Water Science and Engineering
Hydroinformatics - Modelling and Information Systems for Water Management

WSE-HI- Module - 4

Numerical Methods - I

ASSIGNMENT – 1

Water Level in Reservoir

Submitted By;

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1. Task

An irrigation canal is supplied with water from a reservoir along a river. The inflow rate in the reservoir is at a known rate of I(t). The plan cross-section A(h) of the reservoir is variable with the water level h (the variation of A with h are known). The reservoir has a bottom outlet outflowing discharge, denoted by Q(t). A weir regulated the amount of water that is discharged into the irrigation channel. The water level in the reservoir is given by the formula:

$$\frac{dh}{dt} = \frac{I(t) - Q(t) - mb(h-a)^{3/2}}{A(h)}$$

Where m = known weir coefficient that depends on the weir geometry

H = Water level in the reservoir (masl)

I(t) = Inflow in the reservoir (m^3/s)

m = Empirical coefficient for weir

b = Width of the weir

a = Weir height (m)

A = Surface area of the reservoir (m^2) (Varies with h)

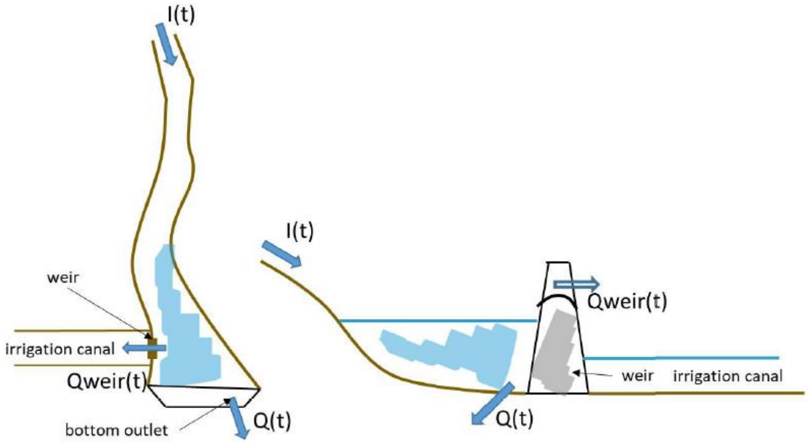

Figure 1: Reservoir

Table 1: dataset for the reservoir

Dataset no	H (masl)	20	40	50		h_0 (masl)	Weir Height a (masl)		B	m	Q (m3/s)	
3	A (km2)	0	1.3	5.4		40	40.5		90	0.5	7	
	Time (Hours)	0	5	10	15	20	25	30	35	40	45	50
	Inflow (m3/s)	11	11	40	76	100	85	60	35	25	20	10

2. Question 6.1

Calculations for the water level in the reservoir and the total outflowing volume for a period of 100 hours using Euler formula and Implicit Euler formula and comment the differences. (Express h^{n+1} as a function h^n)

2.1 Explicit Solution

"In explicit discretization, the values at the grid nodes of each unknown variable at the new time step of computation depend only on known values at the previous time step" (Ioana Popescu, 2014). In this case, the explicit scheme determines the water level in the reservoir using known values of the previous time step. The equation is written as follow when h (water level in reservoir) lesser than weir height (a);

$$h^{n+1} = h^n + \frac{\Delta\mathrm{t}\,(I^n - Q^n)}{A(h^n)}$$

The equation is written as follow when h (water level in reservoir) greater than weir height (a);

$$h^{n+1} = h^n + \frac{\Delta\mathrm{t}\,(I^n - Q - m * b(h^n - a)^{3/2})}{A(h^n)}$$

2.2 Implicit Solution

"The implicit scheme of time discretization determines the values of the unknown function at the grid nodes from values of the function at neighbouring points at the same time level" (Ioana Popescu, 2014). The implicit scheme determines the water level in the reservoir using the same time step. Consider that we have the value of Inflow I(t) with time step. The height of the water in a reservoir can be calculated by the given equations by knowing the height of the previous time step (h^n). For given case cross-section area (A) of the reservoir is changing corresponding to reservoir height (H).

The equation is written as follow when h (water level in reservoir) lesser than weir height (a);

$$h^{n+1} = h^n + \frac{\Delta\mathrm{t}\,(I^{n+1} - Q^{n+1})}{A(h^{n+1})}$$

The equation is written as follows when h (water level in reservoir) greater than weir height (a);

$$h^{n+1} = h^n + \frac{\Delta t\,(I^{n+1} - Q - m * b(h^{n+1} - a)^{3/2})}{A(h^{n+1})}$$

3. Question 6.2

Implementation of the discretization in python file attached to the assignment in .py format.

4. Question 6.3

Graphs of computed water level and the outflowing discharge. Try different time steps and choose the optimal one. Justify the answer.

4.1 Water Level in the Reservoir using Explicit and Implicit Scheme

The cross-section area (a) of the reservoir is changing with the height (H) of the reservoir. In this case, the reservoir has an irrigation canal with a weir and bottom outlet discharge (Q). As per the given data set (Table 1), the following graphs have been generated using an explicit solution (Figure 2) and implicit (Figure 3).

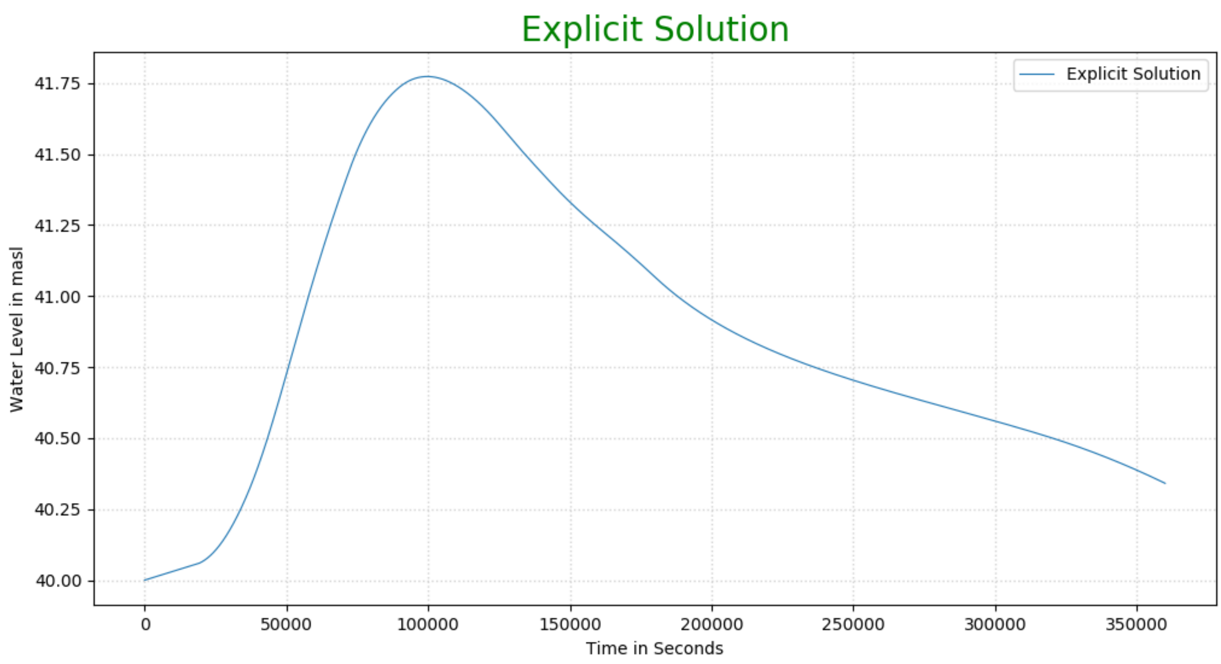

Figure 2: Water level over Time by Explicit Solution

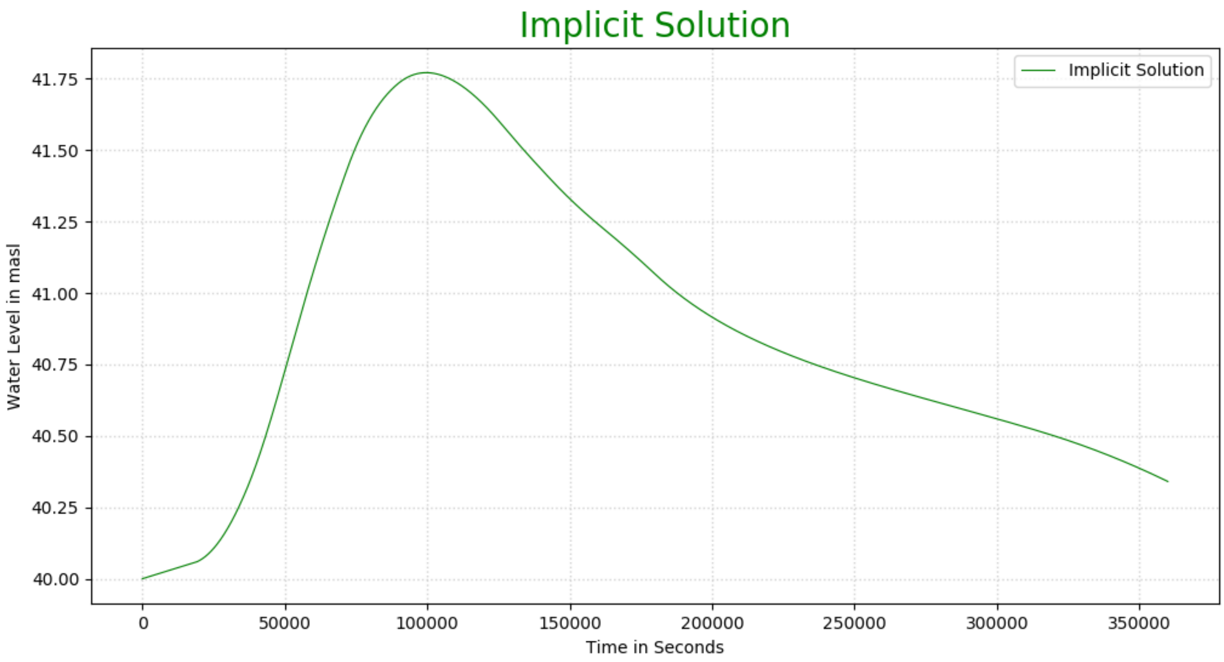

Figure 3: Water level over Time by Implicit Solution

The elevation of the reservoir (H) is 20, 40 & 50 masl and cross-section area (A) is 0, 1.2 & 5.4 respectively. So, the Cross-section area (a) of the reservoir is variable with the height (H) of the reservoir. Provided datasets (Table 1) for the inflow have been used to determine the water level for a period of 100 hours. The initial inflow is 11 m^3/s for the 0 and 5 hours' time steps and then it has started increasing after 10 hours. Peak inflow is given on 20 hours' time step that is 100 m^3/s. After 20 hours inflow starts decreasing continuously with every next hour and after 50 hours there is no inflow. The rate of inflow is assumed to be constant between two conjunctive time intervals. Therefore, the outflow from the weir in the irrigation canal is dependent on the inflow rate in the reservoir. The algorithm has been developed to determine and display the total outflow volume of water in case of overflow.

It can be observed in both explicit and implicit solution based graphs (Figure 2 & 3) initial water level in the reservoir is 40 masl. Δt = 100 seconds has been used for optimal output for the water level over time for both solution and the behaviour of both graphs is the same because it depends on Δt (this will be discussed later). The graph shows the variation in the water level in the reservoir over time. Water level starts increasing gradually from 0 to 20,000 seconds and after that water level graph increasing rapidly, it has increased till 100,000 seconds time steps because the inflow rate going higher in this particular period and the water level reached its maximum level that is computed on 41.78 masl. As shown in the graphs, as per the given inflow rate it is decreasing after 20 hours till 50 hours, therefore the water level is also decreasing continuously. Now by the given condition, there is no more inflow in the reservoir after 50 hours and it can observe that the water level is decreasing in the reservoir because the reservoir also has a bottom discharge and there is a direct relationship between the outflow from the reservoir and the water level.

4.2 Total Outflowing Volume of Water from Reservoir

The initial outflowing volume of the water (Figure 4) from 0 to 43,500 seconds has been observed constant, because of bottom outflow discharge from the reservoir and there is no outflow from the weir to the canal. But after this time step, it has starts overflowing from the weir and increasing continuously till 100,000 seconds at which the outflowing volume of the water has reached on maximum value that is 7160 m^3. Because during this particular time there is the continuous inflow in the reservoir. Same time as in discharge (Figure 5), the outflowing volume of the water also decreasing, it has reached again on the initial level after 320,000 seconds time steps (Figure 4) and there is no outflow from the weir.

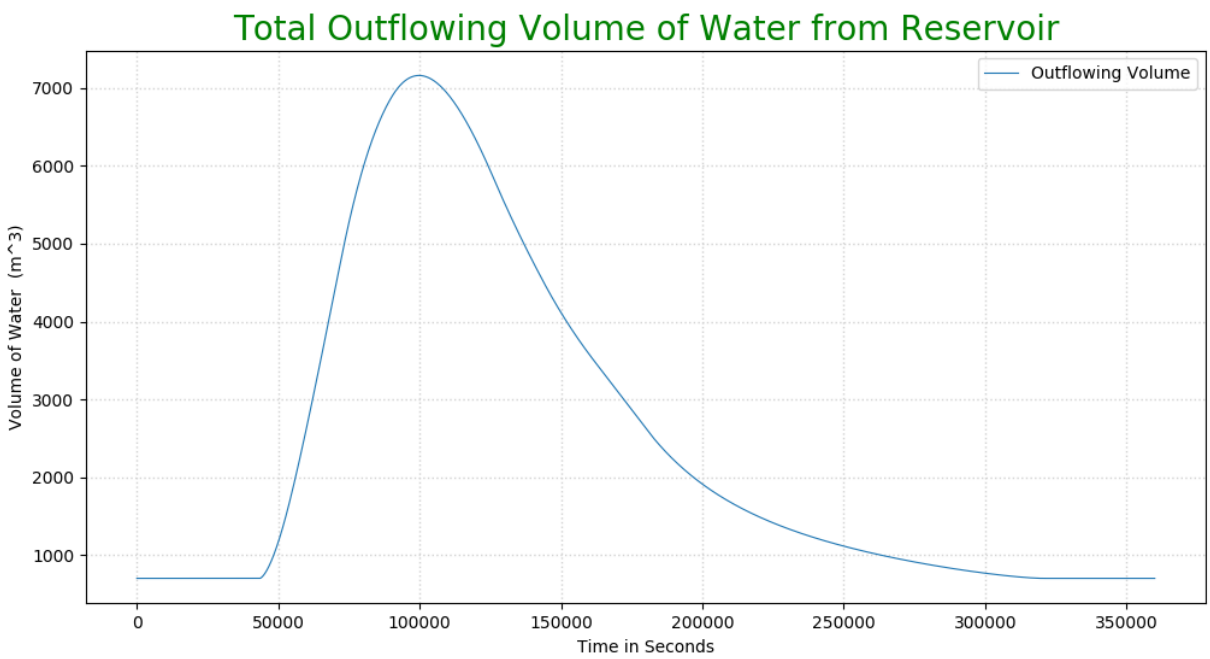

Figure 4: Total Outflowing Volume of Water from Reservoir

4.3 Outflowing Discharge from the Weir

The graph (Figure 5) showing discharge over the weir. It can be observed that it took 43,500 seconds to start overflow from the weir to the canal. Because there is some gap between weir height (a = 40.5 masl) and initial water level in the reservoir (h = 40 masl). And the reservoir also has a bottom discharge. So, the reservoir has taken some time to fill up to reach on weir height level and then after that outflowing discharge has started from the weir to the irrigation canal. The behaviour of the outflowing volume (Figure 4) and discharge (Figure 5) graphs are the same because both are correlated. The peak discharge calculated on 100,000 seconds i.e. 64.5 m^3/s and it has become 0 m^3/s after 320,200 second. The discharge from the weir has been calculated by the following formula;

$$Q_{weir} = (m*b*(h - a)^{1.5}$$

As the water level goes down the discharge also decreases from the weir i.e. less volume of water is outflowing from the reservoir to the canal from the weir with decreasing elevation. On the particular time i.e. approximately 90 hours water level (Figures 2 & 3) has been reached on weir height (a = 40.5 masl) and because of no further inflow and constant bottom discharge from the reservoir, the water level is regularly decreasing. This algorithm (attached .py file) is designed to determine and display the spilled discharge in case of overflow from the weir which depends on inflow water to the reservoir. Results verify that the developed model is working well.

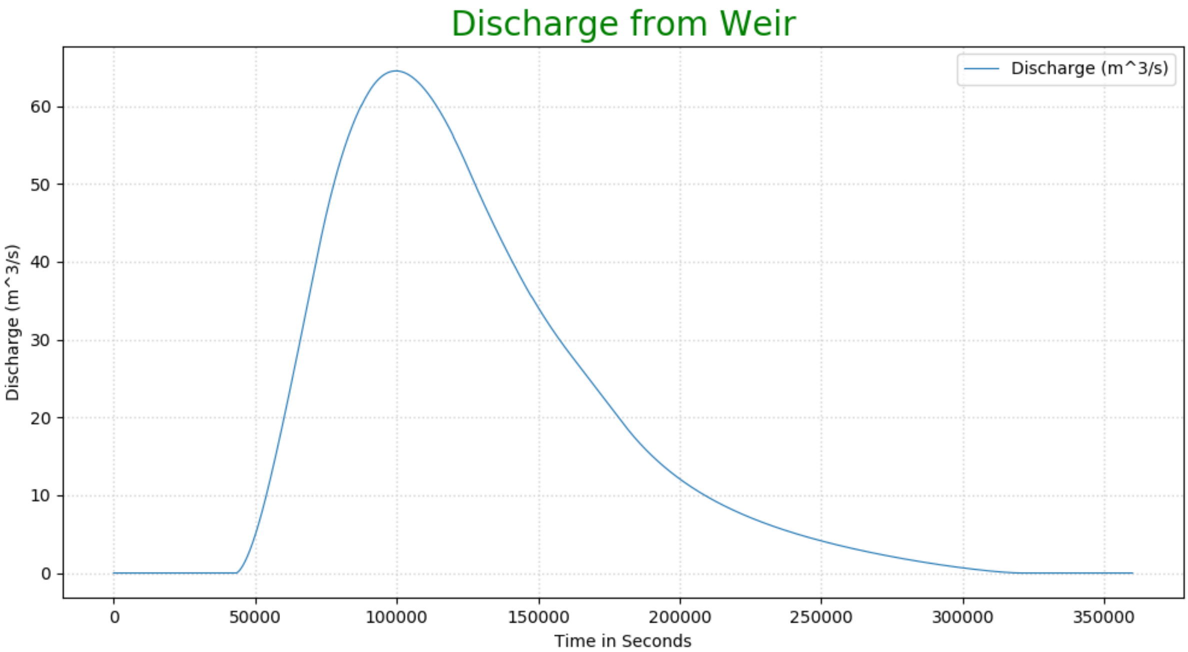

Figure 5: Outflowing Discharge over the Weir

4.4 Test on Different Δt

Three tests have been done to get optimal Δt to determine the water level in the reservoir. It can be observed that in figure 6, the explicit and implicit scheme follows the same pattern when Δt is smaller (Δt = 100 seconds). The behaviour suggests that the time step plays a vital role to get the optimal results. The explicit scheme is conditionally stable (amplification factor should less than equal to 1) which means the explicit scheme is not stable for all values of Δt, accuracy will be limited and implicit scheme is unconditionally stable, when Δt is higher (Figure 7 & 8). The use of an explicit method requires impractically small time steps Δt to keep the error in the result bounded. For such problems, to achieve given accuracy, it takes much less computational time to use an implicit method with larger time steps. In common, it can be concluded that the lower Δt is, the better the results. However, this increases the amount of time and computations required to generate the data. The difference can be seen in the following graphs (Figure 6, 7 & 8).

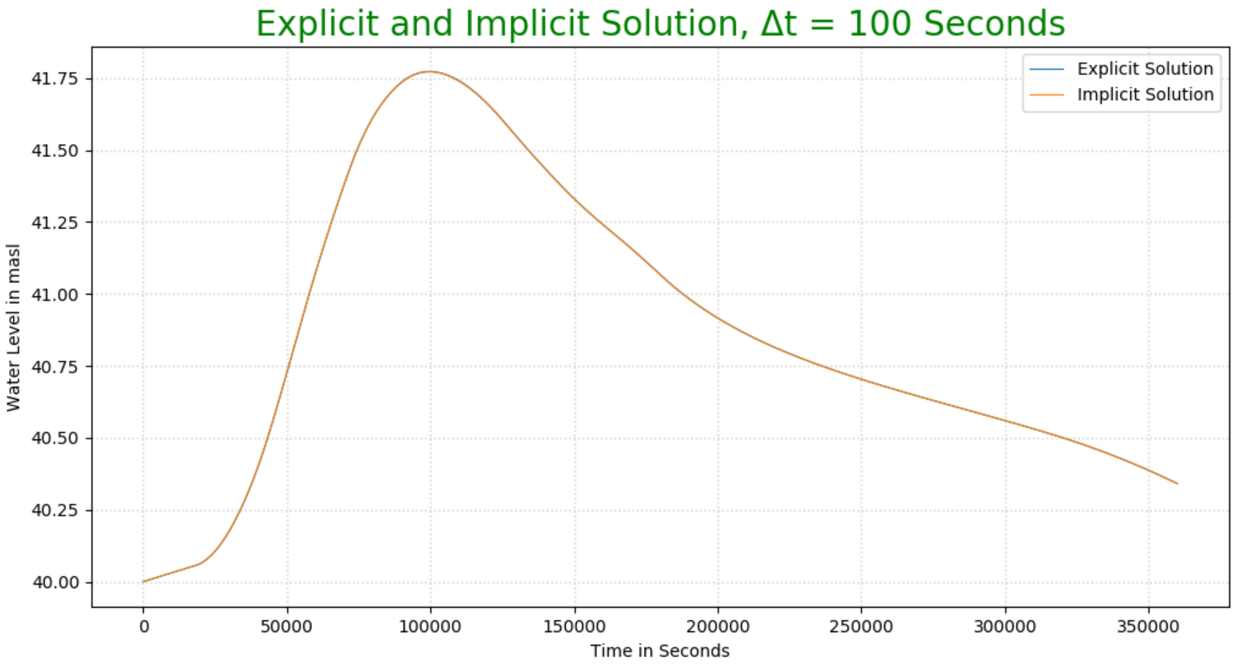

Figure 6: Explicit and Implicit Solution for Water Level in Reservoir, Δt = 100 Seconds

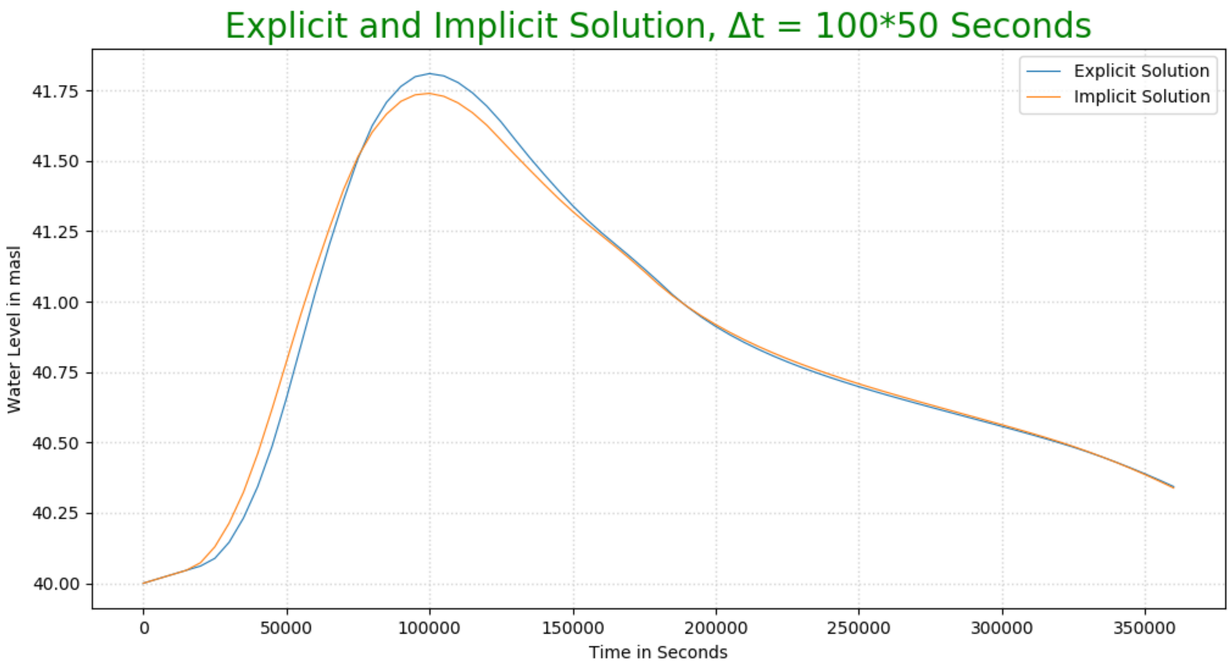

*Figure 7: Explicit and Implicit Solution for Water Level in Reservoir, Δt = 100*50 Seconds*

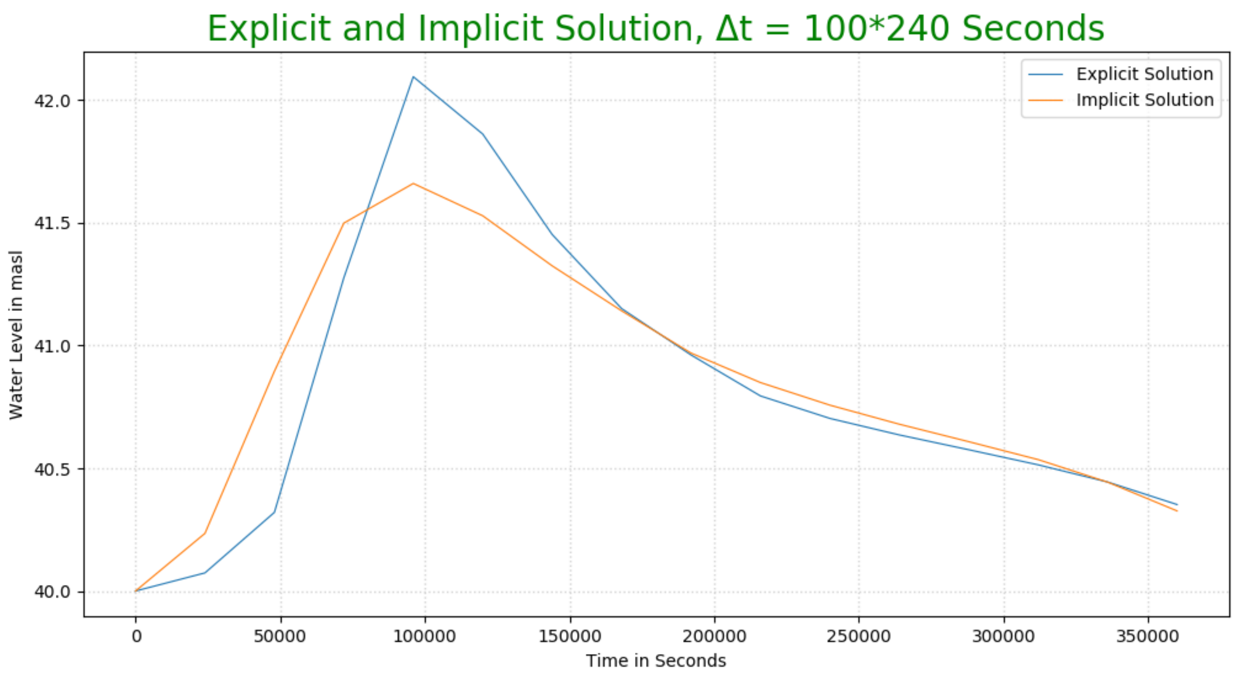

*Figure 8: Explicit and Implicit Solution for Water Level in Reservoir, Δt = 100*240 Seconds*

5. References

Popescu, I, 2014, Computational Hydraulics, IWA Publishing.

Ioana Popescu (2020), WSE-HI-M04-Modelling theory and computational Hydraulics, Numerical Method Part 1, Lecture Notes 1 & 2.

Ioana Popescu (2020), WSE-HI-M04- Numerical Method Part 1, Exercise sessions.