Example 1:
1. Convert **π = 3.1415** to its fractional binary representation with up to four digits after the binary point.
2. Using the result of 1., calculate its decimal representation with up to four digits after the decimal point.
3. What is the conversion error?

1. Conversion from decimal to binary.
Step 1:
Split the number in two parts: the integral part and the fractional part
The integral part is 3 and the fractional part is 0.1415

Step 2:
Convert the integral part from decimal to binary by applying the following steps:

1. Divide the decimal number by 2. This will yield a quotient and a remainder. Take a note of the remainder.

2. While the quotient is not equal to 0, divide it by 2. This will yield a quotient and a remainder. Take a note of the remainders.

3. Reverse the order of all the remainders placing the **last** remainder at the position of the **MSB** and the **first** remainder at the position of the **LSB**, forming the binary equivalent of the integral part.

In this exercise, the integral part is 6. Apply the above algorithm to convert it into binary:

Step Nr	Division	Remainder	Quotient
1	3 / 2	1	1
2	1 / 2	1	0
3	Reverse the order of the remainders, placing the last remainder at the position of the MSB and the first remainder at the position of the LSB.		
4	$\mathbf{3_{10} = 11_2}$		

Step 3:
Convert the fractional part from decimal to binary by applying the following steps:

1. Multiply the fractional number by 2. This results in a new number with an integral part and a fractional part. The integral part will be the MSB of the binary equivalent of the fractional part.

2. While the fractional part of the new number is not zero, multiply it by 2. This results in a new number with an integral part and a fractional part. The integral part will be the next lower significant bit of the binary equivalent of the fractional part.

3. Collect all the integral parts placing the **first** integral part at the position of the **MSB** and the **last** integral part at the position of the **LSB**, forming the binary equivalent of the fractional part.

In this exercise, the fractional part is 0.1415. Apply the above algorithm to convert it into binary:

Step Nr	Multiplication	Integral part	Fractional part
1	0.1415 * 2 = 0.283	0	0.283
2	0.283 * 2 = 0.566	0	0.566
3	0.566 * 2 = 1.132	1	0.132
4	0.132 * 2 = 0.264	0	0.264
	The process is stopped here because we were asked to provide an answer with four digits after the binary point. Otherwise, we would continue until the fractional part would be equal to 0.		
5	Place the first integral part at the position of the MSB and the last integral part at the position of the LSB, forming the binary equivalent of the fractional part.		
6	**$.1415_{10}$ = $.0010_2$**		

Step 4:
Combine the binary equivalents of the integral and fractional numbers to form the binary equivalent of the given number:

$$\mathbf{3.1415_{10} = 11.0010_2}$$

2. Conversion from binary to decimal.

To calculate the represented number, we multiply the value of the digit used by the weight of its position and then add the results of all multiplications.

Example to calculate the decimal value of the binary number 0b11.0010

Digit	1	1	.	0	0	1	0
Weight	$2^1 = 2$	$2^0 = 1$		$2^{-1} = 0.5$	$2^{-2} = 0.25$	$2^{-3} =$ 0.125	$2^{-4} =$ 0.0625
Value of digit in decimal	**1 * 2 = 2**	**1 * 1 = 1**		**0 * 0.5 = 0**	**0 * 0.25 = 0**	**1 * 0.125 = 0.125**	**0 * 0.0625 = 0**
Value of number in decimal: 2 + 1 + 0 + 0 + 0.125 + 0 = 3.125_{10}							

3. Conversion error.

Converting the decimal number to binary and then back to decimal, yielded a different number. The difference between the two numbers is the **conversion error**:

$$\mathbf{3.1415_{10} - 3.125_{10} = 0.0165_{10}}$$

The error occurs because we used only four digits after the binary point. The number of digits used is limited by the resources available by the computer. For example, an 8 bit processor will typically provide resources for 8 bits in a register, thus limiting the accuracy of the conversions it executes.

Example 2:
1. Convert **φ = 1.618** to its binary representation with up to four digits after the binary point.
2. Using the result of 1., calculate its decimal representation with up to four digits after the decimal point.
3. What is the conversion error?

1. Conversion from decimal to binary.
Step 1:
Split the number in two parts: the integral part and the fractional part
The integral part is 1 and the fractional part is 0.618

Step 2:
Convert the integral part from decimal to binary by applying the following steps:

1. Divide the decimal number by 2. This will yield a quotient and a remainder. Take a note of the remainder.

2. While the quotient is not equal to 0, divide it by 2. This will yield a quotient and a remainder. Take a note of the remainders.

3. Reverse the order of all the remainders placing the **last** remainder at the position of the **MSB** and the **first** remainder at the position of the **LSB**, forming the binary equivalent of the integral part.

In this exercise, the integral part is 6. Apply the above algorithm to convert it into binary:

Step Nr	Division	Remainder	Quotient
1	1 / 2	1	0
2	Reverse the order of the remainders, placing the last remainder at the position of the MSB and the first remainder at the position of the LSB.		
3	$\mathbf{1_{10} = 1_2}$		

Step 3:
Convert the fractional part from decimal to binary by applying the following steps:

1. Multiply the fractional number by 2. This results in a new number with an integral part and a fractional part. The integral part will be the MSB of the binary equivalent of the fractional part.

2. While the fractional part of the new number is not zero, multiply it by 2. This results in a new number with an integral part and a fractional part. The integral part will be the next lower significant bit of the binary equivalent of the fractional part.

3. Collect all the integral parts placing the **first** integral part at the position of the **MSB** and the **last** integral part at the position of the **LSB**, forming the binary equivalent of the fractional part.

In this exercise, the fractional part is 0.618. Apply the above algorithm to convert it into binary:

Step Nr	Multiplication	Integral part	Fractional part
1	0.618 * 2 = 1.236	1	0.236
2	0.236 * 2 = 0.472	0	0.472
3	0.472 * 2 = 0.944	0	0.944
4	0.944 * 2 = 1.888	1	0.888
	The process is stopped here because we were asked to provide an answer with four digits after the binary point. Otherwise, we would continue until the fractional part would be equal to 0.		
5	Place the first integral part at the position of the MSB and the last integral part at the position of the LSB, forming the binary equivalent of the fractional part.		
6	**$.618_{10} = .1001_2$**		

Step 4:
Combine the binary equivalents of the integral and fractional numbers to form the binary equivalent of the given number:

$$\mathbf{1.618_{10} = 1.1001_2}$$

2. Conversion from binary to decimal.

To calculate the represented number, we multiply the value of the digit used by the weight of its position and then add the results of all multiplications.

Example to calculate the decimal value of the binary number 0b11.0010

Digit	1	.	1	0	0	1
Weight	$2^0 = 1$		$2^{-1} = 0.5$	$2^{-2} = 0.25$	$2^{-3} =$ 0.125	$2^{-4} =$ 0.0625
Value of digit in decimal	**1 * 1 = 1**		**1 * 0.5 = 0.5**	**0 * 0.25 = 0**	**0 * 0.125 = 0**	**1 * 0.0625 = 0.0625**
Value of number in decimal: 1 + 0.5 + 0 + 0 + 0.0625 = 1.5625_{10}						

3. Conversion error.

Converting the decimal number to binary and then back to decimal, yielded a different number. The difference between the two numbers is the **conversion error**:

$$\mathbf{1.618_{10} - 1.5625_{10} = 0.0555_{10}}$$

Example 3:
1. Convert **78.23** to its fractional binary representation with up to four digits after the binary point.

Note: 78.23°C is the boiling point of Ethanol at 1 Atmosphere (101 kPa) pressure.

2. Using the result of 1., calculate its decimal representation with up to four digits after the decimal point.
3. What is the conversion error?

1. Conversion from decimal to binary.
Step 1:
Split the number in two parts: the integral part and the fractional part
The integral part is 78 and the fractional part is 0.23

Step 2:
Convert the integral part from decimal to binary by applying the following steps:

1. Divide the decimal number by 2. This will yield a quotient and a remainder. Take a note of the remainder.

2. While the quotient is not equal to 0, divide it by 2. This will yield a quotient and a remainder. Take a note of the remainders.

3. Reverse the order of all the remainders placing the **last** remainder at the position of the **MSB** and the **first** remainder at the position of the **LSB**, forming the binary equivalent of the integral part.

In this exercise, the integral part is 6. Apply the above algorithm to convert it into binary:

Step Nr	Division	Remainder	Quotient
1	78 / 2	0	39
2	39 / 2	1	19
3	19 / 2	1	9
4	9 / 2	1	4
5	4 / 2	0	2
6	2 / 2	0	1
7	1 / 2	1	0
8	Reverse the order of the remainders, placing the last remainder at the position of the MSB and the first remainder at the position of the LSB.		
9	$\mathbf{78_{10} = 1001110_2}$		

Step 3:
Convert the fractional part from decimal to binary by applying the following steps:

1. Multiply the fractional number by 2. This results in a new number with an integral part and a fractional part. The integral part will be the MSB of the binary equivalent of the fractional part.

2. While the fractional part of the new number is not zero, multiply it by 2. This results in a new number with an integral part and a fractional part. The integral part will be the next lower significant bit of the binary equivalent of the fractional part.

3. Collect all the integral parts placing the **first** integral part at the position of the **MSB** and the **last** integral part at the position of the **LSB**, forming the binary equivalent of the fractional part.

In this exercise, the fractional part is 0.1415. Apply the above algorithm to convert it into binary:

Step Nr	Multiplication	Integral part	Fractional part
1	0.23 * 2 = 0.46	0	0.46
2	0.46 * 2 = 0.92	0	0.92
3	0.92 * 2 = 1.84	1	0.84
4	0.84 * 2 = 1.68	1	0.68
	The process is stopped here because we were asked to provide an answer with four digits after the binary point. Otherwise, we would continue until the fractional part would be equal to 0.		
5	Place the first integral part at the position of the MSB and the last integral part at the position of the LSB, forming the binary equivalent of the fractional part.		
6	$\mathbf{.23_{10} = .0011_2}$		

Step 4:
Combine the binary equivalents of the integral and fractional numbers to form the binary equivalent of the given number:

$$\mathbf{78.23_{10} = 1001110.0011_2}$$

2. Conversion from binary to decimal.

To calculate the represented number, we multiply the value of the digit used by the weight of its position and then add the results of all multiplications.

Example to calculate the decimal value of the binary number 0b1001110.0011

Digit	1	0	0	1	1	1	0	.	0	0	1	1
Weight	2^6 = 64	2^5 = 32	2^4 = 16	2^3 = 8	2^2 = 4	2^1 = 2	2^0 = 1		2^{-1} = 0.5	2^{-2} = 0.25	2^{-3} = 0.125	2^{-4} = 0.0625
Value of digit in decimal	**1 * 64 = 64**	**0 * 32 = 0**	**0 * 16 = 0**	**1 * 8 = 8**	**1 * 4 = 4**	**1 * 2 = 2**	**0 * 1 = 0**		**0 * 0.5 = 0**	**0 * 0.25 = 0**	**1 * 0.125 = 0.125**	**1 * 0.0625 = 0.0625**
Value of number in decimal: $64 + 0 + 0 + 8 + 4 + 2 + 0 + 0 + 0 + 0.125 + 0.0625 = 78.1875_{10}$												

3. Conversion error.

Converting the decimal number to binary and then back to decimal, yielded a different number. The difference between the two numbers is the **conversion error**:

$$\mathbf{78.23_{10} - 78.1875_{10} = 0.0425_{10}}$$

Example 4:
1. Convert **119.6875** to its fractional binary representation with up to four digits after the binary point.

Note: 119.7 Astronomical Units was the distance between the Sun and the Voyager 2 space probe, when the latter entered the Interstellar Space Medium in 2019, 42 years after its departure from Earth. The number is adjusted above to a form that demonstrates the terminal condition of the conversion loop for the fractional part of the number.

2. Using the result of 1., calculate its decimal representation with up to four digits after the decimal point.
3. What is the conversion error?

1. Conversion from decimal to binary.
Step 1:
Split the number in two parts: the integral part and the fractional part
The integral part is 119 and the fractional part is 0.6875

Step 2:
Convert the integral part from decimal to binary by applying the following steps:

1. Divide the decimal number by 2. This will yield a quotient and a remainder. Take a note of the remainder.

2. While the quotient is not equal to 0, divide it by 2. This will yield a quotient and a remainder. Take a note of the remainders.

3. Reverse the order of all the remainders placing the **last** remainder at the position of the **MSB** and the **first** remainder at the position of the **LSB**, forming the binary equivalent of the integral part.

In this exercise, the integral part is 6. Apply the above algorithm to convert it into binary:

Step Nr	Division	Remainder	Quotient
1	119 / 2	1	59
2	59 / 2	1	29
3	29 / 2	1	14
4	14 / 2	0	7
5	7 / 2	1	3
6	3 / 2	1	1
7	1 / 2	1	0
8	Reverse the order of the remainders, placing the last remainder at the position of the MSB and the first remainder at the position of the LSB.		
9	$\mathbf{119_{10} = 1110111_2}$		

Step 3:
Convert the fractional part from decimal to binary by applying the following steps:

1. Multiply the fractional number by 2. This results in a new number with an integral part and a fractional part. The integral part will be the MSB of the binary equivalent of the fractional part.

2. While the fractional part of the new number is not zero, multiply it by 2. This results in a new number with an integral part and a fractional part. The integral part will be the next lower significant bit of the binary equivalent of the fractional part.

3. Collect all the integral parts placing the **first** integral part at the position of the **MSB** and the **last** integral part at the position of the **LSB**, forming the binary equivalent of the fractional part.

In this exercise, the fractional part is 0.1415. Apply the above algorithm to convert it into binary:

Step Nr	Multiplication	Integral part	Fractional part
1	0.6875 * 2 = 1.375	1	0.375
2	0.375 * 2 = 0.75	0	0.75
3	0.75 * 2 = 1.5	1	0.5
4	0.5 * 2 = 1.00	1	0.00
	The process is stopped here as the fractional part is equal to 0.		
5	Place the first integral part at the position of the MSB and the last integral part at the position of the LSB, forming the binary equivalent of the fractional part.		
6	**$.6875_{10}$ = $.1011_2$**		

Step 4:
Combine the binary equivalents of the integral and fractional numbers to form the binary equivalent of the given number:

$119.6875_{10} = 1110111.1011_2$

2. Conversion from binary to decimal.

To calculate the represented number, we multiply the value of the digit used by the weight of its position and then add the results of all multiplications.

Example to calculate the decimal value of the binary number 0b11.0011

Digit	1	1	1	0	1	1	1 .	1	0	1	1
Weight	2^6 = 64	2^5 = 32	2^4 = 16	2^3 = 8	2^2 = 4	2^1 = 2	2^0 = 1	2^{-1} = 0.5	2^{-2} = 0.25	2^{-3} = 0.125	2^{-4} = 0.0625
Value of digit in decimal	**1 * 64 = 64**	**1 * 32 = 32**	**1 * 16 = 16**	**0 * 8 = 0**	**1 * 4 = 4**	**1 * 2 = 2**	**1 * 1 = 1**	**1 * 0.5 = 0.5**	**0 * 0.25 = 0**	**1 * 0.125 = 0.125**	**1 * 0.0625 = 0.0625**
Value of number in decimal: 64 + 32 + 16 + 0 + 4 + 2 + 1 + 0.5 +0 + 0.125 + 0.0625 = 119.6875_{10}											

3. Conversion error.

Converting the decimal number to binary and then back to decimal, yielded a different number. The difference between the two numbers is the **conversion error**:

$$\mathbf{119.6875_{10} - 119.6875_{10} = 0_{10}}$$