Question 1. Bin Packing:

A1) Fit First Pseudo-Code:

```
Let I = {i_1 ... i_n} be the set of items to packn ≥ 1, n = number of items
Let M = {b_1 ... b_m} be the set of bins, m ≥ 1, m = number of machines

fit − First(I){
  for each item ∈ I{
    for each bin ∈ M{
      if item fits in bin{
        place item in bin
      }
    }
    if item was not placed in bin{
      create a new bin
      place item in bin
    }
  }
  return M
}
```

Running Time:

Worst Case scenario is if for each item a new bin is created. The outer loop will execute n times and the M will be searched n-1 times. $T(n) = O(n^2)$

A2) Fit First Decreasing Pseudo-Code:

```
MergeSort(I)
fit − First(I)
```

Running Time:

Mergesort is $O(nlogn)$; $fit - First\ is\ O(n^2).\ Therefore\ T(n) = O(n^2)$.

A3) Best Fit Pseudo-Code:

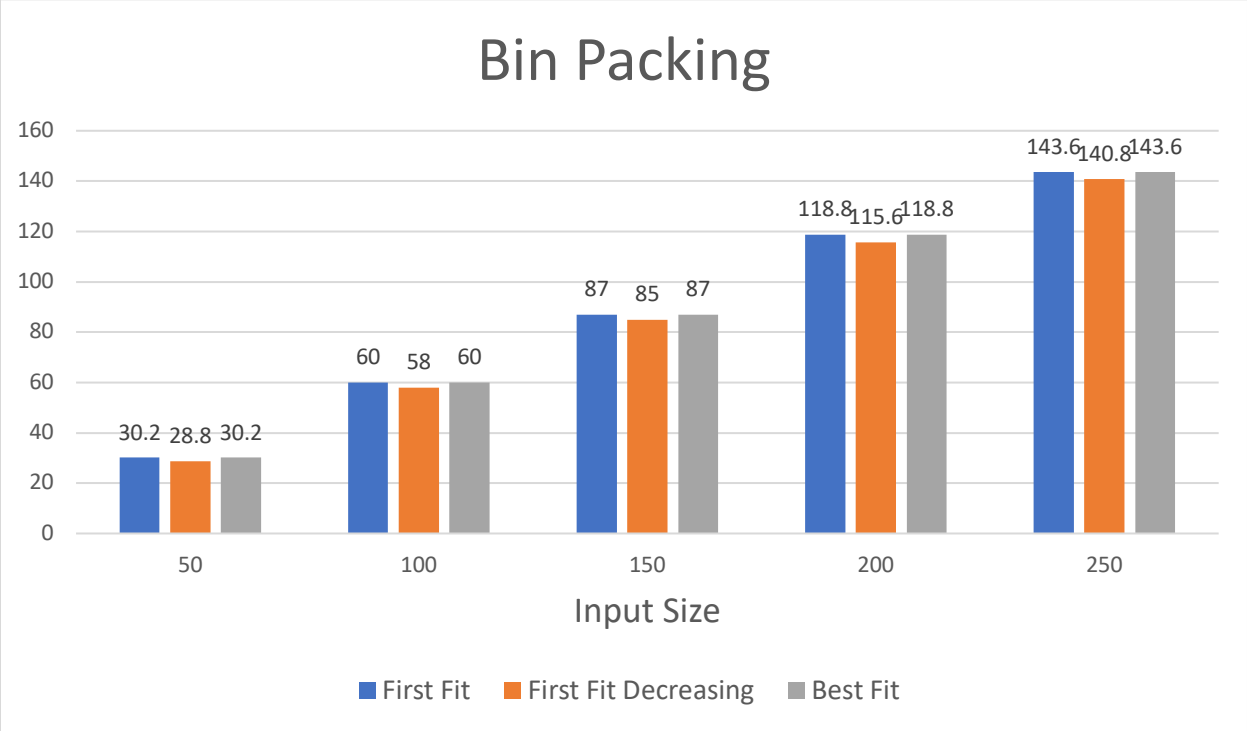
```
fit − First(I){
  let idx be the index of the bin with minimum weight
  idx ← ∅
  minWeight ← ∞
  for each item ∈ I{
    for each bin ∈ M{
      if item fits in bin{
        if bin.remWeight + item ≤ minWeight{
          idx ← index of the current bin
          minWeight ← bin.remWeight + item
        }
      }
    }
    place item in bin whose index = idx
    if item was not placed in bin{
      create a new bin
      place item in bin
    }
  }
  return M
}
```

Running time:

Since each bin has to be searchekidd, $T(n) = O(n^2)$

C) Random Testing

For 20 randomized instances, my input size were 50, 100, 150, 200, and 250. I ran 5 test on each input size. The bin capacity was kept constant at 10. I randomized the weight of the objects within a range (1, 10). The average number of bins for each input size was recorded and graphed:



The results show that Fit-First Decreasing is the optimum algorithm, in all the test cases, it required the least amount of bins.

Question 2. Linear Programming Bin Packing

A) Six item Set, Capacity 10

LINDO CODE:

```
MIN Y1 + Y2 + Y3 + Y4 + Y5 + Y6
ST
  Y1 + Y2 + Y3 + Y4 + Y5 + Y6 >= 1
  4X11 + 4X12 + 4X13 + 6X14 + 6X15 + 6X16 - 10Y1 <= 0
  4X21 + 4X22 + 4X23 + 6X24 + 6X25 + 6X26 - 10Y2 <= 0
  4X31 + 4X32 + 4X33 + 6X34 + 6X35 + 6X36 - 10Y3 <= 0
  4X41 + 4X42 + 4X43 + 6X44 + 6X45 + 6X46 - 10Y4 <= 0
  4X51 + 4X52 + 4X53 + 6X54 + 6X55 + 6X56 - 10Y5 <= 0
  4X61 + 4X62 + 4X63 + 6X64 + 6X64 + 6X66 - 10Y6 <= 0
  X11 + X21 + X31 + X41 + X51 + X61 = 1
  X12 + X22 + X32 + X42 + X52 + X62 = 1
  X13 + X23 + X33 + X43 + X53 + X63 = 1
  X14 + X24 + X34 + X44 + X54 + X64 = 1
  X15 + X25 + X35 + X45 + X55 + X65 = 1
  X16 + X26 + X36 + X46 + X56 + X66 = 1
END
INT Y1
INT Y2
INT Y3
INT Y4
INT Y5
INT Y6
INT X11
INT X21
INT X31
INT X41
INT X51
INT X61
INT X12
INT X22
INT X32
INT X42
INT X52
INT X62
INT X13
INT X23
INT X33
INT X43
INT X53
INT X63
INT X14
INT X24
INT X34
INT X44
INT X54
INT X64
INT X15
INT X25
INT X35
INT X45
INT X55
INT X65
INT X16
INT X26
INT X36
INT X46
INT X56
INT X66
```

Results:

```
LAST INTEGER SOLUTION IS THE BEST FOUND
 RE-INSTALLING BEST SOLUTION...

      OBJECTIVE FUNCTION VALUE

      1)     3.000000

 VARIABLE       VALUE        REDUCED COST
     Y1        1.000000        1.000000
     Y2        0.000000        1.000000
     Y3        1.000000        1.000000
     Y4        0.000000        1.000000
     Y5        1.000000        1.000000
     Y6        0.000000        1.000000
    X11        1.000000        0.000000
    X21        0.000000        0.000000
    X31        0.000000        0.000000
    X41        0.000000        0.000000
    X51        0.000000        0.000000
    X61        0.000000        0.000000
    X12        0.000000        0.000000
    X22        0.000000        0.000000
    X32        1.000000        0.000000
    X42        0.000000        0.000000
    X52        0.000000        0.000000
    X62        0.000000        0.000000
    X13        0.000000        0.000000
    X23        0.000000        0.000000
    X33        0.000000        0.000000
    X43        0.000000        0.000000
    X53        1.000000        0.000000
    X63        0.000000        0.000000
    X14        0.000000        0.000000
    X24        0.000000        0.000000
    X34        0.000000        0.000000
    X44        0.000000        0.000000
    X54        1.000000        0.000000
    X64        0.000000        0.000000
    X15        0.000000        0.000000
    X25        0.000000        0.000000
    X35        0.000000        0.000000
    X45        0.000000        0.000000
    X55        0.000000        0.000000
    X65        1.000000        0.000000
    X16        1.000000        0.000000
    X26        0.000000        0.000000
    X36        0.000000        0.000000
    X46        0.000000        0.000000
    X56        0.000000        0.000000
    X66        0.000000        0.000000
```

Results Discussion:

$Y1..Y2 \in \{1,0\}, a\ value\ of\ 1\ indicates\ that\ the\ bin\ was\ used$

$X_{ij} \in \{1,0\}, a\ value\ of\ 1\ indicates\ that\ item\ i\ was\ placed\ in\ bin\ j$

Bins Y1, Y3, and Y5 were used, a total of 3 Bins. The objective value was 3.

X11 and X16 were both 1, this means that items 1 and 6 were placed into bin Y1.

B) Five Item Set, capacity 20

```
MIN Y1 + Y2 + Y3 + Y4 + Y5
ST
  Y1 + Y2 + Y3 + Y4 + Y5 >= 1
  20X11 + 10X12 + 15X13 + 10X14 + 5X15 - 20Y1 <= 0
  20X21 + 10X22 + 15X23 + 10X24 + 5X25 - 20Y2 <= 0
  20X31 + 10X32 + 15X33 + 10X34 + 5X35 - 20Y3 <= 0
  20X41 + 10X42 + 15X43 + 10X44 + 5X45 - 20Y4 <= 0
  20X51 + 10X52 + 15X53 + 10X54 + 5X55 - 20Y5 <= 0
  X11 + X21 + X31 + X41 + X51 = 1
  X12 + X22 + X32 + X42 + X52 = 1
  X13 + X23 + X33 + X43 + X53 = 1
  X14 + X24 + X34 + X44 + X54 = 1
  X15 + X25 + X35 + X45 + X55 = 1
END
INT Y1
INT Y2
INT Y3
INT Y4
INT Y5
INT X11
INT X12
INT X13
INT X14
INT X15
INT X21
INT X22
INT X23
INT X24
INT X25
INT X31
INT X32
INT X33
INT X34
INT X35
INT X41
INT X42
INT X43
INT X44
INT X45
INT X51
INT X52
INT X53
INT X54
INT X55
```

Results:

```
LP OPTIMUM FOUND AT STEP     39
 OBJECTIVE VALUE =   3.00000000


 NEW INTEGER SOLUTION OF    3.00000000     AT BRANCH     0 PIVOT     39
 RE-INSTALLING BEST SOLUTION...

      OBJECTIVE FUNCTION VALUE

      1)     3.000000

  VARIABLE       VALUE        REDUCED COST
      Y1        1.000000        1.000000
      Y2        1.000000        1.000000
      Y3        1.000000        1.000000
      Y4        0.000000        1.000000
      Y5        0.000000        1.000000
     X11        0.000000        0.000000
     X12        0.000000        0.000000
     X13        1.000000        0.000000
     X14        0.000000        0.000000
     X15        1.000000        0.000000
     X21        1.000000        0.000000
     X22        0.000000        0.000000
     X23        0.000000        0.000000
     X24        0.000000        0.000000
     X25        0.000000        0.000000
     X31        0.000000        0.000000
     X32        1.000000        0.000000
     X33        0.000000        0.000000
     X34        1.000000        0.000000
     X35        0.000000        0.000000
     X41        0.000000        0.000000
     X42        0.000000        0.000000
     X43        0.000000        0.000000
     X44        0.000000        0.000000
     X45        0.000000        0.000000
     X51        0.000000        0.000000
     X52        0.000000        0.000000
     X53        0.000000        0.000000
     X54        0.000000        0.000000
     X55        0.000000        0.000000

 NO. ITERATIONS=      39
 BRANCHES=    0 DETERM.=  1.000E    0
```

Discussion:

Objective Value: 3, 3 bins were used – Y1, Y2, and Y3

Bin Y1: 3, 5

Bin Y2: 1

Bin Y3: 2 4