Robots and Wage Polarization: The Effects of Robot Capital by Occupations*

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Abstract

What are the distributional and aggregate effects of the rising use of industrial robots across occupations? I construct a novel dataset that tracks the cost of robots from Japan by occupations. The dataset reveals a relative one-standard deviation drop of Japan's robot cost induces a 0.2-0.3% drop in the US occupational wages. I develop a general equilibrium model where robots are internationally traded durable goods that may substitute for labor differently across occupations. The elasticities of substitution between robots and labor within an occupation drive the occupation-specific real-wage effects of robotization. I estimate the model using the robot cost shock from my dataset and the optimal instrumental variable implied by the model. I find that the elasticities of substitution between robots and labor are heterogeneous across occupations, and higher than those between general capital goods and labor in production occupations such as welding. The estimated model implies that the industrial robots explain a 0.9 percentage point increase in the 90-50th percentile ratio of US occupational wages, and a 0.2 percentage point increase of the US real income from 1990 to 2007.

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1 Introduction

In the last three decades, the global market size of industrial robots has grown by 12% annually.[1] International trade of robots is also sizable, with 41% of all robots imported. Workers in different occupations are differentially susceptible to robots, raising concerns about the distributional effects of such trends. Motivated by this concern, policymakers have proposed various restrictions on automation, such as a robot tax.[2] An emerging literature has estimated the relative effects of robot penetration on employment and the potential impact of such taxes (e.g., Acemoglu and Restrepo, 2020; Humlum, 2019). However, due to the limited data measuring the cost of robots across occupations and the lack of a model capturing the trade of robots and their dynamic accumulation, our understanding of the distributional and aggregate impacts of industrial robots is still limited.

In this paper, I study how industrial robots affect wage inequality between occupations and aggregate income. First, I assemble a new dataset of the cost of robots by 4-digit occupations that allows me to find stylized facts about the robot cost reduction and its impact on the US occupational labor market. Second, to better interpret these empirical facts, I develop a model where robots are internationally traded durable goods, are endogenously accumulated, and substitutes for labor within occupations. Third, using these data and model, I construct a model-implied optimal instrumental variable and provide the first estimate of the elasticity of substitution (EoS) between robots and workers heterogeneous across occupation groups as well as other model parameters. Finally, counterfactual exercises based on this estimated model reveal the distributional and aggregate implication of robotization in the US since 1990.

[1] Throughout the paper, industrial robots (or robots) are defined as multiple-axes manipulators and are measured by the number of such manipulators, or robot arms, a standard in the literature. A more formal definition given by ISO and example images of robots in such a definition are provided in Section A.2 of the Appendix. Such a definition implies that any automation equipment that does not have multiple axes is out of the scope of the paper, even though some of them are often called "robots" (e.g., Roomba, an autonomous home vacuum cleaner made by iRobot Corporation).

[2] The European Parliament proposed a robot tax on robot owners in 2015, although it eventually rejected the proposal (Delvaux et al., 2016). South Korea revised the corporate tax laws that downsize the "Tax Credit for Investment in Facilities for Productivity Enhancement" for enterprises investing in automation equipment (MOEF, 2018).

My dataset is unique in two ways. First, it tracks the robots' monetary value as well as their number. Second, the observation is disaggregated by the adopting country and the 4-digit occupation in which robots replace labor. To obtain such a dataset, I first use the information from the Japan Robot Association (JARA) about the shipment of Japanese robots to each country and by task, which comprises one-third of the world robot supply. I then combine the JARA data with the O*NET Code Connector's match score and the US Census/ACS data. Finally, I derive the robot cost shock by occupations from the average price variable after controlling for destination-country fixed effects. As a result, I obtain the dataset that links the US occupational labor market outcomes to the cost shock of robots imported from Japan (Japan robot shock).

The dataset reveals two stylized facts. First, over 1990-2007, the Japan robot shock exhibits that the average cost of a Japanese robot reduced and that the cost reduction is heterogeneous by occupations. Second, the Japan robot shock drives the drop in wages and employment by occupations in the US. A relative decrease in one standard deviation of Japanese robots' cost drives an annualized 0.2-0.3 percent decrease in occupational wages. This finding is robust to the control of non-robot occupational demand shocks, like the China trade shock, and thus suggests high responsiveness of relative robot demand to the cost reduction due to the strong substitutability of robots for labor. However, the Japan robot shock measure is subject to a concern that it may reflect robot quality upgrading during the sample period. Furthermore, the reduced-form empirical finding does not fully reveal the distributional and aggregate effects of the Japan robot shock. To overcome these issues and derive more conclusive statement, I employ a dynamic open-economy equilibrium model of automation.

I develop a general equilibrium (GE) model with robotics automation and the following three key features. First, I incorporate the Armington-style trade of robots, which fits well with the sizable robot trade in my dataset about Japanese robot export. Theoretically, trade of robots in a large-open economy implies that a robot tax affects the price of robots traded in the global market. Hence, a country may gain from the aggregate perspective if it can reduce the cost of adopting robots by imposing the robot tax. Second, the model describes the endogenous investment in robots with a convex adjustment cost, which implies sluggish accumulation of robot capital. Therefore, the aggregate income implication of the robot tax is nuanced and different over the time horizon. Finally, the model has a production function with occupation-specific EoS between robots and labor, which varies across occupations. This production function yields rich predictions regarding the real-wage effect of robot capital for the following two reasons. Firstly, the accumulated stock of robots is different across occupations. Secondly, a unit of robots can substitute for workers differentially in each occupation.

To better understand the role of the occupation-specific EoS between robots and workers, I consider an automation shock à la Acemoglu and Restrepo (2020) in which robots can exogenously perform a larger share of tasks compared to labor. I analytically show that, in the equilibrium, the automation shock's effect on occupational real wage is negatively related to the robot-labor EoS. Namely, the higher the EoS, the larger drop of labor demand given the automation shock because of the stronger substitution of labor with robots. The model also features the EoS between occupations, which affects the across-occupation effects of the automation shocks.

To identify the robot-labor EoS, I confront a challenge that the Japan robot shock is correlated with the automation shock, affecting the labor market outcomes simultaneously. To overcome this challenge, I use the GE structure and obtain the structural residual of labor market outcomes, which controls the effect of the automation shock. I then construct a moment condition in which this structural residual is orthogonal to the Japan robot shock. Using this moment condition, I generate an optimal instrumental variable implied by the model, which increases the estimation precision.

I apply this estimation method to the data on occupational labor market outcomes and robot adoption and find that the EoS between robots and workers is heterogeneous across occupation groups. For routine occupations that perform production and material moving, the estimates are as high as around 4. These estimates are significantly higher than the values of the EoS between labor and general capital like structure and equipment estimated in the literature, highlighting one of the main differences between robots and general capital goods. In contrast, the EoS in other occupations is close to 1, or robots and labor are neither substitutes nor complements in these other occupations.

The estimated model and shocks backed out from the model predict occupational US wage changes from 1990-2007. The high EoS between robots and workers in production and material moving occupations implies that the robotization in this period significantly decreased relative wage in these occupations. Since these occupations tend to be in the middle of the occupational wage distribution in 1990, this finding indicates that the automation shock compressed the wage growth of occupations in the middle deciles. Quantitatively, it explains 0.9 percentage point, or 11.7%, of the wage polarization measured by the change in the 90th-50th percentile wage ratio, a measure of wage inequality popularized by Goos and Manning (2007) and Autor et al. (2008). The robotization also explains a 0.2 percentage point increase of the US real income, mostly accounted for by the rise in the firm profit due to the accumulation of robots.

Finally, I examine the counterfactual effect of introducing a tax on robot purchases. Such a robot tax could potentially increase the aggregate income of a country. Due to the trade of robots, a government can exert monopsony power in the global robot market by taxing robot purchases, leading to a decrease in the before-tax price of imported robots in each period. In contrast, the robot tax also disincentivizes the accumulation of robots in the long run, potentially reducing aggregate income. Quantitatively, the latter effect dominates the former in the long-run, and the robot tax decreases aggregate real income.

This paper contributes to the literature that studies the economic impacts of industrial robots by finding a sizable impact of robots on US wage inequality and a short-run positive aggregate effect of a robot tax. The closest papers are Acemoglu and Restrepo (2020) and Humlum (2019). Acemoglu and Restrepo (2020) establishes that the US commuting zones experiencing penetration of robots over 1992-2007 also saw decreased wages and total employment.[3] Humlum (2019) uses firm-level data on robot adoption and firm-

[3] Dauth et al. (2017) and Graetz and Michaels (2018) also use the industry-level aggregate data of robot

worker-level panel data and estimates a model that incorporates a small-open economy of robot importers, a binary decision of robot adoption, and an EoS between occupations. Using these data and model, he studies the distributional effect of robots and a counterfactual robot tax.[4]

In contrast to these papers, my study features the following three elements. First, I use the data about the Japan robot shock by occupation, which empirically reveals impacts on US occupations. Second, I consider the trade of robots in a large-open economy setting, which implies that the US real income effect of robots is positive in the short-run in my counterfactual exercise. Finally, these data and model allow estimating occupation-specific EoS between robots and labor. The estimated model implies that the wage-polarizing effect of the increase in robot use is larger than the prediction of the model with a conventional assumption on the robot-labor EoS, such as Leontief.

Occupations are receiving attention in the literature of automation as they matter when considering the distributional effects. While Jäger et al. (2016) finds no association between industrial robot adoptions and total employment at the firm level, Dinlersoz et al. (2018) report the cost share of workers in the production occupation dropped after the adoption of robots within a firm. Cheng (2018) studies the heterogeneous capital price decrease and its implication on job polarization. Jaimovich et al. (2020) construct a model to study the effect of automation on the labor market of routine and non-routine workers in the steady state. I contribute to this literature by estimating the within-occupation EoS between robots and labor with the occupation-level data of robot costs and labor market outcomes, as well as incorporating the endogenous trade of robots and characterizing the transition dynamics of the effect of robot tax.

Following the seminal work by Autor et al. (2003), there is a growing literature that attempts to detect the task contents of recent technological development. Webb (2019) provides a natural-language-processing method to match technological advances (e.g.,

adoption and its impact on labor markets.

[4]There is also a growing body of studies that use the firm- and plant-level microdata to study the impact on workers in Canada (Dixon et al., 2019), France (Acemoglu et al., 2020; Bonfiglioli et al., 2020), Netherlands (Bessen et al., 2019), Spain (Koch et al., 2019), and the US (Dinlersoz et al., 2018).

robots, software, and artificial intelligence) embodied in the patent title and abstract to occupations. Montobbio et al. (2020) extends this approach to analyzing full patent texts by applying the topic modeling method of machine learning. My matching method between robot application and occupation complements these studies: On one hand, my methodology gives a list of matching scores. Combined with the robot data by application, my dataset yields the number and sales of robots for all 4-digit occupations. On the other hand, I do not provide such detailed textual analysis as the previous literature since I can only observe the title of robot applications.

Since robots are one type of capital goods, my paper is also related to the vast literature of estimating the EoS between capital and labor (to name a few, Arrow et al., 1961; Chirinko, 2008; Oberfield and Raval, 2014). Although the literature yields a set of estimates with a wide range, the upper limit appears around 1.5 (Karabarbounis and Neiman, 2014; Hubmer, 2018). Therefore, the estimates as high as 4 in production and material-moving occupations are significantly higher than this upper limit. In this sense, my estimates highlight one of the main differences between robots and other capital goods: these workers' vulnerability to robots.

The rest of the paper is organized as follows. Section 2 describes my dataset of robots by occupations. I set out the general equilibrium model in 3, and estimate it using the model-implied instrumental variable in model 4. Using the estimated model, I study the effect of robotization and counterfactual robot taxes in Section 5. Section 6 concludes.

2 Data and Stylized Facts

This section begins with setting out two central data sources in Section 2.1: the Japan Robot Association survey and O*NET for matching the robot application code to the labor occupation code at the 4-digit level. Note that Japan has been a major robot innovator, producer, and exporter. For example, the US imports 5 billion-dollar worth of Japanese robots as of 2017, which comprises roughly one-third of robots in the US.[5] Therefore,

[5]Appendix A.3 shows the international robot flows, including Japan, the US, and the rest of the world.

Japanese robot cost reduction significantly affects robot adoption in the US and the world.

Using these data, I describe how to measure the robot cost, provide the matching method to obtain robot measures at the occupation level, and derive the Japan robot shock formally in Section 2.2. Section 2.3 provides stylized facts that suggest substitutability between robots and labor and motivate the model and estimation in later sections.

2.1 Data Sources

The main part of my dataset is provided by Japan Robot Association (JARA), a general incorporated association composed of Japanese robot producing companies. The number of member companies is 381 as of August 2020. JARA annually surveys all these member and several non-member companies about the units and monetary values of robots sold for each destination country and robot application, or specified tasks of robots, which is discussed in detail in Section A.2 of the Appendix. JARA publishes summary cross-tables of the survey, which I digitize and use as one of the main data sources.

I also use Occupational Information Network OnLine (O*NET) Code Connector. O*NET is an online database of occupational definitions sponsored by the US Department of Labor, Employment, and Training Administration. O*NET Code Connector provides an occupational search service that helps workforce professionals determine relevant 4-digit level O*NET-SOC Occupation Codes for job orders. Along with the O*NET-SOC codes, the search algorithm provides (i) the textual description of each code and (ii) a match score that shows the relevance of the search target with the search query term. To match robot applications and labor occupations, I use these textual descriptions and match scores, which are further described in detail in Section A of the Appendix.

2.2 Constructing the Dataset

Using these data, I construct a dataset that matches the cost of Japanese robots to the US labor market outcomes at the occupation level. After clarifying robot cost measurement, I describe the matching process between robot applications and labor occupations.

2.2.1 Measuring the Cost of Robots

To understand the measurement of the robot cost, I clarify how robots work. A modern industrial robot is typically not stand-alone hardware (e.g., robot joints and arms) but an ecosystem that includes the hardware and control units operated by software (e.g., computers and robot-programming language). Due to its complexity, installing robots in the production environment often requires hiring costly system integrators that offer specific engineering knowledge. A relevant cost of robots for adopters, therefore, includes hardware, software, and integration costs.[6]

In this paper, I measure the price of robots by average price, or the total sales divided by the quantity of hardware. In this sense, readers should interpret that my measure of robot price reflects a portion of overall robot costs. Since the literature has not established a method to deal with this issue, I will address this point in the model section by separately defining the observable hardware cost and unobserved components of the cost, and placing assumptions on the latter.

Another issue of this approach is that this price measure includes robot quality upgrading. Namely, innovation in robotics technology could entail both quality upgrading that makes robots perform more tasks at a greater efficiency and cost saving of producing robots that perform the same task as before. Inseparability of these two components poses an identification threat as I describe later in Section 4.2, which none of the previous studies could resolve. To work around this issue, I will use the general equilibrium model to predict the labor market effects of quality upgrading in Section 3.[7]

[6] As Leigh and Kraft (2018) pointed out, the current industry and occupation classifications do not allow separating system integrators, making it hard to estimate the cost from these classifications. Plus, there still remains apparently relevant costs of robot use, like maintenance fee, about which we also lack quantitative evidence. Although understanding these components of the costs is of first-order importance, this paper follows the literature convention and measure robots from market transaction of hardware.

[7] Note that this problem occurs because I consider the price of robots, as the past literature mainly focuses only on the quantity of robots (Acemoglu and Restrepo, 2020) or even firm's binary decision of robot adoption (Humlum, 2019). One of the more data-driven approaches to this issue is to control the quality change by the hedonic approach as in Timmer et al. (2007). However, this strategy requires detailed information about the spec of each robot. Pursuing this direction is the next step of my research agenda, as I collaborate with JARA for retrieving catalog information of robots produced by major producers.

2.2.2 Matching Robot Applications and Labor Occupations

My dataset provides the employment of labor and robots at the occupation level, complementing data in the previous literature at the sector level or, more recently, firm level. This is made possible by having robot application-level data, and converting robot applications to labor occupations. I propose a method to match the JARA data and the Census 4-digit occupation level labor market outcomes.

There has not been formal concordance between application and occupation codes, although robot applications and labor occupations are close concepts. On the one hand, robot application is a task where the robot is applied. On the other hand, labor occupation describes multiple types of tasks the person does on the job. Each task has different requirements for robotics automation. Therefore, a heterogeneous mix of tasks in each occupation generates a difference in the ease of automation across occupations and, thus, heterogeneous penetration of robots (Manyika et al., 2017). I show examples of pairs of robot applications and labor occupations in Section A.2 in the Appendix.

More specifically, let a denote robot application and o labor occupation. JARA data measure robot sales quantity and total monetary transaction values for each application a. I write these as robot measures X_a^R, a generic notation that means both quantity and monetary values. The goal is to convert an application-level robot measure X_a^R to an O*NET-SOC occupation-level one X_o^R. First, I search occupations in O*NET Code Connector by the title of robot application a. Second, I web-scrape the match score m_{oa} between a and o.[8] Finally, I allocate X_a^R to each occupation o according to m_{oa}-weight by

$$X_o^R = \sum_a \omega_{oa} X_a^R \text{ where } \omega_{oa} \equiv \frac{m_{oa}}{\sum_{o'} m_{o'a}}.$$

[8] I focus on consistent occupations between the 1970 Census and the 2007 ACS that cover the sample period and pre-trend analysis period to obtain consistent data across periods. Therefore, this paper focuses on the intensive-margin substitution in occupations as opposed to the extensive-margin effect of automation that creates new labor-intensive tasks and occupations (Acemoglu and Restrepo, 2018). My dataset shows that 88.7 percent of workers in 2007 worked in the occupations that existed in 1990. It is an open question how to attribute the creation of new occupations to different types of automation goods like occupational robots in my case, although Autor and Salomons (2019) explore how to measure the task contents of new occupations.

As a result, X_o^R measures the occupation-level robot measures such as quantity and monetary values. Note $\sum_o \omega_{oa} X_a^R = X_a^R$ since $\sum_o \omega_{oa} = 1$. In other words, occupation-level robot measures sum back to the application level when summed across occupations, as a desired property of the allocation.

I then convert the O*NET-SOC-level occupation codes to OCC2010 occupation codes to match the labor market measures from the US Census, American Community Survey (ACS), retrieved from the Integrated Public Use Microdata Series (IPUMS) USA (Ruggles et al., 2018), described in detail in Appendix A.1.

2.2.3 Japan Robot Shock

To obtain the robot cost variation by occupation, write $p_{i,o,t}^R$ the average price of robots in occupation o in destination country i in year t. I fit the fixed-effect regression

$$\ln\left(p_{i,o,t}^R\right) - \ln\left(p_{i,o,t_0}^R\right) = \psi_{i,t}^D + \psi_{o,t}^J + \epsilon_{i,o,t} \tag{1}$$

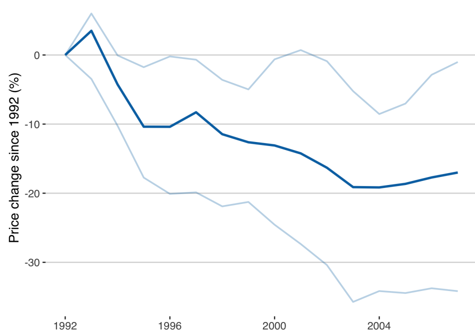
where t_0 is the initial year, $\psi_{i,t}^D$ is destination-year fixed effect, $\psi_{o,t}^J$ is occupation-year fixed effect, and $\epsilon_{i,o,t}$ is the residual. This regression controls any country-year specific effect $\psi_{i,t}^D$, which includes country i's demand shock or trade shock between Japan and i that are independent of occupations. I use the remaining variation across occupations $\psi_{o,t}^J$ as a cost shock of robots by occupations and term it as a "Japan robot shock."

2.3 Stylized Facts

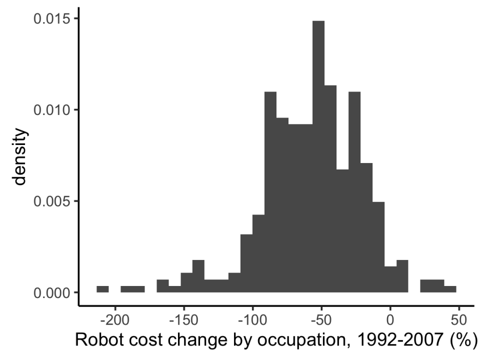
The resulting dataset permits me to study the robot cost variation by occupation, or Japan robot shock, and the corresponding occupation's labor market outcomes with demographic controls, which I explore in the next subsection. Throughout the paper, I define the initial year $t_0 = 1992$ (or for Census data $t_0 = 1990$), in which the JARA data starts tracking the destination-country level variable, and 1992-2007 as the sample period, with notation $t_1 = 2007$.

Figure 1: Distribution of the Cost of Robots

(a) US Robot Price by Occupations



(b) Histogram of the Japan Robot Shock



Note: The author's calculation based on JARA and O*NET. The left panel shows the trend of prices of robots in the US by occupations, $p^R_{USA,o,t}$. The thick and dark line shows the median price in each year, and two thin and light lines are the 10th and 90th percentile. Three-year moving averages are taken to smooth out yearly noises. The right panel shows the histogram of long-run (1992-2007) cost shock of robots measured by the fixed effect ψ^C_{o,t_1} in equation (1).

Fact 1: Trends of the Japan Robot Shock I show the patterns of average prices of robots across occupations that are not intensively studied in the literature. Figure 1a plots the distribution (10th, 50th, and 90th percentile) of the growth rates of the price of robots in the US relative to the initial year. The figure shows two patterns: (i) the robot prices show an overall decreasing trend, with the median growth rate of -17% from 1992 to 2007, or -1.1% annually, and (ii) a significant heterogeneity in the rate of price falls across occupations, with the 10th percentile occupation experienced -34% growth (-2.8% per annum), while the 90th percentile occupation almost did not change the price in the sample period. The price drop is consistent with the decreasing trend of prices of general investment goods since 1980, as Karabarbounis and Neiman (2014) report a 10% decrease per decade from their data sources. The large variation of the changes in prices by occupations persists even after controlling for the destination-year fixed effect $\psi^D_{i,t}$, as Figure 1b shows the distribution of the Japan robot shock in the long-run (1992-2007), or ψ^J_{i,t_1} in equation (1).

There are several interpretations of the price trend, including the reduction in the cost to produce robots and quality changes. First, if the cost of producing robots decreases, the measured prices naturally drop. In the model, I will capture this pattern by positive Hicks-neutral productivity shock to robot producers. Second, if the quality of the robots increased over the period, the quality-adjusted prices may experience a larger decrease than what is observed in the average price measure. They are hard to separate in my data and thus interpreted through the lens of the general equilibrium model in Section 3 by incorporating the quality change and examining its effects on robot prices and quantities. As a result, the differences in the robot cost shock and the quality change may affect the robot adoption and the labor market impacts by occupations.

Fact 2: Effects of the Japan robot shock on US occupations Using the variation of Japan robot shock, I study the effect on US labor market outcomes. Since the labor demand may be affected by the concurrent trade liberalization, notably the China shock, I control for the occupational China shock by the method developed by Autor et al. (2013), namely,

$$IPW_{o,t} \equiv \sum_{s} l_{s,o,t_0} \Delta m^{C}_{s,t}, \tag{2}$$

where l_{s,o,t_0} is sector-s share of employment for occupation o and $\Delta m^{C}_{s,t}$ is the per-worker Chinese export growth to non-US developed countries.[9] An occupation receives a high trade shock if sectors that experienced increased import competition from China intensively employ the occupation. With this measure of the trade shock, I run the following regression

$$\Delta \ln (Y_o) = \alpha_0 + \alpha_1 \times \psi^{J}_{o,t_1} + \alpha_2 \times IPW_{o,t_1} + \boldsymbol{X}_o \cdot \boldsymbol{\alpha} + \varepsilon_o, \tag{3}$$

where Y_o is a labor market outcome by occupations such as hourly wage and employment, $\boldsymbol{X}_o$ is the vector of baseline demographic control variables are the female share, the college-graduate share, the age distribution, and the foreign-born share, and Δ is the long-run time difference between 1990 and 2007.

Table 1 shows the result of regression (3). Columns 1-3 take hourly wages as the outcome, while columns 4-6 do employment. In columns 3 and 6, the main specifications that includes both the Japan robot shock and the China shock, I find that the negative

[9]Specifically, following Autor et al. (2013), I take eight countries: Australia, Denmark, Finland, Germany, Japan, New Zealand, Spain, and Switzerland. Appendix A.1 shows the distribution of occupational employment l_{s,o,t_0} for each sector.

Table 1: Effects of the Japan robot shock on US occupations

VARIABLES	(1) $\Delta \ln(w)$	(2) $\Delta \ln(w)$	(3) $\Delta \ln(w)$	(4) $\Delta \ln(L)$	(5) $\Delta \ln(L)$	(6) $\Delta \ln(L)$
ψ^C	0.0970***		0.0984***	0.0459***		0.0472***
	(0.0263)		(0.0266)	(0.0151)		(0.0142)
IPW		-0.0697**	-0.0748**		-0.0639***	-0.0663***
		(0.0348)	(0.0307)		(0.0143)	(0.0138)
Demographic controls	✓	✓	✓	✓	✓	✓
Observations	324	324	324	324	324	324
R-squared	0.379	0.320	0.394	0.103	0.073	0.178

Note: The table shows the coefficients in regression (3), based on the dataset constructed from JARA, O*NET, and the US Census/ACS. Observations are 4-digit level occupations, and the sample is all occupations that existed throughout 1970 and 2007. ψ^C stands for the Japan robot shock from equation (1) and IPW stands for the occupation-level import penetration measure (in thousand USD) in equation (2). Demographic control variables are the female share, the college-graduate share, the age distribution (shares of age 16-34, 35-49, and 50-64 among workers aged 16-64), and the foreign-born share as of 1990. All time differences, Δ, are taken with a long difference between 1990 and 2007. All regressions are weighted by the employment in the initial year (1990, which is the closest Census year to the initial year that I observe the robot adoption, 1992). Robust standard errors are reported in the parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

Japan robot shock (reduction in the cost of Japanese robots) drives the drop of the labor market outcomes by occupation. Quantitatively, one standard-deviation decrease of the robot cost (annually, 2.8%) implies the fall of occupational wage by 0.2-0.3% in 95% confidence interval. This finding suggests substitutability between robots and workers because when the cost of robots falls in an occupation, the relative demand for robots (resp. labor) increases (resp. decreases) in the same occupation.

In Section A.6 of the Appendix, I compliment the findings in Table 1 by confirming the consistency with the result of Acemoglu and Restrepo (2020). The section also shows that these analyses are robust to a number of sensitivity checks such as measuring robot stocks by quantity, quality adjustment following Khandelwal et al. (2013), and unweighted regression. Although these regressions are informative about the drivers of robot adoption, they do not give an answer to the distributional and aggregate effect of the Japan robot cost shock. To derive such more conclusive statements, I develop and estimate a general equilibrium model.

3 Model

The open-economy dynamic general equilibrium model has three features: (i) occupation-specific substitution of robots for workers, (ii) robot trade in a large-open economy, and (iii) endogenous investment in robots with an adjustment cost. Section 3.1 states the assumptions, agents' optimization problems, and the equilibrium definition. After showing the solution method in Section 3.2, I discuss a key analytical result that shows the occupational wage implication of automation, which underscores the relevance of occupation-specific substitution in Section 3.3.

3.1 Setup

I formalize the model settings, assumptions, and key characterizations. I relegate discussions and comparisons to the literature in Section B.1 of the Appendix. Other standard characterizations of equilibrium conditions are given in Section B.4 of the Appendix.

Environment Time is discrete and has infinite horizon $t = 0, 1, \ldots$. There are N countries and O occupations. To clarify subscripts for countries, I use i, j, and l, where l is a robot-exporting country, i means a robot-importing and non-robot good-exporting country, and j indicates a non-robot good-importing country. There are two types of goods g, a non-robot good $g = G$ and robot $g = R$. Both goods are tradable. The non-robot good G is differentiated by origin countries and can be consumed by households, used as an intermediate good, invested to produce robots, and used as an input for integration, which I will discuss in detail. Robot R is differentiated by origin countries and occupations. There are bilateral and good-specific iceberg trade costs $\tau^g_{ij,t}$ for each $g = G, R$. I use notation Y for the total production, Q for the quantity arrived at the destination. For instance, non-robot good G shipped from i to j in period t satisfies $Y^G_{ij,t} = Q^G_{ij,t}\tau^G_{ij,t}$. There is no intra-country trade cost, thus $\tau^g_{ii,t} = 1$ for all i, g and t.

There are three factors for production of good G: labor by occupation L_o, robot capital by occupation K^R_o, and non-robot capital K. The stock of non-robot capital is exogenously given at any period for each country. There is no international movement of factors. Note that non-robot capital is not occupational. While producers rent non-robot capital from the rental market, they accumulate and own robot capital. All good and factor markets are perfectly competitive.

The government in each country exogenously sets the robot tax. Buyers of robot $Q^R_{li,o,t}$ have to pay ad-valorem robot tax $u_{li,t}$ on top of producer price $p^R_{li,o,t}$ to buy from l. The tax revenue is uniformly rebated to destination country i's workers.

Workers Workers solve a dynamic discrete choice problem to select an occupation (Traiberman, 2019; Humlum, 2019). I follow the discrete sector choice problems in Dix-Carneiro (2014) and Caliendo et al. (2019) in that workers choose the occupations that maximize the lifetime utility based on switching costs and the draw of idiosyncratic shocks. The problem has a closed form solution when the idiosyncratic shocks follow a suitable extreme value distribution (McFadden, 1973).[10] In Section B.2 of the Appendix, I formally define the problem and show that the worker's problem can be characterized by, for each country i and period t, the transition probability $\mu_{i,oo',t}$ from occupation o in period t to occupation o' in period $t+1$, and the exponential expected value $V_{i,o,t}$ for occupation o that satisfy

$$\mu_{i,oo',t} = \frac{\left(\left(1-\chi_{i,oo',t}\right)\left(V_{i,o',t+1}\right)^{\frac{1}{1+\iota}}\right)^{\phi}}{\sum_{o''}\left(\left(1-\chi_{i,oo'',t}\right)\left(V_{i,o'',t+1}\right)^{\frac{1}{1+\iota}}\right)^{\phi}}, \tag{4}$$

$$V_{i,o,t} = \widetilde{\Gamma} C_{i,o,t}\left[\sum_{o'}\left(\left(1-\chi_{i,oo',t}\right)\left(V_{i,o',t+1}\right)^{\frac{1}{1+\iota}}\right)^{\phi}\right]^{\frac{1}{\phi}}, \tag{5}$$

respectively, where $C_{i,o,t+1}$ is the real consumption, $\chi_{i,oo',t}$ is an ad-valorem switching cost from occupation o to o', ϕ is the occupation-switch elasticity, $\widetilde{\Gamma} \equiv \Gamma\left(1-1/\phi\right)$ is a constant that depends on the Gamma function. For each i and t, employment level satisfies the

[10]One of the differences from these past studies is that I characterize the switching cost by an ad-valorem term, which makes the log-linearization simpler when solving the model.

law of motion

$$L_{i,o,t+1} = \sum_{o'} \mu_{i,o'o,t} L_{i,o',t}, \tag{6}$$

with the total employment satisfying an adding-up constraint

$$\sum_{o} L_{i,o,t} = \overline{L}_{i,t}. \tag{7}$$

Production Function I describe a production function in country i in period t. For each good g, there is a given mass of producers. Non-robot good-G producers produce by aggregating the tasks performed by either labor or robots within a given occupation $T^O_{i,o,t}$, intermediate goods $M_{i,t}$, and non-robot capital $K_{i,t}$ by

$$Y^G_{i,t} = A^G_{i,t} \left[\sum_{o} \left(b_{i,o,t}\right)^{\frac{1}{\beta}} \left(T^O_{i,o,t}\right)^{\frac{\beta-1}{\beta}} \right]^{\frac{\beta}{\beta-1}\alpha_L} \left(M_{i,t}\right)^{\alpha_M} \left(K_{i,t}\right)^{1-\alpha_L-\alpha_M}, \tag{8}$$

where $Y^G_{i,t}$ is the production quantity, $A^G_{i,t}$ is a Hicks-neutral total factor productivity (TFP) shock, $b_{i,o,t}$ is the cost share parameter of occupation o, β is the elasticity of substitution between occupations from the production side, and $\alpha_{i,L}$, $\alpha_{i,M}$, and $1-\alpha_{i,L}-\alpha_{i,M}$ are Cobb-Douglas weights on occupations, intermediate goods, and non-robot capital, respectively. Parameters satisfy $b_{i,o,t} > 0$ for all i, o, and t, $\sum_o b_{i,o,t} = 1$, $\beta > 0$, and $\alpha_{i,L}, \alpha_{i,M}, 1-\alpha_{i,L}-\alpha_{i,M} > 0$. For simplification, I assume that robots R for occupation o are produced by investing non-robot goods $I^R_{i,o,t}$ with productivity $A^R_{i,o,t}$:[11]

$$Y^R_{i,o,t} = A^R_{i,o,t} I^R_{i,o,t}. \tag{9}$$

[11]The assumption simplifies the solution of the model because occupations, intermediate goods, and non-robot capital are only used to produce non-robot goods. Furthermore, I can simply use the estimates measured at the unit of output dollar values when taking the budget constraint of the model to the data in log-linearized solution. To conduct the estimation and counterfactual exercises without this simplification, one would need to observe the cost shares of intermediate goods and non-robot capital for robot producers.

Note that the increase in the TFP term $A^R_{i,o,t}$ drives a reduction in the robot prices. To perform each occupation o, producers hire labor $L_{i,o,t}$ and robot capital $K^R_{i,o,t}$

$$T^O_{i,o,t} = \left[(1-a_{o,t})^{\frac{1}{\theta_o}} (L_{i,o,t})^{\frac{\theta_o-1}{\theta_o}} + (a_{o,t})^{\frac{1}{\theta_o}} \left(K^R_{i,o,t} \right)^{\frac{\theta_o-1}{\theta_o}} \right]^{\frac{\theta_o}{\theta_o-1}}, \tag{10}$$

where $\theta_o > 0$ is the elasticity of substitution between robots and labor within occupation o, and $a_{o,t}$ is the cost share of robot capital in tasks performed by occupation o. In the following sections, I use the shift of $a_{o,t}$ as a source of automation. I will discuss real-world examples and the relationship to the models in the literature in Section B.1 The intermediate goods are aggregated by

$$M_{i,t} = \left[\sum_l (M_{li,t})^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{11}$$

where ε the elasticity of substitution. Since intermediate goods are traded across countries and aggregated by equation (11), elasticity parameter ε serves as the trade elasticity. Given the iceberg trade cost $\tau^G_{ij,t}$, the bilateral price of good G that country j pays to i is $p^G_{ij,t} = p^G_{i,t}\tau^G_{ij,t}$.

Discussion–Production Function and Automation It is worth mentioning the relationship between production functions (8) and (10) and the way automation is treated in the literature. A common approach to modeling robots in the literature, called the task-based approach, constructs the production function (task-based production function) based on the producers' allocation problem of production factors (e.g., robot capital, labor) to a set of tasks (e.g., spot welding). A large body of literature develops the task-based approach to model industrial robots (e.g., Acemoglu and Restrepo, 2018) and more general automation (e.g., Autor et al., 2003; Acemoglu and Autor, 2011). I show this task-based approach implies occupation production function (10) with a suitable distributional assumption of the efficiency of task performance for each production factor. Intuitively, one can regard tasks in the occupation o as simply the aggregate of inputs, robot capital and labor, abstracting away from allocating robots and workers to each exact task. More precisely, in Lemma B.1 in Section B, I show that the solution to the factor allocation problem implies production functions (8) and (10).

The cost-share parameter $a_{o,t}$ of equation (10) has several interpretations. First, since the task-based approach consists of the allocation of factors to tasks, the cost-share parameter $a_{o,t}$ is the share of the space of tasks performed by robot capital as opposed to labor. Since automation improvements consist of expansion in the task space, I will log-linearize the equilibrium respect to $a_{o,t}$ and call the change as the *automation shock*. Second, following Khandelwal (2010), quality of goods can be regarded as a non-pecuniary "attribute whose valuation is agreed upon by all consumers." Therefore, the increase in the cost-share parameter $a_{o,t}$ can also be interpreted as quality upgrading of robots, when combined with a suitable adjustment in the TFP term I discuss in Section 3.3. In particular, equation (10) implies that in the long-run (hence dropping the time subscript) the demand for robot capital is

$$K^R_{i,o} = a_o \left(\frac{c^R_{i,o}}{P^O_{i,o}} \right)^{-\theta_o} T^O_{i,o},$$

where $c^R_{i,o}$ is the long-run marginal cost of robot capital formally defined in Section B.4 of the Appendix, and $P^O_{i,o}$ is the unit cost of performing occupation o. In this equation, a_o is the quality term defined above. For this reason, I use terms (positive) automation shocks and robot quality upgrading interchangeably to describe an exogenous increase in a_o.

The robot-labor substitution parameter θ_o is the key elasticity that affects the changes in real wages given the automation shocks. In Section 3.3, I show that θ_o is negatively related to the real wage changes conditional on the initial cost shares. Hence it is critical to know the value of the parameter to answer the welfare and policy questions. To the best of my knowledge, equation (10) is the most flexible formulation of substitution between robots and labor in the literature. For instance, I show that the unit cost function of Acemoglu and Restrepo (2020) can be obtained by $\theta_o \to 0$ for any o under specific assumptions about other parameters in Lemma B.1 in Appendix B.3. I also show that my model can imply the production structure of Humlum (2019) in Lemma B.2.

Producers' Problem The producers' problem comprises two tiers–static optimization about employment for each occupation and dynamic optimization about robot investment. The static optimization is to choose the employment and capital rental conditional on market prices and current stock of robot capital. Namely, for each i and t, conditional on the o-vector of stock of robot capital $\left\{K^R_{i,o,t}\right\}_o$,

$$\pi_{i,t}\left(\left\{K^R_{i,o,t}\right\}_o\right) \equiv \max_{\left\{L_{i,o,t}\right\}_o,\left\{M_{li,t}\right\}_l,K_{i,t}} p^G_{i,t}Y^G_{i,t} - \sum_o w_{i,o,t}L_{i,o,t} - \sum_l p^G_{li,t}M_{li,t} - r_{i,t}K_{i,t}, \tag{12}$$

where $Y^G_{i,t}$ is given by production function (8).

The dynamic optimization is to choose the quantity of new robots to purchase, or robot investment, given the current stock of robot capital. It requires the following three assumptions. First, for each i, o, and t, robot capital $K^R_{i,o,t}$ accumulates according to

$$K^R_{i,o,t+1} = (1-\delta)\, K^R_{i,o,t} + Q^R_{i,o,t}, \tag{13}$$

where $Q^R_{i,o,t}$ is the amount of new robot investment and δ is the depreciation rate of robots. Second, I assume that the new investment is given by CES aggregation of robot arms from country l, $Q^R_{li,o,t}$, and the non-robot good input of integration $I^{int}_{i,o,t}$ that I discussed in Section 2,

$$Q^R_{i,o,t} = \left[\sum_l \left(Q^R_{li,o,t}\right)^{\frac{\varepsilon^R-1}{\varepsilon^R}}\right]^{\frac{\varepsilon^R}{\varepsilon^R-1}\alpha^R} \left(I^{int}_{i,o,t}\right)^{1-\alpha^R} \tag{14}$$

where l denotes the origin of the newly purchased robots, and α^R is the expenditure share of robot arms in the cost of investment. Note that equation (14) implies that the robots are traded because they are differentiated by origin country l. This follows the formulation of capital good trade in Anderson et al. (2019). Furthermore, combined with equation (13), equation (14) implies that the origin-differentiated investment good is aggregated at first, and then added to the stock of capital. This specification helps reduce the number of capital stock variables and is also used in Engel and Wang (2011). Given the iceberg trade cost $\tau^R_{ij,t}$, the bilateral price of robot R is $p^R_{ij,o,t} = p^R_{i,o,t}\tau^R_{ij,t}$. Write the unit investment price of robots as $P^R_{i,o,t}$. Third, installing robots is costly and requires a per-unit convex adjustment cost $\gamma Q^R_{i,o,t}/K^R_{i,o,t}$ measured in units of robots, where γ governs the size of adjustment cost (Cooper and Haltiwanger, 2006). This reflects the technological difficulty and sluggishness of robot adoption, as reviewed in Autor et al. (2020) and discussed in detail in Section B.1.

Given these settings, a producer of non-robot good G in country i solves the dynamic optimization problem

$$\begin{aligned} \max_{\left\{\left\{Q^R_{li,o,t}\right\}_l, I^{int}_{i,o,t}\right\}_o} & \sum_{t=0}^{\infty}\left(\frac{1}{1+\iota}\right)^t \left[\pi_{i,t}\left(\left\{K^R_{i,o,t}\right\}_o\right)\right. \\ & \left. - \sum_{l,o}\left(p^R_{li,o,t}\left(1+u_{li,t}\right)Q^R_{li,o,t} + P^G_{i,t}I^{int}_{i,o,t} + \gamma P^R_{i,o,t}Q^R_{i,o,t}\frac{Q^R_{i,o,t}}{K^R_{i,o,t}}\right)\right], \end{aligned} \tag{15}$$

subject to accumulation equation (13) and (14), and given $\left\{K^R_{i,o,0}\right\}_o$. Because producers are owned by households, the producer uses the household discount rate ι. Since this is a standard dynamic optimization problem, the standard method of Lagrangian multiplier yields the standard investment Euler equations, which I derive in Appendix B.4.

Equilibrium To close the model, the employment level must satisfy an adding-up constraint (7), and robot and non-robot good markets clear as described in Section B.4 of the Appendix. I first define a temporary equilibrium in each period and then a sequential equilibrium, which implies the steady-state definition. Some of the exact expressions are derived in Appendix B.4 to save a space.

Define the bold symbols as vectors of robot capital $\boldsymbol{K}^R_t \equiv \left\{K^R_{i,o,t}\right\}_{i,o}$, marginal values of robot capital $\boldsymbol{\lambda}^R_t \equiv \left\{\lambda^R_{i,o,t}\right\}_{i,o}$, employment $\boldsymbol{L}_t \equiv \left\{L_{i,o,t}\right\}_{i,o}$, workers' value functions $\boldsymbol{V}_t \equiv \left\{V_{i,o,t}\right\}_{i,o}$, non-robot good prices $\boldsymbol{p}^G_t \equiv \left\{p^G_{i,t}\right\}_i$ robot prices $\boldsymbol{p}^R_t \equiv \left\{p^R_{i,o,t}\right\}_{i,o}$, wages, $\boldsymbol{w}_t \equiv \left\{w_{i,o,t}\right\}_{i,o}$, bilateral non-robot good trade levels $\boldsymbol{Q}^G_t \equiv \left\{Q^G_{ij,t}\right\}_{i,j}$, bilateral non-robot good trade levels $\boldsymbol{Q}^R_t \equiv \left\{Q^R_{ij,o,t}\right\}_{i,j,o}$, and occupation transition shares $\boldsymbol{\mu}_t \equiv \left\{\mu_{i,oo',t}\right\}_{i,oo'}$. I write $\boldsymbol{S}_t \equiv \left\{\boldsymbol{K}_t^R, \boldsymbol{\lambda}_t^R, \boldsymbol{L}_t, \boldsymbol{V}_t\right\}$ as state variables.

Definition 1. In each period t, given state variables $\boldsymbol{S}_t$, a *temporary equilibrium* (TE) $\boldsymbol{x}_t$ is the set of prices $\boldsymbol{p}_t \equiv \left\{\boldsymbol{p}_t^G, \boldsymbol{p}_t^R, \boldsymbol{w}_t\right\}$ and flow quantities $\boldsymbol{Q}_t \equiv \left\{\boldsymbol{Q}_t^G, \boldsymbol{Q}_t^R, \boldsymbol{\mu}_t\right\}$ that satisfy: (i) given $\boldsymbol{p}_t$, workers choose occupation optimally by equation (4), (ii) given $\boldsymbol{p}_t$, producers maximize flow profit (12) and optimize investment (B.15), and (iii) markets clear: Labor adds up (7), and goods market clear with trade balances (B.21) and (B.23).

The temporary equilibrium inputs all state variables and outputs other endogenous variables that are determined contemporaneously. The following sequential equilibrium determines all state variables given initial conditions.

Definition 2. Given initial robot capital stocks and employment $\left\{\boldsymbol{K}_0^R, \boldsymbol{L}_0\right\}$, a *sequential equilibrium* (SE) is a sequence of vectors $\boldsymbol{y}_t \equiv \left\{\boldsymbol{x}_t, \boldsymbol{S}_t\right\}_t$ that satisfies the TE conditions and capital accumulation (13), the Euler equation (B.17), employment law of motion (6), value function (5), and the transversality condition: for any j and o,

$$\lim_{t\to\infty} e^{-\iota t}\lambda_{j,o,t}^R K_{j,o,t+1}^R = 0. \tag{16}$$

Finally, I define the steady state as a SE $\boldsymbol{y}$ that does not change over time.

3.2 Solution

I log-linearize around the initial equilibrium in order to solve the model. In particular, I study the effect of shocks on the sequential equilibrium $\boldsymbol{y}_t$. The log-linearization gives a sequence of matrices $\left\{\overline{\boldsymbol{F}}_t\right\}_t$ and a matrix $\overline{\boldsymbol{E}}$ that summarize the first-order effect on sequential equilibrium in transition dynamics and steady state, respectively. The steady state matrix $\overline{\boldsymbol{E}}$ is a key object in estimating the model in Section 4. Section D of the Appendix gives the details of the derivation of these matrices.

In the economy described in Section 3.1, the shocks comprise changes in the economic environment and changes in policy. For instance, consider the increase of the robot task space $a_{o,t}$ in baseline period t_0 by Δ_o percent, or

$$a_{o,t} = \begin{cases} a_{o,t_0} & \text{if } t < t_0 \\ a_{o,t_0} \times (1 + \Delta_o) & \text{if } t \geq t_0 \end{cases}.$$

In this formulation, Δ_o is interpreted as the size of the expansion of the robot task space. I combine all these changes into a column vector $\boldsymbol{\Delta}$. I take the following three steps to solve the model. Write state variables $\boldsymbol{S}_t = \left\{\boldsymbol{K}_t^R, \boldsymbol{\lambda}_t^R, \boldsymbol{L}_t, \boldsymbol{V}_t\right\}$, and use "hat" notation to denote changes from t_0: for any variable z_t, $\widehat{z_t} \equiv \ln(z_t) - \ln(z_{t_0})$.

Step 1. For a given period t, I combine the vector of shocks $\boldsymbol{\Delta}$ and (given) changes in state variables $\widehat{\boldsymbol{S}_t}$ into a (column) vector $\widehat{\boldsymbol{A}_t} = \left\{\boldsymbol{\Delta}, \widehat{\boldsymbol{S}_t}\right\}$. Log-linearizing the TE conditions, I solve for matrices $\overline{\boldsymbol{D}^x}$ and $\overline{\boldsymbol{D}^A}$ such that the log-difference of the TE $\widehat{\boldsymbol{x}_t}$ satisfies

$$\overline{\boldsymbol{D}^x}\widehat{\boldsymbol{x}_t} = \overline{\boldsymbol{D}^A}\widehat{\boldsymbol{A}_t}. \tag{17}$$

In this equation, $\overline{\boldsymbol{D}^x}$ is a substitution matrix and $\overline{\boldsymbol{D}^A}\widehat{\boldsymbol{A}_t}$ is a vector of partial equilibrium shifts in period t. Since the temporary equilibrium vector $\widehat{\boldsymbol{x}_t}$ includes wages $\widehat{\boldsymbol{w}_t}$, equation (17) generalizes the general equilibrium comparative statics formulation in Adao et al. (2019). Note that there exists a block separation of matrix $\overline{\boldsymbol{D}^A} = \left[\overline{\boldsymbol{D}^{A,\Delta}} | \overline{\boldsymbol{D}^{A,S}}\right]$ such that equation (17) can be written as

$$\overline{\boldsymbol{D}^x}\widehat{\boldsymbol{x}_t} - \overline{\boldsymbol{D}^{A,S}}\widehat{\boldsymbol{S}_t} = \overline{\boldsymbol{D}^{A,\Delta}}\boldsymbol{\Delta}. \tag{18}$$

Step 2. Log linearizing laws of motion and Euler equations around the old steady state, I solve for matrices $\overline{\boldsymbol{D}^{y,SS}}$ and $\overline{\boldsymbol{D}^{\Delta,SS}}$ such that $\overline{\boldsymbol{D}^{y,SS}}\widehat{\boldsymbol{y}} = \overline{\boldsymbol{D}^{\Delta,SS}}\boldsymbol{\Delta}$, where superscript SS denotes steady state. Combined with steady state version of equation (18), I have

$$\overline{\boldsymbol{E}^y}\widehat{\boldsymbol{y}} = \overline{\boldsymbol{E}^\Delta}\boldsymbol{\Delta}, \tag{19}$$

where

$$\overline{\boldsymbol{E}^{y}} \equiv \begin{bmatrix} \overline{\boldsymbol{D}^{x}} & -\overline{\boldsymbol{D}^{A,T}} \\ \multicolumn{2}{c}{\overline{\boldsymbol{D}^{y,SS}}} \end{bmatrix}, \text{ and } \overline{\boldsymbol{E}^{\Delta}} \equiv \begin{bmatrix} \overline{\boldsymbol{D}^{A,\Delta}} \\ \overline{\boldsymbol{D}^{\Delta,SS}} \end{bmatrix},$$

which implies the first-order steady state matrix $\overline{\boldsymbol{E}}$ that satisfies $\widehat{\boldsymbol{y}} = \overline{\boldsymbol{E}}\boldsymbol{\Delta}$.

Step 3. Log linearizing laws of motion and Euler equations around the new steady state, I solve for matrices $\overline{\boldsymbol{D}_{t+1}^{y,TD}}$ and $\overline{\boldsymbol{D}_{t}^{y,TD}}$ such that $\overline{\boldsymbol{D}_{t+1}^{y,TD}}\check{\boldsymbol{y}}_{t+1} = \overline{\boldsymbol{D}_{t}^{y,TD}}\check{\boldsymbol{y}}_{t}$, where the superscript TD stands for transition dynamics. Log-linearized sequential equilibrium satisfies the following first-order difference equation

$$\overline{\boldsymbol{F}_{t+1}^{y}}\widehat{\boldsymbol{y}_{t+1}} = \overline{\boldsymbol{F}_{t}^{y}}\widehat{\boldsymbol{y}}_{t} + \overline{\boldsymbol{F}_{t+1}^{\Delta}}\boldsymbol{\Delta}. \tag{20}$$

Combined with the transversality condition, there is a matrix representing the first-order transitional dynamics $\overline{\boldsymbol{F}_{t}}$ such that

$$\widehat{\boldsymbol{y}}_{t} = \overline{\boldsymbol{F}_{t}}\boldsymbol{\Delta}. \tag{21}$$

3.3 Real-wage Effect of Automation

What does the occupation production function (10) imply about the effect of automation? This question is directly related to the distributional and aggregate effects of industrial robots. In this section, I show that the effect of automation on occupational real wages depends negatively on substitution elasticity parameters θ_{o} and β conditional on the changes in input and trade shares. The key insight is that the real wages are relative prices of labor to the bundle of factors, and the relative price changes are related to changes in the input shares and trade trade shares via the demand elasticities. These elasticities are among the target parameters of the estimation in Section 4.

I modify notations in equation (10) to express the result in a concise way. Define

$$A_{i,o,t}^{K} \equiv \left(A_{i,t}^{G}\right)^{\frac{\theta-1}{\alpha_{i,L}}} a_{o,t},\ A_{i,o,t}^{L} \equiv \left(A_{i,t}^{G}\right)^{\frac{\theta-1}{\alpha_{i,L}}} \left(1-a_{o,t}\right). \tag{22}$$

Substituting these into production functions (8) and (10), I have

$$Y_{i,t}^{G} = \left[\sum_{o} (b_{i,o,t})^{\frac{1}{\beta}} \left(\widetilde{T}_{i,o,t}^{O}\right)^{\frac{\beta-1}{\beta}}\right]^{\frac{\beta}{\beta-1}\alpha_{i,L}} (M_{i,t})^{\alpha_{i,M}} (K_{i,t})^{1-\alpha_{i,L}-\alpha_{i,M}},$$

where

$$\widetilde{T}_{i,o,t}^{O} = \left[\left(A_{i,o,t}^{L}\right)^{\frac{1}{\theta_o}} (L_{i,o,t})^{\frac{\theta_o-1}{\theta_o}} + \left(A_{i,o,t}^{K}\right)^{\frac{1}{\theta_o}} \left(K_{i,o,t}^{R}\right)^{\frac{\theta_o-1}{\theta_o}}\right]^{\frac{\theta_o}{\theta_o-1}}.$$

Therefore, one can interpret the newly defined terms $A_{i,o,t}^{K}$ and $A_{i,o,t}^{L}$ as the productivity shock on robots and labor, respectively. The following proposition claims that the long-run real-wage implication of the robot productivity change $\widehat{A_{i,o}^{K}}$ can be expressed by changes in input and trade shares and elasticities of substitutions.[12]

Define the good G-producers' labor share within occupation $\widetilde{x}_{i,o,t}^{L}$, occupation cost share $\widetilde{x}_{i,o,t}^{O}$, and trade shares $\widetilde{x}_{ij,t}^{G}$ as

$$\widetilde{x}_{i,o,t}^{L} \equiv \frac{w_{i,o,t} L_{i,o,t}}{P_{i,o,t}^{O} T_{i,o,t}^{O}},\ \widetilde{x}_{i,o,t}^{O} \equiv \frac{P_{i,o,t}^{O} T_{i,o,t}^{O}}{P_{i,t}^{O} T_{i,t}^{O}},\ \widetilde{x}_{ij,t}^{G} \equiv \frac{p_{i,t}^{G} Q_{ij,t}^{G}}{P_{i,t}^{G} Q_{i,t}^{G}}, \tag{23}$$

where $P_{i,o,t}^{O}$, $P_{i,t}^{O}$, and $P_{i,t}^{G}$ are the price indices of occupation o, aggregated task $T_{i,t}^{O} \equiv \left[\sum_o (b_{i,o,t})^{\frac{1}{\beta}} \left(T_{i,o,t}^{O}\right)^{\frac{\beta-1}{\beta}}\right]^{\frac{\beta}{\beta-1}}$, and non-robot goods consumed in country i, respectively. In Appendix B.4, I discuss how one can compute steady-state labor share $\widetilde{x}_{i,o}^{L}$. Given these, the following proposition characterizes the real-wage changes in the steady state.

Proposition 1. *Suppose robot productivity grows $\widehat{A_{i,o}^{K}} > 0$. For each country i and occupation o,*

$$\widehat{\left(\frac{w_{i,o}}{P_i^G}\right)} = \frac{1}{1-\alpha_{i,M}} \left(\frac{\widehat{\widetilde{x}_{i,o}^{L}}}{1-\theta_o} + \frac{\widehat{\widetilde{x}_{i,o}^{O}}}{1-\beta} + \frac{\widehat{\widetilde{x}_{ii}^{G}}}{1-\varepsilon}\right). \tag{24}$$

[12]By equation (22), robot productivity change $\widehat{A_{i,o,t}^{K}}$ and automation shock $\widehat{a_{o,t}}$ satisfy that $\widehat{A_{i,o,t}^{K}} = \frac{\theta-1}{\alpha_{i,L}}\widehat{A_{i,t}^{G}} + \widehat{a_{o,t}}$. Namely, robot productivity change is the sum of total factor productivity change caused by robotics and the automation shock. I choose to use the automation shock in my main specification in equations (8) and (10) since it has a tight connection to the task-based approach, a common approach in the automation literature (e.g., Acemoglu and Restrepo, 2020), as I discussed in Section 3.1.

Proof. See Appendix B.5. □

Proposition 1 clarifies how the elasticity parameters and change of shares of input and trade affect real wages at the occupation level. Among the elasticity parameters, one can observe that if $\theta_o > 1$, then (i) the larger the fall of the labor share within occupation $\widehat{\widetilde{x}^L_{i,o}}$, the larger the real wage gains, and (ii) pattern (i) is stronger if θ_o is small and close to 1. Therefore, conditional on other terms, the steady state changes of occupational real wages depend on the elasticity of substitution between robots and labor θ_o.

The intuition of Proposition 1 comes from the series of revealed cost reductions, $\widehat{\widetilde{x}^L_{i,o}}$, $\widehat{\widetilde{x}^O_{i,o}}$, and $\widehat{\widetilde{x}^G_{ii}}$. The first term reveals the robot cost reduction relative to labor cost. If $\theta_o > 1$, then the reduction in the price index or cost savings dominates the drop in nominal wage, increasing the real wage. Similar intuition holds for the second and third terms. The second term reveals the relative occupation cost reduction, whereas the last term reveals the relative sectoral cost reduction.

Proposition 1 also extends the welfare sufficient statistic in the trade literature. In particular, Arkolakis et al. (2012, ACR) showed that under a large class of trade models, the welfare effect of the reduction in trade costs can be summarized into the well-known ACR formula, or log-difference of the trade shares times the negative of trade elasticity. In fact, by dropping the robots and non-robot capital and aggregating occupations into one, the model reduces to:

$$\widehat{\left(\frac{w_i}{P^G_i}\right)} = \frac{1}{1-\alpha_{i,M}} \frac{1}{1-\varepsilon} \widehat{\widetilde{x}^G_{ii}},$$

which is a modified ACR formula with intermediate goods as in Caliendo and Parro (2015) and Ossa (2015).

Although Proposition 1 concisely represents the effect of the automation shock on real wages, it is not straightforward to take this equation directly to the data. The reason is that the observed data contain not only the automation shock but also other shocks such as trade shocks that significantly affect $\widehat{\widetilde{x}^G_{ii}}$. To study robotization's role in the occupational wage effect, I estimate the model and back out the automation shock in the following.

4 Estimation

Using the occupation-level Japan robot shock described in Section 2 and the solution to the general equilibrium model in Section 3, I develop an estimation method based on the generalized method of moments (GMM), in particular, the model-implied optimal instrumental variable (MOIV, Adao et al., 2019). To do so, Section 4.1 sets the stage for the structural estimation by giving the implementation detail. I formalize the MOIV estimator in Section 4.2, which gives the structural estimates in Section 4.3.

4.1 Bringing Model to Data

To simplify the notation and tailor to my empirical application, I stick to country labels $i = 1$ as the US (USA), $i = 2$ as Japan (JPN), $i = 3$ as the Rest of the World (ROW). Following my data, I interpret country $i = 1$ as the country of interest in terms of labor market outcome variables, country $i = 2$ as the source country of automation shocks by robots, and country $i = 3$ as the (set of) countries in which the measurement of robots proxies the technological changes in country 2.

In the estimation, I allow heterogeneity across occupations of the within-occupation EoS between robots and labor. To do so, I define the occupation groups as follows. I first separate occupations into three broad occupation groups, Abstract, Service, Routine following Acemoglu and Autor (2011). Routines occupations include production, transportation and material moving, sales, clerical, and administrative support. Abstract occupations are professional, managerial and technical occupations; service occupations are protective service, food preparation, cleaning, personal care and personal services. Given the trend that production and transportation/material moving occupations introduced robots over the sample period, I further divide routine occupations into three sub-categories, Production (e.g., welders), Transportation (indicating transportation and material-moving, e.g., hand laborer), and Others (e.g., repairer), where Others include sales, clerical, and administrative support. As a result, I obtain five occupation groups, for each of which I assume a constant EoS between robots and labor.[13] With each occupation group (or mapping from 4-digit occupation o to the group) represented by g, notation θ_g denotes the robot-labor EoS for occupation group. In Section C.2 of the Appendix, I examine a different choice of occupation grouping.

The vector of structural parameters are denoted as $\mathbf{\Theta}$ and its dimension is $d \equiv \dim(\mathbf{\Theta})$. To formally define $\mathbf{\Theta}$, I fix a subset of parameters of the model at conventional values. In particular, I assume that the annual discount rate is $\iota = 0.05$ and the robot depreciation rate is 10 percent following Graetz and Michaels (2018).[14] I take trade elasticity of $\varepsilon = 4$ from the large literature of trade elasticity estimation (e.g., Simonovska and Waugh, 2014), and $\varepsilon^R = 1.2$ derived from applying the estimation method developed by Caliendo and Parro (2015) to the robot trade data, discussed in detail in Appendix C.1. Following Leigh and Kraft (2018), I assume $\alpha^R = 2/3$. With this parametrization, structural parameters to be estimated are $\mathbf{\Theta} \equiv \left\{\theta_g, \beta, \gamma, \phi\right\}$.

4.2 Estimation Method

I observe changes in endogenous variables, US occupational wages $\widehat{\boldsymbol{w}_1}$, US employment $\widehat{\boldsymbol{L}_1}$, robot shipment from Japan to the US $\widehat{\boldsymbol{Q}_{21}^R}$, and the corresponding unit values $\widehat{\boldsymbol{p}_{21}^R}$ between 1992 and 2007, as well as the initial equilibrium $\boldsymbol{y}_{t_0}$. I approximate the 15-year changes as the steady-state changes. To simplify, I focus on the expansion of robot task space $\widehat{a_o}$ and the efficiency gain to produce robots in Japan $\widehat{A_{2,o}^R}$ as the source of the occupational shocks in this section. Note that the robot production function (9) implies that $\widehat{A_{2,o}^R}$ is negative of the cost shock to produce robots in Japan, I measure the robot efficiency gain by

$$\widehat{A_{2,o}^R} = -\psi_{o,t_1}^J, \tag{25}$$

[13]In terms of OCC2010 codes in the US Census, Routine production occupations are ones in [7700, 8965], Routine transportation are in [9000, 9750], Routine others are in [4700, 6130], Service are in [3700, 4650], and Abstract are in [10, 3540].

[14]For example, see King and Rebelo (1999) for the source of the conventional value of ι who matches the discount rate to the average real return on capital. For ε, see Simonovska and Waugh (2014) or Caliendo and Parro (2015).

where, again, ψ^J_{o,t_1} is the Japan robot shock defined in equation (1) and measured in my dataset.

To discuss the identification challenge and the countermeasure, I decompose the automation shock $\widehat{a_o}$ into observed component $\widehat{a_o^{\text{obs}}}$ and unobserved error component $\widehat{a_o^{\text{err}}}$ such that $\widehat{a_o} = \widehat{a_o^{\text{obs}}} + \widehat{a_o^{\text{err}}}$ for all o. The component $\widehat{a_o^{\text{obs}}}$ is observed conditional on parameter θ_o–namely, it satisfies the steady-state change of relative demand of robots and labor implied by the Euler equation

$$\left(\widehat{\frac{p^R_{i,o}K^R_{i,o}}{w_{i,o}L_{i,o}}}\right) = (1-\theta_o)\left(\widehat{\frac{p^R_{i,o}}{w_{i,o}}}\right) + \frac{\widehat{a_o^{\text{obs}}}}{1-a_{o,t_0}}. \tag{26}$$

Equation (26) highlights the issues in identifying θ. First, the observed relative price change $\left(\widehat{p^R_{i,o}/w_{i,o}}\right)$ does not identify θ_g because $\left(\widehat{p^R_{i,o}/w_{i,o}}\right)$ is endogenous and is correlated with the residual term $\widehat{a_o^{\text{obs}}}/\left(1-a_{o,t_0}\right)$ that represents the task-space expansion of robots (Karabarbounis and Neiman, 2014; Hubmer, 2018). Second, the Japan robot shock ψ^J_{o,t_1} also does not also work as an instrumental variable (IV) in the linear regression model of (26) because of a potential correlation between ψ^J_{o,t_1} and observed task-space expansion shock $\widehat{a_o^{\text{obs}}}$.

To overcome these identification issues, I employ a method based on the full GE model below. Conditional on $\widehat{a_o^{\text{obs}}}$, the error component $\widehat{a_o^{\text{err}}}$ can be inferred from each observed endogenous variable. Take the changes in occupational wages $\widehat{\boldsymbol{w}_1}$ for example. The steady-state solution matrix $\overline{\boldsymbol{E}}$ implies that there is a $O \times O$ sub-matrices $\overline{\boldsymbol{E}}_{w_1,\boldsymbol{a}}$ and $\overline{\boldsymbol{E}}_{w_1,\boldsymbol{A}^R_2}$ such that[15]

$$\widehat{\boldsymbol{w}} = \overline{\boldsymbol{E}}_{w_1,\boldsymbol{a}}\widehat{\boldsymbol{a}} + \overline{\boldsymbol{E}}_{w_1,\boldsymbol{A}^R_2}\widehat{\boldsymbol{A}^R_2}. \tag{27}$$

Since $\widehat{\boldsymbol{a}} = \widehat{\boldsymbol{a}^{\text{obs}}} + \widehat{\boldsymbol{a}^{\text{err}}}$, I have

$$\boldsymbol{\nu}_{\boldsymbol{w}} = \widehat{\boldsymbol{w}} - \overline{\boldsymbol{E}}_{w_1,\boldsymbol{a}}\widehat{\boldsymbol{a}^{\text{obs}}} - \overline{\boldsymbol{E}}_{w_1,\boldsymbol{A}^R_2}\widehat{\boldsymbol{A}^R_2},$$

[15] I use the steady-state matrix $\overline{\boldsymbol{E}}$ instead of the transitional dynamics matrix $\overline{\boldsymbol{F}_t}$ for a computational reason, which is described in Section C.2 in detail.

where $\boldsymbol{\nu}_{w} \equiv \overline{\boldsymbol{E}}_{w_1,\boldsymbol{a}} \widehat{\boldsymbol{a}^{\text{err}}}$ is the O-vector structural residual generated from the linear combination of the unobserved component of the automation shocks. Note that the structural residual depends on the structural parameters $\boldsymbol{\Theta}$. To clarify this, I occasionally write the structural residual as $\boldsymbol{\nu}_{w} = \boldsymbol{\nu}_{w}(\boldsymbol{\Theta})$. For other endogenous variables $\left(\widehat{\boldsymbol{L}_1}, \widehat{\boldsymbol{p}_{21}^R}, \widehat{\boldsymbol{Q}_{21}^R}\right)$, I repeat the same process and obtain corresponding structural errors $\left(\boldsymbol{\nu}_{L}, \boldsymbol{\nu}_{p^R}, \boldsymbol{\nu}_{Q^R}\right)$. Then I stack these vectors into an $O \times 4$ matrix $\boldsymbol{\nu} \equiv \left[\boldsymbol{\nu}_{w}, \boldsymbol{\nu}_{L}, \boldsymbol{\nu}_{p^R}, \boldsymbol{\nu}_{Q^R}\right]$, and from its o-th row define 4×1 vector as $\nu_o = \left[\nu_{w,o}, \nu_{L,o}, \nu_{p^R,o}, \nu_{Q^R,o}\right]^\top$. Given these structural residuals and the Japan robot shock $\boldsymbol{\psi}_{t_1}^J \equiv \left\{\psi_{o,t_1}^J\right\}_o$, I assume the following moment condition.

Assumption 1. *(Moment Condition)*

$$\mathbb{E}\left[\nu_o | \boldsymbol{\psi}_{t_1}^J\right] = 0. \tag{28}$$

Assumption 1 puts restriction on structural residual $\boldsymbol{\nu}$ in that it should not be predicted by the Japan robot shock. Note that it allows that the automation shock $\widehat{a_o}$ may correlate with the robot efficiency change $\widehat{\boldsymbol{A}_2^R}$ which is likely as I discuss in Appendix A.2 in detail. Instead, the structural residual ν_o purges out all the predictions of the impacts of shocks $\widehat{\boldsymbol{a}}$ and $\widehat{\boldsymbol{A}_2^R}$ on endogenous variables, and I place the assumption that the remaining variation should not be predicted by the Japan robot shock from the data.

Under what circumstances does Assumption 1 break? Note that the answer to this question is not the correlation of the structural residuals with other shocks such as trade shocks because I have confirmed controlling for the trade shock does not qualitatively alter the reduced-form findings in Section 2.3. Instead, a candidate answer is a directed technological change, in which the occupational labor demand drives the changes in the cost of robots. Specifically, suppose a positive labor demand shock in an occupation o induces the research and development of robots in occupation o and drives cost down in the long run. This mechanism is not incorporated in my model where robots are produced with production function (9) with exogenous technological change. Therefore, the structural residual ν_o cannot remove this effect and is negatively correlated with ψ_{o,t_1}^J.

In this sense, the positive impact of Japan robot costs found in Section 2.3 still prevails qualitatively even under the directed technological change.[16]

Assumption 1 implies that, for any d-dimensional functions $\boldsymbol{H} \equiv \{H_o\}_o$, $\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nu_o\right] = 0$. The GMM estimator based on $\boldsymbol{H}$ is

$$\boldsymbol{\Theta}_{\boldsymbol{H}} \equiv \arg\min_{\boldsymbol{\Theta}} \sum_{o=1}^{O} \left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nu_o\left(\boldsymbol{\Theta}\right)\right]^\top \left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nu_o\left(\boldsymbol{\Theta}\right)\right], \tag{29}$$

which is consistent under the moment condition (28) if $\boldsymbol{H}$ satisfies the rank conditions in Newey and McFadden (1994). The exact specification of $\boldsymbol{H}$ determines the optimality, or the minimal variance, of estimator (29). To specify $\boldsymbol{H}$, I apply the approach that achieves the asymptotic optimality developed in Chamberlain (1987). Formally, define the instrumental variable Z_o as follows:

$$Z_o \equiv H_o^*\left(\boldsymbol{\psi}_{t_1}^J\right) \equiv \mathbb{E}\left[\nabla_{\boldsymbol{\Theta}}\nu_o\left(\boldsymbol{\Theta}\right)|\boldsymbol{\psi}_{t_1}^J\right]\mathbb{E}\left[\nu_o\left(\boldsymbol{\Theta}\right)\left(\nu_o\left(\boldsymbol{\Theta}\right)\right)^\top|\boldsymbol{\psi}_{t_1}^J\right]^{-1}, \tag{30}$$

and assume the regularity conditions B.1 in Section B.6 of the Appendix.

Proposition 2. *Under Assumptions 1 and B.1, $\boldsymbol{\Theta}_{\boldsymbol{H}^*}$ is asymptotically normal with the minimum variance among the asymptotic variances of the class of estimators in equation* (29)*.*

Proof. See Section B.6. □

To understand the optimality of the IV in equation (30), note that it has two components. The first term is the conditional expected gradient vector $\mathbb{E}\left[\nabla_{\boldsymbol{\Theta}}\nu_o\left(\boldsymbol{\Theta}\right)|\boldsymbol{\psi}_{t_1}^J\right]$, which takes the gradient with respect to the structural parameter vector. Thus, it assigns large weight to occupation that changes the the predicted outcome variable sensitively to the parameters. The second term is the conditional inverse expected variance matrix $\mathbb{E}\left[\nu_o\left(\boldsymbol{\Theta}\right)\left(\nu_o\left(\boldsymbol{\Theta}\right)\right)^\top|\boldsymbol{\psi}_{t_1}^J\right]^{-1}$, which put large weight to occupation that has small variance of the structural residuals.

[16] With increasing returns for robot producers, I could model that the robot demand increase drives cost drop. Estimating such a model requires detailed data on robot producers and is left for future research.

Substituting equation (30) to the general GMM estimator (29), I have an estimator $\mathbf{\Theta}_{\boldsymbol{H}^*} = \arg\min_{\mathbf{\Theta}} \left[\sum_o Z_o \nu_o\left(\mathbf{\Theta}\right)\right]^\top \left[\sum_o Z_o \nu_o\left(\mathbf{\Theta}\right)\right]$. Since Z_o depends on unknown parameters $\mathbf{\Theta}$, I implement the estimation by the two-step feasible method, or the model-implied optimal IV (Adao et al., 2019). I first estimate the first-step estimate $\mathbf{\Theta}_1$ from arbitrary initial values $\mathbf{\Theta}_0$. Since the IV is a function of the Japan robot shock $\boldsymbol{\psi}^J_{t_1}$, $\mathbf{\Theta}_1$ is consistent by Assumption 1. However, it is not optimal. To achieve the optimality, in the second step, I obtain the optimal IV using the consistent estimator $\mathbf{\Theta}_1$. To summarize the discussion so far, define IVs $Z_{o,n}$ where $n = 0, 1$ as follows:

$$Z_{o,n} \equiv H_{o,n}\left(\boldsymbol{\psi}^J_{t_1}\right) = \mathbb{E}\left[\nabla_{\mathbf{\Theta}} \nu_o\left(\mathbf{\Theta}_n\right) | \boldsymbol{\psi}^J_{t_1}\right] \mathbb{E}\left[\nu_o\left(\mathbf{\Theta}_n\right)\left(\nu_o\left(\mathbf{\Theta}_n\right)\right)^\top | \boldsymbol{\psi}^J_{t_1}\right]^{-1}. \tag{31}$$

Then I have the following result.

Proposition 3. *Under Assumptions 1 and B.1, the estimator $\mathbf{\Theta}_2$ obtained in the following procedure is consistent, asymptotically normal, and optimal:*

Step 1: With a guess $\mathbf{\Theta}_0$, estimate $\mathbf{\Theta}_1 = \mathbf{\Theta}_{\boldsymbol{H}_0}$ using $Z_{o,0}$ defined in equation (31).

Step 2: With $\mathbf{\Theta}_1$, estimate $\mathbf{\Theta}_2$ by $\mathbf{\Theta}_2 = \mathbf{\Theta}_{\boldsymbol{H}_1}$ using $Z_{o,1}$ defined in equation (31).

Proof. See Section B.7. □

4.3 Estimation Result

To apply Proposition 3, I need to measure the initial equilibrium $\boldsymbol{y}_{t_0}$, which is an input to the solution matrix $\overline{\boldsymbol{E}}$ in equation (19). I take these data from JARA, IFR, IPUMS USA and CPS, BACI, and World Input-Output Data (WIOD). The measurement of labor market outcomes is standard and relegated to Section A.8 of the Appendix. I set the initial period robot tax to be zero in all countries.

Table 2a gives the estimates of the structural parameters. Panel 2a shows the estimation result when I restrict the EoS between robots and labor to be constant across occupation groups. The estimate of the within-occupation EoS between robots and labor, θ_g, implies that robots and labor are substitutes within an occupation, and rejects

Table 2: Parameter Estimates

(a) All Parameters

Parameter	$\theta_g = \theta$	Free θ_g
θ	2.96 (0.17)	[Table 2b]
β	0.71 (0.23)	0.73 (0.31)
γ	0.30 (0.11)	0.30 (0.14)
ϕ	0.81 (0.26)	0.81 (0.30)

(b) Heterogeneous EoS θ_g

Routine	Production	4.04 (0.24)
	Transportation	4.29 (0.28)
	Others	1.27 (0.53)
Service		1.35 (0.48)
Abstract		0.80 (0.60)

Note: The estimates of the structural parameters based on the estimator in Proposition 3. Standard errors are in parentheses. In the left panel, parameter θ is the within-occupation elasticity of substitution between robots and labor. Parameter β is the elasticity of substitution between occupations. Parameter γ is the capital adjustment cost. Parameter ϕ is the occupation switch elasticity. The column "$\theta_g = \theta$" shows the result with the restriction that θ_o is constant across occupation groups. The column "Free θ_g" shows the result with θ_g allowed to be heterogeneous across five occupation groups. In the right panel, estimates for parameters θ_g with heterogeneity are shown. Transportation indicates "Transportation and Material Moving" occupations in the Census 4-digit occupation codes (OCC2010 from 9000 to 9750). See the main text for other details.

the Cobb-Douglas case $\theta_g = 1$ at a conventional significance level. The point estimate of the EoS between occupations, β, is 0.71, or occupation groups are complementary. The one-standard error bracket covers Humlum's (2019) central estimate of 0.49. The adjustment cost parameter γ is close to the estimate of Cooper and Haltiwanger (2006) when they restrict the model with only quadratic adjustment costs, like in my model. The one-standard error range of occupational dynamic labor supply elasticity ϕ is estimated to be $[0.55, 1.07]$, which contains an estimate of 0.6 in the dynamic occupation choice model in Traiberman (2019) in the case without the specific human capital accumulation.

Panel 2b shows the estimation result when I allow the heterogeneity across occupation groups. The other structural estimates, (β, γ, ϕ), do not change qualitatively. Table 2b shows the estimates of the within-occupation EoS between robots and labor, θ_g. I find that the EoS for routine production occupations and routine transportation occupations is around 4, while those for other occupation groups (other occupations in routine group, service, and abstract occupations) are not significantly different from 1, the case of Cobb-Douglas. The estimates for routine production and transportation indicate the susceptibility of workers in these occupations to accumulated robot capital.

What is the source of identification of these large and heterogeneous EoS between robots and labor identified? As in the literature of estimating the capital-labor substitution elasticity, the positive correlation between the robot price and the wage (labor market outcome) suggests robots and labor are substitutes, or large θ_g. Intuitively, if θ_g is large, then given a percentage decrease in the cost of robots, the steady-state relative robot (resp. labor) demand responds strongly in the positive (resp. negative) direction. Reducing the occupation wage through the labor demand equation, the large robot-labor EoS yields a positive correlation between the robot price trend and the wage trend, as found in Figure A.8. Appendix A.7 further discusses this source of identification of the EoS, the correlation between the Japan robot shock and the US wage change within each occupation group.

4.4 Measuring Shocks and Model Fit

To examine the plausibility of these parameter estimates, I simulate the model and check the model's fit. The simulation process comprises two steps. First, I back out the observed shocks from the estimated model for each year between 1992 and 2007. Namely, with the point estimates in Table 2b, I obtain the efficiency increase of Japanese robots $\widehat{A^R_{2,o,t}}$ using (25), equation the observed automation shock $\widehat{a^{\text{obs}}_{o,t}}$ using (26), and the US occupation demand shock $\widehat{b_{1,o,t}}$. To back out the efficiency shock of robots in the other countries, I assume that $\widehat{A^R_{i,o,t}} = \widehat{A^R_{i,t}}$ for $i = 1, 3$. Then by the robot trade prices $p^R_{ij,t}$ from BACI, I fit fixed effect regression $\Delta \ln \left(p^R_{ij,t}\right) = \widetilde{\psi}^D_{j,t} + \widetilde{\psi}^C_{i,t} + \widetilde{e}_{ij,t}$, and use $\widehat{A^R_{i,t}} = -\widetilde{\psi}^C_{i,t_1}$. The idea to back out the negative efficiency shock $\widetilde{\psi}^C_{i,t_1}$ is similar to the fixed-effect regression in Section 2, but without the occupational variation. Second, applying the backed-out shocks $\widehat{A^R_{i,o,t}}$, $\widehat{a^{\text{obs}}_{o,t}}$, and $\widehat{b_{1,o,t}}$ to the first-order solution of the GE in equation (21), I obtain the prediction of changes of endogenous variables to these shocks to the first-order. Finally, applying the predicted changes to the initial data in $t_0 = 1992$, I obtain the predicted level of endogenous variables.

I run the linear regression model (3) to examine the fit of the model and the role of the automation shock in estimating the robot-labor EoS.[17] First, I hit all the shocks generated in the above paragraph. In this case, the prediction is consistent with the moment condition (28) and thus I predict that the linear regression coefficient α_1 of equation (3) is close to the one in Table 1. I term the predicted wages in this way as the "targeted wage." Second, I hit all the shocks but the automation shock. In this case, the same moment condition is violated since the structural residual fails to incorporate the automation shock. Therefore, this exercise reveals how important taking into account the observed automation shock is in estimation. Namely, the larger the discrepancy of the regression coefficient of equation (3) between the data and this second simulation, the more severe the bias caused by the automation shock. I call the predicted wages in this way as the "untargeted wage."

Table 3 shows the result of these exercises. By comparing the first column that repeats column (3) of Table 1 and the second column based on the targeted wage, I confirm that the targeted moments match well as expected. The third column is the result based on the untargeted wage and shows stronger positive correlation between the simulated wage and the Japan robot shock. This is due to negative correlation between the Japan robot shock ψ_o^J and the observed automation shock $\widehat{a_o^{\text{obs}}}$, which is consistent with that robotic innovations that save cost (thus decreases ψ_o^J) and that upgrade quality (thus increases $\widehat{a_o^{\text{obs}}}$) are likely to happen at the same time, as exemplified in Appendix A.2.4.

More specifically, the real and simulated data with the targeted wage contain the negative bias due to this negative correlation. Since the untargeted wage is free from this bias, the linear regression coefficient α_1 of equation (3) is higher than the one obtained from the real data. In other words, if I have wrongfully assumed that the economy did not experience the automation shock and believed the regression coefficient in Table 1 is bias-free, I would have estimated higher EoS by ignoring the actual negative correlation between

[17] As another model validation exercise, I predict the stock of robots by occupation and find that the model predict the actual robot accumulation dynamics well, described in detail in Appendix C.3. Appendix C.4 gives further discussion about the Japan robot shock and the backed-out observed automation shocks.

Table 3: Model Fit: Linear Regression with Observed and Simulated Data

VARIABLES	(1) $\Delta \ln(w)$	(2) Targeted $\widehat{w}$	(3) Untargeted $\widehat{w}$
ψ^J	0.0984***	0.0980***	0.126***
	(0.0266)	(0.0077)	(0.0009)
Observations	324	324	324
R-squared	0.394	0.532	0.794

Note: The author's calculation based on the dataset generated by JARA, O*NET, and the US Census. Column (1) is the coefficient of the Japan robot shock ψ^J in the reduced-form regression with IPW. Column (2) takes the US wage change predicted by GE with ψ^J as well as other shocks such as the observed automation shock $\widehat{a^{obs}}$. Column (3) takes the US wage change predicted by GE with shocks including the Japan robot shock, but excluding the observed automation shock. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

ψ_o^J and $\widehat{a_o^{\text{obs}}}$. This thought experiment reveals the importance of taking into account the automation shock in estimating the EoS between robots and labor using the robot cost shock.

5 Counterfactual Exercises

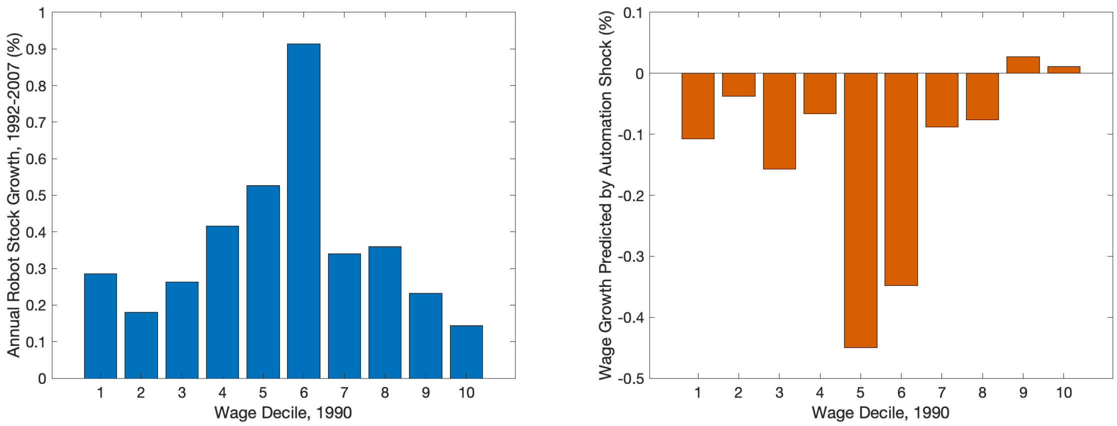
Using the estimated model and backed-out shocks in the previous section, I answer the following questions. The first question is the distributional effects of robots. Autor et al. (2008) argue that the wage inequality measured by the ratio of the wages between the 90th percentile and the 50th percentile (90-50 ratio) steadily increased since 1980.[18] Although my data and model predicts the changes from 1990, can the increased use of industrial robots explain the 90-50 ratio? If so, how much? The second question concerns the policy implication of robot regulation. Due to the fear of automation, policymakers have proposed regulating industrial robots using robot taxes. What would be the effect of taxing on robot purchases?

[18] As Heathcote et al. (2010) argue, a sizable part of the US economic inequality roots in the wage inequality. Furthermore, the polarization is not a unique phenomenon in the US, but found in the other context such as the UK (Goos and Manning, 2007).

Figure 2: Robots, Wage Inequality, and Polarization

(a) Robot growths by Wage Decile

(b) Robots' Effect on Occupational Wages



Notes: The left panel shows the average annual growth rates of the observed robot stock between 1992 and 2007 for every ten deciles of the occupational wage distribution in 1990. The right panel shows the annualized wage growth rates predicted by the backed-out shocks and the estimated model's first-order steady-state solution given in equation (19).

5.1 The Distributional Effects of Robot Adoption

To study the contribution of robots to wage polarization, I begin by showing the pattern of robot accumulations over the occupational wage distribution. Figure 2a shows the average annual growth rates of observed robot stock between 1992 and 2007 for every ten deciles of the occupational wage distribution in 1990. The figure clarifies that the occupations in the middle deciles of the distribution received relatively many robots. Conditional on robot prices, this pattern implies there are relatively large automation shocks on these occupations.

The right panel shows the steady-state annualized predicted wage growths due to the shocks backed out in Section 4.4 and the estimated model with the first-order solution given in equation (21). Consistent with the high growth rate of robot stocks in the middle of the wage distribution and the estimation results that indicate the strong substitutability between robots and labor, I find that the wage effect in the middle deciles of the initial wage distribution is strongly negative. Quantitatively, the 90-50 ratio observed in 1990 and 2007 is, respectively, 1.588 and 1.668. On the other hand, the 90-50 ratio predicted by the initial 1990 data and the first-order solution (21) is 1.597. These findings indicate that

11.7 percent of the observed change in the 90-50 ratio between 1990 and 2007.

These results emerge from aggregating the effects on 4-digit occupational wages and my estimates of the robot-labor EoS. Specifically, relative occupational wages drastically drops in production and transportation (material-moving) occupations. This is a natural consequence of (i) the large quantities robots adopted in these occupations (Figure C.1) and (ii) the high estimates of EoS between robots and labor for these occupations (Table 2b). To confirm these observations, Appendix C.5 describes the wage changes for each of 5 occupation groups and Appendix C.6 performs the robotization exercise in case of low EoS as specified in the literature.

5.2 Robot Tax and Aggregate Income

Next, I consider a counterfactual rise of robot tax as well as the automation shock. In the baseline economy, all countries levied zero robot tax. On the one hand, consider an unexpected, unilateral, and permanent increase in the robot tax by 6% in the US, or the general tax scenario. I also consider the tax on only imported robots by 33.6%, or the import tax scenario, which implies the same amount of tax revenue as in the general tax scenario. Note that the 6% rate of the general tax is more modest than 30% considered in Humlum (2019) for the Danish case, and that the 33.6% import tax would make the tax revenue the same as the 6% general robot tax case, which makes the comparison straightforward between the scenarios. How does these robot tax schemes affect the US real income?[19] In Figure 3a, the solid line tracks the real-income effect of the general robot tax over a 20-year time horizon after the imposition. First, the magnitude of the effect is small because the cost of buying robots and the contribution of robot capital in the aggregate production are small. Second, in the short-run, there is a positive effect while the effect turns negative quickly and continues so in the long-run.

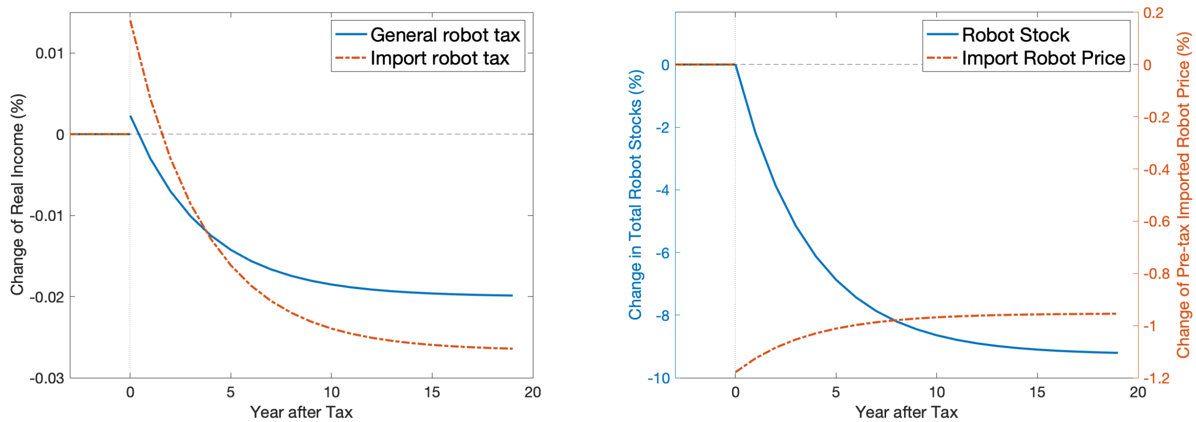
Why is there a short-run positive effect on real income? A country's total income

[19] Section C.7 provides analysis for occupational workers' welfare consequence and concludes that there is no general tax rate that makes all workers better off. There, I numerically show that the 6% general tax would roughly compensate the loss from robotization for production workers.

Figure 3: Effects of the Robot Tax

(a) US Real Income

(b) US Robot Stock and Import Robot Price



Notes: The left panel shows the counterfactual effect on the US real income of the two robot tax scenarios described in the main text over a 20-year time horizon. The right panel shows that of the import robot tax on the US total robot stocks (solid line) and the pre-tax robot price from Japan (dash-dot line) over the same time horizon.

comprises the sum of workers' wages, the non-robot good producer's profit, and tax revenue. Since robots are traded, and the US is a large economy that can affect the robot price produced in other countries, there is a terms-of-trade effect of robot tax in the US. Namely, the robot tax reduces the demand for robots produced in the other country, let the equilibrium robot price go down along the supply curve. This reduction in the robot price contributes to the increase in the firm's profit, raising the real income in the short-run. The short-run positive effect is stronger in the import robot tax scenario because the higher tax rate induces a more substantial drop in the import robot price.

The terms-of-trade manipulation is well-studied in the trade policy literature. This paper offers the upward sloping export supply curve from the general equilibrium, as opposed to the supply curve that is assumed upward sloping (e.g., Broda et al., 2008). Namely, when the demand for robots in a robot exporter country decreases, the resource to produce robots in the exporter country is freed and reallocated to produce the non-robot goods. In my case, the resource is simply the non-robot goods that are input to robot production in equation (9). This increases the supply of non-robot goods in the robot-exporting country, depressing the price of non-robot goods. Again due to robot production function (9), this decrease in the non-robot goods price means the decrease of the cost of producing robots, which in turn reduces the price of robots produced in the exporter country.

Why do I have the different effect on real income in the long-run? The solid line in Figure 3b shows the dynamic impact of the import robot tax on robot stock accumulation. The tax significantly slows the accumulation of robot stocks, and decreases the steady-state stock of robots by 9.7 percent compared to the no-tax case. The smaller quantity of robot stocks reduces the firm profit, which contributes to smaller real income.[20] These results highlight the role of costly robot capital (de-)accumulation in the effect of the robot tax on aggregate income.

In Figure 3b, The dash-dot line shows a distinct dynamic effect: the effect of the robot tax on the price of robots imported from Japan in the US. In the short-run, the price decreases due to the decreased demand from the US. As the sequential equilibrium reaches the new steady state where the US stock of robots is decreased, the marginal value of the robots is higher. This increased marginal value partially offsets the reduced price of robots in the short-run, pushing back the cost of robots imported from Japan. This figure shows the effect of the international trade of robots in a large country as well as the accumulation of robots. As an extreme case, I also consider an alternative model with no trade of robots due to prohibitively high robot cost and give the robot tax counterfactual exercise in Section C.8 of the Appendix.

5.3 Other Exercises

What does the same robot tax do to each occupation? The robot tax rolls back the long-run real wage effect of automation. Workers in occupations that experienced significant automation shocks (e.g., production and transportation in the routine occupation groups) who would have been substituted by accumulated robots benefit from the tax, while the

[20]For each occupation, the counterfactual evolution of robot stocks is similar to each other in percentage and, thus, similar to the aggregate trend in percentage. This is not surprising since the robot tax is ad-valorem and uniform across occupations.

others lose. In Appendix C.9, I discuss the effect of the robot tax on occupational wages in detail. My model also allows the counterfactual exercise regarding robots trade liberalization. As mentioned in Appendix C.10, I find that trade liberalization would benefit all countries in the long-run. The benefit in the US appears immediately after the reduction in the trade cost.

6 Conclusion

In this paper, I study the distributional and aggregate effects of industrial robots, emphasizing that robots perform specified tasks and are internationally traded. I make three contributions. First, I construct a first dataset that tracks the number of robot arms and unit values disaggregated by occupations that robots replace. Second, I develop a general equilibrium model that features the trade of robots in a large-open economy and endogenous robot accumulation with an adjustment cost. When estimating the model, to identify the occupation-specific EoS between robots and labor, I construct a model-implied optimal instrumental variable from the average price of robots in my dataset.

The estimates of within-occupation EoS between robots and labor is heterogeneous and as high as 4 in production and material moving occupations. These estimates are significantly larger than estimates of the EoS of capital goods and workers, with a maximum of about 1.5, revealing the susceptibility of workers in the occupations to robot adaptation. These estimates imply that robots contributed to the wage polarization across occupations in the US from 1990-2007. A commonly advertised robot tax could increase the US real income in the short-run but leads to a decline in the income in the long run due to the small steady-state robot stock. These findings indicate that the accumulated robots may have more massive distributional impacts than is considered in the previous literature, and regulating robots could have a positive effect from the aggregate perspective due to the trade of robots.

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Appendix

A Data Appendix

A.1 Data Sources in Detail

I complement data from JARA data and O*NET data by the ones from IFR, BACI, IPUMS USA and CPS. IFR is a standard data source of industrial robot adoption in several countries (e.g., Graetz and Michaels, 2018; Acemoglu and Restrepo, 2020, AR thereafter), to which JARA provides the robot data of Japan. I use IFR data to show the total robot adoption in each destination country as opposed to the import from Japan. I use Federal Reserve Economic Data (FRED) to convert JARA variables denominated in JPY to USD. BACI provides disaggregated data on trade flows for more than 5000 products and 200 countries and is a standard data source of international trade (Gaulier and Zignago, 2010). I use BACI data to obtain the measure of international trade of industrial robots and baseline trade shares. IPUMS USA collects and harmonizes US census microdata (Ruggles et al., 2018). I use Population Censuses (1970, 1980, 1990, and 2000) and American Community Surveys (ACS, 2006-2008 3-year sample and 2012-2016 5-year sample). I obtain occupational wages, employment, and labor cost shares from these data sources. To obtain the intermediate inputs shares, I take data from the World Input-Output Data (WIOD) in the closest year to the initial year, 1992.

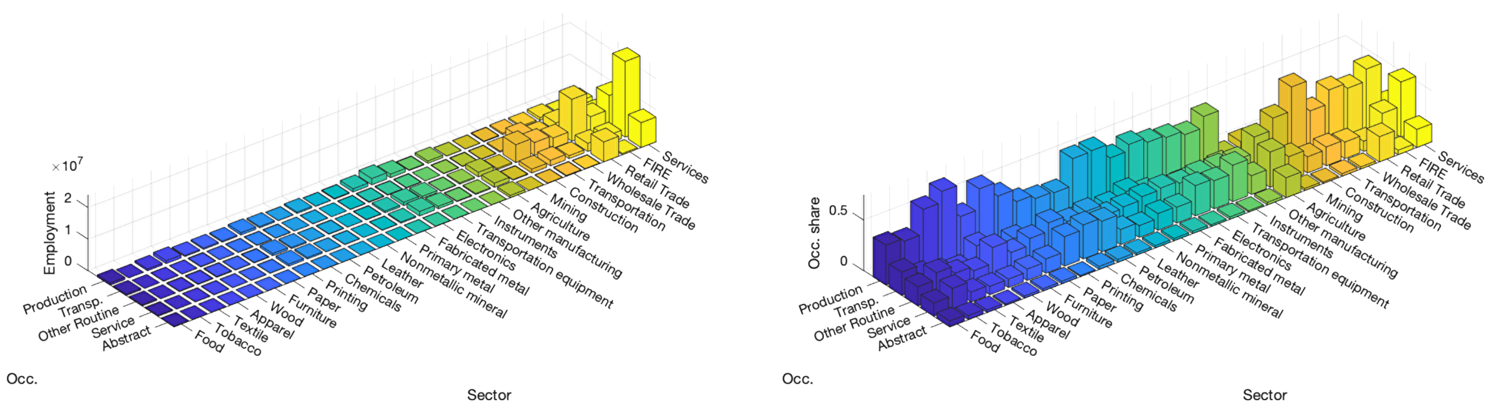
I use the match score from the O*NET Code Connector that contains detailed textual descriptions of 4-digit occupations. The match score is an output of the *weighted search algorithm* used by the O*NET Code Connector, which is the internal search algorithm developed and employed by O*NET and since September 2005. Since then, the O*NET has continually updated the algorithm and improved the quality of the search results. Morris (2019) reports that the updated weighted search algorithm scored 95.9% based on the position and score of a target best 4-digit occupation for a given query.

I followed Autor et al. (2013) for Census/ACS data cleaning procedure. Namely, I extract the 1970, 1980, 1990, 2000 Censuses, the 2006-2008 3-year file of American Community Survey (ACS), and the 2012-2016 5-year file of ACS from Integrated Public Use Micro Samples. For each file, I

Figure A.1: Occupational Employment Distribution

(a) Employment size L_{s,o,t_0}

(b) Employment share l_{s,o,t_0}



Note: The author's calculation from the 1990 US Census. The axis on the left indicates the 5 occupation groups, and the one on the right shows sectors (roughly 4-digit for manufacturing sectors and 2-digit for the non-manufacturing). The left panel shows the size of employment, and the right one indicates the occupation share for each given sector.

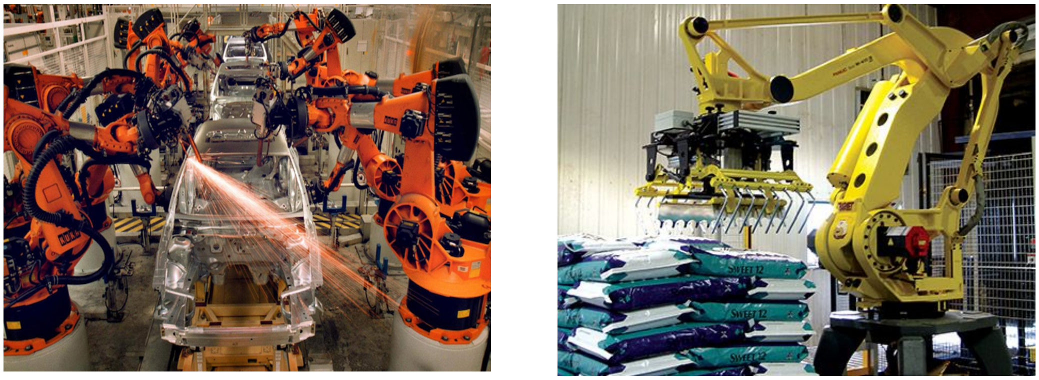
select all workers with the OCC2010 occupation code whose age is between 16 and 64 and who is not institutionalized. I compute education share in each occupation by the share of workers with more than "any year in college," and foreign-born share by the share of workers with BPL (birthplace) variable greater than 150, or those whose birthplace is neither in the US nor in US outlying areas/territories. I compute hours worked by multiplying usual weeks worked and hours worked per week. For 1970, I use the median values in each bin of the usual weeks worked variable and assume all workers worked for 40 hours a week since the hour variable does not exist. To compute hourly wage, I first impute each state-year's top-coded values by multiplying 1.5 and divide by the hours worked. To remove outliers, I take wages below first percentile of the distribution in each year, and set the maximum wage as the top-coded earning divided by 1,500. I compute the real wage in 2000 dollars by multiplying CPI99 variable prepared by IPUMS. I use the person weight variable for aggregating all of these variables to the occupation level. Figure A.1 shows the occupational employment distribution for each sector, a variable used for creating the occupational China shock in equation (2).

To estimate the model with workers' dynamic discrete choice of occupation, I further use the bilateral occupation flow data following the idea of Caliendo et al. (2019). Specifically, I obtain the Annual Social and Economic Supplement (ASEC) of the CPS since 1976. For each year, I select all workers with the 2010 occupation code for the current year (OCC2010) and the last year (OCC10LY) whose age is between 16 and 64 and who is not institutionalized, and treated top-coded wage income, converted nominal wage income to real one, and computed labor hours worked, education, foreign born flag variable with the same method as the one used for Census/ACS above. When computing the occupation switch probability, note that the 4-digit occupations are too disaggregated to precisely estimate with the small sample size of CPS-ASEC, as pointed out by Artuç et al. (2010). Therefore, I assume that the occupations do not flow between 4-digit occupations within the 5 groups defined in Section 4.1, but do between the 5 groups. I assume that workers draw a destination 4-digit occupation occupation from the initial-year occupational employment distribution within the destination group when switching occupations. With these data and assumptions, I compute the occupation switching probability by year.

A.2 Details in Industrial Robots

Figure A.2: Examples of Industrial Robots

(a) Spot Welding (b) Material Handling



Sources: Autobot Systems and Automation (https://www.autobotsystems.com) and PaR Systems (https://www.par.com)

A.2.1 Definition and Examples

As defined in Footnote 1, industrial robots are defined as multiple-axes manipulators. More formally, following International Organization for Standardization (ISO), I define robots as "automatically controlled, reprogrammable, multipurpose manipulator, programmable in three or more axes, which can be either fixed in place or mobile for use in industrial automation applications" (ISO 8373:2012). This section gives a detailed discussion about such industrial robots. Figure A.2 shows the pictures of examples of industrial robots that are intensively used in the production process and considered in this paper. The left panel shows spot-welding robots, while the right panel shows the material-handling robots. The spot welding robots are an example of robots in routine-production occupations, while the material-handling robots are that in routine-transportation (material-moving) occupations.

It is also worthwhile to give an example of technologies that are *not* robots according to the definition in this paper. An example of a growth in technology in the material-handling area is autonomous driving. Mehta and Levy (2020) predicts that such automation will grow strong and result in the reduction of total number of jobs in this area in eight to ten years since 2020. However, since autonomous vehicles do not operate multiple-axes, they are not treated in this paper at all. A similar observation applies for computers or artificial intelligence more generally.

A.2.2 JARA Robot Applications

In addition to applications in Section A.2.1, the full list of robot applications avaliable in JARA data is Die casting; Forging; Resin molding; Pressing; Arc welding; Spot welding; Laser welding; Painting; Load/unload; Mechanical cutting; Polishing and deburring; Gas cutting; Laser cutting; Water jet cutting; General assembly; Inserting; Mounting; Bonding; Soldering; Sealing and gluing; Screw tightening; Picking alignment and packaging; Palletizing; Measurement/inspection/test; and Material handling.

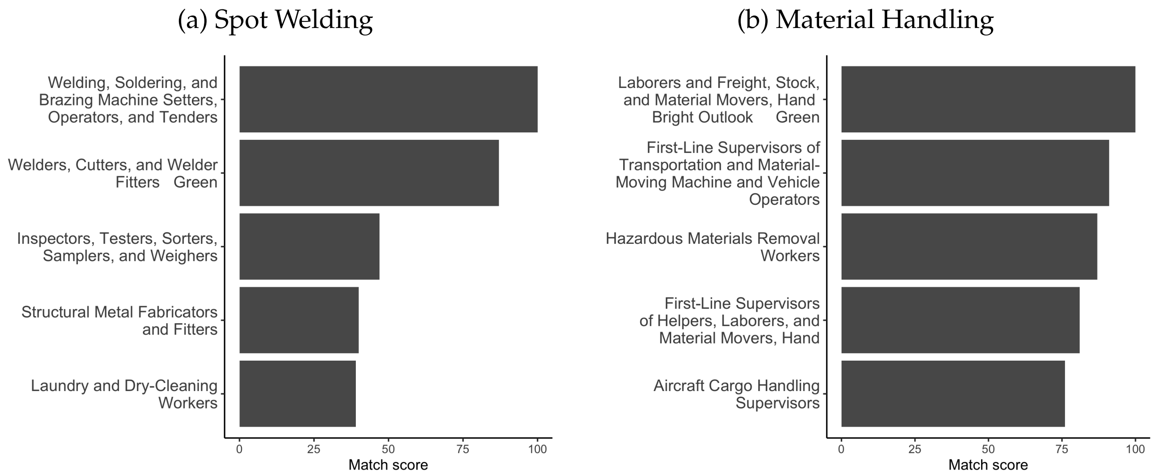
Can robots be classified as one of these applications? If one is familiar with the history of industrial robots, (s)he might wonder that robots are characterized by versatility as opposed to older specified industrial machinery (KHI, 2018). Although it is true that a robot may be reprogrammed to perform more than one task, I claim that robots are well-classified to one of the applications listed above since the layer of dexterity is different. Robots might be able to adjust a model change of the products, but are not supposed to perform different tasks across the 4-digit occupation level. To support this point, recall that "SMEs are mostly high-mix/low-volume producers. Robots are still too inflexible to be switched at a reasonable cost from one task to another" (Autor et al., 2020). These technological bottlenecks still make it hard for producers to have such a versatile robot that can replace a wide range of workers at the 4-digit occupation level even today, all the more for the sample period of my study.

A.2.3 Details in Application-Occupation Matching

Concrete examples of the pairs of an application and an occupation that are close are spot welding and material handling. On the one hand, spot welding is a task of welding two or more metal sheets into one by applying heat and pressure to a small area called spot. In contrast, O*NET-SOC Code 51-4121.06 has the title "Welders, Cutters, and Welder Fitters" ("Welders" below). Therefore, both spot welding robots and welders perform the welding task. On the other hand, Material handling is a short-distance movement of heavy materials. It is another major application of robots. In comparison, ONET-SOC Code 53-7062.00 has the title "Laborers and Freight, Stock, and Material Movers, Hand" ("Material Handler" below). Therefore, both material handling robots and material handlers perform the material handling task.

Figure A.3 shows examples of the O*NET match scores for spot welding and material han-

Figure A.3: Examples of Match Scores



Note: The author's calculation from the search result of O*NET Code Connector. The bars indicate match scores for the search query term "Spot Welding" in panel (a) and "Material Handling" in panel (b). Occupations codes are 2010 O*NET SOC codes. In each panel, occupations are sorted descendingly with the relative relevance scores, and the top 5 occupations are shown. See the main text for the detail of the score.

dling. On the left panel, welding occupations are listed as relevant occupations for spot welding. In contrast, on the right panel, a material-moving laborer is a top occupation in terms of the relevance to the material-handling task, as I described above.

A.2.4 Examples of Robotics Innovation

In the model, I call a change in the robot task space $a_{o,t}$ as the automation shock, and that in robot producer's TFP $A^R_{l,o,t}$ as the cost shock to produce robots. In this section, I show some examples of changes of robot technology and new patents to facilitate understandings of these interpretation. An example of task space expansion is adopting *Programmed Article Transfer* (PAT, Devol, 1961). PAT was machine that moves objects by a method called "teaching and playback". Teaching and playback method needs one-time teaching of how to move, after which the machine playbacks the movement repeatedly and automatically. This feature frees workers of performing repetitive tasks. PAT was intensively introduced in spot welding tasks. KHI (2018) reports that among 4,000 spot welding points, 30% were done be human previously, which PAT took over. Therefore, I interpret the adoption of PAT as the example of the expansion of the robot task space, or increase in $a_{o,t}$. Note that AR also analyze this type of technological change.

An example of cost reduction is adopting *Programmable Universal Manipulator for Assembly*

(PUMA). PUMA was designed to quickly and accurately transport, handle and assemble automobile accessories. A new computer language, *Variable Assembly Language (VAL)*, made it possible because it made the teaching process less work and more sophisticated. In other words, PUMA made tasks previously done by other robots but at cheaper unit cost per unit of task.

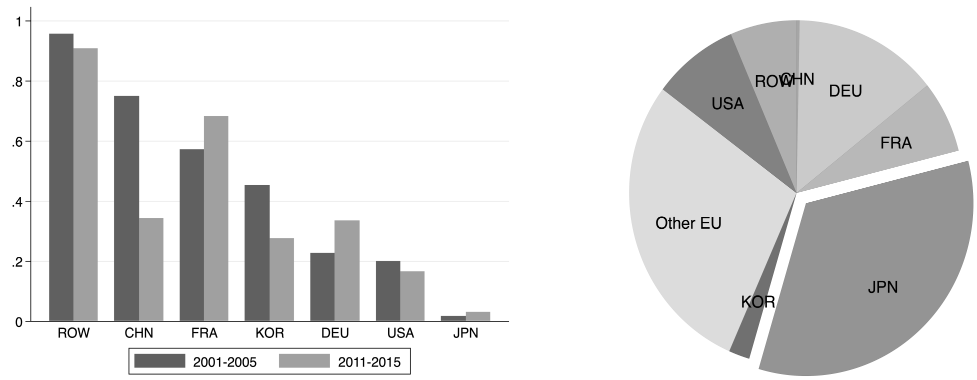
It is also worth mentioning that introduction of a new robot brand typically contains both components of innovation (task space expansion and cost reduction). For example, PUMA also expanded task space of robots. Since VAL allowed the use of sensors and "expanded the range of applications to include assembly, inspection, palletizing, resin casting, arc welding, sealing and research" (KHI, 2018).

A.3 Trade of Industrial Robots

To compute the trade shares of industrial robots, I combine BACI and IFR data. In particular, I use the HS code 847950 ("Industrial Robots For Multiple Uses") to measure the robots, following Humlum (2019). I approximate the initial year value by year of 1998, when the this HS code of robots is first available. To calculate the total absorption value of robots in each country, I use the IFR data's robot units (quantities), combined with the price indices of robots occasionally released by IFR's annual reports for selected countries. These price indices do not give disaggregation by robot tasks or occupations, highlighting the value added of the JARA data. Figure A.4 the pattern of international trade of international robots. In the left panel, I compute the import-absorption ratio. To remove the noise due to yearly observations and focus on a long-run trend, I aggregate by five-year bins 2001-2005 and 2011-2015. The result indicates that many countries import robots as opposed to produce in their countries. Japan's low import ratio is outstanding, revealing that its comparative advantage in this area. It is noteworthy that China largely domesticated the production of robots over the sample period. Another way to show grasp the comparative advantage of the robot industry is to examine the share of exports as in the right panel of Figure A.4. Roughly speaking, the half of the world robot market was dominated by EU and one-third by Japan in 2001-2005. The rest 20% is shared by the rest of the world, mostly by the US and South Korea.

Figure A.4: Trade of Industrial Robots

(a) Robot Import-Absorption Ratio (b) World Robot Export Share, 2001-2005



Note: The author's calculation from the IFR, and BACI data. The left panel show the fraction of import in the total absorption value. The import value is computed by aggregating trade values across origin country in the BACI data (HS-1996 code 847950), and the absorption value is computed by the price index and the quantity variable available for selected countries in the IFR data. The data are five-year aggregated in 2001-2005 and 2011-2015, and countries are sorted according to the import shares in 2001-2005 descendingly. The right panel shows the export share for 2001-2005 aggregates obtained from the BACI data.

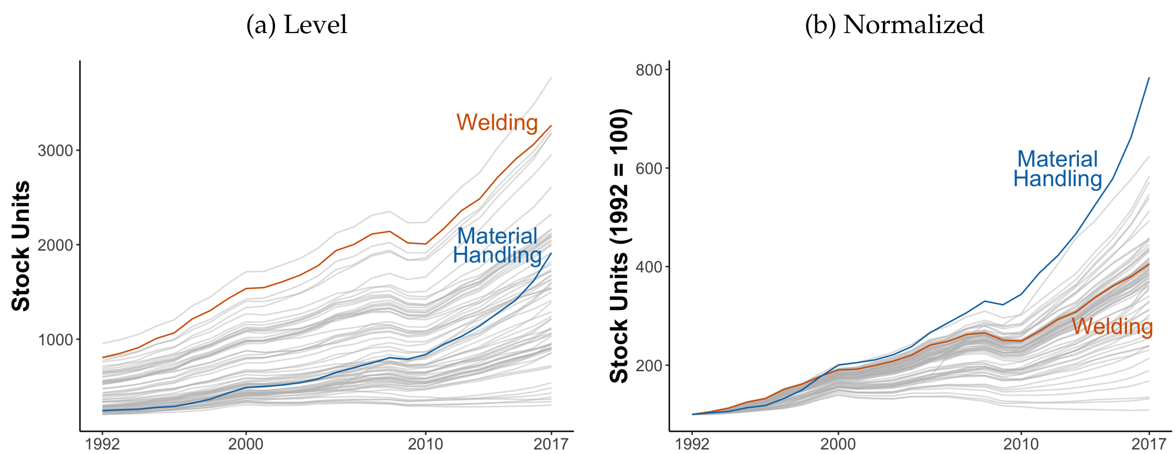
A.4 Trends of Robot Stocks and Prices

I will show that different occupations experienced different trends in robot adoption. Figure A.5 shows the trend of US robot stocks at the occupation level. In the left panel, I show the trend of raw stock. First, the overall robot stocks increased rapidly in the period, as found in the previous literature. The panel also shows that the increase occurred in many occupations, but at differential rates. To highlight such a difference, in the right panel, I plot the normalized trend at 100 in the initial year. There is significant heterogeneity in the growth rates, ranging from a factor of one to eight.

For example, I color in the figure two occupations, robots that correspond to "Welding, Soldering, and Brazing Workers" (or "Welding") and "Laborers and Freight, Stock, and Material Movers, Hand" (or "Material handling"). On the one hand, welding is an occupation where the majority of robots were applied continuously throughout the period, as can be confirmed in the left panel. However, the growth rate of the stocks is not outstanding, but within the range of growth rates of other occupations. On the other hand, material handling was not a majority occupation as of the initial year, but it grew at the most rapid pace in the period.

These findings indicate the difference between the automation shocks each occupation received. Some occupations were already somewhat automated by robotics as of the initial year, and

Figure A.5: US Robot Stocks at the Occupation Level



Notes: The author's calculation based on JARA and O*NET. The figure shows the trend of stocks of robots in the US for each occupation. The left panel shows the level, whereas the right panel shows the normalized trend at 100 in 1992. In both panels, I highlight two occupations. "Welding" corresponds the occupation code in IPUMS USA, OCC2010 = 8140 "Welding, Soldering, and Brazing Workers." "Material Handling" corresponds the occupation code OCC2010 = 9620 "Laborers and Freight, Stock, and Material Movers, Hand." Years are aggregated into five-year bins (with the last bin 2012-2017 being six-year one) to smooth out yearly noises.

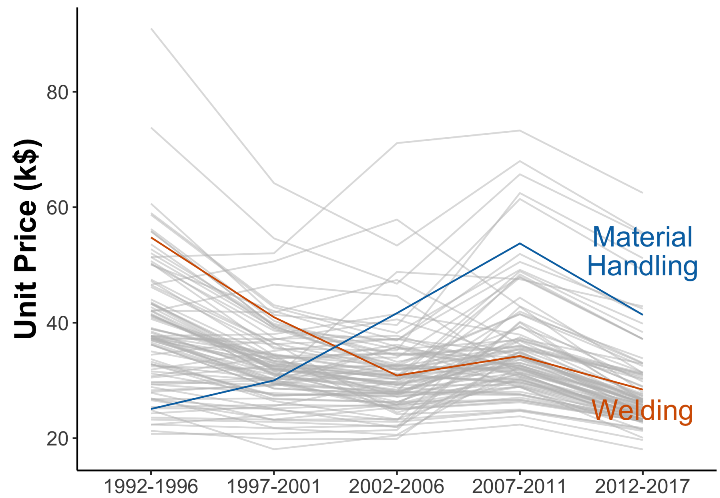
the automation process continued afterward (e.g., welding). There are a few occupations where robotics automation was not achieved initially, but the innovation and adoption occurred rapidly in the period (e.g., material handling). I propose a model that incorporates this heterogeneity and discuss how to exploit it in estimation in the following sections.

Figure A.6 shows the trend of prices of robots in the US for each occupation. In addition to the overall decreasing trend, there is significant heterogeneity in the pattern of price falls across occupations. For instance, although the welding robots saw a large drop in the price during the 1990s, the material handling robots did not but increased the price over the sample period.

A.5 Robots from Japan in the US, Europe, and the Rest of the World

I review the international comparison of the pattern of robot adoption. I generate the growth rates of stock of robots between 1992 and 2017 at the occupation level for each group of destination countries. The groups are the US, the non-US countries, (namely, the world excluding the US and Japan), and five European countries (or "EU-5"), Denmark, Finland, France, Italy, and Sweden used in AR. To calculate the stock of robots, I employ the perpetual inventory method with depreciation rate of $\delta = 0.1$, following Graetz and Michaels (2018).

Figure A.6: Robot Prices at the Occupation Level



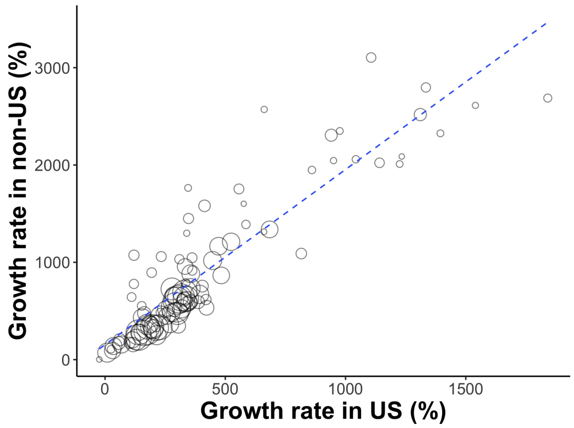
Notes: The author's calculation based on JARA and O*NET. The figure shows the trend of prices of robots in the US for each occupation. I highlight two occupations. "Welding" corresponds the occupation code in IPUMS USA, OCC2010 = 8140 "Welding, Soldering, and Brazing Workers." "Material Handling" corresponds the occupation code OCC2010 = 9620 "Laborers and Freight, Stock, and Material Movers, Hand." Years are aggregated into five-year bins (with the last bin 2012-2017 being six-year one) to smooth out yearly noises. The dollars are converted to 2000 real US dollar using CPI.

Figure A.7 shows scatterplots of the growth rates at the occupation level. The left panel shows the growth rates in the US on the horizontal axis and the ones in non-US countries on the vertical axis. The right panel shows the same measures on the horizontal axis, but the growth rates in the set of EU-5 countries on the vertical axis. These panels show that the stocks of robots at the occupation level grow (1992-2017) between the US and non-US proportionately relative to those between the US and EU-5. This finding is in contrast to AR, who find that the US aggregate robot stocks grew at a roughly similar rate as those did in EU-5. It also indicates that non-US growth patterns reflect growths of robotics technology at the occupation level available in the US. I will use these patterns as the proxy for robotics technology available in the US. In Section 3 and on, I take a further step and solve for the robot adoption quantity and values in non-US countries in general equilibrium including the US and non-US countries.

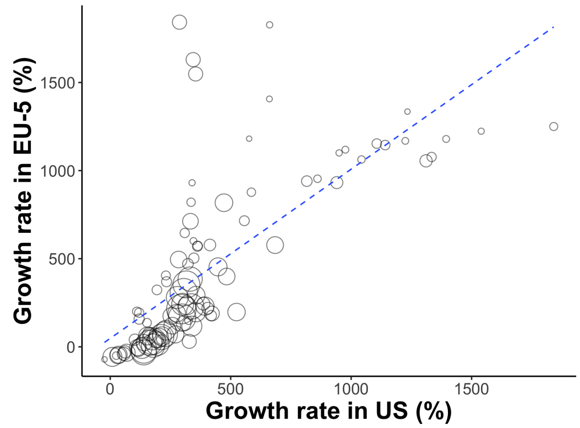
It is worth mentioning that a potential reason for the difference between my finding and AR's is the difference in data sources. In contrast to the JARA data I use, AR use IFR data that include all robot seller countries. Since EU-5 is closer to major robot producer countries other than Japan, including Germany, the robot adoption pattern across occupations may be influenced by

Figure A.7: Growth Rates of Robots at the Occupation Level

(a) Comparison between the US and non-US



(b) Comparison between the US and EU-5



Notes: The author's calculation based on JARA, and O*NET. The left panel shows the correlation between occupation-level growth rates of robot stock quantities from Japan to the US and the growth rates of the quantities to the non-US countries. The right one shows the correlation between growth rates of the quantities to the US and EU-5 countries. Non-US are the aggregate of all countries excluding the US and Japan. EU-5 are the aggregate of Denmark, France, Finland, Italy, and Sweden used in Acemoglu and Restrepo (2020). Each bubble shows an occupation. The bubble size reflects the stock of robot in the US in the baseline year, 1992. See the main text for the detail of the method to create the variables.

their presence. If these close producers have a comparative advantage in producing robots for a specific occupation, then EU-5 may adopt the robots for such occupations intensively from close producers. In contrast, countries out of EU-5, including the US, may not benefit the closeness to these producers. Thus they are more likely to purchase robots from far producers from EU-5, such as Japan.

A.6 Further Analysis about Fact 2

Figure A.8 plots the correlation between the changes in robot measures and the changes in log labor market outcomes in the US at the occupation level, weighted by the size of occupation measured by initial the employment level. The top two panels take the occupational wage as a labor market outcome on the y-axis, while the bottom two take the occupational employment. The left two panels take log monetary value of robot stock in non-US countries as a robot measure on the left panel, and the right two take the log Japan robot shock ψ^J_{o,t_1}.

First, I offer a piece of evidence that robots have replaced workers at the occupation level. To control the demand factor in the US, Acemoglu and Restrepo (2020) used the robot stock changes in the other countries that show a similar trend of robot stocks as a proxy for the robot techno-

Figure A.8: Correlation between Wages and Robot Measures

Note: The author's calculation based on JARA, O*NET, and the US Census/ACS. The figures show the scatterplot, weighted fit line, and the 95 percent confidence interval of the changes in log robot measures and changes in log labor market outcomes. On the y-axis, the top figures take occupational wages, while the bottom figures take occupational employment. The left panels take the change in log robot stocks (measured in monetary value) on the x-axis, while the right panels take the change in log robot average prices on the x-axis. Each bubble represents a 4-digit occupation. The bubble size reflects the employment in the baseline year (1990, which is the closest Census year to the initial year that I observe the robot adoption, 1992).

logical change and find the negative impact on the US regional labor market. Following this approach, using the changes in robot stocks in non-US countries (all countries except for the US and Japan), I find that the robot penetration measure negatively affects and labor market outcomes of wages and employment by occupation.[21] This result provides direct evidence of the substitution of robots for workers who perform the same task as robots, as well as corroborating the finding of

[21] In Section A.5, I show that the robot stock growths are similar between the US and the non-US countries by occupations. In contrast, the occupation-level trend in the five countries Acemoglu and Restrepo (2020) used as comparison (Denmark, Finland, France, Italy, and Sweden) is less similar to the US trend than the non-US countries.

Acemoglu and Restrepo (2020).

Furthermore, it also implies the reallocation of workers across occupations due to technological changes in robotics. Namely, suppose that an occupation experiences a rapid adoption of robots and workers in the occupation switch to other occupations that did not experience the adoption. This mechanism also works as a force that makes the negative correlation found in Figure A.8. In Section 3, I model these points by considering the dynamic occupation choice by workers. The characterizing parameter is the occupation switch elasticity ϕ, which is one of the target elasticity in estimation.

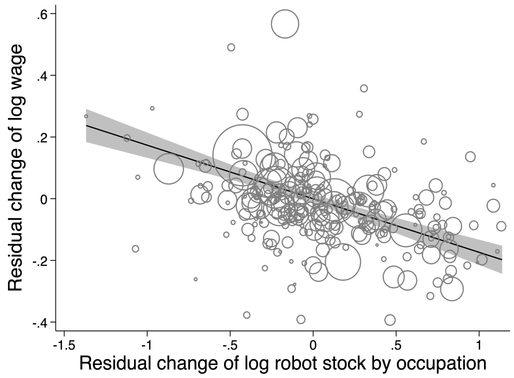
Figure A.9 shows the results of a set of robustness checks with an emphasis on the correlation between the wage changes and the changes in robot measures. Figures A.9a and A.9b show the wage correlation after residualizing the demographic control variables (initial-year female share, college graduates share, age 35-49 share, age 50-64 share, and foreign-born share in each occupation) for the robot stock and robot prices, respectively. Figures A.9c and A.9d show the correlation with the robot measures in the rest of the world (namely, world excluding the US and Japan) after residualizing the demographic control variables. The motivation of this exercise follows the intuition of AR–using the technological change proxied by the rest-of-the-world change in robot measures to move away from the occupational demand shocks since US occupational robot adoption may be affected by occupational demand shocks such as occupational productivity changes. Figure A.9e shows the result with the measurement of the robot stock by quantities of machines as opposed to monetary value, which follows the approach in the past literature such as AR. Figure A.9f shows the result of correlation using quality-adjusted robot prices, where the method of quality adjustment follows the spirit of Khandelwal et al. (2013). Namely, I fit the following equation with the fixed-effect regression:

$$\ln\left(X^R_{JPN\to i,o,t}\right) = -\varsigma \ln\left(p^R_{JPN\to i,o,t}\right) + a^R_{o,t} + e^R_{i,o,t},$$

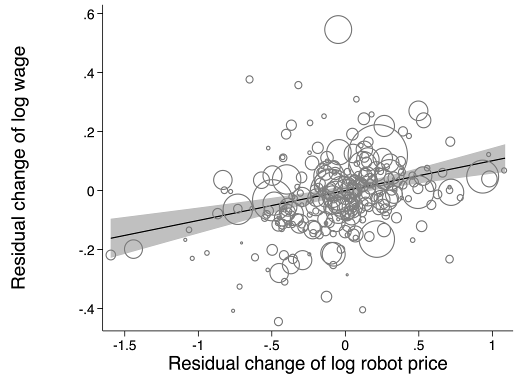
from which I obtain the fixed effect $a^R_{o,t}$, which absorbs the occupation-o specific log sales component that is not explained by the prices. I then proxy the quality change by the change in such fixed effects, $\Delta a^R_{o,t} \equiv a^R_{o,t} - a^R_{o,t_0}$. The (log) quality-adjusted price is then obtained by $\ln\left(p^R_{JPN\to i,o,t}\right) - \Delta a^R_{o,t}$. All the results are robust to these considerations–wage growths are negatively correlated with stock growths, and positively correlated with price growths, both across occupations.

Figure A.9: Correlation between Occupational Wage and Occupational Robot Measures

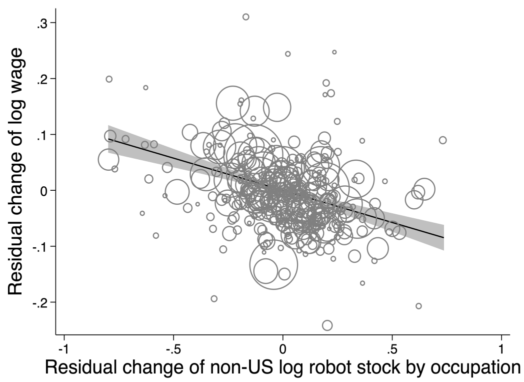
(a) Residualizing Control Variables, Stocks



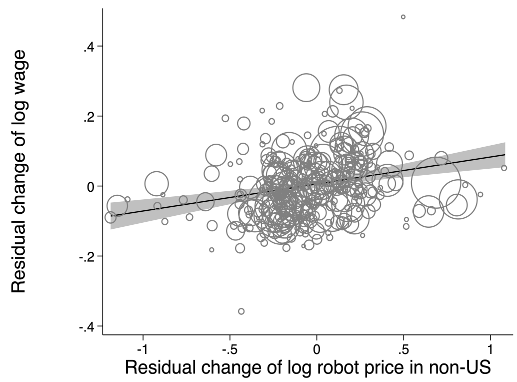
(b) Residualizing Control Variables, Prices



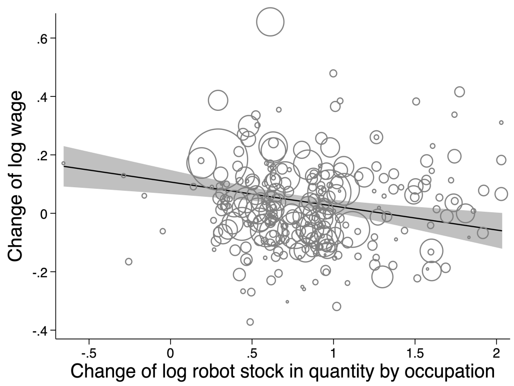
(c) ROW Stocks



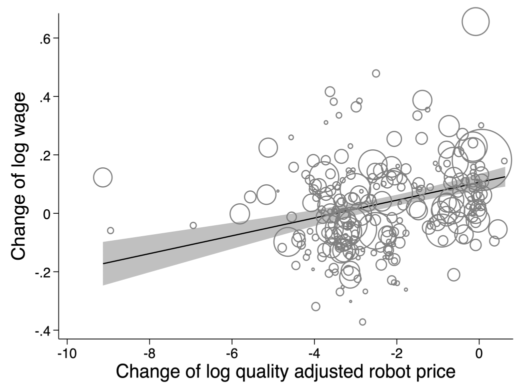
(d) ROW, Prices



(e) Stock Quantity



(f) Quality-adjusted prices



To further check the correlation systematically, I run the following regressions and report the results in Table A.2:

$$\Delta \ln (y_o) = a_R \Delta \ln (R_o) + (X_o)^\top \boldsymbol{a} + e_o,$$

where y_o is labor-market outcome of occupation o (wage and employment), R_o is the measure of robots (stocks and prices), X_o are the demographic control variables, e_o is the regression residual, and Δ indicates the long-difference between 1990 (1992 for $\Delta \ln (R_o)$) and 2007. The coefficient of interest is a_R–I expect negative a_R if I take robot stocks as the explanatory variable, while I expect positive a_R when I take robot price as the right-hand side variable.

A.7 Robot Price Trends by Occupation Groups

In this section, I examine the facts discussed in Section 2.3 for each occupation group described in Section 4.1. First, Figure A.10 shows the plot of the trend of the robot price distribution since 1992 for each occupation group, a version of Figure 1a, disaggregated by occupation groups. The top three panels show the trends for routine occupations, namely, from the left, routine-production, routine-transportation, and routine-others. The bottom two panels show the trends for service occupations and abstract occupations, from the left. All these panels show the overall decreasing trend of robot prices, and the dispersion of prices within each occupation group. Having such a dispersion is important because in Section 4 when I estimate heterogeneous EoS between robots and labor, I use the price variation within each occupation group. Next, Figure A.11 shows the correspondent of Figure A.8 for each occupation group. The alignment of occupation groups is the same as Figure A.10. Interestingly, the positive correlation between occupational wage changes and occupational robot price changes, robustly found in Figure A.8 and Section A.6, is seen only in the group of production occupations and transportation occupations. Given that strong positive correlation yields a high elasticity of substitution, the finding in this figure is consistent with the heterogeneous elasticity of substitution between robots and labor found in Table 2b.

A.8 Initial Share Data

Since the log-linearized sequential equilibrium solution depends on several initial share data generated from the initial equilibrium, I discuss the data sources and methods for measuring these shares. I define $t_0 = 1992$ and the time frequency is annual. I consider the world that consists

Table A.1: List of Data Sources

Variable	Description	Source
$\widetilde{y}^G_{ij,t_0}$, $\widetilde{x}^G_{ij,t_0}$, $\widetilde{y}^R_{ij,t_0}$, $\widetilde{x}^R_{ij,t_0}$	Trade shares of goods and robots	BACI, IFR
$\widetilde{x}^O_{i,o,t_0}$	Occupation cost shares	IPUMS
l_{i,o,t_0}	Labor shares within occupation	JARA, IFR, IPUMS
s^G_{i,t_0}, s^V_{i,t_0}, s^R_{i,t_0}	Robot expenditure shares	BACI, IFR, WIOT
$\alpha_{i,M}$	Intermediate input share	WIOT

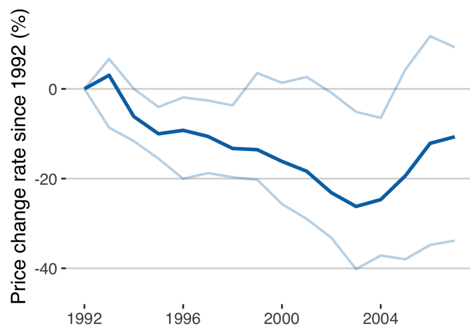
Table A.2: Regression Result of Labor Market Outcome on Robot Measures

VARIABLES	(1) $\Delta \ln(w)$	(2) $\Delta \ln(w)$	(3) $\Delta \ln(w)$	(4) $\Delta \ln(w)$	(5) $\Delta \ln(L)$	(6) $\Delta \ln(L)$	(7) $\Delta \ln(L)$	(8) $\Delta \ln(L)$
$\Delta \ln(K^R_{USA})$	-0.174***				-0.532***			
	(0.0251)				(0.203)			
$\Delta \ln(p^R_{USA})$		0.0969***				0.507***		
		(0.0263)				(0.141)		
$\Delta \ln(K^R_{ROW})$			-0.116***				-0.575***	
			(0.0162)				(0.0953)	
$\Delta \ln(p^R_{ROW})$				0.0999***				0.458***
				(0.0257)				(0.148)
Female share	0.0366	0.0391	0.0383	0.0361	-0.0658	-0.0663	-0.0563	-0.0616
	(0.0320)	(0.0340)	(0.0328)	(0.0335)	(0.175)	(0.178)	(0.175)	(0.181)
Col. grad. share	0.399***	0.379***	0.401***	0.399***	0.114	0.113	0.119	0.107
	(0.0684)	(0.0673)	(0.0707)	(0.0691)	(0.284)	(0.285)	(0.281)	(0.287)
Age 35-49 share	-0.768*	-0.594	-0.697*	-0.672*	0.399	0.449	0.325	0.427
	(0.395)	(0.405)	(0.405)	(0.404)	(1.281)	(1.331)	(1.308)	(1.320)
Age 50-64 share	0.778**	0.797**	0.787**	0.765**	-1.636	-1.650	-1.541	-1.576
	(0.345)	(0.345)	(0.365)	(0.376)	(1.166)	(1.134)	(1.208)	(1.170)
Foreign-born share	-0.0905	-0.0250	-0.0241	-0.00227	-0.255	-0.221	-0.322	-0.197
	(0.225)	(0.213)	(0.230)	(0.221)	(1.142)	(1.073)	(1.074)	(1.044)
Observations	324	324	324	324	324	324	324	324
R-squared	0.467	0.344	0.398	0.367	0.138	0.104	0.199	0.106

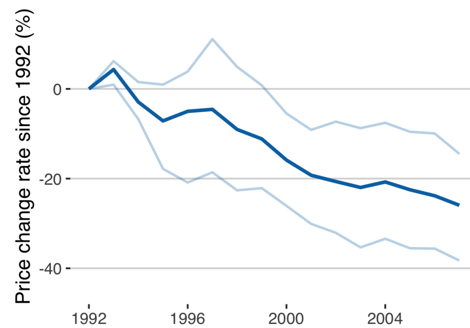
Notes: The author's calculation based on JARA, O*NET, and US Census/ACS. Observations are 4-digit level occupations, and the sample is all occupations that existed throughout 1970 and 2007. In each country $i \in \{USA, ROW\}$, K_i^R stands for the 2000-dollar value of the robot stock in the occupation and p_i^R stands for the average price of robot transacted in each year. All time differences (Δ) are taken with a long difference between 1990 and 2007. All demographic control variables are as of 1990. "Col. Grad. Share" stands for the college graduate share. Robust standard errors are reported in the parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Figure A.10: Robot Price Trends by Occupation Groups

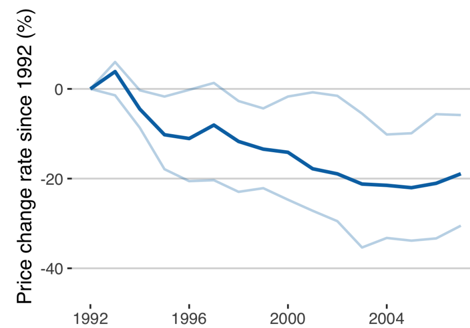
(a) Routine, Production



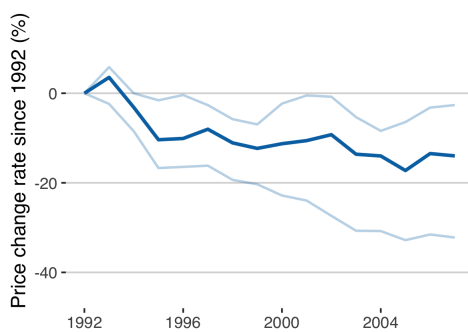
(b) Routine, Transportation



(c) Routine, Others



(d) Service



(e) Abstract

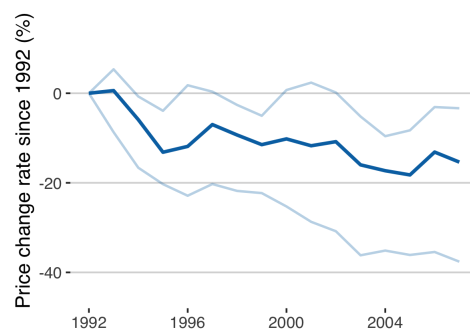


Figure A.11: Correlation between Wage and Robot Prices by Occupation Groups

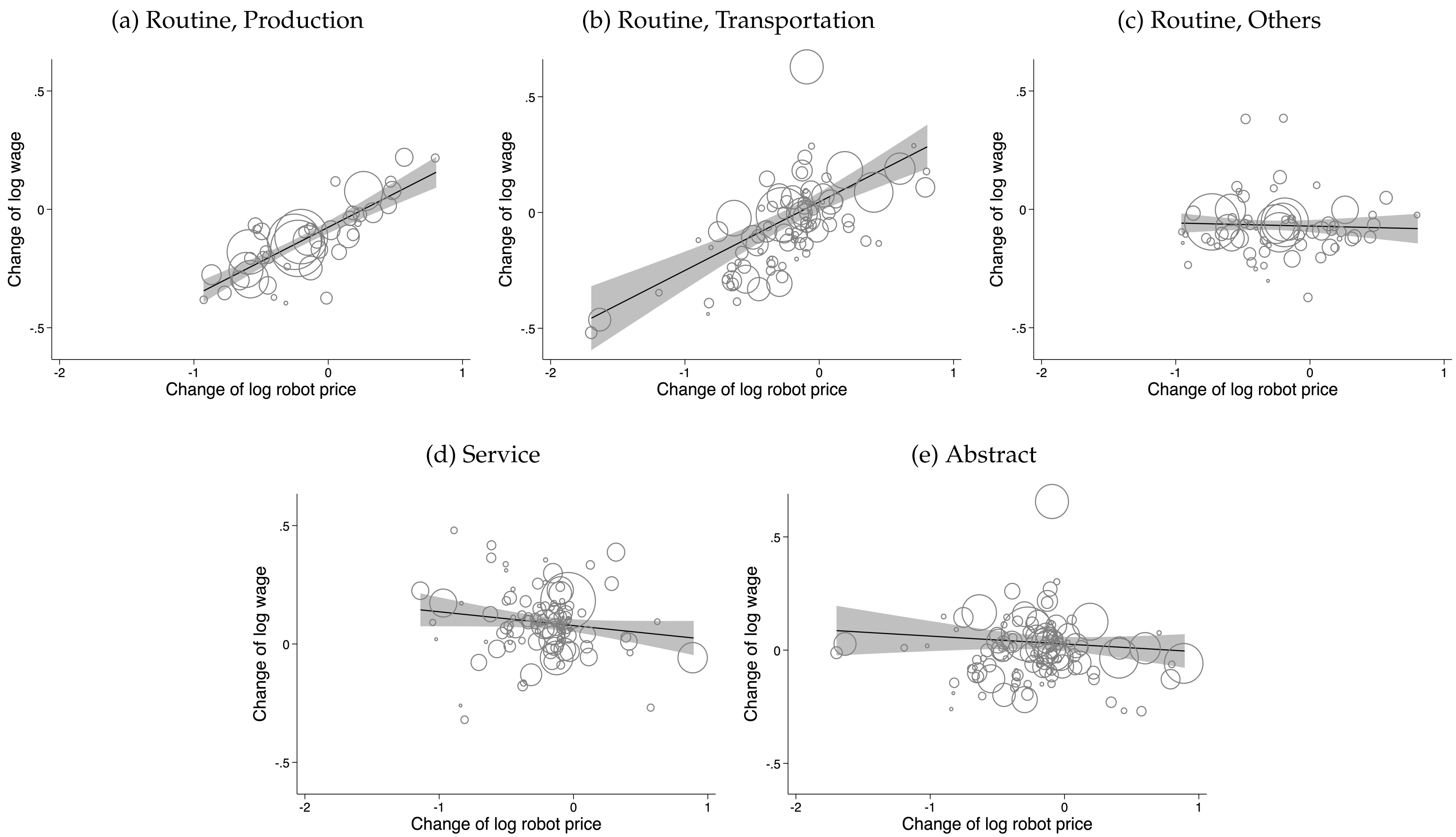
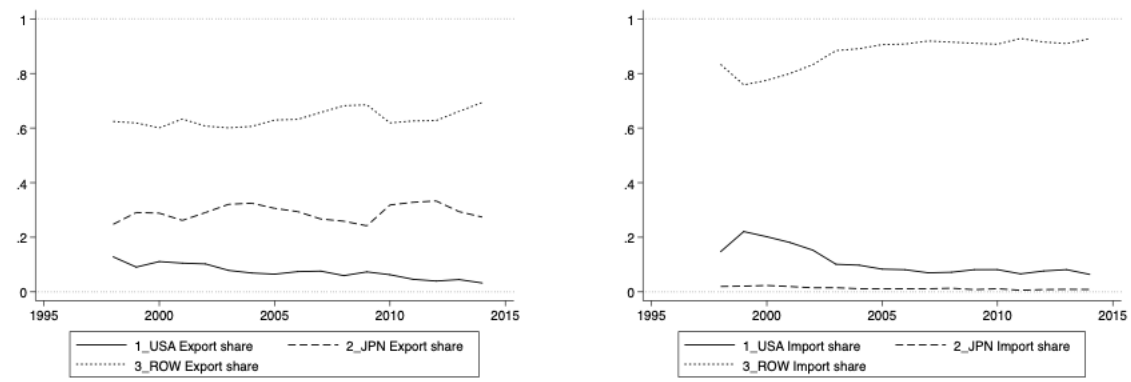


Figure A.12: Robot Trade Share Trends

(a) Exports (b) Imports



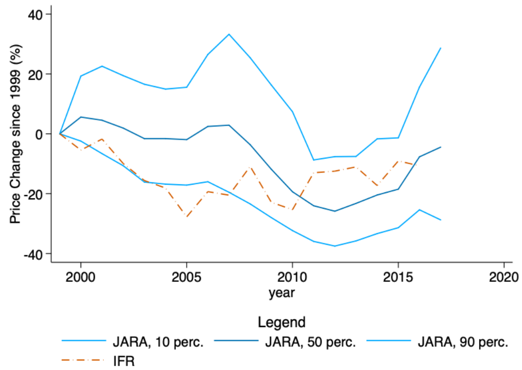
Note: The author's calculation of world trade shares based on the BACI data. Industrial robots are measured by HS code 847950 (Industrial robots for multiple uses).

of three countries $\{USA, JPN, ROW\}$. Table A.1 summarizes overview of the variable notations, descriptions, and data sources.

I take matrices of trade of goods and robots by BACI data. As in Humlum (2019), I measure robots by HS code 847950 ("Industrial Robots For Multiple Uses") and approximate the initial year value by year of 1998, in which the robot HS code is first available. Figure A.12 shows the trend of export and import shares of robots among the world for the US, Japan, and the Rest Of the World. The trends are fairly stable for the three regions of the world, except that the import share of the US has declined relative to the ROW.

To obtain the domestic robot absorption data, I take from IFR data the flow quantity variable and the aggregate price variable for a selected set of countries. I then multiply these to obtain USA and JPN robot adoption value. For robot prices in ROW, I take the simple average of the prices among the set of countries (France, Germany, Italy, South Korea, and the UK, as well as Japan and the US) for which the price is available in 1999, the earliest year in which the price data are available. Graetz and Michaels (2018) discuss prices of robots with the same data source. Figure A.13 shows the comparison of the US price index measure available between JARA and IFR. The JARA measures are disaggregated by 4-digit occupations. The figure shows the 10th, 50th (median), and 90th percentiles each year, as in Figure 1a. All measures are normalized at 1999, the year in which the first price measure is available in the IFR data. Overall, the JARA price trend variation tracks the overall price evolution measured by IFR reasonably well: The long-

Figure A.13: Comparison of US Price Indices between JARA and IFR



Note: The author's calculation of US robot price measures in JARA and IFR. The JARA measures are disaggregated by 4-digit occupations, and the figure shows the 10th, 50th (median), and 90th percentiles each year. All measures are normalized at 1999, the year in which the first price measure is available in the IFR data.

run trends from 1999 to the late 2010s are similar between the JARA median price and the IFR price index. During the 2000s, the IFR price index drops faster than the median price in the JARA data. It compares with the JARA 10th percentile price, which could be due to robotic technological changes in other countries than Japan in the corresponding period.

I construct occupation cost shares $\widetilde{x}^{O}_{i,o,t_0}$ and labor shares within occupation l_{i,o,t_0} as follows. To measure $\widetilde{x}^{O}_{i,o,t_0}$, I aggregate the total wage income of workers that primarily works in each occupation o in year 1990, the Census year closest to t_0. I then take the share of this total compensation measure for each occupation. To measure l_{i,o,t_0}, I take the total compensation as the total labor cost and a measure of the user cost of robots for each occupation. The user cost of robots is calculated with the occupation-level robot price data available in IFR and the set of calibrated parameters in Section 4.1. Table A.3 summarizes these statistics for the aggregated 5 occupation groups in the US. One can see that the cost for production occupations and transportation occupations comprise 18% and 8% of the US economy, respectively, totaling more than one-fourth. Furthermore, the share of robot cost in all occupations is still quite low with the highest share of 0.19% in production occupations, revealing still small-scale adoption of robots from the overall US economy perspective.

To calculate the effect on total income, I also need to compute the sales share of robots by occupations $y^{R}_{i,o,t_0} \equiv Y^{R}_{i,o,t_0} / \sum_o Y^{R}_{i,o,t_0}$ and the absorption share $x^{R}_{i,o,t_0} \equiv X^{R}_{i,o,t_0} / \sum_o X^{R}_{i,o,t_0}$. To obtain y^{R}_{i,o,t_0}, I compute the share of robots by occupations produced in Japan $y^{R}_{2,o,t_0} = Y^{R}_{2,o,t_0} / \sum_o Y^{R}_{2,o,t_0}$ and

Table A.3: Baseline Shares by 5 Occupation Group

Occupation Group	$\widetilde{x}^O_{1,o,t_0}$	l^O_{1,o,t_0}	y^R_{2,o,t_0}	x^R_{1,o,t_0}	x^R_{2,o,t_0}	x^R_{3,o,t_0}
Routine, Production	17.58%	99.81%	64.59%	67.49%	62.45%	67.06%
Routine, Transportation	7.82%	99.93%	12.23%	11.17%	13.09%	11.04%
Routine, Others	28.78%	99.99%	10.88%	9.52%	11.68%	10.40%
Service	39.50%	99.99%	8.87%	8.58%	9.17%	8.32%
Abstract	6.32%	99.97%	3.43%	3.24%	3.60%	3.18%

Note: The author's calculation of initial-year share variables based on the US Census, IFR, and JARA. As in the main text, country 1 indicates the US, country 2 Japan, and country 3 the rest of the world. See the main text for the construction of each variable.

assume the same distribution for other countries due to the data limitation: $y^R_{i,o,t_0} = y^R_{2,o,t_0}$ for all i. To have x^R_{i,o,t_0}, I compute the occupational robot adoption in each country by $X^R_{i,o,t_0} = P^R_{i,t_0} Q^R_{i,o,t_0}$, where Q^R_{i,o,t_0} is the occupation-level robot quantity obtained by the O*NET concordance generated in Section 2.2.2 applied to the IFR application classification. As mentioned above, the robot price index P^R_{i,t_0} is available for a selected set of countries. To compute the rest-of-the-world price index P^R_{3,t_0}, I take the average of all available countries weighted by the occupational robot values each year. The summary table for these variables y^R_{i,o,t_0} and x^R_{i,o,t_0} at 5 occupation groups are shown in Table A.3. All values in Table A.3 are obtained by aggregating 4-digit-level occupations, and raw and disaggregated data are available upon request.

I take a more standard measure, the intermediate input share $\alpha_{i,M}$, from World Input-Output Tables (WIOT Timmer et al., 2015). Finally, I combine the trade matrix generated above and WIOT to construct the good and robot expenditure shares s^G_{i,t_0}, s^V_{i,t_0}, and s^R_{i,t_0}. In particular, with the robot trade matrix, I take the total sales value by summing across importers for each exporter, and total absorption value by summing across exporters for each importers. I also obtain the total good absorption by WIOT. From these total values, I compute expenditure shares. are obtained by aggregating 4-digit occupations, and the disaggregated data are available upon request.

As initial year occupation switching probabilities μ_{i,oo',t_0}, I take 1990 flow Markov transition matrix from the cleaned CPS-ASEC data created in Section A.1. Table A.4 shows this initial-year conditional switching probability. The matrix for the other years are available upon request. As for other countries than the US, although Freeman et al. (2020) has begun to develop occupational wage measures consistent across country, world-consistent occupation employment data are hard to obtain. Therefore, I assign the same flow probabilities for other countries in my estimation.

Table A.4: 1990 Occupation Group Switching Probability

		Production	Routine Transportation	Others	Service	Abstract
Routine	Production	0.961	0.011	0.010	0.006	0.012
	Transportation	0.020	0.926	0.020	0.008	0.025
	Others	0.005	0.006	0.955	0.020	0.014
Service		0.003	0.002	0.020	0.967	0.007
Abstract		0.014	0.014	0.036	0.015	0.922

Note: The author's calculation from the CPS-ASEC 1990 data. The conditional switching probability to column occupation group conditional on being in each row occupation.

B Theory Appendix

B.1 Further Discussion of Model Assumptions

Capital-Skill Complementarity Occupation production function (10) also nest the one in the literature of capital-skill complementarity (Krusell et al., 2000 among others). To simplify, I focus on individual producer's production function in the steady state. Thus I drop subscripts and superscripts of country i and time period t. Suppose the set of occupations is $O \equiv \{R, U\}$ and $a_U = 0$. R stands for the robotized occupation (e.g., spot welding) and U stands for "unrobotized" (e.g., programming). Note that since U is unrobotized $a_U = 0$. Then the unit cost of occupation aggregate (10), P^O, is

$$P^O = \left[(b_R)^{\frac{1}{\beta}} \left((1 - a_R) (w_R)^{1-\theta_R} + a_R (c_R)^{1-\theta_R} \right)^{\frac{1-\beta}{1-\theta_R}} + (b_U)^{\frac{1}{\beta}} (w_U)^{1-\beta} \right]^{\frac{1}{1-\beta}}.$$

Thus different skills R and U are substituted by robots with different substitution parameters θ_R and β, respectively. Since the literature of capital-skill complementarity studies the rising skill premium, the current model also has an ability to discuss the occupation (or skill) premium given the different level of automation across occupations.

Adjustment Cost of Robot Capital To interpret another key feature of the model, the convex adjustment cost of robot adoption, consider the cost of adopting new technology and integration. With the convex adjustment cost, the model predicts the staggered adoption of robots over years that I observe in the data (see Figure 3b), and implies a rich prediction about the short- and long-run effects of robotization.

First, when adopting new technology including robots, it is necessary to re-optimize the overall production process since the production process is typically optimized to employ workers. More generally, the literature of organizational dynamics studies the difficulty, not to say the impossibility, of changing strategies of a company due to complementarities (see Brynjolfsson and Milgrom, 2013 for a review). Such a re-optimization incurs an additional cost of adoption in addition to the purchase of robot arms. Moreover, even within a production unit, there is a variation of this difficulty of adopting robots across production processes. In this case, the part where the adjustment is easy adopts the robots first, and vice versa. This allocation of robot adoptions over years may aggregate to make the robot stock increase slowly (Baldwin and Lin, 2002). Waldman-Brown (2020) also finds that the incremental and sluggish automation is particularly well-observed in small and medium-sized firms, as they add "a machine here or there, rather than installing whole new systems that are more expensive to buy and integrate" (Autor et al., 2020).

The second component of the adjustment cost may come from the cost of integration as I discussed in Section 2.1. The marginal integration cost may increase as the massive upgrading of robotics system may require large-scale overhaul of production process, which increases the complexity and so is costly. The adjustment cost may capture the increasing marginal cost component of the integration cost. It explains an additional component of the integration cost implied by constant returns-to-scale robot aggregation in equation (14).

Another potential choice of modeling a staggered growth of robot stocks is to assume a fixed cost of robot adoption and lumpy investment. Humlum (2019) finds that many plants buy robots only once during the sample period. Since JARA data does not observe plant-level adoptions, I do not separately identify lumpy investment from the staggered growth of robot stocks in the data. To the extent that fixed cost of investment may make the policy intervention less effective (e.g., Koby and Wolf, 2019), the counterfactual analysis in this paper may overestimate the effect of robot taxes since it does not take into account the fixed cost and lumpiness of investment.

B.2 Derivation of Worker's Optimality Conditions

In this section, I formalize the assumptions behind the derivation and show equations (4) and (5). Fix country i and period t. There is a mass $\overline{L}_{i,t}$ of workers. In the beginning of each period, worker

$\omega \in [0, \overline{L}_{i,t}]$ draws a multiplicative idiosyncratic preference shock $\{Z_{i,o,t}(\omega)\}_o$ that follows an independent Fréchet distribution with scale parameter $A^V_{i,o,t}$ and shape parameter $1/\phi$. Note that one can simply extend that the idiosyncratic preference follows a correlated Fréchet distribution to allow correlated preference across occupations, as in Lind and Ramondo (2018). To keep the expression simple, I focus on the case of independent distribution. A worker ω then works in the current occupation, earns income, consumes and derives logarithmic utility, and then chooses the next period's occupation with discount rate ι. When choosing the next period occupation o', she pays an ad-valorem switching cost $\chi_{i,oo',t}$ in terms of consumption unit that depends on current occupation o. She consumes her income in each period. Thus, worker ω who currently works in occupation o_t maximizes the following objective function over the future stream of utilities by choosing occupations $\{o_s\}_{s=t+1}^{\infty}$:

$$E_t \sum_{s=t}^{\infty} \left(\frac{1}{1+\iota}\right)^{s-t} \left[\ln\left(C_{i,o_s,s}\right) + \ln\left(1 - \chi_{i,o_s o_{s+1},s}\right) + \ln\left(Z_{i,o_{s+1},s}(\omega)\right)\right]$$

where $C_{i,o,s}$ is a consumption bundle when working in occupation o in period $s \geq t$, and E_t is the expectation conditional on the value of $Z_{i,o_t,t}(\omega)$. Each worker owns occupation-specific labor endowment $l_{i,o,t}$. I assume that her income is comprised of labor income $w_{i,o,t}$ and occupation-specific ad-valorem government transfer with rate $T_{i,o,t}$. Given the consumption price $P^G_{i,t}$, the budget constraint is

$$P^G_{i,t} C_{i,o,t} = w_{i,o,t} l_{i,o,t} \left(1 + T_{i,o,t}\right)$$

for any worker, with $P^G_{i,t}$ being the price index of the non-robot good G.

By linearity of expectation,

$$\begin{aligned} & E_t \sum_{s=t}^{\infty} \left(\frac{1}{1+\iota}\right)^{s-t} \left[\ln\left(C_{i,o_s,s}\right) + \ln\left(1 - \chi_{i,o_s o_{s+1},s}\right) + \ln\left(Z_{i,o_{s+1},s}(\omega)\right)\right] \\ = & \sum_{s=t}^{\infty} \left(\frac{1}{1+\iota}\right)^{s-t} \left[E_t \ln\left(C_{i,o_s,s}\right) + E_t \ln\left(1 - \chi_{i,o_s o_{s+1},s}\right) + E_t \ln\left(Z_{i,o_{s+1},s}(\omega)\right)\right]. \end{aligned}$$

By monotone transformation with exponential function,

$$\exp\left\{\sum_{s=t}^{\infty}\left(\frac{1}{1+\iota}\right)^{s-t}\left[E_t\ln\left(C_{i,o_s,s}\right)+E_t\ln\left(1-\chi_{i,o_so_{s+1},s}\right)+E_t\ln\left(Z_{i,o_{s+1},s}\left(\omega\right)\right)\right].\right\}$$
$$=\prod_{s=t}^{\infty}\exp\left\{\left(\frac{1}{1+\iota}\right)^{s-t}\left[E_t\ln\left(C_{i,o_s,s}\right)+E_t\ln\left(1-\chi_{i,o_so_{s+1},s}\right)+E_t\ln\left(Z_{i,o_{s+1},s}\left(\omega\right)\right)\right]\right\}.$$

Write the value function conditional on the realization of shocks at period t as follows:

$$V_{i,o_t,t}\left(\omega\right)\equiv\max_{\{o_s\}_{s=t+1}^{\infty}}\prod_{s=t}^{\infty}\exp\left\{\left(\frac{1}{1+\iota}\right)^{s-t}\left[E_t\ln\left(C_{i,o_s,s}\right)+E_t\ln\left(1-\chi_{i,o_so_{s+1},s}\right)+E_t\ln\left(Z_{i,o_{s+1},s}\left(\omega\right)\right)\right]\right\}.$$

I apply Bellman's principle of optimality as follows:

$$\begin{aligned}
&V_{i,o_t,t}\left(\omega\right)\\
&=\max_{\{o_s\}_{s=t+1}^{\infty}}\prod_{s=t}^{\infty}\exp\left\{\left(\frac{1}{1+\iota}\right)^{s-t}\left[E_t\ln\left(C_{i,o_s,s}\right)+E_t\ln\left(1-\chi_{i,o_so_{s+1},s}\right)+E_t\ln\left(Z_{i,o_{s+1},s}\left(\omega\right)\right)\right]\right\}\\
&=\max_{o_{t+1}}\exp\left\{\ln\left(C_{i,o_t,t}\right)+\ln\left(1-\chi_{i,o_to_{t+1},t}\right)+\ln\left(Z_{i,o_{t+1},t}\left(\omega\right)\right)\right\}\times\\
&\max_{\{o_s\}_{s=t+2}^{\infty}}\prod_{s=t+1}^{\infty}\exp\left\{\left(\frac{1}{1+\iota}\right)^{s-(t+1)}\left[E_{t+1}\ln\left(C_{i,o_s,s}\right)+E_{t+1}\ln\left(1-\chi_{i,o_so_{s+1},s}\right)+E_{t+1}\ln\left(Z_{i,o_{s+1},s}\left(\omega\right)\right)\right]\right\}\\
&=\max_{o_{t+1}}\exp\left\{\ln\left(Z_{i,o_t,t}\left(\omega\right)\right)+\ln\left(C_{i,o_t,t}\right)+\ln\left(1-\chi_{i,o_to_{t+1},t}\right)\right\}V_{i,o_{t+1},t+1},
\end{aligned}$$

where $V_{i,o_t,t}$ is the unconditional expected value function $V_{i,o_t,t}\equiv E_{t-1}V_{i,o_t,t}\left(\omega\right)$. Changing the notation from (o_t,o_{t+1}) into (o,o'), I have

$$V_{i,o,t}\left(\omega\right)=\max_{o'}C_{i,o,t}\left(1-\chi_{i,oo',t}\right)Z_{i,o',t}\left(\omega\right)V_{i,o',t+1}.$$

Solving the worker's maximization problem is equivalent to finding:

$$\begin{aligned}
\mu_{i,oo',t}&\equiv\Pr\left(\text{worker }\omega\text{ in }o\text{ chooses occupation }o'\right)\\
&=\Pr\left(\max_{o''}C_{i,o,t}\left(1-\chi_{i,oo'',t}\right)Z_{i,o'',t}\left(\omega\right)V_{i,o'',t+1}\leq C_{i,o,t}\left(1-\chi_{i,oo',t}\right)Z_{i,o',t}\left(\omega\right)V_{i,o',t+1}\right).
\end{aligned}$$

By the independent Fréchet assumption, we have the maximum value distribution

$$\begin{aligned}
\Pr\left(\max_{o''} C_{i,o,t}\left(1-\chi_{i,oo',t}\right) Z_{i,o',t}\left(\omega\right) V_{i,o',t+1} \le v\right) &= \prod_{o'} \Pr\left(Z_{i,o',t}\left(\omega\right) \le \frac{v}{C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}}\right) \\
&= \prod_{o''} \exp\left(\left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi} v^{-\phi}\right) \\
&= \exp\left(\sum_{o''} \left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi} v^{-\phi}\right).
\end{aligned}$$

Therefore, the conditional choice probability satisfies, again by the independent Fréchet assumption,

$$\begin{aligned}
&\mu_{i,oo',t} \\
&= \int_0^{\infty} \Pr\left(\max_{o''\neq o'} C_{i,o,t}\left(1-\chi_{i,oo'',t}\right) Z_{i,o',t}\left(\omega\right) V_{i,o'',t+1} \le v\right) d\Pr\left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) Z_{i,o',t}\left(\omega\right) V_{i,o',t+1} \ge v\right) \\
&= \int_0^{\infty} \exp\left(\sum_{o''\neq o'} \left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi} v^{-\phi}\right) \times \\
&\qquad \left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi} \exp\left(\left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi} v^{-\phi}\right) \times \left(-\phi v^{-\phi-1}\right) dv \\
&= \frac{\left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi}}{\sum_{o''} \left(C_{i,o,t}\left(1-\chi_{i,oo'',t}\right) V_{i,o'',t+1}\right)^{\phi}} \times \\
&\qquad \int_0^{\infty} \exp\left(\sum_{o'''} \left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi} v^{-\phi}\right) \sum_{o''} \left(C_{i,o,t}\left(1-\chi_{i,oo'',t}\right) V_{i,o'',t+1}\right)^{\phi} \times \left(-\phi v^{-\phi-1}\right) dv.
\end{aligned}$$

The last integral term is one by integration and the definition of distribution. Therefore, I arrive at

$$\mu_{i,oo',t} = \frac{\left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi}}{\sum_{o''} \left(C_{i,o,t}\left(1-\chi_{i,oo'',t}\right) V_{i,o'',t+1}\right)^{\phi}} = \frac{\left(\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi}}{\sum_{o''} \left(\left(1-\chi_{i,oo'',t}\right) V_{i,o'',t+1}\right)^{\phi}},$$

$$V_{i,o,t+1} = \widetilde{\Gamma} C_{i,o,t} \left(\sum_{o'} \left(\left(1-\chi_{i,oo',t+1}\right) V_{i,o',t+2}\right)^{\phi}\right)^{\frac{1}{\phi}}.$$

B.3 Relationship with Other Models of Automation

The model in Section 3 is general enough to nest models of automation in the previous literature. In particular, I show how the production functions (8) and (10) imply to specifications in AR and Humlum (2019). Throughout Section B.3, I fix country i and focus on steady states and thus drop subscripts i and t since the discussion is about individual producer's production function.

B.3.1 Relationship with the model in Acemoglu and Restrepo (2020, AR)

Following AR that abstract from occupations, I drop occupations by setting $O = 1$ in this paragraph. Therefore, the EoS between occupations β plays no role, and $\theta_o = \theta$ is a unique value. AR show that the unit cost (hence the price given perfect competition) is written as

$$p^{AR} \equiv \frac{1}{\widetilde{A}} \left[(1-\widetilde{a}) \frac{w}{A^L} + \widetilde{a} \frac{c^R}{A^R} \right]^{\alpha_L} r^{1-\alpha_L},$$

for each sector and location (See AR, Appendix A1, equation A5). In this equation, c^R is the steady state marginal cost of robot capital defined in equation (B.25) and A^L and A^R represent per-unit efficiency of labor and robots, respectively. In Lemma B.1 below, I prove that my model implies a unit cost function that is strict generalization of p^{AR} with proper modification to the shock terms and parameter configuration. I begin with the modification that allows per-unit efficiency terms in my model.

Definition B.1. For labor and robot per-unit efficiency terms $A^L > 0$ and $A^R > 0$ respectively, modified robot task space $\widetilde{a}$ and TFP term $\widetilde{A}$ are

$$\widetilde{a} \equiv \frac{a \left(A^L\right)^{\theta-1}}{a \left(A^L\right)^{\theta-1} + (1-a) \left(A^R\right)^{\theta-1}}, \tag{B.1}$$

$$\widetilde{A} \equiv \frac{A}{\left[(1-\widetilde{a}) \left(A^L\right)^{\theta-1} + \widetilde{a} \left(A^R\right)^{\theta-1}\right]}. \tag{B.2}$$

Lemma B.1. *Set the number of occupations $O = 1$. In the steady state,*

$$p^G = \frac{1}{\widetilde{A}} \left[(1-\widetilde{a}) \left(\frac{w}{A^L}\right)^{1-\theta} + \widetilde{a} \left(\frac{c^R}{A^R}\right)^{1-\theta} \right]^{\frac{\alpha_L}{1-\theta}} \left(p^G\right)^{\alpha_M} r^{1-\alpha_M-\alpha_L}. \tag{B.3}$$

Proof. Note that modified robot task space (B.1) and modified TFP (B.2) can be inverted to have

$$a \equiv \frac{\widetilde{a} \left(A^R\right)^{\theta-1}}{(1-\widetilde{a}) \left(A^L\right)^{\theta-1} + \widetilde{a} \left(A^R\right)^{\theta-1}}, \tag{B.4}$$

$$A \equiv \left[(1-\widetilde{a}) \left(A^L\right)^{\theta-1} + \widetilde{a} \left(A^R\right)^{\theta-1} \right] \widetilde{A}. \tag{B.5}$$

Cost minimization problem with the production functions (8) and (10) and perfect competition imply

$$p^G = \frac{1}{A} \left(P^O\right)^{\alpha_L} p^{\alpha_M} r^{1-\alpha_L-\alpha_M},$$

and

$$P^O = \left[(1-a) w^{1-\theta} + a^{1-\theta} \right]^{\frac{1}{1-\theta}},$$

where P^O is the unit cost of tasks performed by labor and robots. Substituting equations (B.4) and (B.5) and rearranging, I have

$$p^G = \frac{1}{\widetilde{A}} \left(\widetilde{P^O}\right)^{\alpha_L} \left(p^G\right)^{\alpha_M} r^{1-\alpha_L-\alpha_M},$$

where $\widetilde{P^O}$ is the cost of the tasks performed by labor and robots:

$$\widetilde{P^O} = \left[(1-\widetilde{a}) \left(\frac{w}{A^L}\right)^{1-\theta} + a \left(\frac{c^R}{A^R}\right)^{1-\theta} \right]^{\frac{1}{1-\theta}}.$$

□

Lemma B.1 immediately implies the following corollary that shows that the steady state modified unit cost (B.3) strictly nests the unit cost formulation of AR as a special case of Leontief occupation aggregation.

Corollary B.1. *Suppose $\alpha_M = 0$. Then as $\theta \to 0$, $p^G \to p^{AR}$.*

B.3.2 Relationship with the model in Humlum (2019)

I show that production functions (8) and (10) nest the production function used by Humlum (2019). Since the setting of Humlum (2019) does not have non-robot capital, in this section, I simplify the notation for robot capital K^R by dropping the superscript and denote as K. For each firm in each period, Humlum (2019) specifies

$$Q^D = \exp\left[\varphi_H^D + \gamma_H^D K\right] \left[\sum_o \left(\exp\left[\varphi_o^D + \gamma_o^D K\right] \right)^{\frac{1}{\beta}} (L_o)^{\frac{\beta-1}{\beta}} \right]^{\frac{\beta}{\beta-1}}, \tag{B.6}$$

where $K = \{0, 1\}$ is a binary choice, $\varphi_H^D, \gamma_H^D, \varphi_o^D$ and γ_o^D are parameters, and superscript D represents the discrete adoption problem of Humlum (2019). As normalization, suppose that

$$\sum_o \exp\left(\varphi_o^D + \gamma_o^D K\right) = 1.$$

I will start from production function (8) and (10), place restrictions, and arrive at equation (B.6). As a key observation, relative to the discrete choice of robot adoption in Humlum (2019), the continuous choice of robot *quantity* in production function (10) allows significant flexibility. In this paragraph, I assume away with intermediate inputs. This is because Humlum (2019) assumes that intermediate inputs enter in an element of CES, while production function (8) implies that intermediate inputs enter as an element of the Cobb-Douglas function.

Now, given our production functions (8) and (10), suppose producers follow the binary decision rule defined below.

Definition B.2. A binary decision rule of a producer is that producers can choose between two choices: adopting robots $K = 1$ or not $K = 0$. If they choose $K = 1$, they adopt robots at the same unit as labor $K_o = L_o \geq 0$ for all occupation o. If they choose $K = 0$, $K_o = 0$ for all o.

Note that the binary decision rule is nested in the original choice problem from $K_o^R \geq 0$ for each o. Set

$$A_o^D\left(K^R\right) \equiv \begin{cases} A_o\left((1-a_o)^{\frac{1}{\theta}} + (a_o)^{\frac{1}{\theta}}\right)^{\frac{\theta}{\theta-1}(\beta-1)} & \text{if } K^R = L_o \\ A_o\left(1-a_o\right)^{\frac{1}{\theta-1}(\beta-1)} & \text{if } K^R = 0 \end{cases}.$$

Then I have

$$Q = \left[\sum_o \left(A_o^D\left(K_o\right)\right)^{\frac{1}{\beta}} \left(L_o\right)^{\frac{\beta-1}{\beta}}\right]^{\frac{\beta}{\beta-1}}.$$

To normalize, define

$$\widetilde{A_o^D} \equiv \left(\sum_o A_o^D\left(K_o\right)\right)^{\frac{1}{\beta-1}}$$

and

$$a_o^D\left(K_o^R\right) \equiv \frac{A_o^D\left(K_o\right)}{\sum_{o'} A_{o'}^D\left(K_{o'}\right)}.$$

Then I have

$$Q = \widetilde{A_o^D} \left[\sum_o \left(a_o^D \left(K_o \right) \right)^{\frac{1}{\beta}} \left(L_o \right)^{\frac{\beta-1}{\beta}} \right]^{\frac{\beta}{\beta-1}}. \tag{B.7}$$

Finally, let

$$A_{o,0} \equiv \left[\exp \left(\varphi_H^D + \varphi_o^D \right) \right]^{\frac{\theta_o - 1}{\beta - 1}}$$

and

$$A_{o,1} \equiv \left[\left(\exp \left(\varphi_H^D + \varphi_o^D + \gamma_H^D + \gamma_o^D \right) \right)^{\frac{1}{\theta_o} \frac{\theta_o - 1}{\beta - 1}} - \left(\exp \left(\varphi_H^D + \varphi_o^D \right) \right)^{\frac{1}{\theta_o} \frac{\theta_o - 1}{\beta - 1}} \right]^{\theta_o}.$$

and also let A_o and a_o satisfy

$$A_o = \left(A_{o,0} + A_{o,1} \right)^{\frac{\beta-1}{\theta_o - 1}} \tag{B.8}$$

and

$$a_o = \frac{A_{o,1}}{A_{o,0} + A_{o,1}}. \tag{B.9}$$

Then one can substitute equations (B.8) and (B.9) to equation (B.7) and confirm that $Q = Q^D$. Summarizing the discussion above, I have the result that my model can be restricted to produce the production side of the model of Humlum (2019) as follows.

Lemma B.2. *Suppose that (i) producers follow the binary decision rule in Definition B.2 and that (ii) occupation productivity A_o and robot task space a_o satisfy equations (B.8) and (B.9) for each o. Then $Q = Q^D$.*

B.4 Equilibrium Characterization

To characterize the producer problem, I show the static optimization conditions and then the dynamic ones. To solve for the static problem of labor, intermediate goods, and non-robot capital, consider the FOCs of equation (12)

$$p_{i,t}^G \alpha_{i,L} \frac{Y_{i,t}^G}{T_{i,t}^O} \left(b_{i,o,t} \frac{T_{i,t}^O}{T_{i,o,t}^O} \right)^{\frac{1}{\beta}} \left(\left(1 - a_{o,t} \right) \frac{T_{i,o,t}^O}{L_{i,o,t}} \right)^{\frac{1}{\theta_o}} = w_{i,o,t}, \tag{B.10}$$

where $T^O_{i,t}$ is the aggregated occupations $T^O_{i,t} \equiv \left[\sum_o \left(T^O_{i,o,t}\right)^{(\beta-1)/\beta}\right]^{\beta/(\beta-1)}$,

$$p^G_{i,t}\alpha_{i,M}\frac{Y^G_{i,t}}{M_{i,t}}\left(\frac{M_{i,t}}{M_{li,t}}\right)^{\frac{1}{\varepsilon}} = p^G_{li,t}, \tag{B.11}$$

and

$$p^G_{i,t}\alpha_{i,K}\frac{Y^G_{i,t}}{K_{i,t}} = r_{i,t}, \tag{B.12}$$

where $\alpha_{i,K} \equiv 1 - \alpha_{i,L} - \alpha_{i,M}$. Note also that by the envelope theorem,

$$\frac{\partial \pi_{i,t}\left(\left\{K^R_{i,o,t}\right\}\right)}{\partial K^R_{i,o,t}} = p^G_{i,t}\frac{\partial Y_{i,t}}{\partial K^R_{i,o,t}} = p^G_{i,t}\left(\alpha_L \frac{Y^G_{i,t}}{T^O_{i,t}}\left(b_{i,o,t}\frac{T^O_{i,t}}{T^O_{i,o,t}}\right)^{\frac{1}{\beta}}\left(a_{o,t}\frac{T^O_{i,o,t}}{K^R_{i,o,t}}\right)^{\frac{1}{\theta}}\right). \tag{B.13}$$

Another static problem of producers is robot purchase. Define the "before-integration" robot aggregate $Q^{R,BI}_{i,o,t} \equiv \left[\sum_l \left(Q^R_{li,o,t}\right)^{\frac{\varepsilon^R-1}{\varepsilon^R}}\right]^{\frac{\varepsilon^R}{\varepsilon^R-1}}$ and the corresponding price index $P^{R,BI}_{i,o,t}$. By the first order condition with respect to $Q^R_{li,o,t}$ for equation (14), I have $p^R_{li,o,t}Q^R_{li,o,t} = \left(\frac{p^R_{li,o,t}}{P^{R,BI}_{i,o,t}}\right)^{1-\varepsilon^R} P^{R,BI}_{i,o,t}Q^{R,BI}_{i,o,t}$, and $P^{R,BI}_{i,o,t}Q^{R,BI}_{i,o,t} = \alpha P^R_{i,o,t}Q^R_{i,o,t}$. Thus $p^R_{li,o,t}Q^R_{li,o,t} = \alpha\left(\frac{p^R_{li,o,t}}{P^{R,BI}_{i,o,t}}\right)^{1-\varepsilon^R} P^R_{i,o,t}Q^R_{i,o,t}$. Hence

$$Q^R_{li,o,t} = \alpha\left(p^R_{li,o,t}\right)^{-\varepsilon^R}\left(P^{R,BI}_{i,o,t}\right)^{\varepsilon^R-1} P^R_{i,o,t}Q^R_{i,o,t}.$$

Writing $P^R_{i,o,t} = \left(P^{R,BI}_{i,o,t}\right)^{\alpha^R}\left(P_{i,t}\right)^{1-\alpha^R}$, I have

$$Q^R_{li,o,t} = \alpha\left(\frac{p^R_{li,o,t}}{P^{R,BI}_{i,o,t}}\right)^{-\varepsilon^R}\left(\frac{P^{R,BI}_{i,o,t}}{P_{i,t}}\right)^{-\left(1-\alpha^R\right)} Q^R_{i,o,t}.$$

Alternatively, one can define the robot price index by $\widetilde{P}^R_{i,o,t} = \alpha^{\frac{1}{\varepsilon^R}}\left(P^{R,BI}_{i,o,t}\right)^{\frac{\varepsilon^R-\left(1-\alpha^R\right)}{\varepsilon^R}} P_{i,t}^{\frac{1-\alpha^R}{\varepsilon^R}}$ and show

$$Q^R_{li,o,t} = \left(\frac{p^R_{li,o,t}}{\widetilde{P}^R_{i,o,t}}\right)^{-\varepsilon^R} Q^R_{i,o,t}, \tag{B.14}$$

which is a standard gravity representation of robot trade.

To solve the dynamic problem, set up the (current-value) Lagrangian function for non-robot goods producers

$$\mathcal{L}_{i,t} = \sum_{t=0}^{\infty} \left\{ \left(\frac{1}{1+\iota} \right)^t \left[\pi_{i,t} \left(\left\{ K^R_{i,o,t} \right\}_o \right) - \sum_{l,o} \left(p^R_{li,o,t} \left(1 + u_{li,t}\right) Q^R_{li,o,t} + P^G_{i,t} I^{int}_{i,o,t} + \gamma P^R_{i,o,t} Q^R_{i,o,t} \frac{Q^R_{i,o,t}}{K^R_{i,o,t}} \right) \right] \right\}.$$
$$- \lambda^R_{i,o,t} \left\{ K^R_{i,o,t+1} - (1-\delta) K^R_{i,o,t} - Q^R_{i,o,t} \right\}$$

Taking the FOC with respect to the hardware from country l, $Q^R_{li,o,t}$, I have

$$p^R_{li,o,t} \left(1 + u_{li,t}\right) + 2\gamma P^R_{i,o,t} \left(\frac{Q^R_{i,o,t}}{K^R_{i,o,t}} \right) \frac{\partial Q^R_{i,o,t}}{\partial Q^R_{li,o,t}} = \lambda^R_{i,o,t} \frac{\partial Q^R_{i,o,t}}{\partial Q^R_{li,o,t}}. \tag{B.15}$$

Taking the FOC with respect to the integration input $I^{int}_{i,o,t}$, I have

$$P^G_{i,t} + 2\gamma P^R_{i,o,t} \left(\frac{Q^R_{i,o,t}}{K^R_{i,o,t}} \right) \frac{\partial Q^R_{i,o,t}}{\partial I^{int}_{i,o,t}} = \lambda^R_{i,o,t} \frac{\partial Q^R_{i,o,t}}{\partial I^{int}_{i,o,t}}, \tag{B.16}$$

Taking the FOC with respect to $K^R_{i,o,t+1}$, I have

$$\left(\frac{1}{1+\iota} \right)^{t+1} \left[\frac{\partial \pi_{i,t+1} \left(\left\{ K^R_{i,o,t+1} \right\}_o \right)}{\partial K^R_{i,o,t+1}} + \gamma P^R_{i,o,t+1} \left(\frac{Q^R_{i,o,t+1}}{K^R_{i,o,t+1}} \right)^2 + (1-\delta) \lambda^R_{i,o,t+1} \right] - \left(\frac{1}{1+\iota} \right)^t \lambda^R_{i,o,t} = 0.$$

Rearranging, I obtain the following Euler equation.

$$\lambda^R_{i,o,t} = \frac{1}{1+\iota} \left[(1-\delta) \lambda^R_{i,o,t+1} + \frac{\partial}{\partial K^R_{i,o,t+1}} \pi_{i,t+1} \left(\left\{ K^R_{i,o,t+1} \right\} \right) + \gamma p^R_{i,o,t+1} \left(\frac{Q^R_{i,o,t+1}}{K^R_{i,o,t+1}} \right)^2 \right]. \tag{B.17}$$

Turning to the demand for non-robot good, I will characterize bilateral intermediate good trade demand and total expenditure. Write $X^G_{j,t}$ the total purchase quantity (but not value) of good G in country j in period t. By equation (11), the bilateral trade demand is given by

$$p^G_{ij,t} Q^G_{ij,t} = \left(\frac{p^G_{ij,t}}{P^G_{j,t}} \right)^{1-\varepsilon} P^G_{j,t} X^G_{j,t}, \tag{B.18}$$

for any i, j, and t. In this equation, $P^G_{j,t} X^G_{j,t}$ is the total expenditures on non-robot goods. The total expenditure is the sum of final consumption $I_{j,t}$, payment to intermediate goods $\alpha_M p^G_{j,t} Y^G_{j,t}$, input to robot productions $\sum_o P^G_{j,t} I^R_{j,o,t} = \sum_{o,k} p^R_{jk,o,t} Q^R_{jk,o,t}$, and payment to robot integration $\sum_o P^G_{j,t} I^{int}_{j,o,t} = \left(1-\alpha^R\right) \sum_o P^R_{j,o,t} Q^R_{j,o,t}$. Hence

$$P^G_{j,t} X^G_{j,t} = I_{j,t} + \alpha_M p^G_{j,t} Y^G_{j,t} + \sum_{o,k} p^R_{jk,o,t} Q^R_{jk,o,t} + \left(1-\alpha^R\right) \sum_o P^R_{j,o,t} Q^R_{j,o,t}.$$

For country j and period t, by substituting into income $I_{j,t}$ the period cash flow of non-robot good producer that satisfies

$$\Pi_{j,t} \equiv \pi_{j,t}\left(\left\{K^R_{j,o,t}\right\}_o\right) - \sum_{i,o}\left(p^R_{ij,o,t}\left(1+u_{ij,t}\right) Q^R_{ij,o,t} + \sum_o P^G_{j,t} I^{int}_{j,o,t} + \gamma P^R_{j,o,t} Q^R_{j,o,t}\left(\frac{Q^R_{j,o,t}}{K^R_{j,o,t}}\right)\right)$$

and robot tax revenue $T_{j,t} = \sum_{i,o} u_{ij,t} p^R_{ij,o,t} Q^R_{ij,o,t}$, I have

$$I_{j,t} = \left(1-\alpha_M\right) \sum_k p^G_{jk,t} Q^G_{jk,t} - \left(\sum_{i,o} p^R_{ij,o,t} Q^R_{ij,o,t} + \left(1-\alpha^R\right) \sum_o P^R_{j,o,t} Q^R_{j,o,t}\right), \tag{B.19}$$

or in terms of variables in the definition of equilibrium,

$$I_{j,t} = \left(1-\alpha_M\right) \sum_k p^G_{jk,t} Q^G_{jk,t} - \frac{1}{\alpha^R} \sum_{i,o} p^R_{ij,o,t} Q^R_{ij,o,t}.$$

Hence, the total expenditure measured in terms of the production side as opposed to income side is

$$P^G_{j,t} X^G_{j,t} = \sum_k p^G_{jk,t} Q^G_{jk,t} - \sum_{i,o} p^R_{ij,o,t} Q^R_{ij,o,t}\left(1+\gamma \frac{Q^R_{ij,o,t}}{K^R_{j,o,t}}\right). \tag{B.20}$$

Note that this equation embeds the balanced-trade condition. By substituting equation (B.20) into equation (B.18), I have

$$p^G_{ij,t} Q^G_{ij,t} = \left(\frac{p^G_{ij,t}}{P^G_{j,t}}\right)^{1-\varepsilon^G} \left(\sum_k p^G_{jk,t} Q^G_{jk,t} + \sum_{k,o} p^R_{jk,o,t} Q^R_{jk,o,t} - \sum_{i,o} p^R_{ij,o,t} Q^R_{ij,o,t}\right). \tag{B.21}$$

The good and robot-o market-clearing conditions are given by,

$$Y^R_{i,t} = \sum_j Q^G_{ij,t} \tau^G_{ij,t}, \tag{B.22}$$

for all i and t, and

$$p^R_{i,o,t} = \frac{P^G_{i,t}}{A^R_{i,o,t}} \tag{B.23}$$

for all i, o, and t, respectively.

Conditional on state variables $\boldsymbol{S}_t = \left\{\boldsymbol{K}^R_t, \boldsymbol{\lambda}^R_t, \boldsymbol{L}_t, \boldsymbol{V}_t\right\}$, equations (4), (B.10), (B.15), (B.21), (B.22), and (B.23) characterize the temporary equilibrium $\left\{\boldsymbol{p}^G_t, \boldsymbol{p}^R_t, \boldsymbol{w}_t, \boldsymbol{Q}^G_t, \boldsymbol{Q}^R_t, \boldsymbol{L}_t\right\}$. In addition, conditional on initial conditions $\left\{\boldsymbol{K}^R_0, \boldsymbol{L}_0\right\}$, equations (13), (B.17), and transversality condition (16) characterize the sequential equilibrium.

Finally, the steady state conditions are given by imposing the time-invariance condition to equations (13) and (B.17):

$$Q^R_{i,o} = \delta K^R_{i,o}, \tag{B.24}$$

$$\frac{\partial}{\partial K^R_{i,o}} \pi_i\left(\left\{K^R_{i,o}\right\}\right) = (\iota + \delta)\,\lambda^R_{i,o} - \sum_l \gamma p^R_{li,o}\left(\frac{Q^R_{li,o}}{K^R_{i,o}}\right)^2 \equiv c^R_{i,o}. \tag{B.25}$$

Note that equation (B.25) can be interpreted as the flow marginal profit of capital must be equalized to the marginal cost term. Thus I define the steady state marginal cost of robot capital $c^R_{i,o}$ from the right-hand side of equation (B.25). Note that if there is no adjustment cost $\gamma = 0$, the steady state Euler equation (B.25) implies

$$\frac{\partial}{\partial K^R_{i,o}} \pi_i\left(\left\{K^R_{i,o}\right\}\right) = c^R_{i,o} = (\iota + \delta)\,\lambda^R_{i,o},$$

which states that the marginal profit of capital is the user cost of robots in the steady state (Hall and Jorgenson, 1967).

B.5 Proof of Proposition 1

The proof takes the following four conceptual steps. First, I will write the real wage change $\left(\widehat{w_{i,o}/P^G_i}\right)$ in terms of the weighted average of relative price changes, making use of the fact that the sum of shares equals one. Second, I rewrite relative price change into layers of relative price changes with the technique of addition and subtraction. Third, I show that each layer of relative price changes is a change of relevant input or trade shares controlled by elasticity substitution. In other words, an input or trade shares reveals a layer of relative price changes. Finally, I make use of the fact that the sum of shares do not change after the shock to arrive at equation (24).

Cost minimization given production functions (8), (10), and (11) imply

$$\widehat{\left(\frac{w_{i,o}}{P_i^G}\right)} = \frac{1}{1-\alpha_{i,M}} \sum_l \widetilde{x}^G_{li,t_0} \sum_{o'} \widetilde{x}^O_{l,o',t_0} \left[\widetilde{x}^L_{l,o',t_0} \left(\widehat{w_{i,o}} - \widehat{w_{l,o'}}\right) + \left(1-\widetilde{x}^L_{l,o',t_0}\right) \left(\widehat{w_{i,o}} - \left(\frac{\widehat{A^K_{l,o'}}}{1-\theta_o} + \widehat{c^R_{l,o'}}\right)\right)\right]. \tag{B.26}$$

Note that by additions and subtractions, I can rewrite

$$\begin{aligned}\widehat{w_{i,o}} - \widehat{w_{l,o'}} = & \left(\widehat{w_{i,o}} - \widehat{P^O_{i,o}}\right) - \left(\widehat{w_{l,o'}} - \widehat{P^O_{l,o'}}\right) + \left(\widehat{P^O_{i,o}} - \widehat{P^O_i}\right) - \left(\widehat{P^O_{l,o'}} - \widehat{P^O_l}\right) \\ & + \left(\widehat{P^O_i} - \widehat{p^G_i}\right) - \left(\widehat{P^O_l} - \widehat{p^G_l}\right) + \left(\widehat{p^G_i} - \widehat{P^G_i}\right) - \left(\widehat{p^G_l} - \widehat{P^G_i}\right),\end{aligned} \tag{B.27}$$

where $\widehat{P^O_{i,o}}$, $\widehat{P^O_i}$, and $\widehat{P^G_i}$ are the price (cost) index of occupation o, occupation aggregate $T^O_{i,t} \equiv \left[\sum_o \left(T^O_{i,o,t}\right)^{(\beta-1)/\beta}\right]^{\beta/(\beta-1)}$, and consumption of non-rogot good G, and

$$\begin{aligned}\widehat{w_{i,o}} - \left(\frac{\widehat{A^K_{l,o'}}}{1-\theta} + \widehat{c^R_{l,o'}}\right) = & \left(\widehat{w_{i,o}} - \widehat{P^O_{i,o}}\right) - \left(\frac{\widehat{A^K_{l,o'}}}{1-\theta} + \widehat{c^R_{l,o'}} - \widehat{P^O_{l,o'}}\right) + \left(\widehat{P^O_{i,o}} - \widehat{P^O_i}\right) - \left(\widehat{P^O_{l,o'}} - \widehat{P^O_l}\right) \\ & + \left(\widehat{P^O_i} - \widehat{p^G_i}\right) - \left(\widehat{P^O_l} - \widehat{p^G_l}\right) + \left(\widehat{p^G_i} - \widehat{P^G_i}\right) - \left(\widehat{p^G_l} - \widehat{P^G_i}\right).\end{aligned} \tag{B.28}$$

Note that the cost minimizing input and trade shares satisfy

$$\begin{cases} \widehat{\widetilde{x}^L_{i,o}} = (1-\theta_o)\left(\widehat{w_{i,o}} - \widehat{P^O_{i,o}}\right), \ \widehat{1-\widetilde{x}^L_{i,o}} = \widehat{A^K_{i,o}} + (1-\theta_o)\left(\widehat{c^R_{i,o}} - \widehat{P^O_{i,o}}\right) \\ \widehat{\widetilde{x}^O_{i,o}} = (1-\beta)\left(\widehat{P^O_{i,o}} - \widehat{P^O_i}\right), \ \widehat{\widetilde{x}^G_{li}} = (1-\varepsilon)\left(\widehat{p^G_l} - \widehat{P^G_i}\right) \end{cases} \tag{B.29}$$

Combined with the Cobb-Douglas assumption of production function (8), equations (B.27), (B.28), and (B.29) imply

$$\widehat{w_{i,o}} - \widehat{w_{l,o'}} = \frac{\widehat{\widetilde{x}^L_{i,o}}}{1-\theta_o} - \frac{\widehat{\widetilde{x}^L_{l,o'}}}{1-\theta_o} + \frac{\widehat{\widetilde{x}^O_{i,o}}}{1-\beta} - \frac{\widehat{\widetilde{x}^O_{l,o'}}}{1-\beta} + \frac{\widehat{\widetilde{x}^G_{ii}}}{1-\varepsilon} - \frac{\widehat{\widetilde{x}^G_{li}}}{1-\varepsilon}$$

$$\widehat{w_{i,o}} - \left(\frac{\widehat{A^K_{l,o'}}}{1-\theta_o} + \widehat{c^R_{l,o'}}\right) = \frac{\widehat{\widetilde{x}^L_{i,o}}}{1-\theta_o} - \frac{\widehat{\left(1-\widetilde{x}^L_{l,o'}\right)}}{1-\theta_o} + \frac{\widehat{\widetilde{x}^O_{i,o}}}{1-\beta} - \frac{\widehat{\widetilde{x}^O_{l,o'}}}{1-\beta} + \frac{\widehat{\widetilde{x}^G_{ii}}}{1-\varepsilon} - \frac{\widehat{\widetilde{x}^G_{li}}}{1-\varepsilon}.$$

Substituting these in equation (B.26) and using the facts that $\widetilde{x}^L_{i,o,t_0}\widehat{\widetilde{x}^L_{i,o}} + \left(1 - \widetilde{x}^L_{i,o,t_0}\right)\left(\widehat{1 - \widetilde{x}^L_{i,o}}\right) = 0$ for all i and o, $\sum_o \widetilde{x}^O_{i,o,t_0}\widehat{\widetilde{x}^O_{i,o}} = 0$, and $\sum_l \widetilde{x}^G_{li,t_0}\widehat{\widetilde{x}^G_{li}} = 0$ for all i, I have equation (24).

B.6 Proof of Proposition 2

To prove Proposition I follow the arguments made in Sections 2 and 3 of Newey and McFadden (1994). The proof consists of four sub results in the following Lemma. Proposition 2 can be obtained as a combination of the four results. The formal statement requires the following additional assumptions.

Assumption B.1. *(i) A function of* $\widetilde{\mathbf{\Theta}}$, $\mathbb{E}\left[H_o\left(\boldsymbol{\psi}^J_{t_1}\right)\nu_o\left(\widetilde{\mathbf{\Theta}}\right)\right] \neq 0$ *for any* $\widetilde{\mathbf{\Theta}} \neq \mathbf{\Theta}$. *(ii)* $\underline{\theta} \leq \theta_o \leq \overline{\theta}$ *for any* o, $\underline{\beta} \leq \beta \leq \overline{\beta}$, $\underline{\gamma} \leq \gamma \leq \overline{\gamma}$, *and* $\underline{\phi} \leq \phi \leq \overline{\phi}$ *for some positive values* $\underline{\theta}, \underline{\beta}, \underline{\gamma}, \underline{\phi}, \overline{\theta}, \overline{\beta}, \overline{\gamma}, \overline{\phi}$. *(iii)* $\mathbb{E}\left[\sup_{\mathbf{\Theta}} \| H_o\left(\boldsymbol{\psi}^J_{t_1}\right)\nu_o\left(\widetilde{\mathbf{\Theta}}\right) \|\right] < \infty$. *(iv)* $\mathbb{E}\left[\| H_o\left(\boldsymbol{\psi}^J_{t_1}\right)\nu_o\left(\widetilde{\mathbf{\Theta}}\right) \|^2\right] < \infty$ *(v)* $\mathbb{E}\left[\sup_{\mathbf{\Theta}} \| H_o\left(\boldsymbol{\psi}^J_{t_1}\right)\nabla_{\widetilde{\mathbf{\Theta}}}\nu_o\left(\widetilde{\mathbf{\Theta}}\right) \|\right] < \infty$.

Lemma B.3. *Assume Assumptions 1 and B.1(i)-(iii).*

(a) The estimator of the form (29) is consistent.

Additionally, assume Assumptions B.1(iv)-(v).

(b) The estimator of the form (29) is asymptotically normal.

(c) $\sqrt{O}\left(\mathbf{\Theta}_{\boldsymbol{H}^*} - \mathbf{\Theta}\right) \to_d \mathcal{N}\left(0, \left(\boldsymbol{G}^\top \boldsymbol{\Omega}^{-1}\boldsymbol{G}\right)^{-1}\right)$*, and the asymptotic variance is the minimum of that of the estimator of the form (29) for any function* $\boldsymbol{H}$.

Proof. (a) I follow Theorems 2.6 of Newey and McFadden (1994), which implies that it suffices to show conditions (i)-(iv) of this theorem are satisfied. Assumption B.1(i) guarantees condition (i). Condition (ii) is implied by Assumption B.1(ii). Condition (iii) follows because all supply and demand functions in the model is continuous. Condition (iv) is implied by Assumption B.1(iii).

(b) I follow Theorem 3.4 of Newey and McFadden (1994), which implies that it suffices to show conditions (i)-(v) of this theorem are satisfied. Condition (i) is satisfied by Assumption B.1(i). Condition (ii) follows because all supply and demand functions in the model is continuously differentiable. Condition (iii) is implied by Assumption 1 and Assumption B.1(iv). Assumption B.1(v) implies condition (iv). Finally, the gradient vectors of the structural residual is linear independent, guaranteeing the non-singularity of the variance matrix and condition (v).

(c) By Theorem 3.4 of Newey and McFadden (1994), for an arbitrary IV-generating function $\boldsymbol{H}$, the asymptotic variance of the GMM estimator $\boldsymbol{\Theta}_{\boldsymbol{H}}$ is

$$\left(\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right]\right)^{-1}\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nu_o\nu_o^\top\left(H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\right)^\top\right]\left(\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right]\right)^{-1},$$

where $G_o \equiv \mathbb{E}\left[\nabla_{\boldsymbol{\Theta}}\nu_o\left(\boldsymbol{\Theta}\right)|\boldsymbol{\psi}_{t_1}^J\right]$. Therefore, if $H_o\left(\boldsymbol{\psi}_{t_1}^J\right) = Z_o \equiv \mathbb{E}\left[\nabla_{\boldsymbol{\Theta}}\nu_o\left(\boldsymbol{\Theta}\right)|\boldsymbol{\psi}_{t_1}^J\right]\mathbb{E}\left[\nu_o\left(\boldsymbol{\Theta}\right)\left(\nu_o\left(\boldsymbol{\Theta}\right)\right)^\top|\boldsymbol{\psi}_{t_1}^J\right]^{-1}$, then this expression is equal to $\left(\boldsymbol{G}^\top\boldsymbol{\Omega}^{-1}\boldsymbol{G}\right)^{-1}$, where

$$\boldsymbol{G} \equiv \mathbb{E}\left[\nabla_{\boldsymbol{\Theta}}\nu_o\left(\boldsymbol{\Theta}\right)\right] \text{ and } \boldsymbol{\Omega} \equiv \mathbb{E}\left[\nu_o\left(\boldsymbol{\Theta}\right)\left(\nu_o\left(\boldsymbol{\Theta}\right)\right)^\top\right].$$

To show that this variance is minimal, I will check that

$$\begin{aligned}\boldsymbol{\Delta} \equiv &\left(\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right]\right)^{-1}\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nu_o\nu_o^\top\left(H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\right)^\top\right]\left(\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right]\right)^{-1}\\ &-\left(\boldsymbol{G}^\top\boldsymbol{\Omega}^{-1}\boldsymbol{G}\right)^{-1}\end{aligned}$$

is positive semi-definite. In fact, note that

$$\begin{aligned}\boldsymbol{\Delta} = &\left(\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right]\right)^{-1}\times\\ &\left\{\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nu_o\nu_o^\top\left(H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\right)^\top\right]-\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right]\left(\boldsymbol{G}^\top\boldsymbol{\Omega}^{-1}\boldsymbol{G}\right)^{-1}\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right]\right\}\times\\ &\left(\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right]\right)^{-1}.\end{aligned}$$

Define

$$\widetilde{\nu}_o = H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nu_o - \mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nu_o\left(\left(G_o\right)^\top\Omega_o^{-1}\nu_o\right)^{-1}\right]\mathbb{E}\left(\left(G_o\right)^\top\Omega_o^{-1}\nu_o\right)^{-1}\left(G_o\right)^\top\Omega_o^{-1}\nu_o,$$

where $\Omega_o \equiv \mathbb{E}\left[\nu_o\left(\boldsymbol{\Theta}\right)\left(\nu_o\left(\boldsymbol{\Theta}\right)\right)^\top|\boldsymbol{\psi}_{t_1}^J\right]$. Applying Theorem 5.3 of Newey and McFadden (1994), I have

$$\mathbb{E}\left[\widetilde{\nu}_o\left(\widetilde{\nu}_o\right)^\top\right] = \mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nu_o\nu_o^\top\left(H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\right)^\top\right]-\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right]\left(\boldsymbol{G}^\top\boldsymbol{\Omega}^{-1}\boldsymbol{G}\right)^{-1}\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right].$$

Since $\mathbb{E}\left[\widetilde{\nu}_o\left(\widetilde{\nu}_o\right)^\top\right]$ is positive semi-definite, so is $\boldsymbol{\Delta}$, which completes the proof. □

B.7 Proof of Proposition 3

I apply arguments in Section 6.1 of Newey and McFadden (1994). Namely, I define the joint estimator of the first-step and second-step estimator in Proposition 3 that falls into the class of general GMM estimation, and discuss the asymptotic property using the general result about GMM estimation. In the proof, I modify the notation of the set of functions that yield optimal IV, $\boldsymbol{H}^*$, to clarify that it depends on parameters $\boldsymbol{\Theta}$ as follows:

$$H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}\right)=\mathbb{E}\left[\nabla_{\boldsymbol{\Theta}}\nu_o\left(\boldsymbol{\Theta}\right)|\boldsymbol{\psi}_{t_1}^J\right]\mathbb{E}\left[\nu_o\left(\boldsymbol{\Theta}\right)\left(\nu_o\left(\boldsymbol{\Theta}\right)\right)^\top|\boldsymbol{\psi}_{t_1}^J\right]^{-1}.$$

Define the joint estimator as follows:

$$\begin{pmatrix}\boldsymbol{\Theta}_2\\ \boldsymbol{\Theta}_1\end{pmatrix}\equiv\arg\min_{\boldsymbol{\Theta}_2,\boldsymbol{\Theta}_1}\left[\sum_o e_o\left(\boldsymbol{\Theta}_2,\boldsymbol{\Theta}_1\right)\right]^\top\left[\sum_o e_o\left(\boldsymbol{\Theta}_2,\boldsymbol{\Theta}_1\right)\right],$$

where

$$e_o\left(\boldsymbol{\Theta}_2,\boldsymbol{\Theta}_1\right)\equiv\begin{pmatrix}H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_1\right)\nu_o\left(\boldsymbol{\Theta}_2\right)\\ H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_0\right)\nu_o\left(\boldsymbol{\Theta}_1\right)\end{pmatrix}.$$

Since for any $\boldsymbol{\Theta}$, IV-generating function $H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_0\right)$ gives the consistent estimator for $\boldsymbol{\Theta}$, I have $\boldsymbol{\Theta}_1\to\boldsymbol{\Theta}$ and $\boldsymbol{\Theta}_2\to\boldsymbol{\Theta}$. I also have the asymptotic variance

$$\mathrm{Var}\begin{pmatrix}\boldsymbol{\Theta}_2\\ \boldsymbol{\Theta}_1\end{pmatrix}=\left[\left(\widetilde{\boldsymbol{G}}\right)^\top\widetilde{\boldsymbol{\Omega}}\widetilde{\boldsymbol{G}}\right]^{-1},$$

where

$$\begin{aligned}\widetilde{\boldsymbol{G}}&\equiv\mathbb{E}\left[\nabla_{\left(\boldsymbol{\Theta}_2,\boldsymbol{\Theta}_1\right)^\top}e_o\left(\boldsymbol{\Theta}_2,\boldsymbol{\Theta}_1\right)\right]\\ &=\mathbb{E}\begin{bmatrix}H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_1\right)\nabla\nu_o\left(\boldsymbol{\Theta}_2\right) & \nabla H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_1\right)\nu_o\left(\boldsymbol{\Theta}_2\right)\\ \mathbf{0} & H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_0\right)\nabla\nu_o\left(\boldsymbol{\Theta}_1\right)\end{bmatrix}\end{aligned}$$

and

$$\widetilde{\boldsymbol{\Omega}} \equiv \mathbb{E}\left[e_o\left(\boldsymbol{\Theta}_2, \boldsymbol{\Theta}_1\right)\left[e_o\left(\boldsymbol{\Theta}_2, \boldsymbol{\Theta}_1\right)\right]^{\top}\right]$$
$$= \mathbb{E}\left[\begin{array}{cc} H_o^*\left(\boldsymbol{\psi}_{t_1}^J; \boldsymbol{\Theta}_1\right) \nu_o\left(\boldsymbol{\Theta}_2\right)\left[H_o^*\left(\boldsymbol{\psi}_{t_1}^J; \boldsymbol{\Theta}_1\right) \nu_o\left(\boldsymbol{\Theta}_2\right)\right]^{\top} & H_o^*\left(\boldsymbol{\psi}_{t_1}^J; \boldsymbol{\Theta}_1\right) \nu_o\left(\boldsymbol{\Theta}_2\right)\left[H_o^*\left(\boldsymbol{\psi}_{t_1}^J; \boldsymbol{\Theta}_0\right) \nu_o\left(\boldsymbol{\Theta}_1\right)\right]^{\top} \\ H_o^*\left(\boldsymbol{\psi}_{t_1}^J; \boldsymbol{\Theta}_0\right) \nu_o\left(\boldsymbol{\Theta}_1\right)\left[H_o^*\left(\boldsymbol{\psi}_{t_1}^J; \boldsymbol{\Theta}_1\right) \nu_o\left(\boldsymbol{\Theta}_2\right)\right]^{\top} & H_o^*\left(\boldsymbol{\psi}_{t_1}^J; \boldsymbol{\Theta}_0\right) \nu_o\left(\boldsymbol{\Theta}_1\right)\left[H_o^*\left(\boldsymbol{\psi}_{t_1}^J; \boldsymbol{\Theta}_0\right) \nu_o\left(\boldsymbol{\Theta}_1\right)\right]^{\top} \end{array}\right].$$

Note that Assumption 1 implies that any function of $\boldsymbol{\psi}_{t_1}^J$ is orthogonal to ν_o, implying $\mathbb{E}\left[\nabla H_o^*\left(\boldsymbol{\psi}_{t_1}^J; \boldsymbol{\Theta}_1\right) \nu_o\left(\boldsymbol{\Theta}_2\right)\right] = 0$. Therefore, $\widetilde{\boldsymbol{G}}$ is a block-diagonal matrix and thus the marginal asymptotic distribution of $\boldsymbol{\Theta}_2$ is normal with variance $\text{Var}\left(\boldsymbol{\Theta}_2\right) = \left(\boldsymbol{G}^{\top} \boldsymbol{\Omega}^{-1} \boldsymbol{G}\right)^{-1}$, noting that $\boldsymbol{G} = \mathbb{E}\left[H_o^*\left(\boldsymbol{\psi}_{t_1}^J; \boldsymbol{\Theta}\right) \nabla \nu_o\left(\boldsymbol{\Theta}\right)\right]$ and $\boldsymbol{\Omega} \equiv \mathbb{E}\left[H_o^*\left(\boldsymbol{\psi}_{t_1}^J; \boldsymbol{\Theta}\right) \nu_o\left(\boldsymbol{\Theta}\right)\left(H_o^*\left(\boldsymbol{\psi}_{t_1}^J; \boldsymbol{\Theta}\right) \nu_o\left(\boldsymbol{\Theta}\right)\right)^{\top}\right]$. By Proposition 2, this asymptotic variance is minimal among the GMM estimator (29).

C Further Estimation and Simulation Results

C.1 Robot Trade Elasticity

To estimate robot trade elasticity ε^R, I apply and extend the trilateral method of Caliendo and Parro (2015). Namely, decompose the robot trade cost $\tau_{li,t}^R$ into $\ln \tau_{li,t}^R = \ln \tau_{li,t}^{R,T} + \ln \tau_{li,t}^{R,D}$, where $\tau_{li,t}^{R,T}$ is tariff on robots taken from the UNCTAD-TRAINS database and $\tau_{li,t}^{R,D}$ is asymmetric non-tariff trade cost. The latter term is assumed to be $\ln \tau_{li,t}^{R,D} = \ln \tau_{li,t}^{R,D,S} + \ln \tau_{l,t}^{R,D,O} + \ln \tau_{i,t}^{R,D,D} + \ln \tau_{li,t}^{R,D,E}$, where $\tau_{li,t}^{R,D,S}$ captures symmetric bilateral trade costs such as distance, common border, language, and FTA belonging status and satisfies $\tau_{li,t}^{R,D,S} = \tau_{il,t}^{R,D,S}$, $\tau_{l,t}^{R,D,O}$ and $\tau_{i,t}^{R,D,D}$ are the origin and destination fixed effects such as non-tariff barriers respectively, and $\tau_{li,t}^{R,D,E}$ is the random error that is orthogonal to tariffs. From the robot gravity equation (B.14) that I derive in Section B.4, I have

$$\ln\left(\frac{X_{li,t}^R X_{ij,t}^R X_{jl,t}^R}{X_{lj,t}^R X_{ji,t}^R X_{il,t}^R}\right) = \left(1 - \varepsilon^R\right) \ln\left(\frac{\tau_{li,t}^{R,T} \tau_{ij,t}^{R,T} \tau_{jl,t}^{R,T}}{\tau_{lj,t}^{R,T} \tau_{ji,t}^{R,T} \tau_{il,t}^{R,T}}\right) + e_{lij,t}, \tag{C.1}$$

where $X_{li,t}^R$ is the bilateral sales of robots from l to i in year t and $e_{lij,t} \equiv \ln \tau_{li,t}^{R,D,E} + \ln \tau_{ij,t}^{R,D,E} + \ln \tau_{jl,t}^{R,D,E} - \ln \tau_{lj,t}^{R,D,E} - \ln \tau_{ji,t}^{R,D,E} - \ln \tau_{il,t}^{R,D,E}$. The benefit of this approach is that it does not require symmetry for non-tariff trade cost $\tau_{li}^{R,D}$, but only requires the orthogonality for the asymmetric

Table C.1: Coefficient of equation (C.1)

	(1) HS 847950	(2) HS 847950	(3) HS 8479	(4) HS 8479
Tariff	-0.272***	-0.236***	-0.146***	-0.157***
	(0.0718)	(0.0807)	(0.0127)	(0.0131)
FEs	h-i-j-t	ht-it-jt	h-i-j-t	ht-it-jt
N	4610	4521	88520	88441
r2	0.494	0.662	0.602	0.658

Note: The author's calculation based on BACI data from 1996 to 2018 and equation (C.1). The first two columns show the result for the HS code 847950 (Industrial robots for multiple uses), while the last two columns HS code 8479 (Machines and mechanical appliances having individual functions, not specified or included elsewhere in this chapter). The first and third columns control the unilateral fixed effect, while the second and fourth the bilateral fixed effect. See the text for the detail.

component of the trade cost. My method also extends Caliendo and Parro (2015) in using the time-series variation as well as trilateral country-level variation to complement the relatively small number of observations in robot trade data.

When implementing regression of equation (C.1), I further consider controlling for two separate sets of fixed effects. The first set is the unilateral fixed effect indicating if a country is included in the trilateral pair of countries, and the second set is the bilateral fixed effect for the twin of countries is included in the trilateral pair. These fixed effects are relevant in my setting as a few number of countries export robots, and controlling for these exporters' unobserved characteristics is critical.

Table C.1 shows the result of regression of equation (C.1). The first two columns show the result for the HS code 847950 (Industrial robots for multiple uses, the definition of robots used in Humlum, 2019), and the last two columns HS code 8479 (Machines and mechanical appliances having individual functions, not specified or included elsewhere in this chapter). The first and third columns control the unilateral fixed effect, and the second and fourth the bilateral fixed effect. The implied trade elasticity of robots ε^R is fairly tightly estimated and ranges between 1.13-1.34. Given these estimation results, I use $\varepsilon^R = 1.2$ in the estimation and counterfactuals.

To assess the estimation result, note that Caliendo and Parro (2015) show in Table 1 that the regression coefficient of equation (C.1) is 1.52, with the standard error of 1.81, for "Machinery n.e.c", which roughly corresponds to HS 84. Therefore, my estimate for industrial robots falls in the one-standard-deviation range of their estimate for a broader category of goods.

Note that the average trade elasticity across sectors is estimated significantly higher than these values, such as 4 in Simonovska and Waugh (2014). The low trade elasticity for robots ε^R is intuitive given robots are highly heterogeneous and hardly substitutable. This low elasticity implies small gains from robot taxes, with the robot tax incidence almost on the US (robot buyer) side rather than the robot-selling country.

C.2 Estimation at 2 Digit-level Occupation Groups

In this section, I study how the occupation grouping defined in Section 4.1 affects the estimation result. Specifically, I apply the estimation method in Proposition 3 for 2-digit occupation groups provided by the US Census "[f]or users who wish to further aggregate occupation to broader categories[.]"[22] Table C.2 shows the result. I find that the elasticity estimates for "Production" and "Transportation and Material Moving" occupations remain around 4 with small standard errors. However, the estimates for the other occupations become different from the case with the 5-occupation aggregates, and the standard errors are larger and volatile. This exercise reveals that the 5-occupation aggregation in Section 4.1 provides the conservative grouping and tightly estimated elasticities of substitution. The reason is that I use the 4-digit occupational variation for estimation, and 2-digit occupation grouping often yields only a small number of 4-digit occupations, reducing estimation power (see "# Occ." column in Table C.2).

At this point, it is also worth noting the time-series variation. Since I have annual observation for occupational robot costs, it is potentially possible to leverage this rich variation for the structural estimation, which may permit me to estimate the EoS θ_o at a narrower occupation group level. However, the bottleneck of this approach is the computational burden to compute the dynamic solution matrix $\overline{\boldsymbol{F}_t}$. Specifically, the transversality condition (16) requires computing the eigenspace of dynamic substitution matrix $\overline{\boldsymbol{F}_{t+1}^{y}}$ in equation (20), as described in detail in Section D. This is computationally burdensome since we cannot rely on the sparse structure of the matrix $\overline{\boldsymbol{F}_{t+1}^{y}}$. In contrast, the estimation method in Proposition 3 does not involve such computation, but only requires computing the steady-state solution matrix $\overline{\boldsymbol{E}}$. Then I only need to invert steady-state substitution matrix $\overline{\boldsymbol{E}^{y}}$, which is feasible given the sparse structure of $\overline{\boldsymbol{E}^{y}}$. Therefore, a future potential breakthrough on computation technology could make it possible to estimate the model

[22] Further details can be found at https://usa.ipums.org/usa-action/variables/OCC2010 (Accessed on December 6, 2020).

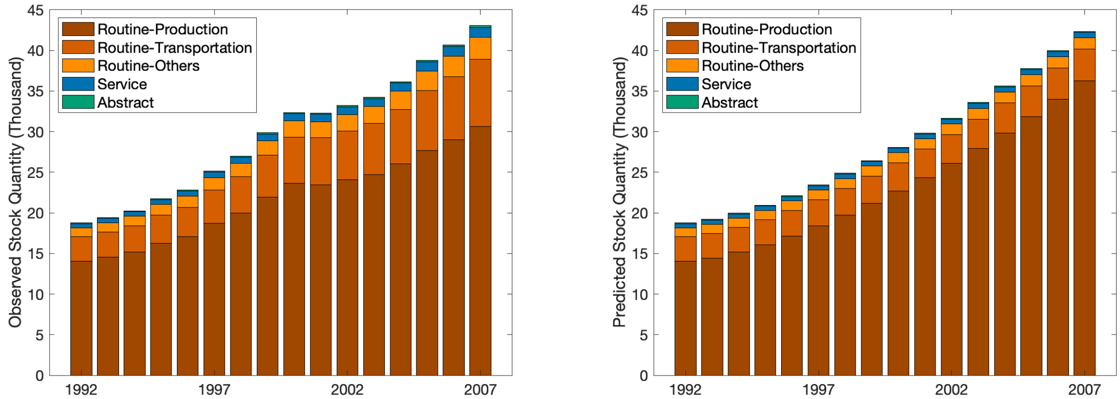
Table C.2: Elasticity of Substitution between Robots and Workers, θ_o, at the 2-digit Occupation Level

OCC2010 2-dig. Label	OCC2010 Range	# Occ.	2-dig. EoS	2-dig. SE	Group	5-group EoS	5-group SE
Management, Business, Science, And Arts	[30, 430]	9	0.15	1.57	Abstract	0.80	0.60
Business Operations Specialists	[500, 730]	9	1.98	0.91			
Financial Specialists	[800, 1240]	8	0.04	1.29			
Architecture And Engineering	[1300, 1560]	15	-0.31	1.48			
Life, Physical, And Social Science	[1600, 1960]	14	-0.05	1.49			
Community And Social Services	[2000, 2140]	6	0.23	0.69			
Education, Training, And Library	[2200, 2540]	9	0.24	0.64			
Arts, Design, Entertainment, Sports, And Media	[2600, 2910]	13	0.98	0.77			
Healthcare Practitioners And Technical	[3010, 3650]	23	0.6	1.48			
Protective Service	[3720, 4130]	13	1.94	0.58	Service	1.35	0.48
Building And Grounds Cleaning And Maintenance	[4200, 4650]	16	0.5	0.9			
Sales And Related	[4700, 4965]	15	2.07	1.49	Routine, Others	1.27	0.53
Office And Administrative Support	[5000, 5940]	38	-0.24	1.06			
Farming, Fishing, And Forestry	[6005, 6130]	7	2.15	1.22			
Construction	[6200, 6765]	22	1.03	0.85			
Extraction	[6800, 6940]	5	1.44	1.22			
Installation, Maintenance, And Repair	[7000, 7610]	22	-0.7	1.36			
Production	[7700, 8965]	55	3.91	0.24	Routine, Production	4.04	0.24
Transportation And Material Moving	[9000, 9750]	25	4.41	0.36	Routine, Transportation	4.29	0.28

Note: The estimates of the structural parameters based on the estimator in Proposition 3. In header, "OCC2010 2-dig. Label" shows the label of 2-digit occupations groups in the OCC2010 coding scheme, "OCC2010 Range" shows the range of OCC2010 codes that fall into the 2-digit occupation group, "Num. Occ." shows the number of 4-digit level occupations in the 2-digit occupation group, "2-dig. EoS" shows the point estimate of the elasticity of substitution between robots and workers in the 2-digit occupation group, "2-dig. SE" shows the standard error estimate of the elasticity of substitution between robots and workers in the 2-digit occupation group, "Group" shows the 5-group I defined in Section 4.1, "5-group EoS" shows the point estimate of the elasticity of substitution in the "5-group", and "5-group SE" shows the the standard error estimate of the elasticity of substitution in the "5-group."

Figure C.1: Trends of Robot Stocks

(a) Data (b) Model



Notes: Figures show the trend of the observed (left) and predicted (right) stock of robots for each occupation group measured by quantities. The predicted robot stocks are computed by shocks backed out from the estimated model and applying the first-order solution to the general equilibrium described in equation (21).

based on the dynamic solution matrix $\overline{\boldsymbol{F}_t}$ and annual observation of my dataset.

C.3 Actual and Predicted Robot Accumulation Dynamics

Figure C.1 shows the trends of robot stock in the US in the data and the model. Although I do not match the overall robot capital stocks, the estimated model tracks the observed pattern well between 1992 and the late 2010s, consistent with the fact that I target the changes between 1992 and 2007. There is a slight over-prediction of the growth of production robots and under-prediction of the growth of transportation (material moving) robots between occupation groups.

C.4 Japan Robot Shock and Observed Automation Shock

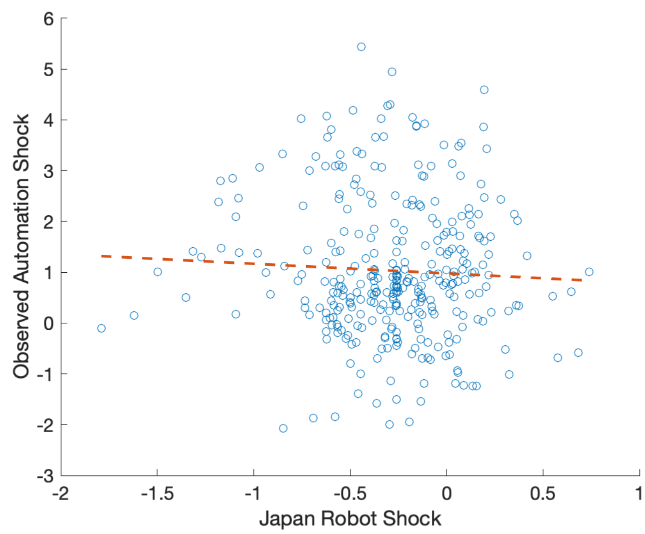
Table C.3 shows the Japan robot shock and observed automation shock backed out from the estimated model. On the one hand, one can see that the Japanese robots' cost declined similarly across occupation groups. On the other hand, the observed automation shock shows a significant variation, namely, larger in production and transportation occupations than other occupations. In turn, Figure C.2 shows a further detailed scatter plot between the two shocks, delivering a mild negative relationship. This negative correlation is consistent with the example of robotic innovations

Table C.3: Shocks Aggregated at 5 groups

Group	ψ^J	$\widehat{a^{obs}}$
Routine, Production	-0.305	2.453
Routine, Transportation	-0.497	3.428
Routine, Others	-0.460	0.335
Service	-0.378	0.623
Abstract	-0.289	0.133

Note: The author's calculation based on JARA, O*NET, and US Census/ACS. The Japan robot shock ψ^J is based on the regression of equation (1). The observed automation shock $\widehat{a_o^{obs}}$ is backed out from equation (26) with the estimated parameters in Table 2. Both measures are aggregated from the 4-digit level to 5 groups using the initial employment weight.

Figure C.2: Correlation between ψ_o^J and $\widehat{a_o^{obs}}$



Note: The author's calculation based on JARA, O*NET, and US Census/ACS. The Japan robot shock is based on the regression of equation (1). The observed automation shock is backed out from equation (26) with the estimated parameters in Table 2. Each circle is 4-digit occupation and dashed line is the fitted line.

in Appendix A.2.4.

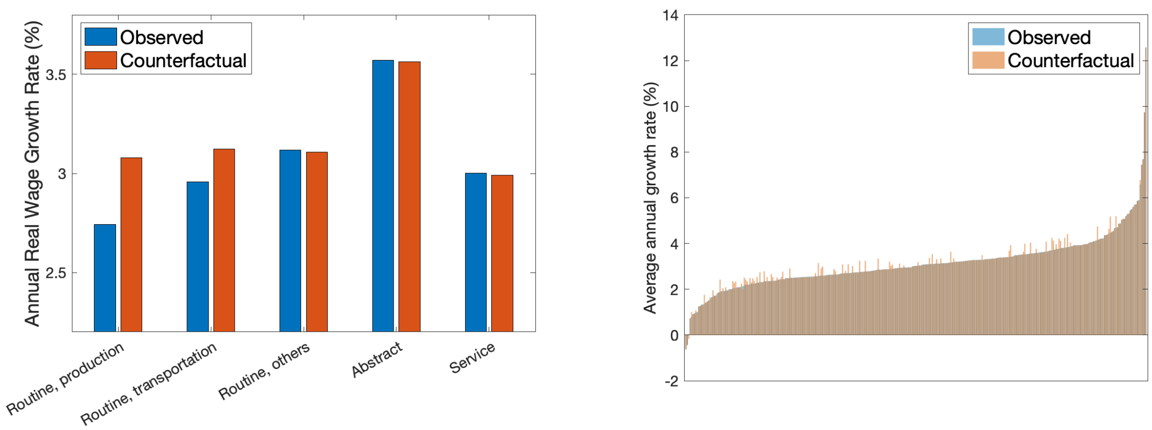
C.5 Automation and Wages at Occupations

Figure C.3 shows the observed and counterfactual growth rate of real wages for each occupation, where the counterfactual change means the simulated change absent the automation shock. Figure C.3a shows the results aggregated at the 5 occupations groups defined in Section 4.1. I compute the counterfactual growth rate from the observed rate of the wage change, subtracted by the change predicted by the first-order steady-state solution $\overline{\boldsymbol{E}}$ and the observed automation shock

Figure C.3: The Steady-state Effect of Robots on Wages

(a) Occupation Groups

(b) All Occupations



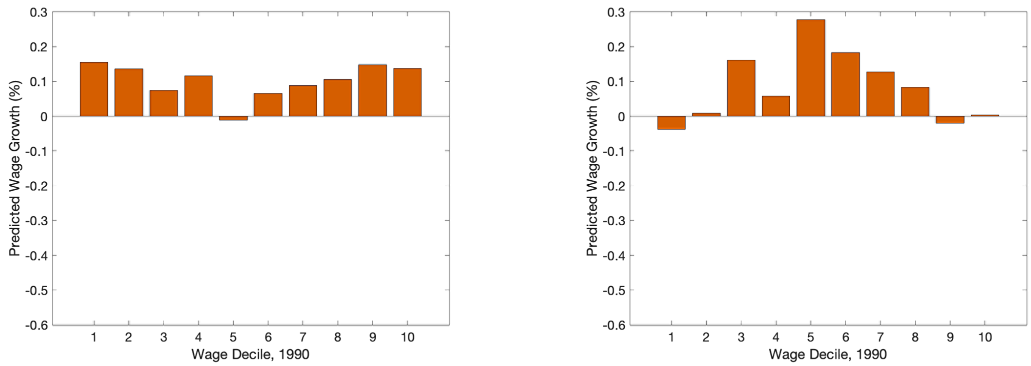
$\widehat{a^{\text{obs}}}$. The result is based on the observed high growth rates of robots in routine production and transportation (material moving) occupations, and these occupations' high EoS estimates between robots and workers. In particular, at the 5-occupation aggregate level, most of the observed differences in the real wage growth rates in the three routine occupation groups are closed absent the automation shock. Applying the similar exercise for all occupations in my sample, Figure C.3b shows a more granular result, where occupations are sorted by the observed changes of wages from 1990-2007.

C.6 Wage Polarization Exercise under Different Robot-labor EoS

To learn the role of my robot-labor EoS estimates in deriving the wage-polarizing effect of robotization, I perform the same robotization exercise as in Section 5.1 under different values of the EoS θ_g and study the occupational wage consequence. Specifically, I consider the following two cases: $\theta_g = 0$ for any occupation group g as in Acemoglu and Restrepo (2020) as described in Section B.3.1, and Cobb-Douglas case of $\theta = 1$ for any g. The results are shown in Figure C.4. Compared with the right panel of Figure 2b, we do not find that the observed robotization shock does not contribute to wage polarization when ϑ is as low as 0 or 1. This finding is because the increased robot use in the middle of the distribution does not reduce wage in such a case. In this sense, it is critical to have a cost shock measure to estimate the robot-labor EoS to derive the wage-polarizing

Figure C.4: Wage Polarization Exercise under Different Elasticity of Substitution, θ_g

(a) $\theta_g = 1$ (b) $\theta_g = 0$



Notes: The annualized wage growth rates predicted by the backed-out shocks and the estimated model's first-order steady-state solution given in equation (19) under specific value of the elasticity of substitution between robots and labor, θ. The left panel shows the case with $\theta = 1$ and the right $\theta = 0$. See Figure 2b for comparison to the case under my parameter estimates.

effect of robotization.

C.7 Robot Tax and Workers' Welfare

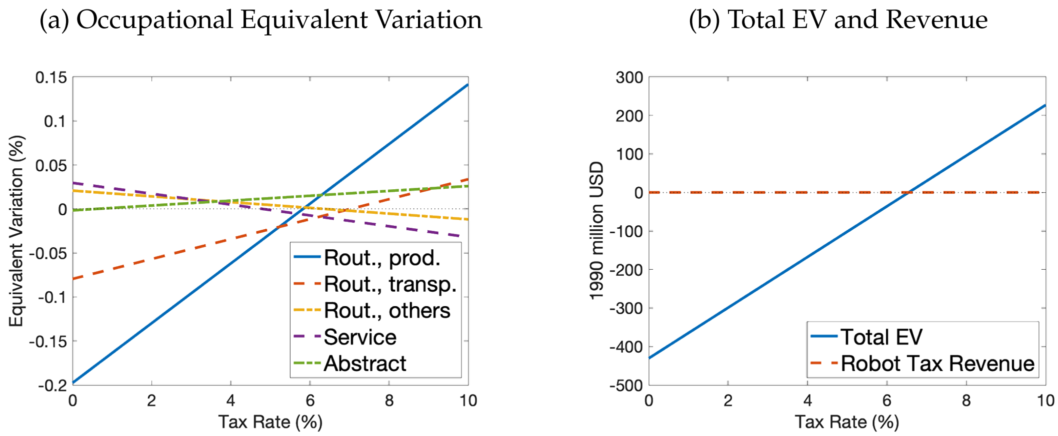
To examine how the robot tax affects workers in different occupations, I define the equivalent variation (EV) implicitly as follows:

$$\sum_{t=t_0}^{\infty} \left(\frac{1}{1+\iota}\right)^t \ln\left(\left[C'_{i,o,t}\right]\right) = \sum_{t=t_0}^{\infty} \left(\frac{1}{1+\iota}\right)^t \ln\left(C_{i,o,t}\left[1 + EV_{i,o}\right]\right). \tag{C.2}$$

Namely, the EV is the fraction of the occupation-specific subsidy that would make the present discounted value (PDV) of the utility in the robotized and taxed equal to the PDV of the utility if the occupation-specific subsidy were exogenously given in the initial equilibrium. On the left-hand side, I hit the robotization shock backed out in Section 4.4. As in Section 5.2, I consider the US unilateral (not inducing a reaction in other countries), unexpected, and permanent tax on robot purchases. By this definition, the worker in occupation o prefers the robotized and taxed world if and only if the EV is positive for o.

Figure C.5a shows this occupation-specific EV as a function of the tax rate. The far-left side of the figure is the case of zero robot tax, thus a case of only the robotization shock. Consistent with the occupational wage effects (cf. Figure C.3), workers in production and transportation occupa-

Figure C.5: Robot Tax and Workers' Welfare

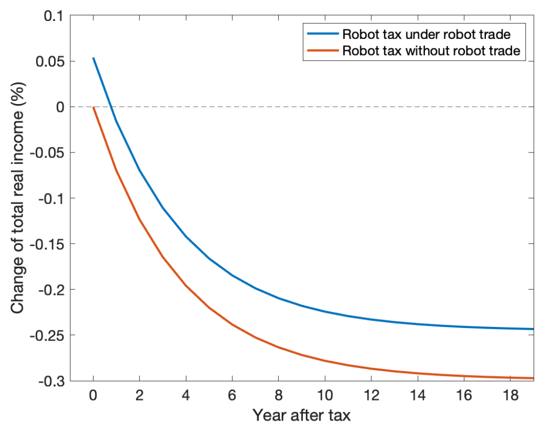


Note: The left panel shows the US workers' equivalent variation defined in equation (C.2) as a function of the US robot tax rate. Labels "Rout., prod.", "Rout., transp.", and "Rout., others" mean routine, production; routine, transportation; and routine, others occupations, respectively. The right panel shows monetary values of equivalent variations aggregated across workers and robot tax revenue as a function of the robot tax rate, measured in 1990 million USD.

tions lose significantly due to robotization. In contrast other workers are roughly indifferent between the robotized world and the non-robotized initial equilibrium or slightly prefer the former world. Going right through the figure, the production and transportation workers' EV improves as the robot tax reduces competing robots. The EV of production workers turns positive when the tax rate is around 6%, and that of transportation workers is positive when the rate is about 7%. However, these tax rates are too high and would make EVs in other occupations negative. In fact, in production and transportation occupations, robots do not accumulate and adversely affect labor demand in the other occupations.

To study if the reallocation policy by robot tax may work, I also compute the equivalent variation in terms of monetary value aggregated by occupation groups (total EV) and compare it with the robot tax revenue, both as a function of robot tax. Figure C.5b shows the result. One can confirm that the marginal robot tax revenue is far from enough to compensate for workers' loss that concentrates on production and transportation workers, at the initial equilibrium with zero robot tax rate. The robot tax revenue is negligible at this margin compared with the workers' loss due to robotization. It is true that as the robot tax rate increases, the total EV rises: When the rate is as large as 6-7%, the sum of the total EV and the robot tax revenue is positive. However, one should be cautious that my solution to the model is to the first order. Thus the approximation error may play an important role when the robot tax rate is significantly higher than the one in the initial

Figure C.6: Effects of the Robot Tax on the US Real Income



equilibrium, zero. Extending my solution to the higher-order or even finding the exact solution is left for future research.

C.8 The Role of Trade that Plays in the Robot Tax Effect

Figure C.6 shows the dynamic effect of the robot tax on the US real income. If the robot trade is not allowed, the robot tax does not increases the real income in any period since the terms-of-trade effect does not show up, but only the long-run capital decumulation effect does. On the other hand, once I allow the robot trade as observed in the data, the robot tax may increase the real income because it decreases the price of imported robots. The effect is concentrated in the short-run before the capital decumulation process matures. In the long run, the negative decumulation effect dominates the positive terms-of-trade effect.

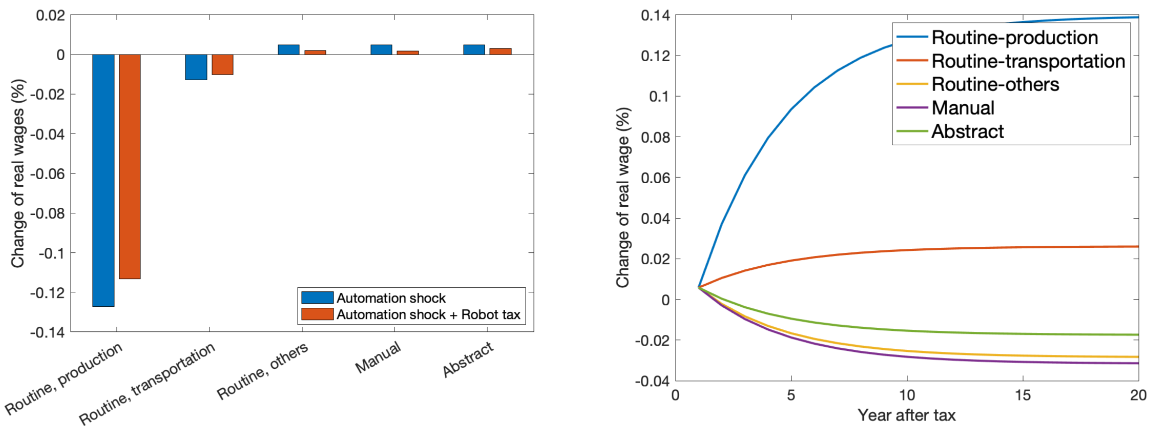
C.9 The Occupational Real Wage Effect of the Robot Tax

In Figure C.7a, I show two scenarios of the steady-state changes in occupational real wages. On the one hand, I shock the economy only with the automation shocks. On the other hand, I shock the economy with both the automation shocks and the robot tax. The result shows heterogenous effects on occupational real wages of the robot tax. The tax mitigates the negative effect of automation on routine production workers and routine transportation workers, while the tax marginally decreases the small gains that workers in the other occupations would have enjoyed. Overall, the

Figure C.7: Effects of the Robot Tax on Occupational Real Wages

(a) Steady-state Comparison

(b) Transitional Effect of Tax

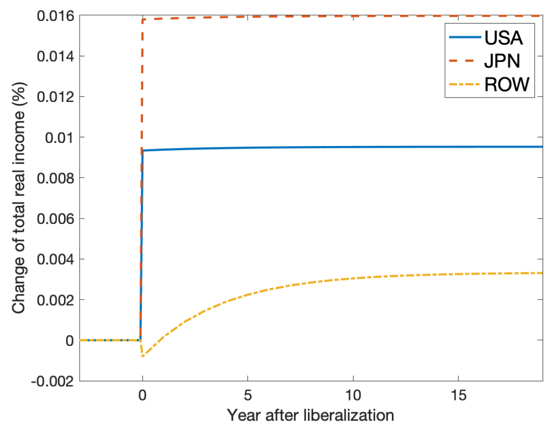


robot tax mitigates the large heterogeneous effects of the automation shocks, that could go negative and positive directions depending on occupation groups, and compresses the effects towards zero. Figure C.7b shows the dynamics of the effects of robot tax, net of the effects of automation shocks. Although the steady-state effects of robot tax were heterogeneous as shown in Figure C.7a, the effect is not immediate but materializes after around 10 years, due to the sluggish adjustment in the accumulation of the robot capital stock. Overall, I find that since the robot tax slows down the adoption of robots, it rolls back the real wage effect of automation–workers in occupations that experienced significant automation shocks (e.g., production and transportation in the routine occupation groups) benefit from the tax, while the others lose.

C.10 Trade Liberalization of Robots

Following Ravikumar et al. (2019), I consider unexpected and permanent 20% reduction in the bilateral trade costs to study the effect trade liberalization of capital good and dynamics gains from trade. Figure C.8 shows the result of such a simulation for a 20-years time horizon. All country groups in the model gain from the trade liberalization. The US gain materialize almost immediately after the trade cost change. A possible explanation is the combination of the following two observation. First, it takes time to accumulate robots after the trade liberalization, which makes the gains from trade liberalization sluggish. Second, by exporting robots to ROW, the US

Figure C.8: The Effect of Robot Trade Cost Reduction



increases the revenue of robot sales immediately after the trade cost drop, improving the short-run real income gain. The real income gain is the largest for Japan, a large net robot exporter. It is noteworthy that ROW loses from the reduction in the robot trade cost, possibly due to the terms-of-trade deterioration in the short-run.

D Detail of the GE Solution

I discuss the derivation log-linearization in equations (17), (19), and (21), so that I can bring the theory with computation. Throughout the section, relational operator $\circ$ is Hadamard product, $\oslash$ indicates Hadamard division, and $\otimes$ means Kronecker product.

It is useful to show that the CES production structure implies the share-weighted log-change expression for both prices and quantities. Namely, I have a formula for the change in destination price index $\widehat{P_{j,t}^G} = \sum_i \tilde{x}_{ij,t_0}^G \widehat{p_{ij,t}^G}$ and one for the change in destination expenditure $\widehat{P_{j,t}^G} + \widehat{Q_{j,t}^G} = \sum_i \tilde{x}_{ij,t_0}^G \left(\widehat{p_{ij,t}^G} + \widehat{Q_{ij,t}^G} \right)$. These imply that

$$\widehat{Q_{j,t}^G} = \sum_i \tilde{x}_{ij,t_0}^G \widehat{Q_{ij,t}^G},$$

or the changes of quantity aggregate $\widehat{Q_{j,t}^G}$ are also share-weighted average of changes of origin quantity $\widehat{Q_{ij,t}^G}$.

By log-linearizing equation (B.22) for any i ,

$$
\begin{aligned}
& -\alpha_M \widehat{p_{i,t}^G} + \alpha_M \sum_l \widetilde{x}_{li,t_0}^G \widehat{p_{l,t}^G} + (1-\alpha_M) \sum_j \widetilde{y}_{ij,t_0}^G \widehat{Q_{ij,t}^G} - \alpha_L \sum_o \widetilde{x}_{i,o,t_0}^O l_{i,o,t_0}^O \widehat{L_{i,o,t}} \\
& = \frac{\alpha_L}{\theta-1} \sum_o \frac{\widetilde{x}_{i,o,t_0}^O}{1-a_{o,t_0}} \left(-a_{o,t_0} l_{i,o,t_0}^O + (1-a_{o,t_0}) \left(1-l_{i,o,t_0}^O\right)\right) \widehat{a_{o,t}} + \alpha_L \sum_o \widetilde{x}_{i,o,t_0}^O \frac{1}{\beta-1} \widehat{b_{i,o,t}} \\
& \quad + \widehat{A_{i,t}^G} + (1-\alpha_L-\alpha_M) \widehat{K_{i,t}} - \alpha_M \sum_l \widetilde{x}_{li,t_0}^G \widehat{\tau_{li,t}^G} - (1-\alpha_M) \sum_j \widetilde{y}_{ij,t_0}^G \widehat{\tau_{ij,t}^G} + \alpha_L \sum_o \widetilde{x}_{i,o,t_0}^O \left(1-l_{i,o,t_0}^O\right) \widehat{K_{i,o,t}^R},
\end{aligned}
$$

To write a matrix notation, write

$$
\overline{\boldsymbol{M}^{yG}} \equiv \begin{bmatrix} \left[\widetilde{y}_{11,t_0}^G, \ldots, \widetilde{y}_{1N,t_0}^G\right] & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \left[\widetilde{y}_{N1,t_0}^G, \ldots, \widetilde{y}_{NN,t_0}^G\right] \end{bmatrix}
$$

a $N \times N^2$ matrix,

$$
\overline{\boldsymbol{M}^{xOl}} \equiv \begin{bmatrix} \left(\widetilde{\boldsymbol{x}}_{1,\cdot,t_0} \circ \widetilde{\boldsymbol{l}}_{1,\cdot,t_0}\right)^\top & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \left(\widetilde{\boldsymbol{x}}_{N,\cdot,t_0} \circ \widetilde{\boldsymbol{l}}_{N,\cdot,t_0}\right)^\top \end{bmatrix}
$$

a $N \times NO$ matrix where

$$
\widetilde{\boldsymbol{x}}_{1,\cdot,t_0} \equiv \left(\widetilde{x}_{1,o,t_0}^O\right)_o \text{ and } \widetilde{\boldsymbol{l}}_{1,\cdot,t_0} \equiv \left(l_{1,o,t_0}^O\right)_o \tag{D.1}
$$

are $O \times 1$ vectors, $\overline{\boldsymbol{M}^{al}}$ as a matrix with its element

$$
M_{i,o}^{al} = \frac{-a_{o,t_0} l_{i,o,t_0}^O + (1-a_{o,t_0}) \left(1-l_{i,o,t_0}^O\right)}{1-a_{o,t_0}},
$$

and a $N \times O$ matrix,

$$
\overline{\boldsymbol{M}^{xO}} \equiv \begin{bmatrix} \left[\widetilde{x}_{1,1,t_0}^O, \ldots, \widetilde{x}_{1,O,t_0}^O\right] & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \left[\widetilde{x}_{N,1,t_0}^O, \ldots, \widetilde{x}_{N,O,t_0}^O\right] \end{bmatrix},
$$

a $N \times NO$ matrix,

$$
\overline{\boldsymbol{M}^{xG}} \equiv \left[\begin{array}{ccc} \operatorname{diag}\left(\widetilde{x}_{1\cdot,t_0}^G\right) & \ldots & \operatorname{diag}\left(\widetilde{x}_{N\cdot,t_0}^G\right) \end{array} \right],
$$

a $N \times N^2$ matrix, and

$$\overline{\boldsymbol{M}^{xOl,2}} \equiv \begin{bmatrix} \left(\widetilde{\boldsymbol{x}}_{1,\cdot,t_0} \circ \left(\mathbf{1}_O - \widetilde{\boldsymbol{l}}_{1,\cdot,t_0}\right)\right)^\top & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \left(\widetilde{\boldsymbol{x}}_{N,\cdot,t_0} \circ \left(\mathbf{1}_O - \widetilde{\boldsymbol{l}}_{N,\cdot,t_0}\right)\right)^\top \end{bmatrix},$$

a $N \times NO$ matrix where $\widetilde{x}_{1,\cdot,t_0}$ and $\widetilde{\boldsymbol{l}}_{1,\cdot,t_0}$ are defined in equation (D.1). Then I have

$$\begin{aligned} &- \alpha_M \left(\overline{\boldsymbol{I}} - \left(\overline{\widetilde{\boldsymbol{x}}^G_{t_0}}\right)^\top\right) \widehat{\boldsymbol{p}^G_t} + (1-\alpha_M)\, \overline{\boldsymbol{M}^{yG}} \widehat{\boldsymbol{Q}^G_t} - \alpha_L \overline{\boldsymbol{M}^{xOl}} \widehat{\boldsymbol{L}}_t \\ &= \frac{\alpha_L}{\theta - 1} \left(\widetilde{\boldsymbol{x}}^O_{t_0} \circ \overline{\boldsymbol{M}^{al}}\right) \widehat{\boldsymbol{a}}_t + \frac{\alpha_L}{\beta - 1} \overline{\boldsymbol{M}^{xO}} \widehat{\boldsymbol{b}}_t + \widehat{\boldsymbol{A}^G_t} + (1 - \alpha_L - \alpha_M)\, \widehat{\boldsymbol{K}_t} \\ &\quad - \left[\alpha_M \overline{\boldsymbol{M}^{xG}} + (1-\alpha_M)\, \overline{\boldsymbol{M}^{yG}}\right] \widehat{\boldsymbol{\tau}^G_t} + \alpha_L \overline{\boldsymbol{M}^{xOl,2}} \widehat{\boldsymbol{K}^R_t}, \end{aligned}$$

By log-linearizing equation (B.23) for any i and o,

$$\widehat{p^R_{i,o,t}} = \widehat{P^G_{i,t}} - \widehat{A^R_{i,o,t}}$$

$$- \sum_l \widetilde{x}^G_{li,t_0} \widehat{p^G_{l,t}} + \widehat{p^R_{i,o,t}} = -\widehat{A^R_{i,o,t}} + \sum_l \widetilde{x}^G_{li,t_0} \widehat{\tau^G_{li,t}}.$$

In matrix notation, write

$$\overline{\boldsymbol{M}^{xG,2}} \equiv \begin{bmatrix} \mathbf{1}_O \left[\widetilde{x}^G_{11,t_0}, \ldots, \widetilde{x}^G_{N1,t_0}\right] \\ \vdots \\ \mathbf{1}_O \left[\widetilde{x}^G_{1N,t_0}, \ldots, \widetilde{x}^G_{NN,t_0}\right] \end{bmatrix}$$

a $NO \times N$ matrix, and

$$\overline{\boldsymbol{M}^{xG,3}} \equiv \begin{bmatrix} \widetilde{x}^G_{11,t_0} & \cdots & \widetilde{x}^G_{N1,t_0} & & & \mathbf{0} & \\ & & & \ddots & & & \\ & \mathbf{0} & & & \widetilde{x}^G_{1N,t_0} & \cdots & \widetilde{x}^G_{NN,t_0} \end{bmatrix} \otimes \mathbf{1}_O$$

a $NO \times N^2$ matrix. Then I have

$$-\overline{\boldsymbol{M}^{xG,2}} \widehat{\boldsymbol{p}^G_t} + \widehat{\boldsymbol{p}^R_t} = -\widehat{\boldsymbol{A}^R_t} + \overline{\boldsymbol{M}^{xG,3}} \widehat{\boldsymbol{\tau}^G_t}.$$

By log-linearizing equations (4), (5), and (6) for any i, o, and o', I have

$$\widehat{\mu_{i,oo',t}} = \phi\left(-d\chi_{i,oo',t} + \frac{1}{1+\iota}\widehat{V_{i,o',t+1}}\right) - \sum_{o''}\mu_{i,oo'',t_0}\left(-d\chi_{i,oo'',t} + \frac{1}{1+\iota}\widehat{V_{i,o'',t+1}}\right), \tag{D.2}$$

$$\widehat{V_{i,o,t+1}} = \widehat{w_{i,o,t+1}} + dT_{i,o,t+1} - \widehat{P_{i,t+1}} + \sum_{o'}\mu_{i,oo',t_0}\left(-d\chi_{i,oo',t+1} + \frac{1}{1+\iota}\widehat{V_{i,o',t+2}}\right), \tag{D.3}$$

and

$$\widehat{L_{i,o,t+1}} = \sum_{o'}\frac{L_{i,o',t_0}}{L_{i,o,t_0}}\mu_{i,o'o,t_0}\left(\widehat{\mu_{i,o'o,t}} + \widehat{L_{i,o',t}}\right). \tag{D.4}$$

In matrix notation, by equation (D.2),

$$\widehat{\boldsymbol{\mu}_t^{\text{vec}}} = -\phi\left(\overline{\boldsymbol{I}_{NO^2}} - \overline{\boldsymbol{M}^{\mu}}\right)d\boldsymbol{\chi}_t^{\text{vec}} + \frac{\phi}{1+\iota}\left(\overline{\boldsymbol{I}_{NO^2}} - \overline{\boldsymbol{M}^{\mu}}\right)\left(\overline{\boldsymbol{I}_{NO}} \otimes \mathbf{1}_O\right)\widehat{\boldsymbol{V}_{t+1}}.$$

where

$$\overline{\boldsymbol{M}^{\mu}} \equiv \overline{\boldsymbol{M}^{\mu,3}} \otimes \mathbf{1}_O,$$

$$\overline{\boldsymbol{M}^{\mu,3}} \equiv \begin{bmatrix} \left(\boldsymbol{\mu}_{i,1\cdot,t_0}\right)^\top & & & & & & \\ & \ddots & & & & \mathbf{0} & \\ & & \left(\boldsymbol{\mu}_{i,O\cdot,t_0}\right)^\top & & & & \\ & & & \ddots & & & \\ & & & & \left(\boldsymbol{\mu}_{N,1\cdot,t_0}\right)^\top & & \\ & \mathbf{0} & & & & \ddots & \\ & & & & & & \left(\boldsymbol{\mu}_{i,O\cdot 1,t_0}\right)^\top \end{bmatrix},$$

$$d\boldsymbol{\chi}_t^{\text{vec}} \equiv \begin{bmatrix} d\boldsymbol{\chi}_{1,1\cdot,t} & \dots & d\boldsymbol{\chi}_{1,O\cdot,t} & \dots & d\boldsymbol{\chi}_{N,1\cdot,t} & \dots & d\boldsymbol{\chi}_{N,O\cdot,t} \end{bmatrix}^\top,$$

and

$$\boldsymbol{\mu}_{i,o\cdot,t_0} \equiv \left(\mu_{i,oo',t_0}\right)_{o'} \text{ and } d\boldsymbol{\chi}_{1,o\cdot,t} \equiv \left(d\chi_{1,oo',t}\right)_{o'} \tag{D.5}$$

are $O \times 1$ vectors. By equation (D.3),

$$\frac{1}{1+\iota}\overline{\boldsymbol{M}^{\mu,2}}\check{\boldsymbol{V}}_{t+2} = \overline{\boldsymbol{M}^{xG,2}}\boldsymbol{p}_{t+1}^{\check{G}} - \check{\boldsymbol{w}}_{t+1} + \check{\boldsymbol{V}}_{t+1}.$$

where

$$\overline{\boldsymbol{M}^{\mu,2}} \equiv \begin{bmatrix} \left(\boldsymbol{\mu}_{1,1\cdot,t_0}\right)^\top & & \\ \vdots & & \mathbf{0} \\ \left(\boldsymbol{\mu}_{1,O\cdot,t_0}\right)^\top & & \\ & \ddots & \\ & & \left(\boldsymbol{\mu}_{N,1\cdot,t_0}\right)^\top \\ \mathbf{0} & & \\ & & \left(\boldsymbol{\mu}_{N,O\cdot,t_0}\right)^\top \end{bmatrix},$$

and $\boldsymbol{\mu}_{i,o\cdot,t_0}$ is given by equation (D.5) for any i and o. By equation (D.3),

$$\check{\boldsymbol{L}_{t+1}} = \overline{\boldsymbol{M}^{\mu L,2}} \boldsymbol{\mu}_t^{\check{\text{vec}}} + \overline{\boldsymbol{M}^{\mu L}} \check{\boldsymbol{L}}_t$$

where $\overline{\boldsymbol{M}^{\mu L}}$ being the $NO \times NO$ matrix

$$\overline{\boldsymbol{M}^{\mu L}} = \overline{\boldsymbol{M}^{\mu,2}} \circ \left(\begin{bmatrix} \left(\boldsymbol{L}_{1,\cdot,t_0}\right)^\top & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \left(\boldsymbol{L}_{N,\cdot,t_0}\right)^\top \end{bmatrix} \otimes \mathbf{1}_O \right) \oslash \left(\begin{bmatrix} \boldsymbol{L}_{1,\cdot,t_0} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \boldsymbol{L}_{N,\cdot,t_0} \end{bmatrix} \otimes \left(\mathbf{1}_O\right)^\top \right)$$

and $\overline{\boldsymbol{M}^{\mu L,2}}$ being the $NO \times NO^2$ matrix

$$\overline{\boldsymbol{M}^{\mu L,2}} = \overline{\boldsymbol{M}^{\mu,4}} \circ \left(\begin{bmatrix} \left(\boldsymbol{L}_{1,\cdot,t_0}\right)^\top & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \left(\boldsymbol{L}_{N,\cdot,t_0}\right)^\top \end{bmatrix} \otimes \overline{\boldsymbol{I}_O} \right) \oslash \\ \begin{pmatrix} \left(\mathbf{1}_O\right)^\top \otimes \operatorname{diag}\left(L_{1,o,t_0}\right) & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \left(\mathbf{1}_O\right)^\top \otimes \operatorname{diag}\left(L_{N,o,t_0}\right) \end{pmatrix},$$

where

$$\overline{\boldsymbol{M}^{\mu,4}} \equiv \begin{bmatrix} \operatorname{diag}\left(\boldsymbol{\mu}_{1,1\cdot,t_0}\right) & \dots & \operatorname{diag}\left(\boldsymbol{\mu}_{i,O\cdot,t_0}\right) & & & \mathbf{0} \\ & & & \ddots & & \\ & \mathbf{0} & & & \operatorname{diag}\left(\boldsymbol{\mu}_{N,1\cdot,t_0}\right) & \dots & \operatorname{diag}\left(\boldsymbol{\mu}_{N,O\cdot,t_0}\right) \end{bmatrix},$$

and $\boldsymbol{\mu}_{i,o\cdot,t_0}$ is given by equation (D.5) for any i and o.

By log-linearizing equation (B.21) for each i and j,

$$\widehat{Q^G_{ij,t}} = -\varepsilon^G \widehat{p^G_{ij,t}} - \left(1-\varepsilon^G\right)\widehat{P^G_{j,t}} + \left[s^G_{j,t_0}\sum_k \widehat{p^G_{jk,t}Q^G_{jk,t}} + s^V_{j,t_0}\sum_{i,o}\widehat{p^R_{ij,o,t}Q^R_{ij,o,t}} + s^R_{j,t_0}\sum_{o,k}\widehat{p^R_{jk,o,t}Q^R_{jk,o,t}}\right]$$

where

$$s^G_{j,t_0} \equiv \frac{\sum_k p^G_{jk,t_0}Q^G_{jk,t_0}}{\sum_k p^G_{jk,t_0}Q^G_{jk,t_0} - \sum_{i,o} p^R_{ij,o,t_0}Q^R_{ij,o,t_0} + \sum_{o,k} p^R_{jk,o,t_0}Q^R_{jk,o,t_0}}$$

is the baseline share of non-robot good production in income,

$$s^R_{j,t_0} \equiv \frac{\sum_{o,k} p^R_{jk,o,t_0}Q^R_{jk,o,t_0}}{\sum_k p^G_{jk,t_0}Q^G_{jk,t_0} - \sum_{i,o} p^R_{ij,o,t_0}Q^R_{ij,o,t_0} + \sum_{o,k} p^R_{jk,o,t_0}Q^R_{jk,o,t_0}},$$

is the baseline share of robot production, and

$$s^V_{j,t_0} \equiv -\frac{\sum_{i,o} p^R_{ij,o,t_0}Q^R_{ij,o,t_0}}{\sum_k p^G_{jk,t_0}Q^G_{jk,t_0} - \sum_{i,o} p^R_{ij,o,t_0}Q^R_{ij,o,t_0} + \sum_{o,k} p^R_{jk,o,t_0}Q^R_{jk,o,t_0}},$$

is the (negative) baseline absorption share of robots. Thus

$$\begin{aligned} &\left[\varepsilon^G \widehat{p^G_{i,t}} + \left(1-\varepsilon^G\right)\sum_l \widetilde{x}^G_{lj,t_0}\widehat{p^G_{l,t}} - s^G_{j,t_0}\widehat{p^G_{j,t}}\right] - \left[s^V_{j,t_0}\sum_{l,o}\widetilde{x}^R_{lj,o,t_0}\widetilde{x}^R_{j,o,t_0}\widehat{p^R_{l,o,t}} + s^R_{t_0}\sum_o \widetilde{y}^R_{j,o,t_0}\widehat{p^R_{j,o,t}}\right] \\ &+ \left(\widehat{Q^G_{ij,t}} - s^G_{j,t_0}\sum_k \widetilde{y}^G_{jk,t_0}\widehat{Q^G_{jk,t}}\right) - \left(s^V_{j,t_0}\sum_{l,o}\widetilde{x}^R_{lj,o,t_0}\widetilde{x}^R_{j,o,t_0}\widehat{Q^R_{lj,o,t}} + s^R_{j,t_0}\sum_{k,o}\widetilde{y}^R_{jk,o,t_0}\widetilde{y}^R_{j,o,t_0}\widehat{Q^R_{jk,o,t}}\right) \\ &= -\left[\varepsilon^G \widehat{\tau^G_{ij,t}} + \left(1-\varepsilon^G\right)\sum_l \widetilde{x}^G_{lj,t_0}\widehat{\tau^G_{lj,t}} - s^G_{j,t_0}\sum_k \widetilde{y}^G_{jk,t_0}\widehat{\tau^G_{jk,t}}\right] + \left[s^V_{j,t_0}\sum_{l,o}\widetilde{x}^R_{lj,t_0}\widehat{\tau^R_{lj,t}} + s^R_{j,t_0}\sum_{k,o}\widetilde{y}^R_{jk,t_0}\widehat{\tau^R_{jk,t}}\right] \end{aligned}$$

where

$$\widetilde{x}^R_{ij,o,t_0} \equiv \frac{p^R_{ij,o,t_0}Q^R_{ij,o,t_0}}{P^R_{j,o,t_0}Q^R_{j,o,t_0}}, \widetilde{x}^R_{j,o,t_0} \equiv \frac{P^R_{j,o,t_0}Q^R_{j,o,t_0}}{P^R_{j,t_0}Q^R_{j,t_0}}, \widetilde{x}^R_{ij,t_0} \equiv \frac{\sum_o p^R_{ij,o,t_0}Q^R_{ij,o,t_0}}{P^R_{j,t_0}Q^R_{j,t_0}},$$

$$\widetilde{y}^R_{ij,o,t_0} \equiv \frac{p^R_{ij,o,t_0} Q^R_{ij,o,t_0}}{\sum_k p^R_{ik,o,t_0} Q^R_{ik,o,t_0}}, \widetilde{y}^R_{i,o,t_0} \equiv \frac{\sum_k p^R_{ik,o,t_0} Q^R_{ik,o,t_0}}{\sum_{k,o'} p^R_{ik,o',t_0} Q^R_{ik,o',t_0}}, \widetilde{y}^R_{ij,t_0} \equiv \frac{\sum_o p^R_{ij,o,t_0} Q^R_{ij,o,t_0}}{\sum_{k,o} p^R_{ik,o,t_0} Q^R_{ik,o,t_0}}.$$

In matrix notation, define

$$\overline{\boldsymbol{M}^{xR}} \equiv \mathbf{1}_N \otimes \begin{bmatrix} \widetilde{\boldsymbol{x}}^R_{t_0} \circ \widetilde{\boldsymbol{x}}^R_{\cdot 1,\cdot,t_0} & \dots & \widetilde{\boldsymbol{x}}^R_{t_0} \circ \widetilde{\boldsymbol{x}}^R_{\cdot N,\cdot,t_0} \end{bmatrix},$$

a $N^2 \times NO$ matrix,

$$\overline{\boldsymbol{M}^{yR}} \equiv \mathbf{1}_N \otimes \begin{bmatrix} \widetilde{y}^R_{1,1} & \dots & \widetilde{y}^R_{1,O} & & \mathbf{0} & \\ & & & \ddots & & \\ & \mathbf{0} & & \widetilde{y}^R_{N,1} & \dots & \widetilde{y}^R_{N,O} \end{bmatrix},$$

a $N^2 \times NO$ matrix,

$$\overline{\boldsymbol{M}^{yG,2}} \equiv \mathbf{1}_N \otimes \overline{\boldsymbol{M}^{yG}}.$$

a $N^2 \times N^2$ matrix,

$$\overline{\boldsymbol{M}^{xR,2}} \equiv \mathbf{1}_N \otimes \begin{bmatrix} \left[\widetilde{x}^R_{1,o,t_0} \widetilde{x}^R_{11,o,t_0}\right]_o & & \mathbf{0} & & \left[\widetilde{x}^R_{1,o,t_0} \widetilde{x}^R_{N1,o,t_0}\right]_o & & \mathbf{0} \\ & \ddots & & \dots & & \ddots & \\ \mathbf{0} & & \left[\widetilde{x}^R_{N,o,t_0} \widetilde{x}^R_{1N,o,t_0}\right]_o & & \mathbf{0} & & \left[\widetilde{x}^R_{N,o,t_0} \widetilde{x}^R_{NN,o,t_0}\right]_o \end{bmatrix}$$

a $N^2 \times N^2O$ matrix ,

$$\overline{\boldsymbol{M}^{yR,2}} \equiv \mathbf{1}_N \otimes \begin{bmatrix} \left[\widetilde{y}^R_{1,o,t_0} \widetilde{y}^R_{11,o,t_0}\right]_o & \dots & \left[\widetilde{y}^R_{N,o,t_0} \widetilde{y}^R_{1N,o,t_0}\right]_o & & & \mathbf{0} & \\ & & & \ddots & & & \\ & \mathbf{0} & & & \left[\widetilde{y}^R_{1,o,t_0} \widetilde{y}^R_{N1,o,t_0}\right]_o & \dots & \left[\widetilde{y}^R_{N,o,t_0} \widetilde{y}^R_{NN,o,t_0}\right]_o \end{bmatrix}$$

a $N^2 \times N^2O$ matrix,

$$\overline{\boldsymbol{M}^{xG,4}} \equiv \mathbf{1}_N \otimes \overline{\boldsymbol{M}^{xG}}$$

a $N^2 \times N^2$ matrix,

$$\overline{\boldsymbol{M}^{xR,3}} \equiv \mathbf{1}_N \otimes \begin{bmatrix} \operatorname{diag}\left(\widetilde{x}^R_{1\cdot,t_0}\right) & \dots & \operatorname{diag}\left(\widetilde{x}^R_{N\cdot,t_0}\right) \end{bmatrix}$$

a $N^2 \times N^2$ matrix,

$$\overline{\boldsymbol{M}^{yR,3}} \equiv \mathbf{1}_N \otimes \begin{bmatrix} \left[\widetilde{y}^R_{11,t_0}, \ldots, \widetilde{y}^R_{1N,t_0}\right] & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \left[\widetilde{y}^R_{N1,t_0}, \ldots, \widetilde{y}^R_{NN,t_0}\right] \end{bmatrix}$$

a $N^2 \times N^2$ matrix, and

$$\overline{\boldsymbol{M}^{xO,2}} \equiv \mathbf{1}_N \otimes \overline{\boldsymbol{M}^{xO}},$$

a $N^2 \times NO$ matrix. Then I have

$$\begin{aligned}
&\left(\varepsilon^G \left[\overline{\boldsymbol{I}_N} \otimes \mathbf{1}_N\right] + \left(1-\varepsilon^G\right)\left[\mathbf{1}_N \otimes \left(\widetilde{\boldsymbol{x}}^G_{t_0}\right)^\top\right] - \mathrm{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^G_{t_0}\right)\left[\mathbf{1}_N \otimes \overline{\boldsymbol{I}_N}\right]\right)\widehat{\boldsymbol{p}^G_t} \\
&- \left(\mathrm{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^V_{t_0}\right)\overline{\boldsymbol{M}^{xR}} + \mathrm{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^R_{t_0}\right)\overline{\boldsymbol{M}^{yR}}\right)\widehat{\boldsymbol{p}^R_t} \\
&+ \left(\overline{\boldsymbol{I}_{N^2}} - \mathrm{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^G_{t_0}\right)\overline{\boldsymbol{M}^{yG,2}}\right)\widehat{\boldsymbol{Q}^G_t} - \left[\mathrm{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^V_{t_0}\right)\overline{\boldsymbol{M}^{xR,2}} + \mathrm{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^R_{t_0}\right)\overline{\boldsymbol{M}^{yR,2}}\right]\widehat{\boldsymbol{Q}^R_t} \\
&= -\left(\varepsilon^G + \left(1-\varepsilon^G\right)\overline{\boldsymbol{M}^{xG,4}} - \mathrm{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^G_{t_0}\right)\overline{\boldsymbol{M}^{yG,2}}\right)\widehat{\boldsymbol{\tau}^G_t} \\
&\qquad + \left(\mathrm{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^V_{t_0}\right)\overline{\boldsymbol{M}^{xR,3}} + \mathrm{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^R_{t_0}\right)\overline{\boldsymbol{M}^{yR,3}}\right)\widehat{\boldsymbol{\tau}^R_t}
\end{aligned}$$

By log-linearizing equation (B.15) for each i, j, and o,,

$$\begin{aligned}
&\left(1-\alpha^R\right)\frac{1+u_{ij,t_0}}{1+u_{ij,t_0}+2\gamma\delta}\sum_l \widetilde{x}^G_{lj,t_0}\widehat{p^G_{l,t}} + \frac{1+u_{ij,t_0}}{1+u_{ij,t_0}+2\gamma\delta}\widehat{p^R_{i,o,t}} \\
&+ \left[\frac{2\gamma\delta}{1+u_{ij,t_0}+2\gamma\delta} - \left(1-\alpha^R\right)\frac{1+u_{ij,t_0}}{1+u_{ij,t_0}+2\gamma\delta}\right]\sum_l \widetilde{x}^R_{lj,o,t_0}\widehat{p^R_{l,o,t}} \\
&+ \frac{1+u_{ij,t_0}}{1+u_{ij,t_0}+2\gamma\delta}\frac{1}{\varepsilon^R}\widehat{Q^R_{ij,o,t}} + \left[-\frac{1}{\varepsilon^R}\frac{1+u_{ij,t_0}}{1+u_{ij,t_0}+2\gamma\delta} + \frac{2\gamma\delta}{1+u_{ij,t_0}+2\gamma\delta}\right]\sum_l \widetilde{x}^R_{lj,o,t_0}\widehat{Q^R_{lj,o,t}} \\
&= -\frac{1+u_{ij,t_0}}{1+u_{ij,t_0}+2\gamma\delta}du_{ij,t} - \left(1-\alpha^R\right)\frac{1+u_{ij,t_0}}{1+u_{ij,t_0}+2\gamma\delta}\sum_l \widetilde{x}^G_{lj,t_0}\widehat{\tau^G_{lj,t}} - \frac{1+u_{ij,t_0}}{1+u_{ij,t_0}+2\gamma\delta}\widehat{\tau^R_{ij,t}} \\
&- \left[\frac{2\gamma\delta}{1+u_{ij,t_0}+2\gamma\delta} - \left(1-\alpha^R\right)\frac{1+u_{ij,t_0}}{1+u_{ij,t_0}+2\gamma\delta}\right]\sum_l \widetilde{x}^R_{lj,o,t_0}\widehat{\tau^R_{lj,t}} + \widehat{\lambda^R_{j,o,t}} + \frac{2\gamma\delta}{1+u_{ij,t_0}+2\gamma\delta}\widehat{K^R_{j,o,t}}.
\end{aligned}$$

In matrix notation, write a preliminary $N \times N$ matrix $\widetilde{\overline{\boldsymbol{u}_{t_0}}}$ as such that the (i, j)-element is

$$\frac{1+u_{ij,t_0}}{1+u_{ij,t_0}+2\gamma\delta}.$$

Then $\mathbf{1}_N\left(\mathbf{1}_N\right)^\top - \overline{\widetilde{\boldsymbol{u}_{t_0}}}$ is a matrix that is filled with $2\gamma\delta / \left(1 + u_{ij,t_0} + 2\gamma\delta\right)$ for its (i, j) element and

$$\overline{\boldsymbol{M}^{u}} \equiv \operatorname{diag}\left(\left[\widetilde{u_{1\cdot,t_0}}, \ldots, \widetilde{u_{N\cdot,t_0}}\right]^\top\right).$$

Using these, write

$$\overline{\boldsymbol{M}^{xG,5}} \equiv \left(\overline{\boldsymbol{M}^{u}} \otimes \overline{\boldsymbol{I}_O}\right)\left(\mathbf{1}_N \otimes \left(\widetilde{\boldsymbol{x}}^G_{t_0}\right)^\top \otimes \mathbf{1}_O\right)$$

a $N^2O \times N$ matrix,

$$\overline{\boldsymbol{M}^{u,2}} \equiv \begin{bmatrix} \widetilde{\boldsymbol{u}_{1\cdot,t_0}} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \widetilde{\boldsymbol{u}_{N\cdot,t_0}} \end{bmatrix} \otimes \overline{\boldsymbol{I}_O},$$

a $N^2O \times NO$ matrix where $\widetilde{\boldsymbol{u}_{i\cdot,t_0}} \equiv \left(\widetilde{u_{i\cdot,t_0}}\right)_j$ is a $N \times 1$ vector,

$$\overline{\boldsymbol{M}^{xR,4}} \equiv \left\{\left[\left(\overline{\boldsymbol{I}_{N^2}} - \overline{\boldsymbol{M}^{u}}\right) - \left(1 - \alpha^R\right)\overline{\boldsymbol{M}^{u}}\right] \otimes \overline{\boldsymbol{I}_O}\right\} \times .$$
$$\left(\mathbf{1}_N \otimes \begin{bmatrix} \operatorname{diag}\left(\left\{\widetilde{x}^R_{11,o,t_0}\right\}_o\right) & \ldots & \operatorname{diag}\left(\left\{\widetilde{x}^R_{N1,o,t_0}\right\}_o\right) \\ \vdots & & \vdots \\ \operatorname{diag}\left(\left\{\widetilde{x}^R_{1N,o,t_0}\right\}_o\right) & \ldots & \operatorname{diag}\left(\left\{\widetilde{x}^R_{NN,o,t_0}\right\}_o\right) \end{bmatrix}\right)$$

a $N^2O \times NO$ matrix,

$$\overline{\boldsymbol{M}^{xR,5}} \equiv \left\{\left[-\frac{1}{\varepsilon^R}\overline{\boldsymbol{M}^{u}} + \left(\overline{\boldsymbol{I}_{N^2}} - \overline{\boldsymbol{M}^{u}}\right)\right] \otimes \overline{\boldsymbol{I}_O}\right\} \times$$
$$\left\{\mathbf{1}_N \otimes \begin{bmatrix} \operatorname{diag}\left(\begin{bmatrix} \widetilde{x}^R_{11,1,t_0} \\ \vdots \\ \widetilde{x}^R_{11,O,t_0} \\ \vdots \\ \widetilde{x}^R_{1N,O,t_0} \end{bmatrix}\right) & \ldots & \operatorname{diag}\left(\begin{bmatrix} \widetilde{x}^R_{N1,1,t_0} \\ \vdots \\ \widetilde{x}^R_{N1,O,t_0} \\ \vdots \\ \widetilde{x}^R_{NN,O,t_0} \end{bmatrix}\right) \end{bmatrix}\right\}$$

a $N^2O \times N^2O$ matrix,

$$\overline{\boldsymbol{M}^{xG,6}} \equiv \left(\overline{\boldsymbol{M}^{u}} \otimes \overline{\boldsymbol{I}_O}\right) \left\{ \mathbf{1}_N \otimes \begin{bmatrix} \operatorname{diag}\left(\begin{bmatrix} \widetilde{x}^{G}_{11,t_0} \\ \vdots \\ \widetilde{x}^{G}_{1N,t_0} \end{bmatrix} \right) & \cdots & \operatorname{diag}\left(\begin{bmatrix} \widetilde{x}^{G}_{N1,t_0} \\ \vdots \\ \widetilde{x}^{G}_{NN,t_0} \end{bmatrix} \right) \end{bmatrix} \otimes \mathbf{1}_O \right\}$$

a $N^2O \times N^2$ matrix,

$$\begin{aligned} \overline{\boldsymbol{M}^{xR,6}} \equiv & \left\{ \left[\left(\overline{\boldsymbol{I}_{N^2}} - \overline{\boldsymbol{M}^{u}}\right) - \left(1-\alpha^R\right) \overline{\boldsymbol{M}^{u}} \right] \otimes \overline{\boldsymbol{I}_O} \right\} \\ & \times \left\{ \mathbf{1}_N \otimes \begin{bmatrix} \left[\widetilde{x}^{R}_{11,o,t_0}\right]_o & \mathbf{0} & \mathbf{0} & \cdots & \left[\widetilde{x}^{R}_{N1,o,t_0}\right]_o & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} & & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \left[\widetilde{x}^{R}_{1N,o,t_0}\right]_o & & \mathbf{0} & \mathbf{0} & \left[\widetilde{x}^{R}_{N3,o,t_0}\right]_o \end{bmatrix} \right\} \end{aligned}$$

a $N^2O \times N^2$ matrix, and

$$\overline{\boldsymbol{M}^{u,3}} \equiv \begin{bmatrix} 1-\widetilde{\widehat{u_{11,t_0}}} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & 1-\widetilde{\widehat{u_{1N,t_0}}} \\ & \vdots & \\ 1-\widetilde{\widehat{u_{N1,t_0}}} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & 1-\widetilde{\widehat{u_{NN,t_0}}} \end{bmatrix} \otimes \overline{\boldsymbol{I}_O}$$

a $N^2O \times NO$ matrix. Finally, I have

$$\begin{aligned} & \left(1-\alpha^R\right) \overline{\boldsymbol{M}^{xG,5}} \widehat{\boldsymbol{p}^{G}_t} + \left[\overline{\boldsymbol{M}^{u,2}} + \overline{\boldsymbol{M}^{xR,4}}\right] \widehat{\boldsymbol{p}^{R}_t} + \left\{ \frac{1}{\varepsilon^R} \left(\overline{\boldsymbol{M}^{u}} \otimes \overline{\boldsymbol{I}_O}\right) + \overline{\boldsymbol{M}^{xR,5}} \right\} \widehat{\boldsymbol{Q}^{R}_t} \\ & = -\left(\overline{\boldsymbol{M}^{u}} \otimes \mathbf{1}_O\right) d\boldsymbol{u}_t - \left(1-\alpha^R\right) \overline{\boldsymbol{M}^{xG,6}} \widehat{\boldsymbol{\tau}^{G}_t} - \left[\left(\overline{\boldsymbol{M}^{u}} \otimes \mathbf{1}_O\right) + \overline{\boldsymbol{M}^{xR,6}} \right] \widehat{\boldsymbol{\tau}^{R}_t} + \left(\mathbf{1}_N \otimes \overline{\boldsymbol{I}_{NO}}\right) \widehat{\boldsymbol{\lambda}^{R}_t} + \overline{\boldsymbol{M}^{u,3}} \widehat{\boldsymbol{K}^{R}_t}. \end{aligned}$$

By log-linearizing equation and (B.10) for each i and o,

$$\begin{aligned}
&\widehat{p^G_{i,t}} + \sum_j \widetilde{y}^G_{ij,t_0}\widehat{Q^G_{ij,t}} - \widehat{w_{i,o,t}} + \left[-\frac{1}{\theta} + \left(-\frac{1}{\beta} + \frac{1}{\theta}\right) l^O_{i,o,t_0}\right]\widehat{L_{i,o,t}} + \left(-1 + \frac{1}{\beta}\right)\sum_{o'} \widetilde{x}^O_{i,o',t_0} l^O_{i,o',t_0}\widehat{L_{i,o',t}} \\
&= -\frac{1}{\beta}\widehat{b_{i,o,t}} + \frac{1}{\theta}\frac{a_{o,t_0}}{1-a_{o,t_0}}\widehat{a_{o,t}} + \left(-\frac{1}{\beta} + \frac{1}{\theta}\right)\frac{1}{\theta - 1}\left[-\left(1 - l^O_{i,o,t_0}\right) + l^O_{i,o,t_0}\frac{a_{o,t_0}}{1-a_{o,t_0}}\right]\widehat{a_{o,t}} \\
&\quad + \left(-1 + \frac{1}{\beta}\right)\frac{1}{\theta - 1}\sum_{o'} \widetilde{x}^O_{i,o',t_0}\left[-\left(1 - l^O_{i,o',t_0}\right) + l^O_{i,o',t_0}\frac{a_{o',t_0}}{1-a_{o',t_0}}\right]\widehat{a_{o',t}} \\
&\quad - \sum_j y^G_{ij,t_0}\widehat{\tau^G_{ij,t}} - \left(-\frac{1}{\beta} + \frac{1}{\theta}\right)\left(1 - l^O_{i,o,t_0}\right)\widehat{K^R_{i,o,t}} - \left(-1 + \frac{1}{\beta}\right)\sum_{o'} \widetilde{x}^O_{i,o',t_0}\left(1 - l^O_{i,o',t_0}\right)\widehat{K^R_{i,o',t}},
\end{aligned}$$

In matrix notation, write

$$\overline{\boldsymbol{M}^{yG,3}} \equiv \overline{\boldsymbol{M}^{yG}} \otimes \mathbf{1}_O$$

a $NO \times N^2$ matrix,

$$\overline{\boldsymbol{M}^{xOl,3}} \equiv \overline{\boldsymbol{M}^{xOl}} \otimes \mathbf{1}_O$$

a $NO \times NO$ matrix,

$$\overline{\boldsymbol{M}^{a}} \equiv \mathbf{1}_N \otimes \operatorname{diag}\left(\frac{a_{o,t_0}}{1-a_{o,t_0}}\right)$$

a $NO \times O$ matrix,

$$\overline{\boldsymbol{M}^{al,2}} \equiv \begin{bmatrix} \operatorname{diag}\left(-\left(1 - l^O_{1,o,t_0}\right) + l^O_{1,o,t_0}\frac{a_{o,t_0}}{1-a_{o,t_0}}\right) \\ \vdots \\ \operatorname{diag}\left(-\left(1 - l^O_{N,o,t_0}\right) + l^O_{N,o,t_0}\frac{a_{o,t_0}}{1-a_{o,t_0}}\right) \end{bmatrix}$$

a $NO \times O$ matrix,

$$\overline{\boldsymbol{M}^{al,3}} \equiv \left(\widetilde{\boldsymbol{x}}^O_{t_0} \circ \overline{\boldsymbol{M}^{al}}\right) \otimes \mathbf{1}_O$$

a $NO \times O$ matrix,

$$\overline{\boldsymbol{M}^{xOl,4}} \equiv \overline{\boldsymbol{M}^{xOl,2}} \otimes \mathbf{1}_O,$$

a $NO \times NO$ matrix. I have

$$\left(\boldsymbol{I}_N \otimes \boldsymbol{1}_O\right) \widehat{\boldsymbol{p}_t^G} - \widehat{\boldsymbol{w}_t} + \overline{\boldsymbol{M}^{yG,3}} \widehat{\boldsymbol{Q}_t^G} + \left(-\frac{1}{\theta}\overline{\boldsymbol{I}_{NO}} + \left(-\frac{1}{\beta}+\frac{1}{\theta}\right) \operatorname{diag}\left(\boldsymbol{l}_{t_0}^O\right) + \left(-1+\frac{1}{\beta}\right)\overline{\boldsymbol{M}^{xOl,3}}\right) \widehat{\boldsymbol{L}}_t$$
$$= -\frac{1}{\beta}\widehat{\boldsymbol{b}_t} + \left[\frac{1}{\theta}\overline{\boldsymbol{M}^a} + \left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\frac{1}{\theta-1}\overline{\boldsymbol{M}^{al,2}} + \left(-1+\frac{1}{\beta}\right)\frac{1}{\theta-1}\overline{\boldsymbol{M}^{al,3}}\right]\widehat{\boldsymbol{a}}_t - \overline{\boldsymbol{M}^{yG,3}}\widehat{\boldsymbol{\tau}_t^G}$$
$$+ \left[-\left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\operatorname{diag}\left(1-l_{i,o,t_0}^O\right) - \left(-1+\frac{1}{\beta}\right)\overline{\boldsymbol{M}^{xOl,4}}\right]\widehat{\boldsymbol{K}_t^R}.$$

Hence the log-linearized temporary equilibrium system is

$$\overline{\boldsymbol{D}^x}\widehat{\boldsymbol{x}} = \overline{\boldsymbol{D}^A}\widehat{\boldsymbol{A}}$$

where matrices $\overline{\boldsymbol{D}^x}$ and $\overline{\boldsymbol{D}^A}$ are defined as

$$\overline{\boldsymbol{D}^x} \equiv \begin{bmatrix} \overline{\boldsymbol{D}_{11}^x} & \boldsymbol{0} & \boldsymbol{0} & \overline{\boldsymbol{D}_{14}^x} & \boldsymbol{0} & \overline{\boldsymbol{D}_{16}^x} \\ -\overline{\boldsymbol{M}^{xG,2}} & \overline{\boldsymbol{I}_{NO}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \phi\overline{\boldsymbol{M}^{xG,2}} & \boldsymbol{0} & -\phi\overline{\boldsymbol{I}_{NO}} & \boldsymbol{0} & \boldsymbol{0} & \overline{\boldsymbol{M}^l} \\ \overline{\boldsymbol{D}_{41}^x} & \overline{\boldsymbol{D}_{42}^x} & \boldsymbol{0} & \overline{\boldsymbol{D}_{44}^x} & \overline{\boldsymbol{D}_{45}^x} & \boldsymbol{0} \\ \overline{\boldsymbol{D}_{51}^x} & \overline{\boldsymbol{D}_{52}^x} & \boldsymbol{0} & \boldsymbol{0} & \overline{\boldsymbol{D}_{55}^x} & \boldsymbol{0} \\ \overline{\boldsymbol{D}_{61}^x} & \boldsymbol{0} & -\overline{\boldsymbol{I}_{NO}} & \overline{\boldsymbol{M}^{yG,3}} & \boldsymbol{0} & \overline{\boldsymbol{D}_{66}^x} \end{bmatrix},$$

where

$$\overline{\boldsymbol{D}_{11}^x} \equiv -\alpha_M\left(\overline{\boldsymbol{I}_N} - \left(\widetilde{\boldsymbol{x}}_{t_0}^G\right)^\top\right),\ \overline{\boldsymbol{D}_{14}^x} \equiv \left(1-\alpha_M\right)\overline{\boldsymbol{M}^{yG}},\ \overline{\boldsymbol{D}_{16}^x} \equiv -\alpha_L\overline{\boldsymbol{M}^{xOl}},$$

$$\overline{\boldsymbol{D}_{41}^x} \equiv \varepsilon^G\left[\overline{\boldsymbol{I}_N}\otimes\boldsymbol{1}_N\right] + \left(1-\varepsilon^G\right)\left[\boldsymbol{1}_N\otimes\left(\widetilde{\boldsymbol{x}}_{t_0}^G\right)^\top\right] - \operatorname{diag}\left(\boldsymbol{1}_N\otimes\boldsymbol{s}_{t_0}^G\right)\left[\boldsymbol{1}_N\otimes\overline{\boldsymbol{I}_N}\right],$$

$$\overline{\boldsymbol{D}_{42}^x} \equiv \operatorname{diag}\left(\boldsymbol{1}_N\otimes\boldsymbol{s}_{t_0}^V\right)\overline{\boldsymbol{M}^{xR}} + \operatorname{diag}\left(\boldsymbol{1}_N\otimes\boldsymbol{s}_{t_0}^R\right)\overline{\boldsymbol{M}^{yR}},$$

$$\overline{\boldsymbol{D}_{44}^x} \equiv \overline{\boldsymbol{I}_{N^2}} - \operatorname{diag}\left(\boldsymbol{1}_N\otimes\boldsymbol{s}_{t_0}^G\right)\overline{\boldsymbol{M}^{yG,2}},$$

$$\overline{\boldsymbol{D}_{45}^x} \equiv -\operatorname{diag}\left(\boldsymbol{1}_N\otimes\boldsymbol{s}_{t_0}^V\right)\overline{\boldsymbol{M}^{xR,2}} - \operatorname{diag}\left(\boldsymbol{1}_N\otimes\boldsymbol{s}_{t_0}^R\right)\overline{\boldsymbol{M}^{yR,2}},$$

$$\overline{\boldsymbol{D}_{51}^x} \equiv \left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xG,5}},\ \overline{\boldsymbol{D}_{52}^x} \equiv \overline{\boldsymbol{M}^{u,2}} + \overline{\boldsymbol{M}^{xR,4}},\ \overline{\boldsymbol{D}_{55}^x} \equiv \frac{1}{\varepsilon^R}\left(\overline{\boldsymbol{M}^u}\otimes\overline{\boldsymbol{I}_O}\right) + \overline{\boldsymbol{M}^{xR,5}},$$

$$\overline{\boldsymbol{D}_{61}^x} \equiv \boldsymbol{I}_N\otimes\boldsymbol{1}_N,\ \overline{\boldsymbol{D}_{66}^x} \equiv -\frac{1}{\theta} + \left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\operatorname{diag}\left(\boldsymbol{l}_{t_0}^O\right) + \left(-1+\frac{1}{\beta}\right)\overline{\boldsymbol{M}^{xOl,3}},$$

and

$$\overline{\boldsymbol{D}^{A}} \equiv \begin{bmatrix} \mathbf{0} & \overline{\boldsymbol{D}_{12}^{A}} & \overline{\boldsymbol{D}_{13}^{A}} & \overline{\boldsymbol{I}_{N}} & \mathbf{0} & \overline{\boldsymbol{D}_{16}^{A}} & \overline{\boldsymbol{D}_{17}^{A}} & \mathbf{0} & \alpha_{L}\overline{\boldsymbol{M}^{xOl,2}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\overline{\boldsymbol{I}_{NO}} & \mathbf{0} & \overline{\boldsymbol{M}^{xG}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\phi\overline{\boldsymbol{M}^{xG,3}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \overline{\boldsymbol{D}_{47}^{A}} & \overline{\boldsymbol{D}_{48}^{A}} & \mathbf{0} & \mathbf{0} \\ \overline{\boldsymbol{D}_{51}^{A}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \overline{\boldsymbol{D}_{57}^{A}} & \overline{\boldsymbol{D}_{58}^{A}} & \overline{\boldsymbol{M}^{u,3}} & \overline{\boldsymbol{D}_{5,10}^{A}} \\ \mathbf{0} & \overline{\boldsymbol{D}_{62}^{A}} & -\frac{1}{\beta}\overline{\boldsymbol{I}_{NO}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\overline{\boldsymbol{M}^{yG,3}} & \mathbf{0} & \overline{\boldsymbol{D}_{69}^{A}} & \mathbf{0} \end{bmatrix},$$

where

$$\overline{\boldsymbol{D}_{12}^{A}} \equiv \frac{\alpha_L}{\theta-1}\left(\widetilde{\boldsymbol{x}}_{t_0}^{O} \otimes \overline{\boldsymbol{M}^{al}}\right),\ \overline{\boldsymbol{D}_{13}^{A}} \equiv \frac{\alpha_L}{\beta-1}\overline{\boldsymbol{M}^{xO}},$$

$$\overline{\boldsymbol{D}_{16}^{A}} \equiv (1-\alpha_L-\alpha_M)\,\overline{\boldsymbol{I}_N},\ \overline{\boldsymbol{D}_{17}^{A}} \equiv -\left[\alpha_M\overline{\boldsymbol{M}^{xG}} + (1-\alpha_M)\,\overline{\boldsymbol{M}^{yG}}\right],$$

$$\overline{\boldsymbol{D}_{47}^{A}} \equiv -\varepsilon^{G} + \left(1-\varepsilon^{G}\right)\overline{\boldsymbol{M}^{xG,4}} + \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}_{t_0}^{G}\right)\overline{\boldsymbol{M}^{yG,2}},$$

$$\overline{\boldsymbol{D}_{48}^{A}} \equiv \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}_{t_0}^{V}\right)\overline{\boldsymbol{M}^{xR,3}} + \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}_{t_0}^{R}\right)\overline{\boldsymbol{M}^{yR,3}},$$

$$\overline{\boldsymbol{D}_{51}^{A}} \equiv -\left(\overline{\boldsymbol{M}^{u}} \otimes \mathbf{1}_O\right),\ \overline{\boldsymbol{D}_{57}^{A}} \equiv -\left(1-\alpha^{R}\right)\overline{\boldsymbol{M}^{xG,6}},$$

$$\overline{\boldsymbol{D}_{58}^{A}} \equiv -\left[\left(\overline{\boldsymbol{M}^{u}} \otimes \mathbf{1}_O\right) + \overline{\boldsymbol{M}^{xR,6}}\right],\ \overline{\boldsymbol{D}_{5,10}^{A}} \equiv \mathbf{1}_N \otimes \overline{\boldsymbol{I}_{NO}},$$

$$\overline{\boldsymbol{D}_{62}^{A}} \equiv \frac{1}{\theta}\overline{\boldsymbol{M}^{a}} + \left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\frac{1}{\theta-1}\overline{\boldsymbol{M}^{al,2}} + \left(-1+\frac{1}{\beta}\right)\frac{1}{\theta-1}\overline{\boldsymbol{M}^{al,3}},$$

and

$$\overline{\boldsymbol{D}_{69}^{A}} \equiv -\left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\operatorname{diag}\left(1-l_{i,o,t_0}^{O}\right) - \left(-1+\frac{1}{\beta}\right)\overline{\boldsymbol{M}^{xOl,4}}.$$

Note that to normalize the price, one of the good-demand equation must be replaced with log-linearized numeraire condition $\widehat{P_{1,t}^{G}} = \sum_i x_{i1,t_0}^{G}\left(\widehat{p_{i,t}^{G}} + \widehat{\tau_{i1,t}^{G}}\right) = 0$, or

$$\overline{\boldsymbol{M}^{xG,num}}\widehat{\boldsymbol{p}_t^{G}} = -\overline{\boldsymbol{M}^{xG,num}}\widehat{\boldsymbol{\tau}_t^{G}},$$

where $\overline{\boldsymbol{M}^{xG,num}} \equiv \left[x_{11,t_0}^{G}, x_{21,t_0}^{G}, x_{31,t_0}^{G}\right]$.

To analyze the steady state conditions, first note that the steady state accumulation condition (B.24) implies $\widehat{Q_{i,o}^{R}} = \widehat{K_{i,o}^{R}}$. Using robot integration function, integration demand and unit cost formula, I have

$$\widehat{Q^R_{i,o}} = \sum_l x^R_{li,o,t_0}\widehat{Q^R_{li,o}} + \left(1-\alpha^R\right)\left(\sum_l \widetilde{x}^R_{ij,o,t_0}\widehat{p^R_{li,o}} - \sum_l \widetilde{x}^G_{li,t_0}\widehat{p^G_{li,t}}\right) \tag{D.6}$$

Thus the condition is

$$\begin{aligned}
&\sum_l \widetilde{x}^R_{li,o,t_0}\widehat{Q^R_{li,o}} + \left(1-\alpha^R\right)\sum_l \widetilde{x}^R_{li,o,t_0}\widehat{p^R_{l,o}} - \left(1-\alpha^R\right)\sum_l \widetilde{x}^G_{li,t_0}\widehat{p^G_l} - \widehat{K^R_{i,o}} \\
&= \left(1-\alpha^R\right)\sum_l \widetilde{x}^G_{li,t_0}\widehat{\tau^G_{li}} - \left(1-\alpha^R\right)\sum_l \widetilde{x}^R_{li,o,t_0}\widehat{\tau^R_{li}}.
\end{aligned}$$

In a matrix form, write

$$\overline{\boldsymbol{M}^{xR,7}} \equiv \begin{bmatrix} \operatorname{diag}\left(\widetilde{x}^R_{1\cdot,\cdot,t_0}\right) & \dots & \operatorname{diag}\left(\widetilde{x}^R_{N\cdot,\cdot,t_0}\right) \end{bmatrix}$$

a $NO \times N^2O$ matrix,

$$\overline{\boldsymbol{M}^{xR,8}} \equiv \begin{bmatrix} \operatorname{diag}\left(\widetilde{x}^R_{11,\cdot,t_0}\right) & \dots & \operatorname{diag}\left(\widetilde{x}^R_{N1,\cdot,t_0}\right) \\ \vdots & & \vdots \\ \operatorname{diag}\left(\widetilde{x}^R_{1N,\cdot,t_0}\right) & \dots & \operatorname{diag}\left(\widetilde{x}^R_{NN,\cdot,t_0}\right) \end{bmatrix}$$

a $NO \times NO$ matrix, and

$$\overline{\boldsymbol{M}^{xG,7}} \equiv \begin{bmatrix} \widetilde{x}^G_{11,t_0} & & & \dots & \widetilde{x}^G_{N1,t_0} & & \mathbf{0} \\ & \ddots & & & & \ddots & \\ \mathbf{0} & & \widetilde{x}^G_{1N,t_0} & & & & \widetilde{x}^G_{NN,t_0} \end{bmatrix} \otimes \mathbf{1}_O$$

a $NO \times N^2$ matrix.

$$\overline{\boldsymbol{M}^{xR,9}} \equiv \begin{bmatrix} \widetilde{\boldsymbol{x}}^R_{11,\cdot,t_0} & & \mathbf{0} & & \widetilde{\boldsymbol{x}}^R_{N1,\cdot,t_0} & & \mathbf{0} \\ & \ddots & & \dots & & \ddots & \\ \mathbf{0} & & \widetilde{\boldsymbol{x}}^R_{1N,\cdot,t_0} & & \mathbf{0} & & \widetilde{\boldsymbol{x}}^R_{NN,\cdot,t_0} \end{bmatrix},$$

a $NO \times N^2$ matrix, where $\widetilde{\boldsymbol{x}}^R_{ij,\cdot,t_0} \equiv \left(\widetilde{x}^R_{ij,o,t_0}\right)_o$ is an $O \times 1$ vector for any i and j. Then I have

$$-\left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xG,2}}\widehat{\boldsymbol{p}^G}+\left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xR,8}}\widehat{\boldsymbol{p}^R}+\overline{\boldsymbol{M}^{xR,7}}\widehat{\boldsymbol{Q}^R}-\widehat{\boldsymbol{K}^R}=\left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xG,7}}\widehat{\boldsymbol{\tau}^G}-\left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xR,9}}\widehat{\boldsymbol{\tau}^R}$$

Next, to study the steady state Euler equation (B.25) , note that by equation (B.13),

$$\begin{aligned}
\frac{\partial \pi_{i,t}\left(\widehat{\left\{K^R_{i,o,t}\right\}}\right)}{\partial K^R_{i,o,t}} &= \sum_j \widetilde{y}^G_{ij,t}\left(\widehat{p^G_{ij,t}}+\widehat{Q^G_{ij,t}}\right)+\left[-\frac{1}{\beta}\sum_{o'} x^O_{i,o',t_0}\widehat{b_{i,o',t}}+\frac{1}{\beta}\widehat{b_{i,o,t}}\right] \\
&+\left\{\left(-1+\frac{1}{\beta}\right)\frac{1}{\theta-1}\sum_{o'}\frac{\widetilde{x}^O_{i,o',t_0}}{1-a_{o,t_0}}\left[-l^O_{i,o',t_0}a_{o,t_0}+\left(1-l^O_{i,o',t_0}\right)\left(1-a_{o,t_0}\right)\right]\widehat{a_{o',t}}\right. \\
&\left.+\left\{\left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\frac{1}{\theta-1}\frac{-l^O_{i,o,t_0}a_{o,t_0}+\left(1-l^O_{i,o,t_0}\right)\left(1-a_{o,t_0}\right)}{1-a_{o,t_0}}+\frac{1}{\theta}\right\}\widehat{a_{o,t}}\right\} \\
&+\left[\left(-1+\frac{1}{\beta}\right)\sum_{o'} x^O_{i,o',t_0}l^O_{i,o',t_0}\widehat{L_{i,o',t}}+\left(-\frac{1}{\beta}+\frac{1}{\theta}\right)l^O_{i,o,t_0}\widehat{L_{i,o,t}}\right] \\
&+\left[\left(-1+\frac{1}{\beta}\right)\sum_{o'}\widetilde{x}^O_{i,o',t_0}\left(1-l^O_{i,o',t_0}\right)\widehat{K^R_{i,o',t}}+\left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\left(1-l^O_{i,o,t_0}\right)\widehat{K^R_{i,o,t}}+\left(-\frac{1}{\theta}\right)\widehat{K^R_{i,o,t}}\right].
\end{aligned} \tag{D.7}$$

Note that by the steady state accumulation condition (B.24), $Q^R_{i,o,t_0}/K^R_{i,o,t_0} = \delta$. Note also that investment function implies that, in the steady state,

$$\frac{\lambda^R_{j,o}}{P^R_{j,o}} = \left(\sum_i \frac{x^R_{ij,o}}{\left(1+u_{ij}\right)^{1-\varepsilon^R}}\right)^{\frac{1}{1-\varepsilon^R}\alpha^R} + 2\gamma\delta. \tag{D.8}$$

To simplify the notation, set

$$\widetilde{u}^{SS}_{j,o,t_0} \equiv \frac{(\iota+\delta)\left[\left(\sum_i x^R_{ij,o,t_0}\left(1+u_{ij,t_0}\right)^{-\left(1-\varepsilon^R\right)}\right)^{\frac{1}{1-\varepsilon^R}\alpha^R}+2\gamma\delta\right]}{(\iota+\delta)\left[\left(\sum_i x^R_{ij,o,t_0}\left(1+u_{ij,t_0}\right)^{-\left(1-\varepsilon^R\right)}\right)^{\frac{1}{1-\varepsilon^R}\alpha^R}+2\gamma\delta\right]-\gamma\delta^2},$$

Then by log-linearizing equation (B.25) implies, after rearranging,

$$\left[\widehat{p_i^G}+2\left(1-\alpha^R\right)\left(1-\widetilde{u}_{i,o,t_0}^{SS}\right)\sum_l \widetilde{x}_{li,t_0}^G\widehat{p_{l,t}^G}\right]-\left(1-\widetilde{u}_{i,o,t_0}^{SS}\right)\widehat{p_{i,o}^R}-2\left(1-\alpha^R\right)\left(1-\widetilde{u}_{i,o,t_0}^{SS}\right)\sum_l \widetilde{x}_{ij,o,t_0}^R\widehat{p_{l,o}^R}$$
$$+\sum_j \widetilde{y}_{ij,t_0}^G\widehat{Q_{ij}^G}-2\left(1-\widetilde{u}_{i,o,t_0}^{SS}\right)\sum_l \widetilde{x}_{li,o,t_0}^R\widehat{Q_{li,o}^R}+\left[\left(-1+\frac{1}{\beta}\right)\sum_{o'}\widetilde{x}_{i,o',t_0}^O l_{i,o',t_0}^O\widehat{L_{i,o'}}+\left(-\frac{1}{\beta}+\frac{1}{\theta}\right)l_{i,o,t_0}^O\widehat{L_{i,o}}\right]$$
$$+\left[\left(-1+\frac{1}{\beta}\right)\sum_{o'}\widetilde{x}_{i,o',t_0}^O\left(1-l_{i,o',t_0}^O\right)\widehat{K_{i,o'}^R}+\left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\left(1-l_{i,o,t_0}^O\right)\widehat{K_{i,o}^R}+\left(-\frac{1}{\theta}\right)\widehat{K_{i,o}^R}+2\left(1-\widetilde{u}_{i,o,t_0}^{SS}\right)\widehat{K_{i,o}^R}\right]$$
$$-\widetilde{u}_{i,o,t_0}^{SS}\widehat{\lambda_{i,o}^R}$$
$$=-\left(-1+\frac{1}{\beta}\right)\frac{1}{\theta-1}\sum_{o'}\frac{\widetilde{x}_{i,o',t_0}^O}{1-a_{o,t_0}}\left[\left(1-l_{i,o',t_0}^O\right)\left(1-a_{o',t_0}\right)-l_{i,o',t_0}^O a_{o',t_0}\right]\widehat{a_{o'}}$$
$$-\left\{\left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\frac{1}{\theta-1}\frac{1}{1-a_{o,t_0}}\left[\left(1-l_{i,o,t_0}^O\right)\left(1-a_{o,t_0}\right)-l_{i,o,t_0}^O a_{o,t_0}\right]+\frac{1}{\theta}\right\}\widehat{a_o}$$
$$-\left[-\frac{1}{\beta}\sum_{o'}\widetilde{x}_{i,o',t_0}^O\widehat{b_{i,o'}}+\frac{1}{\beta}\widehat{b_{i,o}}\right]+\left[-\sum_j \widetilde{y}_{ij,t_0}^G\widehat{\tau_{ij}^G}-2\left(1-\alpha^R\right)\left(1-\widetilde{u}_{i,o,t_0}^{SS}\right)\sum_l \widetilde{x}_{li,t_0}^G\widehat{\tau_{li,t}^G}\right]$$
$$+2\left(1-\alpha^R\right)\left(1-\widetilde{u}_{i,o,t_0}^{SS}\right)\sum_l \widetilde{x}_{ij,o,t_0}^R\widehat{\tau_{li}^R}$$

In matrix notation, write

$$\overline{\boldsymbol{M}^{xO,3}}\equiv\overline{\boldsymbol{M}^{xO}}\otimes\mathbf{1}_O$$

a $NO\times N^2$ matrix. Then

$$\left[\left(\overline{\boldsymbol{I}_N}\otimes\mathbf{1}_O\right)+2\left(1-\alpha^R\right)\operatorname{diag}\left(1-\widetilde{u}_{\cdot,\cdot,t_0}^{SS}\right)\overline{\boldsymbol{M}^{xG,2}}\right]\widehat{\boldsymbol{p}^G}-\operatorname{diag}\left(1-\widetilde{u}_{\cdot,\cdot,t_0}^{SS}\right)\left(\overline{\boldsymbol{I}_{NO}}-2\left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xR,8}}\right)\widehat{\boldsymbol{p}^R}$$
$$+\overline{\boldsymbol{M}^{yG,3}}\widehat{\boldsymbol{Q}^G}-2\operatorname{diag}\left(1-\widetilde{u}_{\cdot,\cdot,t_0}^{SS}\right)\overline{\boldsymbol{M}^{xR,7}}\widehat{\boldsymbol{Q}^R}+\left[\left(-1+\frac{1}{\beta}\right)\overline{\boldsymbol{M}^{xOl,3}}+\left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\operatorname{diag}\left(l_{\cdot,\cdot,t_0}^O\right)\right]\widehat{\boldsymbol{L}}$$
$$+\left[\left(-1+\frac{1}{\beta}\right)\overline{\boldsymbol{M}^{xOl,4}}+\left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\operatorname{diag}\left(1-l_{\cdot,\cdot,t_0}^O\right)-\frac{1}{\theta}\overline{\boldsymbol{I}_{NO}}+2\operatorname{diag}\left(1-\widetilde{u}_{\cdot,\cdot,t_0}^{SS}\right)\right]\widehat{\boldsymbol{K}^R}-\operatorname{diag}\left(\widetilde{u}_{\cdot,\cdot,t_0}^{SS}\right)\widehat{\boldsymbol{\lambda}^R}$$
$$=-\left[\left(-1+\frac{1}{\beta}\right)\frac{1}{\theta-1}\overline{\boldsymbol{M}^{al,3}}-\left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\frac{1}{\theta-1}\overline{\boldsymbol{M}^{al,2}}\right]\widehat{\boldsymbol{a}}-\frac{1}{\beta}\left(\overline{\boldsymbol{I}_{NO}}-\overline{\boldsymbol{M}^{xO,3}}\right)\widehat{\boldsymbol{b}}$$
$$+\left[-\overline{\boldsymbol{M}^{yG,3}}-2\left(1-\alpha^R\right)\operatorname{diag}\left(1-\widetilde{u}_{\cdot,\cdot,t_0}^{SS}\right)\overline{\boldsymbol{M}^{xG,7}}\right]\widehat{\boldsymbol{\tau}^G}+2\left(1-\alpha^R\right)\operatorname{diag}\left(1-\widetilde{u}_{\cdot,\cdot,t_0}^{SS}\right)\overline{\boldsymbol{M}^{xR,9}}\widehat{\boldsymbol{\tau}^R}$$

In the steady state, I write equations (D.3) and (D.4) as

$$\overline{\boldsymbol{M}^{xG,2}}\widehat{\boldsymbol{p}^G}-\widehat{\boldsymbol{w}}+\left[\overline{\boldsymbol{I}_{NO}}-\frac{1}{1+\iota}\overline{\boldsymbol{M}^{\mu,2}}\right]\widehat{\boldsymbol{V}}=-\overline{\boldsymbol{M}^{xG,7}}\widehat{\boldsymbol{\tau}^G}+d\boldsymbol{T}-\overline{\boldsymbol{M}^{\mu,3}}d\boldsymbol{\chi}^{\text{vec}}$$

and

$$\left[\overline{\boldsymbol{I}_{NO}} - \overline{\boldsymbol{M}^{\mu L}}\right] \widehat{\boldsymbol{L}} - \overline{\boldsymbol{M}^{\mu L,2}}\widehat{\boldsymbol{\mu}^{\text{vec}}} = \boldsymbol{0}.$$

respectively.

Hence the log-linearized steady state system is

$$\overline{\boldsymbol{E}^{y}}\widehat{\boldsymbol{y}} = \overline{\boldsymbol{E}^{\Delta}}\boldsymbol{\Delta},$$

where

$$\overline{\boldsymbol{E}^{y}} \equiv \begin{bmatrix} \overline{\boldsymbol{D}^{x}} & -\overline{\boldsymbol{D}^{A,T}} \\ \multicolumn{2}{c}{\overline{\boldsymbol{D}^{y,SS}}} \end{bmatrix}, \text{ and } \overline{\boldsymbol{E}^{\Delta}} \equiv \begin{bmatrix} \overline{\boldsymbol{D}^{A,\Delta}} \\ \overline{\boldsymbol{D}^{\Delta,SS}} \end{bmatrix},$$

$\overline{\boldsymbol{D}^{A}} \equiv \begin{bmatrix} \overline{\boldsymbol{D}^{A,T}} & \overline{\boldsymbol{D}^{A,\Delta}} \end{bmatrix}$, and matrices $\overline{\boldsymbol{D}^{y,SS}}$ and $\overline{\boldsymbol{D}^{\Delta,SS}}$ are defined as

$$\overline{\boldsymbol{D}^{y,SS}} \equiv \begin{bmatrix} \overline{\boldsymbol{D}^{y,SS}_{11}} & \overline{\boldsymbol{D}^{y,SS}_{12}} & \boldsymbol{0} & \boldsymbol{0} & \overline{\boldsymbol{M}^{xR,7}} & \boldsymbol{0} & -\overline{\boldsymbol{I}_{NO}} & \boldsymbol{0} \\ \overline{\boldsymbol{D}^{y,SS}_{21}} & \overline{\boldsymbol{D}^{y,SS}_{22}} & \boldsymbol{0} & \overline{\boldsymbol{M}^{yG,3}} & \overline{\boldsymbol{D}^{y,SS}_{25}} & \overline{\boldsymbol{D}^{y,SS}_{26}} & \overline{\boldsymbol{D}^{y,SS}_{27}} & \overline{\boldsymbol{D}^{y,SS}_{28}} \end{bmatrix},$$

where

$$\overline{\boldsymbol{D}^{y,SS}_{11}} \equiv -\left(1-\alpha^{R}\right)\overline{\boldsymbol{M}^{xG,2}},$$

$$\overline{\boldsymbol{D}^{y,SS}_{12}} \equiv \left(1-\alpha^{R}\right)\overline{\boldsymbol{M}^{xR,8}},$$

$$\overline{\boldsymbol{D}^{y,SS}_{21}} \equiv \left(\overline{\boldsymbol{I}_{N}} \otimes \boldsymbol{1}_{O}\right) + 2\left(1-\alpha^{R}\right)\text{diag}\left(1-\widetilde{u}^{SS}_{\cdot,\cdot,t_0}\right)\overline{\boldsymbol{M}^{xG,2}},$$

$$\overline{\boldsymbol{D}^{y,SS}_{22}} \equiv -\text{diag}\left(1-\widetilde{u}^{SS}_{\cdot,\cdot,t_0}\right)\left(\overline{\boldsymbol{I}_{NO}} + 2\left(1-\alpha^{R}\right)\overline{\boldsymbol{M}^{xR,8}}\right),$$

$$\overline{\boldsymbol{D}^{y,SS}_{25}} \equiv -2\text{diag}\left(1-\widetilde{u}^{SS}_{\cdot,\cdot,t_0}\right)\overline{\boldsymbol{M}^{xR,7}},$$

$$\overline{\boldsymbol{D}^{y,SS}_{26}} \equiv \left(-1+\frac{1}{\beta}\right)\overline{\boldsymbol{M}^{xOl,3}} + \left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\text{diag}\left(l^{O}_{\cdot,\cdot,t_0}\right),$$

$$\overline{\boldsymbol{D}^{y,SS}_{27}} \equiv \left(-1+\frac{1}{\beta}\right)\overline{\boldsymbol{M}^{xOl,4}} + \left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\text{diag}\left(1-l^{O}_{\cdot,\cdot,t_0}\right) - \frac{1}{\theta}\overline{\boldsymbol{I}_{NO}} + 2\text{diag}\left(1-\widetilde{u}^{SS}_{\cdot,\cdot,t_0}\right),$$

$$\overline{\boldsymbol{D}^{y,SS}_{28}} \equiv -\text{diag}\left(\widetilde{u}^{SS}_{\cdot,\cdot,t_0}\right),$$

and

$$\overline{\boldsymbol{D}^{\Delta,SS}} \equiv \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \overline{\boldsymbol{D}_{17}^{\Delta,SS}} & \overline{\boldsymbol{D}_{18}^{\Delta,SS}} \\ \mathbf{0} & \overline{\boldsymbol{D}_{22}^{\Delta,SS}} & \overline{\boldsymbol{D}_{23}^{\Delta,SS}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \overline{\boldsymbol{D}_{27}^{\Delta,SS}} & \overline{\boldsymbol{D}_{28}^{\Delta,SS}} \end{bmatrix},$$

where

$$\overline{\boldsymbol{D}_{17}^{\Delta,SS}} \equiv \left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xG,7}},$$

$$\overline{\boldsymbol{D}_{18}^{\Delta,SS}} \equiv -\left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xR,9}},$$

$$\overline{\boldsymbol{D}_{22}^{\Delta,SS}} \equiv \left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\frac{1}{\theta-1}\overline{\boldsymbol{M}^{al,2}} - \left(-1+\frac{1}{\beta}\right)\frac{1}{\theta-1}\overline{\boldsymbol{M}^{al,3}},$$

$$\overline{\boldsymbol{D}_{23}^{\Delta,SS}} \equiv -\frac{1}{\beta}\left(\overline{\boldsymbol{I}_{NO}} - \overline{\boldsymbol{M}^{xO,3}}\right),$$

$$\overline{\boldsymbol{D}_{27}^{\Delta,SS}} \equiv -\overline{\boldsymbol{M}^{yG,3}} - 2\left(1-\alpha^R\right)\operatorname{diag}\left(1-\widetilde{u}^{SS}_{\cdot,\cdot,t_0}\right)\overline{\boldsymbol{M}^{xG,7}},$$

and

$$\overline{\boldsymbol{D}_{28}^{\Delta,SS}} \equiv 2\left(1-\alpha^R\right)\operatorname{diag}\left(1-\widetilde{u}^{SS}_{\cdot,\cdot,t_0}\right)\overline{\boldsymbol{M}^{xR,9}}.$$

If $\overline{\boldsymbol{E}^y}$ is invertible, I have $\overline{\boldsymbol{E}} \equiv \left(\overline{\boldsymbol{E}^y}\right)^{-1}\overline{\boldsymbol{E}^\Delta}$ such that $\widehat{\boldsymbol{y}} = \overline{\boldsymbol{E}}\boldsymbol{\Delta}$. Write dimensions of $\boldsymbol{y}$ and $\boldsymbol{\Delta}$ as $n_y \equiv N + 3NO + N^2 + N^2O$ and $n_\Delta \equiv 3N^2 + O + 2NO + 2N$, respectively.

Finally, to study the transitional dynamics, the capital accumulation dynamics (13) implies

$$\check{K^R_{i,o,t+1}} = -\delta\left(1-\alpha^R\right)\sum_l \widetilde{x}^G_{li,t_0}\check{p}^G_{l,t} + \delta\left(1-\alpha^R\right)\sum_l \widetilde{x}^R_{li,o}\check{p}^R_{l,o,t} + \delta\sum_l \widetilde{x}^R_{li,o}\check{Q}^R_{li,o,t} + (1-\delta)\check{K}^R_{i,o,t}.$$

In a matrix form, write

$$\check{\boldsymbol{K}}^R_{t+1} = -\delta\left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xG,2}}\check{\boldsymbol{p}}^G_t + \delta\left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xR,8}}\check{\boldsymbol{p}}^R_t + \delta\overline{\boldsymbol{M}^{xR,7}}\check{\boldsymbol{Q}}^R_t + (1-\delta)\overline{\boldsymbol{I}_{NO}}\check{\boldsymbol{K}}^R_t.$$

Next, to study the Euler equation, define

$$\widetilde{u}^{TD,1}_{i,o} \equiv \frac{-(\iota+\delta)\left[\left(\sum_l x^R_{li,o}\left(1+u_{li}\right)^{-\left(1-\varepsilon^R\right)}\right)^{\frac{1}{1-\varepsilon^R}\alpha^R} + 2\gamma\delta\right] + \gamma\delta^2}{(1-\delta)\left[\left(\sum_l x^R_{li,o}\left(1+u_{li}\right)^{-\left(1-\varepsilon^R\right)}\right)^{\frac{1}{1-\varepsilon^R}\alpha^R} + 2\gamma\delta\right]}$$

and

$$\widetilde{u}_{i,o}^{TD,2} \equiv \frac{-\gamma\delta^2}{(1-\delta)\left[\left(\sum_l x_{li,o}^R \left(1+u_{li}\right)^{-\left(1-\varepsilon^R\right)}\right)^{\frac{1}{1-\varepsilon^R}\alpha^R} + 2\gamma\delta\right]}.$$

Then I have

$$\begin{aligned}
&\left[-\widetilde{u}_{i,o}^{TD,1} p_{i,t+1}^{\check{G}} + 2\left(1-\alpha^R\right)\widetilde{u}_{i,o}^{TD,2}\sum_l \widetilde{x}_{li}^G p_{l,t+1}^{\check{G}}\right] + \left[-\widetilde{u}_{i,o}^{TD,2} p_{i,o,t+1}^{\check{R}} - 2\left(1-\alpha^R\right)\widetilde{u}_{i,o}^{TD,2}\sum_l \widetilde{x}_{li,o}^R p_{l,o,t+1}^{\check{R}}\right] \\
&- \widetilde{u}_{i,o}^{TD,1}\sum_j \widetilde{y}_{ij}^G Q_{ij,t+1}^{\check{G}} - 2\widetilde{u}_{i,o}^{TD,2}\sum_l \widetilde{x}_{li,o}^R Q_{li,o,t+1}^{\check{R}} - \widetilde{u}_{i,o}^{TD,1}\left(-1+\frac{1}{\beta}\right)\sum_{o'} x_{i,o'}^O \left(1-l_{i,o'}^O\right) K_{i,o',t+1}^{\check{R}} \\
&- \widetilde{u}_{i,o}^{TD,1}\left[\left(-1+\frac{1}{\beta}\right)\sum_{o'} x_{i,o'}^O l_{i,o'}^O L_{i,o',t+1}^{\check{}} + \left(-\frac{1}{\beta}+\frac{1}{\theta}\right) l_{i,o}^O L_{i,o,t+1}^{\check{}}\right] \\
&- \left[\widetilde{u}_{i,o}^{TD,1}\left\{\left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\left(1-l_{i,o}^O\right) + \left(-\frac{1}{\theta}\right)\right\} - 2\widetilde{u}_{i,o}^{TD,2}\right] K_{i,o,t+1}^{\check{R}} + \lambda_{i,o,t+1}^{\check{R}} = \frac{1+\iota}{1-\delta}\lambda_{i,o,t}^{\check{R}}
\end{aligned}$$

In a matrix form, write

$$\overline{\boldsymbol{M}^{u,4}} = \begin{bmatrix} \widetilde{\boldsymbol{u}}_{1,\cdot}^{TD,1} & & \boldsymbol{0} \\ & \ddots & \\ \boldsymbol{0} & & \widetilde{\boldsymbol{u}}_{N,\cdot}^{TD,1} \end{bmatrix},$$

a $NO \times N$ matrix where $\widetilde{\boldsymbol{u}}_{i,\cdot}^{TD,1} \equiv \left(\widetilde{u}_{i,o}^{TD,1}\right)_o$ is an $O \times 1$ vector for any i. Then

$$\begin{aligned}
&\left(-\overline{\boldsymbol{M}^{u,4}} + 2\left(1-\alpha^R\right)\mathrm{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,2}\right)\overline{\boldsymbol{M}^{xG,2}}\right)\boldsymbol{p}_{t+1}^{\check{G}} - \mathrm{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,2}\right)\left(\overline{\boldsymbol{I}_{NO}} + 2\left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xR,8}}\right)\boldsymbol{p}_{t+1}^{\check{R}} \\
&\quad - \left[\left(\overline{\boldsymbol{M}^{u,4}} \otimes \left(\boldsymbol{1}_N\right)^\top\right) \circ \overline{\boldsymbol{M}^{yG,3}}\right]\boldsymbol{Q}_{t+1}^{\check{G}} - 2\left(\left(\boldsymbol{1}_N\right)^\top \otimes \mathrm{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,2}\right)\right) \circ \overline{\boldsymbol{M}^{xR,7}}\boldsymbol{Q}_{t+1}^{\check{R}} \\
&\quad + \left[-\left(-1+\frac{1}{\beta}\right)\left(\left(\overline{\boldsymbol{M}^{u,4}} \otimes \left(\boldsymbol{1}_O\right)^\top\right) \circ \overline{\boldsymbol{M}^{xOl,3}}\right) - \left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\mathrm{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,1} l_{\cdot,\cdot}^O\right)\right]\boldsymbol{L}_{t+1}^{\check{}} \\
&\quad + \left\{\left(-1+\frac{1}{\beta}\right)\left(\left(\overline{\boldsymbol{M}^{u,4}} \otimes \left(\boldsymbol{1}_O\right)^\top\right) \circ \overline{\boldsymbol{M}^{xOl,4}}\right) - \left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\mathrm{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,1}\left(1-l_{\cdot,\cdot}^O\right)\right)\right. \\
&\quad \left. + \frac{1}{\theta}\mathrm{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,1}\right) + 2\mathrm{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,2}\right)\right\}\boldsymbol{K}_{t+1}^{\check{R}} + \overline{\boldsymbol{I}_{NO}}\boldsymbol{\lambda}_{t+1}^{\check{R}} = \frac{1+\iota}{1-\delta}\overline{\boldsymbol{I}_{NO}}\boldsymbol{\lambda}_t^{\check{R}}.
\end{aligned}$$

Hence the log-linearized transitional dynamic system is $\overline{\boldsymbol{D}_{t+1}^{y,TD}}\check{\boldsymbol{y}}_{t+1} = \overline{\boldsymbol{D}_t^{y,TD}}\check{\boldsymbol{y}}_t$, where matrices $\overline{\boldsymbol{D}_{t+1}^{y,TD}}$ and $\overline{\boldsymbol{D}_t^{y,TD}}$ are defined as

$$\overline{\boldsymbol{D}_{t+1}^{y,TD}} = \left[\begin{array}{cccccccc} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \overline{\boldsymbol{I}_{NO}} & \boldsymbol{0} \\ \hline \boldsymbol{D}_{21,t+1}^{y,TD} & \boldsymbol{D}_{22,t+1}^{y,TD} & \boldsymbol{0} & \boldsymbol{D}_{24,t+1}^{y,TD} & \boldsymbol{D}_{25,t+1}^{y,TD} & \boldsymbol{D}_{26,t+1}^{y,TD} & \boldsymbol{D}_{27,t+1}^{y,TD} & \overline{\boldsymbol{I}_{NO}} \end{array}\right],$$

where

$$\overline{\boldsymbol{D}_{21,t+1}^{y,TD}} \equiv -\overline{\boldsymbol{M}^{u,4}} + 2\left(1-\alpha^{R}\right)\operatorname{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,2}\right)\overline{\boldsymbol{M}^{xG,2}},$$

$$\overline{\boldsymbol{D}_{22,t+1}^{y,TD}} \equiv -\operatorname{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,2}\right)\left(\overline{\boldsymbol{I}_{NO}} + 2\left(1-\alpha^{R}\right)\overline{\boldsymbol{M}^{xR,8}}\right),$$

$$\overline{\boldsymbol{D}_{24,t+1}^{y,TD}} \equiv -\left(\overline{\boldsymbol{M}^{u,4}} \otimes (\mathbf{1}_N)^{\top}\right) \circ \overline{\boldsymbol{M}^{yG,3}},$$

$$\overline{\boldsymbol{D}_{25,t+1}^{y,TD}} \equiv -2\left((\mathbf{1}_N)^{\top} \otimes \operatorname{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,2}\right)\right) \circ \overline{\boldsymbol{M}^{xR,7}},$$

$$\overline{\boldsymbol{D}_{26,t+1}^{y,TD}} \equiv -\left(-1+\frac{1}{\beta}\right)\left(\left(\overline{\boldsymbol{M}^{u,4}} \otimes (\mathbf{1}_O)^{\top}\right) \circ \overline{\boldsymbol{M}^{xOl,3}}\right) - \left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\operatorname{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,1} l_{\cdot,\cdot}^{O}\right),$$

$$\begin{aligned}\overline{\boldsymbol{D}_{27,t+1}^{y,TD}} \equiv & \left(-1+\frac{1}{\beta}\right)\left(\left(\overline{\boldsymbol{M}^{u,4}} \otimes (\mathbf{1}_O)^{\top}\right) \circ \overline{\boldsymbol{M}^{xOl,4}}\right) \\ & - \left(-\frac{1}{\beta}+\frac{1}{\theta}\right)\operatorname{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,1}\left(1-l_{\cdot,\cdot}^{O}\right)\right) + \frac{1}{\theta}\operatorname{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,1}\right) + 2\operatorname{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,2}\right),\end{aligned}$$

and

$$\overline{\boldsymbol{D}_{t}^{y,TD}} = \begin{bmatrix} -\delta\left(1-\alpha^{R}\right)\overline{\boldsymbol{M}^{xG,2}} & \delta\left(1-\alpha^{R}\right)\overline{\boldsymbol{M}^{xR,8}} & \mathbf{0} & \mathbf{0} & \delta\overline{\boldsymbol{M}^{xR,7}} & \mathbf{0} & (1-\delta)\overline{\boldsymbol{I}_{NO}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \frac{1+\iota}{1-\delta}\overline{\boldsymbol{I}_{NO}} \end{bmatrix}. \tag{D.9}$$

Since $\check{\boldsymbol{y}}_t = \widehat{\boldsymbol{y}}_t - \widehat{\boldsymbol{y}}$ for any $t \geq t_0$ and $\widehat{\boldsymbol{y}} = \overline{\boldsymbol{E}}\boldsymbol{\Delta}$, I have

$$\overline{\boldsymbol{D}_{t+1}^{y,TD}}\left(\widehat{\boldsymbol{y}_{t+1}} - \widehat{\boldsymbol{y}}\right) = \overline{\boldsymbol{D}_{t}^{y,TD}}\left(\widehat{\boldsymbol{y}}_t - \widehat{\boldsymbol{y}}\right)$$

$$\iff \overline{\boldsymbol{D}_{t+1}^{y,TD}}\widehat{\boldsymbol{y}_{t+1}} = \overline{\boldsymbol{D}_{t}^{y,TD}}\widehat{\boldsymbol{y}}_t - \left(\overline{\boldsymbol{D}_{t+1}^{y,TD}} - \overline{\boldsymbol{D}_{t}^{y,TD}}\right)\overline{\boldsymbol{E}}\boldsymbol{\Delta}.$$

Recall the temporary equilibrium condition $\overline{\boldsymbol{D}^{x}}\widehat{\boldsymbol{x}}_t - \overline{\boldsymbol{D}^{A,S}}\widehat{\boldsymbol{S}}_t = \overline{\boldsymbol{D}^{A,\Delta}}\widehat{\boldsymbol{\Delta}}$ for any t. Thus

$$\overline{\boldsymbol{F}_{t+1}^{y}}\widehat{\boldsymbol{y}_{t+1}} = \overline{\boldsymbol{F}_{t}^{y}}\widehat{\boldsymbol{y}}_t + \overline{\boldsymbol{F}_{t+1}^{\Delta}}\boldsymbol{\Delta},$$

where

$$\overline{\boldsymbol{F}_{t+1}^{y}} \equiv \begin{bmatrix} \overline{\boldsymbol{D}^{x}} \quad -\overline{\boldsymbol{D}^{A,T}} \\ \overline{\boldsymbol{D}_{t+1}^{y,TD}} \end{bmatrix}, \ \overline{\boldsymbol{F}_{t}^{y}} \equiv \begin{bmatrix} \mathbf{0} \\ \overline{\boldsymbol{D}_{t}^{y,TD}} \end{bmatrix}, \ \overline{\boldsymbol{F}_{t+1}^{\Delta}} \equiv \begin{bmatrix} \overline{\boldsymbol{D}^{A,\Delta}} \\ \left(\overline{\boldsymbol{D}_{t+1}^{y,TD}} - \overline{\boldsymbol{D}_{t}^{y,TD}}\right)\overline{\boldsymbol{E}} \end{bmatrix},$$

or with $\overline{F^y} \equiv \left(\overline{F^y_{t+1}}\right)^{-1}\overline{F^y_t}$ and $\overline{F^\Delta} \equiv \left(\overline{F^y_{t+1}}\right)^{-1}\overline{F^\Delta_{t+1}}$, one can write

$$\widehat{y_{t+1}} = \overline{F^y}\widehat{y}_t + \overline{F^\Delta}\boldsymbol{\Delta}. \tag{D.10}$$

It remains to find the initial values of the system (D.10) that satisfies the transversality condition. To this end, I apply a standard method in Stokey and Lucas (1989). In particular, I first homogenize the system: Note that equation (D.10) can be rewritten as $\widehat{y_{t+1}} = \overline{F^y}\widehat{y}_t + \left(\overline{I} - \overline{F^y}\right)\left(\overline{I} - \overline{F^y}\right)^{-1}\overline{F^\Delta}\boldsymbol{\Delta}$ and thus

$$\widehat{z_{t+1}} = \overline{F^y}\widehat{z}_t \tag{D.11}$$

where

$$\widehat{z}_t \equiv \widehat{y}_t - \left(\overline{I} - \overline{F^y}\right)^{-1}\overline{F^\Delta}\boldsymbol{\Delta}. \tag{D.12}$$

Next, for the transversality condition to be satisfied, the system (D.11) must not explode. Thus it must be that $\widehat{z}_t \to \mathbf{0}$ or $\widehat{y}_t \to \left(\overline{I} - \overline{F^y}\right)^{-1}\overline{F^\Delta}\boldsymbol{\Delta}$. To find the condition, write Jordan decomposition of $\overline{F^y}$ as $\overline{F^y} = \overline{B}^{-1}\overline{\Lambda B}$. Then Theorem 6.4 of Stokey and Lucas (1989) implies that it must be that out of n_y vector of $\overline{B}\widehat{z_{t_0}}$, n-th element must be zero if $|\lambda_n| > 1$. Since $\widehat{K^R_{t_0}} = \mathbf{0}$, I can write

$$\widehat{z_{t_0}} = \overline{F^\Delta_{t_0}}\boldsymbol{\Delta} + \overline{F^\lambda_{t_0}}\widehat{\lambda^R_{t_0}},$$

where

$$\overline{F^\Delta_{t_0}} \equiv \begin{bmatrix} \left(\overline{D^x}\right)^{-1}\overline{D^{A,\Delta}} \\ \mathbf{0}_{2NO\times n_\Delta} \end{bmatrix} - \left(\overline{I} - \overline{F^y}\right)^{-1}\overline{F^\Delta} \text{ and } \overline{F^\lambda_{t_0}} \equiv \begin{bmatrix} \left(\overline{D^x}\right)^{-1}\overline{D^{A,\lambda}} \\ \mathbf{0}_{NO\times NO} \\ \overline{I_{NO}} \end{bmatrix}$$

and $\overline{D^{A,\lambda}}$ is the right block matrix of $\overline{D^A} \equiv \begin{bmatrix} \overline{D^{A,K}} & \overline{D^{A,\lambda}} \end{bmatrix}$ that corresponds to vector $\widehat{\lambda^R}$. Extracting n-th row from $\overline{F^\Delta_{t_0}}$ and $\overline{F^\lambda_{t_0}}$ where $|\lambda_n| > 1$ and writing them as a $NO \times n_\Delta$ matrix $\overline{G^\Delta_{t_0}}$ and $NO \times NO$ matrix $\overline{G^\lambda_{t_0}}$, the condition of the Theorem is

$$\mathbf{0} = \overline{G^\Delta_{t_0}}\boldsymbol{\Delta} + \overline{G^\lambda_{t_0}}\widehat{\lambda^R_{t_0}},$$

or $\widehat{\lambda^R_{t_0}} = \overline{G_{t_0}}\boldsymbol{\Delta}$ where $\overline{G_{t_0}} \equiv -\left(\overline{G^\lambda_{t_0}}\right)^{-1}\overline{G^\Delta_{t_0}}$. Finally, tracing back to obtain the initial conditions for

$\widehat{\boldsymbol{y}}_t$, it must be $\widehat{\boldsymbol{y}_{t_0}} = \overline{\boldsymbol{F}^{y}_{t_0}}\boldsymbol{\Delta}$,where

$$\overline{\boldsymbol{F}^{y}_{t_0}} \equiv \begin{bmatrix} \left(\overline{\boldsymbol{D}^{x}}\right)^{-1}\left(\overline{\boldsymbol{D}^{A,\Delta}} + \overline{\boldsymbol{D}^{A,\lambda}}\,\overline{\boldsymbol{G}_{t_0}}\right) \\ \mathbf{0}_{NO\times n_{\Delta}} \\ \overline{\boldsymbol{G}_{t_0}} \end{bmatrix}.$$