

On Modeling of Fluctuations in Quasi-Static Approach Describing the Temporal Evolution of Retry Traffic

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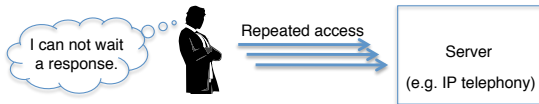
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- Significant problems with the Internet include node failure due to congestion or overload.
- One of the key factors behind overload is the generation of **retry traffic**.
- We consider **user impatience** which is one of the cause of retry traffic.

Retry traffic due to user impatience

- Users who can not endure their waiting time before starting the service might generate duplicate service requests.



Background (2)

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Background

Quasi-static retry
traffic model

Quasi-static
approach

Verification of the
quasi-static
approach

Conclusion

- An evaluation method that can accurately model retry traffic is important.
- In previous work, we proposed the **quasi-static approach** that replicates the temporal evolution of these traffic.
 - We separate a timescale of the transitions of user and system behavior in the approach.
 - The quasi-static approach can evaluate an high-speed system in which we can not use simulation approach.

The aim of this study

In this study, we confirm the validity of the quasi-static approach by comparing the traditional Monte-Carlo simulations.

Quasi-static retry traffic model

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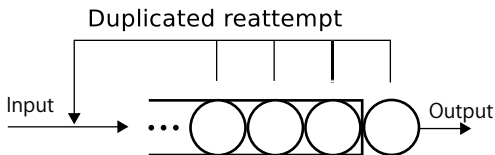
Quasi-static retry
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approach

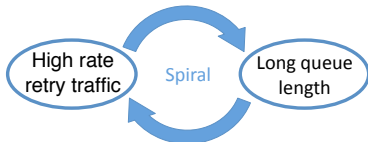
Verification of the
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approach

Conclusion

- We focus on one of the simplest model as an example:
M/M/1 with retry traffic.
- The rate of the retry traffic is depend on the queue length.



- Long queue length bring high rate retry traffic, high rate retry traffic bring more long queue.



Quasi-static retry traffic model (2)

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Quasi-static retry traffic model

Quasi-static approach

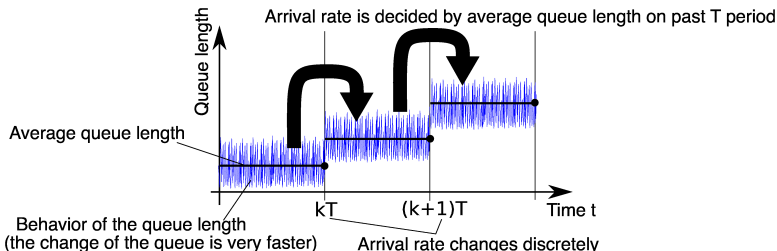
Verification of the quasi-static approach

Conclusion

- We modeled the behavior of retry traffic as the **quasi-static retry traffic model**.

Character of quasi-static retry traffic model

- The change of the queue is very faster compared with the user responses (We can **separate the timescales**).
- The change of the traffic rate is proportional to average queue length on past T period.



Quasi-static approach

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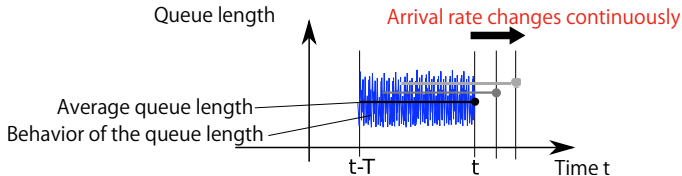
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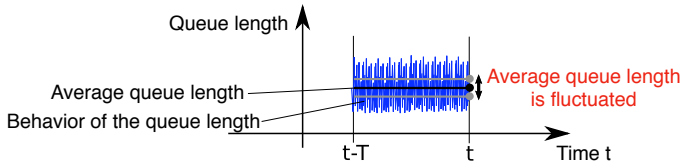
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- In actual system, the traffic changes not discretely but continuously.



- On finite speed system, an average queue length contains stochastic fluctuations.
- On infinite high speed system, the fluctuation is 0.



Quasi-static approach (2)

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- By considering the continuity and fluctuation, we describe the traffic behavior as the following Langevin equation:

$$\underbrace{\frac{d}{dt}X(t) = F(X(t))}_{\text{Behavior of the infinite high speed system}} + \underbrace{\sqrt{D(X(t))}\xi(t)}_{\text{Stochastic fluctuation}} \quad \begin{array}{l} X(t) : \text{Number of arrivals in} \\ \text{past } T \text{ period} \\ \xi(t) : \text{White Gaussian noise} \end{array}$$

$$\text{Average behavior} : F(X) = \lambda_0 - \frac{X}{T} + \varepsilon \frac{X/(\mu T)}{1 - X/(\mu T)}$$

$$\text{Magnitude of the fluctuation} : D(X) = \lambda_0 + \varepsilon \frac{X(t)/(\mu T)}{1 - X(t)/(\mu T)}$$

Quasi-static approach (3)

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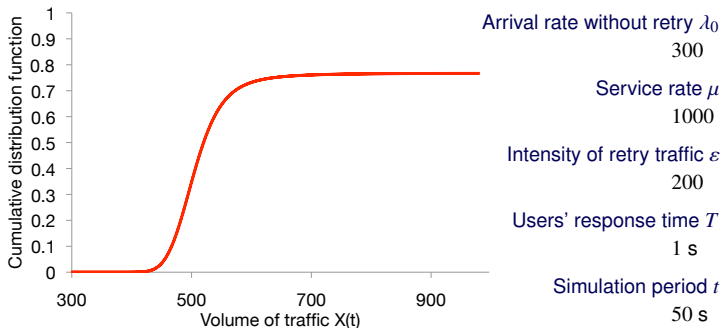
Verification of the
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Conclusion

- It is well known that the Langevin equation is equivalent to the Fokker-Planck equation as shown by

$$\frac{\partial}{\partial t} p(x, t) = -\frac{\partial}{\partial x} F(x) p(x, t) + \frac{1}{2} \frac{\partial^2}{\partial x^2} D(x) p(x, t)$$

- We can calculate the pdf $p(x, t)$ of $X(t)$, and can get the distribution.



Verification of the quasi-static approach

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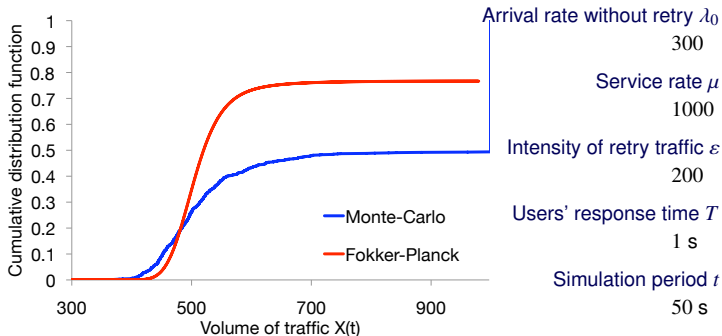
Quasi-static
approach

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Conclusion

- To confirm the validity of our approach, we compare the result with that of Monte-Carlo simulations.

$$F(X) = \lambda_0 - \frac{X(t)}{T} + \varepsilon \frac{X(t)/(\mu T)}{1 - X(t)/(\mu T)}, \quad D(X) = \lambda_0 + \varepsilon \frac{X(t)/(\mu T)}{1 - X(t)/(\mu T)}$$



- The distributions that are computed by the Monte-Carlo and our approach are not corresponding.

Verification of the quasi-static approach (2)

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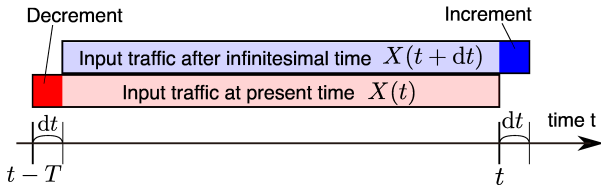
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- We reconsider $F(X(t))$ and $D(X(t))$.
 - $X(t)$ is a number of arrivals in past T period.
 - Infinitesimal change $dX(t)/dt$ is a random variables.
 - $F(X(t))$ and $D(X(t))$ indicate mean and variance of $dX(t)/dt$.
- $dX(t)/dt$ is composed of increment and decrement of $X(t)$.



- Mean and variance of each element are as follows.
 - Mean of increment : λ_0
 - Mean of decrement : $-X/T$
 - Variance of increment : λ_0
 - Variance of decrement : X/T

Verification of the quasi-static approach (3)

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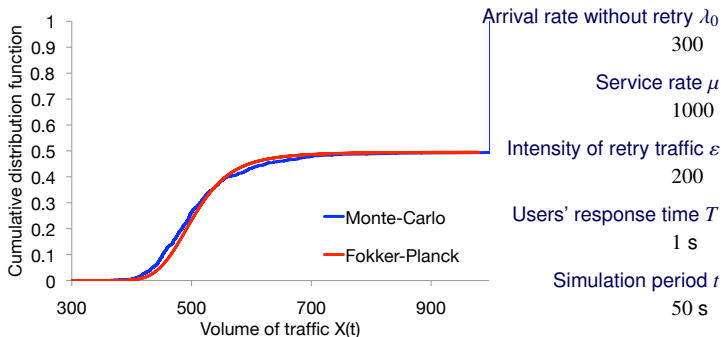
Quasi-static
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Conclusion

- We modified $F(X(t))$ and $D(X(t))$, and recalculate the distribution of the traffic.

$$F(X) = \lambda_0 - \frac{X(t)}{T} + \varepsilon \frac{X(t)/(\mu T)}{1 - X(t)/(\mu T)}, \quad D(X) = \lambda_0 + \frac{X(t)}{T} + \varepsilon \frac{X(t)/(\mu T)}{1 - X(t)/(\mu T)}$$



- We can confirm that the quasi-static approach yields result similar to that of the Monte-Carlo simulation.

In this study, we verified the **validity of quasi-static approach** that describes the behavior of input traffic including retry traffic.

- We computed the temporal evolution of input traffic on a M/M/1 based system with retry traffic by using the quasi-static approach.
- We were able to confirm that the results are corresponding to that of Monte-Carlo simulations.
- Therefore, we confirmed that the quasi-static approach can appropriately evaluate a system with retry traffic.

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Thank you very much for your kind attention.

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