectively. The buffer capacity of the router was set to 50 packets.

To obtain conditional loss probability,  $l_c(\tau, x)$ , we first ran a 50,000-s simulation in advance where 64-byte probe packets were sent with an exponential interval with a mean of 10 ms. Extra traffic caused by these probe packets is about 50 Kbps.

This Poisson sampling was recommended in [20] to measure unbiased statistics. The  $l_c(\tau, x)$  were calculated using the loss and delay information of these probe packets. After calculating  $l_c(\tau, x)$ , we ran 10 simulations each lasting 1,000 s and calculated *ELR* and *EULR*. In each run, probe and user packets were sent in the same way as in the first 50,000-s simulation. Time-average loss rate, *LR*, was calculated using all probe packets, and user-average loss rate, *ULR*, was calculated using all 1,000-byte packets. To estimate loss rate, we only used one probe packet per 100 probe

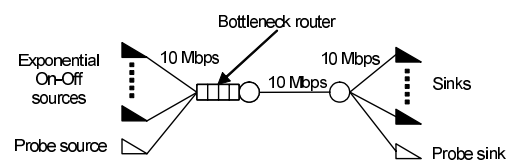
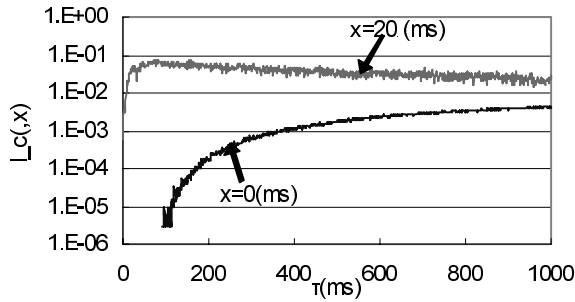
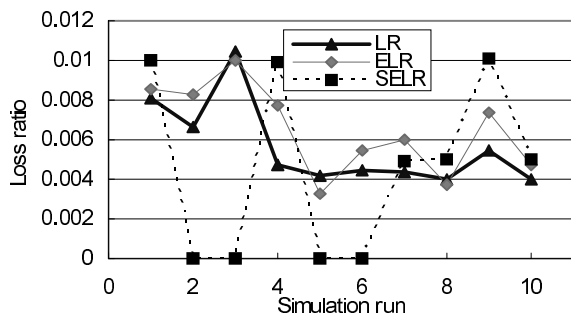


Fig. 2 Network configuration for simulation.



**Fig. 3** Conditional loss probability ( $l_c(\tau, x)$ ) for  $x = 0$  and 20 (ms).

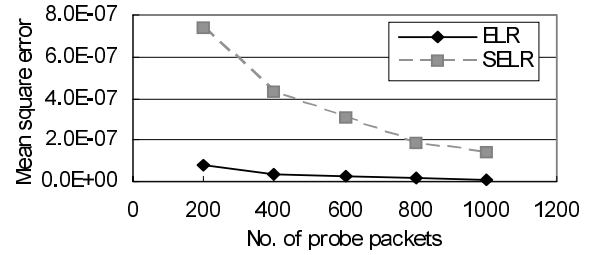


**Fig. 4** Loss rate and its estimation for ten simulations.

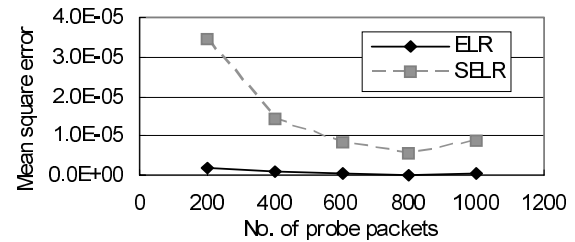
packets. In other words, we only sent one probe packet every second on average during actual measurements. We set  $(\delta_+, \delta_-) = (100, 100)$  (ms) in this paper. The maximum delay was 29 ms. Figure 3 shows  $l_c(\tau, 0)$  and  $l_c(\tau, 20)$  with 140 hosts for the  $\delta_+$  side. As expected,  $l_c(\tau, 20)$  decreased as  $\tau$  increased. We can also see that  $l_c(\tau, 0)$  increased as  $\tau$  increased and converged to a constant, which is expected to be the time-average loss rate (unconditional loss probability).

### 3.1.1 Time-Averaged Loss Rate

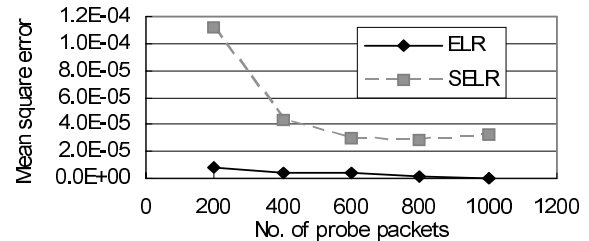
First, we tested *ELR*, the estimator for time-average loss rate. Figure 4 shows *LR* (time-average loss rate), *SELR* (estimates using only the loss information of probe packets), and *ELR* (estimates using both delay and loss information of probe packets) for ten simulations with 140 hosts. We see that while *SELRs* deviated from *LRs*, *ELR* could estimate *LR* with high accuracy. Note that while we used the same  $l_c(\tau, x)$  for all estimates, *ELR* could estimate different values for *LR* for ten simulations, by reflecting differences in probe delays. We can also see that no probe packets were lost in simulations 2, 3, 5, and 6, while *ELR* could estimate loss rate by using delay information. We also compared mean square errors,  $E[(ELR - LR)^2]$  and  $E[(SELR - LR)^2]$  by varying  $t_2 = 200, 400, 600, 800$ , and 1,000 s (we fixed  $t_1$  at 0). Figures 5-7 show the results for the case of 100, 140, and 180 hosts, respectively. The accuracy of our method was higher than



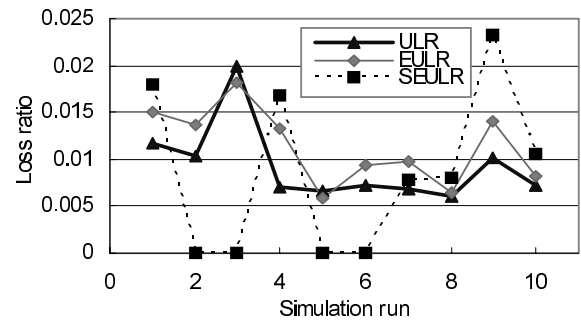
**Fig. 5** Mean square errors for time-average loss rate for 100 hosts.



**Fig. 6** Mean square errors for time-average loss rate for 140 hosts.



**Fig. 7** Mean square errors for time-average loss rate for 180 hosts.

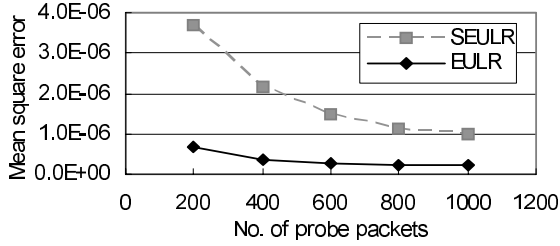


**Fig. 8** Loss and its estimations results for ten simulations.

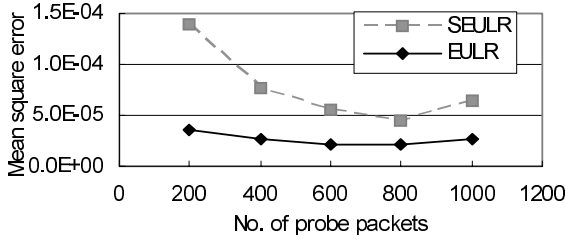
that for the simple loss rate estimation as (7).

### 3.1.2 User-Packet Loss Rate

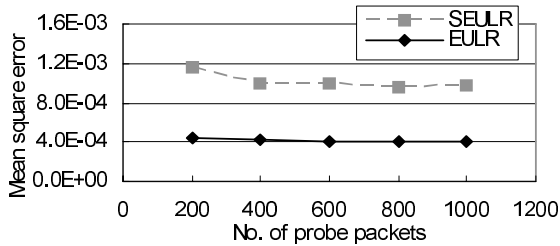
Next, we tested *EULR*, the estimator of the loss rate in user packets. Figure 8 shows the *ULR* (user-experienced loss rate), *SEULR* (estimates of *ULR* using only loss information of probe packets and number of user packets), and *EULR* (estimates of *ULR* using



**Fig. 9** Mean square errors for user-experienced loss rate for 100 hosts.



**Fig. 10** Mean square errors for user-experienced loss rate for 140 hosts.

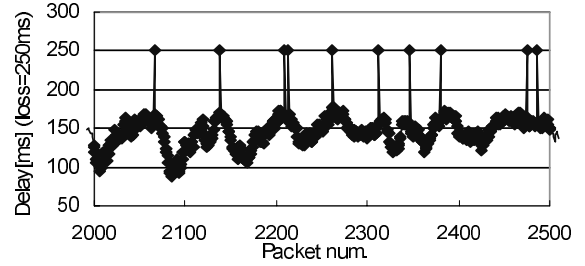


**Fig. 11** Mean square errors for user-experienced loss rate for 180 hosts.

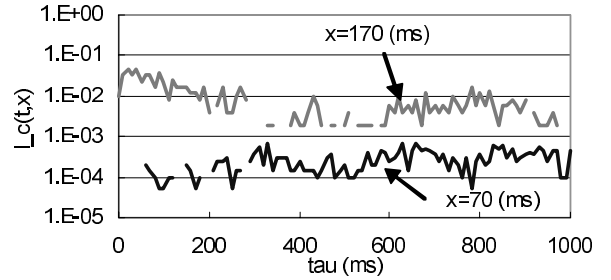
both delay and loss information of probe packets and number of user packets) for 140 hosts case. We can also see from this figure that our estimation could follow the loss rate for user packets, which was about 1%, *i.e.* higher than the time-average loss rate shown in Fig. 4. This was due to the correlation between the number of user packets and the loss rate, which was discussed in Sect. 1. By weighting the loss rate with the number of user packets sent near probe packets, our method could convert the time-average loss rate to an user-average loss-rate. Figures 9-11 show the mean square error for the case of 100, 140, and 180 hosts, respectively. Our estimator achieved a lower mean square error compared with the simple change-of-measure method.

### 3.2 Actual Measurement Results

We did end-to-end loss and delay measurements on an actual network to evaluate our method. We measured one-way delay and loss from an asymmetric digital subscriber line (ADSL) customer LAN to a company LAN during office hours (10:00-18:00) in February 2003.



**Fig. 12** Sample path of delay and loss in measurement (Losses are shown as 250 ms delay).



**Fig. 13** Conditional loss probability ( $L_c(\tau, x)$ ) for  $x = 70$  and  $170$  (ms).

The two LANs were connected via two ISPs, where the path consisted of 15 hops. The narrowest link along the path was the ADSL up-link, with a bandwidth of about 400 Kbps<sup>†</sup>. Here, we only evaluated the estimator for time-average loss rate because there were no user packets on the link.

We sent 64-byte UDP packets between two GPS-synchronized PCs in both LANs as a Poisson process where the mean interval was 20 ms. We ran thirty measurements each lasting 900 s and used the first twenty to calculate conditional loss probability, and the last ten to estimate loss rate. The maximum, mean and minimum delay were 207, 60, and 20 ms. The time-average loss rate for the whole measurement was 0.07%.

Figure 12 shows a sample path for the delay and loss of the probe packets (Here, loss is shown as 250 ms). We can see the fluctuations in delay and packet losses during some peaks of fluctuation. This indicates that there is a correlation between loss and delay even in the actual network. To provide more direct evidence of this correlation, Fig. 13 has conditional loss probability for delays of 70 ms and 170 ms (Broken parts of the line indicate that the measured conditional loss probability is zero). There is a clear correlation between loss and delay the same as in simulation.

#### 3.2.1 Time-Averaged Loss Rate

Figure 14 shows  $LR$ ,  $SELR$ , and  $ELR$  for 10 measure-

<sup>†</sup>During the measurements, because no packets except active probe packets were sent to the link, it could not cause queueing delays or losses for the probe packets on the link.

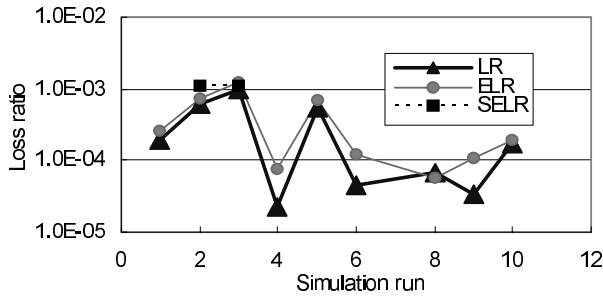


Fig. 14 Loss rate and its estimations for ten measurements.

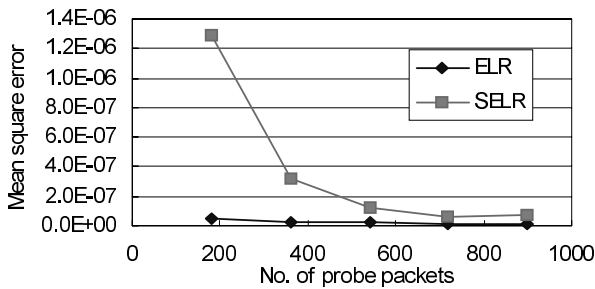


Fig. 15 Mean square errors for time-average loss rate.

ments where  $LR$  was calculated using all probe packets, and  $ELR$  and  $SELR$  was estimated by using one probe packets per 50 packets. In these simulations, as the loss rate varied from  $10^{-5}$  to  $10^{-3}$ , we plotted the loss rate semi-logarithmically. Except for measurements 2 and 3, no probe packets used for calculating  $ELR$  and  $SELR$  were lost, and the  $SELR$  was zero, which cannot be shown in the figure. Even so, our method could estimate time-average loss rate accurately especially for loss rates over  $10^{-4}$ . Figure 15 shows the mean square errors of proposed estimator  $ELR$  and simple loss estimator  $SELR$ . The errors for  $ELR$  are smaller than those for  $SELR$  for every number of probe packets. Because our method can estimate the average loss rate of few minutes, our method is well-adopted to Internet streaming applications such as movie trailers whose duration is reported as about few minutes [21].

### 3.2.2 Number of Pairs to Calculate the Conditional Loss Probability

To see the effect of the number of pairs used to calculate conditional loss probability to the accuracy of estimates, we varied the number of measurements to calculate  $l_c(\tau, x)$  and compared these with the mean square error of  $ELR$  (Fig. 16). We can see that with conditional loss probability calculated with ten measurements, the MSE converged to a constant. Thus, in this environment, 10,000-s measurements or 5,000,000 delay and loss pairs sufficed to calculate conditional loss probability.

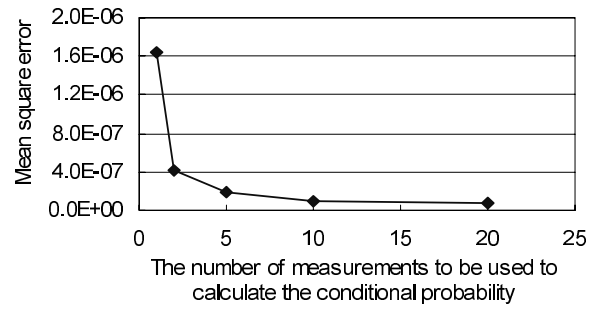


Fig. 16 Mean square errors varying the number of measurements to calculate the conditional loss probability.

## 4. Conclusion

We proposed a two-step method of estimating the loss rate in user packets that involved: 1) estimating the time-average loss rate and 2) converting that rate to the user-experienced loss rate. We used the delay in probe packets and the conditional loss probability given by their delay in Step 1. In Step 2, the time-average loss rate estimated in Step 1 was converted to the user-experienced loss rate through our previously proposed change-of-measure based method. Our method can be used to estimate both the time-average and user-experienced loss rate accurately with a limited number of probe packets. It can be used to determine the average loss rate over short periods of time such as duration of a streaming videos or phone calls.

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## References

- [1] M. Aida, N. Miyoshi, and K. Ishibashi, "A scalable and lightweight QoS monitoring technique combining passive and active approaches," Proc. IEEE INFOCOM 2003, pp.125–133, March 2003.
- [2] K. Ishibashi, T. Kanazawa, and M. Aida, "Active/passive combination-type performance measurement method using change-of-measure framework," Proc. IEEE GLOBECOM 2002, pp.2538–2542, Nov. 2002.
- [3] T.J. Kostas, M.S. Borella, I. Sidhu, G.M. Schuster, J. Grabiec, and J. Mahler, "Real-time voice over packet switched networks," IEEE Network, vol.12, no.1, pp.18–27, Jan./Feb. 1998.
- [4] D. Loguinov and H. Radha, "Large-scale experimental study of Internet performance using video traffic," ACM SIGCOMM Computer Communication Review, vol.32, no.1, pp.7–19, Jan. 2002.

- [5] J. Padhye, V. Firoiu, D. Towsley, and J. Kurose, "Modeling TCP throughput: A simple model and its empirical validation," Proc. ACM SIGCOMM '98, pp.303-314, 1998.
- [6] K. Papagiannaki, S. Moon, C. Fraleigh, P. Thiran, F. Tobagi, and C. Diot, "Analysis and measured single-hop delay from an operational backbone network," Proc. IEEE INFOCOM 2002, pp.908-921, June 2002.
- [7] "Caida: Internet measurement tool taxonomy," <http://www.caida.org/tools/taxonomy/index.xml>
- [8] V. Paxson, "End-to-end Internet packet dynamics," IEEE/ACM Trans. Netw., vol.7, no.3, pp.277-292, 1999.
- [9] J.-C. Bolot, "Characterizing end-to-end packet delay and loss in the Internet," Proc. ACM SIGCOMM'93, pp.289-298, Sept. 1993.
- [10] S.B. Moon, J. Kurose, and D. Towsley, "Correlation of packet delay and loss in the Internet," Technical Report 98-11, Department of Computer Science, University of Massachusetts, Amherst, MA, 2003.
- [11] M. Yajnik, S. Moon, J. Kurose, and D. Towsley, "Measurement and modelling of the temporal dependence in packet loss," Proc. INFOCOM '99, pp.345-352, March 1999.
- [12] M.S. Borella, D. Swider, S. Uludag, and G.B. Brewster, "Internet packet loss: Measurement and implications for end-to-end QoS," Proc. International Conference on Parallel Processing, pp.3-14, Aug. 1998.
- [13] K. Mochalski, J. Miceel, and S. Donnelly, "Packet delay and loss at the auckland Internet access path," Proc. PAM2002, March 2002.
- [14] Y. Matsumoto, K. Kawahara, and Y. Oie, "Proposal and evaluation of packet loss rate inference scheme by using packet train method for active measurement," IEICE Technical Report, IN2002-188, Feb. 2003.
- [15] S. Shioda and H. Saito, "Real-time cell loss ratio estimation and its applications to ATM traffic controls," Proc. IEEE INFOCOM '97, pp.1072-1079, April 1997.
- [16] G. Almes, S. Kalidindi, and M. Zekauskas, "A one-way packet loss metric for IPPM," RFC2680, Sept. 1999.
- [17] R.W. Wolff, Stochastic Modeling and the Theory of Queues, Prentice-Hall, 1988.
- [18] V. Paxson and S. Floyd, "Wide-area traffic: The failure of Poisson modeling," IEEE/ACM Trans. Netw., vol.3, no.3, pp.226-244, 1995.
- [19] UCB/LBNL/VINT Network Simulator - ns (version 2), <http://www.isi.edu/nsnam/ns>
- [20] V. Paxson, G. Almes, J. Mahdavi, and M. Mathis, "Framework for IP performance metrics," RFC2330, May 1998.
- [21] A. Koike, S. Takigawa, K. Takeda, A. Kobayashi, M. Morimoto, and K. Kawashima, "Characteristics of movie contents and its impact for traffic design," IEICE Trans. Commun., vol.E86-B, no.6, pp.1839-1848, June 2003.

## Appendix: Proof of Equation (7)

By conditioning the value of  $V(0)$ , we have

$$\begin{aligned}
 & E[(ELR(1, \delta_+, \delta_-) - LR(-\delta_-, \delta_+))^2] \\
 &= \int_0^{D_l} E \left[ \left( R(x) - \frac{\int_{-\delta_-}^{\delta_+} \mathbf{1}_{\{V(t)=D_l|V(0)=x\}} dt}{\delta_+ + \delta_-} \right)^2 \right] dF_1(x) \\
 &= \int_0^{D_l} \text{Var} \left[ R(x) - \frac{\int_{-\delta_-}^{\delta_+} \mathbf{1}_{\{V(t)=D_l|V(0)=x\}} dt}{\delta_+ + \delta_-} \right] dF_1(x)
 \end{aligned}$$

$$+ \int_0^{D_l} E \left[ R(x) - \frac{\int_{-\delta_-}^{\delta_+} \mathbf{1}_{\{V(t)=D_l|V(0)=x\}} dt}{\delta_+ + \delta_-} \right]^2 dF_1(x) \quad (\text{A.1})$$

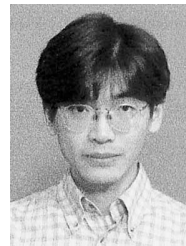
Because  $R(x)$  is constant if  $V(0)$  is specified, and is same as the mean of  $\int_{-\delta_-}^{\delta_+} \mathbf{1}_{\{V(t)=D_l|V(0)=x\}} / (\delta_+ + \delta_-)$ , we have

$$\begin{aligned}
 & E[(ELR(1, \delta_+, \delta_-) - LR(-\delta_-, \delta_+))^2] \\
 &= \int_0^{D_l} \text{Var} \left[ \frac{\int_{-\delta_-}^{\delta_+} \mathbf{1}_{\{V(t)=D_l|V(0)=x\}} dt}{\delta_+ + \delta_-} \right] dF_1(x). \quad (\text{A.2})
 \end{aligned}$$

The mean square error for  $SELR$  is also obtained by conditioning of the value  $V(0)$ . However, then,  $SELR(1)$  is either 1 ( $x = D_l$ ) or 0 (otherwise), so we can only consider the situation for  $x = D_l$  for the corresponding second term in (A.1). Then,

$$\begin{aligned}
 & E[(SELR(1) - LR(-\delta_-, \delta_+))^2] \\
 &= \int_0^{D_l} \text{Var} \left[ \frac{\int_{-\delta_-}^{\delta_+} \mathbf{1}_{\{V(t)=D_l|V(0)=x\}} dt}{\delta_+ + \delta_-} \right] dF_1(x) \\
 &+ f_1(D_l) + \int_0^{D_l} E \left[ \frac{\int_{-\delta_-}^{\delta_+} \mathbf{1}_{\{V(t)=D_l|V(0)=x\}} dt}{\delta_+ + \delta_-} \right]^2 dF_1(x) \\
 &- 2f_1(D_l) E \left[ \frac{\int_{-\delta_-}^{\delta_+} \mathbf{1}_{\{V(t)=D_l|V(0)=D_l\}} dt}{\delta_+ + \delta_-} \right] \\
 &= \int_0^{D_l} \text{Var} \left[ \frac{\int_{-\delta_-}^{\delta_+} \mathbf{1}_{\{V(t)=D_l|V(0)=x\}} dt}{\delta_+ + \delta_-} \right] dF_1(x) \\
 &+ f_1(D_l) + \int_0^{D_l} R(x)^2 dF_1(x) \\
 &- 2f_1(D_l) R(D_l). \quad (\text{A.3})
 \end{aligned}$$

Subtracting (A.3) from (A.2) yields equation (7).



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