PAPER

# Packet Delay Estimation That Transcends a Fundamental Accuracy Bound due to Bias in Active Measurements*

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**SUMMARY** For network researchers and practitioners, active measurement, in which probe packets are injected into a network, is a powerful tool to measure end-to-end delay. It is, however, suffers the intrusiveness problem, where the load of the probe traffic itself affects the network QoS. In this paper, we first demonstrate that there exists a fundamental accuracy bound of the conventional active measurement of delay. Second, to transcend that bound, we propose INTrusiveness-aware ESTimation (INTEST), an approach that compensates for the delays produced by probe packets in wired networks. Simulations of M/M/1 and MMPP/M/1 show that INTEST enables a more accurate estimation of end-to-end delay than conventional methods. Furthermore, we extend INTEST for multi-hop networks by using timestamps or multi-flow probes.

***key words:*** *active measurement, probe packets, bias, intrusiveness, delay*

## 1. Introduction

It is important to accurately estimate end-to-end delay and evaluate the path quality when we design networks and applications. Real-time applications, such as audio/video conferencing and IP telephony, require low end-to-end delay compared to traditional applications such as e-mail, web browsing, etc. ITU-T Recommendation G.114 [1] mentions that an end-to-end delay of more than or equal to 150 [msec] adversely affects the communication quality of interactive Voice over IP (VoIP) applications. It is especially important to accurately estimate not only average delay, but also high quantile delays, since the communication quality of a VoIP application is characterized by them. Active measurement [2], in which probe packets are injected into a network, is one representative measurement technique for end-to-end delay.

Prior works on active measurement have left us with a rich collection of literature. Many tools that estimate end-to-end delay [3]–[5], packet loss [6], [7], available bandwidth [8], [9], link delay [10], traffic characteristics [11], node failure [12], [13] have been proposed in prior works, and large-scale measurements in real networks have allowed a better understanding of variable characteristics of the networks [14], [15]. Additionally, some works have focused on the accuracy or overhead of active measurements [16]–[21].

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A problem with active measurement, however, is that when we inject many probe packets into network (so as to improve accuracy), the load given by this probe traffic itself acts to impair path quality [22], [23]. In the active measurement of end-to-end delay, probe packet injection is treated as a method of sampling of the delay process of target path. If there were no such intrusive effect, estimation accuracy would be improved as the probe load increases. In reality, however, the transmission of probe packets itself consumes network resources. Thereby, the delays experienced by probe packets in a network with probe load tend to be larger than the virtual delay in the same network without probe load.

Needless to say, network researchers and practitioners are not interested in the delay of a network with probe load, but rather in the delay of that network without probe load. Nonetheless, because of this intrusiveness, conventional estimators based on the delay experienced by probe packets estimates larger value than the true value (i.e., the virtual delay of the network without probe load). In other words, conventional estimators of delay are biased. Baccelli et al. [22] clarified the utility of Poisson Arrivals See Time Average (PASTA) when we measure the delay without bias. They indicated that the delay experienced by probe packets corresponds to the virtual delay of the network if the injecting time of probe packets follows Poisson arrivals. Note, however, that under this treatment, the obtained estimation is the mean delay of a network with probe load. Roughan [23] discusses accuracy in terms of the variances of conventional estimators but does not consider the bias. Accordingly, discussion is needed in which such bias is taken into consideration.

In this paper, we first show that the conventional estimation of delay has a fundamental accuracy bound due to bias. To evaluate the effect of bias on accuracy, we calculate the Mean Squared Error (MSE) of a conventional estimator for the number of packets in a router modeled by M/M/1. Through this evaluation, we show that MSE has a lower bound varying the probing rate.

To transcend this MSE bound, we next propose INTrusiveness-aware ESTimation (INTEST), an approach that allows us to estimate the virtual delay of a wired network without probe load from the delay of that same network with probe load. INTEST achieves the estimation by compensating for the increased delay brought about by the load imposed by probe traffic. Performing simulations with single M/M/1 queuing, MMPP/M/1, and a multi-hop network, we show that INTEST enables more accurate estimation of delay quantiles than conventional estimator.

The remainder of this paper is organized as follows. Section 2 summarizes previous works regarding active measurements of delay. We formulate the intrusiveness problem of active measurement in Sect. 3. In Sect. 4, by theoretically calculating MSE, we show that there exists a lower bound of MSE when estimating the number of packets in a router modeled by M/M/1 queuing. In Sect. 5, we summarize INTEST, and in Sect. 6, we extend it for multi-hop networks. We evaluate it through simulations in Sect. 7. Finally, we conclude the paper and present issues for future research in Sect. 8.

## 2. Related Works

As mentioned above, it is important to estimate high quantile of end-to-end delay of networks, though accurate estimation is a difficult task. Some prior works [17], [20] have tried to estimate high quantile of end-to-end delay. Choi et al. [17] has proposed a scheme that estimates a high quantile with bounded error. The scheme allows us to know the minimum number of probe packets needed to bound the error of quantile estimation within a prescribed accuracy. In [17], the error of the quantile estimation is theoretically obtained as a function of the number of probe packets. Therefore, we can optimize the measurement overheads of active measurement using the function. Sommers [20] also have proposed an estimator of high quantile. Since the estimator provides confidence intervals, we can tune the number of probe packets to achieve the required accuracy. The load of probe packets is, however, not taken into consideration in [17], [20]. These works cannot transcend the fundamental accuracy bound due to bias.

There are a few works that mention the effect of the probe traffic on the path quality. Baccelli et al. [22] have shown that an arrival process of the probe packets affects an estimator. In [22], Baccelli et al. indicated that the mean delay experienced by probe packets is equal to the mean delay of the network when the injected probe packets follow Poisson arrival. If we inject the probe packets upon the other arrival processes, the mean delay experienced by probe packets does not correspond to the mean virtual delay of the network. The estimator in the work, however, cannot estimate the delay of a network without probe load though it estimates the delay of a network with probe load. Roughan [23] has indicated that the variance of an estimator is increased by both excessive and insufficient numbers of probe packets. The excess of the number of probe packets increases the total traffic, and the deficiency of that leads lack of samples. The optimal probe rate is theoretically derived for the router that is modeled by M/M/1 queuing in [23]. Roughan [23] has not taken the accuracy bound due to the bias into consideration, though he has shown the accuracy bound of active measurement due to the variance.

Aida et al. [24] proposed a technique that transforms virtual delay into the delay experienced by user packets. Aside from reference [24], the works [17], [20], [22], [23] measure virtual delay of the network. If the probe packets follow Poisson arrival, the mean delay experienced by probe packets corresponds to the mean virtual delay of the network [22]. Even if we inject the probe packets upon Poisson arrival, the delay experienced by probe packets does not correspond to the delay experienced by user packets unless user packets follow Poisson arrival. By using the technique of [24], we can estimate delay experienced by user packets from virtual delay measured by probe packets. Needless to say, accuracy of a delay estimation of user packets improves as the accuracy in estimating the virtual delay improves.

## 3. Formulation of the Intrusiveness Problem

First of all, we formulate the intrusiveness problem of active measurement, and show how we evaluate the effect of bias on accuracy. We define the target traffic as a traffic that streams from an edge node $a$ to an edge node $b$ of a network. We let $D_{\mathrm{g}}(t)$ [sec] denote a virtual one-way delay from $a$ to $b$ at time $t$. The subscript g indicates that the process represents the so-called ground truth of the delay. We define cross traffic as the traffic that excludes the target traffic from the whole traffic in the network. $D_{\mathrm{g}}(t)$ is the virtual delay of the network without probe load. If applying an active measurement to determine a characteristic of the virtual delay $D_{\mathrm{g}}(t)$, we add probe traffic to the traffic that streams from the edge node $a$ to $b$.

Similarly, we let $D_{\mathrm{gp}}(t)$ [sec] denote a virtual one-way delay from $a$ to $b$ on the network with probe load at time $t$, where the subscript gp stands for ground truth and probe traffic. Note that $D_{\mathrm{gp}}(t)$ differs from $D_{\mathrm{g}}(t)$. Our goal is to accurately estimate a characteristic of $D_{\mathrm{g}}(t)$ from the active measurement.

Based on delays that are experienced by $n$ probe packets, we estimate the mean, quantile, and other statistics about the one-way delay. We let $T_i\,(i = 1, \ldots, n)$ denote the time at which $i$th probe packet is injected into the network (i.e., the packet is dequeued from the queue of source side edge node $a$). The delay that is experienced by $i$th probe packet is expressed by $D_{\mathrm{gp}}(T_i)$. When estimating the mean virtual delay, the conventional estimator is

$$\hat{D}_{\mathrm{ave}} = \frac{1}{n}\sum_{i=1}^{n} D_{\mathrm{gp}}(T_i). \tag{1}$$

Furthermore, when we estimate the $q$-quantile of delay, we define $\kappa(q) = \lceil (1-q)n \rceil$, where $\lceil\cdot\rceil$ denotes the ceiling function. Then, the conventional estimator for the $q$-quantile is the $\kappa(q)$th largest value $\hat{D}_{\mathrm{qua}}(q)$ among the delays $D_{\mathrm{gp}}(T_i)$ experienced by the probe packets [20].

In this paper, we consider the case that network researchers and practitioners estimate the characteristics of $D_{\mathrm{g}}(t)$. When they measure characteristics of a network, they perform active measurement temporally. In this situation, they want to estimate the characteristics in an usual state of the network. Conversely, in a part of applications where probe packets are always injected into a network (e.g., SLA monitoring), we may consider that they want to estimate the characteristics of $D_{\mathrm{gp}}(t)$. This type of continuous measurements are out of scope of this paper.

Since $D_{\mathrm{gp}}(t) \neq D_{\mathrm{g}}(t)$, $\hat{D}_{\mathrm{ave}}$ and $\hat{D}_{\mathrm{qua}}(q)$ are generally not unbiased. A delay experienced by a probe packet is not much affected by other probe packets when the number of probe packets $n$ is small. However, if $n$ is small, the estimator variance is large (and accurate estimation is difficult) since there are only a few samples. On the other hand, if $n$ is large, the accuracy of the estimator increasingly suffers from bias because of the delays experienced by other probe packets. In short, there is a trade-off between the variance and bias of an estimator.

A biased estimator can be assessed by MSE, which is a statistic composed of the variance and bias of an estimator. The MSE of an arbitrary estimator $\hat{P}$ is defined as follows:

$$\mathrm{Var}[\hat{P}] + \{\mathrm{E}[\hat{P}] - P^*\}^2 = \mathrm{E}[(\hat{P} - P^*)^2], \qquad (2)$$

where $P^*$ denotes the true value of the virtual delay.

## 4. A Fundamental Bound on the Accuracy of Active Measurement

By analyzing a router modeled by M/M/1 queuing, we show that the conventional estimation by active measurement of delay is restricted by a fundamental bound on the accuracy due to bias. Here, we evaluate the accuracy of estimation by MSE. We verify the dependence of the accuracy on probing rate by plotting the MSE of the estimator as a function of probing rate.

To theoretically verify the dependence of accuracy on probing rate while taking such bias into consideration, we consider MSE regarding the number of packets in a router modeled by M/M/1 queuing. In this first analysis, we estimate the mean or quantile of the number of packets in a router, and next we will estimate the delay. We assume that target traffic packets and probe packets are generated according to Poisson processes with rate $\lambda_{\mathrm{g}}$ [packet/sec] and $\lambda_{\mathrm{p}}$ [packet/sec], respectively. Because the superposition of two Poisson process is also Poisson, the queue is modeled by M/M/1. The service rate of the M/M/1 queue is $\mu$ [packet/sec]. We estimate the mean number of packets in the router without probe load by the sample average $\hat{M}_{\mathrm{ave}} = n^{-1} \sum_{i=1}^{n} M_{\mathrm{gp}}(T_i)$, where $M_{\mathrm{gp}}(T_i)$ [packet] denotes the number of packets in the router at time $T_i$. Note that the true value of the estimator is the mean number of packets in a router without probe load, though $M_{\mathrm{gp}}(T_i)$ is the number of the packets in a router with probe load. Under active measurement, we cannot observe the number of packets in a router. However, we can analytically derive the MSE of the estimator regarding the number of packets in a router, and the results provide good insights on accuracy characteristics.

We will calculate the MSE by using the autocovariance function $r(\rho, \tau)$ of the process $M(t)$ of the number of packets of M/M/1 queue. $r(\rho, \tau)$ is given by

$$\begin{aligned} r(\rho, \tau) &= \mathrm{E}\left[(M(t) - \mathrm{E}[M(t)])(M(t+\tau) - \mathrm{E}[M(t)])\right] \\ &\simeq \frac{\rho}{2(1-\rho)^2}\left(\mathrm{e}^{-A(\rho)|\tau|} + \mathrm{e}^{-B(\rho)|\tau|}\right), \end{aligned}$$

$$A(\rho) = \frac{(1-\rho)^2}{1+\rho+\sqrt{\rho}}, \qquad B(\rho) = \frac{(1-\rho)^2}{1+\rho-\sqrt{\rho}},$$

where $\rho$ denotes the utilization of M/M/1 queue [23], [25].

The variance $\sigma^2 = \mathrm{Var}[\hat{M}_{\mathrm{ave}}]$ that corresponds to $\mathrm{Var}[\hat{P}]$ in Eq. (2) can be expressed by $r(\rho, \tau)$ and $\rho_{\mathrm{gp}} \equiv (\lambda_{\mathrm{g}} + \lambda_{\mathrm{p}})/\mu$ as

$$\begin{aligned} \sigma^2 &= \frac{1}{n} r(\rho_{\mathrm{gp}}, 0) + \frac{1}{n^2} \sum_{i \neq j} \mathrm{E}[r(\rho_{\mathrm{gp}}, |T_i - T_j|)] \\ &= \frac{1}{n} r(\rho_{\mathrm{gp}}, 0) + \frac{1}{n^2} \sum_{i \neq j} \int_0^\infty r(\rho_{\mathrm{gp}}, t) f(t, |i-j|, 1/\lambda_{\mathrm{p}}) \mathrm{d}t \\ &\simeq \frac{\rho_{\mathrm{gp}}}{n(1-\rho_{\mathrm{gp}})^2} + \frac{2}{n^2} \sum_{k=1}^{n} \left\{ \frac{\rho_{\mathrm{gp}}(n-k)}{2(1-\rho_{\mathrm{gp}})} \right. \\ &\qquad \left. \times \left( \frac{(\lambda_{\mathrm{p}})^k}{\lambda_{\mathrm{p}} + A(\rho_{\mathrm{gp}})^k} + \frac{(\lambda_{\mathrm{p}})^k}{\lambda_{\mathrm{p}} + B(\rho_{\mathrm{gp}})^k} \right) \right\}, \end{aligned}$$

where $f(t, k, \alpha)$ denotes the probability density function of an Erlang distribution with shape parameter $k$ and scale parameter $\alpha$ [23]. On the other hand, the bias $\varepsilon$ that corresponds to $\mathrm{E}[\hat{P}] - P^*$ in Eq. (2) can be expressed in terms of $\rho_{\mathrm{g}} \equiv \lambda_{\mathrm{g}}/\mu$ and $\rho_p \equiv \lambda_{\mathrm{p}}/\mu$ as

$$\varepsilon = \frac{\rho_{\mathrm{gp}}}{1-\rho_{\mathrm{gp}}} - \frac{\rho_{\mathrm{g}}}{1-\rho_{\mathrm{g}}} = \frac{\rho_p}{(1-\rho_{\mathrm{g}})(1-\rho_{\mathrm{gp}})}.$$

As a result, we can calculate MSE as $\sigma^2 + \varepsilon^2$.

Substituting concrete values into the MSE variables and varying the number $n$ of probe packets, we derive the MSE relation shown in Fig. 1. Assuming the link capacity and the mean packet length to be 155.52 [Mbps] and 600 [byte], respectively, we have $\mu = 32400$ [packet/sec]. The link utilization $\rho_{\mathrm{g}}$ of the target traffic is set to 0.9. We set the mean measurement period $l$ to 1.0 [sec]. To inject $n$ probe packets in the period, we set probing rate $\lambda_{\mathrm{p}}$ to $n/l$. We will finish the measurement by the $n$th probe packet. Note that injecting time of the $n$th probe packet is not always $l$ since injecting time follows Poisson arrivals though the mean time is $l$. Note that in Fig. 1, the horizontal axis represents the ratio of probe load to the link capacity.

In Fig. 1, we see that MSE has a lower bound (i.e., a limit of accuracy). When the ratio $\rho_{\mathrm{p}}$ of probe load to the link capacity is low, the variance $\sigma^2$ is large, hence MSE is large. This is because we cannot obtain sufficient sampling. When $\rho_{\mathrm{p}}$ is large, MSE also becomes large, even though the variance is low, because the bias $\varepsilon$ is large. The lower bound of MSE is around 1.5 [packet$^2$]. We cannot obtain a more accurate estimation by increasing nor decreasing the number of probe packets.

Furthermore, in practical measurements, it is very difficult to determine the optimal probe load because of a dependency on the ratio $\rho_{\mathrm{g}}$ of traffic load to link capacity. The

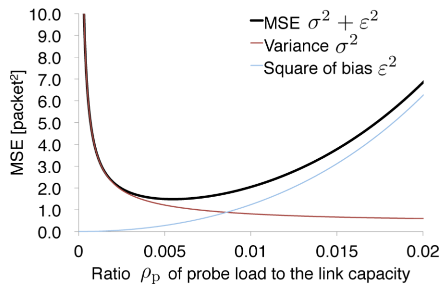


**Fig. 1** MSE of an estimator for the number of packets in a router modeled by M/M/1 queuing.

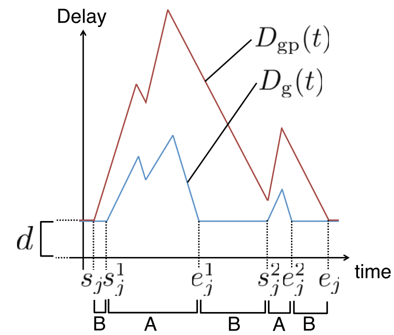


**Fig. 2** Relationship between $D_{\mathrm{g}}(t)$ and $D_{\mathrm{gp}}(t)$.

process of the number of packets in a router may not correspond to that of delay, as mentioned above. The delay is, however, proportional to the number of packets, in the queue and we confirm that similar results are obtained upon delay estimation based simulation (see Sect. 7).

## 5. Intrusiveness-Aware Estimation

We here propose INTEST, an approach that compensates for delays produced by probe packets on wired networks and transcends the accuracy bound mentioned in Sect. 4. INTEST estimates the delay $D_{\mathrm{g}}(t)$ of the network without probe load from the delay $D_{\mathrm{gp}}(T_i)$ of the network with probe load. This estimation is done by subtracting the delay produced by the load given by probe traffic. Note that $D_{\mathrm{gp}}(t)$ is always larger than $D_{\mathrm{g}}(t)$ as shown in Fig. 2.

### 5.1 Delay in Busy Periods

We first consider a network in which a single router with FIFO queuing is connected between two edges $a$ and $b$, and the delay from the egress port of $a$ to the ingress port of $b$ is measured. We clarify the relationship between the delay $D_{\mathrm{g}}(t)$ of the network without probe load and the delay $D_{\mathrm{gp}}(t)$ of the network with probe load. Letting $c$ [bps] denote the link capacity, we define the amount of data $B_{\mathrm{gp}}(t)$ [bit] in the queue at time $t$ as

$$B_{\mathrm{gp}}(t) = \begin{cases} \lim\limits_{\tau\to t-0} \sum\limits_{h=1}^{M_{\mathrm{gp}}(\tau)} x_h - c(t-u_0(t)), & (M_{\mathrm{gp}}(t)>0), \\ 0, & (M_{\mathrm{gp}}(t)=0), \end{cases} \tag{3}$$

where $x_h$ [bit] ($h = 1,2,\ldots,M_{\mathrm{gp}}(t)$) denotes the length of $h$th packets in the queue and $u_0(t)$ denotes the transmission start time of a packet transmitting at time $t$. A delay $D_{\mathrm{gp}}(t)$ that is experienced by a packet injected at time $t$ into the network is related to $B_{\mathrm{gp}}(t)$ as $D_{\mathrm{gp}}(t) = B_{\mathrm{gp}}(t)/c + d$, where $d$ denotes a propagation delay of end-to-end path (i.e., $d$ is the sum of the propagation delays on the two link).

Letting an interval $[s,e)$ denote a busy period (i.e., a maximal interval where the number of packets in the router is positive), amounts of data $B_{\mathrm{gp}}(t_1)$ and $B_{\mathrm{gp}}(t_2)$ at times $t_1$ and $t_2$ ($s \le t_1 \le t_2 < e$) are related as follows.

$$B_{\mathrm{gp}}(t_2) = B_{\mathrm{gp}}(t_1) + X_{\mathrm{gp}}(t_1,t_2) - c(t_2-t_1),$$

where $X_{\mathrm{gp}}(t_1,t_2)$ [bit] denotes the total amount of traffic arrived in interval $[t_1,t_2)$. We let $[s_j,e_j)$ denote the start and end time of the $j$th busy period of the router with probe load (see Fig. 2). We derive the following equation regarding the amount of data $B_{\mathrm{gp}}(t)$ [bit] in the network with probe load.

$$B_{\mathrm{gp}}(T_i) = B_{\mathrm{gp}}(T_{i-1}) + X_{\mathrm{g}}(T_{i-1},T_i) + x_{i-1}^{\mathrm{p}} - c(T_i-T_{i-1}), \quad (i \in \{\iota \,|\, s_j \le T_{\iota-1}, T_\iota < e_j\}).$$

$$X_{\mathrm{g}}(T_{i-1},T_i) = B_{\mathrm{gp}}(T_i) - B_{\mathrm{gp}}(T_{i-1}) - x_{i-1}^{\mathrm{p}} + c(T_i-T_{i-1}), \quad (i \in \{\iota \,|\, s_j \le T_{\iota-1}, T_\iota < e_j\}). \tag{4}$$

Here, $X_{\mathrm{g}}(t_1,t_2)$ [bit] denotes the total amount of cross and target traffic that stream the same path with the probe packets in an interval $[t_1,t_2)$ and $x_i^{\mathrm{p}}$ [bit] denotes the length of $i$th probe packet. Note that $B_{\mathrm{gp}}(T_i)$ does not include probe packet injected at time $T_i$ since $B_{\mathrm{gp}}(t)$ is defined by Eq. (3). The last term on the right-hand side of Eq. (4) represents the amount of transmitted data in $[T_{i-1},T_i)$. Since we assume that $[s_j,e_j)$ is a busy period, the amount of transmitted data can be expressed as the product of the capacity $c$ and time $T_i - T_{i-1}$.

We can clarify relationship between $D_{\mathrm{g}}(T_i)$ and $D_{\mathrm{gp}}(T_i)$ since a similar relationship holds for $B_{\mathrm{g}}(t)$ [bit] in $[s_j^k, e_j^k)$ ($k = 1,2,\ldots,m_j$) (intervals A in Fig. 2). Here, we let $s_j^k (k = 1,2,\ldots,m_j)$ and $e_j^k$ denote the start and end times of the $k$th busy period of the router without probe load within an interval $[s_j,e_j)$. $m_j$ denotes the number of the busy periods of the router without probe load within an interval $[s_j,e_j)$. Then, we have

$$B_{\mathrm{g}}(T_i) = B_{\mathrm{g}}(T_{i-1}) + X_{\mathrm{g}}(T_{i-1},T_i) - c(T_i-T_{i-1}), \tag{5}$$

for $i \in \{\iota \,|\, s_j^k \le T_{\iota-1}, T_\iota < e_j^k\}$ ($T_i$ and $T_{i-1}$ are in one of the intervals A in Fig. 2). The amount of transmitted data can be expressed as the product of the capacity $c$ and time $T_i - T_{i-1}$ as similar to Eq. (4), since $T_{i-1}$ and $T_i$ are in a busy period $[s_j^k, e_j^k)$. By substituting Eq. (4) into Eq. (5), we obtain

$$B_{\mathrm{g}}(T_i) = B_{\mathrm{g}}(T_{i-1}) + B_{\mathrm{gp}}(T_i) - B_{\mathrm{gp}}(T_{i-1}) - x_{i-1}^{\mathrm{p}}, \tag{6}$$

for $i \in \{\iota \,|\, s_j^k \le T_{\iota-1}, T_\iota < e_j^k\}$. Dividing the both sides of Eq. (6) by $c$,

$$D_g(T_i) = D_g(T_{i-1}) + D_{gp}(T_i) - D_{gp}(T_{i-1}) - \frac{x_{i-1}^p}{c} \tag{7}$$

for $i \in \{\iota \,|\, s_j^k \le T_{\iota-1}, T_\iota < e_j^k\}$. Note that each busy period for the router without probe traffic is included in a busy period for the router with probe traffic.

$D_g(T_i)$ is equal to $d$ for $i \in \{\iota \,|\, s_j \le T_\iota < s_j^1\} \bigcup \{\iota \,|\, e_j^k \le T_\iota < s_j^{k+1}\} \bigcup \{\iota \,|\, e_j^{m_j} \le T_\iota < e_j\}$ ($T_i$ is in one of the intervals B in Fig. 2). Since the right hand side of Eq. (5) is less than 0 when $T_i$ is in one of the intervals, the right hand side of Eq. (7) is less than $d$. Therefore, when $T_i$ is in one of the intervals A or $T_i$ is in the intervals B, the equation,

$$D_g(T_i) = \max\Big(D_g(T_{i-1}) + D_{gp}(T_i) - D_{gp}(T_{i-1}) - \frac{x_{i-1}^p}{c}, d\Big), \tag{8}$$

holds. We consider that Eq. (8) approximately holds when $T_i$ is in A but $T_{i-1}$ is in B. In this case, the relation between $B_g(T_i)$ and $B_g(T_{i-1})$ cannot express as Eq. (5) by using total amount of traffic $X_g(T_{i-1}, T_i)$. If $T_{i-1}$ is in $[e_j^k, s_j^{k+1})$ and $T_i$ is in $[s_j^{k+1}, e_j^{k+1})$,

$$B_g(T_i) = B_g(T_{i-1}) + X_g(s_j^{k+1}, T_i) - c(T_i - s_j^{k+1}). \tag{9}$$

Eq. (9) does not correspond to Eq. (5). The approximation is reasonable when we can assume that the traffic including cross and target traffic have a constant rate $X_g(T_{i-1}, T_i)/(T_i - T_{i-1}) > c$ in an interval $[T_{i-1}, T_i)$. If the traffic have a constant rate, $B_g(t)$ increases $X_g(T_{i-1}, T_i)/(T_i - T_{i-1}) - c$ bit per second. Hence, $B_g(T_i) = X_g(T_{i-1}, T_i) - c(T_i - T_{i-1})$ and Eq. (5) hold since $B_g(T_{i-1}) = 0$. As a result, Eq. (8) means that we can estimate $D_g(T_i)$ from delays $D_{gp}(T_i)$ and $D_{gp}(T_{i-1})$ experienced by probe packets.

### 5.2 Estimator

In practical measurements, we must estimate a start time $s_j$ of a busy period, an end time $e_j$ of the busy period, and a propagation delay $d$. $s_j$, $e_j$, and $d$ are also possible to obtain $D_{gp}(T_i)$ as a delay experienced by a probe packet. A propagation delay $d$ can be estimated by $\hat{d}$, the minimum delay of delays experienced by probe packets. By using threshold $\delta$, we can detect start of busy periods by $U = \{T_i \,|\, D_{gp}(T_i) \le \hat{d} + \delta < D_{gp}(T_{i+1})\}$. Hence, $\hat{s}_j$ that is a $j$th smallest element of $U$ is an estimator of $s_j$. Similarly, $\hat{e}_j$ that is $j$th smallest element of $V = \{T_i \,|\, D_{gp}(T_{i-1}) \ge \hat{d} + \delta > D_{gp}(T_i)\}$ is an estimator of $e_j$. The threshold $\delta$ affects the performance of INTEST. Since we should tune the value of $\delta$ appropriately, we discuss the sensitivity of $\delta$ to the performance later.

The link capacity $c$ should also be estimated if the INTEST is used by a person who is not a network manager. The difference $(T_{i+1} + D_{gp}(T_{i+1})) - (T_i + D_{gp}(T_i))$ between receiving times of consecutive probe packets is longer than the transmission time of the traffic that arrives between the arrival times of the two packets. Hence, with regards to $c$ in one hop networks, the following inequality holds:

$$\frac{x_i^p + X_g(T_i, T_{i+1})}{c} \le (T_{i+1} + D_{gp}(T_{i+1})) - (T_i + D_{gp}(T_i)).$$

Since $X_g(T_i, T_{i+1}) \ge 0$, the estimator $\hat{c}$ of $c$ is

$$c \ge \frac{x_i^p}{(T_{i+1} + D_{gp}(T_{i+1})) - (T_i + D_{gp}(T_i))}$$

$$\hat{c} = \max_{1 \le i \le n-1}\left(\frac{x_i^p}{T_{i+1} - T_i + D_{gp}(T_{i+1}) - D_{gp}(T_i)}\right). \tag{10}$$

Needless to say, we should $\hat{c} = c$ when we can use the information of the link capacity.

Based on Eq. (8), $\hat{s}_j$, $\hat{e}_j$, $\hat{d}$, and $\hat{c}$, we can estimate a delay $D_g(T_i)$ of a network without probe load from a delay $D_{gp}(T_i)$ of a network with probe load. Considering the estimator of $D_g(T_i)$ should be $D_{gp}(T_i)$ if $T_i$ is in an idle period (i.e., a maximal interval where the number of packets in a router is 0), the estimator of $D_g(T_i)$ $(1 < i \le n)$ is given by

$$\hat{D}_g(T_i) = \begin{cases} \max\Big(\hat{D}_g(T_{i-1}) + D_{gp}(T_i) - D_{gp}(T_{i-1}) - \dfrac{x_{i-1}^p}{\hat{c}}, \hat{d}\Big), \\ \qquad\qquad \hat{s}_j < T_i \le \hat{e}_j \text{ for some } j \\ D_{gp}(T_i), \qquad\qquad \text{otherwise.} \end{cases} \tag{11}$$

Note that the first case of the estimator is the equation in which $D_g(t)$, $d$, and $c$ are replaced by $\hat{D}_g(t)$, $\hat{d}$, and $\hat{c}$ in Eq. (8), respectively. According to the definition of $\hat{s}_j$, $D_g(T_i) = D_{gp}(T_i)$ when $i = 1$. We can estimate an average delay of a network without probe load by $\sum_{i=1}^n \hat{D}_g(T_i)/n$. The $\kappa(q)$th largest value $\hat{D}_{qua}(q)$ within the values of $\hat{D}_g(T_i)$ is the $q$-quantile estimator of a network without probe load.

INTEST is applicable for any model of probe and target traffic. INTEST can provide non-intrusive samples $\hat{D}_g(T_i)$ of a virtual delay process if an arrival process of probe packets satisfies $T_1 < T_2 < \cdots < T_n$. Therefore, we can use any arrival process in a large set of arrival processes that satisfy the assumption of NIMASTA (Non-Intrusive Mixing Arrivals See Time Averages) [22] (e.g., Poisson arrivals, a process with inter-arrival time that follows Gamma distribution, etc.). Moreover, we can estimate a characteristic of virtual delay process $D_g(t)$ for any arrival process of packets of target traffic. Our purpose is to accurately estimate a characteristic of a virtual delay process. A characteristic of virtual delay process does not correspond to that of target traffic if an arrival process of packets of target traffic is not Poisson arrivals [22]. However, the technique that transforms virtual delay to delay of target traffic has been proposed in a previous work [24].

## 6. Extension for Multi-Hop Network

We can extend INTEST to estimate the delay in multi-hop networks composed of multiple routers. In Eq. (11), we use the information regarding link capacity $\hat{c}$ in the case of busy periods, i.e., $[s_j, e_j)$. When we use INTEST in multi-hop network, we have to find the congested link that causes each busy period since link capacity is different in each link. There are two options to extend INTEST to multi-hop networks.

### 6.1 Use of Timestamp

First option is to utilize timestamps provided by each router. Letting $T_i^k$ denote a timestamp of $k$th router ($k = 1, \ldots, K$) recorded on $i$th probe packet, we can estimate a queuing delay of $k$th router by $D_{\rm gp}^k(T_i) = T_i^{k-1} - T_i^k - \min_{1 \le l \le n}(T_l^k - T_l^{k-1})$, where $n$ denotes the total number of packets. As we defined in Sect. 3, $T_i$ is the time at which $i$th probe packet is dequeued from the queue of source side edge node. Note that we do not require time synchronization of routers since we measure queueing delay $D_{\rm gp}^k(T_i)$. Replacing $D_{\rm gp}(T_i)$ in Eqs. (10) and (11) with $D_{\rm gp}^k(T_i)$ and setting $\hat{d}$ in Eq. (11) to 0, we can estimate a queuing delay of $k$th router without probe load.

$$c_k \ge \frac{x_i^{\rm p}}{(T_{i+1}^k + D_{\rm gp}^k(T_{i+1})) - (T_i^k + D_{\rm gp}^k(T_i))}$$

$$\hat{c}_k = \max_{1 \le i \le n-1} \left( \frac{x_i^{\rm p}}{T_{i+1}^k - T_i^k + D_{\rm gp}^k(T_{i+1}) - D_{\rm gp}^k(T_i)} \right)$$

$$\hat{D}_{\rm g}^k(T_i) = \begin{cases} \max\left(\hat{D}_{\rm g}^k(T_{i-1}) + D_{\rm gp}^k(T_i) \right. \\ \quad \left. - D_{\rm gp}^k(T_{i-1}) - \frac{x_{i-1}^{\rm p}}{\hat{c}_k}, 0\right), & \hat{s}_j^k < T_i \le \hat{e}_j^k \\ D_{\rm gp}^k(T_i), & \text{otherwise,} \end{cases}$$

where $c_k$ [bit] and $\hat{c}_k$ [bit] denote link capacity of a link between $k$th and $(k+1)$th routers and its estimator. $\hat{D}_{\rm gp}^k(T_i)$ [sec] denotes an estimator of queueing delay of the $k$th router recorded on the $i$th probe packet. $\hat{s}_j^k$ and $\hat{e}_j^k$ are the estimators of the start and end time of the $j$th busy period of $k$th router, and they are estimated in a similar manner as $\hat{s}_j$ and $\hat{e}_j$. The estimator $\hat{D}_{\rm g}^{\rm ts}(T_i)$ of end-to-end delay of a network without probe load when we utilize timestamp is as follows:

$$\hat{D}_{\rm g}^{\rm ts}(T_i) = \hat{d} + \sum_{k=1}^{K} \hat{D}_{\rm g}^k(T_i). \tag{12}$$

By using Eq. (12), we can estimate an end-to-end delay of a multi-hop network without probe load. Note that $\hat{D}_{\rm g}^k(T_i)$ [bit] denotes an estimator of queueing delay, and it does not include propagation delay. Ping tool used ICMP packet provides an option to record a timestamp of intermediate routers using header space (The number of hops is limited to 9 hops in Linux ping tool). On the other hand, we can record timestamps at intermediate routers or capture cards on network.

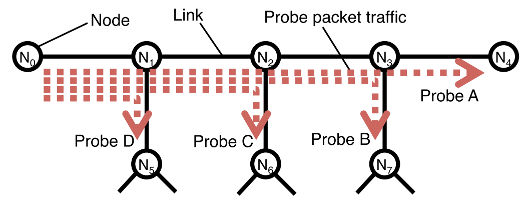


**Fig. 3** Probe flow design for a multi-hop measurement.

### 6.2 Use of a Multi-Flow Probe

To perform a measurement of multi-hop networks using INTEST, we have a second option in which measurement results of the multi-flow probe are utilized. In multi-flow approach, we design paths of probe packets as a tree (see Fig. 3). This type of path design of probe flow is commonly used in network tomography [26], [27]. Since Eq. (11) contains link capacity $\hat{c}$, we have to specify a congested link to compensate for the increased delay brought about by the load. We can specify the congested link by comparing delay patterns of probe flows, assuming that a path contains at most one congested link at the same time. For example, in Fig. 3, when large delay is observed by probe $A$ and $B$ (probe $C$ and $D$ do not observe a large delay), we can specify congested link is the link between $N_2$-$N_3$.

In multi-flow approach, we measure end-to-end delay of each paths of probe flows, and estimate start and end time of congestion for each paths. Let $r_L$ denote $L$ hop path that we want to measure end-to-end delay by INTEST. We let $r_l$ $(1 \le l < L)$ denote paths that share first $l$ hop path with $r_L$. We measure end-to-end delay on $r_1, r_2, \ldots, r_L$ by probe packets. Note that these end-to-end delays are increased by the load of probe traffic. Here, we define busy periods on a path as periods in which at least one router on the path is in a busy period. In the same manner as estimator of $\hat{s}_j$ and $\hat{e}_j$, we estimate start time $\hat{s}_{j,l}$ and end time $\hat{e}_{j,l}$ of $j$th busy period on path $r_l$.

Comparing the estimated busy periods on each paths, we specify congested link corresponds to each busy periods on path $r_L$. Let $l_{\rm min}^j$ denote minimum $l$ that satisfies $|\hat{s}_{j,L} - \hat{s}_{j',l}| < \Delta$ and $|\hat{e}_{j,L} - \hat{e}_{j',l}| < \Delta$ $(1 \le j')$ for a $j$th busy period on the path $r_L$. The threshold $\Delta$ should be proportional on the mean of probe generation intervals since large probe interval degrades the estimation accuracy of start and end time of busy periods. It is expected that $r_l$ that satisfies $|\hat{s}_{j,L} - \hat{s}_{j',l}| < \Delta$ and $|\hat{e}_{j,L} - \hat{e}_{j',l}| < \Delta$ $(1 \le j')$ have a common busy period and a common router of congested link. The router of congested link is in common routers on these paths. We can specify congested link is $l_{\rm min}^j$th link for $j$th busy period on $r_L$. Let $c^j$ [bps] denote bandwidth $c_{l_{\rm min}^j}$ of $l_{\rm min}^j$th link.

Therefore, we can obtain the following estimator for multi-hop network.

$$\hat{D}_{\rm g}(T_i) = \begin{cases} \max\left(\hat{D}_{\rm g}(T_{i-1}) + D_{\rm gp}(T_i) - D_{\rm gp}(T_{i-1}) - \dfrac{X^{\rm p}_{i-1}}{c^j}, \hat{d}\right), \\ \qquad\qquad \hat{s}_{j,L} < T_i \leq \hat{e}_{j,L} \text{ for some } j \\ D_{\rm gp}(T_i), \qquad\qquad \text{otherwise}, \end{cases} \tag{13}$$

where $X^{\rm p}_i$ [bit] denotes the sum of the length of probe packets that were injected in $[T_i, T_{i+1})$.

In some cases, it is difficult to measure end-to-end delay on the path that share first $l'$ hops with $r_L$. If we cannot measure end-to-end delay on the path that share first $l'$ hop path with $r_L$, we consider the $l'$th link and $(l'+1)$th link as an virtual link with bandwidth $\max(c_{l'+1}, c_{l'})$. In this case, if $l^j_{\rm min} = l' + 1$ for $j$th busy period, $c^j = \max(c_{l'+1}, c_{l'})$ in Eq. (13). We cannot distinguish which one is the congested link between $l'$th and $l'+1$th links without delay of $r_{l'}$. By using $\max(c_{l'+1}, c_{l'})$ as $c^j$, since both of $X^{\rm p}_{i-1}/c_{l'}$ and $X^{\rm p}_{i-1}/c_{l'+1}$ are greater than or equal to $X^{\rm p}_{i-1}/\max(c_{l'+1}, c_{l'})$, we underestimate the compensated delay, thereby returning closer estimator to the conventional estimator.

## 7. Evaluation

We next evaluate the performance of INTEST through simulations to confirm that it can produce more accurate estimates than conventional active measurement. Through simulation, we first evaluate INTEST in a single-hop network modeled by M/M/1. Because the theoretical value of MSE of an estimator can be derived in M/M/1 queueing, we estimate the number of packets in a router in addition to delay when evaluating the single-hop network. We then move on to a valuation of INTEST as applied to a multi-hop network.

### 7.1 A Network Composed of Single Router

#### 7.1.1 A Estimation of the Number of Packets in a Router

We evaluate INTEST in a single-hop network composed of a router that is modeled by M/M/1 queuing, and show that INTEST can produce more accurate estimations than conventional active measurement can. So as to clarify the fundamental characteristics of INTEST through a simple scenario, we focus here on the average number of packets in a router and on quantiles of delay. In a simulation of M/M/1 model, we can compare the simulation results with the theoretical results shown in Sect. 4, though M/M/1 model is not a realistic model of a router. By comparing both results, we can confirm the estimator behaves as expected. Note that the estimator shown in Sect. 5 does not assume M/M/1 model, and we later evaluate the estimation on a more realistic model in Sect. 7.2.

We first perform a simulation under an assumption that we can observe the number of packets in a router at the time of the probe packet injection. The simulation is useful in evaluating the fundamental characteristics of INTEST. Note that in practice, we cannot observe the number of packets in a router by a probe packet injection. We can, however, theoretically calculate the MSE, as described in Sect. 4. Parameters for this simulation are the same as those for the simulation shown in Sect. 4.

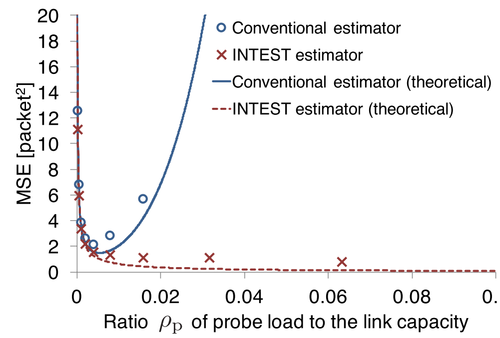


**Fig. 4** MSEs of the conventional and INTEST estimators when we estimate the number of packets in a router modeled by M/M/1 queuing.

To estimate the mean number of packets in a router, we modify the INTEST estimator of Eq. (11) as follows.

$$\hat{M}_{\rm g}(T_i) = \begin{cases} \max\left(\hat{M}_{\rm g}(T_{i-1}) + M_{\rm gp}(T_i) - M_{\rm gp}(T_{i-1}) - 1, 0\right), \\ \qquad\qquad \hat{s}'_j < T_i \leq \hat{e}'_j \text{ for arbitrary } j \\ M_{\rm gp}(T_i), \qquad\qquad \text{otherwise}, \end{cases} \tag{14}$$

where $\hat{M}_{\rm g}(t)$ [packet] is the estimator of the number of packets in a router without probe load at time $t$. Also, $\hat{s}'_j$ and $\hat{e}'_j$ that are $j$th smallest elements of $U' = \{T_i \,|\, M_{\rm gp}(T_i) < \theta \leq M_{\rm gp}(T_{i+1})\}$ and $V' = \{T_i \,|\, M_{\rm gp}(T_{i-1}) > \theta \geq M_{\rm gp}(T_i)\}$ are estimators of $s'_j$ and $e'_j$, respectively. We can obtain Eq. (14) by replacing $D_{\rm gp}(t)$, $\hat{D}_{\rm g}(t)$, $\hat{d}$, $x^{\rm p}_i/\hat{c}$, $\hat{s}_j$, and $\hat{e}_j$ with $M_{\rm gp}(t)$, $\hat{M}_{\rm g}(t)$, 0, 1, $\hat{s}'_j$, and $\hat{e}'_j$ in Eq. (11). In estimation of the number of packets in a router, we do not have to consider a propagation delay. The number of packets in a router increases by 1 if a probe packet is injected though the delay in a router increases by $x^{\rm p}_i/\hat{c}$. We set threshold $\theta$ to 7.5 [packets]. We should tune the threshold carefully to bring out the INTEST performance (we confirm the sensitivity of the accuracy on the threshold later).

By varying the number of probe packets for each simulation from 2 [packets] to 2048 [packets] (i.e., the probe load is changed from 0.0062% to 6.3%), we calculate MSE for the conventional estimator and INTEST estimator of Eq. (14). Results are shown in Fig. 4. In the calculation of MSE, we repeated the simulation 5,000 times at each number of probe packets. The theoretical values for the conventional estimator in Fig. 4 are the same as the MSE values shown in Fig. 1, and the theoretical values for the INTEST estimator are $\sigma^2$, which is the MSE for an unbiased estimator. We can confirm from Fig. 4 that the minimum value of the MSE of the INTEST estimator is smaller than that of the conventional estimator. This shows that INTEST achieves highly accurate measurement that broke the bound of conventional active measurement.

We also calculate MSE for the conventional estimator and INTEST estimator for the simulation with low link uti-

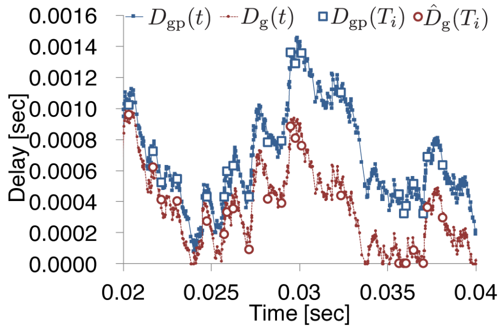


**Fig. 5** A comparison of a delay process of the network without probe load and samples $\hat{D}_g(T_i)$ by INTEST.

lization. In the above simulation, we changes the target traffic so that the link utilization is 0.5. The other conditions of the simulation are the same as the above simulation. In the simulation with low link utilization, MSEs of INTEST estimator are similar to those of conventional estimator. The deferences are lower than 0.16% of MSE of conventional estimators. When a link utilization is low, INTEST estimator behaves as a conventional estimator since the number of packets in a router is almost always lower than the threshold $\theta$. INTEST estimator can give similar performance with conventional estimator in uncongested networks in which delay measurement is not necessarily important.

### 7.1.2 A Estimation of the Delay

Next, we evaluate the network delay under the same conditions as the above simulation. To confirm that $\hat{D}_g(T_i)$—a sample of a delay process obtained by compensation with INTEST in Eq. (11)—corresponds to the delay of the network without probe load at time $T_i$, we derive a delay process of the network without probe load and compare $\hat{D}_g(T_i)$ to the process. We set the threshold $\delta$ to 0.2 [msec] when deriving $\hat{D}_g(T_i)$. We should tune the threshold carefully to bring out the INTEST performance (we confirm the sensitivity of the accuracy on the threshold later). In Fig. 5, we show $\hat{D}_g(T_i)$ and delay processes over the interval [0.02, 0.04) [sec]. In the example, the number $n$ of probe is 1024. From the figure, we note that samples $\hat{D}_g(T_i)$ of the INTEST estimator are very close to delay process $D_g(t)$ of the network without probe load, although samples $D_{gp}(T_i)$ of the conventional estimator differ greatly from the process.

Repeating similar simulations 5,000 times for each probe load, we derive the bias and MSE of the estimator at 95%-quantile of delay and evaluate the accuracy. It is well known that the cumulative distribution function of M/M/1 queuing delay is

$$F(t) = 1 - \frac{\lambda}{\mu} e^{-(\mu-\lambda)t}, \qquad (0 \leq t).$$

Then, the true value $D^*_{\rm qua}(q)$ of $q$-quantile is

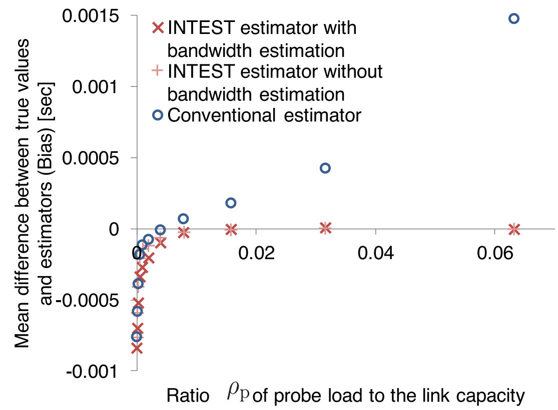


**Fig. 6** Biases of the conventional and INTEST estimators when we estimate 95%-quantile of delay of a router modeled by M/M/1 queuing.

$$D^*_{\rm qua}(q) = \begin{cases} -\dfrac{1}{\mu-\lambda} \log \dfrac{\mu}{\lambda}(1-q), & 1 - \dfrac{\lambda}{\mu} \leq q \\ 0, & \text{otherwise.} \end{cases} \qquad (15)$$

We show the bias (i.e., mean difference between the value of Eq. (15) and an estimator) of the conventional and INTEST estimator in Fig. 6. We find that the INTEST estimator can provide an unbiased estimation when the probe load is sufficiently large. The conventional estimator and INTEST estimator both show a bias when the ratio of probe load to the link capacity is small. This is because the number of probe packets, at 2 to 64 [packets], is too small. The results of INTEST estimator without bandwidth estimation is also shown in the figure for comparison. The biases of INTEST estimator without bandwidth estimation are slightly better than those of INTEST estimator with bandwidth estimation when the probe load is small. When the probe rate is too small, it is difficult to accurately estimate bandwidth since probe packet intervals are large. As for MSE, we obtained results similar to those shown in Fig. 4. We also calculate MSE for the conventional estimator and INTEST estimator for the simulation with low link utilization. In the simulation with low link utilization, MSEs of INTEST estimator are similar to those of conventional estimator.

We compare Eq. (8) and virtual delay to confirm the validity of the approximation on Eq. (8). In the simulation of M/M/1 queueing, we derive virtual delay (i.e., true values), and calculate the mean error of Eq. (8) for $i \in \{\iota | T_{\iota-1} < s^k_j \leq T_\iota < e^k_j\}$ (i.e., $T_i$ is in A but $T_{i-1}$ is in B). Note that a bandwidth is not estimated, but given for the calculation. The results are shown in Fig. 7. In the figure, mean error of a sample increase as probing rate increases. When probing rate is large, a busy period $[s_j, e_j)$ of the router with probe load includes many busy period $[s^k_j, e^k_j)$ of the router without probe load. The error can be raised for every busy period $[s^k_j, e^k_j)$ of the router without probe load. As a result, error of Eq. (8) is accumulated in a busy period $[s_j, e_j)$ of the router with probe load. Though the results of INTEST estimator in Fig. 6 include the error, the estimator is accurate than the conventional estimator even if the probe load is high. The results in Fig. 6 indicate that the error can be negligible with

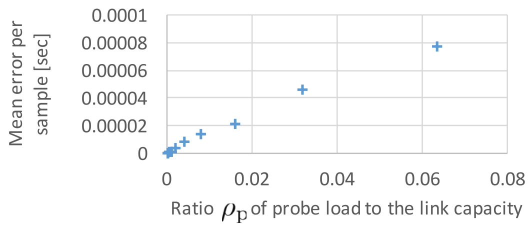


**Fig. 7** Mean error of Eq. (8).

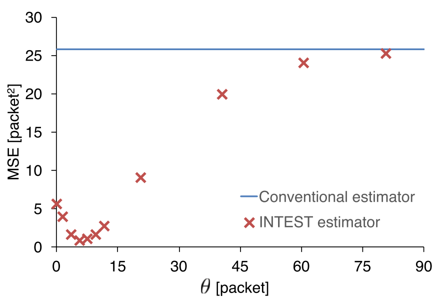


**Fig. 8** Sensitivity of the performance of INTEST on the threshold $\theta$ when we estimate the number of packets in a router modeled by M/M/1 queuing.

respect to the bias of conventional estimators.

### 7.1.3 The Sensitivity of Performance on the Threshold

We verify the sensitivity of performance on the threshold $\theta$ and $\delta$. Changing the threshold $\theta$ and $\delta$, we perform two simulations regarding the average number of packets in a router and quantiles of delay. Except the threshold $\theta$ and $\delta$, the parameters for these simulations are almost the same as the simulation with high link utilization in Sect. 7.1.2. For the estimation of the average number of packets in a router, we change $\theta$ between 0 and 80.5, and we show the results of the simulation in Fig. 8. For the estimation of quantiles of delay, we change $\delta$ between 0 and 0.0064, and we show the results of the simulation in Fig. 9. We can confirm that MSE of INTEST estimator converges to that of the conventional estimator when $\theta \to \infty$ and $\delta \to \infty$. As $\theta$ and $\delta$ become large, periods in $(\hat{s}_j, \hat{e}_j]$ became short. Therefore, Eq. (11) approaches $\hat{D}_g(T_i) = D_{gp}(T_i)$. Namely, INTEST estimator approaches conventional estimator. On the other hand, $\theta = 0$ and $\delta = 0$ do not achieve the best performance. When the threshold $\theta$ and $\delta$ are too small, it is difficult to estimate $\hat{s}_j$ and $\hat{e}_j$ since $D_{gp}(t)$ frequently exceeds the threshold. Hence, there are the optimal values for the thresholds $\theta$ and $\delta$.

### 7.1.4 An Arrival Process Other than Poisson Process

Finaly, to confirm that INTEST can be applied to an arrival process other than Poisson processes, we also calculate MSE of conventional and INTEST estimators under a Markov-Modulated Poisson Process (MMPP) [28]. We change the arrival process of the target process in the above simulation to a two-state MMPP. We set the threshold $\delta$ to 0.2 [msec]. The transition rate matrix $Q$ of the MMPP we

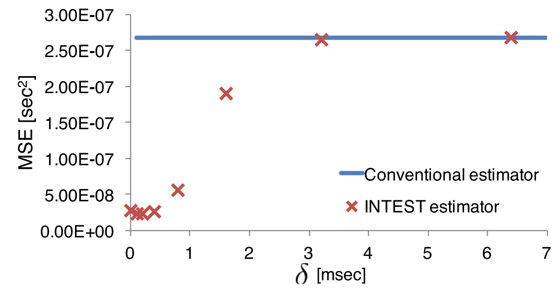


**Fig. 9** Sensitivity of the performance of INTEST on the threshold $\delta$ when we estimate 95%-quantile of delay of a router modeled by M/M/1 queuing.

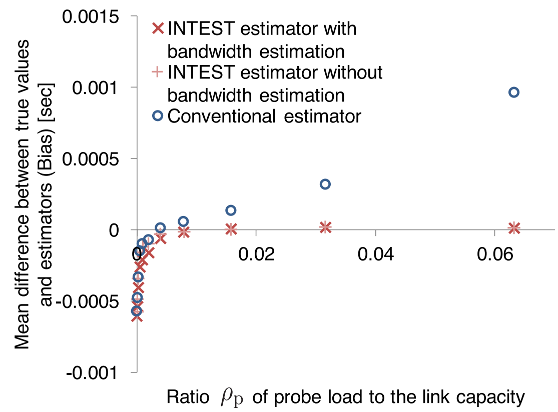


**Fig. 10** Biases of the conventional and INTEST estimators when we estimate 95%-quantile of delay of a router modeled by MMPP/M/1 queuing.

use is

$$Q = \begin{pmatrix} -5 & 5 \\ 5 & -5 \end{pmatrix}.$$

The link utilization on states 1 and 2 are 0.5 and 0.9, respectively. The results are shown in Fig. 10, and we got the similar results with the simulation of M/M/1 queueing.

From the results of the above simulations, we confirm that INTEST transcends the fundamental accuracy bound pointed out in Sect. 4 and enables highly accurate measurements of delay in a single-hop network. In INTEST, when the probing rate increases, accuracy does not suffer but instead improves. In conventional estimation, the parameter area (in which both bias and variance are small) is itself very small. With INTEST, however, we can conduct unbiased and small variance estimations as long as the probing rate is fairly high.

### 7.2 A Network Composed of Multiple Routers

To confirm that INTEST enables accurate estimation for a network composed of multiple routers, we perform a simulation with ns-3 simulator [29]. We perform a simulation of the network shown in Fig. 11, and estimate end-to-end delay of the network without probe load, using probe flows from node $N_0$ to node $P_1$ and from $N_0$ to $P_2$. Note that the router model in the simulation is not M/M/1 model in ns-3. The capacities of links $N_1$-$N_2$ and $N_2$-$N_3$ are taken as

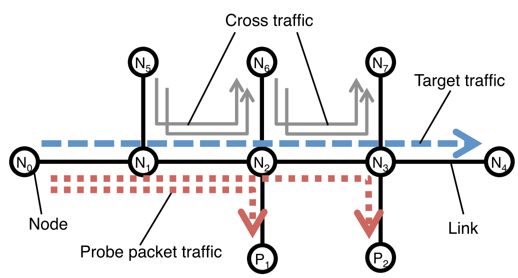


**Fig. 11** Simulation model.

15.552 [Mbps] and 31.104 [Mbps], respectively. That of the other links as 62.208 [Mbps]. In the simulation, the target traffic streams from node $N_0$ to node $N_4$. The packet length of the target traffic is 600 [bytes]. The sending time of the packets follow a Poisson arrival; here, we tuned the sending rate so as to occupy 10% of the link capacity of the bottleneck link $N_1$-$N_2$. As mentioned above, INTEST is not restricted by a Poisson arrival of the target traffic. However, it is difficult to exactly know the true value of the virtual delay on the path by measuring delay of target traffic unless the sending time of the packets follows a Poisson arrival [22]. As for cross traffic, traffic streams along two routes (i.e., from $N_5$ to $N_6$ and from $N_6$ to $N_7$), and there are two flows for each route. The cross traffic streams over repetitive ON/OFF intervals, with the constant bit rate of 8 [Mbps] during ON periods. The ON and OFF periods follow exponential distribution with mean 1.0 and 4.0, respectively. In ON periods, the cross traffic generated with constant bit rate. The packet length of the cross traffic is 600 [byte]. When two flows sharing the same route are both in an ON interval, the total amount of traffic exceeds the bottleneck link capacity, thereby producing packet delays on node $N_1$ or $N_2$. Packet loss does not occur since the size of the buffers on node $N_1$ or $N_2$ is supposed to be sufficiently large. If there is no probe packets in the network, the total link utilization of the bottleneck link $N_1$-$N_2$ is about 0.306. The length of the probe packets is 64 [byte]. The injecting time of the probe packets follows a Poisson arrival, and we tuned the probing rate so as to occupy 0.125%, 0.25%, 1.0%, 2.0%, and 4.0% of the link capacity of the bottleneck link. All of the packets in the simulation are UDP packets, and the simulation time is 50.0 [sec]. On an assumption that the link capacity is already known, we did not estimate the link capacity with Eq. (10).

In Fig. 12, we show the differences between estimators and the true value of 95%-quantile end-to-end delay. From the figure, the conventional estimators become increasingly separated from the true value with an increasing ratio of probe load to link capacity. We note that the INTEST estimators are closer to the true value than the conventional estimators. Through this simulation, we confirm that INTEST can accurately estimate the end-to-end delay of a network composed of multiple routers.

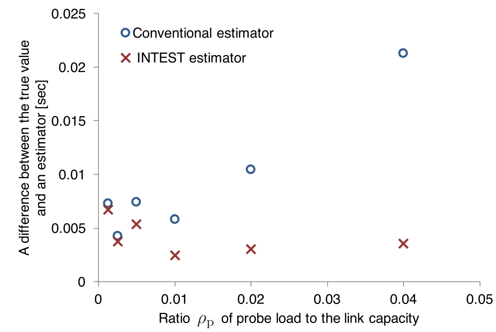


**Fig. 12** Estimation by INTEST and conventional estimators of end-to-end delay of a network composed of multiple routers.

## 8. Conclusion

In this paper, we demonstrated that there exists a fundamental accuracy bound to conventional active measurement of delay in the M/M/1 case and proposed INTEST to transcend the bound.

We evaluated the accuracy in terms of MSE, taking bias into consideration in the case of estimating the number of packets in a router modeled by M/M/1 queuing, and showed that MSE has a lower bound across varying probing rates. Performing simulations of a single M/M/1 queuing, MMPP/M/1, and a multi-hop network, we demonstrated that our INTEST estimator provides unbiased and small variance estimation of high quantile end-to-end delay, whereas the conventional estimator does not provide unbiased estimation.

We plan to evaluate INTEST on a real network and extend it to packet loss estimation and to wireless networks in future works.

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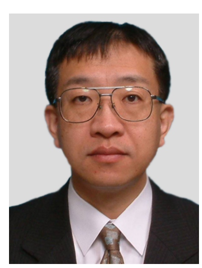

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