Roughly one page essay: Symmetry breaking in AdS/CMT[a)]

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(Dated: February 11, 2018)

The lectureintroduces the concept of spontaneous symmetry breaking. For instance, a generic Hamiltonian will be translationally invariant. The symmetry can be broken spontaneously by adding a potential energy, such as the periodic potential for a toy-model crystal. In this roughly one page essay, I take a quick plunge into the manifestations of spontaneous symmetry breaking in AdS/CMT by looking at holographic superconductors (properly superfluids) which should manifest $U(1)$ symmetry breaking.

It is the current state of affairs that any paper regarding the application of the anti-de Sitter/conformal field theory correspondence (AdS/CFT) to condensed matter physics requires a short introduction. The scope of this essay is insufficient for any kind of introduction beyond a simple description. For an actual introduction, I refer to Zaanen *et al.*[1].

I. ADS / CMT

Formally, the AdS/CFT correspondence is nested deep within higher-dimensional string theories being mathematical duals to each other. This story isn't about string theory, so we have to take a limit to lower dimensions and realistic energies. Upon doing so, we find the AdS/CFT correspondence. It is still exact in a number of cases, such as matrix large N theories. Conformal Field theory, however, seldom finds any relation to condensed matter physics. As it turns out, a number of examples have shown that this doesn't matter in some cases.

A modern approach is called AdS/CMT or sometimes phenomelogical AdS/CFT. Here, we take a more practical approach where we take systems to which it might be applicable, such as highly-entangled quantum matter, construct their AdS dual and find out about numerous properties. This works out sufficiently that it can be published in Nature Physics: Bhaseen *et al.*[2].

I must remind readers that I lack space to do a proper introduction, and that the approach taken above is the one taken when speaking shortly to someone you studied undergraduate quantum physics with. For a full, serious introduction you can check out any of the references of this essay.

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II. SUPERCONDUCTORS

Following ref.[3], we quickly recap the features of superconductors. The most striking feature is zero resistivity below a certain critical temperature. A second class of superconductors exist that knows two critical temperatures, in which the transition to zero temperature is visible in between the two critical temperatures. A second independent property is the expulsion of the magnetic field when below critical temperature.

In 1950, Landau and Ginzburg described superconductivity in terms of a second order phase transition with a complex scalar field order parameter ϕ related to the superconducting electron density $n_s = |\phi|^2$. The contribution of ϕ to the free energy is assumed to take the form:

$$\mathscr{F} = \alpha (T - T_c) |\phi|^2 + \frac{\beta}{2} |\phi|^4 + \dots \tag{1}$$

Clearly the minimum of free energy is at $\phi = 0$ above the critical temperature, while it is at a nonzero energy when below the critical temperature. Therefore the density of superconducting electrons is only nonzero below the critical temperature. This is allegedly just like the Higgs mechanism in particle physics[4] and is associated with spontaneously breaking a $U(1)$ symmetry.

III. HOLOGRAPHIC SUPERCONDUCTORS

We will now construct a holographic dual, i.e. an AdS spacetime, for the superconductor. The minimal ingredients follow from the holographic dictionary[1]. First, a notion of temperature requires a black hole which introduces Temperature as the Hawking Temperature. The superconducting condensate is described by some field coupled to gravity - a black hole "hair". As a result we need the usual Schwarzschild or alternatively Reissner-Nordstrom AdS black holes.

In Horowitz[3], they describe a "surprisingly simple solution" found by Gubser. A charged scalar field around a charged black hole in AdS would have the desired property. Consider the following action:

$$S = \int d^4x\sqrt{-g}\left(R + \frac{6}{L^2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right. \tag{2}$$

$$\left. - \left|\nabla\Psi - \imath qA\Psi\right|^2 - m^2\left|\Psi\right|^2\right) \tag{3}$$

This is general relativity with a negative cosmological constant $\Lambda = -\frac{3}{L^2}$ coupled to a Maxwell field and a charged scalar with mass m and charge q. The theory will develop scalar hair at low temperature.

Horowitz [3] Points out that scalar hair breaks a local $U(1)$ symmetry in the bulk, the dual description consists of a condensate breaking a global $U(1)$ symmetry in the boundary. As a result, we're properly describing superfluids rather than superconductors. There is a different aspect puzzling me. As I understand it, the scalar field is a continuous object (a field) and as such it should obey Elitzur's theorem[5].

IV. CONDENSATE

After rescaling $A_\mu \to \frac{A_\mu}{q}$ and $\Psi \to \frac{\Psi}{q}$, we surpress backreaction of the matter fields on the metric. This is called the probe limit when $q \to \infty$.

Starting from a planar Schwarzschild anti-de Sitter black hole in four dimensions as the fixed background, the Maxwell scalar-equations can be solved. Assume a planar ansatz, $\Psi = \psi(r)$ and $A_t = \phi(r)$. If the other components of A_μ are zero, then the phasemust be constant and we can take ψ to be real. The Maxwell-scalar field equations reduce to a set of coupled, nonlinear, ordinary differential equations. Horowitz [3] then takes the reader through the boundary conditions.

As is apparently generally the case, the local gauge symmetry corresponds to a global $U(1)$ symmetry in the CFT. I'm not sure that gauge freedom is meant in this particular case or something else entirely. Either way, the solutions to the set of ordinary differential equations are $\psi^+ \propto (T_c - T)^{\frac{1}{2}}$, which is the behaviour we would expect from Landua-Ginzburg (recall the absolute squared is proportional to the superfluid density).

V. CONCLUSION

My short venture into holographic superfluids has shown a number of properties related to spontaneous symmetry breaking. There is still some confusion, such as the "local gauge symmetry" being dual to the global $U(1)$ symmetry that is later broken. I am not at all sure how this ties into the comment that "Gauge symmetries cannot be broken". However, the comment that "local ... symmetry is dual to global .. symmetry in the field theory" is what I was looking for.

I'm sure this will be illuminated in future lectures.

REFERENCES

[1] J. Zaanen, Y. Liu, Y.-W. Sun, and K. Schalm, *Holographic duality in condensed matter physics* (Cambridge University Press, Cambridge, United Kingdom, 2015) iSBN 9781107080089.

[2] M. J. Bhaseen, B. Doyon, A. Lucas, and K. Schalm, "Energy flow in quantum critical systems far from equilibrium," Nature Physics **11**, 509–514 (2015).

[3] G. T. Horowitz, "Introduction to holographic superconductors," hep-th **arXiv**, 1002.1722 (2010).

[4] We learn about the Higgs mechanism in a future lecture.

[5] Continuous local symmetries cannot be spontanously broken.