Probing for Loss: The Case Against Probe Trains

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***Abstract*—We assess the bias and variance of active probing measurement techniques for loss in networks. We show that loss probability is best measured using isolated probes rather than with probe 'trains', which contrasts with prior work which found that probe trains better estimate statistics of lossy periods.**

***Index Terms*—Probing, probe trains, loss, optimal variance, NIMASTA.**

I. Introduction

PACKET loss is a fundamental measure of performance in computer networks. Loss and delay are the two sources of raw information available to end-to-end measurement by probing. In contrast to delay, loss data is scarce as loss probabilities are typically low and loss yields coarser binary information than delay. It is therefore crucial to use probing methods and statistical estimators which make the most of every recorded loss, and which are robust to details of how they occur.

We investigate the best sampling strategy for loss measurement, i.e., how to choose probe sending times. Previous work concluded that trains are superior because they better estimate statistics such as the duration of lossy episodes. Here we focus on the overall *loss probability*. We begin by extending to loss our prior work [4] on rigorous unbiased sampling for delay, and then compare approaches via estimation variance. We focus on variance since that is the best distinguishing factor among approaches that are unbiased (recall that mean square error is the sum of the variance and squared bias).

Our main result is that the variance for loss probability is significantly lower when probes are isolated than when they are grouped into trains. Our result is based on the assumption that losses are concentrated during generally well separated *congestion episodes*, but is otherwise general. This assumption is well-motivated given that Internet queues inherently have memory and that most traffic is TCP-based. Using simulation experiments, we verify that our result holds true more widely and that the variance improvement obtained by using isolated probes is significant.

II. Preliminaries

A. What to Measure?

A natural target of loss measurement is the probability p_x that a packet of size x (bytes), injected into the network at some source, would fail to arrive at the receiver. As described in [1], p_x is well defined using $I_x(t)$, a binary stochastic process, which is 1 if a packet of size x, *if it were* injected at time t, would be lost, else 0. Assuming the *loss process* $I_x(t)$ is stationary, $p_x = \mathrm{P}(I_x(t) = 1)$. This notion of a loss process corresponds to ideal non-intrusive probing, but is otherwise very general, applying even to complex multi-hop networks with diverse dropping policies (as in [1] for delay).

Ideally, to estimate statistics related to loss, we would need to know the value taken by the loss process $I_x(t)$ over continuous time. In practice, we send probes at discrete times $\{T_n\}$ and observe their losses to estimate loss-related statistics including p_x. Our goal is to provide insights into how the $\{T_n\}$ should be chosen so that p_x can be estimated optimally.

B. Related Work

Early measurements of end-to-end loss sent periodic probes along multiple Internet paths to estimate loss probability. Later papers used the PASTA principle to motivate exponentially-separated probe packets for loss estimation. Poisson sampling was used in to study the time-varying nature of loss, and other metrics such as loss episode durations. The IETF IP Performance Metrics (IPPM) Group [2] also recommends Poisson sampling both for measuring loss probability as well as characteristics of loss episodes.

Recent work [3] questioned the use of Poisson probes to estimate loss episodes, proposing a train-based scheme instead, and the estimation of loss probability as the product of loss episode probability and duration. In prior work, we and others have questioned the logic and utility of Poisson sampling for delay on theoretical grounds ([4], [1] and references therein).

C. Sampling Results for Loss

We consider non-intrusive probing in order to focus on sampling issues in isolation. As in [4], [1], our first concern is to use those probe streams that measure loss without 'asymptotic sample bias' that is when the following holds:

$$\lim_{N\to\infty} \frac{1}{N}\sum_{n=1}^{N} f(I_x(T_n)) = E[f(I_x(0))], \tag{1}$$

where f is any positive loss-based function. E.g. $f(x) = x$ corresponds to the empirical loss probability ($\hat{p}_x$) converging to the true mean $\mathbb{E}[I_x(0)] = p_x$.

We can show (proof omitted) that our delay theorems from [4] also apply to loss estimation provided suitable definitions of loss process and sampling, such as those above, are adopted. In particular, NIMASTA (Non-mixing Arrivals See Time Averages) applies to loss estimation and (1) holds whenever the probing stream sending times are chosen from a mixing process (e.g., the non-periodic streams in Fig. 1).

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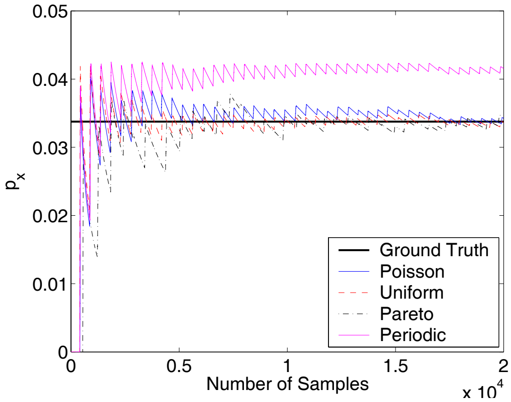

Fig. 1. Loss probability estimation in a 3-hop system, cross-traffic [TCP,Poisson,Periodic]. Because of the phase locking on hop 3, the periodic sampling stream has sample-path bias, whereas for all probe streams sent according to a mixing process, NIMASTA applies.

NIMASTA applies to a very general class of networks, and includes numerous scheduling disciplines such as processor sharing and Weighted Fair Queueing as well as FIFO, networks with non-trivial topologies, and load balancing. As an illustration, we show in Fig. 1 the sample-path convergence of loss probability estimation (1) in a multi-hop case. This was generated using ns-2 with three hops each having different kinds of cross-traffic, including a TCP flow over all three hops with feedback oscillations, and a periodic stream.

NIMASTA can be extended to include probe trains, via a train-loss process $I_N(t)$ which counts the number of probes lost in an N-probe train sent in at time t. As before, we assume non-intrusiveness. If the inter-arrival times of probes within a train of size $k+1$ are $\tau_1, \ldots \tau_k$, we can write

$$\lim_{N\to\infty} \frac{1}{N}\sum_{n=1}^{N} g(I_x(T_n), ..I_x(T_n+\tau_k)) = E[g(I_x(0), ..I_x(\tau_k))].$$

If g is taken to be the mean loss probability within a train, the above implies that unbiased train estimation is possible.

III. Sampling for Optimal Variance

We consider how isolated probes compare to *probe trains*. Empirical averages will be used to estimate the mean p_t of the loss process $I_N(t)$, whose variance we aim to reduce.

A. System Model

Our model is based on the idea that networks may undergo periods of congestion where loss probabilities are locally (or conditionally) higher. Any definition of a congestion episode gives rise naturally to an *episode process*, a continuous time process with alternating *congested* and *quiet* episodes. Such an episode process will inherit the stationarity and ergodicity of the underlying system on which it depends. As an example, $I_x(t)$ itself could be seen as such an episode process.

To formulate our result on variance, we assume the following

- Timescale separation: trains falls within a single episode,
- Conditional episode stationarity: constant loss probabilities within each episode type: Congested: p Quiet: p',
- Weak intra-train correlations: intra-train independence.

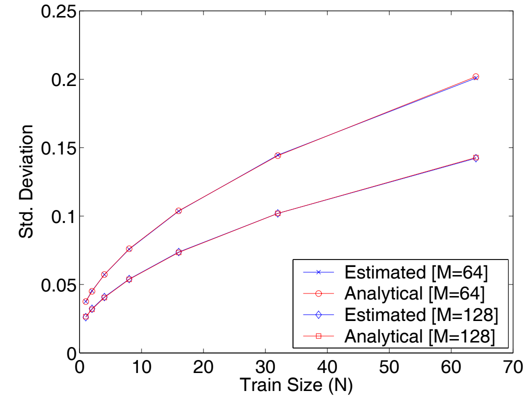

Fig. 2. Assumption-obeying simulation of train sampling.

These assumptions constitute a simplifying abstraction overlaying an otherwise general episode process.

Let α be the probability of encountering a congestion period. Then using the above, the average loss probability is

$$p_t = \alpha p + (1-\alpha)p'. \tag{2}$$

Under the constraint of sharing the same 'probe budget' of M probes, we want to compare the following strategies:

1. Isolated Probes: $T = M$ trains ($N = 1$).
2. Probes Trains: T trains of $N = M/T$ probes each.

The estimator can be written as

$$\hat{p}_t = \frac{1}{T}\left(\sum_{l=1}^{C} A_l + \sum_{l=1}^{Q} B_l\right) \tag{3}$$

where Q (resp. C) denotes the random number of Quiet (resp. Congested) trains, $Q + C = T$, and where

$$A_l = \frac{1}{N}\sum_{i=1}^{N} H_{i,l} \tag{4}$$

denotes the empirical mean of the losses in the l-th congested train (with a symmetrical definition for B_l). where $H_{i,l} = 1$ if the i-th probe of train l is lost and 0 otherwise.

B. Comparing Variances

Conditional on a train being in a given episode type, probe losses within the train are a sequence of N independent $\{0,1\}$ variables. It follows that $\{A_l\}$ is the mean of N Bernoulli random variables and has a mean p and variance $p(1-p)/N$ (similarly for $\{B_l\}$).

For isolated probes, the probe loss process reduces to a set of M i.i.d. $\{0,1\}$ variables with variance

$$V_1 = \frac{1}{M}p_t(1-p_t). \tag{5}$$

For the train case, we use the episode type separation of (3). We calculate the variance V_2 of the train-based estimator using the *law of total expectation*, namely $\mathbb{E}$(X)=$\mathbb{E}$($\mathbb{E}$(X|Y)), and conditioning on $C = n$, we get

$$\begin{aligned} V_2 &= \mathbb{E}\left[(\hat{p}_t - p_t)^2\right] = \mathbb{E}\left[\mathbb{E}\left[(\hat{p}_t - p_t)^2 | C\right]\right] \\ &= \sum_{n=0}^{T} g(\alpha; n; T)\,\mathbb{E}\left[\frac{1}{T^2}\left(\sum_{l=1}^{n} A_l + \sum_{l=1}^{T-n} B_l - Tp_t\right)^2\right] \end{aligned}$$

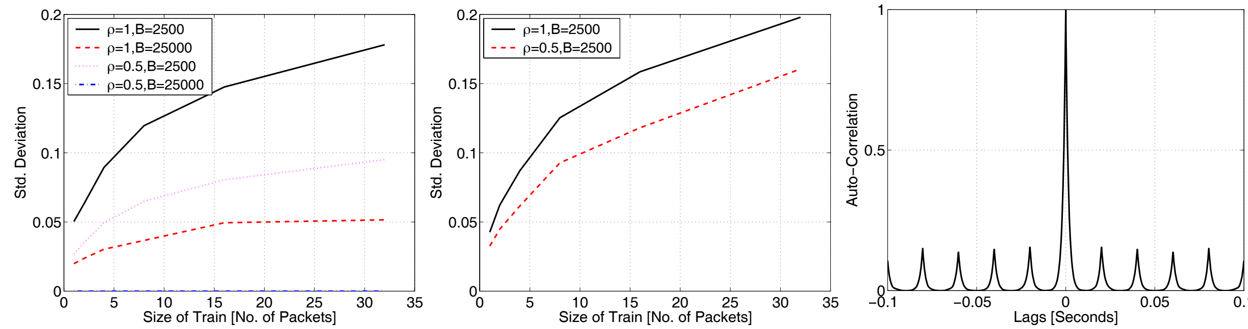

Fig. 3. **(Left)**: Standard deviation of isolated probes and probe trains as a function of train size in a single hop for different cross-traffic load (ρ) and buffer sizes (B). The $\rho = 0.5$, $B = 25000$ bytes simulation has no loss and standard deviation is always zero, otherwise variance increases with train size. **(Middle)**: Std. dev. as a function of train size for a non-convex system, for two different cross-traffic loads. **(Right)**: oscillating auto-correlation of the loss process indicates lack of convexity of the $\rho = 1$ system.

where $g(\alpha; n; T) = \alpha^n(1-\alpha)^{T-n}\binom{T}{n}$. Rewriting $Tp_t = T(\alpha p + (1-\alpha)p')$ to separate out the individual means: $Tp_t = np + (T-n)p' + (T\alpha - n)(p-p')$, the above expectation can be rewritten as

$$\mathbb{E}\left[\frac{1}{T^2}\left(\sum_{l=1}^{n}(A_l - p) + \sum_{l=1}^{T-n}(B_l - p') - (T\alpha - n)(p - p')\right)^2\right]$$

Using A, B to denote centered random variables $\sum_{l=1}^{n}(A_l - p)$ and $\sum_{l=1}^{T-n}(B_l - p')$ resp., we can expand the above square. Since A and B have zero expectation, the cross-multiplied terms are zero due to independence and we get:

$$\frac{1}{T^2}\mathbb{E}\left[A^2\right] + \frac{1}{T^2}\mathbb{E}\left[B^2\right] + \frac{1}{T^2}\mathbb{E}\left[\left((T\alpha - n)(p - p')\right)^2\right]$$

However, the centering of A also implies that $E[A^2]$ is equal to its variance, $np(1-p)/N$ and,

$$\sum_{n=0}^{T} g(\alpha; n; T)E[A^2] = \frac{p(1-p)}{NT}\sum_{n=0}^{T}\frac{n}{T}g(\alpha; n; T) = \frac{\alpha p(1-p)}{M}$$

Using a similar expression involving B, we obtain:

$$\begin{aligned} V_2 &= \frac{1}{M}(\alpha p(1-p) + (1-\alpha)p'(1-p')) + \xi && (6)\\ &= V_1 + (p-p')^2\alpha(1-\alpha)\frac{N-1}{M} && (7) \end{aligned}$$

where

$$\begin{aligned} \xi &= \frac{(p-p')^2}{T^2}\sum_{n=0}^{T}\alpha^n(1-\alpha)^{T-n}\binom{T}{n}(T\alpha - n)^2 \\ &= \frac{(p-p')^2\alpha(1-\alpha)}{T}. \end{aligned}$$

For $N = 1$, trains reduce to probes and $V_1 = V_2$. The significant result is that for any non-trivial train ($N \geq 2$), variance is not reduced by using trains. On the contrary it is always *increased*, and monotonically so with N at fixed M.

IV. Simulation Results

We first simulate a loss process according to our assumed model, and sample it according to different probing processes. The results agree with those derived earlier as we would expect, as seen in Fig. 2, which plots the estimated and analytically derived standard deviation against train size.

In reality, trains can overlap congested and quiet states. To investigate the breaking of the no-overlap assumption, we simulate a single hop of capacity 10Mbps using small (2500 bytes) and large (25000 bytes) buffers under low ($\rho = 0.5$) and high ($\rho = 1$) cross-traffic load with average packet size 500 bytes. In Fig. 3 **(Left)**, cross-traffic arrives according to a Poisson process of rate 100 packets/second, with exponential packet sizes. The standard deviation shown of loss probability estimates is based on $M = 64$ probes, assembled into probing trains of different sizes (including isolated probes). The variance increases significantly with train size.

In [1], we showed that periodic probing, a special kind of isolated probing, is superior to trains of any kind if the loss process autocorrelation is convex. We next simulate another single hop system with cross-traffic arrivals chosen such that the loss process autocorrelation is not convex, to verify that our result holds true even without the aid of convexity. We use cross-traffic that arrives periodically in bunches of 50 packets, resulting in a dramatically non-convex autocorrelation Fig. 3 **(Right)**, and exponential packet sizes (others yielded the same results). The results in Fig. 3 **(Middle)** show that isolated probes continue to outperform.

V. Conclusion

We considered the estimation of loss probability assuming that the loss process followed a simplified, but well-motivated model. We showed that isolated probes have provably smaller variance than probe trains for the estimation of loss probability, and that the variance of probing trains increases significantly with train size. We used simple proof-of-concept simulations to illustrate our result, test it when a key assumption is broken, and to prove that it is complementary to a recent alternative approach based on 'convex' networks.

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