Binary subtraction

Binary subtraction follows the same steps as decimal subtraction. To help clarify the process, the four rules governing it are summarised below.

$$0 - 0 = 0$$
$$1 - 0 = 1$$
$$1 - 1 = 0$$
$$0 - 1 = 1 \text{ (a borrow is required)}$$

Consider the subtraction of number B from number A, where A = 0b1010 and B = 0b0110. The steps taken to calculate the result are:

Step 1: We subtract the least significant bit (bit 0) of each of the two numbers: 0 - 0 = 0. So bit 0 of the result is equal to 0.

Step 2: We subtract the bit 1 of each of the two numbers: 1 - 1 = 0. So bit 1 of the result is equal to 0.

Step 3: We subtract the bit 2 of each of the two numbers: 0 - 1 = 1. Because 1 > 0, a 1 must be borrowed from the subtraction of the next significant bit. So, bit 2 of the result is equal to 1 and a 1 is borrowed from the next step.

Step 4: We subtract the bit 3 of each of the two numbers, but this time we also consider the bit that was borrowed to step 3. The 0 of number B is subtracted from the 1 of number A, and we also subtract the 1 that was borrowed to step 3: 0 - 1 - 1 = 0. . So, bit 3 of the result is 0

Step 5: we put the above results together and we get the answer:

$$0b1010 - 0b0110 = 0b0100$$

Example 1:

Subtract number B: 0b01001110 from number A: 0b01110111:

	Bit 7	Bit 6	Bit 5	Bit 4	Bit 3	Bit 2	Bit 1	Bit 0
Number A bit	0	1	1	1	0	1	1	1
Number B bit	0	1	0	0	1	1	1	0
Carry bit	**0**	**0**	**0**	**0**	**1**	**0**	**0**	**0**	**0**
Result bit	0	0	0	1	0	1	0	0	1

The steps taken to calculate the result are:

Step 1: We subtract the least significant bit (bit 0) of each of the two numbers:

1 - 0 = 1, or Bit 0 from number A – Bit 0 from number B

Bit 0 of the result is equal to 1.

Step 2: We subtract the bit 1 of each of the two numbers:

1 - 1 - 0 = 0, or Bit 1 from number A – Bit 1 from number B

Bit 1 of the result is equal to 0.

Step 3: We subtract the bit 2 of each of the two numbers:

1 - 1 - 0 = 0, or Bit 2 from number A – Bit 2 from number B

Bit 2 of the result is equal to 0.

Step 4: We subtract the bit 3 of each of the two numbers:

0 - 1 - 0 = 1, or Bit 3 from number A – Bit 3 from number B

Because 1 > 0, a 1 must be borrowed from the subtraction of the next significant bit.
Bit 3 of the result is equal to 1 and a 1 is borrowed from the next step.

Step 5: We subtract the bit 4 of each of the two numbers, but this time we also consider the bit that was borrowed to step 4:

1 - 0 - 1 = 0, or Bit 4 from number A – Bit 4 from number B – bit borrowed by previous step.

Bit 4 of the result is 0

Step 6: We subtract the bit 5 of each of the two numbers:

1 - 0 - 0 = 1, or Bit 5 from number A – Bit 5 from number B

Bit 5 of the result is 1

Step 7: We subtract the bit 6 of each of the two numbers:

1 - 1 - 0 = 0, or Bit 6 from number A – Bit 6 from number B

Bit 6 of the result is 0

Step 8: We subtract the bit 7 of each of the two numbers:

0 - 0 - 0 = 1, or Bit 7 from number A – Bit 7 from number B

Bit 7 of the result is 0

Result: 0b00101001

Example 2:

Subtract number B: 0b11010111 from number A: 0b01001110

	Bit 7	Bit 6	Bit 5	Bit 4	Bit 3	Bit 2	Bit 1	Bit 0
Number A bit	0	1	0	0	1	1	1	0
Number B bit	1	1	0	1	0	1	1	1
Carry bit	**1** **1**	**1**	**1**	**1**	**0**	**1**	**1**	**1** **0**
Result bit	1 1	0	1	1	1	0	1	1 1

The steps taken to calculate the result are:

Step 1: We subtract the least significant bit (bit 0) of each of the two numbers:

0 - 1 = 1, or Bit 0 from number A – Bit 0 from number B

Because 1 > 0, a 1 must be borrowed from the subtraction of the next significant bit.
Bit 0 of the result is equal to 1 and a 1 is borrowed from the next step.

Step 2: We subtract the bit 1 of each of the two numbers, but we also consider the bit that was borrowed:

1 - 1 - 1 = 1, or Bit 1 from number A – Bit 1 from number B – bit borrowed by previous step.

Bit 1 of the result is equal to 1 and a 1 is borrowed from the next step.

Step 3: We subtract the bit 2 of each of the two numbers, but we also consider the bit that was borrowed:

1 - 1 - 1 = 1, or Bit 2 from number A – Bit 2 from number B – bit borrowed by previous step.

Bit 2 of the result is equal to 1 and a 1 is borrowed from the next step.

Step 4: We subtract the bit 3 of each of the two numbers, also considering the bit that was borrowed:

1 - 0 - 1 = 0, or Bit 3 from number A – Bit 3 from number B – bit borrowed by previous step.

Bit 3 of the result is equal to 0.

Step 5: We subtract the bit 4 of each of the two numbers:

0 – 1 - 0 = 1, or Bit 4 from number A – Bit 4 from number B

Bit 4 of the result is equal to 1 and a 1 is borrowed from the next step.

Step 6: We subtract the bit 5 of each of the two numbers:
0 - 0 - 1 = 1, or Bit 5 from number A – Bit 5 from number B – bit borrowed by previous step.

Bit 5 of the result is 1 and a 1 is borrowed from the next step.

Step 7: We subtract the bit 6 of each of the two numbers:
1 - 1 - 1 = 1, or Bit 6 from number A – Bit 6 from number B – bit borrowed by previous step.

Bit 6 of the result is 1 and a 1 is borrowed from the next step.

Step 8: We subtract the bit 7 of each of the two numbers, also considering the bit that was borrowed:

0 - 1 - 1 = 0, or Bit 7 from number A – Bit 7 from number B – bit borrowed by previous step.

Bit 7 of the result is 0 and a 1 would have been borrowed if there was another step following this one. Because this is a subtraction, the borrowed bit indicates an underflow.

Result: 0b(1)01110111

Example 3:

Subtract number B: 0b10101010 from number A: 0b01010101

	Bit 7	Bit 6	Bit 5	Bit 4	Bit 3	Bit 2	Bit 1	Bit 0
Number A bit	0	1	0	1	0	1	0	1
Number B bit	1	0	1	0	1	0	1	0
Carry bit	**1**	**0**	**1**	**0**	**1**	**0**	**1**	**0**	**0**
Result bit	1	1	0	1	0	1	0	1	1

The steps taken to calculate the result are:

Step 1: We subtract the least significant bit (bit 0) of each of the two numbers:

1 - 0 = 1, or Bit 0 from number A – Bit 0 from number B

Bit 0 of the result is equal to 1.

Step 2: We subtract the bit 1 of each of the two numbers, but we also consider the bit that was borrowed:

0 - 1 - 0 = 1, or Bit 1 from number A – Bit 1 from number B – bit borrowed by previous step.

Bit 1 of the result is equal to 1 and a 1 is borrowed from the next step.

Step 3: We subtract the bit 2 of each of the two numbers, but we also consider the bit that was borrowed:

1 - 0 - 1 = 0, or Bit 2 from number A – Bit 2 from number B – bit borrowed by previous step.

Bit 2 of the result is equal to 0.

Step 4: We subtract the bit 3 of each of the two numbers, also considering the bit that was borrowed:

0 - 1 - 0 = 1, or Bit 3 from number A – Bit 3 from number B.

Bit 3 of the result is equal to 1 and a 1 is borrowed from the next step.

Step 5: We subtract the bit 4 of each of the two numbers:

1 - 0 - 1 = 1, or Bit 4 from number A – Bit 4 from number B – bit borrowed by previous step.

Bit 4 of the result is equal to 0.

Step 6: We subtract the bit 5 of each of the two numbers:
0 - 1 - 0 = 1, or Bit 5 from number A – Bit 5 from number B.

Bit 5 of the result is 1 and a 1 is borrowed from the next step.

Step 7: We subtract the bit 6 of each of the two numbers:
1 - 0 - 1 = 0, or Bit 6 from number A – Bit 6 from number B – bit borrowed by previous step.

Bit 6 of the result is 0.

Step 8: We subtract the bit 7 of each of the two numbers, also considering the bit that was borrowed:

0 - 1 - 0 = 1, or Bit 7 from number A – Bit 7 from number B – bit borrowed by previous step.

Bit 7 of the result is 0 and a 1 would have been borrowed if there was another step following this one. Because this is a subtraction, the borrowed bit indicates an underflow.

Result: 0b(1)01110111