Design of Probe Intervals to Improve Accuracy of CoMPACT Monitor

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Abstract—**We have proposed CoMPACT monitor that achieves scalable measurement of one-way delay distribution for each flow. CoMPACT monitor is technique that transforms one-way delay data obtained by active measurement by using passively monitored traffic data of the target flow. In a recent study, it was reported that by using an inter-probe time that has a Gamma distribution can improve the accuracy of simple active measurement. The improvement is in terms of the ensemble mean of stationary stochastic process. In this paper, to improve accuracy of CoMPACT monitor, we apply Gamma-probing as an active measurement of CoMPACT monitor. The significant issue in this application is in the difference between objects to measure; CoMPACT monitor estimates not the ensemble mean but the time average of sample path. We investigate the characteristics of CoMPACT monitor and Gamma-probing, and verify the accuracy improvement of CoMPACT monitor through simulations.**

Keywords-**CoMPACT monitor, change-of-measure, Gamma distribution, QoS measurement.**

I. INTRODUCTION

As the Internet has grown larger over the last several years, it came to play an important role as infrastructure. Various applications provide new services including telephony and live video, and the traffic they stream exhibits complex characteristics. These various applications require the various quality of service (QoS) that differs from traditional e-mail and web browsing. Then, since the new application will be developed in the future, the diversification of QoS requirements will progress even more.

In order to meet such varied requirements for network control, we need a measurement technology to produce detailed QoS information. Measuring the QoS for each of multiple flows (e.g., users, applications, or organizations) is important since these are used as key parameters in service level agreements (SLAs) between an Internet service provider (ISP) and users. One-way packet delay is one of the most important QoS metrics. This paper focuses on the measurement of one-way delay for each flow.

Conventional means of measuring QoS can be classified into two types: passive and active measurements.

Passive measurement monitors the target user packet directly, by capturing the packets, including the target information. Passive measurement is used to measure the volume of traffic, one-way delay, round-trip time (RTT), loss, etc. and can get any desired information about the traffic since it observes the actual traffic. Passive measurement can be categorized into two-point monitoring with data-matching processes (to measure one-way delay etc.) and one-point monitoring (to measure volume of traffic etc.).

Passive measurement has the advantage of accuracy. However if we perform passive measurement in large-scale networks, the number of monitored packets is enormous and network resources are wasted by gathering the monitored data at a data center. Moreover, in order to measure delay, it is necessary to determine the difference in arrival time of a particular packet at different points in the network. This requires searching for the same packet pairs monitored at the different points in the monitored packet data. The passive measurement lacks scalability due to this packet matching.

Active measurement monitors QoS by injecting probe packets into a network path and monitoring them. Active measurement can be used to measure one-way delay, RTT, loss, etc. It cannot obtain the per-flow QoS, though it is easy for the end user to carry out. Unfortunately, the QoS data obtained by active measurement does not represent the QoS for user packets, but only QoS for the probe packets.

By complementary use of the advantages of active and passive measurements, the authors propose a new technique of scalable measurement called *c*hange-*of-m*easure-based *p*assive/*act*ive *monitor*ing (CoMPACT monitor) to measure per-flow QoS [1], [2], [3], [4].

The idea of CoMPACT monitor is as follows. The direct measurement of QoS of the target flow by passive measurement is difficult due to the scalability problem. So, we try to obtain QoS of the target flow by using a transformation of QoS data (obtained by active measurement). The transformation can be determined by passively monitored traffic data for the target flow. The problem of scalability does not arise, because the volume of traffic can be measured by one-point passive measurement without requiring data-matching processes.

We have believed Poisson arrivals (intervals according to exponential distribution) is appropriate to a policy of probe packets arrivals since we can apply PASTA (Poisson Arrivals See Time Averages) property to it.

However, recent work [5] indicates that many distributions exist that are more accurate than an exponential distribution if a non-intrusive context (ignoring the effect of probe packets) can be assumed. Moreover, we can find a distribution
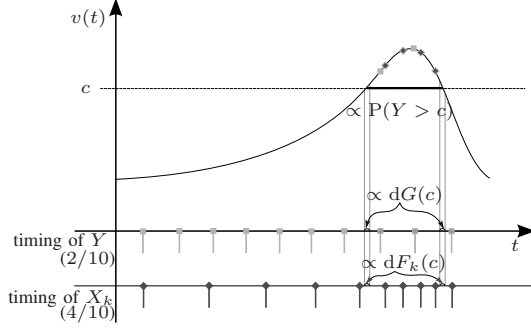
Figure 1. The relation between timing of arrivals and empirical QoS

that is suboptimal in accuracy by selecting an inter-probe time according to the parameterized Gamma distribution.

This paper confirms that applying Gamma-probing to CoMPACT monitor can improve the accuracy of measurement of one-way delay distribution for individual flows. The significant issue in this application is in the difference between objects to measure. In [5], the process observed by probe packets is assumed to be a stationary and ergodic stochastic process. So, the accuracy improvement is guaranteed only when we estimate the ensemble mean of stationary stochastic process. However, CoMPCT monitor estimates the time average of sample path. Therefore, we should carefully investigate the characteristics of CoMPACT monitor and Gamma-probing.

The rest of the paper is organized as follows. We describe the summary of CoMPACT monitor in Section II. Next, we briefly summarize the theory of Gamma-probing for active measurement, shown in [5], in Section III. Section IV discusses the difference of objects we want to estimate between Gamma-probing [5] and CoMPACT monitor. To confirm that CoMPACT monitor is able to be improved in accuracy by Gamma-probing, we execute simulation in Section V. We conclude the paper in Section VI.

II. SUMMARY OF CoMPACT MONITOR

CoMPACT monitor estimates an empirical QoS for the target flow by converting observed values of network performance at timing of probe packet arrivals into a measure of the target flow timing. Now, let $v(t)$ denote the network process under observation (e.g. the virtual one-way delay at time t), and X_k denote a random variable which is observed $v(t)$ with a certain timing (e.g. the timing of user packet arrivals). The probability for X_k to exceed c is

$$P(X_k > c) = \int 1_{\{x>c\}} dF_k(x) = E_{F_k}[1_{\{x>c\}}]$$

where $F_k(x)$ is the distribution function of X_k.

If we can directly monitor X_k, its distribution can be estimated by $\sum_{n=1}^{m} 1_{\{X_k(n)>c\}}/m$, where $X_k(n)\,(n = 1, 2, \cdots, m)$ denote the nth observed value.

Now, let us consider the situation that X_k cannot be directly monitored. Let Y denote a random variable that is observed $v(t)$ at a different timing (e.g. timing of probe packet arrivals) independent of X_k. Then we consider the relationship between X_k and Y.

Characters of X_k and Y are different if their timing is different, even if they observe a common process $v(t)$ (see Fig. 1). X_k and Y can be related by each distribution functions F_k and G, and $P(X_k > c)$ expressed by measure of X_k can be transformed into measure of Y as follows.

$$P(X_k > c) = \int 1_{\{x>c\}} dF_k(x)$$

$$= \int 1_{\{y>c\}} \frac{dF_k(y)}{dG(y)} dG(y)$$

$$= E_G \left[1_{\{Y>c\}} \frac{dF_k(Y)}{dG(Y)} \right]$$

Therefore, $P(X_k > c)$ can be estimated by

$$\frac{1}{m} \sum_{n=1}^{m} 1_{\{Y(n)>c\}} \frac{dF_k(Y(n))}{dG(Y(n))}, \qquad (1)$$

for sufficiently large m, where $Y(n)\ (n = 1, 2, \cdots, m)$ denote the nth observed value. Note that this estimator does not need to monitor the timing of X_k, if we can get $dF_k(Y(n))/dG(Y(n))$. This means the QoS of a specific flow (as decided by k) can be estimated by just one probe packet train that arrives with a timing of Y.

In the following, we briefly summarizes the mathematical formulation of CoMPACT Monitor [4]. We assume the traffic in the target flow can be treated as a fluid. In other words, we assume packets of the target flow are more numerous than active probe packets.

Let $a(t)$ and $v(t)$, respectively, denote the traffic in the target flow at time t and the virtual one-way delay on the path that we want to measure. $a(t)$ and $v(t)$ are a nonnegative deterministic processes assumed to be right-continuous with left limits and bounded on $t \geq 0$. We can consider $a(t)$ and $v(t)$ are sample paths of corresponding stochastic processes.

Considering to measure the empirical one-way delay distribution $\pi(c)$, the value we want to measure is the ratio to all traffic of the target flow of traffic for which the delay to exceeds c, which is given by

$$\pi(c) = \lim_{t \to \infty} \frac{\int_0^t 1_{\{v(s)>c\}} a(s) ds}{\int_0^t a(s) ds}. \qquad (2)$$

This can be estimated through m times monitoring by

$$Z_m(c) = \frac{1}{m} \sum_{n=1}^{m} 1_{\{v(T_n)>c\}} \frac{a(T_n)}{\sum_{l=1}^{m} a(T_l)/m} \qquad (3)$$

for sufficiently large m (see [4] for details), where T_n $(n = 1, 2, \cdots, m)$ denotes the nth sampling time, and each time of sampling corresponds to a time of probe packet

arrival. Active and one-point passive measurement are used respectively to observe $v(T_n)$ and $a(T_n)$. Note that one-point passive measurement can be conducted very easily here, compared with two-point passive measurement for measuring the one-way delay.

If we extract the quantity $\sum_{n=1}^{m} 1_{\{v(T_n)>c\}}/m$ from (3), this quantity is a simple active estimator that counts the packets for the delay to exceeds c. However, (3) is weighted by $a(T_n)/(\sum_{l=1}^{m} a(T_l)/m)$, which is decided by the traffic in the target flow when probe packets arrive. This means that the one-way delay distribution (measured by active measurement without bias) is corrected to the empirical one-way delay distribution by the bias of the target flow (observed by passive measurement). $a(T_n)/(\sum_{l=1}^{m} a(T_l)/m)$ in (3) corresponds to $\mathrm{d}F_k(Y(n))/\mathrm{d}G(Y(n))$ in (1).

III. Suboptimal probe intervals

Since the PASTA property is good for non-biased measurement, Poisson arrivals have been widely used as policy of probe packets arrivals for active measurement. However, if arrival process of the probe packets is stationary and mixing, under non-intrusive conditions, the following equation holds and we can also ignore the effects of probe packets under non-intrusive conditions.

$$\lim_{m\to\infty} \frac{1}{m}\sum_{n=1}^{m} f(Y(T_n)) = \lim_{t\to\infty} \frac{1}{t}\int_0^t f(Y(t))\mathrm{d}t$$
$$= \mathrm{E}[f(Y(0))] \qquad \text{a.s.,} \qquad (4)$$

where f is an arbitrary positive function and the second equality follows from the stationary and ergodicity of the target process $Y(t)$. [6] proved (4) and named this property NIMASTA (Non-Intrusive Mixing Arrivals See Time Averages).

Mixing is the requirement to guarantee jointly ergodicity between probe packet process and the target process. For example, there are processes whose intervals obey the Gamma distribution, the uniform distribution, etc [6]. Note that periodic-probing, with determinate intervals is not a mixing process, and does not satisfy (4).

The recent study [5] also reported that NIMASTA-based probing is suitable for measurement. That provides an improvement in the accuracy of the measurement. We can select a suboptimal probing process in terms of accuracy under the specific assumption by using an inter-probe time given by the parameterized Gamma distribution.

If we estimate the mean of $Y(0)$ by using active measurement, estimator $\hat{p}$ is

$$\hat{p} = \frac{1}{m}\sum_{n=1}^{m} Y(T_n). \qquad (5)$$

Then variance of $\hat{p}$ is

$$\mathrm{Var}[\hat{p}] = \frac{1}{m^2}\mathrm{Var}\left[\sum_{n=1}^{m} Y(T_n)\right]$$
$$= \frac{1}{m^2}\sum_{n=1}^{m}\mathrm{Var}[Y(T_n)] + \frac{2}{m^2}\sum_{n\neq l}\mathrm{Cov}\left(Y(T_n), Y(T_l)\right)$$
$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad (6)$$
$$= \frac{1}{m}\mathrm{Var}[Y(0)] + \frac{2}{m^2}\sum_{n\neq l}\int R(\tau)f_{|n-l|}(\tau)\mathrm{d}\tau \qquad (7)$$

where f_k is probability density function (pdf) of T_k, $R(\tau) = \mathrm{Cov}\left(Y(t), Y(t-\tau)\right)$ is the autocovariance function of the target process $Y(t)$ and the last equality follows from the stationary of $Y(t)$ and probe packet process.

If $R(\tau)$ is convex, the following can be proven by using Jensen's inequality. No other probing process with an average interval of μ has a variance that is lower than that of periodic-probing (see [5]). A lower variance of the estimator is connected with accuracy. Therefore, periodic-probing is the best policy if we focus only on variance.

On the other hand, periodic-probing does not satisfy the assumptions of NIMASTA due to non-mixing, so periodic-probing is not necessarily the best. This is because a phase-lock phenomenon may occur and the estimator may converge on a false value when the cycle of the target process corresponds to the cycle of the probing process.

To tune the tradeoff between traditional PASTA-based probing and periodic-probing, [5] proposes a suboptimal policy that gives an inter-probe time that obeys the parameterized Gamma distribution. The pdf that is used as the intervals between probe packets is given by

$$g(x) = \frac{x^{\beta-1}}{\Gamma(\beta)}\left(\frac{\beta}{\mu}\right)^{\beta}\mathrm{e}^{-x\beta/\mu} \qquad (x>0), \qquad (8)$$

where $g(x)$ is the Gamma distribution whose shape and scale parameters are β and μ/β, respectively. μ denotes the mean, and β is the parameter. When $\beta = 1$, $g(x)$ reduces to the exponential distribution. When $\beta \to \infty$, the policy reduces to periodic-probing because $g(x)$ converges on $\delta(x-\mu)$.

If the autocovariance function is convex, it is proven that variance of estimator $\hat{p}$ sampled by intervals according to (8) monotonically decreases with increase of β. This is caused by decrease of the covariance part that is the second term on the right-hand side of (7). We can achieve near-optimal variance of periodic-probing, since (8) corresponds to periodic-probing towards limit $\beta \to \infty$.

The problem of incorrectness due to phase-lock phenomenon can be avoided if we tune β to a limited value (this gamma-probing satisfy the mixing assumption). We can get a suboptimal probing if we give β an appropriate value.

IV. The application to CoMPACT monitor

This section discusses the application of Gamma-probing to CoMPACT monitor. Comparing (5) with (3), we can con-
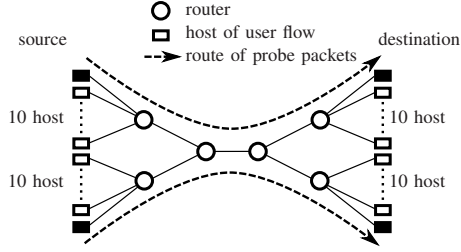
Figure 2. Network model

sider that stochastic process $Y(t)$ observed by CoMPACT monitor is

$$Y(t) = 1_{\{V(t)>c\}} \frac{A(t)}{\mathrm{E}[A(t)]} \ , \qquad (9)$$

where $V(t)$ and $A(t)$ are stochastic processes corresponding to sample path $v(t)$ and $a(t)$, respectively. If it is confirmed that the autocovariance function $R(\tau)$ of stochastic process $Y(t)$ is convex, we can guarantee accuracy improvement due to Gamma-probing by applying the theory in Section III.

The convexity of the autocovariance function has be verified in cases of simple virtual delay process and loss process by data of large-scale passive measurement and simulations [5]. However, $Y(t)$ observed by CoMPACT monitor is weighted by traffic $A(t)$ of a specific flow, so the property of $Y(t)$ obviously differs from delay/loss processes that are influenced by all flows on the network. Therefore, we must confirm the convexity of $R(\tau)$ in the CoMPACT monitor case. In Section V, we will show that the assumption ($R(\tau)$ is convex) is appropriate to CoMPACT monitor.

As described in Section II, we consider $v(t)$ and $a(t)$ are sample paths of corresponding stochastic processes (without stationary and ergodicity) since CoMAPCT monitor estimates the time average of sample path given by (2). However, as explained in Section III, the target process $Y(t)$ is stationary stochastic process [5], and the decrease of variance of $\hat{p}$ is guaranteed for the estimation of the ensemble mean of $Y(t)$. The variance of $\hat{p}$ depends on both stochastic variations of $Y(t)$ and T_n. On the other hand, CoMPACT monitor observe sample path $y(t)$ rather than stochastic process $Y(t)$, because CoMPACT monitor estimates one-way delay that is experienced by user flows.

Therefore, it is desired in CoMPACT monitor that variance of estimator $\hat{p}$ which depends only on sampling time T_n on certain sample path rather than $Y(t)$ decreases. In other words, variance we want to decrease is given by

$$\mathrm{Var}[\hat{p}] = \frac{1}{m^2} \sum_{n=1}^{m} \mathrm{Var}[y(T_n)] + \frac{2}{m^2} \sum_{n \neq l} \mathrm{Cov}(y(T_n), y(T_l)).$$
$$(10)$$

Note that (10) corresponds to (6). If the autocovariance function of $Y(t)$ is convex, we can consider that the covariance part that is the second term on the right-hand side of (10)

tends to decrease with increase of β. Then variance of inter-probe time given by (8) is μ^2/β, and decreases with increase of β. Hence, we can also expect that the variance part that is the first term on the right-hand side of (10) decreases with increase of β since each observed value $y(T_n)$ is not varied. In Section V, we present the result of simulation that variance of estimator which depends only on T_n also decreases with increase of β by using Gamma-probing.

V. THE EFFECTIVENESS OF GAMMA-PROBING

A. Simulation model

We investigated the effectiveness of Gamma-probing in the framework of CoMPACT monitor. The network model we used in the simulation is shown in Fig. 2.

There are 20 pairs of source and destination end hosts. Each source end host transfers packets by UDP to the corresponding destination end host. User flows are given as ON/OFF processes and categorized into the four types listed in TABLE I, with there are five flows in each type.

Probe packet trains are categorized into the five types listed in TABLE II. Note that *Exp* and *periodic* in TABLE II are special cases of Gamma distribution, and parameters of *Exp* and *periodic* are parameters of the Gamma distribution corresponding to each probing. 300 trains of each type are streamed on the two routes shown in Fig. 2, so the total number of probe packet trains in the network is 3000. To analyze the variance of the estimator, we streamed a large number of probe packet trains. Of course we can estimate the empirical delay from only one probe packet train.

User flow packets and probe packets are 1500 bytes and 64 bytes, respectively. Link capacities are identical at 64 Mbps. Delay occurs mainly in the link between the core routers, since it is a bottleneck, but no loss occures, because there is sufficient buffering.

Our simulation model is simple. However, we have confirmed that CoMPACT monitor (without Gamma-probing) is effective in an actual environment. Therefore, this model is enough to confirm the combination between CoMPACT monitor and Gamma-probing.

We ran the simulation for 500 s. The non-intrusive requirement was satisfied, since the ratio of the probe packet traffic (bytes) to all traffic (bytes) is about 0.00197%.

B. The convexity of the autocovariance function

In this subsection, we will discuss whether the autocovariance function of the target process $Y(t)$ is convex. Note that target process is treated as not sample path $y(t)$ but stochastic process $Y(t)$.

The (standardized) autocovariance function for $c = 0.1$ for flow #1 with 95% confidence intervals by 10 experiments is depicted in Fig. 4. To represent at each flow type, we plotted flows #6, #11 and #16. This permitted the conclusion that none of these results contradicted the assumption that the autocovariance function is convex.

Table I
TYPE OF USER FLOWS

Flow type	Flow ID	Mean ON/OFF period	Distribution of ON/OFF length	Shape parameter	Rate at ON period
type 1	#1-5	10s/5s	Exp	-	6 Mbps
type 2	#6-10	5s/10s	Exp	-	6 Mbps
type 3	#11-15	5s/10s	Parete	1.5	9 Mbps
type 4	#16-20	1s/19s	Parete	1.5	9 Mbps

Table II
TYPE OF PROBING

Distribution of intervals	Parameter	Mean probe interval
Exp	$(\beta = 1)$	0.5 s
Gamma	$\beta = 5$	0.5 s
Gamma	$\beta = 25$	0.5 s
Gamma	$\beta = 125$	0.5 s
Periodic	$(\beta \to \infty)$	0.5 s
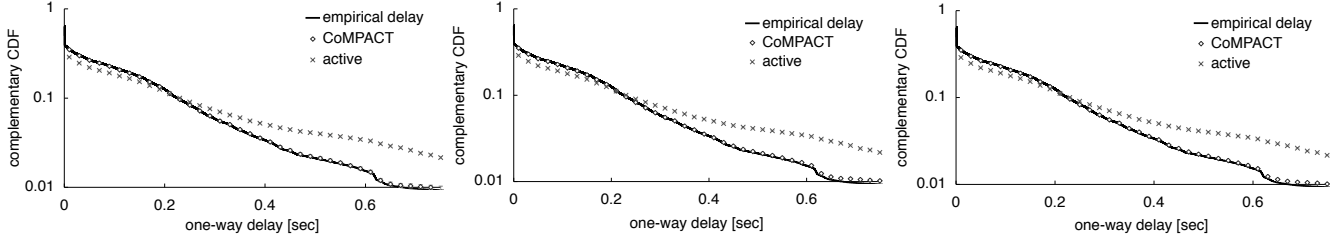
Figure 3. The estimation of complementary CDF for (Left) $\beta = 1$, (Middle) $\beta = 25$ and (Right) $\beta \to \infty$ (flow #1)

C. One-way delay distribution

In this subsection, we show that CoMPACT monitor can estimate the empirical one-way delay when using Gamma-probing. The estimation of the complementary cumulative distribution function (CDF) of the one-way delay experienced by user flows (as given by (2)) will be shown below. Note that $v(t)$ and $a(t)$ are both sample paths in (2).

To estimate the complementary CDF of the one-way delay experienced by flow #1, we use probe packet trains with parameter $\beta = 1$, 25 and $\beta \to \infty$ respectively. Each result is shown in Fig. 3. To compare the empirical delay with the estimate from CoMPACT monitor, we include the estimate from active measurement in the plot.

In Fig. 3, we can see that the CoMPACT monitor gives good estimates of the true value. We cannot judge the superiority or inferiority of any type of probe packet trains. To represent each flow type, we have plotted for flows #6, #11 and #16, getting results similar to Fig. 3.

D. Accuracy Improvement of CoMPACT monitor

In this subsection, we verify the relationship between the parameter β and the variance of estimator. Note that we consider the variance of $\hat{p}$ which depends only on probe packets timing (namely, sampling time T_n).

We plot the standard deviation of each point of the complementary CDF in Fig. 5. Error bars indicate the 95% confidence interval when the standard deviation calculated from 30 probe packet trains is considered to be a single data point.

The standard deviation clearly decreases as β increases from $\beta = 1$ to $\beta = 125$. In periodic-probing corresponding to $\beta \to \infty$, the standard deviation is often larger than that for $\beta = 125$ and 25. This reversal is a sign of the phase-lock.

Hence, it is confirmed that we can obtain adequate accuracy on variance which depends only on sampling time T_n

with a suboptimal probing process if we tune the parameter of Gamma distribution that we use as the inter-probe time.

E. The upper bounds of variance

Adding to the simulation results in previous subsection, we prove that periodic-probing is surely more superior than the traditional PASTA-based probing.

Let us idealize the traffic process $a(t)$ to ON/OFF process. We assume $a(t)$ is the binary process which takes a value α if the target flow streams, and 0 otherwise. Then $\hat{p}$ is expressed by

$$\hat{p} = \sum_{n=1}^{m} 1_{\{v(T_n)>c\}} \frac{a(T_n)}{\sum_{l=1}^{m} a(T_l)}$$
$$= \frac{\sum_{n=1}^{m} 1_{\{v(T_n)>c \wedge a(T_n)>0\}}}{\sum_{l=1}^{m} 1_{\{a(T_l)>0\}}}. \qquad (11)$$

Since UDP does not control the volume of traffic, the assumption is appropriate to the UDP flow.

Now, we consider the range of (11). $1_{\{v(t)>c \wedge a(t)>0\}}$ and $1_{\{a(t)>0\}}$ are the binary processes, and we can divide them into the periods of 0 or 1. If they are observed by periodic-probing, the difference between the maximum and minimum observation frequency is 1 at most in one period (see Fig. 6).

The maximum range Δ of (11) is given by

$$\Delta = \frac{k_2}{mq - k_1}, \qquad (12)$$

where k_1 denote the number of periods of $1_{\{a(t)>0\}} = 1$, q denote the time average of $1_{\{a(t)>0\}}$ and k_2 denote the number of periods of $1_{\{v(t)>c \wedge a(t)>0\}} = 1$.

The distribution that has the maximum variance with the range Δ has pdf $h(x) = \delta(x-a)/2 + \delta(x-a-\Delta)/2$, where $\delta(\cdot)$ is Dirac δ function and a is an arbitrary constant. Hence, the upper bounds of estimator variance is as follows.

$$\text{Var}[\hat{p}] \le \left(\frac{\Delta}{2}\right)^2 = \frac{k_2^2}{4(mq - k_1)^2}. \qquad (13)$$
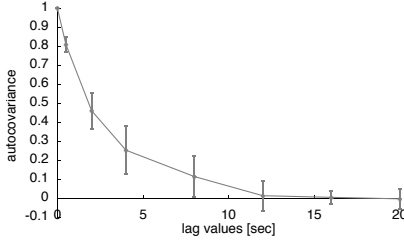
Figure 4.   Autocovariance function (flow #1)
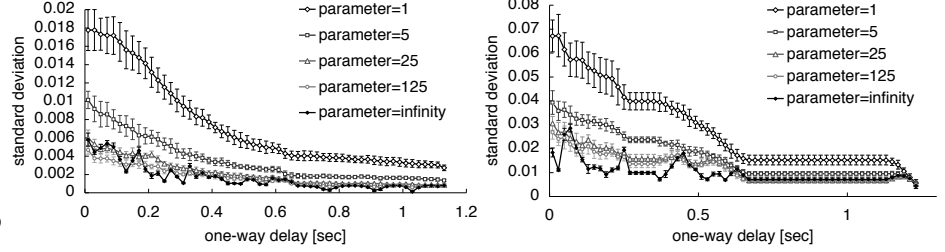

Figure 5.   Standard deviation of estimator for (Left) flow #1 and (Right) flow #16
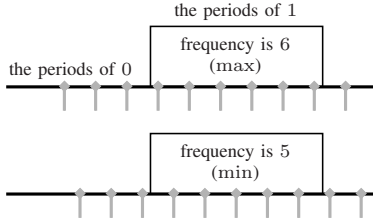

Figure 6.   The max/min observation frequency of periodic-probing
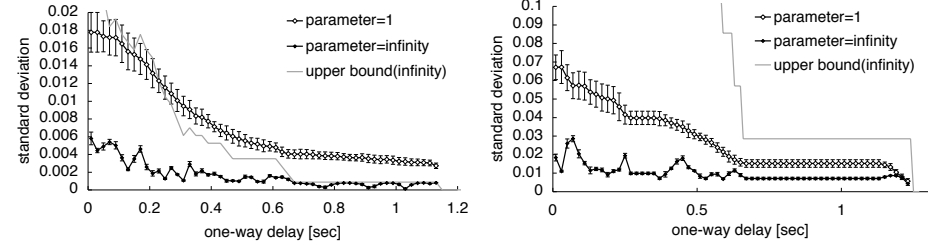

Figure 7.   The upper bounds of periodic-probing for (Left) flow #1 and (Right) flow #16

Calculating k_1, k_2 and q in above simulation, we plot the upper bounds of periodic-probing in Fig. 7. Note that standard deviations of PASTA-based probing and periodic-probing (the same data as we plot in Fig. 5) are displayed.

In results of flow #1, #6 and #11, we can confirm that the upper bounds of periodic-probing are smaller than standard deviation of PASTA-based probing in the domain of interest (the domain with large delay). Therefore, it is guaranteed that periodic-probing is surely more superior than PASTA-based probing.

In result of flow #16, the upper bounds of periodic-probing are larger than standard deviation of PASTA-based probing because the combination of flow type 4 (e.g. flow #16) and (13) is bad. The flows of type 4 has short ON periods and long OFF periods as shown in TABLE I. So the maximum range given by (12) becomes large because the denominator becomes small. Therefore, the appropriate upper bounds cannot be given by (13).

VI. CONCLUSION

In a non-intrusive context where the effect of probe packets can be ignored, it was confirmed that the accuracy of estimating the complementary CDF of one-way delay can be improved by using Gamma-probing as part of applying CoMPACT monitor estimates. This means that Gamma-probing proposed in [5] is able to apply to CoMPACT monitor that estimates the time average of sample path.

Then, we were able to confirm that the autocovariance function of process observed by CoMACT monitor is convex. The convexity contributes to the accuracy improvement when we estimate not only the ensemble mean of stochastic process but also the time average of sample path.

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