Action Model Learning with Guarantees (Appendix)

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Abstract

This document is the appendix of Aineto and Scala (2024), containing additional results and proofs.

1 Additional Results

The following corollaries, derived by theorems 2 and 3, interpret the update rules in an offline fashion, after any number of demonstrations.

Corollary 1. *$\mathcal{L}_{\mathcal{H}^a_p,D_p} = \{hp_\mathcal{L}\}$ s.t. $hp_\mathcal{L} = \bigcap_{(s,1)\in D_p} s$.*

Corollary 2. *$\mathcal{L}_{\mathcal{H}^a_e,D_e} = \{he_\mathcal{L}\}$ s.t. $he_\mathcal{L} = \bigcup_{(s,s')\in D_e} s' \setminus s$*

Corollary 3. *$\mathcal{U}_{\mathcal{H}^a_e,D_e} = \{he_\mathcal{U}\}$ s.t. $he_\mathcal{U} = \bigcap_{(s,s')\in D_e} s'$*

Corollaries 1 and 3 show that learning a consistent action model is as easy as intersecting all pre-states for the preconditions and all post-states for the effects. In addition, Corollary 2 states that tighter effects can be obtained by joining all pre-state to post-state deltas. Note that, $he_\mathcal{L}$ can also be computed using $hp_\mathcal{L}$ and $he_\mathcal{U}$. Formally:

Lemma 2. *Let $\mathcal{L}_{\mathcal{H}^a_p,D_p} = \{hp_\mathcal{L}\}$, $\mathcal{L}_{\mathcal{H}^a_e,D_e} = \{he_\mathcal{L}\}$ and $\mathcal{U}_{\mathcal{H}^a_e,D_e} = \{he_\mathcal{U}\}$, we have that $he_\mathcal{L} = he_\mathcal{U} \setminus hp_\mathcal{L}$.*

Thanks to Lemma 2, we do not need to maintain the $\mathcal{L}_{\mathcal{H}^a_e}$ boundary which can be used to slightly improve the performance of the VSLAM algorithm. Moreover, we leverage it in the next lemma, which implies that all consistent models using the lower bound $hp_\mathcal{L}$ for their preconditions induce the same transition system.

Lemma 3. *Let $\mathcal{L}_{\mathcal{H}^a_p,D_p} = \{hp_\mathcal{L}\}$ and $\mathcal{V}_{\mathcal{H}^a_e,D_e}$. The following holds $\forall he, he' \in \mathcal{V}_{\mathcal{H}^a_e,D_e} : he \setminus he' \subseteq hp_\mathcal{L}$.*

This result from Lemma 3 is also used in the proof of Theorem 5.

2 Technical Proofs

Here we present the full proofs of all our lemmas, including Lemma 1 from the main paper, and lemmas 2 and 3 from this appendix.

Lemma 1. *Let $d = \langle s, a, s'\rangle$ be a positive demonstration, and $h \in \mathcal{H}^a_e$. h is consistent with d, i.e., $s' = (s \setminus \overline{h}) \cup h$, iff $s' \setminus s \subseteq h \subseteq s'$.*

Proof. Starting with $s' \setminus s \subseteq h \subseteq s' \implies s' = (s \setminus \overline{h}) \cup h$. By contradiction, we assume the antecedent is true while the consequent is false. If $s' \neq (s \setminus \overline{h}) \cup h$ then either (1) $s' \not\subseteq (s \setminus \overline{h}) \cup h$ or (2) $s' \not\supseteq (s \setminus \overline{h}) \cup h$.

For (1), it means that

$$s' \setminus ((s \setminus \overline{h}) \cup h) \neq \emptyset$$
$$(s' \setminus (s \setminus \overline{h})) \cap (s' \setminus h) \neq \emptyset$$
$$((s' \cap \overline{h}) \cup (s' \setminus s)) \cap (s' \setminus h) \neq \emptyset$$

Since $h \subseteq s'$ then $s' \cap \overline{h} = \emptyset$

$$(s' \setminus s) \cap (s' \setminus h) \neq \emptyset$$

Since $s' \setminus s \subseteq h$, it follows that $(s' \setminus s) \cap (s' \setminus h) = \emptyset$ and we arrive at a contradiction $\emptyset \neq \emptyset$.

For (2), it means that

$$((s \setminus \overline{h}) \cup h) \setminus s' \neq \emptyset$$
$$((s \setminus \overline{h}) \setminus s') \cup (h \setminus s') \neq \emptyset$$

Since $h \subseteq s'$ then $h \setminus s' = \emptyset$

$$(s \setminus \overline{h}) \setminus s' \neq \emptyset$$

$$(s \setminus s') \setminus \overline{h} \neq \emptyset$$

Before continuing, observe that a state represents a full assignment of fluents F and can be understood as $|F|$-sized bit vectors. Under this interpretation, $s - s' = {\sim}({\sim}s + s')$ where $\sim$ denotes the bitwise complement (logical negation on each bit). Coming back to our set representation, this means that $\overline{s' \setminus s} = s \setminus s'$ since $s \setminus s'$ and $s' \setminus s$ contain the literals associated to fluents that are evaluated differently in s and s' which given the Boolean domain of fluents can only be their complementary literals. Resuming our proof, note that, since $\overline{s' \setminus s} = s \setminus s'$ and $s' \setminus s \subseteq h$, we have that $s \setminus s' \subseteq \overline{h}$ and we arrive at a contradiction $\emptyset \neq \emptyset$

Now, we move on to the proof of $s' = (s \setminus \overline{h}) \cup h \implies s' \setminus s \subseteq h \subseteq s'$. Again, we assume the antecedent is true while the consequent is false. Meaning, either (3) $s' \setminus s \not\subseteq h$ or (4) $h \not\subseteq s'$.

For (3), consider that $s' = (s \setminus \overline{h}) \cup h$ implies $s' \subseteq (s \setminus \overline{h}) \cup h$.

$$s' \subseteq (s \setminus \overline{h}) \cup h$$
$$s' \setminus (s \setminus \overline{h}) \subseteq h$$
$$(s' \setminus s) \cup (s' \cap \overline{h}) \subseteq h$$

Hence, $(s' \setminus s) \subseteq h$ and (3) cannot be true.

Finally, (4) is very obviously false given that s' is the union of $(s \setminus \overline{h})$ and h.

$\square$

Lemma 2. *Let $\mathcal{L}_{\mathcal{H}_p^a, D_p} = \{hp_{\mathcal{L}}\}$, $\mathcal{L}_{\mathcal{H}_e^a, D_e} = \{he_{\mathcal{L}}\}$ and $\mathcal{U}_{\mathcal{H}_e^a, D_e} = \{he_{\mathcal{U}}\}$, we have that $he_{\mathcal{L}} = he_{\mathcal{U}} \setminus hp_{\mathcal{L}}$.*

Proof. For simplicity, we develop this segment of the proof assuming only two learning examples, i.e., $D_e = \{(s_1, s_1'), (s_2, s_2')\}$.
Starting with

$(s_1' \cap s_2') \setminus (s_1 \cap s_2)$

and applying De Morgan's Law we arrive at

$((s_1' \cap s_2') \setminus s_1) \cup ((s_2' \cap s_1') \setminus s_2)$

Intersection with set difference is set difference with intersection

$((s_1' \setminus s_1) \cap s_2') \cup ((s_2' \setminus s_2) \cap s_1')$

From Lemma 1, we know that for all $he \in \mathcal{V}_{\mathcal{H}_e^a, D_e}$ it holds that $(s_1' \setminus s_1) \subseteq he$, $(s_2' \setminus s_2) \subseteq he$, $he \subseteq s_1'$ and $he \subseteq s_2'$, so it follows that $(s_1' \setminus s_1) \subseteq s_2'$ and $(s_2' \setminus s_2) \subseteq s_1'$ and we arrive at

$(s_1' \setminus s_1) \cup (s_2' \setminus s_2)$

Generalizing this procedure to any D_e, we have that $he_{\mathcal{L}} = \bigcup_{(s,s') \in D_e} s' \setminus s = (\bigcap_{(s,s') \in D_e} s') \setminus (\bigcap_{(s,s') \in D_e} s)$ and, therefore, that $he_{\mathcal{L}} = he_{\mathcal{U}} \setminus hp_{\mathcal{L}}$.

$\square$

Lemma 3. *Let $\mathcal{L}_{\mathcal{H}_p^a, D_p} = \{hp_{\mathcal{L}}\}$ and $\mathcal{V}_{\mathcal{H}_e^a, D_e}$. The following holds $\forall he, he' \in \mathcal{V}_{\mathcal{H}_e^a, D_e} : he \setminus he' \subseteq hp_{\mathcal{L}}$.*

Proof. Let $\mathcal{L}_{\mathcal{H}_e^a, D_e} = \{he_{\mathcal{L}}\}$ and $\mathcal{U}_{\mathcal{H}_e^a, D_e} = \{he_{\mathcal{U}}\}$ be the boundaries of $\mathcal{V}_{\mathcal{H}_e^a, D_e}$. From Lemma 2, we know that $he_{\mathcal{L}} = he_{\mathcal{U}} \setminus hp_{\mathcal{L}}$. Since $\forall he \in \mathcal{V}_{\mathcal{H}_e^a, D_e} : he_{\mathcal{L}} \subseteq he \subseteq he_{\mathcal{U}}$, it follows that $he \setminus he_{\mathcal{L}} \subseteq he_{\mathcal{U}} \setminus he_{\mathcal{L}}$. Substituting, $he \setminus he_{\mathcal{L}} \subseteq he_{\mathcal{U}} \setminus (he_{\mathcal{U}} \setminus hp_{\mathcal{L}})$ simplifying to $he \setminus he_{\mathcal{L}} \subseteq he_{\mathcal{U}} \cap hp_{\mathcal{L}}$. $\square$

References

Aineto, D., and Scala, E. 2024. Action model learning with guarantees.