Question 1: Shortest Path using LP

a) Shortest Path from G to C:

Answer: 16

LINDO CODE

Note: dc is read as distance to vertex c, dg is distance to vertex g, so on and so forth

```
max dc
ST
  dg = 0
  dd - dg <= 2
  dh - dg <= 3
  db - dh <= 9
  da - dh <= 4
  db - da <= 8
  df - da <= 10
  db - df <= 7
  da - df <= 5
  dc - df <= 3
  de - df <= 2
  dd - dc <= 3
  df - dd <= 18
  de - dd <= 25
  dd - de <= 9
  dg - de <= 7
  dc - db <= 4
  de - db <= 10
  da >= 0
  db >= 0
  dc >= 0
  dd >= 0
  de >= 0
  df >= 0
  dg >= 0
  dh >= 0
END
```

LINDO Results:

LP OPTIMUM FOUND AT STEP 6

OBJECTIVE FUNCTION VALUE

1) 16.00000

VARIABLE	VALUE	REDUCED COST
DC	16.000000	0.000000
DG	0.000000	0.000000
DD	0.000000	0.000000
DH	3.000000	0.000000
DB	12.000000	0.000000
DA	4.000000	0.000000
DF	13.000000	0.000000
DE	0.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	1.000000
3)	2.000000	0.000000
4)	0.000000	1.000000
5)	0.000000	1.000000
6)	3.000000	0.000000
7)	0.000000	0.000000
8)	1.000000	0.000000
9)	8.000000	0.000000
10)	14.000000	0.000000
11)	0.000000	0.000000
12)	15.000000	0.000000
13)	19.000000	0.000000
14)	5.000000	0.000000
15)	25.000000	0.000000
16)	9.000000	0.000000
17)	7.000000	0.000000
18)	0.000000	1.000000
19)	22.000000	0.000000
20)	4.000000	0.000000
21)	12.000000	0.000000
22)	16.000000	0.000000
23)	0.000000	0.000000
24)	0.000000	0.000000
25)	13.000000	0.000000
26)	0.000000	0.000000
27)	3.000000	0.000000

NO. ITERATIONS= 6

b) Distance of the shortest paths from G to all other vertices:

Answer:

A – 7; B – 12; C – 16; D – 2; E – 19; F – 17; G – 0; H – 3;

LINDO CODE:

```
max da + db + dc + dd + de + df + dg + dh
ST
  dg = 0
  dd - dg <= 2
  dh - dg <= 3
  db - dh <= 9
  da - dh <= 4
  db - da <= 8
  df - da <= 10
  db - df <= 7
  da - df <= 5
  dc - df <= 3
  de - df <= 2
  dd - dc <= 3
  df - dd <= 18
  de - dd <= 25
  dd - de <= 9
  dg - de <= 7
  dc - db <= 4
  de - db <= 10
  da >= 0
  db >= 0
  dc >= 0
  dd >= 0
  de >= 0
  df >= 0
  dg >= 0
  dh >= 0
END
```

LINDO results:

LP OPTIMUM FOUND AT STEP 7

OBJECTIVE FUNCTION VALUE

1) 76.00000

VARIABLE	VALUE	REDUCED COST
DA	7.000000	0.000000
DB	12.000000	0.000000
DC	16.000000	0.000000
DD	2.000000	0.000000
DE	19.000000	0.000000
DF	17.000000	0.000000
DG	0.000000	0.000000
DH	3.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	8.000000
3)	0.000000	1.000000
4)	0.000000	6.000000
5)	0.000000	2.000000
6)	0.000000	3.000000
7)	3.000000	0.000000
8)	0.000000	2.000000
9)	12.000000	0.000000
10)	15.000000	0.000000
11)	4.000000	0.000000
12)	0.000000	1.000000
13)	17.000000	0.000000
14)	3.000000	0.000000
15)	8.000000	0.000000
16)	26.000000	0.000000
17)	26.000000	0.000000
18)	0.000000	1.000000
19)	3.000000	0.000000
20)	7.000000	0.000000
21)	12.000000	0.000000
22)	16.000000	0.000000
23)	2.000000	0.000000
24)	19.000000	0.000000
25)	17.000000	0.000000
26)	0.000000	0.000000
27)	3.000000	0.000000

NO. ITERATIONS= 7

Question 2. Product Mix:
A) Setup:
Profit per silk tie:
6.70s – 20 * (0.125s) – 0.75s = 3.45s

Profit per polyester tie:
3.55p – 6 * (0.08p) – 0.75p = 2.32p

Profit per blend1 tie:
4.31b – 9*(0.05b) – 6*(0.05b) – 0.75b = 2.81b

Profit per blend2 tie:
4.81c – 9*(0.07c) – 6*(0.03c) – 0.75c = 3.25 c

LINDO Code:

```
max 3.45s + 2.32b + 2.81p + 3.25c
ST
  0.05b + 0.07c <= 1250
  0.08p + 0.05 b + 0.03c <= 2000
  0.125s <= 1000
  s >= 6000
  s <= 7000
  p >= 10000
  p <= 14000
  b >= 13000
  b <= 16000
  c >= 6000
  c <= 8500
END
```

Answer:
Objective: $120453.2
# of silk ties: 7000
# of blend 1 ties: 13100
# of polyester ties: 13625
# of blend 2 ties: 8500

LINDO results:

```
LP OPTIMUM FOUND AT STEP      7

     OBJECTIVE FUNCTION VALUE

     1)     120453.2

 VARIABLE       VALUE          REDUCED COST
      S     7000.000000         0.000000
      B    13100.000000         0.000000
      P    13625.000000         0.000000
      C     8500.000000         0.000000


     ROW   SLACK OR SURPLUS     DUAL PRICES
      2)        0.000000        11.275000
      3)        0.000000        35.125000
      4)      125.000000         0.000000
      5)     1000.000000         0.000000
      6)        0.000000         3.450000
      7)     3625.000000         0.000000
      8)      375.000000         0.000000
      9)      100.000000         0.000000
     10)     2900.000000         0.000000
     11)     2500.000000         0.000000
     12)        0.000000         1.407000
     13)     7000.000000         0.000000
     14)    13100.000000         0.000000
     15)    13625.000000         0.000000
     16)     8500.000000         0.000000

 NO. ITERATIONS=      7
```

Question 3: Transshipment.

Part A:

Objective: $17100.00

Optimal Shipping from plants to warehouses:

P1 -> W1: 150 units

P2 -> W1: 200 units

P2 -> W2: 250 units

P3 -> W2: 150 units

P3 -> W3: 100 units

P4 -> W3: 150 units

Optimal shipping from warehouses to retailers:

W1 -> R1: 100 units

W1 -> R2: 150 units

W1 -> R3: 100 units

W2 -> R4: 200 units

W2 -> R5: 200 units

W3 -> R6: 150 units

W3 -> R7: 100 units

```
MIN 10P11 + 15P12 + 11P21 + 8P22 + 13P31 + 8P32 + 9P33 + 14P42 + 8P43 + 5W11 + 6W12 + 7W13 + 10W14 + 12W23 + 8W24 + 10W25 +
14W26 + 14W34 + 12W35 +12W36 + 6W37
ST
  P11 + P12 <= 150
  P21 + P22 <= 450
  P31 + P32 + P33 <= 250
  P42 + P43 <= 150
  P11 + P21 + P31 - W11 - W12 - W13 - W14 = 0
  P12 + P22 + P32 + P42 - W23 - W24 - W25 - W26 = 0
  P33 + P43 - W34 - W35 - W36 - W37 = 0
  W11 >= 100
  W12 >= 150
  W13 + W23 >= 100
  W14 + W24 + W34 >= 200
  W25 + W35 >= 200
  W26 + W36 >= 150
  W37 >= 100
  P11 >= 0
  P12 >= 0
  P21 >= 0
  P22 >= 0
  P31 >= 0
  P32 >= 0
  P33 >= 0
  W11 >= 0
  W12 >= 0
  W13 >= 0
  W14 >= 0
  W23 >= 0
  W24 >= 0
  W25 >= 0
  W26 >= 0
  W34 >= 0
  W35 >= 0
  W36 >= 0
  W37 >= 0
END
GIN P11
GIN P12
```

```
GIN P21
GIN P22
GIN P31
GIN P32
GIN P33
GIN P42
GIN P43
GIN W11
GIN W12
GIN W13
GIN W14
GIN W23
GIN W24
GIN W25
GIN W26
GIN W34
GIN W35
GIN W36
GIN W37
```

Results:

```
LP OPTIMUM FOUND AT STEP      6

        OBJECTIVE FUNCTION VALUE

        1)     17100.00

 VARIABLE        VALUE          REDUCED COST
     P11      150.000000         0.000000
     P12        0.000000         8.000000
     P21      200.000000         0.000000
     P22      250.000000         0.000000
     P31        0.000000         2.000000
     P32      150.000000         0.000000
     P33      100.000000         0.000000
     P42        0.000000         7.000000
     P43      150.000000         0.000000
     W11      100.000000         0.000000
     W12      150.000000         0.000000
     W13      100.000000         0.000000
     W14        0.000000         5.000000
     W23        0.000000         2.000000
     W24      200.000000         0.000000
     W25      200.000000         0.000000
     W26        0.000000         1.000000
     W34        0.000000         0.000000
     W35        0.000000         3.000000
     W36      150.000000         0.000000
     W37      100.000000         0.000000
```

Part B:
It is not feasible with current requirements.
By inspection:
Warehouse 3, W3, is the only warehouse that can supply retailers R5, R6, and R7 if W2 is closed.
The total demand of R5, R6, and R7:
200 + 150 + 100 = 450

In the best case scenario, warehouse 3, would receive all the refrigerators from plants P3 and P4:
250 + 150 = 400
The total supply from P3 and P4 cannot meet the demands of R5, R6, and R7. Therefore this infeasible.

```
min 10p11 + 11p21 + 13p31 + 9p33 + 8p43 + 5w11 + 6w12 + 7w13 +  10w14 + 14w34 + 12w35 + 12w36 + 6w37
ST
  p11 = 150
  p21 = 450
  p31 + p33 = 250
  p43 = 150
  w11 >= 100
  w12 >= 150
  w13 >= 100
  w14 + w34 >= 200
  w35 >= 200
  w36 >= 150
  w37 >= 100
  p11 + p21 + p31 - w11 - w12 - w13 - w14 = 0
  p33 + p43 - w34 - w35 - w36 - w37 = 0
  p11 >= 0
  p21 >= 0
  p31 >= 0
  p33 >= 0
  p43 >= 0
  w11 >= 0
  w12 >= 0
  w13 >= 0
  w14 >= 0
  w34 >= 0
END
```

Part C:

Yes it's possible.

Answers:

Objective: $18300

LINDO Code:

```
min 10p11 + 15p12 + 11p21 + 8p22+ 13p31 + 8p32 + 9p33 + 14p42 + 8p43 + 5w11 + 6w12 + 7w13 + 10w14 + 12w23 + 8w24 + 10w25 +
14w26 + 14w34 + 12w35 + 12w36 + 6w37
ST
  p11 + p12 = 150
  p21 + p22 = 450
  p31 + p32 + p33 = 250
  p42 + p43 = 150
  w11 >= 100
  w12 >= 150
  w13 >= 100
  w14 + w24 + w34 >= 200
  w25 + w35 >= 200
  w26 + w36 >= 150
  w37 >= 100
  p11 + p21 + p31 - w11 - w12 - w13 - w14 = 0
  p33 + p43 - w34 - w35 - w36 - w37 = 0
  w23 + w24 + w25 + w26 <= 100
  p11 >= 0
  p21 >= 0
  p31 >= 0
  p33 >= 0
  p43 >= 0
  w11 >= 0
  w12 >= 0
  w13 >= 0
  w14 >= 0
  w34 >= 0
END
```

```
min 10p11 + 15p12 + 11p21 + 8p22+ 13p31 + 8p32 + 9p33 + 14p42 + 8p43 + 5w11 + 6w12 + 7w13 + 10w14 + 12w23 + 8w24 + 10w25 +
14w26 + 14w34 + 12w35 + 12w36 + 6w37
ST
  p11 + p12 = 150
  p21 + p22 = 450
  p31 + p32 + p33 = 250
  p42 + p43 = 150
  w11 >= 100
  w12 >= 150
  w13 >= 100
  w14 + w24 + w34 >= 200
  w25 + w35 >= 200
  w26 + w36 >= 150
  w37 >= 100
  p11 + p21 + p31 - w11 - w12 - w13 - w14 = 0
  p33 + p43 - w34 - w35 - w36 - w37 = 0
  w23 + w24 + w25 + w26 <= 100
  p11 >= 0
  p21 >= 0
  p31 >= 0
  p33 >= 0
  p43 >= 0
  w11 >= 0
  w12 >= 0
  w13 >= 0
  w14 >= 0
  w34 >= 0
END
```

Question 4: Coin Change

A) Objective Value: 10 coins

# of 25-value coins: 8

# of 1-value coins: 2

# of 5-value coins: 0

# of 10-value coins: 0

LINDO Code:

```
MIN C1 + C5 + C10 + C25
ST
  1C1 + 5C5 + 10C10 + 25C25 = 202
  C1 >= 0
  C5 >= 0
  C10 >= 0
  C25 >= 0
END
GIN C1
GIN C5
GIN C10
GIN C25
```

Results:

```
LAST INTEGER SOLUTION IS THE BEST FOUND
 RE-INSTALLING BEST SOLUTION...

      OBJECTIVE FUNCTION VALUE

      1)     10.00000

 VARIABLE       VALUE        REDUCED COST
      C1        2.000000        1.000000
      C5        0.000000        1.000000
     C10        0.000000        1.000000
     C25        8.000000        1.000000


     ROW   SLACK OR SURPLUS     DUAL PRICES
      2)        0.000000        0.000000
      3)        2.000000        0.000000
      4)        0.000000        0.000000
      5)        0.000000        0.000000
      6)        8.000000        0.000000

 NO. ITERATIONS=      32
 BRANCHES=    6 DETERM.=  1.000E    0
```

B)Objective Value: 14 coins
# of 27-value coins: 9
# of 12-value coins: 3
# of 7-value coins: 2
# of 1-value coins: 0
# of 3-value coins: 0
LINDO Code:

```
MIN C1 + C3 + C7 + C12 + C27
ST
  1C1 + 3C3 + 7C7 +12C12 + 27C27 = 293
  C1 >= 0
  C3 >= 0
  C7 >= 0
  C12 >= 0
  C27 >= 0
END
GIN C1
GIN C3
GIN C7
GIN C12
GIN C27
```

Results

```
     OBJECTIVE FUNCTION VALUE

     1)     14.00000

 VARIABLE       VALUE        REDUCED COST
     C1        0.000000        1.000000
     C3        0.000000        1.000000
     C7        2.000000        1.000000
    C12        3.000000        1.000000
    C27        9.000000        1.000000


    ROW   SLACK OR SURPLUS     DUAL PRICES
     2)        0.000000        0.000000
     3)        0.000000        0.000000
     4)        0.000000        0.000000
     5)        2.000000        0.000000
     6)        3.000000        0.000000
     7)        9.000000        0.000000
```