Queueing Syst (2006) 52: 193–198
DOI 10.1007/s11134-006-5223-1

Tail asymptotics for the queue length in an M/G/1 retrial queue

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Received: 22 September 2004 / Accepted: 24 June 2005

Abstract In this paper, we study the tail behavior of the stationary queue length of an M/G/1 retrial queue. We show that the subexponential tail of the stationary queue length of an M/G/1 retrial queue is determined by that of the corresponding M/G/1 queue, and hence the stationary queue length in an M/G/1 retrial queue is subexponential if the stationary queue length in the corresponding M/G/1 queue is subexponential. Our results for subexponential tails also apply to regularly varying tails, and we provide the regularly varying tail asymptotics for the stationary queue length of the M/G/1 retrial queue.

Keywords M/G/1 retrial queue · Queue length · Subexponentiality · Regular variation · Tail asymptotics

AMS subject classifications: 60J25, 60K25

1. Introduction

Retrial queues have applications in a wide range of practical systems, such as telephone switching systems, cellular networks, and computer networks; and have drawn much attention in the research community in the last two to three decades. In addition to a large number of research papers published on retrial queues, the survey papers by Yang and Templeton [23], Falin [8], Kulkarni and Liang [14], and Li et al. [16], a book by Falin and Templeton [9], and a bibliography by Artalejo [1] testify their significance.

The seminal work by Leland et al. [15] in 1993 revealed that queues with heavy-tailed distributions may generate self-similar or long range dependent traffics in networks. Thereafter the asymptotic relation between the service times and performance measures of queueing systems, including queue length, busy period, and waiting time, became an important research topic. For heavy-tailed asymptotics of the stationary queue length, readers may refer to Resnick [19], Roughan et al. [20], Jelenković [12], Asmussen et al. [2], Foss and Korshunov [10], and Li and Zhao [17]. For asymptotic analysis of the busy period, one can find results in Meyer and Teugels [18], Zwart [24] and Boxma and Dumas [5]. For tail behavior of the waiting time, works by Erramilli et al. [7], Boxma and Cohen [4], Whitt [22] and Takine [21] can be noted.

The subexponential asymptotics for queue length in standard queues with subexponential service times have been discussed by Asmussen et al. [2] and Foss and Korshunov [10]. We extend the results in [2, 10] to a retrial system using a different approach. While Asmussen et al. [2] and Foss and Korshunov [10] obtained their results via distributional Little's law, we carry out the asymptotic analysis by showing that the subexponential tail of the M/G/1 queue determines that of the M/G/1 retrial queue using stochastic decomposition.

This paper consists of 5 sections. In Section 2, we introduce the M/G/1 retrial queue, and express the distribution of the stationary queue length. We present the definition of subexponentiality, and list some important properties which relate to our later discussion in this paper. In Section 3, we

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study the subexponentiality of the stationary queue length. Using stochastic decomposition, we show that the stationary queue length in an M/G/1 retrial queue is subexponential if that of the corresponding standard M/G/1 queue is subexponential, and the tails of the queue length in these two systems are asymptotically equivalent. It is clear that the results for subexponential tails can be applied to regularly varying tails, and we briefly discuss this conclusion in Section 4. In the last section, we briefly summarize our results and point out some future research directions.

2. Preliminary

In this section, we introduce the retrial queueing system and provide a few existing results needed in our later discussion.

2.1. M/G/1 retrial queue

Consider an M/G/1 retrial queue with a general service time distribution function $B(x)$. New customers arrive to the system in a Poisson stream with intensity λ. They are called primary calls. When a primary call enters the system, it is served immediately if the server is idle. Otherwise, it exits from the service area and waits in an orbit. While in orbit, each customer spends an exponential time with rate μ before re-visiting the server again. We call in-orbit customers repeated calls. When it arrives at the server, a repeated call behaves in the same manner as a primary call. A customer, repeated or primary, exits from the system immediately upon the completion of its service.

The M/G/1 retrial queue is stable if and only if $\rho = \lambda\beta_1 < 1$ (e.g., see chapter 1 in Falin and Templeton [9]), where β_1 is the mean service time. This stable condition implies that the mean service time $\beta_1 = -\beta'(0)$ must be finite, where $\beta(s)$ is the Laplace-Stieltjes transform (LST) of $B(x)$.

Let $N_\mu(t)$ be the total number of customers in the system (either in orbit or under service) at time t. We call $\{N_\mu(t), t \geq 0\}$ the queue length process. If the system is stable, we use N_μ to represent $N_\mu(t)$ as $t \to +\infty$. We use $q_m = P(N_\mu = m)$ to denote the stationary distribution of the queue length and $Ez^{N_\mu} = \sum_{m=0}^{\infty} q_m z^m$ to denote the corresponding probability generating function. It is worth noting that for the queue length process, the standard M/G/1 queue can be viewed as a special case of the M/G/1 retrial queue. When the retrial rate $\mu \to +\infty$, N_μ described above reduces to N_∞, which is the stationary queue length in the corresponding standard M/G/1 queue.

The following Lemma shows the relation between the stationary queue length of an M/G/1 retrial queue and that of a standard M/G/1 queue (see, for example, Theorem 1.3 in Falin and Templeton [9]).

Lemma 1 ((Stochastic Decomposition)). *The total number of customers in an M/G/1 retrial queue, N_μ, can be represented as the sum of two independent random variables: one is the total number of customers in the corresponding standard M/G/1 queueing system, N_∞, and the other is R_μ, with its distribution coincides with the conditional distribution of the number of repeated calls in orbit given that the server is free. That is,*

$$N_\mu = N_\infty + R_\mu. \tag{1}$$

Moreover, the probability generating functions of N_∞ and R_μ are given by

$$Ez^{N_\infty} = (1-\rho)\frac{1-z}{k(z)-z}k(z), \tag{2}$$

$$Ez^{R_\mu} = \exp\left\{\frac{\lambda}{\mu}\int_1^z \frac{1-k(u)}{k(u)-u}du\right\}, \tag{3}$$

respectively, where $k(z) = \beta(\lambda - \lambda z)$.

2.2. Subexponentiality

A distribution function $F(x)$ is heavy-tailed, if $\bar{F}(x) = 1 - F(x) > 0$ for $x \geq 0$, and

$$\lim_{x\to+\infty} \frac{\bar{F}(x+y)}{\bar{F}(x)} = 1, \quad \forall\, y \geq 0.$$

We use $\mathcal{L}$ to denote the class of heavy-tailed distributions. The attention of the research community has been focused on the subclass of subexponential distributions $\mathcal{S} \subset \mathcal{L}$ for heavy-tailed analysis. The definition below for subexponential distributions can be found in Goldie and Klüppelberg [11].

Definition 1. A distribution function $F(x)$ with support $[0, +\infty)$ is subexponential, if for all $n \geq 2$,

$$\lim_{x\to+\infty} \frac{\overline{F^{n*}}(x)}{\bar{F}(x)} = n,$$

where F^{n*} denotes the nth convolution of F.

Examples of subexponential distributions include heavy-tailed Weibull and regularly varying distributions introduced below.

Heavy-tailed Weibull: $\bar{F}(x) = e^{-\theta x^\alpha}$, $x \geq 0$, $\theta > 0$ and $0 < \alpha < 1$.

Regularly varying distributions: $\bar{F}(x) = x^{-\alpha}L(x)$ as $x \to +\infty$, for $\alpha > 0$ and some slowly varying function $L(x)$ that satisfies $\lim_{x\to+\infty} L(yx)/L(x) = 1$, $\forall\, y > 0$.

More examples can be found in Goldie and Klüppelberg [11]. The following two Lemmas are important in our later discussion. The first can be found in Theorem A3.20 in Embrechts et al. [6], and the second can be found in Lemma A3.23 in Embrechts et al. [6].

Lemma 2. *Suppose (p_m) is a probability measure on $\mathbb{N}_0$ such that $\sum_{m=0}^{\infty} p_m(1+\varepsilon)^m < +\infty$, for some $\varepsilon > 0$. We set*

$$G(x) = \sum_{m=0}^{\infty} p_m F^{m*}(x), \quad x \geq 0.$$

If $F \in \mathcal{S}$, then $G \in \mathcal{S}$, and

$$\lim_{x\to+\infty} \frac{\bar{G}(x)}{\bar{F}(x)} = \sum_{m=1}^{\infty} m p_m.$$

Lemma 3. *For two distribution functions $F(x)$ and $G(x)$ on $[0, +\infty)$; let $H = F * G$ be their convolution. If $F \in \mathcal{S}$ and $\bar{G}(x) = o(\bar{F}(x))$ as $x \to +\infty$, then $H \in \mathcal{S}$ with $\bar{H}(x) \sim \bar{F}(x)$ as $x \to +\infty$.*

For technical reasons, in studying subexponential distributions, we usually need to restrict this class further to the class of $\mathcal{S}^* \subset \mathcal{S}$, which was introduced by Klüppelberg [13].

Definition 2. Let $F(x)$ be a distribution function on $[0, +\infty)$ such that $\bar{F}(x) > 0$ for $x \geq 0$. Then $F \in \mathcal{S}^*$ if F has finite mean $1/\sigma$ and

$$\lim_{x\to+\infty} \int_0^x \frac{\bar{F}(x-y)}{\bar{F}(x)} \bar{F}(y)dy = \frac{2}{\sigma}.$$

For a standard M/G/1 queue, if the service time is subexponential, then the stationary queue length N_∞ may not be subexponential. Additional requirements on the distribution of the service time are needed to guarantee that the stationary queue length is subexponential, as shown in the following Lemma (see, for example, Proposition 3.3 in Asmussen et al. [2]). For a distribution F with finite mean $1/\sigma$, we define its equilibrium distribution as $F_e(x) = \sigma \int_0^x \bar{F}(y)dy$ for $x \geq 0$.

Lemma 4. *If the equilibrium distribution of the service time $B_e \in \mathcal{S}$, and satisfies*

$$\lim_{y\to+\infty} \frac{\bar{B}_e(ye^{x/\sqrt{y}})}{\bar{B}_e(y)} = \lim_{y\to+\infty} \frac{\bar{B}_e(y + x\sqrt{y})}{\bar{B}_e(y)} = 1, \quad \text{locally uniformly in } x \geq 0, \tag{4}$$

then $N_\infty \in \mathcal{S}$, and

$$P(N_\infty > m) \sim \frac{\rho}{1-\rho} \bar{B}_e(m/\lambda), \quad m \to \infty. \tag{5}$$

It is worth pointing out that if $B \in \mathcal{S}^*$ and $\bar{B}$ satisfies Eq. (4), then $B_e \in \mathcal{S}$ and $\bar{B}_e$ also satisfies Eq. (4). Hence by Lemma 4, it is easy to check that when the service time follows either a heavy-tailed Weibull distribution with $\alpha \in (0, 1/2)$ or a regularly varying distribution with $\alpha > 1$, then the stationary queue length of the standard M/G/1 queue is subexponential and its tail can be estimated using Eq. (5). Foss and Korshunov [10] derived similar conditions as follows to guarantee the subexponentiality of the stationary queue length in a standard M/G/1 queue.

Lemma 5. *Let $\gamma \in (1/2, 1]$ and $g(x) = -\log \bar{B}_e(x)$. For any function $d(x) = o(x^{1/2})$ such that $d(x) \to +\infty$ as $x \to +\infty$, if*

$$g(x + x^\gamma/d(x)) = g(x) + o(1), \quad x \to +\infty, \tag{6}$$

then $N_\infty \in \mathcal{S}$ and its tail can be estimated using Eq. (5).

3. Subexponentiality of the stationary queue length

In this section, we discuss subexponential asymptotics for N_μ, the total number of customers in the retrial queue system. For the stochastic decomposition $N_\mu = N_\infty + R_\mu$, it is interesting to discuss how the tails of the two variables N_∞ and R_μ determine that of N_μ. We aim at finding a sufficient condition for the stationary queue length to be subexponential, and our analysis is summarized in the following Theorem.

Theorem 1. *If $N_\infty \in \mathcal{S}$ (e.g., the conditions in Lemma 4 or Lemma 5 are satisfied), then $N_\mu \in \mathcal{S}$ and $P(N_\mu > m) \sim P(N_\infty > m)$ as $m \to \infty$.*

Proof: Let $p_m = P(N_\infty = m)$ for $m \geq 0$ and $Ez^{N_\infty} = y(z) = \sum_{m=0}^{\infty} p_m z^m$. Then from

$$y(z) = (1-\rho)\frac{1-z}{k(z)-z}k(z),$$

we obtain

$$k(z) = \frac{y(z)z}{y(z) - (1-\rho)(1-z)}.$$

Let $Ez^{R_\mu} = \exp(t(z))$, where

$$\begin{aligned} t(z) &= \frac{\lambda}{\mu}\int_1^z \frac{1-k(u)}{k(u)-u}du \\ &= \frac{\lambda}{\mu}\left[(1-z)+\int_1^z \frac{1-u}{k(u)-u}du\right] \\ &= \frac{\lambda}{\mu}\left[(1-z)+\int_1^z \frac{y(u)-(1-\rho)(1-u)}{(1-\rho)u}du\right] \\ &= \frac{\lambda}{\mu}\left[-\log z+\frac{1}{1-\rho}\int_1^z \sum_{m=0}^{\infty} p_m u^{m-1}du\right] \\ &= \frac{\lambda}{\mu(1-\rho)}\sum_{m=1}^{\infty} p_m \frac{z^m-1}{m}. \end{aligned}$$

Then

$$Ez^{R_\mu} = \sum_{n=0}^{\infty}\frac{1}{n!}\left[\frac{\lambda}{\mu(1-\rho)}\sum_{m=1}^{\infty} p_m\frac{z^m-1}{m}\right]^n. \tag{7}$$

Let V be a nonnegative random variable with $P(V=m)=v_m=p_m/m$ for $m\ge 1$, and $P(V=0)=v_0=1-\sum_{m=1}^{\infty} v_m>0$. We write $Ez^V=d(z)=\sum_{m=0}^{\infty} v_m z^m$, and $c=\lambda/\mu(1-\rho)$. From (7), we have

$$\begin{aligned} Ez^{R_\mu} &= \sum_{n=0}^{\infty}\frac{1}{n!}\left[c\,(d(z)-1)\right]^n \\ &= \sum_{n=0}^{\infty}\frac{c^n}{n!}\left[\sum_{m=0}^{n}\frac{n!}{m!(n-m)!}(-1)^{n-m}d^m(z)\right] \\ &= \sum_{m=0}^{\infty} d^m(z)\left[\sum_{n=m}^{\infty}\frac{c^n}{n!}\frac{n!}{m!(n-m)!}(-1)^{n-m}\right] \\ &= \sum_{m=0}^{\infty} d^m(z)\left[\sum_{l=0}^{\infty}\frac{c^{(m+l)}}{m!l!}(-1)^l\right] \\ &= \sum_{m=0}^{\infty}\frac{c^m d^m(z)}{m!}\left[\sum_{l=0}^{\infty}\frac{(-c)^l}{l!}\right] \\ &= \sum_{m=0}^{\infty}\frac{c^m e^{-c}}{m!}d^m(z). \end{aligned} \tag{8}$$

Let M denote a Poisson random variable with $P(M=m)=w_m=e^{-c}c^m/m!$ for $m\ge 0$. Then from Eq. (8), we can obtain

$$R_\mu = \sum_{n=1}^{M} V(n),$$

where $V(1), V(2), \ldots$ are i.i.d. as V. □

We denote $V^{(m)}=\sum_{n=1}^{m} V(n)$ for $m\ge 1$, and $N_\infty^{(m)}=\sum_{n=1}^{m} N_\infty(n)$ for $m\ge 1$, where $N_\infty(1), N_\infty(2);\ldots$ are i.i.d. as N_∞. It is easy to see that for $m>1$ and $l\ge 0$,

$$\begin{aligned} &P\big(V^{(m)}>l\big) \\ &= v_0P\big(V^{(m-1)}>l\big)+\sum_{n=1}^{l} v_nP\big(V^{(m-1)}>l-n\big)+P(V>l), \\ &P\big(N_\infty^{(m)}>l\big) \\ &= p_0P\big(N_\infty^{(m-1)}>l\big)+\sum_{n=1}^{l} p_nP\big(N_\infty^{(m-1)}>l-n\big)+P(N_\infty>l). \end{aligned}$$

Based on the above two equations, we obtain by induction,

$$P\big(V^{(m)}>l\big)\le \frac{d^{m-1}}{l+1}P\big(N_\infty^{(m)}>l\big), \quad m\ge 1,\ l\ge 0,$$

where $d=1/p_0+2$. We notice that

$$\frac{P(R_\mu>l)}{P(N_\infty>l)}=\frac{\sum_{m=1}^{\infty} w_mP\big(V^{(m)}>l\big)}{P(N_\infty>l)}\le \Delta_1(l), \tag{9}$$

where

$$\Delta_1(l)=\frac{\sum_{m=1}^{\infty} w_m\big(\frac{d^{m-1}}{l+1}\big)P\big(N_\infty^{(m)}>l\big)}{P(N_\infty>l)}.$$

We further define

$$\Delta_2(l)=\frac{\sum_{m=1}^{\infty} w_m d^{m-1}P\big(N_\infty^{(m)}>l\big)}{P(N_\infty>l)},$$

then $\Delta_1(l)\le\Delta_2(l)$. By our assumption, N_∞ is subexponential. Apply Lemma 2 and we have

$$\lim_{l\to\infty}\Delta_2(l)=ce^{c(d-1)}<+\infty.$$

By the dominated convergence Theorem,

$$\lim_{l\to\infty}\Delta_1(l)=ce^{c(d-1)}\lim_{l\to\infty}\left(\frac{1}{l+1}\right)=0. \tag{10}$$

Equation (9) and (10) imply

$$\lim_{l\to\infty}\frac{P(R_\mu>l)}{P(N_\infty>l)}=0.$$

This, together with Lemmas 1 and 3, leads to our conclusion.

4. Regularly varying tails of the stationary queue length

It is clear that our analysis for subexponential tails is applicable to regularly varying tail asymptotics. We present the regularly varying tail behavior of the stationary queue length in the following Corollary.

Corollary 2. *If the service time is regularly varying with* $\bar{B}(x) \sim x^{-\alpha} L(x)$ *as* $x \to +\infty$ *for* $\alpha > 1$, *then*

$$P(N_\mu > m) \sim P(N_\infty > m) \sim \frac{\lambda^\alpha}{(\alpha - 1)(1 - \rho)} m^{-(\alpha-1)} L(m), \quad \text{as } m \to \infty. \tag{11}$$

Proof: By Theorem 1, we only need to show the second tail equivalence. From Proposition 1.5.10 in Bingham et al. [3], we have for $\alpha > 1$, $\int^{+\infty} y^{-\alpha} L(y) dy$ converges and

$$\frac{x^{-(\alpha-1)} L(x)}{\int_x^{+\infty} y^{-\alpha} L(y) dy} \to \alpha - 1, \quad \text{as } x \to +\infty.$$

This implies as $x \to +\infty$,

$$\bar{B}_e(x) = \frac{1}{\beta_1} \int_x^{+\infty} y^{-\alpha} L(y) dy \sim \frac{x^{-(\alpha-1)} L(x)}{\beta_1(\alpha - 1)}. \tag{12}$$

Equation (5) and (12) lead to Eq. (11). □

Based on the characteristic of the LSTs for regularly varying distributions, Roughan et al. [20] have derived the second tail equivalence in Eq. (11) when the regular variation index of the service time $-\alpha$ is in the interval $(-2, -1)$. With Corollary 2, we can establish Eq. (11) for $-\alpha \in (-\infty, -1)$, which significantly strengthens the result of Roughan et al. [20].

5. Conclusion

Asmussen et al. [2] and Foss and Korshunov [10] derived conditions under which the stationary queue length of a standard M/G/1 queue is subexponential. With a stochastic decomposition, we find that the tail asymptotics of the stationary queue length in an M/G/1 retrial queue are determined by the tail of the stationary queue length in the corresponding standard M/G/1 queue. Thus we conclude that under the same conditions shown in Asmussen et al. [2] and Foss and Korshunov [10], the stationary queue length of an M/G/1 retrial queue is subexponential. Our major results are also applicable to discussing the tail behavior of the stationary queue in an M/G/1 retrial queue when the service time is regularly varying.

It is worthwhile to note that the asymptotic analysis for the busy period and the waiting time of an M/G/1 retrial queue are still open issues. Our preliminary analysis shows that it is very challenging. We conjecture that, unlike the queue length, the tail behaviors of the waiting time and the busy period may not be entirely determined by that of the standard M/G/1 queue.

Acknowledgments

We thank an anonymous referee for his detailed comments and helpful suggestions. In particular, we are grateful to the referee and the AE for suggesting the possibility of generalizing our original results for regularly varying tails to subexponential tails, and hence they deserve as much credit for this work as the authors. This research is supported in part by Hong Kong Research Grant Council through a grant HKUST6133/02E. The third author also thanks the Hong Kong University of Science and Technology (HKUST) for providing opportunity and support for his visit to HKUST.

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