Homework 2

236860: Digital Image Processing

Winter 2018-19

Finding the optimal blur kernel

In the first homework we dealt with correcting the motion blur occurring in the images acquired through burst mode – where the motion of camera is faster than the shutter speed of the lens. In this homework, we will look at another instance of image blur called the *out-of-focus* blur – observed when the camera's focus is not properly adjusted by the user.

The point spread function (PSF) describes the distribution of light in the camera focal plane for a point source. This function strictly relates to the resolution and blur of the optical device. In the absence of any imaging noise, the action of the camera can be mathematically expressed as follows

$$y = p * f$$

where f and y correspond to the continuous scene and the resulting discrete image respectively. p corresponds to PSF of the camera.

Super resolution (SR) relates to the task of estimating a high-resolution image from a low-resolution one. It can be modeled similar to the above equation where f and y correspond to the high and low resolution images respectively and p is the blur kernel relating them. This problem is usually studied in two flavors: non-blind and blind. Non-blind SR is the classical inverse problem scenario where the kernel p and the blurred image y are known and the task is to estimate f. As we learned in the class, several priors can be induced on f for better reconstruction. *Blind* SR refers to the case where the kernel p is unknown and the task is to first estimate p and then estimate a high-resolution image f using a non-blind SR method.

In this homework, we are interested in the *non-blind SR* problem. We will assume that we know the ideal PSFs through which the low and high resolution images are obtained. Using this information, we will first derive a kernel that "correctly" translates the low-resolution image to a high-resolution one. We will then implement a standard inverse problem solver to obtain the high-resolution image.

Notation: $f(\mathbf{x})$ denotes the "continuous scene". PSF_L and PSF_H corresponds to the PSF of the low and high images respectively. See Figure 1 for an illustration.

Figure 1: Problem setting

Dry part

- Let $f(\mathbf{x})$ be the "continuous" scene. Write an expression for the low-resolution image $l[\mathbf{n}]$ using PSF_L.
- Write an expression for constructing an α times higher resolution discrete image $h[\mathbf{n}]$ from the continuous image $f(\mathbf{x})$ using PSF_H.
- Let us assume that h and l are related by a blur kernel k (refer to Figure 1). Express $l[\mathbf{n}]$ in terms of h and k.
- Using the answers derived from the above three questions, write an expression for PSF_L in terms PSF_H and k.
- Solve the above derived expression relating PSF_H and PSF_L as a least squares problem and write an expression for k.
- Given k (from the above question) and the low-resolution image l, (i) write an expression to reconstruct h using the Wiener filter; and (ii) write the objective function to estimate h as a least squares problem with a total-variation (TV) prior on h.

Wet part

Please download the DIPSourceHW2.png image that is provided. Assume this is the "continuous" scene, i.e. $f(\mathbf{x})$. Assume $\alpha = 2.5$ for all the experiments.

- If the PSF of the camera is a Gaussian kernel, construct PSF_L and PSF_H.
- If the PSF of the camera is a box function, construct PSF_L and PSF_H
- Using PSF_L and PSF_H, construct the corresponding low l and high resolution h images in both the above cases (Gaussian and box).
- Using the results obtained from the dry part, construct the blur kernel k in both cases (Gaussian and box).
- Use this kernel k obtained in the above step to estimate the high-resolution image h by using (i) Wiener filter and (ii) least squares estimation with a TV prior. (for both Gassian and box kernels).
- Upsample l using bilinear and bicubic kernels to estimate h. Compare the results with that of using k.

Deliverables

- Dry part: Write answers to all the questions in a digital file.
- Wet part:
  - PSF_H, PSF_L, k, h, l for both Gaussian and box kernels.
  - The code.

Please submit all the deliverables by **February 6, 2019. 11:55PM**

Submission email: sanketh@cs.technion.ac.il

Thanks and enjoy!
Alex and Sanketh