nature physics

ARTICLES

PUBLISHED ONLINE: 4 MAY 2015 | DOI: 10.1038/NPHYS3320

Energy flow in quantum critical systems far from equilibrium

M. J. Bhaseen[1]*, Benjamin Doyon[2], Andrew Lucas[3] and Koenraad Schalm[3,4]

Characterizing the behaviour of strongly coupled quantum systems out of equilibrium is a cardinal challenge for both theory and experiment. With diverse applications ranging from the dynamics of the quark–gluon plasma to transport in novel states of quantum matter, establishing universal results and organizing principles out of equilibrium is crucial. We present a universal description of energy transport between quantum critical heat baths in arbitrary dimension. The current-carrying non-equilibrium steady state (NESS) is a Lorentz-boosted thermal state. In the context of gauge/gravity duality this reveals an intimate correspondence between far-from-equilibrium transport and black hole uniqueness theorems. We provide analytical expressions for the energy current and the generating function of energy current fluctuations, together with predictions for experiment.

In recent years there has been significant interest in the behaviour of strongly correlated systems out of equilibrium. Experiments on cold atomic gases have raised fundamental questions ranging from the nature of thermalization in one spatial dimension ($d=1$; refs 1,2) to the dynamics of spontaneous symmetry breaking[3–5]. Time-resolved experiments have also revealed the light-cone spreading of correlations in many-body systems[6] and the dynamics of quantum critical points[7]. Recent experiments have opened up new directions for the investigation of thermal transport[8] and the thermal expansion of degenerate quantum gases[9].

Stimulated by these advances, theoretical attention has focused on the behaviour of many-body systems following time-dependent protocols such as rapid quenches[10]. A strong motivation is the possibility of establishing universal results for the far-from-equilibrium response[11]. In particular, in exactly solvable $d=1$ models, analytical and numerical methods have given considerable insight. A key finding is that integrable models do not typically thermalize owing to the large number of conservation laws. Instead, quantum statistical averages may be governed by a Generalized Gibbs Ensemble (GGE; refs 12–15). However, much less is known about the dynamics of non-integrable or higher-dimensional models. This is a major challenge owing to a lack of non-perturbative techniques.

In this manuscript we address this challenge. By combining insights from quantum many-body systems, with explicit computations using gauge/gravity duality and hydrodynamics, we present results for thermal transport for quantum critical systems in arbitrary dimensions. For simplicity we focus on Lorentz-invariant quantum critical points with a relativistic linear dispersion relation. We argue that a universal non-equilibrium steady state (NESS) emerges when two strongly interacting, infinitely large quantum critical heat baths, independently thermalized at different temperatures, are brought into thermal contact along a perfectly transmitting interface at time $t=0$ (see Fig. 1). We compute the transport of energy across the contact interface, including all of its large fluctuations. A key finding is that the NESS is a Lorentz-boosted thermal state at a temperature $T=\sqrt{T_L T_R}$, in arbitrary dimension. The steady-state energy current is evaluated from both gauge/gravity duality and hydrodynamics. The second key finding is that the cumulant generating function of the fluctuations within the steady state then follows from energy conservation and parity/time-reversal (PT) symmetry. Our results provide quantitative perspectives on the nature of thermal transport in a much larger class of systems than has previously been explored. Although we focus on Lorentz-invariant quantum critical points, our methodology is applicable to other strongly interacting systems, including systems with charge flow.

Non-equilibrium steady states

Non-equilibrium steady states (NESS) are an important class of far-from-equilibrium states. In classical systems, their study has led to important results on generalized fluctuation relations[16]. Unfortunately, much less is known about NESS in strongly correlated quantum systems, in spite of their relevance for transport. So far, exact results are available only in low dimensions. In $d=1$, a universal regime of thermal transport emerges when two isolated quantum critical systems are brought into instantaneous contact[17–19]. The predicted steady-state energy flow across the interface has been recently observed[20] using time-dependent density matrix renormalization group (DMRG) simulations at finite temperature[21–23], in agreement with experiments on quantum heat flow using noise thermometry[24]. Generalizing such results to higher dimensions is crucial and is the focus of this work.

Let us consider the partitioning set-up in Fig. 1. Two semi-infinite halves ($x<0$ and $x>0$) of a quantum critical system in d spatial dimensions are independently thermalized at temperatures T_L and T_R. They are then brought into instantaneous contact along a $d-1$-dimensional hypersurface at $x=0$. Equivalently, the homogeneous system is subject to an initial temperature distribution with a step profile. A key question is whether a NESS emerges, with a non-vanishing energy flow $J_E=\langle T^{tx}\rangle_s$. Here, $T^{\mu\nu}$ is the energy–momentum tensor and the subscript s denotes the steady state.

We argue that it does, because coherent ballistic energy transport occurs at criticality. In our set-up, there is no external bath to drive a current. Instead, the far regions of the system serve this purpose,

[1]Department of Physics, King's College London, Strand, London WC2R 2LS, UK. [2]Department of Mathematics, King's College London, Strand, London WC2R 2LS, UK. [3]Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA. [4]Institute Lorentz, Leiden University, PO Box 9506, Leiden 2300 RA, The Netherlands. *e-mail: joe.bhaseen@kcl.ac.uk

NATURE PHYSICS DOI: 10.1038/NPHYS3320

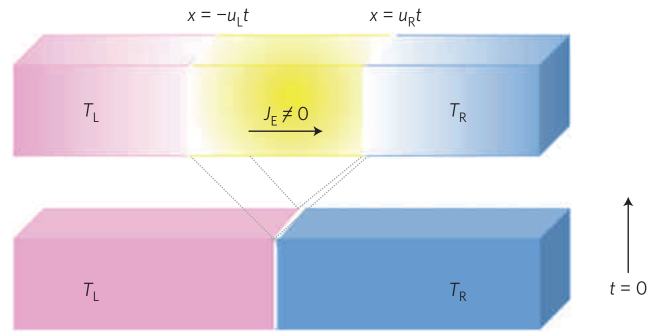

Figure 1 | Thermal transport set-up. Two isolated quantum critical systems at temperatures T_L and T_R are brought into instantaneous thermal contact. For large systems, a spatially homogeneous non-equilibrium steady state develops across the interface. This carries an energy current $J_E \equiv \langle T^{tx} \rangle_s \neq 0$.

so that diffusive currents are suppressed. In Lorentz-invariant critical systems, the energy current is conserved and the associated Drude peak[25] indicates ballistic transport, hence a current-carrying steady state. As transport is ballistic, the steady-state region is spatially homogeneous and is not accompanied by an energy density gradient. We will confirm these predictions using a hydrodynamic analysis.

Next, we ask about the nature of the steady state. What is the value of the energy current J_E and the distribution of its fluctuations? We will argue that the NESS is fully characterized in arbitrary dimension by a non-equilibrium density matrix ρ_s such that correlation functions of $T^{\mu\nu}$ and its derivatives are given by

$$\langle \mathcal{O} \ldots \rangle_s = \mathrm{Tr}(\rho_s \mathcal{O} \ldots)/\mathrm{Tr}(\rho_s)$$

The steady-state density matrix takes the universal form

$$\rho_s = \mathrm{e}^{-\beta \cosh\theta H + \beta \sinh\theta P^x} \tag{1}$$

where H is the total energy of the system and P^x is the x-component of total momentum; β and θ are parameters that will be determined. The density matrix (1) is closely related to the Boltzmann distribution, and is interpreted as the density matrix of a thermal state at temperature $T = \beta^{-1}$, with $k_B = 1$, boosted by a rapidity θ. That is, the NESS is a Lorentz-boosted equilibrium state, after suitable identifications of T, θ in terms of $T_{L,R}$. As a consequence, the universal form of the steady-state stress tensor is constrained. In a Lorentz-invariant critical system, scale invariance demands that $T^{\mu\nu}$ be traceless, while its scaling with T is fixed by dimensional analysis. Lorentz invariance further constrains the stress tensor to take the form

$$\langle T^{\mu\nu} \rangle_s = a_d \, T^{d+1} \left(\eta^{\mu\nu} + (d+1) u^\mu u^\nu \right) \tag{2}$$

where $\eta^{\mu\nu} = \mathrm{diag}(-1, 1, \ldots, 1)$ is the spacetime metric and $u^\mu = (\cosh\theta, \sinh\theta, 0, \ldots, 0)$ is the boost velocity in natural units, where the dispersion velocity is $v = 1$. The constant a_d is not fixed by symmetries; it counts the effective number of degrees of freedom of the CFT and characterizes its rest-frame energy density, $\langle T^{tt} \rangle = d a_d \, T^{d+1}$, analogous to the Stefan–Boltzmann law[26]. In particular, the steady-state energy current equals

$$\langle T^{tx} \rangle_s = \frac{a_d}{2} T^{d+1} (d+1) \sinh 2\theta$$

In this manuscript we will determine T and θ in terms of T_L and T_R, and we will provide further justification for the above claims and the universality of our results.

Gauge/gravity duality

To confirm the above insights into the Lorentz-boosted nature of the steady state we begin with the anti-de Sitter/conformal field theory correspondence (AdS/CFT); for reviews see refs 27–29. This holographic approach offers unique opportunities for advancing our understanding of far-from-equilibrium dynamics in arbitrary dimension. The emergent behaviour is encoded in the real-time evolution of a gravity theory in one more spatial dimension, which asymptotically approaches an AdS spacetime (see Fig. 2). AdS/CFT allows access to the dynamical behaviour of conformally invariant theories in $d > 1$, where there are very few tractable microscopic theories[30–38]. Although the correspondence holds only for strongly interacting theories with 'large-N matrix' degrees of freedom, holographic results obey all conformal Ward identities[27–29]. Quantities that are heavily constrained by these symmetries may have universal forms, valid beyond the large-N limit. By combining insights from holography with other approaches, we will obtain predictions with broad validity.

As we are concerned with energy transport, we need only the simplest holographic theory. We consider the Einstein–Hilbert action, familiar from general relativity:

$$S = \frac{1}{16\pi G_N} \int \mathrm{d}^{d+2}x \, \sqrt{-g} (R - 2\Lambda) \tag{3}$$

Here $\Lambda = -d(d+1)/2L^2$ is a negative cosmological constant and L is the radius of AdS. The model (3) is dual to a strongly coupled CFT in d spatial dimensions (see Fig. 2). In particular, the metric $g^{\mu\nu}$ is dual to the energy–momentum tensor $T^{\mu\nu}$ of the CFT. On the gravitational side, L should be large in units of Newton's constant: $L^d \gg G_N$, to use classical gravity. L^d/G_N encodes the number of degrees of freedom of the CFT, which is large. In equilibrium, a thermal CFT in flat space is dual to a planar black hole in asymptotically AdS space. The temperature of the CFT is the Hawking temperature of the black hole. The geometry dual to the set-up considered in Fig. 1 therefore corresponds to two black holes at different temperatures, each defined on only half of AdS and joined together at $t = 0$. This initial configuration is not a static solution of the Einstein equations, and it will evolve with time. In the Supplementary Information we solve the dynamical Einstein equations in $d = 1$ analytically, although in general this requires numerical general relativity (NGR). Instead, by focusing on the NESS, we circumvent this difficulty and use gauge/gravity duality to gain analytical insights into the nature of the NESS in arbitrary dimension.

The homogeneous, stationary nature of the NESS is reflected in its gravitational dual. As we discuss in the Supplementary Information, the only regular homogeneous solutions to Einstein's equations, dual to theories on flat spacetime, which encode a homogeneous constant stress tensor are the Lorentz-boosted black branes:

$$\mathrm{d}s^2 = \frac{L^2}{z^2} \left[\frac{\mathrm{d}z^2}{f(z)} - f(z) (\mathrm{d}t \cosh\theta - \mathrm{d}x \sinh\theta)^2 \right.$$
$$\left. + (\mathrm{d}x \cosh\theta - \mathrm{d}t \sinh\theta)^2 + \mathrm{d}y_\perp^2 \right]$$

where $f(z) = 1 - (z/z_0)^{d+1}$ and $z_0 = (d+1)/4\pi T$. Here θ is the boost parameter corresponding to a boost in the x-direction, $y_\perp$ parameterizes the transverse spatial coordinates, T is the unboosted temperature of the black hole, and z_0 is the position of the planar horizon. We conclude that in the large-N limit the NESS is a boosted equilibrium state, where T, θ are to be determined in terms of $T_{L,R}$. The correlation functions of the stress tensor are governed by the boosted density matrix in equation (1).

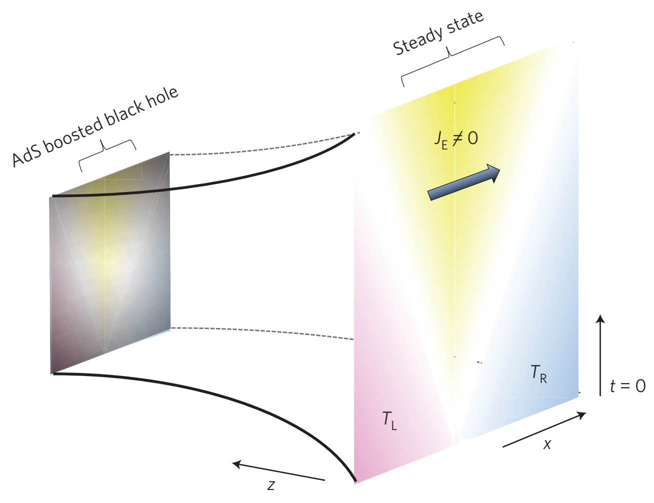

Figure 2 | AdS/CFT correspondence. The Hawking temperature of the unboosted black hole corresponds to the equilibrium temperature of the dual CFT. Lorentz-boosted solutions describe non-equilibrium steady states. This is in agreement with exact results in $d=1$ and hydrodynamics in $d>1$.

Using the AdS/CFT correspondence, the one-point function $\langle T^{\mu\nu}\rangle_s$ is obtained from the metric of the gravitational dual. One finds the Lorentz-boosted stress tensor given by equation (2) with $a_d=(4\pi/(d+1))^{d+1}L^d/16\pi G_N$; see Supplementary Information.

To see that these insights generalize beyond the large-N limit, it is instructive to consider the Lorentz-boosted density matrix from the perspective of local quantum field theory. According to ideas first expressed by Deutsch and Srednicki[39,40], in a thermodynamic quantum many-body system with local interactions, averages of local observables depend on the state in a very limited way: they may only depend on the eigenvalues of local conserved quantities, including the energy. This is sometimes referred to as the eigenstate thermalization hypothesis (ETH) and forms the basis for the GGE in integrable models[12–15]. In d-dimensional critical systems, for $d>1$, there are very few local conserved quantities, and those of interest here are the energy and the momenta. Using the symmetries of our quench set-up, the steady-state density matrix depends only on H and P^x. The exponential dependence of the non-equilibrium density matrix on H and P^x may be inferred from the requirement that the cluster decomposition property holds. Hence equation (1) is the proposed universal form of the steady state in arbitrary dimension. It is striking that general thermalization arguments, concerning local conserved charges and averages of local observables, are equivalent, within the large-N limit, to a uniqueness result in a gravity theory. For recent work examining energy flows dual to boosted black holes in different set-ups see refs 41–43.

We now describe the dynamical evolution and determine T, θ in terms of $T_{L,R}$. We first focus on $d=1$, and make contact with the exact results obtained in refs 17,18 using the scattering formalism and the Virasoro algebra. We then turn to $d>1$ using hydrodynamics.

One-dimension

First, in $d=1$ the homogeneity of the steady state follows by conservation of $T^{\mu\nu}$ and stationarity: $\partial_x\langle T^{xx}\rangle_s=-\partial_t\langle T^{tx}\rangle_s=0$. Tracelessness yields $\langle T^{xx}\rangle_s=\langle T^{tt}\rangle_s$ and so $\langle T^{tt}\rangle_s$ is homogeneous, in agreement with ballistic transport. Second, we explicitly prove the existence of a non-trivial steady state by performing the full time evolution. In the spirit of refs 17,18, conservation of $T^{\mu\nu}$ and tracelessness in $d=1$ implies that the dynamics can be factorized into left- and right-moving components: $\langle T^{tx}(x,t)\rangle=F(x-t)-F(x+t)$ and $\langle T^{tt}(x,t)\rangle=F(x-t)+F(x+t)$, where the function F depends on the initial conditions. The initial thermal form of the energy density on the left and the right gives $F(x)=(c\pi/12)T_L^2\Theta(-x)+(c\pi/12)T_R^2\Theta(x)$, where Θ is the Heaviside step function and c is the central charge which, like a_d, counts the effective number of degrees of freedom of the CFT. Hence, a steady-state energy current is established within the light-cone $|x|<t$ and is given by $\langle T^{tx}\rangle_s=(c\pi/12)(T_L^2-T_R^2)$. This corresponds to the difference of independently 'thermalized' left- and right-moving densities. These independent densities produce sharp 'shock waves' emanating from the interface at unit speed (see Fig. 1). Equivalently, $\langle T^{tx}\rangle_s=cg\Delta T$, where $\Delta T\equiv T_L-T_R$, $g=\pi^2k_B^2T_{ave}/3h$ and $T_{ave}=(T_L+T_R)/2$. Here g is the quantum of thermal conductance as measured in experiments on quantum wires[44–46]; for recent advances see ref. 24. Numerical simulations[20] have shown that the NESS is rapidly established and the CFT predictions govern the behaviour over a broad range of timescales and parameters. For experiments confirming the light-cone spreading of correlations in quantum many-body systems see ref. 6.

Third, following similar arguments to those in refs 17,18 the full density matrix may be obtained. As the left and right movers are independently thermalized, we have $\rho_s=e^{-\beta_L H_+-\beta_R H_-}$, where $H_\pm=\sum_k(|k|\pm k)/2$ are the total energies of the right- and left-moving excitations and $\beta_{L,R}=1/T_{L,R}$. With $H_\pm=(H\pm P^x)/2$, one obtains $\rho_s=e^{-\beta_+H-\beta_-P^x}$, where $\beta_\pm=(\beta_R\pm\beta_L)/2$. This is equivalent to a boosted thermal state (1) with $\beta=\sqrt{\beta_L\beta_R}$ and $e^{2\theta}=\beta_R/\beta_L$. The NESS in $d=1$ is therefore also a boosted thermal state. The identification of β and θ is consistent, within holography, with results for BTZ (Bañados–Teitelboim–Zanelli) black holes[43,47]: in $d=1$, the boosted black hole corresponds to a state with its left and right movers thermally populated at temperatures $T_L=Te^{\theta}$ and $T_R=Te^{-\theta}$. $T_{L,R}$ may also be regarded as arising from the Doppler shift of the Stefan–Boltzmann radiation.

Using $a_1=L\pi/4G_N$ with the holographic relation $c=3L/2G_N$, one finds $a_1=c\pi/6$ and thus $\langle T^{tx}\rangle_s=(c\pi/12)(T_L^2-T_R^2)$, as required. This highlights the direct connection between holography and other approaches. In the Supplementary Information we show that the full time evolution in $d=1$ follows from Einstein's equations, with initial conditions dual to the step profile.

The steady-state density matrix ρ_s allows one to compute the generating function of the energy current fluctuations, as shown in $d=1$ in refs 17,18. We provide a derivation in arbitrary dimension below.

Hydrodynamics and $d>1$

For $d>1$ there is no holomorphic factorization and thermalization is now an important aspect of the dynamical evolution. The guiding principle is that in a strongly interacting theory, the dynamics of $\langle T^{\mu\nu}\rangle$ is rapidly described by relativistic hydrodynamics[48]. Equation 2 corresponds to the stress tensor of a perfect conformal fluid, where u^μ is the local fluid velocity. The hydrodynamic equations are the conservation law $\partial_\mu\langle T^{\mu\nu}\rangle=0$ for this stress tensor. When the hydrodynamic description starts to be valid, the full quantum evolution will have produced some 'initial' non-universal distribution for $\langle T^{\mu\nu}\rangle$. By causality, this initial condition embodies asymptotic heat baths outside the light-cone. We may therefore consider different initial conditions that interpolate between T_L and T_R. By rescaling x and t, the symmetries of perfect hydrodynamics allow us to map the dynamics on large length and timescales to the solution of the Riemann problem: T_L for $x<0$ and T_R for $x>0$. We will observe numerically that the resulting solution is not sensitive to the details of the interpolating initial conditions.

The Riemann problem for conformal hydrodynamics does not have a unique solution in general; see Supplementary Information. However, a natural solution that is realized in our simulations contains two planar shock waves emanating from the contact.

NATURE PHYSICS DOI: 10.1038/NPHYS3320

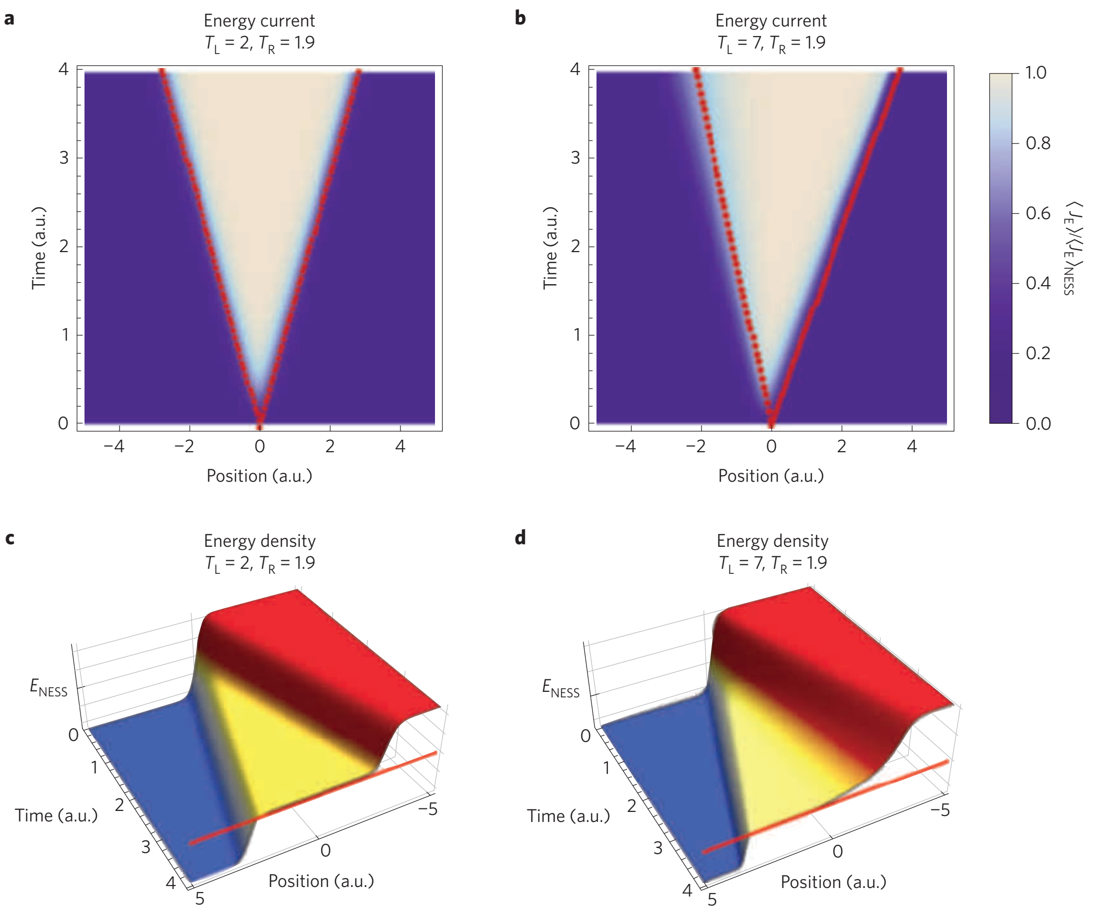

Figure 3 | Conformal hydrodynamics. Numerical solution of $\partial_\mu \langle T^{\mu\nu} \rangle = 0$ with the stress tensor given by equation (2) in $d=2$. We take an initial temperature profile $T(x) = (T_L + T_R)/2 + [(T_R - T_L)/2] \tanh(x/x_0)$ with $x_0 = 1/3$. **a**,**b**, Density plots of the energy current $\langle T^{tx} \rangle$ showing outgoing shockwaves for $T_L = 2$, $T_R = 1.9$ (**a**) and $T_L = 7$, $T_R = 1.9$ (**b**) with speeds $u_{L,R}$ given by equation (5), as indicated by the red dotted lines. For $T_L \gg T_R$ (**b**) the asymmetry in $u_{L,R}$ is clearly visible. **c**,**d**, Time evolution of the energy density $\langle T^{tt} \rangle$ showing the approach to the steady-state solution for $T_L = 2$, $T_R = 1.9$ (**c**) and $T_L = 7$, $T_R = 1.9$ (**d**). The red line is the predicted value of $\langle T^{tt} \rangle_s$. The results confirm the validity of the two-shock wave description. a.u., arbitrary units.

We consider left- and right-moving shocks that are homogeneous in the transverse spatial directions and move at constant speeds u_L and u_R respectively; given the Riemann conditions, $u_{L,R}$ are uniquely determined by the relativistic Rankine–Hugoniot equations, corresponding to energy and momentum conservation across the shocks. These further constrain the form of the steady-state stress tensor:

$$\langle T^{tx} \rangle_s = a_d \left(\frac{T_L^{d+1} - T_R^{d+1}}{u_L + u_R} \right) \tag{4}$$

Invoking (2) gives explicitly $u_{L,R}$ in terms of $T_{L,R}$:

$$u_L = \frac{1}{d} \sqrt{\frac{\chi + d}{\chi + d^{-1}}}, \quad u_R = \sqrt{\frac{\chi + d^{-1}}{\chi + d}} \tag{5}$$

where $\chi \equiv (T_L/T_R)^{(d+1)/2}$. The steady-state region is a boosted thermal state, with temperature $T = \sqrt{T_L T_R}$ and boost velocity $(\chi - 1)/\sqrt{(\chi + d)(\chi + d^{-1})}$. Remarkably, the rest-frame temperature coincides with the temperature attained by two heat baths after extracting the maximum amount of work allowed by reversible thermodynamics using a Carnot engine. As we discuss in the Supplementary Information, the two-shock solution seems to emerge from a related minimization principle, which supports its universality as a description of non-equilibrium quantum many-body steady states. The validity of the two-shock solution and the predictions in $d > 1$ are confirmed by conformal fluid numerics in Fig. 3 and in the Supplementary Information.

The shock waves emanating from the contact are nonlinear generalizations of sound waves. Using energy and momentum conservation, the shock speeds satisfy $u_L u_R = c_s^2$, where $c_s = 1/\sqrt{d}$ is the speed of sound. As a function of T_L/T_R, greater than unity, u_R interpolates between c_s and the linear dispersion speed $v = 1$, whilst u_L interpolates between c_s and c_s^2/v. The asymmetric speeds $u_{L,R}$ are quantitative predictions for experiment.

In $d > 1$ the stress tensor for a conformal fluid generically admits derivative corrections[48], including viscous contributions. In general, these corrections will lead to diffusion, and a broadening of the shock fronts. In the linear response regime, $|T_L - T_R| \ll T_L + T_R$, we can solve the viscous hydrodynamic equations; the two 'shocks' propagate at the speed of sound, and have a width growing diffusively as $\sqrt{t}$. On long length scales this reduces to the sharp shock dynamics discussed above, because the width of the shocks is sublinear in t. Therefore, the steady-state regime exists at times larger than the diffusion constant, and $\langle T^{\mu\nu} \rangle_s$ is independent of viscosity. Similarly, we expect our nonlinear results for $\langle T^{\mu\nu} \rangle_s$ to remain valid in the presence of derivative corrections.

Furthermore, realistic initial conditions may include spatial inhomogeneities at the contact interface. In the Supplementary Information we present simulations which test the stability of our results to perturbations of the initial conditions in both the longitudinal and transverse directions. Our numerics are reliable up to $T_L/T_R \sim 5$ in

$d=2$, and $T_L/T_R \sim 2.5$ in $d=3$. In both cases, large longitudinal deviations in the fluid temperature or velocity at the contact interface rapidly propagate away and leave behind a homogeneous steady state. In $d=2$, conservation of enstrophy protects the homogeneity of the steady state from transverse perturbations. In $d=3$, transverse perturbations create spatial inhomogeneities that persist at the contact interface over the duration of our simulations. Nonetheless, the total energy current, integrated over the contact area, rapidly approaches our predictions. For more extreme initial conditions we cannot rule out the possibility of turbulent flows[49,50] in $d=3$. This may change the total energy current, and it would be interesting to investigate this in more detail.

Fluctuation relations

One of the most important goals in non-equilibrium systems is a complete description of quantum and thermal fluctuations, including generalizations of the fluctuation–dissipation relation[16]. This is challenging in strongly correlated systems, where non-perturbative techniques are absent in $d>1$. Of particular interest is the probability distribution of the total transfer of energy, $J_{\rm tot}=\int {\rm d}t{\rm d}^{d-1}y_\perp T^{tx}(x=0,y_\perp,t)$, in the steady state. Scaling out the transverse area and the integration time, the cumulants are given by the connected correlation functions, $c_n \equiv \langle J_{\rm tot}^{n-1} T^{tx}(x^\mu=0)\rangle_s^c$. Non-equilibrium steady states are expected to give rise to non-trivial relations between these cumulants[16]. However, explicit predictions in $d>1$ are lacking. Here, we exploit symmetries to compute these cumulants directly, using the generating function $F(z)\equiv\sum_{n=1}^{\infty} z^n c_n/n!$. As we demonstrate in the Supplementary Information, $F(z)$ obeys the extended fluctuation relations (EFR; ref. 19):

$$dF(z)/dz = J_E(\beta_L - z, \beta_R + z) \tag{6}$$

where $J_E(\beta_L,\beta_R)=\langle T^{tx}\rangle_s$ is the steady-state energy current for left and right inverse temperatures β_L and β_R respectively, and the arguments are continued by z. Hence, the knowledge of J_E as a function of β_L, β_R, as given by equation (4), fixes $F(z)$. As noted in ref. 19, using the parity symmetry $J_E(\beta_L,\beta_R)=-J_E(\beta_R,\beta_L)$, the EFR also implies the fluctuation relations $F(\beta_L-\beta_R-z)=F(z)$ (ref. 16). Combining equation (6) with the hydrodynamic expression equation (4) for $J_E=\langle T^{tx}\rangle_s$, we obtain the entire fluctuation spectrum of a strongly interacting quantum system in $d>1$; see Supplementary Information for details.

Conclusions

We have presented universal results for the far-from-equilibrium energy flow in strongly coupled quantum critical systems in $d>1$. We have argued that the emergent steady state is a boosted thermal state, and have provided analytical expressions for the large fluctuations of energy transport. In the context of gauge/gravity duality this exposes an intimate link between NESS and black hole uniqueness theorems. The dynamical evolution is described in terms of shock waves with asymmetric propagation speeds. Although we have focused on CFTs, non-trivial steady states may emerge under broader conditions, provided energy and momentum are conserved. We look forward to tests of these predictions in experiments using cold atomic gases and in simulations.

Received 24 July 2014; accepted 25 March 2015;
published online 4 May 2015; corrected online 3 September 2015

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Acknowledgements

We thank B. Benenowski, D. Bernard, P. Chesler, D. Haldane, C. Herzog, D. Marolf, B. Najian, C-A. Pillet, S. Sachdev and A. Starinets for helpful comments; we especially thank A. Green for suggesting the interpretation of $T_{L,R}$ in $d=1$ as Doppler-shifted radiation. M.J.B. and K.S. thank the Kavli Royal Society Center Chicheley Hall and the Isaac Newton Institute, Cambridge for hospitality. M.J.B. and B.D. thank The Galileo Galilei Institute for Theoretical Physics. B.D. thanks Université Paris Diderot, where part of this work was done, for financial support through a visiting professorship. A.L. is supported by the Smith Family Science and Engineering Graduate Fellowship and thanks the Perimeter Institute for hospitality. Research at Perimeter Institute is supported by the Government of Canada through Industry Canada and by the Province of Ontario through the Ministry of Economic Development & Innovation. This work was supported in part by a VICI grant of the Netherlands Organization for Scientific Research (NWO), by the Netherlands Organization for Scientific Research/Ministry of Science and Education (NWO/OCW) and by the Foundation for Research into Fundamental Matter (FOM).

Author contributions

M.J.B. initiated and coordinated the project. B.D. led the field theory and fluctuation analysis. A.L. led the hydrodynamic analysis, and wrote and performed numerical simulations. K.S. led the gauge-gravity analysis. M.J.B. wrote the manuscript with input from all the authors.

Additional information

Supplementary information is available in the online version of the paper. Reprints and permissions information is available online at www.nature.com/reprints. Correspondence and requests for materials should be addressed to M.J.B.

Competing financial interests

The authors declare no competing financial interests.

Erratum: Energy flow in quantum critical systems far from equilibrium

M. J. Bhaseen, Benjamin Doyon, Andrew Lucas and Koenraad Schalm

Nature Physics **11,** 509–514 (2015); published online 4 May 2015; corrected after print 3 September 2015.

In the print and PDF versions of this Article originally published the digital object identifier contained a typographical error and should have read 10.1038/nphys3320. This error has now been corrected in the online versions.