A Comparison of Poisson and Uniform Sampling for Active Measurements

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Abstract— Active probes of network performance represent samples of the underlying performance of a system. Some effort has gone into considering appropriate sampling patterns for such probes: i.e. there has been significant discussion of the importance of sampling using a Poisson process to avoid biases introduced by synchronization of system and uniformly spaced probes. However, there are unanswered questions about whether Poisson probing has costs in terms of sampling efficiency, and there is some misinformation about what types of inferences are possible based on different probe patterns. This paper provides a quantitative comparison of different sampling methods, both in terms of efficiency. The paper also shows that the irregularity in probing patterns is useful not just in avoiding synchronization, but also in determining frequency domain properties of a system. The paper provides a firm basis for practitioners or researchers to use in making such a decision about the type of sampling they should use in a particular application, along with methods for the analysis of their outputs.

Index Terms— Network performance, Internet measurement, error estimation, measurement planning.

I. INTRODUCTION

Active measurement of system performance is an integral part of Network Quality Assurance (NQA). Fault detection and Service Level Agreement (SLA) verification are just two applications. A number of companies offer active measurement services or devices (e.g. Matrix NetSystems, Keynote, Niksun, Brix Networks, etc), and there are now a number of Internet Engineering Task Force (IETF) Requests for Comments (RFCs) [1], [2], [3], [4], [5] describing standards for such measurement. However, the basic statistical properties of such measurements are still poorly understood.

Active probing is notionally simple. One wishes to understand the performance of a packet network, and so one sends *probe* packets into the network, and measures the performance of these probes. These packets are samples of the performance of the underlying network. They are usually performed from end-to-end across a network, this being one of the chief advantages of such an approach, as they require no special access to the network in question, and so can be easily made by customers, or researchers.

Many samples may be needed, both to provide better sampled statistics, and also to allow observations of changes in the network over time. However, there are clear scalability issues: given N end hosts, there are N^2 end-to-end paths, and if each path requires many measurements, this could create a significant network load. Hence, the pattern of probing must be carefully planned to gain the most out of such measurements, without them creating problems.

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Naively, the probe packets would sample periodically (with probe packets separated by a fixed sampling interval t_s). However, there is the possibility that periodic samples may be synchronized with a periodicity in the system under observation, either by accident, or deliberate malfeasance. In such a circumstance, the probes might not observe the true system behaviour.

It is unknown to this author how prevalent periodicities are in the modern Internet. However, there is certainly theoretical grounds for their existence, for instance due to auto-synchronization of network protocols [6], [7], or large volumes of streaming traffic using a codec that results in periodic traffic, or due to other applications that periodically make requests (such as NTP). Furthermore, regular samples are easy to predict, and therefore manipulate, perhaps by deliberately scheduling all data packets away from times when probes are anticipated. In addition, the probe packets themselves alter the network, and there is the potential that they cause the network to synchronize around their own period (in addition to other impacts, for instance see [8]). Hence there is reasonable cause to fear synchronization of probes and network.

Moreover, some problems with samples can even occur where the samples and system periodicities are not at all synchronized, or even where the system is a-periodic, but contains high-frequency periodic components, resulting in *aliasing*.

Poisson sampling steps away from the naive uniform sampling used above by sending its probes at the epochs of a Poisson Process. A result known by the acronym PASTA (Poisson Arrivals See Time Averages) ensures that (under relatively weak conditions) the average state observed by such probes will converge to the true average of the system under observation. Furthermore, when Poisson sampling is used, the next arriving packet cannot be anticipated, and so such measurements are harder to manipulate.

This approach therefore avoids some of the problems of uniform sampling. However, little work has considered the cost of this type of sampling. A number of comments have been made about difficulties introduced, for instance, non-fixed interval length for sampling, and the problems of making some types of measurements, such as those of reordering. At least one IETF RFC [9] has been written in response to these issues, however, in many cases it is hard for a practitioner to decide which type of measurement would be best for their application. This paper is aimed at providing quantitative, and qualitative advice towards making such decision by providing a thorough analysis of the advantages of each approach.

For instance, little work has considered implications for measuring higher-order statistics of the network (not just averages). Can Poisson sampling measure properties such as the delay variation or the power spectrum of a system? This

paper demonstrates that Poisson sampling can in fact be used to perform most of the tasks one might reasonably wish to perform with Internet measurements. For instance we can estimate

1) the mean, variance, and variance of differences (e.g. the delay variation),
2) the frequency content via a periodogram,
3) the autocorrelation,

without any interpolation being required to construct the estimates. This paper also presents results describing the advantages and disadvantages of such sampling. In summary

- Poisson sampling results in a loss of efficiency, i.e. more measurements are needed to obtain results of the same accuracy, however the efficiency loss is small, being much smaller than the natural variation of measurement efficiency as a result of changing network load.
- Both periodic and Poisson sampling can be used to perform harmonic analysis of the network to examine it for evidence of network periodicities. Poisson sampling actual has an advantage in this regard — the Lomb-Scargle Periodogram (Section IV) based on irregular sampling can detect frequencies greater than the periodic-sampling Nyquist frequency, allowing one to perform analyses using such streams that would be otherwise impossible.
- Autocovariances of the processes of interest can be estimated, and from these one can infer the delay variation metrics that one would obtain from arbitrary uniform probes.
- Poisson sampling is a special case of irregular sampling methods, but other types of irregular sampling would have many of the same benefits, and similar analysis could be applied. This is important because it is hard to produce a genuinely Poisson traffic stream.

This paper is concerned with comparing the two main types of probing, so that practitioners will know which to use in a particular circumstance, and researchers will have a better foundation for research in more complicated active probing problems such as network tomography, or bandwidth estimation. Following the introduction the paper presents some background, notation and related work in Section II. This is followed by Section III that provides results for the accuracy and efficiency of the two probing methods. Section IV compares the two methods on the basis of how they may be used to estimate time-series properties of the observed system, such as power spectra and autocorrelation functions. Such measures have been frequently used to characterize Internet systems (for instance see the large literature on self-similarity in network traffic). Section V discusses some of the other more intractable issues in comparing the two types of probing: for instance, the issue of measuring properties such as reordering of packets. Finally the paper concludes in Section VI.

II. BACKGROUND AND RELATED WORK

A. Performance measurement technologies

There are many measurements one may collect from a network, for instance, traffic, topology, or performance measurements. We focus here on measurements that provide measures of network performance, though note that other supplemental measures may be required (e.g. network topology) to make sense of this data. Network performance can mean many things, for instance: reachability, delay, loss, jitter, reordering, and bulk throughput. One can also form more complex functions of these metrics to attain measures such as the subjective performance of an application, e.g. VoIP. Many examples of tools to perform such measurements may be found at [10].

There are a number of ways in which one may collect data about network performance:

Direct measurements: it is possible for a router, or switch to maintain information about its own performance. For instance, to maintain data on the number of packets or bytes in buffers, or the number of packets dropped for various reasons. Such information can then be collected at regular intervals through a mechanism such as SNMP [11], [12]. In principle, such information could be collected without sampling (i.e. it could be collected in enough detail to reconstruct the exact sample path). However, in practice there are limits on the (fast) memory required to store such information, and also on the rate at which it is collected (SNMP is not a particularly efficient mechanism). Hence, such data might be collected typically every five minutes, and despite its potential to be one of the best sources of performance measurements available, it is often one of the worst.

Active probes: The more common method used to infer network performance is the well developed approach of active probing, for instance see [13], [14], [1], [2], [3], [4], [5]. In this approach, one deliberately sends *probe* packets into the network with precisely controlled departure times, and measures their arrival times elsewhere in the network. Such probes require installation of probe equipment into a network, but this equipment is typically fairly cheap, and it does not require special access to a network (for instance, the boxes could be installed by a customer, or researchers). There are many possible types of probe: e.g. ICMP echo probes, TCP SYN/ACK probes, DNS probes, HTTP page downloads, as well as dedicated probe protocols, and many types of measurements: e.g. round-trip or one-way measurements. These factors have led to active probing being the most widely deployed form of IP network performance measurement methodology. For one-way measurements, there are significant issues in clock-synchronization, though we shall not discuss these issues in detail here.

Passive traffic measurements: Where active probing is impractical, passive traffic measurements can be used to infer network performance through multiple measurements of the same packet [15], [16]. By measuring the arrival time of a packet (or its acknowledgement) at multiple points, we can infer the delay between these points. This approach can also provide data of very fine detail, however, it also has limits. Firstly, dedicated packet monitors are typically cheap, but involve non-negligible installation and maintenance cost, and so are not usually installed everywhere. Hence, ones ability to perform such measurements is limited to the locations of packet monitors, though implementing packet sampling at routers would remove this issue. More importantly, passive monitoring of this type can only infer performance on paths that carry traffic. More generally, passive measurements cannot control the sampling pattern, and so we cannot guarantee either Poisson of periodic samples, but note that considerable parts of this paper still apply in these cases.

For a practical comparison of some of the above techniques see [17]. In this paper we will be mainly concerned with active probing because of its advantages, and the fact that precisely timed samples are impossible with passive sampling.

Even simple measurements such as delays are comprised of a number of components:

1) *Packet processing delay* is the delay to perform tasks such as forwarding table lookup, and is very small in modern high-speed routers (e.g. $\ll 1$ ms).

2) *Packet transmit time* is the time from starting to send the first bit of a packet onto the wire, until the last bit is finished. It is given by the packet size (including framing bits) divided by the link bandwidth, a small time for high bandwidth links, e.g. $\sim 4.8\mu s$ for a 1500 byte packet on an OC48 (2.5 Gbps) link.

3) *Propagation delay* is the delay a packet experiences on the wire, and is given by the physical distance divided by the speed of light in fibre (roughly 200,000 km per sec). e.g. $\sim$30 ms for a direct East to West Coast in North America.

4) *Queueing delay* is the time spent by a packet in queues, which depends on load, and can potentially be quite large, e.g. 0.2 seconds, even on single OC48 line cards.

The two components that are significant, and therefore of primary interest in most networks are propagation delay and queueing delay. Propagation delay is determined by network topology and routing, and for the purpose of this paper we shall consider it to be a constant (see [18] for a more realistic view), which is derivable from other network data (topology and routing information), or from long-term measurements of the network. The queueing delays may be seen as drawn from a random process, and one goal of this paper is to estimate the behaviour of this process.

In addition to statistical variations in queueing delays, there are measurement errors. Any set of performance measurements contains errors and artifacts, for instance, delays in time-stamping a packet once it is received at a monitor. In a well designed measurement system they should be an order of magnitude smaller than the queueing variations, and so we shall not consider these in detail here.

This paper focusses on network performance measurements, but the analysis in this paper could equally be applied to measurements of server performance. In principle, any system that can be modelled as a queueing system is susceptible to this type of analysis.

B. Statistical Notation

We shall denote a continuous-time stochastic process, by $X(t)$. We denote the mean of the process at time t by $E[X(t)]$ (where $E[\cdot]$ denotes the expectation, or average), its variance by $\text{Var}(X(t))$, and its autocovariance by $\text{Cov}(X(t), X(t+s))$. A stationary process is one whose multi-dimensional statistics are all time-shift invariant, but we shall use a more tractable definition of stationarity, namely that of wide-sense stationary, which means that the mean, variance and auto-covariance are all constant with respect to t, and can consequently be written $E[X(t)] = \mu_X$, $\text{Var}(X(t)) = \sigma_X^2$, and $\text{Cov}(X(t), X(t+s)) = R_X(s)$, respectively, where we refer to s as the *lag*. We also define the AutoCorrelation Function (ACF) of the process by $r_X(s) = R_X(s)/\sigma_X^2$. Note that the term ACF is

used (in some texts) to denote $W_X(s) = E[X(t)X(t+s)]$, but we shall refer to this as the Mean Zero AutoCovariance Function (MZACF), as it is equivalent to the autocovariance $R_X(s)$ for a mean zero process. In general the relationship is

$$W_X(s) = R_X(s) + \mu_X^2 = \sigma_X^2 r_X(s) + \mu_X^2. \qquad (1)$$

Another useful notation is an indicator function of the process, for instance

$$I(X(t) \in B) = \begin{cases} 1, & \text{if } X(t) \in B, \\ 0, & \text{otherwise.} \end{cases} \qquad (2)$$

Indicators can be used to *indicate* when arbitrary events occur. Note that the expected value of the indicator gives the probability, i.e. $E[I(X(t) \in B)] = P\{X(t) \in B\}$, and we may therefore use indicators to construct expectations, and higher moments of the process.

1) Sampling: We shall assume that we can measure the process $X(t)$ at sample epochs $T_1, T_2, \ldots, T_N$. The measurements form a discrete-time random process (or time series) $X(T_1), X(T_2), \ldots, X(T_N)$, which we will sometimes denote $X_1, X_2, \ldots, X_N$. The two main cases considered in this paper are periodic sampling, where $T_n = nt_s$ for some inter-sampling interval t_s, and Poisson sampling, where the T_n form a Poisson Process (described below). Where we apply periodic sampling, we shall equivalently use the term *uniform* sampling to be consistent with the related signal-processing literature.

We wish to estimate some parameter s of the stochastic process $X(t)$, and so construct an estimate $\hat{s} = f(X_0, X_1, \ldots, X_N)$. We shall measure the accuracy of this estimate by the Mean Squared Error (MSE) $E[(\hat{s} - s)^2]$. Standard arguments show us that we can decompose this error into a variance term, and a bias term, i.e.

$$E[(\hat{s} - s)^2] = \text{Var}(\hat{s}) + (E[\hat{s}] - s)^2, \qquad (3)$$

where the variance of the estimator $\text{Var}(\hat{s})$ is of prime interest here, as we shall consider unbiased estimators, i.e. estimators for which the bias $E[\hat{s}] - s = 0$, at least asymptotically as the number of measurements $N \to \infty$. For instance, the sample mean is defined by

$$\hat{X}_N = \frac{1}{N} \sum_{i=1}^{N} X_i, \qquad (4)$$

and it is an unbiased estimate of the true mean of the stochastic process $X_1, X_2, \ldots, X_N$ given that this process is stationary and ergodic (where ergodicity is a technical condition we shall not discuss in detail here, but it ensures that a time average such as the sample mean above will converge to the true mean of the process). The variance of $\hat{X}_N$ depends on the autocovariance of the sampled measurements X_i (we will consider this in more detail below).

It is simple to show that $\hat{X}_N$ may not converge to the mean of a non-stationary process, for instance, consider the process $X(t) = \cos(t)$ sampled at time points $T_n = 2n\pi$, so that $X_n = 1, \forall n$, whereas the average value $E[X(t)] = 0$. This highlights the property alluded to earlier, namely that uniform sampling, when precisely synchronized with a periodicity in the observed process may result in biased estimates. Should one worry about such a synchronization happening in reality? Certainly periodic behaviour can happen in networks, but what are the chances that we inadvertently synchronize exactly

to these phenomena? The problem is that active probes are just that, active. It is possible that they stimulate activity at precisely the frequency at which they arrive, thus causing the network to synchronize around these probes.

There are two methods used to avoid such issues: firstly, in signal processing, one passes a signal to be measured through a low-pass filter with cut-off threshold f_c, and then samples (uniformly) with sample frequency $f_s > 2f_c$. We shall discuss the reasoning behind this, and in particular the problem of aliasing (of which the above is a special case) in Section IV, but note that with the active probing methodology discussed above, prefiltering of the signal is not possible. The second method for avoiding the problem is to randomize the sampling intervals such that the epochs T_n form a Poisson process, which is described below. Figure 1 (a) illustrates sampling times for the two cases of primary interest, while Figure 1 (b) shows an example of samples from a queueing process.
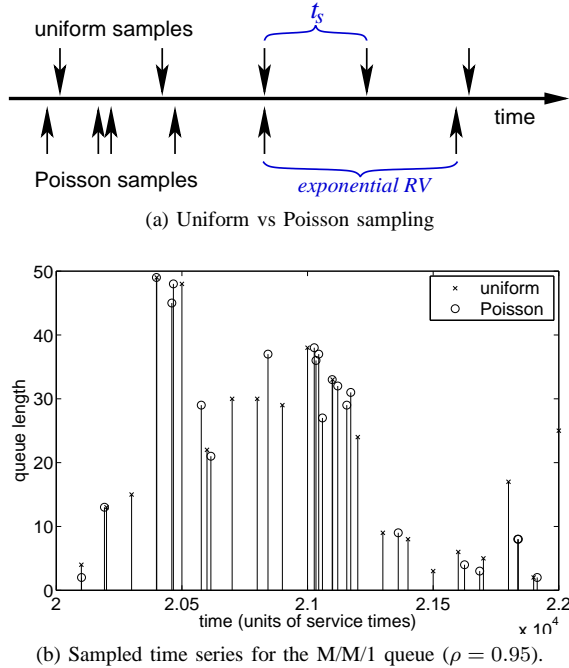

(a) Uniform vs Poisson sampling

(b) Sampled time series for the M/M/1 queue ($\rho = 0.95$).

Fig. 1.   Sampling examples.

Note that a large part of this paper is applicable to other sampling processes T_n. This is important because

- Even in uniform sampling there is often some jitter in the measurements (it is difficult to ensure probe packets are sent at precise intervals).
- Where Poisson sampling is used, there is some minimum time between packet probes (at least because of the finite size of packets), and this means no probe stream is perfectly Poisson.

2) Frequency Domain properties: We often wish to analyse a set of data to discover if it has periodic components. Such analysis is most easily accomplished in the frequency domain. A typical approach to obtaining such information is using the Fourier Transform (FT), defined here with its inverse for a function $g(t)$ by

$$FT_g(f) = \int_{-\infty}^{\infty} g(t)e^{-i2\pi ft}\, dt, \quad g(t) = \int_{-\infty}^{\infty} FT_g(f)e^{i2\pi ft}\, df, \tag{5}$$

or for discrete-time data, we use the Discrete Fourier Transform (DFT)

$$FT_X(k) = \sum_{n=0}^{N-1} X_n e^{\frac{-i2\pi kn}{N}}, \quad X_n = \frac{1}{N}\sum_{n=0}^{N-1} FT_X(k)e^{\frac{i2\pi kn}{N}}. \tag{6}$$

To detect sinusoidal signals, one often plots the periodogram (though note there are many variations on this method, one of which we discuss below), which is proportional to the squared magnitude of the FT, for instance in the discrete case

$$P_X(k) = \frac{1}{N}|FT_X(k)|^2 = \frac{1}{N}\left|\sum_{n=0}^{N-1} X_n e^{-i2\pi kn/N}\right|^2. \tag{7}$$

It is common to report the discrete spectra in terms of the index k, for which we can convert to a frequency by taking $f = k/(t_s N)$. The more data we have the finer the minimum frequency resolution, while the faster the sampling rate (smaller t_s) the wider a frequency band we can study (see Section IV-A for more detail on the frequency band we can study given by the Nyquist frequency for uniform sampling). Note that often frequencies are reported using $\omega = 2\pi f$ in units of radians per cycle.

Given a stochastic process $X(t)$, we typically talk about the *spectral density* rather than periodogram. Loosely, it gives the expected periodogram, though its actual values for a given random signal would be random variables. We shall abuse notation here and use P to indicate the spectral density. An important result in this context is the Wiener-Khintchine (WK) theorem, which relates the spectral density to $W_X(s)$, by the Fourier transform, i.e. in the continuous case,

$$P_g(f) = \int_{-\infty}^{\infty} W_X(t)e^{-i2\pi ft}\, dt. \tag{8}$$

In the discrete case it is only true as the number of samples goes to infinity, but for finite N we can take the approximation

$$P_X(k) \simeq \frac{1}{2N+1}\sum_{n=-N}^{N} W_X(t_s n)e^{-i2\pi kn/N}, \tag{9}$$

and obviously we can determine W_X via the inverse transform. For both deterministic and stochastic signals we refer to the term at $f = 0$ as the DC term, and note that it is directly related to the square of the mean of the process.

C. The Poisson Sampling Process

The Poisson Process (PP) is the renewal process formed by taking Independent, Identically Distributed (IID) exponential Random Variables (RVs) $\{S_i\}_{i=1}^{\infty}$, and forming times $T_n = \sum_{i=1}^{n} S_i$. Formally, the PP is defined by the process that counts the number of renewals up to time t, i.e.

$$N(t) = \sum_{i=1}^{\infty} I(T_n < t), \tag{10}$$

where $I(\cdot)$ is an indicator function.

The inter-renewal times S_i have an exponential distribution $P\{S_i < s\} = 1 - \exp(-\lambda s)$ where $\mu = 1/\lambda$ is the mean inter-renewal time, and for the homogeneous PP, λ is a constant

called the *rate*. The mean of the PP is given by $E[N(t)] = \lambda t$, and its autocovariance function is given [19] by

$$R_N(t, s) = \begin{cases} \lambda s, & \text{if } t > s, \\ \lambda t, & \text{if } t < s. \end{cases} \qquad (11)$$

Given the PP is clearly non-stationary we are not primarily interested in it, but rather in samples taken at the epochs of this process. We shall define a new process, namely the *Poisson Sampling Process* (PSP) as the derivative of the PP, i.e.

$$S(t) = \sum_{i=0}^{\infty} \delta(t - T_i) = \frac{dN}{dt}, \qquad (12)$$

where $\delta(t)$ is the Dirac Delta function. Expectation is a linear operator, and so

$$E[S(t)] = E\left[\frac{dN}{dt}\right] = \frac{d\,E[N(t)]}{dt} = \frac{d}{dt}(\lambda t) = \lambda,$$

and we can likewise derive the autocovariance of $S(t)$ using the double derivative $E[S(t)S(s)] = E\left[\frac{dN}{dt}\frac{dN}{ds}\right] = \frac{\partial}{\partial t}\frac{\partial}{\partial s} E[N(t)N(s)]$, resulting in

$$R_S(t, s) = \lambda \delta(t - s), \qquad (13)$$

Note that the sampling process $S(t)$ is stationary, and so we write the autocovariance, and ACF as a function only of the lag $u = t - s$, e.g.

$$r_S(u) = \delta(u). \qquad (14)$$

The delta function is (by definition) zero for non-zero lags u, and so the PSP is completely uncorrelated. Another way of stating this is to state that the PP has independent increments, a feature sometimes used in alternative definitions. This lack of correlation of the sampling process results in the key properties of interest here. Namely, that the number of samples to occur in non-overlapping time intervals will be independent random variables. Hence, given information about the past of the process, we gain no information about its future — it is impossible to anticipate the samples.

It is also simple to derive the fact that the number of samples to occur in any arbitrary interval $[t, t + s]$ will be a discrete Poisson random variable, with mean λs, and the events in the interval will be uniformly distributed over that interval.

1) PSP in the Frequency Domain: We can also represent a PSP in the frequency domain through the WK theorem (8). Note that for the Poisson sampling process $W_S(u) = \sigma_S^2 r_S(u) + \mu_S^2 = \lambda \delta(u) + \lambda^2$. We can easily take the FT of this function to obtain spectral density

$$P_S(f) = \lambda^2 \delta(f) + \lambda. \qquad (15)$$

The discrete spectra is directly analogous, both resulting in a flat spectrum (except for the DC term, which relates to the mean value of the process). This property of the spectral density of the samples is often referred to as "white". White noise sometimes suggests a Gaussian process, but the term actually refers to the spectral density of the process containing equal power in all bands (the metaphor refers to white light despite this not containing an equal spectral content). The PSP is perhaps the simplest *sampling* process to have this property.

2) PASTA: The fact that the system cannot anticipate the number of arrivals is the feature of the PSP, which here results in the PASTA property. It seems instructive to consider why this is an important factor in PASTA, and so we shall present a sketch of the key results here.

Formally, we define an indicator function $U_B(t) = I(X(t) \in B)$ for measurable set B, and then we define the time spent in state B, and the number of arrivals that see state B over interval $[0, t)$ by

$$V(t) = \int_0^t U_B(u)\,du, \qquad Z(t) = \sum_{i=1}^{N(t)} U_B(T_i), \qquad (16)$$

respectively, where $N(t)$ is the PP defined above. Although technical conditions such as left-continuity of sample paths are required, the PASTA theorem intuitively comes from one result: $Z(t)$ can be written as a Riemann-Stieltjes integral, and this integral can then be approximated by finite sums

$$\begin{aligned} Z(t) &= \int_0^\infty U_B(u)\,dN(u) \\ &\simeq \sum_{k=0}^{n-1} U_B(kt/n)\left[N\left(\frac{(k+1)t}{n}\right) - N\left(\frac{kt}{n}\right)\right], \end{aligned} \qquad (17)$$

for sufficiently large n. For any arrival process which cannot be anticipated the number of arrivals in interval $\left[\frac{kt}{n}, \frac{(k+1)t}{n}\right)$ is independent of the state of the system at time t, and so

$$E[Z(t)] \simeq \sum_{k=0}^{n-1} E[U_B(kt/n)]\,E\left[N\left(\frac{(k+1)t}{n}\right) - N\left(\frac{kt}{n}\right)\right],$$

Now, in the case of Poisson samples, the number of arrivals in time $\left[\frac{kt}{n}, \frac{(k+1)t}{n}\right)$ is $\lambda t/n$, and so we can write

$$E[Z(t)] \simeq \lambda t \frac{1}{n} \sum_{k=0}^{n-1} E[U_B(kt/n)] \simeq \lambda t \int_0^\infty U_B(u)\,du, \qquad (18)$$

for sufficiently large n (a real proof requires use of dominated convergence theorems to deal with the limit as $n \to \infty$). It is then a technical matter to show that convergence occurs for averages as $t \to \infty$.

The above proof relies on arrivals being Poisson, but it is important to note that the Arrivals See Time Averages (ASTA) property applies for other arrival processes [20]. The key detail is the lack of anticipation. For instance, note that none of the above requires stationarity of the process $X(t)$. If it is, and the increments of the sampling process $N(t + s) - N(t)$ are independent of the state of the process $X(t)$, then $E[U_B(t)] = E[U_B(0)]$, and we can write

$$\begin{aligned} E[Z(t)] &\simeq E[U_B(0)] \sum_{k=0}^{n-1} E\left[N\left(\frac{(k+1)t}{n}\right) - N\left(\frac{kt}{n}\right)\right], \\ &\simeq E[U_B(0)]\,E[N(t)], \end{aligned} \qquad (19)$$

and once again we can prove ASTA. Note that here the arrival process $N(t)$ is no longer necessarily Poisson, however the lack of anticipation property must still hold, and so we cannot use this result for an arbitrary arrival process. The lack of anticipation requires the system does not try (deliberately, or through accidental synchronization) to anticipate the samples.

This is true in many systems, but it is dangerous to assume so without any evidence, and so Poisson sampling is considered to be more safe.

The above proof sketch is instructive because it shows that the ASTA property applies to point estimates of properties of the system. It does *not* apply where we have a property that depends on the process at more than one point in time, such as the spectral density, or delay variation of the process. Such measures are derived for multiple time points in the system, bringing the system correlations into the estimates.

3) Other properties of the PP: The inter-renewal times of the PP, S_i, take an exponential distribution, i.e. $P\{S_i < s\} = 1 - \exp(-\lambda s)$, and therefore the PP is easy to generate by generating the S_i, but we could also generate a PP by exploiting the fact that the number of events to occur in any arbitrary interval $[t, t + s]$ will be a discrete Poisson random variable (with mean λs) and the events in the interval will be uniformly distributed over that interval. Also note that the sum of two PPs also forms a PP, as does a PP which is thinned by randomly removing points with probability p.

The time between n samples will be the sum of n exponential distributions, which is given by an Erlang-n distribution [19], with distribution function

$$P\left\{\sum_{i=1}^{n} S_i\right\} = \frac{\lambda(\lambda s)^{n-1}e^{-\lambda s}}{(n-1)!}. \qquad (20)$$

This property is important to consider in this context because the Erlang-n distribution approaches a Normal distribution $N(n/\lambda, n/\lambda^2)$ for large n. As n becomes large, we could consider the Erlang times to be uniformly spaced with some small random jitter. Hence, while at one level the PP is completely random, if we take every nth probe, and only use this data, it is close to uniform sampling with probes spaced at n/λ intervals. So periodic subsamples from a PP can approximate uniform sampling.

D. Related Work

The PASTA result [21] is well known in the communications networking community, as are the problems of synchronization of probes with network periodicities (see [6], [7] and the references therein) with a result that Poisson sampling is discussed in a number of IETF RFCs [3], [4], [5], [22], [23]. The idea of Poisson probing is therefore well founded on strong theory.

However, there has been little work on the topic since these foundations. One recent paper [24] starts to consider the issue. The paper uses a large number of experiments to consider whether the use of Poisson sampling makes any difference in practice. The conclusions are largely negative, i.e. they find little or no statistically significant differences between Poisson and uniform samples of network delays, losses, or packet pair dispersion.

There are intrinsic difficulties to such a study primarily because one can only compare the difference between the two, not how accurate they are with respect to the underlying process. Further, experiments cannot prove a negative — that is, they cannot show that there is never a problem with network and probe synchronization.

However, given the negative evidence is substantial, this paper shall not try to make claims about the necessity for Poisson probing. The paper will show the quantitative differences between the two approaches, and reduce some of the mysticism surrounding them. The correct choice of probe will depend on the application, but this paper should allow a correct assessment of the true restrictions imposed by each method.

This paper will also drawn on the work contained in [8]. In particular, that paper presented methods for estimating the accuracy of measured network performance metrics. We shall use this approach to compare the accuracy of the two types of sampling.

There are a number of works, in other contexts, on irregular sampling. We discuss these in more detail in Section IV below.

III. ACCURACY AND EFFICIENCY

We wish to measure properties of a stochastic process describing the network behaviour, and so it is natural to ask how accurately we can do so. Note that this is not accuracy as specified in the relevant RFCs, where it means the accuracy of the devices used for measurement. Here we mean accuracy with which we can determine the parameters of a stochastic process (the observed delays), and we shall here use MSE as a measure of accuracy. Alternatively, we might ask how many samples are required to obtain a given accuracy. In this section we seek to measure the properties of a (wide-sense) *stationary* stochastic processes (measuring the random delays of probe packets transiting a network). It is valid to measure a non-stationary process, for instance to perform change detection for the purpose of detecting anomalies, however, when doing so, the concept of accuracy of the measurements has less meaning.

Given we shall typically apply (asymptotically) unbiased estimators, the variance of the estimator is the key quantity of interest. Variance is usually monotonically decreasing with the number of probes N, and so an alternative way of describing the problem is using statistical efficiency, which describes how efficiently each data point is used in estimating parameters. In general, this depends on the particular parameter to be estimated, and the estimator used. We shall focus on a simple parameter (the average delay), and use a simple statistic to estimate this parameter (the sample mean). This is not necessarily the best linear estimate of the mean of the data, but it is known to have high relative efficiency ($> 98\%$) compared to the best linear estimate (which is unknown *a priori*) [25].

The PASTA property [21] ensures that $E[X(t)] = E[X_n]$, at least in a limiting sense. However PASTA says nothing about the rate of convergence, and it is intuitive that the variance would increase with Poisson sampling, given we are adding variability to our measurement process, but we show this is not necessarily true below.

Take stationary stochastic process $X(t)$ and further simplify (without loss of generality) by shifting the process so that it is mean zero, and consider the sample mean of the process.

$$\begin{aligned}
\text{Var}\left(\hat{X}_N\right) &= E\left[\hat{X}_N^2\right] = E\left[\left(\frac{1}{N}\sum_{i=1}^{N} X(T_i)\right)^2\right] \\
&= \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N} E[X(T_i)X(T_j)]. \qquad (21)
\end{aligned}$$

Hence the variance of the estimator depends on correlations between samples. For uniform samples $T_j - T_i = t_s(j-i)$, and

hence for a mean zero process $E[X(T_i)X(T_j)] = R_X(t_s(j-i))$. For Poisson samples the times $T_j - T_i$ are Erlang-$(j-i)$ random variables, and so the expectation is given by a probability integral over the autocovariance. It is commonly assumed when making measurements that correlations drop to zero if one spaces ones measurements sufficiently apart, and so we need not include these terms in our calculations of variance. However, in the Internet, the correlations may extend further than we can naturally space measurements [8], and furthermore, when performing Poisson sampling, the intervals between probes are random, and so some intervals may be very short. Hence, we must take account of the correlations in the system under observation.

The formula above is computable, but more instructive results regarding the above variance can be derived in the limit as $N \to \infty$. Limiting formula have been derived in the form of Central Limit Theorems. For uniform samples, the standard theorem states that

$$\sqrt{N}\left(\hat{X}_N - \bar{X}\right) \to N(0, s_X^2), \qquad (22)$$

where $N(0, \sigma^2)$ denotes a normal distribution with zero mean, and variance σ_X^2, and s_X^2 is referred to (see [26]) as the *asymptotic variance* of the process X, and it is defined by

$$s_X^2 \equiv \lim_{N \to \infty} N \operatorname{Var}\left(\hat{X}_N\right). \qquad (23)$$

One may compute the asymptotic variance for uniform samples using the following relationship (from [27]),

$$s_{\text{uniform}}^2 = \sigma_X^2 \left(1 + 2\sum_{i=1}^{\infty} r_X(it_s)\right), \qquad (24)$$

where t_s is the time between the samples, and the sum is finite (convergence of the sum is not guaranteed, with alternative results in that case).

A similar theorem for Poisson samples is derived in [8]

$$s_{\text{Poisson}}^2 = \sigma_X^2 \left(1 + 2\lambda \int_0^{\infty} r_X(u)\, du,\right), \qquad (25)$$

where the integral is finite. Note that if we take $\lambda = 1/t_s$ then the average rate of probes for both methods is the same. Given $t_s \to 0$, then $\frac{1}{\lambda}\sum_{i=1}^{\infty} \rho(it_s) \to \int_0^{\infty} \rho(u) du$, so the two methods are consistent for high sampling rates.

The result is that for certain types of autocovariance, either Poisson or uniform sampling could be superior. For instance, where the autocovariance has oscillatory behaviour we could construct a function such that the sum over r_X at regular points is either greater or smaller than the integral over the whole autocovariance function. Given that either case is possible, what is likely? We shall consider this in the case of a simple queueing model, the M/M/1 model.

A. Accuracy of measurements of a simple queueing model

One of the simplest queueing systems is the M/M/1 queue, i.e. a queue with a Poisson arrival process (of rate λ) of packets whose service times are exponential (with mean $1/\mu$). We shall initially use this queue to model the effect of the two sampling regimes, because of its simplicity, and two of its properties:

- The sum of two PPs is also a PP, and we can simply model a system including Poisson probe traffic by adding the rate of probe traffic to data traffic, with the result still having Poisson arrivals.
- The output of a M/M/1 queue is also Poisson, allowing one to place them in series, to obtain results for concatenated queues with relative ease.

Figure 1 (b) shows samples from the queue length of a simulation of the M/M/1 queue.

We denote the traffic intensity $\rho = \lambda/\mu$. The queue is stable in the sense that it is positive recurrent for $\rho \le 1$, but that the expected length of the busy period is infinite for $\rho = 1$ (because the distribution of the length of the busy period has a heavy-tail in the critical transition between stability and instability), and so we only consider queues with $\rho < 1$. The M/M/1 queue is very well studied, with many text book results, e.g. see [28]. For instance, the mean and variance of the number of packets in the system are

$$\mu_Q = E[Q] = \frac{\rho}{1-\rho}, \quad \sigma_Q^2 = \operatorname{Var}(Q) = \frac{\rho}{(1-\rho)^2}.$$

The mean and variance of the waiting time are

$$E[W] = \frac{1}{\mu} E[Q], \quad \operatorname{Var}(W) = \frac{1}{\mu^2}\left(\mu_Q + \sigma_Q^2\right).$$

The M/M/1 auto-covariance results are not as simple, because of their dependence on the transient behaviour of the M/M/1 queue. One can find the auto-covariance of this queue in [29], [30]. The autocorrelation function is given by

$$r_Q(s) = \frac{(\mu - \lambda)^3}{\pi} \int_0^{2\pi} \sin^2 \theta \frac{e^{-w|s|}}{w^3} d\theta, \qquad (26)$$

where $w = \lambda + \mu - 2\sqrt{\lambda\mu}\cos\theta$. This is not quite the same as the auto-correlation of the delays, but it is close enough to give us the required insight for this paper, whereas the formula for waiting times are considerably more complicated, without providing any additional insight. Morse [29] gives the integral of the ACF $r_Q(s)$ over s as does [26, (22)], but note the time scaling of $1/\mu$ in [26]. Given such a scaling, the observation time is measured in units of number of (average) service times. Given this integral we can analytically compute the asymptotic variance for Poisson sampled measurements of the number of packets in the system, and this is closely approximated by

$$s_{\hat{B}}^2(p, \lambda, \mu) \simeq \frac{\rho^2}{(1-\rho)^2} + p\frac{4\rho^3}{(1-\rho)^4}, \qquad (27)$$

where we note that the Poisson samples compose proportion p of the traffic sent to the queue, so that $\lambda_s = p\lambda$, where λ is the total arrival rate of packets to the system.

We do not have a closed form for the discrete summation, but it can be computed accurately by summing over the above autocorrelation, which we can evaluate using numerical integration. Alternatively, we can apply the approximation given in [31, (3.7)], i.e.

$$r_Q(s) \simeq \frac{1}{2}\left[e^{-A|s|} + e^{-B|s|}\right],$$

where

$$A = \frac{(1-\rho)^2}{1 + \rho + \sqrt{\rho}}, \qquad B = \frac{(1-\rho)^2}{1 + \rho - \sqrt{\rho}}. \qquad (28)$$

This form can be analytically integrated to give

$$\int_0^{\infty} r_Q(s)\, ds \simeq \frac{2(1+\rho)}{2(1-\rho)^2}, \qquad (29)$$

and summed to give

$$\sum_{n=1}^{\infty} r_N(nt_s) \simeq \frac{1}{2}\left[\frac{e^{-At_s}}{1-e^{-At_s}} + \frac{e^{-Bt_s}}{1-e^{-Bt_s}}\right]. \qquad (30)$$

Figure 2 shows a comparison of the asymptotic variances for Poisson and uniform sampling. The figures show both the approximation (lines), and the exact numerical results (markers). Figure 2 (a) shows the impact of varying the sampling interval. The figure shows that Poisson sampling has a higher asymptotic variance, which should be expected for a monotonically decreasing ACF. Note also that the figure seems to indicate that the two asymptotic variances converge for $t_s \to 0$ as we would expect. Figure 2 (a) shows the results for a moderate load $\rho = 0.75$, but the results above show a dependence on ρ, which we explore in Figures 2 (b) and (c), by showing (for a fixed sample rate) the results for a range of values of ρ. For $t_s = 100$, we note significant variation between the two asymptotic variances, but that the differences decrease for heavy and light loads.

However, the most interesting thing to note from Figures 2 (b) and (c) is the dramatic dependence of the asymptotic variance on the system load ρ. As noted in [8], the total traffic rate has a very significant impact on measurement accuracy. This was argued in [8] to be a much more general effect than we have demonstrated here (for the M/M/1 queue). It is important here because it shows that there is *much* more variation in measurement accuracy between measurements taken at different loads, than we might see using different sampling methodologies. The variations for different sampling methods should be seen as a minor effect in comparison.

Figure 2 also shows that for large sampling rates the asymptotic variance increases. This results from the fact that the samples are more closely spaced, and so the are more correlated, and less additional information is added per sample.

B. Fixed time-interval sampling

One issue that arises in using Poisson sampling is that fact that the time taken to obtain a fixed number N of samples varies. Alternatively, given a fixed time interval T, the number of samples will be random. The results above tell us the asymptotic variance for a fixed number of samples. We should also consider the case of a fixed time interval.

Given a fixed time interval, the number of samples N_T will be distributed as a Poisson random variable, i.e.

$$P\{N_T = n\} = \frac{(\lambda T)^n \exp(-\lambda T)}{n!}, \qquad (31)$$

which has mean λT and variance λT. For large values of λT we may approximate N_T by a normal distribution, i.e., $N_T \sim N(\lambda T, \lambda T)$. The variance of N_T is an additional source of variance in the results.

To analyze this, let us consider the simple case of uncorrelated measurements. In this case the sample mean, conditional on N_T is has variance σ_X^2/N_T, for $N_T > 0$. Note that for true Poisson sampling there is a non-negligible chance of zero samples occuring, so we condition on at least one sample, resulting in a distribution

$$P\{N_T = n\} = \frac{(\lambda T)^n \exp(-\lambda T)}{(1 - \exp(-\lambda T))n!}. \qquad (32)$$

Take expectation over the conditional variance, and use the notation $\hat{X}_T$ to denote the sample mean over a fixed time interval, we can see that

$$\mathrm{Var}\left(\hat{X}_T\right) = E\left[\mathrm{Var}\left(\hat{X}_{N_T}\right)\right] = \sigma_X^2 \frac{e^{-\lambda T}}{(1 - e^{-\lambda T})} \sum_{i=1}^{\infty} \frac{(\lambda T)^n}{n.n!}.$$

Figure 3 shows the relative variance of the sample mean for IID RVs generated by uniform versus Poisson samples, i.e. the figure shows $\mathrm{Var}\left(\hat{X}_T\right) / \mathrm{Var}\left(\hat{X}_{N_T}\right)$, where N_T is fixed so that $N_T = \lambda T$. The figures shows that for small λT, the variance in the Poisson samples increases the variance of the estimator by up to 32%, but that this increase quickly becomes negligible. After only around 50 samples ($\lambda T = 50$), the loss in efficiency of the two becomes around 2%, and given the number of samples collected in many experiments, this can be considered to be a negligible source of error.

The drop below 100% for $\lambda T = 1$ arises because in this case, uniform sampling results in exactly one sample, but for Poisson samples we are conditioning on at least one sample, with some probability of more, and so there is a notional advantage here, but it arises due to the conditioning, and so should not be given much weight.

C. Impact of active probes

The results above neglect the impact of the active probes themselves (results are reported for traffic intensities, irrespective of what proportion of this traffic is probe traffic, or genuine data traffic). The work of [8] showed that the impact of active probes themselves was substantial (in that it increased load, and therefore correlation scale, and hence the asymptotic variance). We will not repeat this discussion here, due to limitations in space.

However, there is one respect in which this work differs from [8]. When Poisson probes are combined with Poisson traffic, the resulting input traffic is still Poisson. However, when uniform probes are combined with Poisson traffic, the result now deviates from Poisson, which will distort the behaviour of the queue. This type of effect should be minor for low sampling rates, but could still be interesting in future work.

D. Long-range dependence

No-one would claim that the M/M/1 queue is a good model for Internet performance. A more realistic model would include features such as Long-Range Dependence (LRD). As noted in [8], LRD changes the quantitative results described above (though qualitatively similar phenomena are observed). Most importantly, where we have LRD, the autocovariance function is not integrable, and so the CLT used above in (24) and (25) will not hold. In fact we need to use a generalized CLT, which will generally predict a much slower rate of convergence. See [8] for some informative simulations. We leave the development of theory to future work.

IV. TIME-SERIES PROPERTIES

Many time-series algorithms are designed with uniformly-sampled data in mind. For instance, a problem of interest in the Internet is the detection of synchronization effects. These have

(a) Asymptotic variance for $\rho = 0.75$ and a range of sampling rates.

(b) Asymptotic variance for range of traffic intensities ρ with $t_s = 100$.

(c) Asymptotic variance for range of traffic intensities ρ with $t_s = 1$.
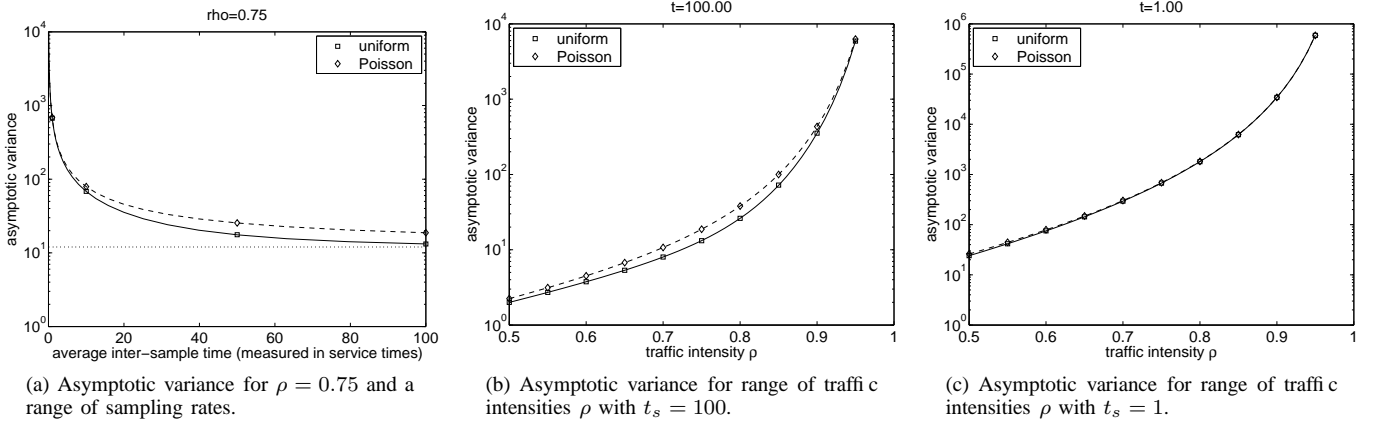
Fig. 2. Asymptotic variance for Poisson and uniform sampling of the M/M/1 queue. The lines show results derived from approximations (29) and (30), whereas the markers show results derived from direct numerical integration. Note time is given in units of average service times.
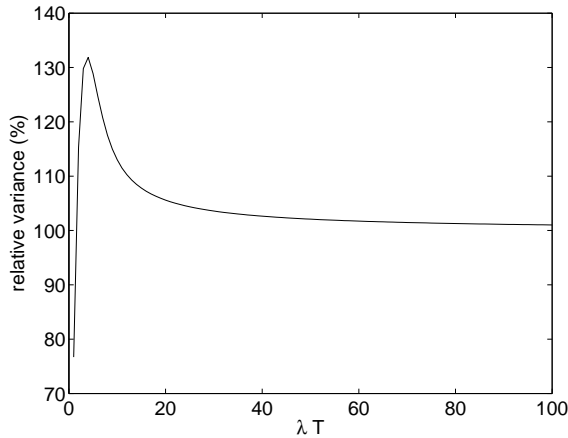

Fig. 3. The relative variance of the sample mean for IID random variables generated by uniform versus Poisson samples.

had an impact of sampling methods, but moreover, synchronized oscillations often result in reductions in performance. A case in point is TCP congestion-control synchronization [6], but it has been noted in other contexts such as routing [7]. However, there are few papers concerning detection of such oscillations in practice (for one example see [32]), perhaps in part because detection of such frequency components using irregularly sampled data seemed to be a hard problem. This paper demonstrates that this should not be a major issue, i.e. that we can detect periodic components of signals even where the data is irregularly sampled.

The problem of detection of periodic components of uniformly sampled data has been extensively studied. There are a number of approaches, but the simplest (conceptually) is to take the Discrete Fourier Transform (DFT), and examine the corresponding periodogram for peaks that would correspond to a periodic signal. There are a number of issues related to this method, many of which we shall omit in this paper (e.g., those concerning numerical properties of the periodogram). However, there is one critical issue, that of the Nyquist frequency, which we shall discuss further in Section IV-A.

One method to apply time-series algorithms to Poisson data would be to re-sample the data onto a uniform grid through interpolation. However, in doing so one runs the risk of introducing unknown interpolation artifacts. However, there are other contexts (such as astronomy) where observations

are perforce made irregularly. In these areas considerable work has been devoted to analysis of such time series. In particular, we can determine periodicities present in such data using the Lomb-Scargle Periodogram (LSP). This is a natural generalization of the standard periodogram defined in (7). It can be seen as analogous if we expand the complex exponential in the periodogram to give

$$P_X(\omega) = \frac{1}{N} \left[\left(\sum_{k=1}^{N} X_k \cos(\omega T_k) \right)^2 + \left(\sum_{k=1}^{N} X_k \sin(\omega T_k) \right)^2 \right].$$

The LSP [33], [34], [35] is instead defined by

$$P_X^{(LS)}(\omega) = \frac{1}{2} \left[\frac{\left(\sum_{k=1}^{N} (X(T_k) - \bar{X}) \cos(\omega(T_k - \tau)) \right)^2}{\sum_{k=1}^{N} \cos^2(\omega(T_i - \tau))} + \frac{\left(\sum_{k=1}^{N} (X(T_k) - \bar{X}) \sin(\omega(T_k - \tau)) \right)^2}{\sum_{k=1}^{N} \sin^2(\omega(T_i - \tau))} \right],$$

$$(33)$$

where τ is a frequency-dependent time shift (included to make the periodogram time-shift invariant) defined by

$$\tan(2\omega\tau) = \frac{\sum_{k=1}^{N} \sin(2\omega T_k)}{\sum_{k=1}^{N} \cos(2\omega T_k)}.$$

This new $P_X^{(LS)}$ gives us information about the frequency content of the signal at frequency ω. There are several methods for deriving, or justifying the LSP. For instance, as a periodogram that guarantees a flat spectral density when applied to irregularly sampled white noise [34], or which results from performing a least-squares fit of sinusoids to the irregularly sampled data (the standard periodogram can be thought of in the same way for regularly sampled data) [33]. The statistics of the LSP are known, and so one can determine simple hypothesis tests for the presence, or absence of particular frequencies in the measured signal.

As describe above the algorithm is not particularly efficient. However, fast $O(N \log N)$ algorithms for the computations of the LSP exist [36], [35], much as the Fast Fourier Transform exists for the DFT.

The LSP is not just applicable to Poisson sampled data. For instance, passive sampling of packet delays will also result in an irregularly-sampled times series, as in this type of measurement, we do not control the times of the packets, and moreover cannot even control the average rate of samples. However, note that the LSP has been explicitly applied to Poisson sampled data, for instance in [37].

In this paper we shall again use the M/M/1 queue for comparison because of its tractability. In particular, consider the ACF of the M/M/1 queue, given in (26). The WK theorem (8) allows us to compute the spectra of the M/M/1 queue from this relationship by taking the FT of $W_Q(s)$. To gain some insight into the shape of the spectra, we can use the FT of approximation (28) to analytically derive an approximation for the spectra. Note that the continuous FT of $e^{-A|t|}$ is given by

$$FT\left\{e^{-A|t|}\right\} = \frac{2A}{A^2 + 4\pi^2 f^2}, \tag{34}$$

at frequency f. Given that the FT is a linear operator, we can write

$$FT\left\{r_Q(t)\right\} \simeq \frac{A}{A^2 + 4\pi^2 f^2} + \frac{B}{B^2 + 4\pi^2 f^2}. \tag{35}$$

The discrete spectra turn out to be a little different. Taking the DFT of $r_W(nt_s)$ we get two terms of the form

$$\sum_{n=-N}^{N} e^{-A|nt_s|} e^{-in\omega}$$

$$= 1 + \sum_{n=0}^{N} e^{-nAt_s} e^{-in\omega} + \sum_{n=0}^{N} e^{-nAt_s} e^{in\omega}$$

$$= 1 + \sum_{n=0}^{N} \left(e^{-(At_s - i\omega)}\right)^n + \sum_{n=0}^{N} \left(e^{-(At_s + i\omega)}\right)^n$$

$$\rightarrow 1 + \frac{e^{-(At_s - i\omega)}}{1 - e^{-(At_s - i\omega)}} + \frac{e^{-(At_s - i\omega)}}{1 - e^{-(At_s + i\omega)}}, \tag{36}$$

in the limit as $N \rightarrow \infty$ (NB: $\omega = 2\pi f$). In addition to this term, we add a similar one with A replaced by B, and the mean queue results in an additional $\mu_Q^2 \delta(k)$ term.

Figure 4 shows the standard and LSPs for uniform and Poisson sampling of a simulated M/M/1 queueing process with $t_s = 100$ unit service times, $\rho = 0.95$, and $N = 100,000$ arrivals (note the first 5000 are discarded to avoid transients). The total number of samples is therefore near 1000 in each case. We deliberately choose a high-load example to illustrate the spectrum more clearly. The figure also shows the theoretical spectral density as described above.

Figure 4 (a) shows the periodogram for uniform sampling (top plot), which matches what we would expect from the theoretical spectral density (bottom plot). Surprisingly, the standard periodogram for Poisson sampling (middle plot) also appears to match the true spectral density as well. We will discuss the reasons for this in Section IV-B, but note that it is not a generic property for Poisson sampling, but one peculiar to the M/M/1 queue.

Figure 4 (b) shows the LSPs for uniform and Poisson sampling. We will discuss these further in Section IV-A but note that the LSP for Poisson sampling (middle plot) differs noticeably from the theoretical spectral density. The main difference is that the noise floor for the LSP is around zero

dB, masking the true spectrum of the process for frequencies greater than 1 cycle per 1000 service times. This may be thought of as the result of a general rule for signal processing: you don't get something for nothing. In this case, the penalty is an increase in the noise we must content with. We shall discuss the gain in the following section.

A. Aliasing

When data is uniformly sampled at frequency f_s (that is, the samples are separated by times $t_s = 1/f_s$), we loose data (concerning the signal between samples), and this in turn may lead to an ambiguity. Signals with different frequencies may result in identical samples, e.g. Figure 5 shows an example of two sinusoids which could generate the same set of sample data points (shown in the figure as circles). This problem is generally referred to as aliasing. Aliasing can be avoided if the highest frequency present in the signal has frequency $< f_s/2$, the so called Nyquist frequency, with the result that a signal $f(t)$ can unambiguously be reconstructed from its samples if sampled at twice the rate of the highest frequency present.

Figure 4 (b) illustrates the problem of aliasing in our context. The top plot shows the LSP for uniform samples. We can see that the central peak at $f = 0$ is aliased to create to additional peaks outside the Nyquist band (shown by vertical dashed lines). This is what one typically expects to see in the frequency domain — periodic repetitions of the spectra of the process at intervals of the Nyquist band.

In fact, the bias introduced when we uniformly sample a system synchronized with its periodicities is a special case of aliasing. A peak at frequency $f = n/t_s$ is exactly aliased to the DC term in the spectrum, which is equivalent to an estimate of the mean, hence distorting the estimate of the mean.

In order to ensure that aliasing not occur, signals should be filtered prior to sampling, typically with a low-pass filter with cut-off below the Nyquist frequency. For instance, when encoding audio information into the digital format used by CD (which are sampled at 44.1kHz) one would first pass the signal through a filter that removed any frequencies above around 22kHz. Given such filtering, one can unambiguously detect periodic components of the signal, but not over an arbitrary range — signals above the cut-off are effectively removed from the signal, and so are invisible.

However, there are a number of applications where the sampling methodology makes pre-filtering the signal inherently hard. In recording a CD, the filtering can be done through an analogue electrical circuit hooked to a microphone input. However, in many astronomical observations, the estimate is taken at a point in time. Likewise, for Internet measurements, the probe packets are point estimates, and we have no opportunity to low-pass these samples prior to making the measurements (note that post-processing the samples will not work). This results in a problem for Internet measurements. How can we prevent aliasing?

As it turns out, irregular sampling has its key benefit here. It has been shown [38] that the analogue of the Nyquist frequency is considerably higher for irregularly sampled data than for uniformly sampled data. The naive intuition is that the increased Nyquist frequency arises because with irregular sampling, some samples will be closer together than the average sampling distance, thereby removing ambiguity. However, it is

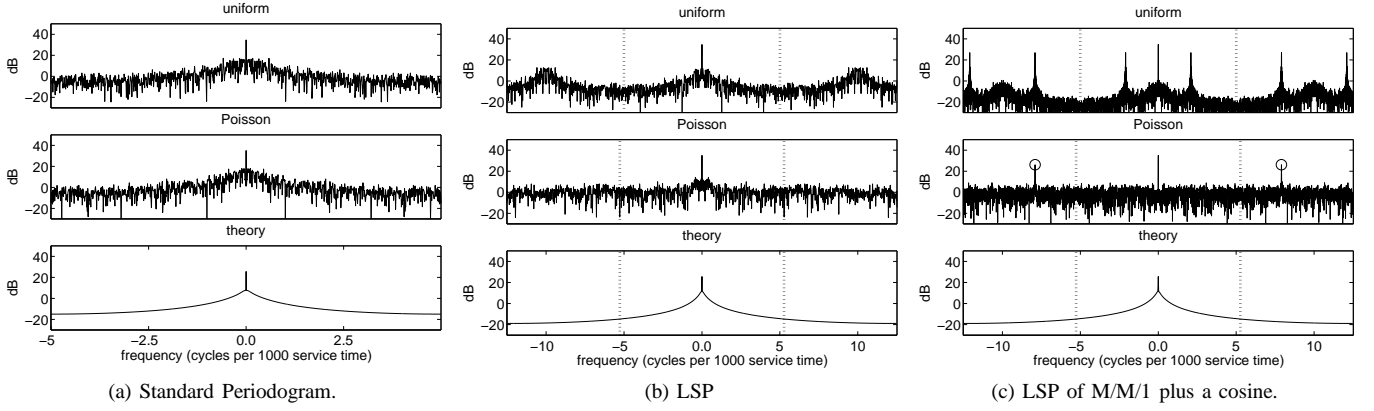(a) Standard Periodogram.   (b) LSP   (c) LSP of M/M/1 plus a cosine.

Fig. 4. Periodograms of the M/M/1 queue with $\rho = 0.95$ to show a wider-band spectrum. In each figure the top plot shows the periodogram using uniform sampling, the middle shows Poisson sampling, and the lower plot shows the theoretical spectral density of the queueing process. In plot (c) an additional cosine with frequency $f = 1.5/(2\,E[t_s])$ is added, clearly illustrating aliasing for uniform sampling. The signal is detected cleanly using Poisson sampling (see circles in middle plot).

not true that the new Nyquist frequency is $1/2\min(T_{i+1} - T_i)$. In fact [38] shows considerably higher frequencies may be resolved (it uses 122 data points of astronomical data, and successfully finds periodic behaviour with frequency of the order of a 100 times the uniform sampling Nyquist limit).
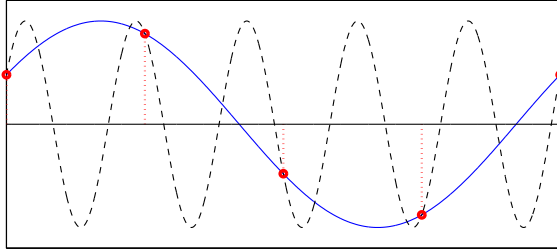

Fig. 5. Aliasing in the time and frequency domains. The figure shows two sinusoids that generate the same set of sample points (circles).

Figure 4 (b) illustrates this property. Note that when Poisson sampling is used (middle plot), the aliasing of the central peak to higher frequencies is eliminated. Figure 4 (c) further illustrates aliasing. In this case, we add a cosine wave with frequency $1.5\times$ the uniform sampling Nyquist frequency to the queueing process before sampling. When uniform sampling is used (top plot), we see multiple aliased peaks resulting from the sine function. When Poisson sampling is applied (middle plot), the two genuine peaks appear at the correct frequency. Notably, the detected sine has frequency outside the uniform sampling Nyquist band.

To correctly understand the result, we must correctly understand the impact of sampling in the Fourier domain, often referred to as the *window function*. We may think of sampling a continuous function as the process of multiplying this function by a sample process, i.e.

$$(X_n)(t) = X(t)S(t) = X(t)\sum_{k=-\infty}^{\infty} \delta(t - T_k). \quad (37)$$

The FT of the product of two functions is the convolution of the respective FTs, and so

$$FT\{(X_n)\} = FT\{X(t)\} * FT\{S(t)\}, \quad (38)$$

where $*$ denotes the convolution operation. In the case of uniform sampling,

$$FT\left\{\sum_{k=-\infty}^{\infty} \delta(t - kt_s)\right\} = \sum_{k=-\infty}^{\infty} t_s \delta(\omega - k/t_s), \quad (39)$$

which, when convolved with the true spectrum, results in repetitions of the spectrum at intervals of $1/t_s$. These repetitions do not overlap if the spectral density of the spectra is isolated into a band $1/t_s$ in width. Note also that the FT of a real process is even, and so there is only half this band available for non-redundant information.

On the other hand, the spectral density of the Poisson sampling process is given in (15). We can see that it is a constant, plus a δ DC term. Hence, when we convolve with this spectral density, the δ term results in the correct spectra (without repetitions), while the constant results in even spreading of the spectral content of the signal across all bands, albeit at a lower level than the signal of interest. Hence, this explanation provides an intuitive understanding of the origin of the noise in the LSP for the Poisson sampling cases. It also suggest that we can find periodic components of arbitrary frequency in a signal (i.e. there is no Nyquist limit), but note that this property only applies in the limit as the number of samples goes to infinity. However, even for finite samples we can certainly detect frequencies above $1/2t_s$.

Figure 4 (b) shows the increase in background noise that occurs in the LSP. Although methods exist to reduce this, it is still an issue if one is attempting to characterize the spectral density of the process. On the other hand if one wishes to detect the presence (or absence) of a limited number of sinusoids in the time series, then it is quite practical to do so, and standard statistical tests exist [37]. Hence, this approach is quite viable should one wish to, for instance, detect network synchronization effects.

A perhaps more intuitive explanation for the difference lies in the impact of randomization of the sample times. For the purpose of illustration, imagine that we start with uniform samples, but these are subject to some random jitter (perhaps as a result of irregularities in the instrumentation). Figure 6 shows two possible examples, for different frequency signals. When there is a change in the sampling time for the low-frequency case, we can see the jitter in time, as an error in the sample value, of similar order. However, a similar change

in the sampling time for the high-frequency case can result in an error of the order of magnitude of the signal itself, so the change in sampling has dramatically different effects at different frequencies.

B. Autocorrelations

The standard estimator of autocovariance for regularly sampled data is well known. Obviously, given such samples we do not get access to the autocovariance of the continuous process $X(t)$ from which we are sampling, but we can obtain asymptotically unbiased estimates at lags that form integer multiples of the sampling interval t_s using the estimator

$$\hat{R}_X(nt_s) = \frac{1}{N} \sum_{k=1}^{N-n} \left[X_k - \hat{X}_N \right] \left[X_{n+k} - \hat{X}_N \right], \quad (40)$$

where $\hat{X}_N$ is the sample mean of the process. Note the factor $1/N$ is correct, and results in a smaller variance than $1/(N-n)$ (which we might choose if we wished an unbiased estimate) and in addition this choice guarantees the estimates are non-negative definite.

This estimator is predicated on uniform sampling. Consider its behaviour when Poisson sampling is applied. We shall do so for a mean-zero sampled process $X(T_k)$, in order to make the analysis simple. For such a process denote our estimate (not necessarily of R_X) by

$$\hat{Q}(n) = \frac{1}{N} \sum_{k=1}^{N-n} X(T_k)X(T_{n+k}), \quad (41)$$

then we can rearrange to find

$$
\begin{aligned}
E\left[\hat{Q}(n)\right] &= \frac{1}{N} \sum_{k=1}^{N-n} E[X(T_k)X(T_{n+k})] \\
&= \frac{1}{N} \sum_{k=1}^{N-n} E[R(T_{n+k} - T_k)] \\
&= \frac{1}{N} \sum_{k=1}^{N-n} \int_0^\infty R(s) P\{T_{n+k} - T_k = s\}\, ds \\
&= \frac{1}{N} \sum_{k=1}^{N-n} \int_0^\infty R(s) \frac{\lambda(\lambda s)^{n-1} e^{-\lambda s}}{(n-1)!}\, ds, \quad (42)
\end{aligned}
$$

given that $T_{n+k} - T_k$ is the sum of n exponential RVs and therefore distributed as an Erlang-n RV. Note that the terms inside the summation do not depend on k, and so

$$E\left[\hat{Q}(n)\right] = \frac{N-n}{N} \int_0^\infty R(s) \frac{\lambda(\lambda s)^{n-1} e^{-\lambda s}}{(n-1)!}\, ds. \quad (43)$$

Clearly $\hat{Q}(n)$ may not be a good estimator of $R_X(nt_s)$.

It is interesting to consider the case of Poisson sampling of the M/M/1 queue. We shall do so by placing the approximation to $R(s)$, e.g. (28), for the M/M/1 queue into (43), such that we get two integrals of the form

$$
\begin{aligned}
\left(\frac{N-n}{N}\right) &\frac{\lambda \sigma^2}{2} \int_0^\infty \frac{(\lambda s)^{n-1} e^{-(\lambda+A)s}}{(n-1)!}\, ds \\
&= \left(\frac{N-n}{N}\right) \frac{\sigma^2}{2} \frac{1}{(1+A/\lambda)^n} \\
&\to \frac{\sigma^2}{2} \exp(-An/\lambda), \quad (44)
\end{aligned}
$$

for large n and N. Hence we can see that for the M/M/1 queue, the Poisson sampled ACF approaches the true ACF in the limit for a large number of samples, and large lags. See Figure 7 below to see how close the approximation is in a real case. In conjunction with the WK theorem (8), we can now see why the standard periodogram reported in Figure 4 (a) (middle plot) gives almost correct results for Poisson sampling. Given the ACF is estimated reasonably using Poisson sampling, we should expect the spectra to be equally correct.

The results show two things. Firstly, PASTA as a result is not necessarily true for higher-order statistics of a process. This is important to realize when we consider measurement of statistics such as delay variation. In this case the distortion is minimal, but could perhaps be larger for other processes, though in general we might expect estimates for very large lags to be reasonable, given higher-order Erlang distributions tend towards a deterministic distribution.

To obtain a true estimate of the ACF we could use interpolation of our original data set, but as before interpolation may introduce unknown artifacts. Alternatively, one might bin the time differences $t_i - t_j$ and compute an approximation to the autocovariance over such intervals. However, in this approach, information is lost.

On the other hand, the WK theorem, which relates the spectral density to $W_X(s)$, gives a method for computing the ACF by applying an inverse FT to the periodogram [37]. Starting from a periodogram plotted with the standard range and resolution for uniform sampling, we obtain estimates at discrete lags $\hat{W}_X(nt_s)$ (as we would with the time-domain estimator). We can plot a LSP at higher frequencies, and with finer resolution than the standard periodogram, resulting in finer resolution when we produce the resulting ACF, however, note that the same information is represented in the periodogram, and so this represents a form of implicit interpolation [34].

There is one important detail to correct when applying the inverse FT. As noted above, the FT of the sampled data is a convolution of the signal of interest with the FT of the window function. When we compute the periodogram, we take the magnitude of this function, and the result is a distortion of the spectra. The distortion is passed back to the time domain by the inverse DFT and so the measured ACF will be

$$\hat{r}(nt_s) = \hat{r}_X(nt_s) r_g(nt_s), \quad (45)$$

where r_g is introduced by the sampling. This effect must be removed to obtain $\hat{r}_X(nt_s)$, by dividing through by $r_g(nt_s)$.

Figure 7 shows ACF results for various approaches to estimation. The figure compares samples from simulations of the M/M/1 queue with the theoretical results described above. The results use somewhat more samples (of the order of 10,000), as one might expect when trying to estimate higher-order moments of a process. Figure 7 (a) shows time-domain estimates based on uniform and Poisson samples. We can see that over the range plotted they are both reasonably close to the true ACF, though note that (as described above) this is not a guaranteed property of Poisson samples, but rather a property that arise when sampling the M/M/1 queue. Figure 7 (a) shows frequency domain estimates derived by estimating the periodogram, and inverting. Three techniques are displayed, the first two being based on the standard periodogram of both uniform and Poisson samples to illustrate that these produce

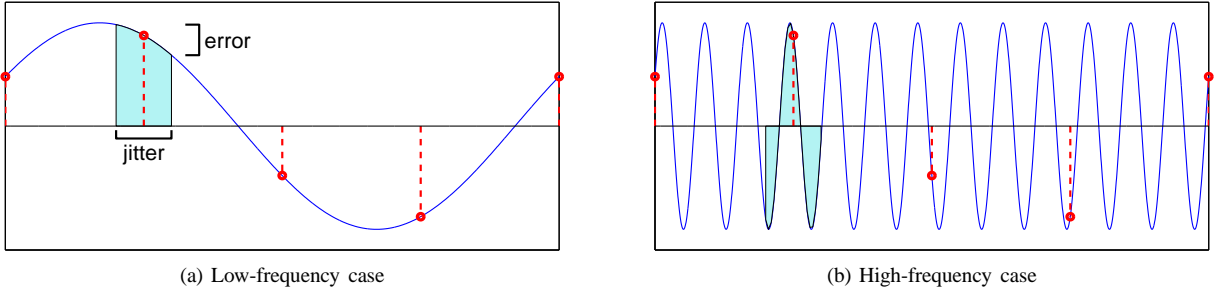(a) Low-frequency case

(b) High-frequency case

Fig. 6. The impact of irregularities in sampling times. The shaded region illustrates a potential range of jitter in the sampling times converts into an error in the sample value. For low frequencies the error is the same as the order of the jitter, but for high frequencies the error is of the order of the signal.

(a) Time domain estimates using the estimator (40).

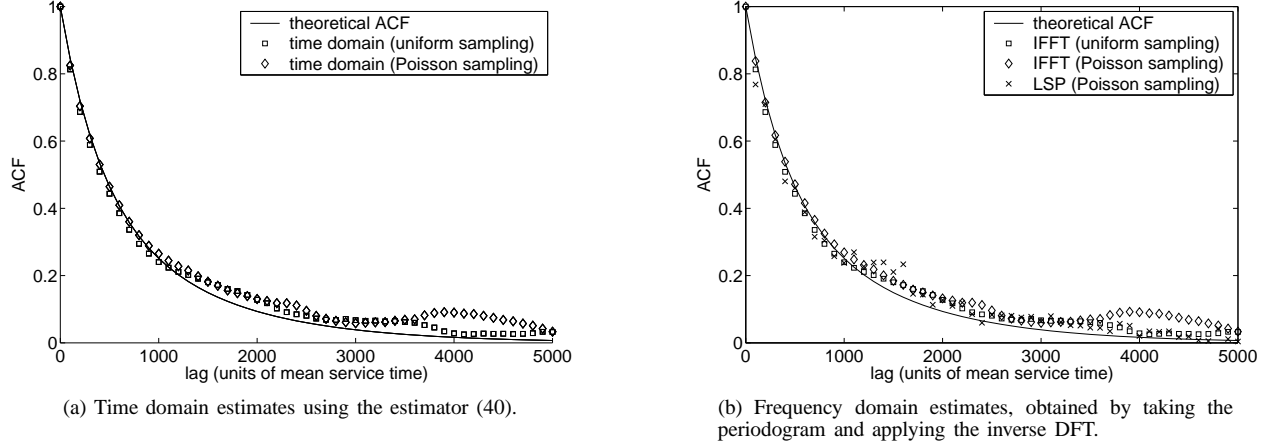(b) Frequency domain estimates, obtained by taking the periodogram and applying the inverse DFT.

Fig. 7. The estimated and theoretical ACF for $\rho = 0.95$ and $t_s = 100$. Note that most of the methods produce estimates close to the theoretical ACF, though the time-domain, and standard IFFT methods perform slightly more poorly for Poisson sampling. In particular, time domain and standard IFFT methods produce almost exactly the same results (for a given sampling). Inverting the LSP periodogram, however, underestimates the correlations.

almost identical results to their time domain equivalents. The last is based on a LSP, and shows that we could estimate the ACF for an arbitrary process, given Poisson sampling. Of the methods, the LSP appears to have the largest errors, possibly due to the additional noise in the spectra for Poisson sampling.

A similar method (inverting Fourier spectra) can also be used to derive cross-correlations for two sets of measurements [37].

C. Other Second Order Properties

Delay variation (as defined by the IETF [23]) is intended to capture a notion of the variability of delay measurements. A possible application is the adaptive design of playout buffers, which must buffer enough data so that even if some packets are delayed, a steady stream of data can be provided to an application. RFC 3393 [23] specifies delay variation be measured by examining differences in delays, i.e. we consider the process $Y_i = X_i - X_{i-1}$. Such a process gives us some idea of the variation of the delays. The RFC describes a number of statistics that one might collect to collect a group of such measurements — we shall consider its variance here (i.e. $\mathrm{Var}\,(Y_i)$), because of the numerical tractability of this measure, though similar findings will apply to other statistics.

In the case of uniform sampling, the interpretation of this metric is typically related to an application which sends traffic at the same intervals as we sample. However, it should be obvious from our discussion of correlations in queueing that $\mathrm{Var}\,(Y_i)$ depends on the sampling interval. In fact, we can

derive the variance explicitly in terms of the autocovariance of $X(t)$. First note that

$$E[Y_i] = E[X_i] - E[X_{i-1}] = 0, \qquad (46)$$

for a stationary process $X(t)$. Now, given $E[Y_i] = 0$

$$
\begin{aligned}
\mathrm{Var}\,(Y_i) &= E\big[Y_i^2\big] = E\big[(X_i - X_{i-1})^2\big] \\
&= E\big[X_i^2 - 2X_iX_{i-1} + X_{i-1}^2\big] \\
&= 2\,E\big[X_i^2\big] - 2\,E[X_iX_{i-1}]. \qquad (47)
\end{aligned}
$$

Let us further simplify (without loss of generality) to consider X_i to be mean zero, and we see that (as before)

$$\mathrm{Var}\,(Y_i) = 2\sigma_X^2 - 2\,E[R(T_i - T_{i-1})]. \qquad (48)$$

For uniform sampling this reduces to

$$\mathrm{Var}\,(Y_i) = 2[\sigma_X^2 - R(t_s)]. \qquad (49)$$

So clearly, for uniform sampling, the delay variation depends on the sampling interval t_s. It is therefore believed that to measure a delay-variation of relevance to a particular application, that the measurements must be made at the same intervals as the applications traffic, if these measures are to be of interest for studying the performance of that application.

For Poisson sampled measurements $X_i = X(T_i)$, and

$$E[R(T_i - T_{i-1})] = \lambda \int_0^\infty R(s)e^{-s\lambda}\,ds, \qquad (50)$$

resulting in different metrics for Poisson and uniform samples.

However, we can invert the LSP (with Poisson samples) to derive an ACF for the delay measurements, and from this, we could easily substitute values directly into (49) in order to obtain an estimate of the delay variation for an arbitrary sampling interval (whereas uniform sampling will provide this only for a fixed sampling interval and integer multiples thereof). Figure 8 shows a comparison of the direct computation from uniform samples with estimates obtained from inverting the LSP for Poisson samples. One can see that both provide a reasonable approximation to the true delay variation.
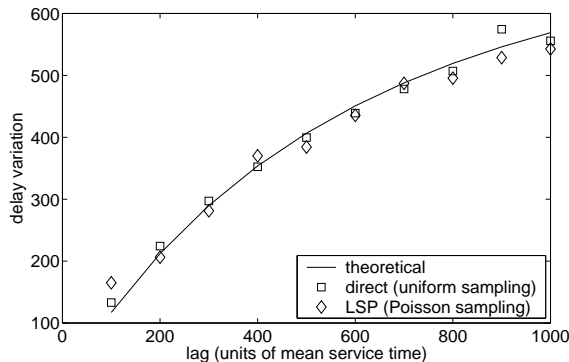

Fig. 8. A comparison of the direct computation of delay variation from uniform samples with estimates obtained from inverting the LSP for Poisson samples for the M/M/1 queue with $\rho = 0.95$, and $t_s = 100$.

V. OTHER ISSUES

There are many other issues that impact the choice of sampling used for performance measurements. We will discuss these, primarily for completeness, though the quantitative analysis of these issues will follow in future work.

1) Sampling intervals: Firstly, note that where random spacing between samples is used, we can no longer guarantee that we would observe a particular "event" of fixed finite duration. It is often assumed that uniform sampling avoids this issue by constraining the maximum time between probes to be t_s. However, when we consider a stationary process the notion of an event of a certain time is spurious — an "event" implies a non-stationarity in the data. Where the data are stationary, and we wish to observe a "micro-burst" of traffic, we will always do so with some probability determined by the sampling rate, and type of stochastic event (typically a level crossing). Where the events to be observed are of longer duration (e.g. a DoS attack over a few minutes), then a non-stationary model may well be more accurate, and therefore the issue of sampling times becomes relevant in determining time to detection, but note that the time scale of probes is typically an order of magnitude faster than the sort of events we might wish to model as non-stationarities. In fact, as noted earlier, periodic subsamples of the PP can closely approximate uniform samples spaced at times nt_s for large n, and so we can still use Poisson samples to detect such non-stationarities with very high probability.

2) Distorted sampling intervals: An alternative is to cap the maximum time between probes. This introduces similar issues to the fact that Poisson probes must have a minimum spacing (due at least to non-zero packet sizes but also because of maximum send rates at end points), whereas the exponential distribution allows arbitrarily small gaps between probes. It is even more true for measurements that require a period of time, such as bandwidth probes. Both distortions to the inter-arrival distribution away from a pure PP break the PASTA property, thereby allowing the opportunity for some distortion of the traffic measurements. However, note that the work reported on spectra of irregularly sampled data would still apply, and hence detecting synchronization effects is quite possible using near-Poisson sampling, and we can then determine whether there is any risk of synchronization, and hence avoid possible bias.

3) Component performance: Another point of importance is the fact that a network is really made up of many queues. PASTA applies to a system as a whole, and so a single queueing model such as used here may work for modelling end-to-end measurements of delay through a series of queues. However, the sample sequences of probes at each link in the network will not be Poisson. They will be distorted by, for example, queueing with cross-traffic at previous links. The PASTA property will therefore not hold at individual components of an end-to-end path. Does this matter? Not if we only consider end-to-end measurements. However, by performing probes across a set of end-to-end paths, one can form an inverse problem whose solution gives the performance of the network at individual components (for example, see [39] and the papers therein). However, PASTA will not hold for these individual components, and so there is an apparent reliance in such approaches on synchronizing effects being absent from the network.

4) Unamenable measurements: Some measurements are intrinsically hard to make using Poisson probes. For instance, reordering metrics are critically dependent on the spacing between probe packets. Packets far apart in time are very unlikely to be reordered. One could model this via delays in multiple queues, and attempt to infer reordering from Poisson samples, however, the estimate would be highly dependent on how accurate a model was used for the reordering process. Hence, for metrics such as this, uniform probing seems to make more sense. On the other hand, the same affect noted above — the distortion of probes away from their ideal probe stream at downstream links — also applies to uniform samples. Hence, if the reordering occurs late along a probe path, the samples will not be uniformly spaced, and the reordering metric will not measure the same thing as it would were the reordering happening early in the path. This seems to be a difficulty in the notion of defining a reordering metric, rather than the sampling method used.

In some cases, one wishes to either obtain measurements that are similar to a genuine applications traffic, or from a probe stream that is actually generated by the application. While it may be possible in some cases to infer the expected results of such an experiment indirectly from Poisson probes, it is often conceptually easier, and more convincing (to management) to just use the application sampled probes. These may be uniform, or have some random characteristics. In these cases, the approaches used above could be applied to provide quantitative understanding of the characteristics of the measurements.

VI. CONCLUSION

This paper has quantitatively compared the major differences between Poisson and uniform sampling when performing active performance measurements, and more generally presented techniques for understanding the properties of any sampling approach.

The key findings of the paper are:

- Poisson sampling results in a reduction in efficiency, and hence more Poisson probes would be required to provide estimates of the same statistical accuracy. However, the reduction in efficiency is dwarfed by the natural variation as a result of different system loads, and so this seems not to be the major consideration in choosing probing strategies.

- Irregular probes can be used to find time series properties of a system, such as its periodogram, or ACF. Such techniques could be used to detect periodicities in the system, and warn of potential bias due to synchronization (if pure Poisson sampling is not used).

It is clear that these findings do not rule out either approach. Rather a practitioner should choose the most appropriate measurements for their application, but this should be a choice informed by the above results.

There is significant future work to extend these results, in particular, to other types of sampling, for instance Poisson sampling where measurements are separated by an additional minimum time interval. This type of problem could be modelled using alternating renewal processes an associated theory.

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