Problem Set 8 (due 6/5)
(Physics 115A, Spring 2015, H. W. Jiang)

1. Derive the following inverse Fourier transform relation using the Dirac bra-ket notation.

$$\Phi(p) = \tfrac{1}{\sqrt{2\pi\hbar}} \int dx \Psi(x) e^{-ipx/\hbar}$$

2. Consider a particle in a one-dimensional infinite potential box of length a. Its wave function has the most general form

$$\psi(x) = \sum_{n=1}^{\infty} c_n \sqrt{2/a} \sin \tfrac{n\pi x}{a}$$

Write this in the Dirac Notation using the ket-bra sum for the box base kets denoted by |n>. Show the correspondence between the two notations. Interpret $\langle x|\Psi\rangle$, $\langle x|n\rangle$, and $\langle n|\Psi\rangle$ that will arise as you do the problem.

3. If A and B are hermitian operators, prove that (1) the operator of AB is only hermitian if A and B commute, and (2) the operator $(A+B)^n$ is hermitian.

4. The initial state $|\Psi_i\rangle$ of a quantum system is given in an orthonormal basis of three states $|\alpha\rangle$, $|\beta\rangle$, and $|\gamma\rangle$ that form a complete set:

$\langle\alpha|\Psi_i\rangle = i/(3)^{1/2}$, $\langle\beta|\Psi_i\rangle = (2/3)^{1/2}$, $\langle\gamma|\Psi_i\rangle = 0$

Calculate the probability of finding the system in a state $|\Psi_f\rangle$ given in the same basis as

$\langle\alpha|\Psi_f\rangle = (1+i)/(3)^{1/2}$, $\langle\beta|\Psi_f\rangle = 1/(6)^{1/2}$, $\langle\gamma|\Psi_f\rangle = 1/(6)^{1/2}$

5. Show that if the operator H is hermitian, then $\exp(-iH)$ is unitary.

6. Use the commutation relations between the momentum p and the position x to obtain the equations describing the time dependence of $\langle x\rangle$ and $\langle p\rangle$ given the Hamiltonians

(a) $H = \dfrac{p^2}{2m} + \dfrac{1}{2} m(\omega_1^2 x^2 + \omega_2 x + \varepsilon)$

(b) $H = \dfrac{p^2}{2m} + \dfrac{1}{2} m\omega^2 x^2 - \dfrac{A}{x^2}$