Online Appendix

F Data Appendix

F.1 Data Sources in Detail

I complement data from JARA data and O*NET data by the ones from IFR, BACI, IPUMS USA and CPS. IFR is a standard data source of industrial robot adoption in several countries (e.g., Graetz and Michaels 2018; Acemoglu and Restrepo, 2020, AR thereafter), to which JARA provides the robot data of Japan. I use IFR data to show the total robot adoption in each destination country as opposed to the import from Japan. I use Federal Reserve Economic Data (FRED) to convert JARA variables denominated in JPY to USD. BACI provides disaggregated data on trade flows for more than 5000 products and 200 countries and is a standard data source of international trade (Gaulier and Zignago 2010). I use BACI data to obtain the measure of international trade of industrial robots and baseline trade shares. IPUMS USA collects and harmonizes US census microdata (Ruggles et al. 2018). I use Population Censuses (1970, 1980, 1990, and 2000) and American Community Surveys (ACS, 2006-2008 3-year sample and 2012-2016 5-year sample). I obtain occupational wages, employment, and labor cost shares from these data sources. To obtain the intermediate inputs shares, I take data from the World Input-Output Data (WIOD) in the closest year to the initial year, 1992.

I use the match score from the O*NET Code Connector that contains detailed textual descriptions of 4-digit occupations. The match score is an output of the *weighted search algorithm* used by the O*NET Code Connector, which is the internal search algorithm developed and employed by O*NET and since September 2005. Since then, the O*NET has continually updated the algorithm and improved the quality of the search results. Morris (2019) reports that the updated weighted search algorithm scored 95.9% based on the position and score of a target best 4-digit occupation for a given query.

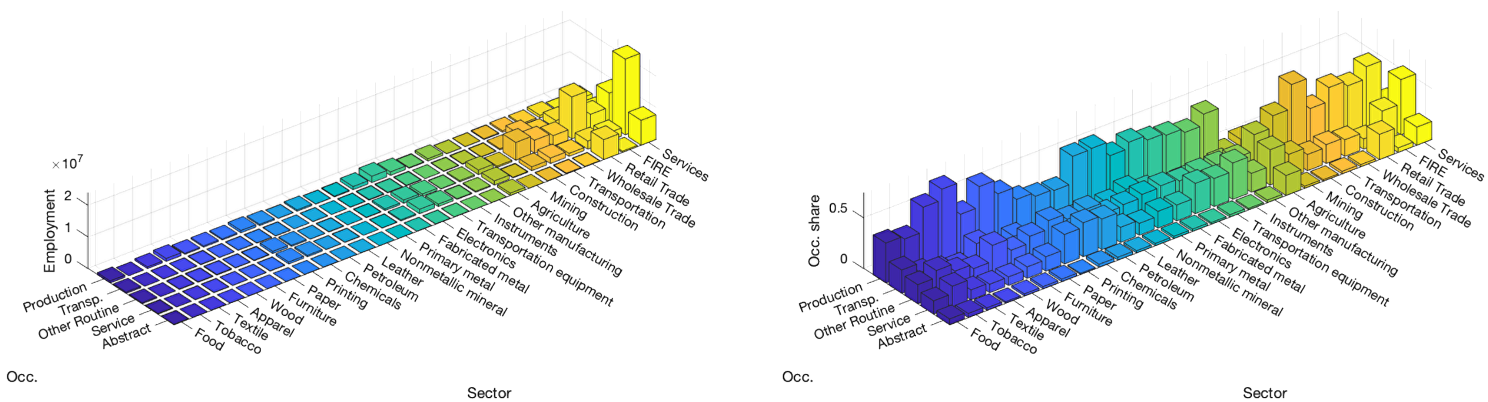
I focus on consistent occupations between the 1970 Census and the 2007 ACS that cover the sample period and pre-trend analysis period to obtain consistent data across periods. Therefore, this paper focuses on the intensive-margin substitution in occupations as opposed to the extensive-margin effect of automation that creates new labor-intensive tasks and occupations (Acemoglu and Restrepo 2018). My dataset shows that 88.7 percent of workers in 2007 worked in the occupations that existed in 1990. It is an open question how to attribute the creation of new occupations to different types of automation goods like occupational robots in my case, although Autor and Salomons (2019) explore how to measure the task contents of new occupations.

I follow Autor, Dorn, and Hanson (2013) for Census/ACS data cleaning procedure. Namely, I extract the 1970, 1980, 1990, 2000 Censuses, the 2006-2008 3-year file of American Community Survey (ACS), and the 2012-2016 5-year file of ACS from Integrated Public Use Micro Samples. For each file, I select all workers with the OCC2010 occupation code whose age is between 16 and 64 and who is not institutionalized. I compute education share in each occupation by the share of workers with more than "any year in college," and foreign-born share by the share of workers with BPL (birthplace) variable greater than 150, or those whose birthplace is neither in the US nor in US outlying areas/territories. I compute hours worked by multiplying usual weeks worked and hours worked per week. For 1970, I use the median values in each bin of the usual weeks worked variable and assume all workers worked for 40 hours a week since the hour variable does not exist. To compute hourly wage, I first impute each state-year's top-coded values by multiplying 1.5 and divide by the hours worked. To remove outliers, I take wages below first percentile of the distribution in each year, and set the maximum wage as the top-coded earning divided by 1,500. I compute the real wage in 2000 dollars by multiplying CPI99 variable prepared by IPUMS. I use the person weight variable for aggregating all of these variables to the occupation level. Figure F.1 shows the occupational employment distribution for each sector, a variable used for creating the occupational China shock in equation (2).

To estimate the model with workers' dynamic discrete choice of occupation, I further use the bilateral occupation flow data following the idea of Caliendo, Dvorkin, and Parro (2019). Specifically, I obtain the Annual Social and Economic Supplement (ASEC) of the CPS since 1976. For each year, I select all workers with the 2010 occupation code for the current year (OCC2010) and the last year (OCC10LY) whose age is between 16 and 64 and who is not institutionalized, and treated top-coded wage income, converted nominal wage income to real one, and computed labor hours worked, education, foreign born flag variable with the same method as the one used for Census/ACS above. When computing the occupation switch probability, note that the 4-digit oc-

Figure F.1: Occupational Employment Distribution

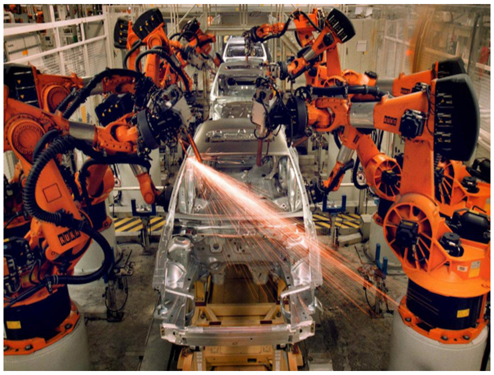
(a) Employment size L_{s,o,t_0} (b) Employment share l_{s,o,t_0}



Note: The author's calculation from the 1990 US Census. The axis on the left indicates the 5 occupation groups defined in Section 4.1, and the one on the right shows sectors (roughly 4-digit for manufacturing sectors and 2-digit for the non-manufacturing). The left panel shows the size of employment, and the right one indicates the occupation share for each given sector.

Figure F.2: Examples of Industrial Robots

(a) Spot Welding



(b) Material Handling

Sources: Autobot Systems and Automation (https://www.autobotsystems.com) and PaR Systems (https://www.par.com)

cupations are too disaggregated to precisely estimate with the small sample size of CPS-ASEC, as pointed out by Artuç, Chaudhuri, and McLaren (2010). Therefore, I assume that the occupations do not flow between 4-digit occupations within the 5 groups defined in Section 4.1, but do between the 5 groups. I assume that workers draw a destination 4-digit occupation occupation from the initial-year occupational employment distribution within the destination group when switching occupations. With these data and assumptions, I compute the occupation switching probability by year.

F.2 Details in Industrial Robots

F.2.1 Definition and Examples

As defined in Footnote 1, industrial robots are defined as multiple-axes manipulators. More formally, following International Organization for Standardization (ISO), I define robots as "automatically controlled, reprogrammable, multipurpose manipulator, programmable in three or more axes, which can be either fixed in place or mobile for use in industrial automation applications" (ISO 8373:2012). This section gives a detailed discussion about such industrial robots. Figure F.2 shows the pictures of examples of industrial robots that are intensively used in the production process and considered in this paper. The left panel shows spot-welding robots, while the right panel shows the material-handling robots. The spot welding robots are an example of robots in routine-production occupations, while the material-handling robots are that in routine-transportation (material-moving) occupations.

It is also worthwhile to give an example of technologies that are *not* robots according to the definition in this paper. An example of a growth in technology in the material-handling area is autonomous driving. Mehta and Levy (2020) predicts that such automation will grow strong and result in the reduction of total number of jobs in this area in eight to ten years since 2020. However, since autonomous vehicles do not operate multiple-axes, they are not treated in this paper at all. A similar observation applies for computers or artificial intelligence more generally.

F.2.2 JARA Robot Applications

In addition to applications in Section F.2.1, the full list of robot applications avaliable in JARA data is Die casting; Forging; Resin molding; Pressing; Arc welding; Spot welding; Laser welding; Painting; Load/unload; Mechanical cutting; Polishing and deburring; Gas cutting; Laser cutting; Water jet cutting; General assembly; Inserting; Mounting; Bonding; Soldering; Sealing and gluing; Screw tightening; Picking alignment and packaging; Palletizing; Measurement/inspection/test; and Material handling.

Can robots be classified as one of these applications? If one is familiar with the history of industrial robots, (s)he might wonder that robots are characterized by versatility as opposed to older specified industrial machinery (KHI 2018). Although it is true that a robot may be reprogrammed to perform more than one task, I claim that robots are well-classified to one of the applications listed above since the layer of dexterity is different. Robots might be able to adjust a model change of the products, but are not supposed to perform different tasks across the 4-digit occupation level. To support this point, recall that "SMEs are mostly high-mix/low-volume producers. Robots are still too inflexible to be switched at a reasonable cost from one task to another" (Autor, Mindell, and Reynolds 2020). These technological bottlenecks still make it hard for producers to have such a versatile robot that can replace a wide range of workers at the 4-digit occupation level even today, all the more for the sample period of my study.

F.2.3 Examples of Robotics Innovation

In the model, I call a change in the robot task space $a_{o,t}$ as the automation shock, and that in robot producer's TFP $A^R_{l,o,t}$ as the cost shock to produce robots. In this section, I show some examples of changes of robot technology and new patents to facilitate understandings of these interpretation.

An example of task space expansion is adopting *Programmed Article Transfer* (PAT, Devol 1961). PAT was machine that moves objects by a method called “teaching and playback”. Teaching and playback method needs one-time teaching of how to move, after which the machine playbacks the movement repeatedly and automatically. This feature frees workers of performing repetitive tasks. PAT was intensively introduced in spot welding tasks. KHI (2018) reports that among 4,000 spot welding points, 30% were done be human previously, which PAT took over. Therefore, I interpret the adoption of PAT as the example of the expansion of the robot task space, or increase in $a_{o,t}$. Note that AR also analyze this type of technological change.

An example of cost reduction is adopting *Programmable Universal Manipulator for Assembly (PUMA)*. PUMA was designed to quickly and accurately transport, handle and assemble automobile accessories. A new computer language, *Variable Assembly Language (VAL)*, made it possible because it made the teaching process less work and more sophisticated. In other words, PUMA made tasks previously done by other robots but at cheaper unit cost per unit of task.

It is also worth mentioning that introduction of a new robot brand typically contains both components of innovation (task space expansion and cost reduction). For example, PUMA also expanded task space of robots. Since VAL allowed the use of sensors and “expanded the range of applications to include assembly, inspection, palletizing, resin casting, arc welding, sealing and research” (KHI 2018).

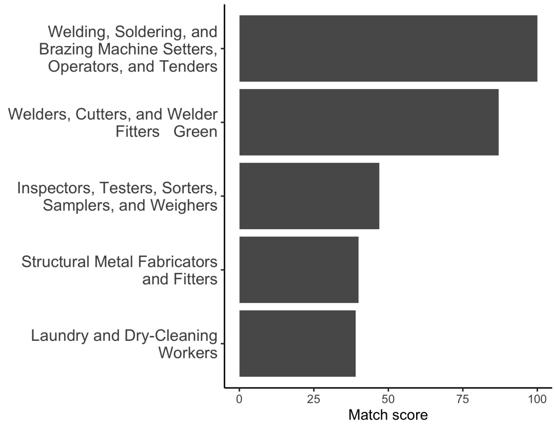
F.3 Details in Application-Occupation Matching

Concrete examples of the pairs of an application and an occupation that are close are spot welding and material handling. On the one hand, spot welding is a task of welding two or more metal sheets into one by applying heat and pressure to a small area called spot. In contrast, O*NET-SOC Code 51-4121.06 has the title “Welders, Cutters, and Welder Fitters” (“Welders” below). Therefore, both spot welding robots and welders perform the welding task. On the other hand, Material handling is a short-distance movement of heavy materials. It is another major application of robots. In comparison, ONET-SOC Code 53-7062.00 has the title “Laborers and Freight, Stock, and Material Movers, Hand” (“Material Handler” below). Therefore, both material handling robots and material handlers perform the material handling task.

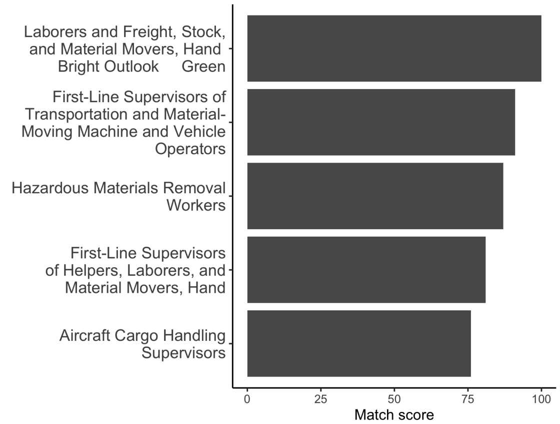
Figure F.3 shows examples of the O*NET match scores for spot welding and material handling.

Figure F.3: Examples of Match Scores

(a) Spot Welding



(b) Material Handling



Note: The author's calculation from the search result of O*NET Code Connector. The bars indicate match scores for the search query term "Spot Welding" in panel (a) and "Material Handling" in panel (b). Occupations codes are 2010 O*NET SOC codes. In each panel, occupations are sorted descendingly with the relative relevance scores, and the top 5 occupations are shown. See the main text for the detail of the score.

On the left panel, welding occupations are listed as relevant occupations for spot welding. In contrast, on the right panel, a material-moving laborer is a top occupation in terms of the relevance to the material-handling task, as I described above.

F.4 Methods for Adjusting the Robot Prices

In the paper, I use the general equilibrium model to control for the quality component of robot prices. However, there are other methods proposed in the literature of measuring the price of capital goods. In this subsecsion, I briefly describe these methods and their limitations.

Another approach to solving this issue is to control for the quality change by the hedonic approach as in Timmer, Van Ark, et al. (2007), and in the application to digital capital in Tambe et al. (2019). The hedonic approach requires detailed information about the detailed specification of each robot. Unfortunately, it is difficult to keep track of the detailed specifications of commonly used robots as the robotics industry is rapidly changing.

Another method is a more data-driven one. Specifically, the Bank of Japan (BoJ) provides the quality-controlled price index. However, the method is not clearly declared. In fact, it is claimed to be "cost-evaluation method," in which the BoJ asks producer firms to measure the component of quality upgrading for price changes between periods. Unfortunately, I do not know the survey firms and quality components. Therefore, it is hard for me to determine better measures, and so I stick to use my raw price measure based on the representativeness of my data.

F.5 Trade of Industrial Robots

To compute the trade shares of industrial robots, I combine BACI and IFR data. In particular, I use the HS code 847950 ("Industrial Robots For Multiple Uses") to measure the robots, following Humlum (2019). I approximate the initial year value by year of 1998, when the this HS code of robots is first available. To calculate the total absorption value of robots in each country, I use the IFR data's robot units (quantities), combined with the price indices of robots occasionally released by IFR's annual reports for selected countries. These price indices do not give disaggregation by robot tasks or occupations, highlighting the value added of the JARA data. Figure F.4 the pattern of international trade of international robots. In the left panel, I compute the import-absorption ratio. To remove the noise due to yearly observations and focus on a long-run trend, I aggregate by five-year bins 2001-2005 and 2011-2015. The result indicates that many countries import robots as opposed to produce in their countries. Japan's low import ratio is outstanding, revealing that its comparative advantage in this area. It is noteworthy that China largely domesticated the production of robots over the sample period. Another way to show grasp the comparative advantage of the robot industry is to examine the share of exports as in the right panel of Figure F.4. Roughly speaking, the half of the world robot market was dominated by EU and one-third by Japan in 2001-2005. The rest 20% is shared by the rest of the world, mostly by the US and South Korea.

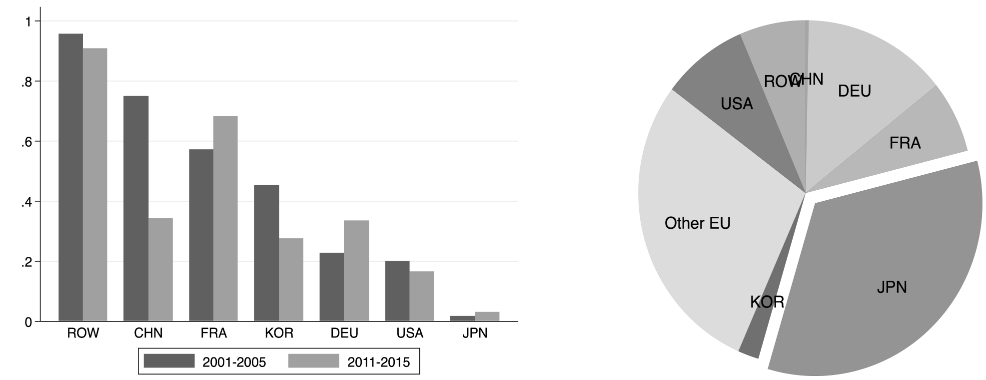
F.6 Trends of Robot Stocks and Prices

In this section, I show that each occupation experienced different trends in robot adoption. Figure F.5 shows the trend of US robot stocks at the occupation level. In the left panel, I show the trend of raw stock, which reveals the following two facts. Firstly, it shows that the overall robot stocks increased rapidly in the period, as found in the previous literature. Second, the panel also depicts that the increase occurred at different speeds across occupation. To highlight such a difference, in the right panel, I plot the normalized trend at 100 in the initial year. There is significant heterogeneity in the growth rates, ranging from a factor of one to eight.

Figure F.4: Trade of Industrial Robots

(a) Robot Import-Absorption Ratio

(b) World Robot Export Share, 2001-2005

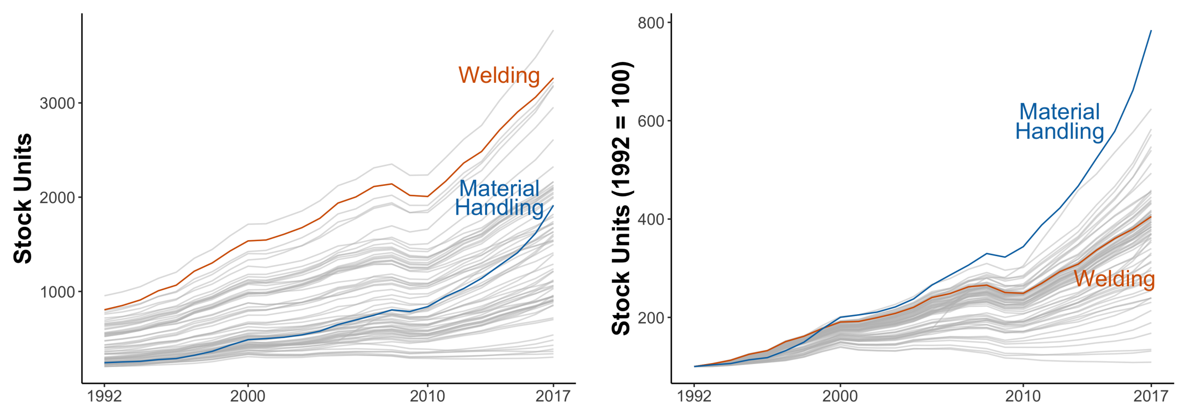


Note: The author's calculation from the IFR, and BACI data. The left panel show the fraction of import in the total absorption value. The import value is computed by aggregating trade values across origin country in the BACI data (HS-1996 code 847950), and the absorption value is computed by the price index and the quantity variable available for selected countries in the IFR data. The data are five-year aggregated in 2001-2005 and 2011-2015, and countries are sorted according to the import shares in 2001-2005 in the descending order. The right panel shows the export share for 2001-2005 aggregates obtained from the BACI data.

Figure F.5: US Robot Stocks at the Occupation Level

(a) Level

(b) Normalized



Note: The author's calculation based on JARA and O*NET. The figure shows the trend of stocks of robots in the US for each occupation. The left panel shows the level, whereas the right panel shows the normalized trend at 100 in 1992. In both panels, I highlight two occupations. "Welding" corresponds the occupation code in IPUMS USA, OCC2010 = 8140 "Welding, Soldering, and Brazing Workers." "Material Handling" corresponds the occupation code OCC2010 = 9620 "Laborers and Freight, Stock, and Material Movers, Hand." Years are aggregated into five-year bins (with the last bin 2012-2017 being six-year one) to smooth out yearly noises.

To further emphasize the different speed, in these two figures, I color the following two occupations: "Welding, Soldering, and Brazing Workers" (or "Welding") and "Laborers and Freight, Stock, and Material Movers, Hand" (or "Material handling"). On the one hand, the stock of welding robots grew continuously throughout the period, as can be confirmed in the left panel. However, the growth rate is not outstanding but within the range of growth rates of other occupations. On the other hand, material handling was not a majority occupation as of the initial year, but it grew at the most rapid pace in the sample period. These findings indicate the difference between the automation shocks to each occupation. Some occupations were already somewhat automated by robots as of the initial year, and the automation process continued afterward (e.g., welding). There are a few occupations where robotics automation had not occurred initially, but the adoption proceeded rapidly in the sample period (e.g., material handling). This observation bases the model that incorporates the heterogeneity across occupations in the Model section.

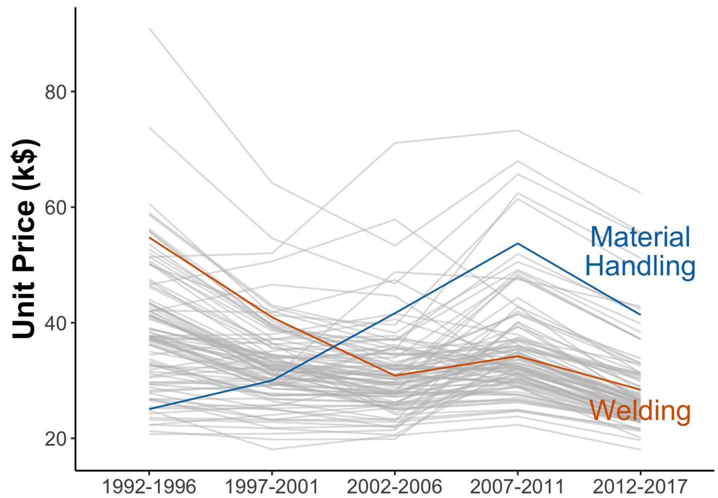
Next, Figure F.6 shows the trend of prices of robots in the US for each occupation. In addition to the overall decreasing trend, there is significant heterogeneity in the pattern of price falls across occupations. For instance, although the welding robots saw a large drop in the price during the 1990s, the material handling robots did not but increased the price over the sample period. These patterns are strongly correlated across countries, as indicated by the correlation coefficient of 0.968 between the US and non-US prices at the occupation-year level. Based on this finding, I use the non-US countries' prices as the Japan robot shock to the US in the Data section.

In Figure F.6, one might wonder if there is an anomaly to the overall decreasing trend in the 2007-2011 period, in which the trends of robot prices halt dropping across the board. This pattern emerges because of the method for generating these series. The average price is measured by total sales divided by total units. During the Great Recession period, the total units decreased more than the total sales did. The relative pattern caused a temporary increase in average robot prices. In addition, after the Great Recession, both the growth of sales and units of robots accelerated. These observations suggest the structural break of the robot industry during the Great Recession, which is out of the scope of the paper.

F.7 Robots from Japan in the US, Europe, and the Rest of the World

I review the international comparison of the pattern of robot adoption. I generate the growth rates of stock of robots between 1992 and 2017 at the occupation level for each group of destination countries. The groups are the US, the non-US countries, (namely, the world excluding the US and Japan), and five European countries (or "EU-5"), Denmark, Finland, France, Italy, and Sweden used in AR. To calculate the stock of robots, I employ the perpetual inventory method with

Figure F.6: Robot Prices at the Occupation Level



Note: The author's calculation based on JARA and O*NET. The figure shows the trend of prices of robots in the US for each occupation. I highlight two occupations. "Welding" corresponds the occupation code in IPUMS USA, OCC2010 = 8140 "Welding, Soldering, and Brazing Workers." "Material Handling" corresponds the occupation code OCC2010 = 9620 "Laborers and Freight, Stock, and Material Movers, Hand." Years are aggregated into five-year bins (with the last bin 2012-2017 being six-year one) to smooth out yearly noises. The dollars are converted to 2000 real US dollar using CPI.

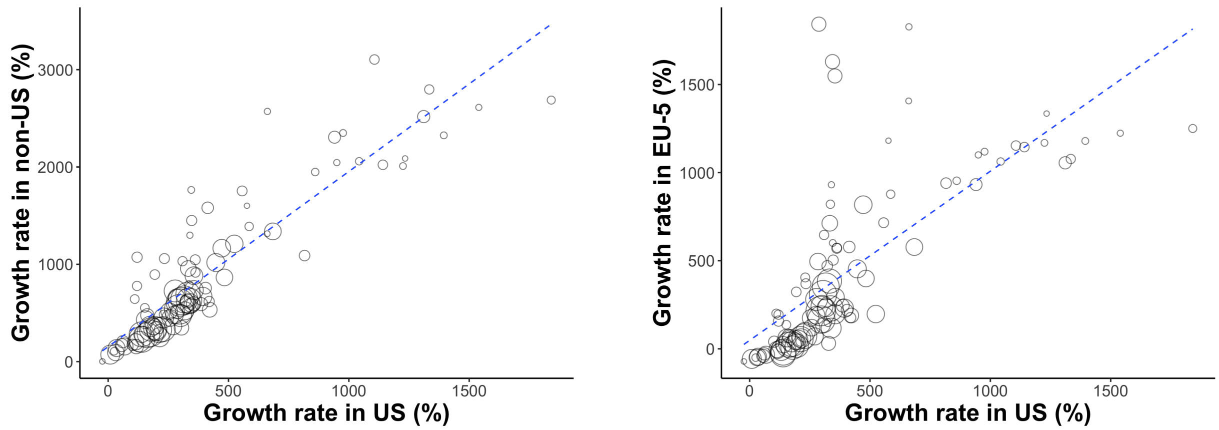
depreciation rate of $\delta = 0.1$, following Graetz and Michaels (2018).

Figure F.7 shows scatterplots of the growth rates at the occupation level. The left panel shows the growth rates in the US on the horizontal axis and the ones in non-US countries on the vertical axis. The right panel shows the same measures on the horizontal axis, but the growth rates in the set of EU-5 countries on the vertical axis. These panels show that the stocks of robots at the occupation level grow (1992-2017) between the US and non-US proportionately relative to those between the US and EU-5. This finding is in contrast to AR, who find that the US aggregate robot stocks grew at a roughly similar rate as those did in EU-5. It also indicates that non-US growth patterns reflect growths of robotics technology at the occupation level available in the US. I will use these patterns as the proxy for robotics technology available in the US. In Section 3 and on, I take a further step and solve for the robot adoption quantity and values in non-US countries in general equilibrium including the US and non-US countries.

It is worth mentioning that a potential reason for the difference between my finding and AR's is the difference in data sources. In contrast to the JARA data I use, AR use IFR data that include all robot seller countries. Since EU-5 is closer to major robot producer countries other than

Figure F.7: Growth Rates of Robots at the Occupation Level

(a) Comparison between the US and non-US (b) Comparison between the US and EU-5



Note: The author's calculation based on JARA, and O*NET. The left panel shows the correlation between occupation-level growth rates of robot stock quantities from Japan to the US and the growth rates of the quantities to the non-US countries. The right one shows the correlation between growth rates of the quantities to the US and EU-5 countries. Non-US are the aggregate of all countries excluding the US and Japan. EU-5 are the aggregate of Denmark, France, Finland, Italy, and Sweden used in Acemoglu and Restrepo (2020). Each bubble shows an occupation. The bubble size reflects the stock of robot in the US in the baseline year, 1992. See the main text for the detail of the method to create the variables.

Japan, including Germany, the robot adoption pattern across occupations may be influenced by their presence. If these close producers have a comparative advantage in producing robots for a specific occupation, then EU-5 may adopt the robots for such occupations intensively from close producers. In contrast, countries out of EU-5, including the US, may not benefit the closeness to these producers. Thus they are more likely to purchase robots from far producers from EU-5, such as Japan.

F.8 Further Analysis about Fact 2

We provide series of evidence about the robotics substitution of existing occupations in this section. First, Figure F.8 plots the correlation between the changes in robot measures and the changes in log labor market outcomes in the US at the occupation level, weighted by the size of occupation measured by initial the employment level. The top two panels take the occupational wage as a labor market outcome on the y-axis, while the bottom two take the occupational employment. The left two panels take log monetary value of robot stock in US as a robot measure on the left panel, and the right two take the log robot prices from Japan. As expected, the correlation between the two labor market outcomes and the robot stock measure is negative, while that with the Japanese

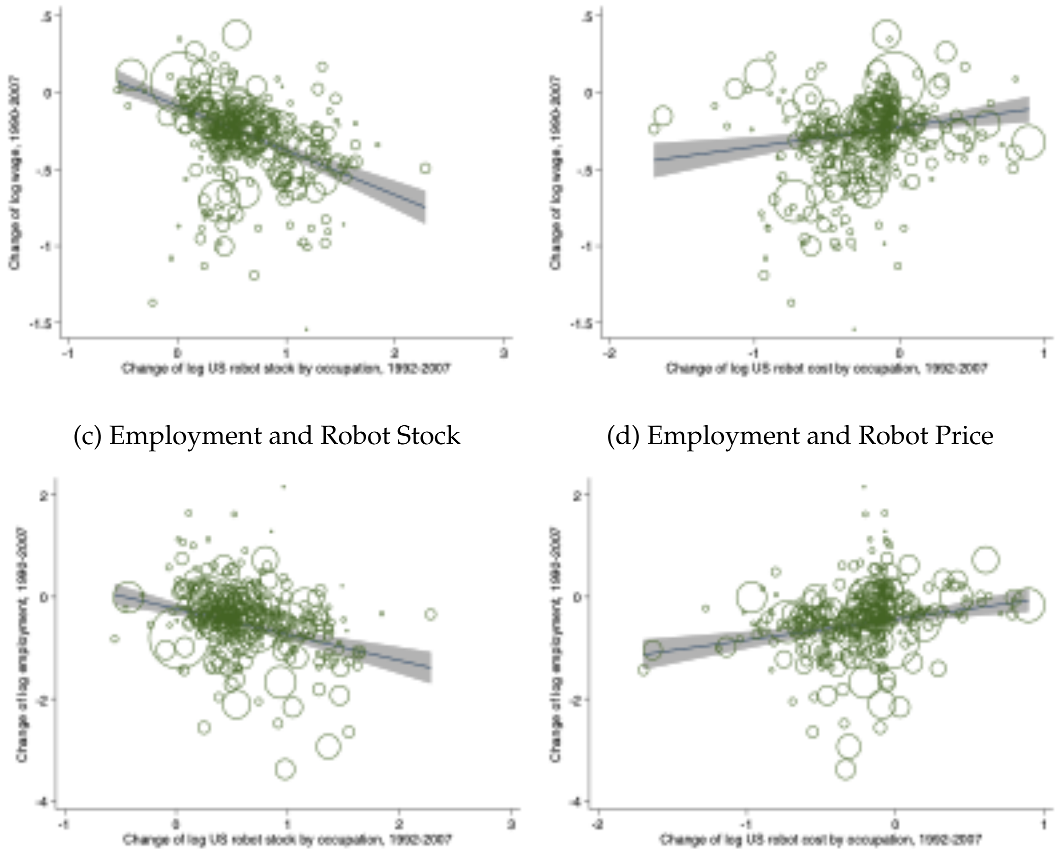
Figure F.8: Correlation between US Occupational Wage and US Robot Measures (Raw)

(a) Wage and Robot Stock

(b) Wage and Robot Price

(c) Employment and Robot Stock

(d) Employment and Robot Price



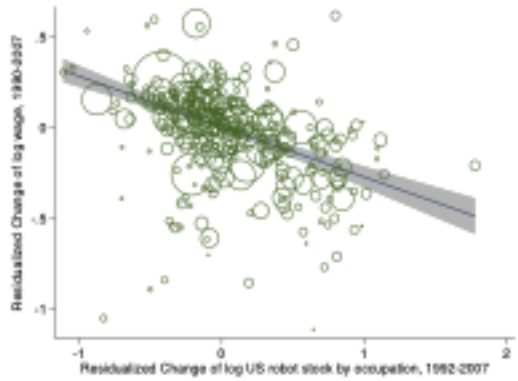
Note: The author's calculation based on JARA, O*NET, and the US Census/ACS. The figures show the scatterplot, weighted fit line, and the 95 percent confidence interval of the changes in log robot measures and changes in log labor market outcomes. Each bubble represents a 4-digit occupation. The bubble size reflects the employment in the baseline year (1990, which is the closest Census year to the initial year that I observe the robot adoption, 1992). All variables are partialled out by control variables (the occupational female share, college share, age distribution, and foreign born share). In panel (a), y-axis and x-axis are log occupational wages and the change of log US robot stock measured by the monetary value. In panel (b), y-axis and x-axis are log occupational wages and the change of log US robot price. In panel (c), y-axis and x-axis are log occupational wages and the change of log US robot stock measured by the monetary value. In panel (d), y-axis and x-axis are log occupational wages and the change of log US robot prices.

robot price is positive.

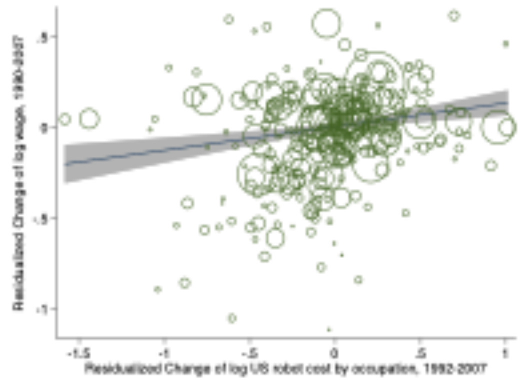
Second, I consider the role of control variables. Figure F.9 shows the results of a set of robustness checks. Each panel shows the same result as the ones corresponding to the same panel in Figure F.8, but after residualizing all variables with respect to the demographic control variables (initial-year female share, college graduates share, age 35-49 share, age 50-64 share, and foreign-born share in each occupation). The main result is robust to the control of these demographic variables.

Figure F.9: Correlation between US Occupational Wage and US Robot Measures (Controlled)

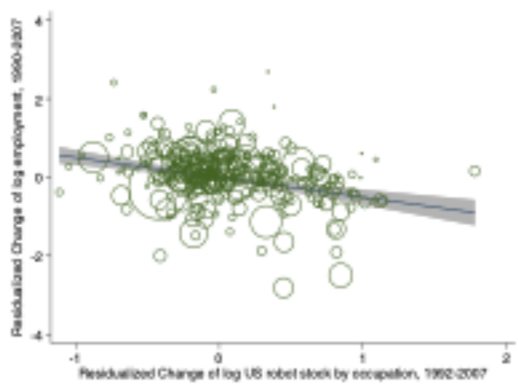
(a) Wage and Robot Values



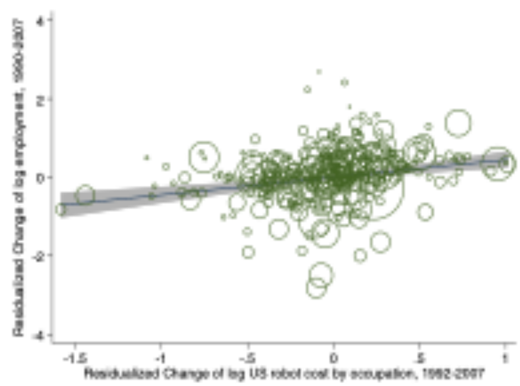
(b) Wage and Robot Prices



(c) Employment and Robot Values



(d) Employment and Robot Prices



Note: The author's calculation based on JARA, O*NET, and the US Census/ACS. The figures show the scatterplot, weighted fit line, and the 95 percent confidence interval of the changes in log robot measures and changes in log labor market outcomes. Each bubble represents a 4-digit occupation. The bubble size reflects the employment in the baseline year (1990, which is the closest Census year to the initial year that I observe the robot adoption, 1992). All variables are partialled out by control variables (the occupational female share, college share, age distribution, and foreign born share). In panel (a), y-axis and x-axis are log occupational wages and the change of log US robot stock measured by the monetary value. In panel (b), y-axis and x-axis are log occupational wages and the change of log US robot price. In panel (c), y-axis and x-axis are log occupational wages and the change of log US robot stock measured by the monetary value. In panel (d), y-axis and x-axis are log occupational wages and the change of log US robot prices.

Third, the robot adoption in the US, as used in Figure F.8 and F.9, might suffer from the endogeneity bias due to the demand factors that drive both the labor market outcome and robot adoption. To control for the demand factor in the US, Acemoglu and Restrepo (2020) used the robot stock changes in the other countries that show a similar trend of robot stocks as a proxy for the robot technological change and find the negative impact on the US regional labor market. Following this approach, I use the changes in robot stocks and the Japan robot shocks in non-US countries (all countries except for the US and Japan), which are defined in the main text. Note that the robot stock growths are similar between the US and the non-US countries by occupations. In contrast, the occupation-level trend in the five countries Acemoglu and Restrepo (2020) used as comparison (Denmark, Finland, France, Italy, and Sweden) is less similar to the US trend than the non-US countries. These facts are shown in Section F.7.

Figure F.10 show the result of this analysis in graph. As in Figure F.8, each dot represents 4 digit-level occupations and the size of the bubble is based on the initial employment. The top two panels take the occupational wage as a labor market outcome on the y-axis, while the bottom two take the occupational employment. The left two panels take log monetary value of robot stock in non-US countries as a robot measure on the left panel, and the right two take the Japan robot shock. These results provides more direct evidence of the substitution of robots for workers who perform the same task as robots, corroborating the finding of Acemoglu and Restrepo (2020). The similar results can be found when I control for the demographic variables described above as well, as shown in Figure F.11.

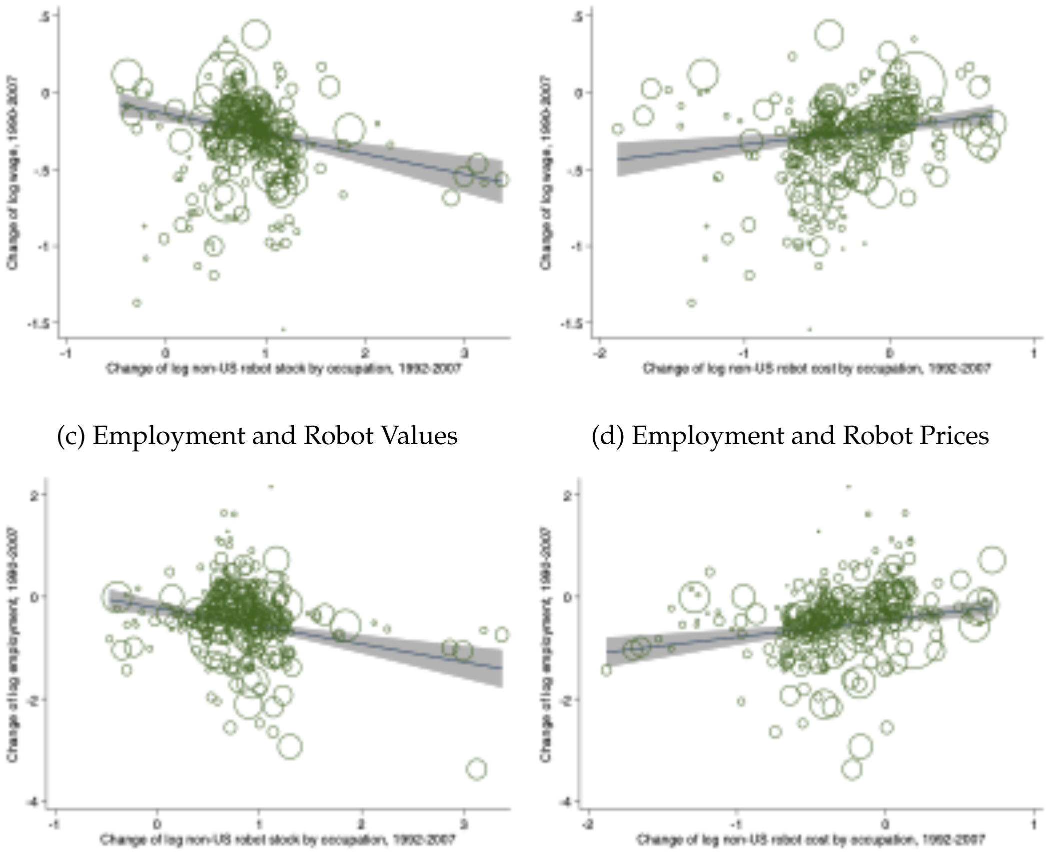
Fourth, one may be concerned that robot quality changes over year. Specifically, if the per-unit efficiency of robots increases over year, the average unit price may understate the decrease in the price of robots. To deal with this concern, I consider the following method of quality adjustment, based on the spirit of Khandelwal, Schott, and Wei (2013). Namely, I fit the following equation with the fixed-effect regression:

$$\ln\left(X^{R}_{JPN\to i,o,t}\right) = -\varsigma \ln\left(p^{R}_{JPN\to i,o,t}\right) + a^{R}_{o,t} + e^{R}_{i,o,t},$$

from which I obtain the fixed effect $a^{R}_{o,t}$, which absorbs the occupation-o specific log sales component that is not explained by the prices. I then proxy the quality change by the change in such fixed effects, $\Delta a^{R}_{o,t} \equiv a^{R}_{o,t} - a^{R}_{o,t_0}$. The (log) quality-adjusted price is then obtained by $\ln\left(p^{R}_{JPN\to i,o,t}\right) - \Delta a^{R}_{o,t}$. Figure F.12 shows the result of correlation using quality-adjusted robot prices All the results are robust to these considerations–wage growths are negatively correlated with stock growths, and positively correlated with price growths, both across occupations.

Finally, one might be concerned that the correlation between the US labor market outcomes and robot measures was simply an artifact of the long-run trend that has nothing to do with the robotization. To mitigate such a concern, I examine the pre-trend correlation between them. To do so, I take the 20-year difference of occupational wage since 1970 to 1990 as the outcome

Figure F.10: Correlation between US Occupational Wage and Non-US Robot Measures (Raw)



(a) Wage and Robot Values (b) Wage and Robot Prices

(c) Employment and Robot Values (d) Employment and Robot Prices

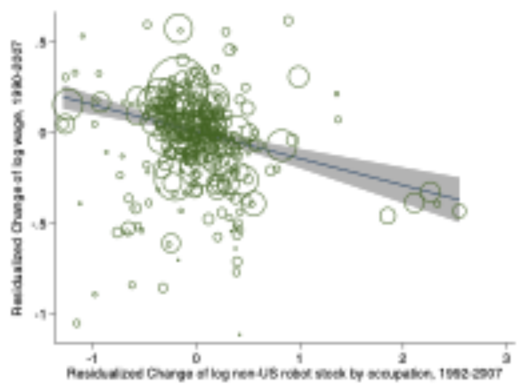
Note: The author's calculation based on JARA, O*NET, and the US Census/ACS. The figures show the scatterplot, weighted fit line, and the 95 percent confidence interval of the changes in log robot measures and changes in log labor market outcomes. Each bubble represents a 4-digit occupation. The bubble size reflects the employment in the baseline year (1990, which is the closest Census year to the initial year that I observe the robot adoption, 1992). In panel (a), y-axis and x-axis are log occupational wages and the change of log non-US robot stock measured by the monetary value. In panel (b), y-axis and x-axis are log occupational wages and the change of log non-US robot price. In panel (c), y-axis and x-axis are log occupational wages and the change of log non-US robot stock measured by the monetary value. In panel (d), y-axis and x-axis are log occupational wages and the change of log non-US robot prices.

variable and run the main reduced-form regression (3). The result is shown in Figure F.13. I have considered wage and employment as an outcome variable, robot stock and robot prices measured in the US and non-US countries. The interaction of the choice of these variables yield eight panels in Figure F.13, while none of them shows significant relationship between (lagged) the changes in labor market outcome variables and the changes in robot measures. After these thorough analysis, I have concluded that the risk of capturing the pretrend in the main analysis is minimal.

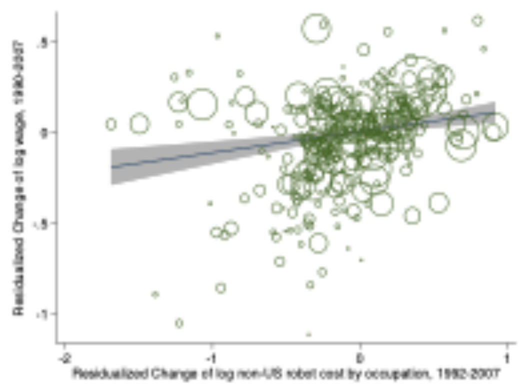
To further check the correlation systematically, I run the following regressions and report the

Figure F.11: Correlation between US Occupational Wage and Non-US Robot Measures (Controlled)

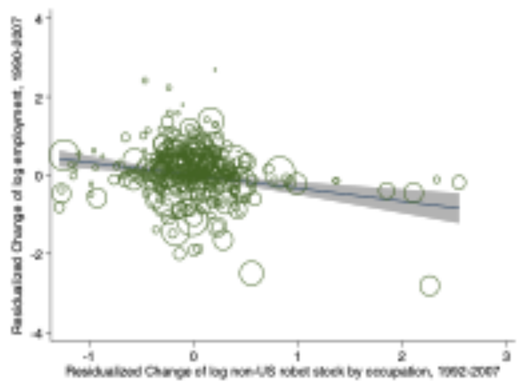
(a) Wage and Robot Values



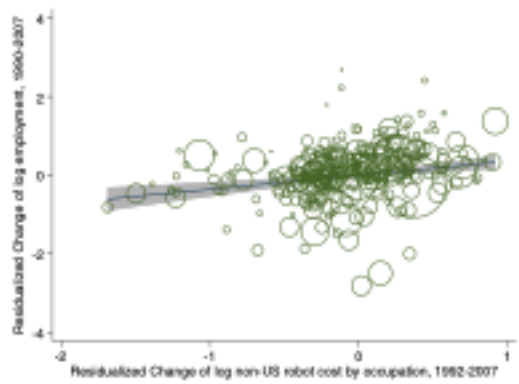
(b) Wage and Robot Prices



(c) Employment and Robot Values



(d) Employment and Robot Prices



Note: The author's calculation based on JARA, O*NET, and the US Census/ACS. The figures show the scatterplot, weighted fit line, and the 95 percent confidence interval of the changes in log robot measures and changes in log labor market outcomes. Each bubble represents a 4-digit occupation. The bubble size reflects the employment in the baseline year (1990, which is the closest Census year to the initial year that I observe the robot adoption, 1992). All variables are partialled out by control variables (the occupational female share, college share, age distribution, and foreign born share). In panel (a), y-axis and x-axis are log occupational wages and the change of log non-US robot stock measured by the monetary value. In panel (b), y-axis and x-axis are log occupational wages and the change of log non-US robot price. In panel (c), y-axis and x-axis are log occupational wages and the change of log non-US robot stock measured by the monetary value. In panel (d), y-axis and x-axis are log occupational wages and the change of log non-US robot prices.

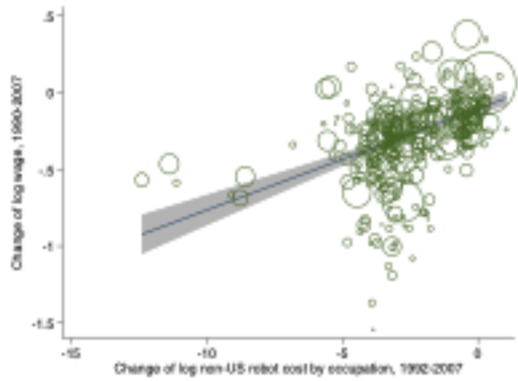
results in Table F.1:

$$\Delta \ln (y_o) = a_R \Delta \ln (R_o) + (X_o)^\top \boldsymbol{a} + e_o,$$

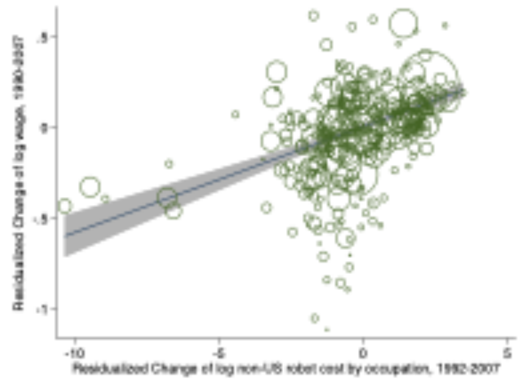
where y_o is labor-market outcome of occupation o (wage and employment), R_o is the measure of robots (stocks and prices), X_o are the demographic control variables, e_o is the regression residual, and Δ indicates the long-difference between 1990 (1992 for $\Delta \ln (R_o)$) and 2007. The coefficient of interest is a_R. I expect negative a_R if I take robot stocks as the explanatory variable, while I expect

Figure F.12: Correlation between US Occupational Wage and Non-US Quality-adjusted Robot Prices

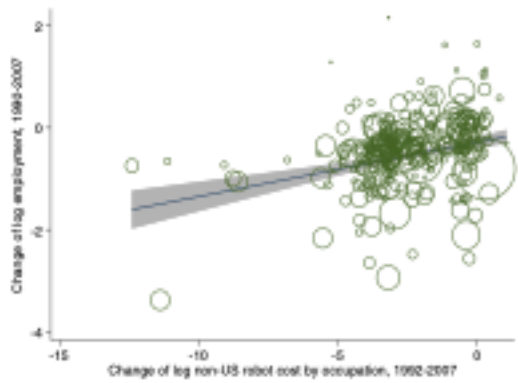
(a) Wage and Robot Price, Raw



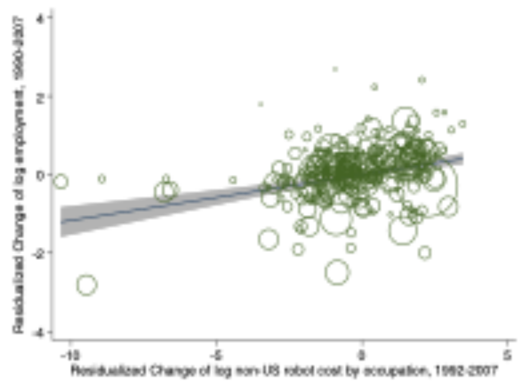
(b) Wage and Robot Price, Controlled



(c) Employment and Robot Price, Raw



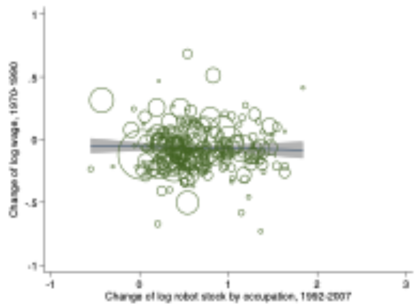
(d) Employment and Robot Price, Control



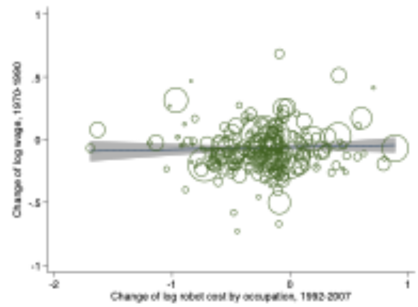
Note: The author's calculation based on JARA, O*NET, and the US Census/ACS. The figures show the scatterplot, weighted fit line, and the 95 percent confidence interval of the changes in log robot measures and changes in log labor market outcomes. Each bubble represents a 4-digit occupation. The bubble size reflects the employment in the baseline year (1990, which is the closest Census year to the initial year that I observe the robot adoption, 1992). In panel (a), y-axis and x-axis are log occupational wages and the change of log non-US quality-adjusted robot price measured in raw values. In panel (b), y-axis and x-axis are log occupational wages and the change of log non-US quality-adjusted robot price, both partialled out by control variables (the occupational female share, college share, age distribution, and foreign born share). In panel (c), y-axis and x-axis are log occupational wages and the change of log non-US quality-adjusted robot price in raw values. In panel (d), y-axis and x-axis are log occupational wages and the change of log non-US quality-adjusted robot prices, both partialled out by control variables.

Figure F.13: Pretrend Analysis

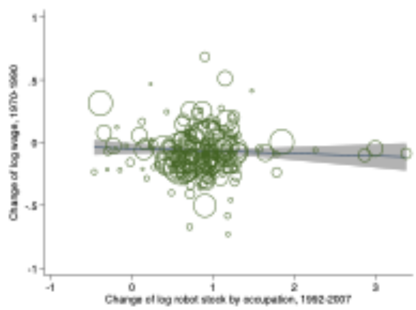
(a) Wage and US Robot Stock



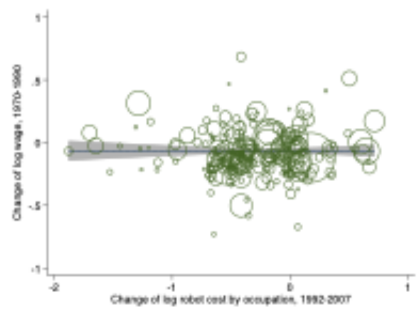
(b) Wage and US Robot Price



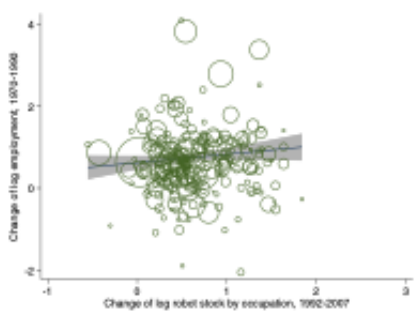
(c) Wage and non-US Robot Stock



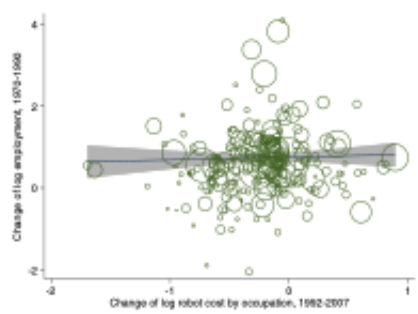
(d) Wage and non-US Robot Price



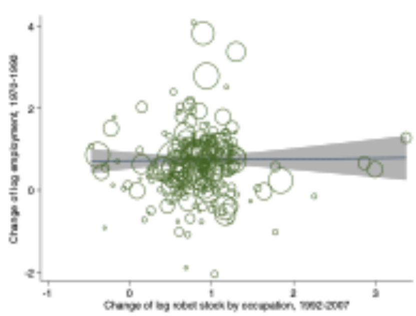
(e) Employment and US Robot Stock



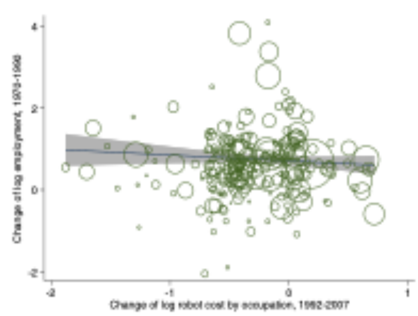
(f) Employment and US Robot Price



(g) Employment and non-US Robot Stock



(h) Employment and non-US Robot Price



Note: The author's calculation based on JARA, O*NET, and the US Census/ACS. The figures show the scatterplot, weighted fit line, and the 95 percent confidence interval of the 1990-2007 changes in log robot measures and the lagged (1970-1990) changes in log labor market outcomes. Each bubble represents a 4-digit occupation. The bubble size reflects the employment in the baseline year (1990, which is the closest Census year to the initial year that I observe the robot adoption, 1992). In panel (a), y-axis and x-axis are log occupational wages and the change of log US robot stock. In panel (b), y-axis and x-axis are log occupational wages and the change of log US robot price. In panel (c), y-axis and x-axis are log occupational wages and the change of log non-US robot stock. In panel (d), y-axis and x-axis are log occupational wages and the change of log non-US robot price. In panel (e), y-axis and x-axis are log occupational employment and the change of log US robot stock. In panel (f), y-axis and x-axis are log occupational employment and the change of log US robot price. In panel (g), y-axis and x-axis are log occupational employment and the change of log non-US robot stock. In panel (h), y-axis and x-axis are log occupational employment and the change of log non-US robot price.

Table F.1: Regression Result of Labor Market Outcome on Robot Measures

VARIABLES	(1) $\Delta\ln(w)$	(2) $\Delta\ln(w)$	(3) $\Delta\ln(w)$	(4) $\Delta\ln(w)$	(5) $\Delta\ln(L)$	(6) $\Delta\ln(L)$	(7) $\Delta\ln(L)$	(8) $\Delta\ln(L)$
$\Delta\ln(K^R_{USA})$	-0.276***				-0.495***			
	(0.0407)				(0.183)			
$-\Delta\ln(p^R_{USA})$		-0.129**				-0.445***		
		(0.0571)				(0.137)		
$\Delta\ln(K^R_{ROW})$			-0.146***				-0.319**	
			(0.0284)				(0.159)	
Japan Robot Shock, $-\psi^J$				-0.118**				-0.371***
				(0.0569)				(0.142)
Female share	0.106*	0.0824	0.123**	0.0828	-0.0516	-0.118	-0.0104	-0.113
	(0.0567)	(0.0588)	(0.0584)	(0.0617)	(0.170)	(0.190)	(0.192)	(0.198)
Col. grad. share	0.526***	0.501***	0.546***	0.520***	-0.390	-0.455	-0.341	-0.392
	(0.112)	(0.108)	(0.114)	(0.114)	(0.313)	(0.318)	(0.316)	(0.324)
Age 35-49 share	-0.880**	-0.317	-0.732	-0.411	1.208	2.470**	1.373	2.129*
	(0.416)	(0.498)	(0.463)	(0.501)	(1.319)	(1.167)	(1.315)	(1.173)
Age 50-64 share	0.971**	0.692	1.084**	0.678	-1.318	-2.113*	-1.031	-2.107*
	(0.403)	(0.467)	(0.445)	(0.506)	(1.246)	(1.137)	(1.268)	(1.119)
Foreign-born share	0.468	0.706*	0.609	0.778*	1.852**	2.084**	2.020**	2.344***
	(0.386)	(0.386)	(0.416)	(0.401)	(0.935)	(0.847)	(0.950)	(0.801)
Exposure to China Trade	-0.293	-0.459	-0.324	-0.582	-3.206**	-3.475**	-3.203**	-3.868**
	(0.843)	(0.773)	(0.886)	(0.763)	(1.380)	(1.514)	(1.514)	(1.495)
Observations	324	324	324	324	324	324	324	324
R-squared	0.421	0.275	0.319	0.279	0.125	0.101	0.096	0.096

Note: The author's calculation based on JARA, O*NET, and US Census/ACS. Observations are 4-digit level occupations, and the sample is all occupations that existed throughout 1970 and 2007. In each country $i \in \{USA, ROW\}$, K^R_i stands for the 2000-dollar value of the robot stock in the occupation and p^R_i stands for the average price of robot transacted in each year. All time differences (Δ) are taken with a long difference between 1990 and 2007. All demographic control variables are as of 1990. "Col. Grad. Share" stands for the college graduate share. Robust standard errors are reported in the parentheses. *** p<0.01, ** p<0.05, * p<0.1.

positive a_R when I take robot price as the right-hand side variable.

F.9 Robot Price Trends by Occupation Groups

In this section, I examine the facts discussed in Section 2.3 for each occupation group described in Section 4.1. Figure F.14 shows the plot of the trend of the robot price distribution since 1992 for each occupation group, a version of Figure 1a, disaggregated by occupation groups. The top three panels show the trends for routine occupations, namely, from the left, routine-production, routine-transportation, and routine-others. The bottom two panels show the trends for service occupations and abstract occupations, from the left. All these panels show the overall decreasing trend of robot prices, and the dispersion of prices within each occupation group. Having such a dispersion is important because in Section 4 when I estimate heterogeneous EoS between robots and labor, I use the price variation within each occupation group.

F.10 The Effect of Robots from Japan and Other Countries

A potential concern for my empirical setting is the selection issue regarding the robot origin country of Japan. Specifically, robots from Japan may be different from those from other countries, so the labor market implication may also differ between them. Unfortunately, it is hard to directly compare the effects of these two different groups of robots due to the data limitation, so I will focus on the best comparable measures of robotization between Japan-sourced robots and robots from all countries, which is the quantity of robot stock. Specifically, I take the total stock of robots in the US from the IFR data. This measure does not contain the monetary value at the occupation level for all the sample periods, but it is the number of units. Note also that the IFR variable is the total number that does not specify the source country. I then convert the IFR application codes to the JARA application codes to use the allocation rule for matching the JARA application codes and the occupation codes. As a result, I obtain the robots used in the US and sourced from any countries at the occupation level. I then run the following regression to examine the correlations between labor market outcomes and the robot measures and compare them depending on if the measure is robots from Japan or robots from all over the world:

$$\Delta Y_o = \beta^Q \Delta K_o^{R,Q} + X_o \gamma^Q + \varepsilon_o^Q, \tag{F.1}$$

Figure F.14: Robot Price Trends by Occupation Groups

Note: The author's calculation based on JARA and O*NET. Panels show the trend of robot prices by occupation groups defined in the main text.

where ΔY_o is either the changes in wages or employment at the occupation-o level, ΔK_o^Q is the measure of the number of robots taken either from JARA (so robots from Japan) or IFR (so robots from the world), and ε_o^Q is the error term. The coefficient of interest is β^Q, which gives us an insight about the correlation between the changes in labor market outcomes and the changes in robot quantity, depending on if the robots are conditioned to be sourced from Japan or not. Specifically, if robots from Japan may substitute workers stronger than robots from the other countries, coefficient β^Q is expected to be larger when we use the JARA robot measure than IFR.

Table F.2 shows the regression result of equation (F.1). Columns 1-4 considers the changes in occupational wage in the outcome variable, while columns 5-8 take occupational employment. Columns 1, 2, 5, and 6 do not include the demographic control variables (female share, age distribution, college-graduate share, and foreign-born share), while columns 3, 4, 7, and 8 do. Columns

Table F.2: Regression Result of Labor Market Outcome on JARA and IFR Robot Stocks

VARIABLES	(1) $\Delta\ln(w)$	(2) $\Delta\ln(w)$	(3) $\Delta\ln(w)$	(4) $\Delta\ln(w)$	(5) $\Delta\ln(L)$	(6) $\Delta\ln(L)$	(7) $\Delta\ln(L)$	(8) $\Delta\ln(L)$
$\Delta\ln(K^{R,Q}_{JPN\to USA})$	-0.372***		-0.264***		-0.765***		-0.636***	
	(0.0311)		(0.0317)		(0.0903)		(0.0830)	
$\Delta\ln(K^{R,Q}_{USA})$		-0.144***		-0.110***		-0.311***		-0.461***
		(0.0161)		(0.0196)		(0.0447)		(0.0461)
Observations	324	324	324	324	324	324	324	324
R-squared	0.307	0.200	0.334	0.262	0.182	0.131	0.179	0.260
Demographic controls			✓	✓			✓	✓

Note: The author's calculation based on JARA, IFR, O*NET, and US Census/ACS. Observations are 4-digit level occupations, and the sample is all occupations that existed throughout 1970 and 2007. Columns 1-4 considers the changes in occupational wage in the outcome variable, while columns 5-8 take occupational employment. Columns 1, 2, 5, and 6 do not include the demographic control variables (female share, age distribution, college-graduate share, and foreign-born share), while columns 3, 4, 7, and 8 do. Columns 1, 3, 5, and 7 take the robots from Japan from JARA data, while columns 2, 4, 6, and 8 take the robots from the world from IFR data. *** p<0.01, ** p<0.05, * p<0.1.

Table F.3: List of Data Sources

Variable	Description	Source
$\widetilde{y}^G_{ij,t_0}, \widetilde{x}^G_{ij,t_0}, \widetilde{y}^R_{ij,t_0}, \widetilde{x}^R_{ij,t_0}$	Trade shares of goods and robots	BACI, IFR
$\widetilde{x}^O_{i,o,t_0}$	Occupation cost shares	IPUMS
l_{i,o,t_0}	Labor shares within occupation	JARA, IFR, IPUMS
$s^G_{i,t_0}, s^V_{i,t_0}, s^R_{i,t_0}$	Robot expenditure shares	BACI, IFR, WIOT
$\alpha_{i,M}$	Intermediate input share	WIOT

1, 3, 5, and 7 take the robots from Japan from JARA data, while columns 2, 4, 6, and 8 take the robots from the world from IFR data. Table F.2 reveals that both the JARA- and IFR-based robot measures capture the substitution of workers with robots. The result for the IFR data is in line with the previous finding, such as Acemoglu and Restrepo (2020). In contrast, comparing the size of coefficients, one can find that the coefficient is somewhat stronger for JARA robot measures than for IFR. Overall, I find some evidence that Japanese robots substitute workers stronger than other countries' robots, while all sorts of robots do seem to have some substitution effect on workers.

F.11 Data about Initial Shares

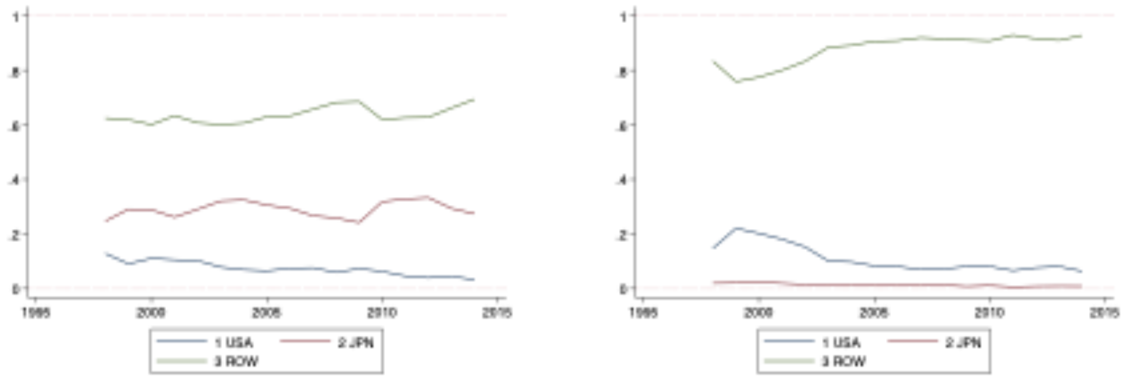
Since the log-linearized sequential equilibrium solution depends on several initial share data generated from the initial equilibrium, I discuss the data sources and methods for measuring these shares. I define $t_0 = 1992$ and the time frequency is annual. I consider the world that consists of three countries $\{USA, JPN, ROW\}$. Table F.3 summarizes overview of the variable notations, descriptions, and data sources.

I take matrices of trade of goods and robots by BACI data. As in Humlum (2019), I measure robots by HS code 847950 ("Industrial Robots For Multiple Uses") and approximate the initial year value by year of 1998, in which the robot HS code is first available. Figure F.15 shows the trend of export and import shares of robots among the world for the US, Japan, and the Rest Of the World. The trends are fairly stable for the three regions of the world, except that the import share of the US has declined relative to the ROW.

To obtain the domestic robot absorption data, I take from IFR data the flow quantity variable and the aggregate price variable for a selected set of countries. I then multiply these to obtain USA and JPN robot adoption value. For robot prices in ROW, I take the simple average of the

Figure F.15: Robot Trade Share Trends

(a) Exports (b) Imports

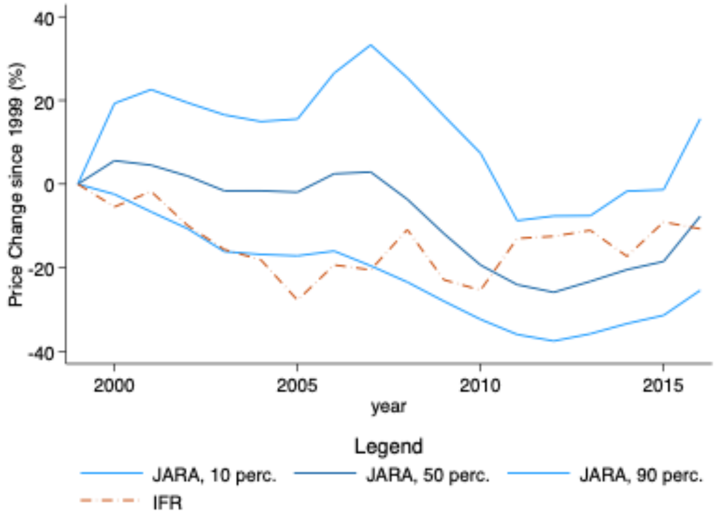


Note: The author's calculation of world trade shares based on the BACI data. Industrial robots are measured by HS code 847950 (Industrial robots for multiple uses).

prices among the set of countries (France, Germany, Italy, South Korea, and the UK, as well as Japan and the US) for which the price is available in 1999, the earliest year in which the price data are available. Graetz and Michaels (2018) discuss prices of robots with the same data source. Figure F.16 shows the comparison of the US price index measure available between JARA and IFR. The JARA measures are disaggregated by 4-digit occupations. The figure shows the 10th, 50th (median), and 90th percentiles each year, as in Figure 1a. All measures are normalized at 1999, the year in which the first price measure is available in the IFR data. Overall, the JARA price trend variation tracks the overall price evolution measured by IFR reasonably well: The long-run trends from 1999 to the late 2010s are similar between the JARA median price and the IFR price index. During the 2000s, the IFR price index drops faster than the median price in the JARA data. It compares with the JARA 10th percentile price, which could be due to robotic technological changes in other countries than Japan in the corresponding period.

I construct occupation cost shares $\widetilde{x}^{O}_{i,o,t_0}$ and labor shares within occupation l_{i,o,t_0} as follows. To measure $\widetilde{x}^{O}_{i,o,t_0}$, I aggregate the total wage income of workers that primarily works in each occupation o in year 1990, the Census year closest to t_0. I then take the share of this total compensation measure for each occupation. To measure l_{i,o,t_0}, I take the total compensation as the total labor cost and a measure of the user cost of robots for each occupation. The user cost of robots is calculated with the occupation-level robot price data available in IFR and the set of calibrated parameters in Section 4.1. Table F.4 summarizes these statistics for the aggregated 5 occupation groups in the

Figure F.16: Comparison of US Price Indices between JARA and IFR



Note: The author's calculation of US robot price measures in JARA and IFR. The JARA measures are disaggregated by 4-digit occupations, and the figure shows the 10th, 50th (median), and 90th percentiles each year. All measures are normalized at 1999, the year in which the first price measure is available in the IFR data.

Table F.4: Baseline Shares by 5 Occupation Group

Occupation Group	$\widetilde{x}^O_{1,o,t_0}$	l^O_{1,o,t_0}	y^R_{2,o,t_0}	x^R_{1,o,t_0}	x^R_{2,o,t_0}	x^R_{3,o,t_0}
Routine, Production	17.58%	99.81%	64.59%	67.49%	62.45%	67.06%
Routine, Transportation	7.82%	99.93%	12.23%	11.17%	13.09%	11.04%
Routine, Others	28.78%	99.99%	10.88%	9.52%	11.68%	10.40%
Service	39.50%	99.99%	8.87%	8.58%	9.17%	8.32%
Abstract	6.32%	99.97%	3.43%	3.24%	3.60%	3.18%

Note: The author's calculation of initial-year share variables based on the US Census, IFR, and JARA. As in the main text, country 1 indicates the US, country 2 Japan, and country 3 the rest of the world. See the main text for the construction of each variable.

US. One can see that the cost for production occupations and transportation occupations comprise 18% and 8% of the US economy, respectively, totaling more than one-fourth. Furthermore, the share of robot cost in all occupations is still quite low with the highest share of 0.19% in production occupations, revealing still small-scale adoption of robots from the overall US economy perspective.

To calculate the effect on total income, I also need to compute the sales share of robots by occupations $y^R_{i,o,t_0} \equiv Y^R_{i,o,t_0} / \sum_o Y^R_{i,o,t_0}$ and the absorption share $x^R_{i,o,t_0} \equiv X^R_{i,o,t_0} / \sum_o X^R_{i,o,t_0}$. To obtain y^R_{i,o,t_0}, I compute the share of robots by occupations produced in Japan $y^R_{2,o,t_0} = Y^R_{2,o,t_0} / \sum_o Y^R_{2,o,t_0}$ and assume the same distribution for other countries due to the data limitation: $y^R_{i,o,t_0} = y^R_{2,o,t_0}$ for all

i. To have x^R_{i,o,t_0}, I compute the occupational robot adoption in each country by $X^R_{i,o,t_0} = P^R_{i,t_0} Q^R_{i,o,t_0}$, where Q^R_{i,o,t_0} is the occupation-level robot quantity obtained by the O*NET concordance generated in Section 2.2 applied to the IFR application classification. As mentioned above, the robot price index P^R_{i,t_0} is available for a selected set of countries. To compute the rest-of-the-world price index P^R_{3,t_0}, I take the average of all available countries weighted by the occupational robot values each year. The summary table for these variables y^R_{i,o,t_0} and x^R_{i,o,t_0} at 5 occupation groups are shown in Table F.4. All values in Table F.4 are obtained by aggregating 4-digit-level occupations, and raw and disaggregated data are available upon request.

I take a more standard measure, the intermediate input share $\alpha_{i,M}$, from World Input-Output Tables (WIOT Timmer, Dietzenbacher, et al. 2015). Finally, I combine the trade matrix generated above and WIOT to construct the good and robot expenditure shares s^G_{i,t_0}, s^V_{i,t_0}, and s^R_{i,t_0}. In particular, with the robot trade matrix, I take the total sales value by summing across importers for each exporter, and total absorption value by summing across exporters for each importers. I also obtain the total good absorption by WIOT. From these total values, I compute expenditure shares. are obtained by aggregating 4-digit occupations, and the disaggregated data are available upon request.

As initial year occupation switching probabilities μ_{i,oo',t_0}, I take 1990 flow Markov transition matrix from the cleaned CPS-ASEC data created in Section F.1. Table F.5 shows this initial-year conditional switching probability. The matrix for the other years are available upon request. As for other countries than the US, although Freeman, Ganguli, and Handel (2020) has begun to develop occupational wage measures consistent across country, world-consistent occupation employment data are hard to obtain. Therefore, I assign the same flow probabilities for other countries in my estimation.

G Theory Appendix

G.1 Further Discussion of Model Assumptions

Capital-Skill Complementarity Occupation production function (5) also nest the one in the literature of capital-skill complementarity (Krusell et al. 2000 among others). To simplify, I focus on individual producer's production function in the steady state. Thus I drop subscripts and super-

Table F.5: 1990 Occupation Group Switching Probability

		Routine Production	Routine Transportation	Routine Others	Service	Abstract
Routine	Production	0.961	0.011	0.010	0.006	0.012
	Transportation	0.020	0.926	0.020	0.008	0.025
	Others	0.005	0.006	0.955	0.020	0.014
Service		0.003	0.002	0.020	0.967	0.007
Abstract		0.014	0.014	0.036	0.015	0.922

Note: The author's calculation from the CPS-ASEC 1990 data. The conditional switching probability to column occupation group conditional on being in each row occupation.

scripts of country i and time period t. Suppose the set of occupations is $O \equiv \{R, U\}$ and $a_U = 0$. R stands for the robotized occupation (e.g., spot welding) and U stands for "unrobotized" (e.g., programming). Note that since U is unrobotized $a_U = 0$. Then the unit cost of occupation aggregate (5), P^O, is

$$P^O = \left[(b_R)^{\frac{1}{\beta}} \left((1 - a_R) (w_R)^{1-\theta_R} + a_R (c_R)^{1-\theta_R} \right)^{\frac{1-\beta}{1-\theta_R}} + (b_U)^{\frac{1}{\beta}} (w_U)^{1-\beta} \right]^{\frac{1}{1-\beta}}.$$

Thus different skills R and U are substituted by robots with different substitution parameters θ_R and β, respectively. Since the literature of capital-skill complementarity studies the rising skill premium, the current model also has an ability to discuss the occupation (or skill) premium given the different level of automation across occupations.

Adjustment Cost of Robot Capital To interpret another key feature of the model, the convex adjustment cost of robot adoption, consider the cost of adopting new technology and integration. With the convex adjustment cost, the model predicts the staggered adoption of robots over years that I observe in the data (see Figure 3b), and implies a rich prediction about the short- and long-run effects of robotization.

First, when adopting new technology including robots, it is necessary to re-optimize the overall production process since the production process is typically optimized to employ workers. More generally, the literature of organizational dynamics studies the difficulty, not to say the impossibility, of changing strategies of a company due to complementarities (see Brynjolfsson and Milgrom 2013 for a review). Such a re-optimization incurs an additional cost of adoption in addition to the purchase of robot arms. Moreover, even within a production unit, there is a variation of this difficulty of adopting robots across production processes. In this case, the part where the adjustment is easy adopts the robots first, and vice versa. This allocation of robot adoptions over years may aggregate to make the robot stock increase slowly (Baldwin and Lin 2002). Waldman-Brown (2020) also finds that the incremental and sluggish automation is particularly well-observed in small and medium-sized firms, as they add "a machine here or there, rather than installing whole new systems that are more expensive to buy and integrate" (Autor, Mindell, and Reynolds 2020).

The second component of the adjustment cost may come from the cost of integration as I discussed in Section 2.1. The marginal integration cost may increase as the massive upgrading of robotics system may require large-scale overhaul of production process, which increases the complexity and so is costly. The adjustment cost may capture the increasing marginal cost component of the integration cost. It explains an additional component of the integration cost implied by constant returns-to-scale robot aggregation in equation (9).

Another potential choice of modeling a staggered growth of robot stocks is to assume a fixed cost of robot adoption and lumpy investment. Humlum (2019) finds that many plants buy robots only once during the sample period. Since JARA data does not observe plant-level adoptions, I do not separately identify lumpy investment from the staggered growth of robot stocks in the data. To the extent that fixed cost of investment may make the policy intervention less effective (e.g., Koby and Wolf 2019), the counterfactual analysis in this paper may overestimate the effect of robot taxes since it does not take into account the fixed cost and lumpiness of investment.

G.2 Derivation of Worker's Optimality Conditions

In this section, I formalize the assumptions behind the derivation and show equations (B.2) and (B.3). One trick new to the below discussion is that I characterize the switching cost by an ad-valorem term, which makes the log-linearization simpler when solving the model.

Fix country i and period t. There is a mass $\overline{L}_{i,t}$ of workers. In the beginning of each period, worker $\omega \in \left[0, \overline{L}_{i,t}\right]$ draws a multiplicative idiosyncratic preference shock $\left\{Z_{i,o,t}\left(\omega\right)\right\}_o$ that follows an independent Fréchet distribution with scale parameter $A^V_{i,o,t}$ and shape parameter $1/\phi$. Note that one can simply extend that the idiosyncratic preference follows a correlated Fréchet distribution to allow correlated preference across occupations, as in Lind and Ramondo (2018). To keep the expression simple, I focus on the case of independent distribution. A worker ω then works in the current occupation, earns income, consumes and derives logarithmic utility, and then chooses the next period's occupation with discount rate ι. When choosing the next period occupation o', she pays an ad-valorem switching cost $\chi_{i,oo',t}$ in terms of consumption unit that depends on current occupation o. She consumes her income in each period. Thus, worker ω who currently works in occupation o_t maximizes the following objective function over the future stream of utilities by choosing occupations $\{o_s\}_{s=t+1}^{\infty}$:

$$E_t \sum_{s=t}^{\infty} \left(\frac{1}{1+\iota}\right)^{s-t} \left[\ln\left(C_{i,o_s,s}\right) + \ln\left(1-\chi_{i,o_s o_{s+1},s}\right) + \ln\left(Z_{i,o_{s+1},s}\left(\omega\right)\right)\right]$$

where $C_{i,o,s}$ is a consumption bundle when working in occupation o in period $s \geq t$, and E_t is the expectation conditional on the value of $Z_{i,o_t,t}\left(\omega\right)$. Each worker owns occupation-specific labor endowment $l_{i,o,t}$. I assume that her income is comprised of labor income $w_{i,o,t}$ and occupation-specific ad-valorem government transfer with rate $T_{i,o,t}$. Given the consumption price $P_{i,t}^{G}$, the budget constraint is

$$P_{i,t}^{G} C_{i,o,t} = w_{i,o,t} l_{i,o,t} \left(1+T_{i,o,t}\right)$$

for any worker, with $P_{i,t}^{G}$ being the price index of the non-robot good G.

By linearity of expectation,

$$\begin{aligned}
& E_t \sum_{s=t}^{\infty} \left(\frac{1}{1+\iota}\right)^{s-t} \left[\ln\left(C_{i,o_s,s}\right) + \ln\left(1-\chi_{i,o_s o_{s+1},s}\right) + \ln\left(Z_{i,o_{s+1},s}\left(\omega\right)\right)\right] \\
&= \sum_{s=t}^{\infty} \left(\frac{1}{1+\iota}\right)^{s-t} \left[E_t \ln\left(C_{i,o_s,s}\right) + E_t \ln\left(1-\chi_{i,o_s o_{s+1},s}\right) + E_t \ln\left(Z_{i,o_{s+1},s}\left(\omega\right)\right)\right].
\end{aligned}$$

By monotone transformation with exponential function,

$$\begin{aligned}
& \exp\left\{\sum_{s=t}^{\infty} \left(\frac{1}{1+\iota}\right)^{s-t} \left[E_t \ln\left(C_{i,o_s,s}\right) + E_t \ln\left(1-\chi_{i,o_s o_{s+1},s}\right) + E_t \ln\left(Z_{i,o_{s+1},s}\left(\omega\right)\right)\right].\right\} \\
&= \prod_{s=t}^{\infty} \exp\left\{\left(\frac{1}{1+\iota}\right)^{s-t} \left[E_t \ln\left(C_{i,o_s,s}\right) + E_t \ln\left(1-\chi_{i,o_s o_{s+1},s}\right) + E_t \ln\left(Z_{i,o_{s+1},s}\left(\omega\right)\right)\right]\right\}.
\end{aligned}$$

Write the value function conditional on the realization of shocks at period t as follows:

$$V_{i,o_t,t}(\omega) \equiv \max_{\{o_s\}_{s=t+1}^{\infty}} \prod_{s=t}^{\infty} \exp\left\{ \left(\frac{1}{1+\iota}\right)^{s-t} \left[E_t \ln\left(C_{i,o_s,s}\right) + E_t \ln\left(1-\chi_{i,o_s o_{s+1},s}\right) + E_t \ln\left(Z_{i,o_{s+1},s}(\omega)\right)\right] \right\}.$$

I apply Bellman's principle of optimality as follows:

$$\begin{aligned}
&V_{i,o_t,t}(\omega) \\
&= \max_{\{o_s\}_{s=t+1}^{\infty}} \prod_{s=t}^{\infty} \exp\left\{ \left(\frac{1}{1+\iota}\right)^{s-t} \left[E_t \ln\left(C_{i,o_s,s}\right) + E_t \ln\left(1-\chi_{i,o_s o_{s+1},s}\right) + E_t \ln\left(Z_{i,o_{s+1},s}(\omega)\right)\right] \right\} \\
&= \max_{o_{t+1}} \exp\left\{ \ln\left(C_{i,o_t,t}\right) + \ln\left(1-\chi_{i,o_t o_{t+1},t}\right) + \ln\left(Z_{i,o_{t+1},t}(\omega)\right) \right\} \times \\
&\max_{\{o_s\}_{s=t+2}^{\infty}} \prod_{s=t+1}^{\infty} \exp\left\{ \left(\frac{1}{1+\iota}\right)^{s-(t+1)} \left[E_{t+1} \ln\left(C_{i,o_s,s}\right) + E_{t+1} \ln\left(1-\chi_{i,o_s o_{s+1},s}\right) + E_{t+1} \ln\left(Z_{i,o_{s+1},s}(\omega)\right)\right] \right\} \\
&= \max_{o_{t+1}} \exp\left\{ \ln\left(Z_{i,o_t,t}(\omega)\right) + \ln\left(C_{i,o_t,t}\right) + \ln\left(1-\chi_{i,o_t o_{t+1},t}\right) \right\} V_{i,o_{t+1},t+1},
\end{aligned}$$

where $V_{i,o_t,t}$ is the unconditional expected value function $V_{i,o_t,t} \equiv E_{t-1} V_{i,o_t,t}(\omega)$. Changing the notation from (o_t, o_{t+1}) into (o, o'), I have

$$V_{i,o,t}(\omega) = \max_{o'} C_{i,o,t}\left(1-\chi_{i,oo',t}\right) Z_{i,o',t}(\omega) V_{i,o',t+1}.$$

Solving the worker's maximization problem is equivalent to finding:

$$\begin{aligned}
\mu_{i,oo',t} &\equiv \Pr\left(\text{worker } \omega \text{ in } o \text{ chooses occupation } o'\right) \\
&= \Pr\left(\max_{o''} C_{i,o,t}\left(1-\chi_{i,oo'',t}\right) Z_{i,o'',t}(\omega) V_{i,o'',t+1} \leq C_{i,o,t}\left(1-\chi_{i,oo',t}\right) Z_{i,o',t}(\omega) V_{i,o',t+1} \right).
\end{aligned}$$

By the independent Fréchet assumption, I have the maximum value distribution

$$\begin{aligned}
\Pr\left(\max_{o''} C_{i,o,t}\left(1-\chi_{i,oo',t}\right) Z_{i,o',t}(\omega) V_{i,o',t+1} \leq v \right) &= \prod_{o'} \Pr\left(Z_{i,o',t}(\omega) \leq \frac{v}{C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}} \right) \\
&= \prod_{o''} \exp\left(\left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi} v^{-\phi} \right) \\
&= \exp\left(\sum_{o''} \left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi} v^{-\phi} \right).
\end{aligned}$$

Therefore, the conditional choice probability satisfies, again by the independent Fréchet assumption,

$$
\begin{aligned}
&\mu_{i,oo',t} \\
&= \int_0^\infty \Pr\left(\max_{o''\neq o'} C_{i,o,t}\left(1-\chi_{i,oo'',t}\right) Z_{i,o',t}\left(\omega\right) V_{i,o'',t+1} \leq v\right) d\Pr\left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) Z_{i,o',t}\left(\omega\right) V_{i,o',t+1} \geq v\right) \\
&= \int_0^\infty \exp\left(\sum_{o''\neq o'} \left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi} v^{-\phi}\right) \times \\
&\qquad \left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi} \exp\left(\left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi} v^{-\phi}\right) \times \left(-\phi v^{-\phi-1}\right) dv \\
&= \frac{\left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi}}{\sum_{o''}\left(C_{i,o,t}\left(1-\chi_{i,oo'',t}\right) V_{i,o'',t+1}\right)^{\phi}} \times \\
&\qquad \int_0^\infty \exp\left(\sum_{o'''} \left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi} v^{-\phi}\right) \sum_{o''} \left(C_{i,o,t}\left(1-\chi_{i,oo'',t}\right) V_{i,o'',t+1}\right)^{\phi} \times \left(-\phi v^{-\phi-1}\right) dv.
\end{aligned}
$$

The last integral term is one by integration and the definition of distribution. Therefore, I arrive at

$$
\mu_{i,oo',t} = \frac{\left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi}}{\sum_{o''}\left(C_{i,o,t}\left(1-\chi_{i,oo'',t}\right) V_{i,o'',t+1}\right)^{\phi}} = \frac{\left(\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi}}{\sum_{o''}\left(\left(1-\chi_{i,oo'',t}\right) V_{i,o'',t+1}\right)^{\phi}},
$$

$$
V_{i,o,t+1} = \widetilde{\Gamma} C_{i,o,t} \left(\sum_{o'} \left(\left(1-\chi_{i,oo',t+1}\right) V_{i,o',t+2}\right)^{\phi}\right)^{\frac{1}{\phi}}.
$$

G.3 Relationship with Other Models of Automation

The model in Section 3 is general enough to nest models of automation in the previous literature. In particular, I show how the production functions (4) and (5) imply to specifications in AR and Humlum (2019). Throughout Section G.3, I fix country i and focus on steady states and thus drop subscripts i and t since the discussion is about individual producer's production function.

G.3.1 Relationship with the model in Acemoglu and Restrepo (2020, AR)

Following AR that abstract from occupations, I drop occupations by setting $O = 1$ in this paragraph. Therefore, the EoS between occupations β plays no role, and $\theta_o = \theta$ is a unique value. AR show that the unit cost (hence the price given perfect competition) is written as

$$p^{AR} \equiv \frac{1}{\widetilde{A}} \left[(1-\widetilde{a}) \frac{w}{A^L} + \widetilde{a} \frac{c^R}{A^R} \right]^{\alpha_L} r^{1-\alpha_L},$$

for each sector and location (See AR, Appendix A1, equation A5). In this equation, c^R is the steady state marginal cost of robot capital defined in equation (G.27) and A^L and A^R represent per-unit efficiency of labor and robots, respectively. In Lemma G.1 below, I prove that my model implies a unit cost function that is strict generalization of p^{AR} with proper modification to the shock terms and parameter configuration. I begin with the modification that allows per-unit efficiency terms in my model.

Definition G.1. For labor and robot per-unit efficiency terms $A^L > 0$ and $A^R > 0$ respectively, modified robot task space $\widetilde{a}$ and TFP term $\widetilde{A}$ are

$$\widetilde{a} \equiv \frac{a \left(A^L\right)^{\theta-1}}{a \left(A^L\right)^{\theta-1} + (1-a) \left(A^R\right)^{\theta-1}}, \tag{G.1}$$

$$\widetilde{A} \equiv \frac{A}{\left[(1-\widetilde{a}) \left(A^L\right)^{\theta-1} + \widetilde{a} \left(A^R\right)^{\theta-1} \right]}. \tag{G.2}$$

Lemma G.1. *Set the number of occupations $O = 1$. In the steady state,*

$$p^G = \frac{1}{\widetilde{A}} \left[(1-\widetilde{a}) \left(\frac{w}{A^L} \right)^{1-\theta} + \widetilde{a} \left(\frac{c^R}{A^R} \right)^{1-\theta} \right]^{\frac{\alpha_L}{1-\theta}} \left(p^G\right)^{\alpha_M} r^{1-\alpha_M-\alpha_L}. \tag{G.3}$$

Proof. Note that modified robot task space (G.1) and modified TFP (G.2) can be inverted to have

$$a \equiv \frac{\widetilde{a} \left(A^R\right)^{\theta-1}}{(1-\widetilde{a}) \left(A^L\right)^{\theta-1} + \widetilde{a} \left(A^R\right)^{\theta-1}}, \tag{G.4}$$

$$A \equiv \left[(1-\widetilde{a}) \left(A^L\right)^{\theta-1} + \widetilde{a} \left(A^R\right)^{\theta-1} \right] \widetilde{A}. \tag{G.5}$$

Cost minimization problem with the production functions (4) and (5) and perfect competition imply

$$p^G = \frac{1}{A} \left(P^O\right)^{\alpha_L} p^{\alpha_M} r^{1-\alpha_L-\alpha_M},$$

and

$$P^O = \left[(1-a)\, w^{1-\theta} + a^{1-\theta}\right]^{\frac{1}{1-\theta}},$$

where P^O is the unit cost of tasks performed by labor and robots. Substituting equations (G.4) and (G.5) and rearranging, I have

$$p^G = \frac{1}{\widetilde{A}} \left(\widetilde{P^O}\right)^{\alpha_L} \left(p^G\right)^{\alpha_M} r^{1-\alpha_L-\alpha_M},$$

where $\widetilde{P^O}$ is the cost of the tasks performed by labor and robots:

$$\widetilde{P^O} = \left[(1-\widetilde{a}) \left(\frac{w}{A^L}\right)^{1-\theta} + a \left(\frac{c^R}{A^R}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}.$$

□

Lemma G.1 immediately implies the following corollary that shows that the steady state modified unit cost (G.3) strictly nests the unit cost formulation of AR as a special case of Leontief occupation aggregation.

Corollary G.1. *Suppose $\alpha_M = 0$. Then as $\theta \to 0$, $p^G \to p^{AR}$.*

G.3.2 Relationship with the model in Humlum (2019)

I show that production functions (4) and (5) nest the production function used by Humlum (2019). Since the setting of Humlum (2019) does not have non-robot capital, in this section, I simplify the notation for robot capital K^R by dropping the superscript and denote as K. For each firm in each period, Humlum (2019) specifies

$$Q^D = \exp\left[\varphi_H^D + \gamma_H^D K\right] \left[\sum_o \left(\exp\left[\varphi_o^D + \gamma_o^D K\right]\right)^{\frac{1}{\beta}} (L_o)^{\frac{\beta-1}{\beta}}\right]^{\frac{\beta}{\beta-1}}, \tag{G.6}$$

where $K = \{0, 1\}$ is a binary choice, $\varphi_H^D, \gamma_H^D, \varphi_o^D$ and γ_o^D are parameters, and superscript D represents the discrete adoption problem of Humlum (2019). As normalization, suppose that

$$\sum_o \exp\left(\varphi_o^D + \gamma_o^D K\right) = 1.$$

I will start from production function (4) and (5), place restrictions, and arrive at equation (G.6). As a key observation, relative to the discrete choice of robot adoption in Humlum (2019), the continuous choice of robot *quantity* in production function (5) allows significant flexibility. In this paragraph, I assume away with intermediate inputs. This is because Humlum (2019) assumes that intermediate inputs enter in an element of CES, while production function (4) implies that intermediate inputs enter as an element of the Cobb-Douglas function.

Now, given my production functions (4) and (5), suppose producers follow the binary decision rule defined below.

Definition G.2. A binary decision rule of a producer is that producers can choose between two choices: adopting robots $K = 1$ or not $K = 0$. If they choose $K = 1$, they adopt robots at the same unit as labor $K_o = L_o \geq 0$ for all occupation o. If they choose $K = 0$, $K_o = 0$ for all o.

Note that the binary decision rule is nested in the original choice problem from $K_o^R \geq 0$ for each o. Set

$$A_o^D\left(K^R\right) \equiv \begin{cases} A_o\left(\left(1-a_o\right)^{\frac{1}{\theta}} + \left(a_o\right)^{\frac{1}{\theta}}\right)^{\frac{\theta}{\theta-1}(\beta-1)} & \text{if } K^R = L_o \\ A_o\left(1-a_o\right)^{\frac{1}{\theta-1}(\beta-1)} & \text{if } K^R = 0 \end{cases}.$$

Then I have

$$Q = \left[\sum_o \left(A_o^D\left(K_o\right)\right)^{\frac{1}{\beta}}\left(L_o\right)^{\frac{\beta-1}{\beta}}\right]^{\frac{\beta}{\beta-1}}.$$

To normalize, define

$$\widetilde{A_o^D} \equiv \left(\sum_o A_o^D\left(K_o\right)\right)^{\frac{1}{\beta-1}}$$

and

$$a_o^D\left(K_o^R\right) \equiv \frac{A_o^D\left(K_o\right)}{\sum_{o'} A_{o'}^D\left(K_{o'}\right)}.$$

Then I have

$$Q = \widetilde{A_o^D}\left[\sum_o \left(a_o^D\left(K_o\right)\right)^{\frac{1}{\beta}}\left(L_o\right)^{\frac{\beta-1}{\beta}}\right]^{\frac{\beta}{\beta-1}}. \tag{G.7}$$

Finally, let

$$A_{o,0} \equiv \left[\exp\left(\varphi_H^D + \varphi_o^D\right)\right]^{\frac{\theta_0-1}{\beta-1}}$$

and

$$A_{o,1} \equiv \left[\left(\exp\left(\varphi_H^D + \varphi_o^D + \gamma_H^D + \gamma_o^D \right) \right)^{\frac{1}{\theta_o} \frac{\theta_O - 1}{\beta - 1}} - \left(\exp\left(\varphi_H^D + \varphi_o^D \right) \right)^{\frac{1}{\theta_o} \frac{\theta_O - 1}{\beta - 1}} \right]^{\theta_o}.$$

and also let A_o and a_o satisfy

$$A_o = \left(A_{o,0} + A_{o,1} \right)^{\frac{\beta - 1}{\theta_O - 1}} \tag{G.8}$$

and

$$a_o = \frac{A_{o,1}}{A_{o,0} + A_{o,1}}. \tag{G.9}$$

Then one can substitute equations (G.8) and (G.9) to equation (G.7) and confirm that $Q = Q^D$. Summarizing the discussion above, I have the result that my model can be restricted to produce the production side of the model of Humlum (2019) as follows.

Lemma G.2. *Suppose that (i) producers follow the binary decision rule in Definition G.2 and that (ii) occupation productivity A_o and robot task space a_o satisfy equations (G.8) and (G.9) for each o. Then $Q = Q^D$.*

G.4 Equilibrium Characterization

To characterize the producer problem, I show the static optimization conditions and then the dynamic ones. For simplicity, I focus on the case with $\vartheta = 1$, or Cobb-Douglas in the mix of occupation aggregates, intermediates, and non-robot capital. To solve for the static problem of labor, intermediate goods, and non-robot capital, consider the FOCs of equation (7)

$$p_{i,t}^G \alpha_{i,L} \frac{Y_{i,t}^G}{T_{i,t}^O} \left(b_{i,o,t} \frac{T_{i,t}^O}{T_{i,o,t}^O} \right)^{\frac{1}{\beta}} \left((1 - a_{o,t}) \frac{T_{i,o,t}^O}{L_{i,o,t}} \right)^{\frac{1}{\theta_o}} = w_{i,o,t}, \tag{G.10}$$

where $T_{i,t}^O$ is the aggregated occupations $T_{i,t}^O \equiv \left[\sum_o \left(T_{i,o,t}^O \right)^{(\beta-1)/\beta} \right]^{\beta/(\beta-1)}$,

$$p_{i,t}^G \alpha_{i,M} \frac{Y_{i,t}^G}{M_{i,t}} \left(\frac{M_{i,t}}{M_{li,t}} \right)^{\frac{1}{\varepsilon}} = p_{li,t}^G, \tag{G.11}$$

and

$$p_{i,t}^G \alpha_{i,K} \frac{Y_{i,t}^G}{K_{i,t}} = r_{i,t}, \tag{G.12}$$

where $\alpha_{i,K} \equiv 1 - \alpha_{i,L} - \alpha_{i,M}$. Note also that by the envelope theorem,

$$\frac{\partial \pi_{i,t}\left(\left\{K^R_{i,o,t}\right\}\right)}{\partial K^R_{i,o,t}} = p^G_{i,t}\frac{\partial Y_{i,t}}{\partial K^R_{i,o,t}} = p^G_{i,t}\left(\alpha_L \frac{Y^G_{i,t}}{T^O_{i,t}}\left(b_{i,o,t}\frac{T^O_{i,t}}{T^O_{i,o,t}}\right)^{\frac{1}{\beta}}\left(a_{o,t}\frac{T^O_{i,o,t}}{K^R_{i,o,t}}\right)^{\frac{1}{\theta}}\right). \tag{G.13}$$

Another static problem of producers is robot purchase. Define the "before-integration" robot aggregate $Q^{R,BI}_{i,o,t} \equiv \left[\sum_l \left(Q^R_{li,o,t}\right)^{\frac{\varepsilon^R-1}{\varepsilon^R}}\right]^{\frac{\varepsilon^R}{\varepsilon^R-1}}$ and the corresponding price index $P^{R,BI}_{i,o,t}$. By the first order condition with respect to $Q^R_{li,o,t}$ for equation (9), I have $p^R_{li,o,t}Q^R_{li,o,t} = \left(\frac{p^R_{li,o,t}}{P^{R,BI}_{i,o,t}}\right)^{1-\varepsilon^R} P^{R,BI}_{i,o,t}Q^{R,BI}_{i,o,t}$, and $P^{R,BI}_{i,o,t}Q^{R,BI}_{i,o,t} = \alpha P^R_{i,o,t}Q^R_{i,o,t}$. Thus $p^R_{li,o,t}Q^R_{li,o,t} = \alpha\left(\frac{p^R_{li,o,t}}{P^{R,BI}_{i,o,t}}\right)^{1-\varepsilon^R} P^R_{i,o,t}Q^R_{i,o,t}$. Hence

$$Q^R_{li,o,t} = \alpha\left(p^R_{li,o,t}\right)^{-\varepsilon^R}\left(P^{R,BI}_{i,o,t}\right)^{\varepsilon^R-1} P^R_{i,o,t}Q^R_{i,o,t}.$$

Writing $P^R_{i,o,t} = \left(P^{R,BI}_{i,o,t}\right)^{\alpha^R}\left(P_{i,t}\right)^{1-\alpha^R}$, I have

$$Q^R_{li,o,t} = \alpha\left(\frac{p^R_{li,o,t}}{P^{R,BI}_{i,o,t}}\right)^{-\varepsilon^R}\left(\frac{P^{R,BI}_{i,o,t}}{P_{i,t}}\right)^{-\left(1-\alpha^R\right)} Q^R_{i,o,t}.$$

Alternatively, one can define the robot price index by $\widetilde{P}^R_{i,o,t} = \alpha^{\frac{1}{\varepsilon^R}}\left(P^{R,BI}_{i,o,t}\right)^{\frac{\varepsilon^R-\left(1-\alpha^R\right)}{\varepsilon^R}} P_{i,t}^{\frac{1-\alpha^R}{\varepsilon^R}}$ and show

$$Q^R_{li,o,t} = \left(\frac{p^R_{li,o,t}}{\widetilde{P}^R_{i,o,t}}\right)^{-\varepsilon^R} Q^R_{i,o,t}, \tag{G.14}$$

which is a standard gravity representation of robot trade.

To solve the dynamic problem, set up the (current-value) Lagrangian function for non-robot goods producers

$$\begin{aligned}\mathcal{L}_{i,t} = \sum_{t=0}^{\infty}\Bigg\{\left(\frac{1}{1+\iota}\right)^t\Bigg[\pi_{i,t}\left(\left\{K^R_{i,o,t}\right\}_o\right) - \sum_{l,o}\left(p^R_{li,o,t}\left(1+u_{li,t}\right)Q^R_{li,o,t} + P^G_{i,t}I^{int}_{i,o,t} + \gamma P^R_{i,o,t}Q^R_{i,o,t}\frac{Q^R_{i,o,t}}{K^R_{i,o,t}}\right)\Bigg]\Bigg\}.\\ -\lambda^R_{i,o,t}\left\{K^R_{i,o,t+1} - (1-\delta)K^R_{i,o,t} - Q^R_{i,o,t}\right\}\end{aligned}$$

Taking the FOC with respect to the hardware from country l, $Q^R_{li,o,t}$, I have

$$p^R_{li,o,t}\left(1+u_{li,t}\right)+2\gamma P^R_{i,o,t}\left(\frac{Q^R_{i,o,t}}{K^R_{i,o,t}}\right)\frac{\partial Q^R_{i,o,t}}{\partial Q^R_{li,o,t}}=\lambda^R_{i,o,t}\frac{\partial Q^R_{i,o,t}}{\partial Q^R_{li,o,t}}. \tag{G.15}$$

Taking the FOC with respect to the integration input $I^{int}_{i,o,t}$, I have

$$P^G_{i,t}+2\gamma P^R_{i,o,t}\left(\frac{Q^R_{i,o,t}}{K^R_{i,o,t}}\right)\frac{\partial Q^R_{i,o,t}}{\partial I^{int}_{i,o,t}}=\lambda^R_{i,o,t}\frac{\partial Q^R_{i,o,t}}{\partial I^{int}_{i,o,t}}, \tag{G.16}$$

Taking the FOC with respect to $K^R_{i,o,t+1}$, I have

$$\left(\frac{1}{1+\iota}\right)^{t+1}\left[\frac{\partial \pi_{i,t+1}\left(\left\{K^R_{i,o,t+1}\right\}_o\right)}{\partial K^R_{i,o,t+1}}+\gamma P^R_{i,o,t+1}\left(\frac{Q^R_{i,o,t+1}}{K^R_{i,o,t+1}}\right)^2+\left(1-\delta\right)\lambda^R_{i,o,t+1}\right]-\left(\frac{1}{1+\iota}\right)^t\lambda^R_{i,o,t}=0, \tag{G.17}$$

and the transversality condition: for any j and o,

$$\lim_{t\to\infty}e^{-\iota t}\lambda^R_{j,o,t}K^R_{j,o,t+1}=0. \tag{G.18}$$

Rearranging equation (G.17), I obtain the following Euler equation.

$$\lambda^R_{i,o,t}=\frac{1}{1+\iota}\left[\left(1-\delta\right)\lambda^R_{i,o,t+1}+\frac{\partial}{\partial K^R_{i,o,t+1}}\pi_{i,t+1}\left(\left\{K^R_{i,o,t+1}\right\}\right)+\gamma p^R_{i,o,t+1}\left(\frac{Q^R_{i,o,t+1}}{K^R_{i,o,t+1}}\right)^2\right]. \tag{G.19}$$

Turning to the demand for non-robot good, I will characterize bilateral intermediate good trade demand and total expenditure. Write $X^G_{j,t}$ the total purchase quantity (but not value) of good G in country j in period t. By equation (B.1), the bilateral trade demand is given by

$$p^G_{ij,t}Q^G_{ij,t}=\left(\frac{p^G_{ij,t}}{P^G_{j,t}}\right)^{1-\varepsilon}P^G_{j,t}X^G_{j,t}, \tag{G.20}$$

for any i, j, and t. In this equation, $P^G_{j,t}X^G_{j,t}$ is the total expenditures on non-robot goods. The total expenditure is the sum of final consumption $I_{j,t}$, payment to intermediate goods $\alpha_M p^G_{j,t}Y^G_{j,t}$, input to robot productions $\sum_o P^G_{j,t}I^R_{j,o,t}=\sum_{o,k}p^R_{jk,o,t}Q^R_{jk,o,t}$, and payment to robot integration $\sum_o P^G_{j,t}I^{int}_{j,o,t}=$

$\left(1-\alpha^{R}\right)\sum_{o}P_{j,o,t}^{R}Q_{j,o,t}^{R}$. Hence

$$P_{j,t}^{G}X_{j,t}^{G}=I_{j,t}+\alpha_{M}p_{j,t}^{G}Y_{j,t}^{G}+\sum_{o,k}p_{jk,o,t}^{R}Q_{jk,o,t}^{R}+\left(1-\alpha^{R}\right)\sum_{o}P_{j,o,t}^{R}Q_{j,o,t}^{R}.$$

For country j and period t, by substituting into income $I_{j,t}$ the period cash flow of non-robot good producer that satisfies

$$\Pi_{j,t}\equiv\pi_{j,t}\left(\left\{K_{j,o,t}^{R}\right\}_{o}\right)-\sum_{i,o}\left(p_{ij,o,t}^{R}\left(1+u_{ij,t}\right)Q_{ij,o,t}^{R}+\sum_{o}P_{j,t}^{G}I_{j,o,t}^{int}+\gamma P_{j,o,t}^{R}Q_{j,o,t}^{R}\left(\frac{Q_{j,o,t}^{R}}{K_{j,o,t}^{R}}\right)\right)$$

and robot tax revenue $T_{j,t}=\sum_{i,o}u_{ij,t}p_{ij,o,t}^{R}Q_{ij,o,t}^{R}$, I have

$$I_{j,t}=\left(1-\alpha_{M}\right)\sum_{k}p_{jk,t}^{G}Q_{jk,t}^{G}-\left(\sum_{i,o}p_{ij,o,t}^{R}Q_{ij,o,t}^{R}+\left(1-\alpha^{R}\right)\sum_{o}P_{j,o,t}^{R}Q_{j,o,t}^{R}\right),\tag{G.21}$$

or in terms of variables in the definition of equilibrium,

$$I_{j,t}=\left(1-\alpha_{M}\right)\sum_{k}p_{jk,t}^{G}Q_{jk,t}^{G}-\frac{1}{\alpha^{R}}\sum_{i,o}p_{ij,o,t}^{R}Q_{ij,o,t}^{R}.$$

Hence, the total expenditure measured in terms of the production side as opposed to income side is

$$P_{j,t}^{G}X_{j,t}^{G}=\sum_{k}p_{jk,t}^{G}Q_{jk,t}^{G}-\sum_{i,o}p_{ij,o,t}^{R}Q_{ij,o,t}^{R}\left(1+\gamma\frac{Q_{ij,o,t}^{R}}{K_{j,o,t}^{R}}\right).\tag{G.22}$$

Note that this equation embeds the balanced-trade condition. By substituting equation (G.22) into equation (G.20), I have

$$p_{ij,t}^{G}Q_{ij,t}^{G}=\left(\frac{p_{ij,t}^{G}}{P_{j,t}^{G}}\right)^{1-\varepsilon^{G}}\left(\sum_{k}p_{jk,t}^{G}Q_{jk,t}^{G}+\sum_{k,o}p_{jk,o,t}^{R}Q_{jk,o,t}^{R}-\sum_{i,o}p_{ij,o,t}^{R}Q_{ij,o,t}^{R}\right).\tag{G.23}$$

The good and robot-o market-clearing conditions are given by,

$$Y_{i,t}^{R}=\sum_{j}Q_{ij,t}^{G}\tau_{ij,t}^{G},\tag{G.24}$$

for all i and t, and

$$p^R_{i,o,t} = \frac{P^G_{i,t}}{A^R_{i,o,t}} \tag{G.25}$$

for all i, o, and t, respectively.

Conditional on state variables $\boldsymbol{S}_t = \left\{\boldsymbol{K}^R_t, \boldsymbol{\lambda}^R_t, \boldsymbol{L}_t, \boldsymbol{V}_t\right\}$, equations (B.2), (G.10), (G.15), (G.23), (G.24), and (G.25) characterize the temporary equilibrium $\left\{\boldsymbol{p}^G_t, \boldsymbol{p}^R_t, \boldsymbol{w}_t, \boldsymbol{Q}^G_t, \boldsymbol{Q}^R_t, \boldsymbol{L}_t\right\}$. In addition, conditional on initial conditions $\left\{\boldsymbol{K}^R_0, \boldsymbol{L}_0\right\}$, equations (8), (G.19), and (G.18) characterize the sequential equilibrium.

Finally, the steady state conditions are given by imposing the time-invariance condition to equations (8) and (G.19):

$$Q^R_{i,o} = \delta K^R_{i,o}, \tag{G.26}$$

$$\frac{\partial}{\partial K^R_{i,o}} \pi_i \left(\left\{K^R_{i,o}\right\}\right) = (\iota + \delta)\, \lambda^R_{i,o} - \sum_l \gamma p^R_{li,o} \left(\frac{Q^R_{li,o}}{K^R_{i,o}}\right)^2 \equiv c^R_{i,o}. \tag{G.27}$$

Note that equation (G.27) can be interpreted as the flow marginal profit of capital must be equalized to the marginal cost term. Thus I define the steady state marginal cost of robot capital $c^R_{i,o}$ from the right-hand side of equation (G.27). Note that if there is no adjustment cost $\gamma = 0$, the steady state Euler equation (G.27) implies

$$\frac{\partial}{\partial K^R_{i,o}} \pi_i \left(\left\{K^R_{i,o}\right\}\right) = c^R_{i,o} = (\iota + \delta)\, \lambda^R_{i,o},$$

which states that the marginal profit of capital is the user cost of robots in the steady state (Hall and Jorgenson 1967).

G.5 On the Choice of the Steady-State Matrix in Equation (22)

In equation (22), I use the steady-state matrix $\overline{\boldsymbol{E}}$ instead of the transitional dynamics matrix $\overline{\boldsymbol{F}_t}$ for a computational reason. Since I have annual observation for occupational robot costs, it is potentially possible to leverage this rich variation for the structural estimation, which may permit me to estimate the EoS θ_o at a narrower occupation group level. However, the bottleneck of this approach is the computational burden to compute the dynamic solution matrix $\overline{\boldsymbol{F}_t}$. Specifically, dynamic substitution matrix $\overline{\boldsymbol{F}^y_{t+1}}$ in equation (15) is based on the conditions of Blanchard and

Kahn (1980). This requires computing the eigenspace, as described in detail in Section I. This is computationally hard since I cannot rely on the sparse structure of the matrix $\overline{F^y_{t+1}}$. In contrast, the estimation method in Proposition C.1 does not involve such computation, but only requires computing the steady-state solution matrix $\overline{E}$. Then I only need to invert steady-state substitution matrix $\overline{E^y}$, which is feasible given the sparse structure of $\overline{E^y}$.

G.6 Proof of Proposition 1

The proof takes the following four conceptual steps. First, I will write the real wage change $(\widehat{w_{i,o}/P^G_i})$ in terms of the weighted average of relative price changes, making use of the fact that the sum of shares equals one. Second, I rewrite relative price change into layers of relative price changes with the technique of addition and subtraction. Third, I show that each layer of relative price changes is a change of relevant input or trade shares controlled by elasticity substitution. In other words, an input or trade shares reveals a layer of relative price changes. Finally, I make use of the fact that the sum of shares do not change after the shock to arrive at equation (19).

Cost minimization given production functions (4), (5), and (B.1) imply

$$\widehat{\left(\frac{w_{i,o}}{P^G_i}\right)} = \frac{1}{1-\alpha_{i,M}} \sum_l \widetilde{x}^G_{li,t_0} \sum_{o'} \widetilde{x}^O_{l,o',t_0} \left[\widetilde{x}^L_{l,o',t_0} \left(\widehat{w_{i,o}} - \widehat{w_{l,o'}}\right) + \left(1-\widetilde{x}^L_{l,o',t_0}\right) \left(\widehat{w_{i,o}} - \left(\frac{\widehat{A^K_{l,o'}}}{1-\theta_o} + \widehat{c^R_{l,o'}}\right)\right)\right]. \tag{G.28}$$

Note that by additions and subtractions, I can rewrite

$$\begin{aligned} \widehat{w_{i,o}} - \widehat{w_{l,o'}} = {} & \left(\widehat{w_{i,o}} - \widehat{P^O_{i,o}}\right) - \left(\widehat{w_{l,o'}} - \widehat{P^O_{l,o'}}\right) + \left(\widehat{P^O_{i,o}} - \widehat{P^O_i}\right) - \left(\widehat{P^O_{l,o'}} - \widehat{P^O_l}\right) \\ & + \left(\widehat{P^O_i} - \widehat{p^G_i}\right) - \left(\widehat{P^O_l} - \widehat{p^G_l}\right) + \left(\widehat{p^G_i} - \widehat{P^G_i}\right) - \left(\widehat{p^G_l} - \widehat{P^G_l}\right), \end{aligned} \tag{G.29}$$

where $\widehat{P^O_{i,o}}$, $\widehat{P^O_i}$, and $\widehat{P^G_i}$ are the price (cost) index of occupation o, occupation aggregate $T^O_{i,t} \equiv \left[\sum_o \left(T^O_{i,o,t}\right)^{(\beta-1)/\beta}\right]^{\beta/(\beta-1)}$, and consumption of non-rogot good G, and

$$\begin{aligned} \widehat{w_{i,o}} - \left(\frac{\widehat{A^K_{l,o'}}}{1-\theta} + \widehat{c^R_{l,o'}}\right) = {} & \left(\widehat{w_{i,o}} - \widehat{P^O_{i,o}}\right) - \left(\frac{\widehat{A^K_{l,o'}}}{1-\theta} + \widehat{c^R_{l,o'}} - \widehat{P^O_{l,o'}}\right) + \left(\widehat{P^O_{i,o}} - \widehat{P^O_i}\right) - \left(\widehat{P^O_{l,o'}} - \widehat{P^O_l}\right) \\ & + \left(\widehat{P^O_i} - \widehat{p^G_i}\right) - \left(\widehat{P^O_l} - \widehat{p^G_l}\right) + \left(\widehat{p^G_i} - \widehat{P^G_i}\right) - \left(\widehat{p^G_l} - \widehat{P^G_l}\right). \end{aligned} \tag{G.30}$$

Note that the cost minimizing input and trade shares satisfy

$$\begin{cases} \widehat{\tilde{x}_{i,o}^{L}} = (1-\theta_o)\left(\widehat{w_{i,o}} - \widehat{P_{i,o}^{O}}\right), \; \widehat{1-\tilde{x}_{i,o}^{L}} = \widehat{A_{i,o}^{K}} + (1-\theta_o)\left(\widehat{c_{i,o}^{R}} - \widehat{P_{i,o}^{O}}\right) \\ \widehat{\tilde{x}_{i,o}^{O}} = (1-\beta)\left(\widehat{P_{i,o}^{O}} - \widehat{P_{i}^{O}}\right), \; \widehat{\tilde{x}_{i}^{T}} = (1-\vartheta)\left(\widehat{P_{i}^{O}} - \widehat{p_{i}^{G}}\right), \; \widehat{\tilde{x}_{li}^{G}} = (1-\varepsilon)\left(\widehat{p_{l}^{G}} - \widehat{P_{i}^{G}}\right) \end{cases} \tag{G.31}$$

Combined with the Cobb-Douglas assumption of production function (4), equations (G.29), (G.30), and (G.31) imply

$$\widehat{w_{i,o}} - \widehat{w_{l,o'}} = \frac{\widehat{\tilde{x}_{i,o}^{L}}}{1-\theta_o} - \frac{\widehat{\tilde{x}_{l,o'}^{L}}}{1-\theta_o} + \frac{\widehat{\tilde{x}_{i,o}^{O}}}{1-\beta} - \frac{\widehat{\tilde{x}_{l,o'}^{O}}}{1-\beta} + \frac{\widehat{\tilde{x}_{i}^{T}}}{1-\vartheta} - \frac{\widehat{\tilde{x}_{l}^{T}}}{1-\vartheta} + \frac{\widehat{\tilde{x}_{ii}^{G}}}{1-\varepsilon} - \frac{\widehat{\tilde{x}_{li}^{G}}}{1-\varepsilon}$$

$$\widehat{w_{i,o}} - \left(\frac{\widehat{A_{l,o'}^{K}}}{1-\theta_o} + \widehat{c_{l,o'}^{R}}\right) = \frac{\widehat{\tilde{x}_{i,o}^{L}}}{1-\theta_o} - \frac{\widehat{\left(1-\tilde{x}_{l,o'}^{L}\right)}}{1-\theta_o} + \frac{\widehat{\tilde{x}_{i,o}^{O}}}{1-\beta} - \frac{\widehat{\tilde{x}_{l,o'}^{O}}}{1-\beta} + \frac{\widehat{\tilde{x}_{i}^{T}}}{1-\vartheta} - \frac{\widehat{\tilde{x}_{l}^{T}}}{1-\vartheta} + \frac{\widehat{\tilde{x}_{ii}^{G}}}{1-\varepsilon} - \frac{\widehat{\tilde{x}_{li}^{G}}}{1-\varepsilon}.$$

Substituting these in equation (G.28) and using the facts that $\tilde{x}_{i,o,t_0}^{L}\widehat{\tilde{x}_{i,o}^{L}} + \left(1-\tilde{x}_{i,o,t_0}^{L}\right)\left(\widehat{1-\tilde{x}_{i,o}^{L}}\right) = 0$ for all i and o, $\sum_o \tilde{x}_{i,o,t_0}^{O}\widehat{\tilde{x}_{i,o}^{O}} = 0$, and $\sum_l \tilde{x}_{li,t_0}^{G}\widehat{\tilde{x}_{li}^{G}} = 0$ for all i, I have equation (19).

G.7 Details of the Two-step Estimator

I use notation $d \equiv \dim(\mathbf{\Theta})$ to denote the dimension of parameters. Assumption 1 implies that, for any d-dimensional functions $\boldsymbol{H} \equiv \{H_o\}_o$, $\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^{J}\right)\nu_o\right] = 0$. The GMM estimator based on $\boldsymbol{H}$ is

$$\mathbf{\Theta}_{\boldsymbol{H}} \equiv \arg\min_{\mathbf{\Theta}} \sum_{o=1}^{O}\left[H_o\left(\boldsymbol{\psi}_{t_1}^{J}\right)\nu_o(\mathbf{\Theta})\right]^{\top}\left[H_o\left(\boldsymbol{\psi}_{t_1}^{J}\right)\nu_o(\mathbf{\Theta})\right], \tag{G.32}$$

which is consistent under the moment condition (23) if $\boldsymbol{H}$ satisfies the rank conditions in Newey and McFadden (1994). The exact specification of $\boldsymbol{H}$ determines the optimality, or the minimal variance, of estimator (G.32). To specify $\boldsymbol{H}$, I apply the approach that achieves the asymptotic optimality developed in Chamberlain (1987). Formally, define the instrumental variable Z_o as follows:

$$Z_o \equiv H_o^{*}\left(\boldsymbol{\psi}_{t_1}^{J}\right) \equiv \mathbb{E}\left[\nabla_{\mathbf{\Theta}}\nu_o(\mathbf{\Theta})\,|\boldsymbol{\psi}_{t_1}^{J}\right]\mathbb{E}\left[\nu_o(\mathbf{\Theta})\left(\nu_o(\mathbf{\Theta})\right)^{\top}|\boldsymbol{\psi}_{t_1}^{J}\right]^{-1}, \tag{G.33}$$

and assume the regularity conditions (Assumption G.1) in Appendix G.7.1.

Proposition G.1. *Under Assumptions 1 and G.1, $\mathbf{\Theta}_{\boldsymbol{H}^*}$ is asymptotically normal with the minimum variance among the asymptotic variances of the class of estimators in equation* (G.32).

Proof. See Appendix G.7.1. □

To understand the optimality of the IV in equation (G.33), note that it has two components. The first term is the conditional expected gradient vector $\mathbb{E}\left[\nabla_{\mathbf{\Theta}} \nu_o\left(\mathbf{\Theta}\right) | \boldsymbol{\psi}_{t_1}^J\right]$, which takes the gradient with respect to the structural parameter vector. Thus, it assigns large weight to occupation that changes the the predicted outcome variable sensitively to the parameters. The second term is the conditional inverse expected variance matrix $\mathbb{E}\left[\nu_o\left(\mathbf{\Theta}\right)\left(\nu_o\left(\mathbf{\Theta}\right)\right)^\top | \boldsymbol{\psi}_{t_1}^J\right]^{-1}$, which put large weight to occupation that has small variance of the structural residuals.

Substituting equation (G.33) to the general GMM estimator (G.32), I have an estimator $\mathbf{\Theta}_{\boldsymbol{H}^*} = \arg\min_{\mathbf{\Theta}}\left[\sum_o Z_o \nu_o\left(\mathbf{\Theta}\right)\right]^\top\left[\sum_o Z_o \nu_o\left(\mathbf{\Theta}\right)\right]$. Since Z_o depends on unknown parameters $\mathbf{\Theta}$, I implement the estimation by the two-step feasible method, or the model-implied optimal IV (Adao, Arkolakis, and Esposito 2019). I first estimate the first-step estimate $\mathbf{\Theta}_1$ from arbitrary initial values $\mathbf{\Theta}_0$. Since the IV is a function of the Japan robot shock $\boldsymbol{\psi}_{t_1}^J$, $\mathbf{\Theta}_1$ is consistent by Assumption 1. However, it is not optimal. To achieve the optimality, in the second step, I obtain the optimal IV using the consistent estimator $\mathbf{\Theta}_1$. These arguments lead to the IV definition given in equation C.5 and Proposition (C.1).

G.7.1 Proof of Proposition G.1

To prove Proposition I follow the arguments made in Sections 2 and 3 of Newey and McFadden (1994). The proof consists of four sub results in the following Lemma. Proposition G.1 can be obtained as a combination of the four results. The formal statement requires the following additional assumptions.

Assumption G.1. *(i) A function of $\widetilde{\mathbf{\Theta}}$, $\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nu_o\left(\widetilde{\mathbf{\Theta}}\right)\right] \neq 0$ for any $\widetilde{\mathbf{\Theta}} \neq \mathbf{\Theta}$. (ii) $\underline{\theta} \leq \theta_o \leq \overline{\theta}$ for any o, $\underline{\beta} \leq \beta \leq \overline{\beta}$, $\underline{\gamma} \leq \gamma \leq \overline{\gamma}$, and $\underline{\phi} \leq \phi \leq \overline{\phi}$ for some positive values $\underline{\theta}, \underline{\beta}, \underline{\gamma}, \underline{\phi}, \overline{\theta}, \overline{\beta}, \overline{\gamma}, \overline{\phi}$. (iii) $\mathbb{E}\left[\sup_{\mathbf{\Theta}} \| H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nu_o\left(\widetilde{\mathbf{\Theta}}\right) \|\right] < \infty$. (iv) $\mathbb{E}\left[\| H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nu_o\left(\widetilde{\mathbf{\Theta}}\right) \|^2\right] < \infty$ (v) $\mathbb{E}\left[\sup_{\mathbf{\Theta}} \| H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nabla_{\widetilde{\mathbf{\Theta}}}\nu_o\left(\widetilde{\mathbf{\Theta}}\right) \|\right] < \infty$.*

Lemma G.3. *Assume Assumptions 1 and G.1(i)-(iii).*

(a) The estimator of the form (G.32) *is consistent.*

Additionally, assume Assumptions G.1(iv)-(v).

(b) The estimator of the form (G.32) is asymptotically normal.

(c) $\sqrt{O}\left(\boldsymbol{\Theta}_{\boldsymbol{H}^*}-\boldsymbol{\Theta}\right)\to_d \mathcal{N}\left(0,\left(\boldsymbol{G}^\top\boldsymbol{\Omega}^{-1}\boldsymbol{G}\right)^{-1}\right)$*, and the asymptotic variance is the minimum of that of the estimator of the form (G.32) for any function* $\boldsymbol{H}$*.*

Proof. (a) I follow Theorems 2.6 of Newey and McFadden (1994), which implies that it suffices to show conditions (i)-(iv) of this theorem are satisfied. Assumption G.1(i) guarantees condition (i). Condition (ii) is implied by Assumption G.1(ii). Condition (iii) follows because all supply and demand functions in the model is continuous. Condition (iv) is implied by Assumption G.1(iii).

(b) I follow Theorem 3.4 of Newey and McFadden (1994), which implies that it suffices to show conditions (i)-(v) of this theorem are satisfied. Condition (i) is satisfied by Assumption G.1(i). Condition (ii) follows because all supply and demand functions in the model is continuously differentiable. Condition (iii) is implied by Assumption 1 and Assumption G.1(iv). Assumption G.1(v) implies condition (iv). Finally, the gradient vectors of the structural residual is linear independent, guaranteeing the non-singularity of the variance matrix and condition (v).

(c) By Theorem 3.4 of Newey and McFadden (1994), for an arbitrary IV-generating function $\boldsymbol{H}$, the asymptotic variance of the GMM estimator $\boldsymbol{\Theta}_{\boldsymbol{H}}$ is

$$\left(\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right]\right)^{-1}\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nu_o\nu_o^\top\left(H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\right)^\top\right]\left(\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right]\right)^{-1},$$

where $G_o \equiv \mathbb{E}\left[\nabla_{\boldsymbol{\Theta}}\nu_o\left(\boldsymbol{\Theta}\right)|\boldsymbol{\psi}_{t_1}^J\right]$. Therefore, if $H_o\left(\boldsymbol{\psi}_{t_1}^J\right) = Z_o \equiv \mathbb{E}\left[\nabla_{\boldsymbol{\Theta}}\nu_o\left(\boldsymbol{\Theta}\right)|\boldsymbol{\psi}_{t_1}^J\right]\mathbb{E}\left[\nu_o\left(\boldsymbol{\Theta}\right)\left(\nu_o\left(\boldsymbol{\Theta}\right)\right)^\top|\boldsymbol{\psi}_{t_1}^J\right]^{-1}$, then this expression is equal to $\left(\boldsymbol{G}^\top\boldsymbol{\Omega}^{-1}\boldsymbol{G}\right)^{-1}$, where

$$\boldsymbol{G}\equiv\mathbb{E}\left[\nabla_{\boldsymbol{\Theta}}\nu_o\left(\boldsymbol{\Theta}\right)\right] \text{ and } \boldsymbol{\Omega}\equiv\mathbb{E}\left[\nu_o\left(\boldsymbol{\Theta}\right)\left(\nu_o\left(\boldsymbol{\Theta}\right)\right)^\top\right].$$

To show that this variance is minimal, I will check that

$$\begin{aligned}\boldsymbol{\Delta}\equiv&\left(\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right]\right)^{-1}\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nu_o\nu_o^\top\left(H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\right)^\top\right]\left(\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right]\right)^{-1}\\&-\left(\boldsymbol{G}^\top\boldsymbol{\Omega}^{-1}\boldsymbol{G}\right)^{-1}\end{aligned}$$

is positive semi-definite. In fact, note that

$$\begin{aligned}\boldsymbol{\Delta} = &\left(\mathbb{E}\left[H_o\left(\boldsymbol{\psi}^J_{t_1}\right)G_o\right]\right)^{-1}\times\\&\left\{\mathbb{E}\left[H_o\left(\boldsymbol{\psi}^J_{t_1}\right)\nu_o\nu_o^\top\left(H_o\left(\boldsymbol{\psi}^J_{t_1}\right)\right)^\top\right]-\mathbb{E}\left[H_o\left(\boldsymbol{\psi}^J_{t_1}\right)G_o\right]\left(\boldsymbol{G}^\top\boldsymbol{\Omega}^{-1}\boldsymbol{G}\right)^{-1}\mathbb{E}\left[H_o\left(\boldsymbol{\psi}^J_{t_1}\right)G_o\right]\right\}\times\\&\left(\mathbb{E}\left[H_o\left(\boldsymbol{\psi}^J_{t_1}\right)G_o\right]\right)^{-1}.\end{aligned}$$

Define

$$\widetilde{\nu}_o = H_o\left(\boldsymbol{\psi}^J_{t_1}\right)\nu_o - \mathbb{E}\left[H_o\left(\boldsymbol{\psi}^J_{t_1}\right)\nu_o\left(\left(G_o\right)^\top\Omega_o^{-1}\nu_o\right)^{-1}\right]\mathbb{E}\left(\left(G_o\right)^\top\Omega_o^{-1}\nu_o\right)^{-1}\left(G_o\right)^\top\Omega_o^{-1}\nu_o,$$

where $\Omega_o \equiv \mathbb{E}\left[\nu_o\left(\boldsymbol{\Theta}\right)\left(\nu_o\left(\boldsymbol{\Theta}\right)\right)^\top|\boldsymbol{\psi}^J_{t_1}\right]$. Applying Theorem 5.3 of Newey and McFadden (1994), I have

$$\mathbb{E}\left[\widetilde{\nu}_o\left(\widetilde{\nu}_o\right)^\top\right] = \mathbb{E}\left[H_o\left(\boldsymbol{\psi}^J_{t_1}\right)\nu_o\nu_o^\top\left(H_o\left(\boldsymbol{\psi}^J_{t_1}\right)\right)^\top\right]-\mathbb{E}\left[H_o\left(\boldsymbol{\psi}^J_{t_1}\right)G_o\right]\left(\boldsymbol{G}^\top\boldsymbol{\Omega}^{-1}\boldsymbol{G}\right)^{-1}\mathbb{E}\left[H_o\left(\boldsymbol{\psi}^J_{t_1}\right)G_o\right].$$

Since $\mathbb{E}\left[\widetilde{\nu}_o\left(\widetilde{\nu}_o\right)^\top\right]$ is positive semi-definite, so is $\boldsymbol{\Delta}$, which completes the proof. □

G.7.2 Proof of Proposition C.1

I apply arguments in Section 6.1 of Newey and McFadden (1994). Namely, I define the joint estimator of the first-step and second-step estimator in Proposition C.1 that falls into the class of general GMM estimation, and discuss the asymptotic property using the general result about GMM estimation. In the proof, I modify the notation of the set of functions that yield optimal IV, $\boldsymbol{H}^*$, to clarify that it depends on parameters $\boldsymbol{\Theta}$ as follows:

$$H_o^*\left(\boldsymbol{\psi}^J_{t_1};\boldsymbol{\Theta}\right) = \mathbb{E}\left[\nabla_{\boldsymbol{\Theta}}\nu_o\left(\boldsymbol{\Theta}\right)|\boldsymbol{\psi}^J_{t_1}\right]\mathbb{E}\left[\nu_o\left(\boldsymbol{\Theta}\right)\left(\nu_o\left(\boldsymbol{\Theta}\right)\right)^\top|\boldsymbol{\psi}^J_{t_1}\right]^{-1}.$$

Define the joint estimator as follows:

$$\begin{pmatrix}\boldsymbol{\Theta}_2\\\boldsymbol{\Theta}_1\end{pmatrix} \equiv \arg\min_{\boldsymbol{\Theta}_2,\boldsymbol{\Theta}_1}\left[\sum_o e_o\left(\boldsymbol{\Theta}_2,\boldsymbol{\Theta}_1\right)\right]^\top\left[\sum_o e_o\left(\boldsymbol{\Theta}_2,\boldsymbol{\Theta}_1\right)\right],$$

where

$$e_o\left(\mathbf{\Theta}_2, \mathbf{\Theta}_1\right) \equiv \left(\begin{array}{c} H_o^*\left(\boldsymbol{\psi}_{t_1}^J; \mathbf{\Theta}_1\right) \nu_o\left(\mathbf{\Theta}_2\right) \\ H_o^*\left(\boldsymbol{\psi}_{t_1}^J; \mathbf{\Theta}_0\right) \nu_o\left(\mathbf{\Theta}_1\right) \end{array}\right).$$

Since for any $\mathbf{\Theta}$, IV-generating function $H_o^*\left(\boldsymbol{\psi}_{t_1}^J; \mathbf{\Theta}_0\right)$ gives the consistent estimator for $\mathbf{\Theta}$, I have $\mathbf{\Theta}_1 \to \mathbf{\Theta}$ and $\mathbf{\Theta}_2 \to \mathbf{\Theta}$. I also have the asymptotic variance

$$\operatorname{Var}\left(\begin{array}{c} \mathbf{\Theta}_2 \\ \mathbf{\Theta}_1 \end{array}\right) = \left[\left(\widetilde{\boldsymbol{G}}\right)^\top \widetilde{\mathbf{\Omega}} \widetilde{\boldsymbol{G}}\right]^{-1},$$

where

$$\begin{aligned} \widetilde{\boldsymbol{G}} &\equiv \mathbb{E}\left[\nabla_{\left(\mathbf{\Theta}_2, \mathbf{\Theta}_1\right)^\top} e_o\left(\mathbf{\Theta}_2, \mathbf{\Theta}_1\right)\right] \\ &= \mathbb{E}\left[\begin{array}{cc} H_o^*\left(\boldsymbol{\psi}_{t_1}^J; \mathbf{\Theta}_1\right) \nabla \nu_o\left(\mathbf{\Theta}_2\right) & \nabla H_o^*\left(\boldsymbol{\psi}_{t_1}^J; \mathbf{\Theta}_1\right) \nu_o\left(\mathbf{\Theta}_2\right) \\ \mathbf{0} & H_o^*\left(\boldsymbol{\psi}_{t_1}^J; \mathbf{\Theta}_0\right) \nabla \nu_o\left(\mathbf{\Theta}_1\right) \end{array}\right] \end{aligned}$$

and

$$\begin{aligned} \widetilde{\mathbf{\Omega}} &\equiv \mathbb{E}\left[e_o\left(\mathbf{\Theta}_2, \mathbf{\Theta}_1\right)\left[e_o\left(\mathbf{\Theta}_2, \mathbf{\Theta}_1\right)\right]^\top\right] \\ &= \mathbb{E}\left[\begin{array}{cc} H_o^*\left(\boldsymbol{\psi}_{t_1}^J; \mathbf{\Theta}_1\right) \nu_o\left(\mathbf{\Theta}_2\right)\left[H_o^*\left(\boldsymbol{\psi}_{t_1}^J; \mathbf{\Theta}_1\right) \nu_o\left(\mathbf{\Theta}_2\right)\right]^\top & H_o^*\left(\boldsymbol{\psi}_{t_1}^J; \mathbf{\Theta}_1\right) \nu_o\left(\mathbf{\Theta}_2\right)\left[H_o^*\left(\boldsymbol{\psi}_{t_1}^J; \mathbf{\Theta}_0\right) \nu_o\left(\mathbf{\Theta}_1\right)\right]^\top \\ H_o^*\left(\boldsymbol{\psi}_{t_1}^J; \mathbf{\Theta}_0\right) \nu_o\left(\mathbf{\Theta}_1\right)\left[H_o^*\left(\boldsymbol{\psi}_{t_1}^J; \mathbf{\Theta}_1\right) \nu_o\left(\mathbf{\Theta}_2\right)\right]^\top & H_o^*\left(\boldsymbol{\psi}_{t_1}^J; \mathbf{\Theta}_0\right) \nu_o\left(\mathbf{\Theta}_1\right)\left[H_o^*\left(\boldsymbol{\psi}_{t_1}^J; \mathbf{\Theta}_0\right) \nu_o\left(\mathbf{\Theta}_1\right)\right]^\top \end{array}\right]. \end{aligned}$$

Note that Assumption 1 implies that any function of $\boldsymbol{\psi}_{t_1}^J$ is orthogonal to ν_o, implying $\mathbb{E}\left[\nabla H_o^*\left(\boldsymbol{\psi}_{t_1}^J; \mathbf{\Theta}_1\right) \nu_o\left(\mathbf{\Theta}_2\right)\right] = 0$. Therefore, $\widetilde{\boldsymbol{G}}$ is a block-diagonal matrix and thus the marginal asymptotic distribution of $\mathbf{\Theta}_2$ is normal with variance $\operatorname{Var}\left(\mathbf{\Theta}_2\right) = \left(\boldsymbol{G}^\top \mathbf{\Omega}^{-1} \boldsymbol{G}\right)^{-1}$, noting that $\boldsymbol{G} = \mathbb{E}\left[H_o^*\left(\boldsymbol{\psi}_{t_1}^J; \mathbf{\Theta}\right) \nabla \nu_o\left(\mathbf{\Theta}\right)\right]$ and $\mathbf{\Omega} \equiv \mathbb{E}\left[H_o^*\left(\boldsymbol{\psi}_{t_1}^J; \mathbf{\Theta}\right) \nu_o\left(\mathbf{\Theta}\right)\left(H_o^*\left(\boldsymbol{\psi}_{t_1}^J; \mathbf{\Theta}\right) \nu_o\left(\mathbf{\Theta}\right)\right)^\top\right]$. By Proposition G.1, this asymptotic variance is minimal among the GMM estimator (G.32).

H Further Estimation and Simulation Results

H.1 Robot Trade Elasticity

To estimate robot trade elasticity ε^R, I apply and extend the trilateral method of Caliendo and Parro (2015). Namely, decompose the robot trade cost $\tau^R_{li,t}$ into $\ln \tau^R_{li,t} = \ln \tau^{R,T}_{li,t} + \ln \tau^{R,D}_{li,t}$, where $\tau^{R,T}_{li,t}$ is tariff on robots taken from the UNCTAD-TRAINS database and $\tau^{R,D}_{li,t}$ is asymmetric non-tariff trade cost. The latter term is assumed to be $\ln \tau^{R,D}_{li,t} = \ln \tau^{R,D,S}_{li,t} + \ln \tau^{R,D,O}_{l,t} + \ln \tau^{R,D,D}_{i,t} + \ln \tau^{R,D,E}_{li,t}$, where $\tau^{R,D,S}_{li,t}$ captures symmetric bilateral trade costs such as distance, common border, language, and FTA belonging status and satisfies $\tau^{R,D,S}_{li,t} = \tau^{R,D,S}_{il,t}$, $\tau^{R,D,O}_{l,t}$ and $\tau^{R,D,D}_{i,t}$ are the origin and destination fixed effects such as non-tariff barriers respectively, and $\tau^{R,D,E}_{li,t}$ is the random error that is orthogonal to tariffs. From the robot gravity equation (G.14) that I derive in Section G.4, I have

$$\ln\left(\frac{X^R_{li,t}X^R_{ij,t}X^R_{jl,t}}{X^R_{lj,t}X^R_{ji,t}X^R_{il,t}}\right) = \left(1-\varepsilon^R\right)\ln\left(\frac{\tau^{R,T}_{li,t}\tau^{R,T}_{ij,t}\tau^{R,T}_{jl,t}}{\tau^{R,T}_{lj,t}\tau^{R,T}_{ji,t}\tau^{R,T}_{il,t}}\right) + e_{lij,t}, \tag{H.1}$$

where $X^R_{li,t}$ is the bilateral sales of robots from l to i in year t and $e_{lij,t} \equiv \ln \tau^{R,D,E}_{li,t} + \ln \tau^{R,D,E}_{ij,t} + \ln \tau^{R,D,E}_{jl,t} - \ln \tau^{R,D,E}_{lj,t} - \ln \tau^{R,D,E}_{ji,t} - \ln \tau^{R,D,E}_{il,t}$. The benefit of this approach is that it does not require symmetry for non-tariff trade cost $\tau^{R,D}_{li}$, but only requires the orthogonality for the asymmetric component of the trade cost. My method also extends Caliendo and Parro (2015) in using the time-series variation as well as trilateral country-level variation to complement the relatively small number of observations in robot trade data.

When implementing regression of equation (H.1), I further consider controlling for two separate sets of fixed effects. The first set is the unilateral fixed effect indicating if a country is included in the trilateral pair of countries, and the second set is the bilateral fixed effect for the twin of countries is included in the trilateral pair. These fixed effects are relevant in my setting as a few number of countries export robots, and controlling for these exporters' unobserved characteristics is critical.

Table H.1 shows the result of regression of equation (H.1). The first two columns show the result for the HS code 847950 (Industrial robots for multiple uses, the definition of robots used in Humlum 2019), and the last two columns HS code 8479 (Machines and mechanical appliances having individual functions, not specified or included elsewhere in this chapter). The first and third columns control for the unilateral fixed effect, and the second and fourth the bilateral fixed effect. The implied trade elasticity of robots ε^R is fairly tightly estimated and ranges between 1.13-

Table H.1: Coefficient of equation (H.1)

	(1) HS 847950	(2) HS 847950	(3) HS 8479	(4) HS 8479
Tariff	-0.272***	-0.236***	-0.146***	-0.157***
	(0.0718)	(0.0807)	(0.0127)	(0.0131)
Constant	-0.917***	-0.893***	-1.170***	-1.170***
	(0.0415)	(0.0381)	(0.00905)	(0.00853)
FEs	h-i-j-t	ht-it-jt	h-i-j-t	ht-it-jt
N	4610	4521	88520	88441
r2	0.494	0.662	0.602	0.658

Note: The author's calculation based on BACI data from 1996 to 2018 and equation (H.1). The first two columns show the result for the HS code 847950 (Industrial robots for multiple uses), while the last two columns HS code 8479 (Machines and mechanical appliances having individual functions, not specified or included elsewhere in this chapter). The first and third columns control the unilateral fixed effect, while the second and fourth the bilateral fixed effect. See the text for the detail.

1.34. Given these estimation results, I use $\varepsilon^R = 1.2$ in the estimation and counterfactuals.

To assess the estimation result, note that Caliendo and Parro (2015) show in Table 1 that the regression coefficient of equation (H.1) is 1.52, with the standard error of 1.81, for "Machinery n.e.c", which roughly corresponds to HS 84. Therefore, my estimate for industrial robots falls in the one-standard-deviation range of their estimate for a broader category of goods.

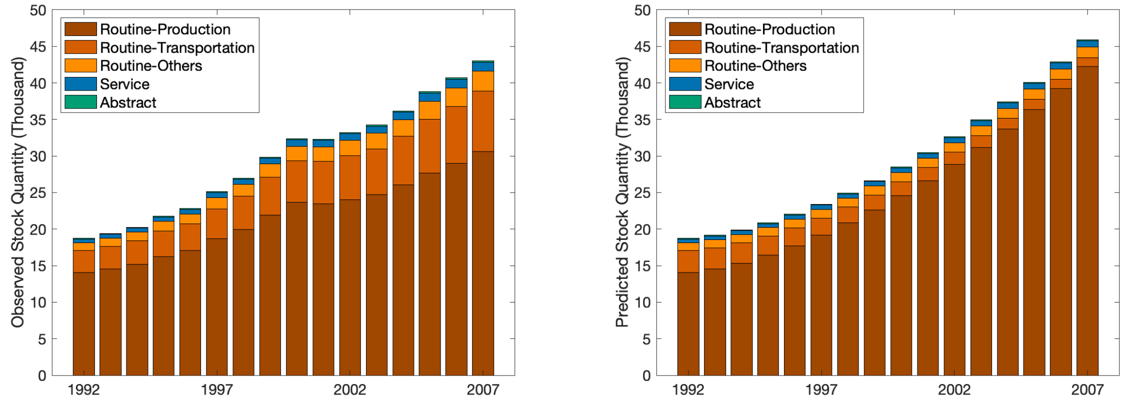
Note that the average trade elasticity across sectors is estimated significantly higher than these values, such as 4 in Simonovska and Waugh (2014). The low trade elasticity for robots ε^R is intuitive given robots are highly heterogeneous and hardly substitutable. This low elasticity implies small gains from robot taxes, with the robot tax incidence almost on the US (robot buyer) side rather than the robot-selling country.

H.2 Actual and Predicted Robot Accumulation Dynamics

Figure H.1 shows the trends of robot stock in the US in the data and the model. Although I do not match the overall robot capital stocks, the estimated model tracks the observed pattern well between 1992 and the late 2010s, consistent with the fact that I target the changes between 1992 and 2007. Although there is over-prediction of the growth of production robots and under-prediction of the growth of transportation (material moving) robots between occupation groups, the overall predicted stock of robots matches well with the actual data.

Figure H.1: Trends of Robot Stocks

(a) Data (b) Model



Note: Figures show the trend of the observed (left) and predicted (right) stock of robots for each occupation group measured by quantities. The predicted robot stocks are computed by shocks backed out from the estimated model and applying the first-order solution to the general equilibrium described in equation (16).

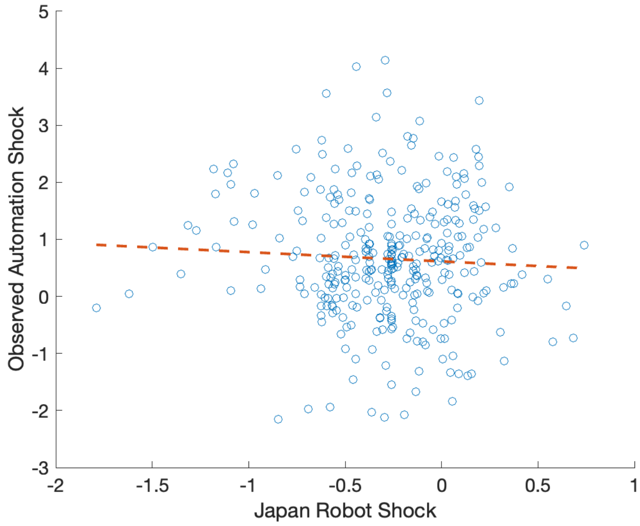
H.3 The Japan Robot Shock and The Implied Automation Shock

In turn, Figure H.2 shows a further detailed scatter plot between the two shocks, delivering a mild negative relationship. This negative correlation is consistent with the example of robotic innovations in Appendix F.2.3.

H.4 Automation and Wages for each Occupation

Figure H.3 shows the observed and counterfactual growth rate of real wages for each occupation, where the counterfactual change means the simulated change when there is no the automation shock. Figure H.3a shows the results aggregated at the 5 occupations groups defined in Section 4.1. I compute the counterfactual growth rate from the observed rate of the wage change, subtracted by the change predicted by the first-order steady-state solution $\overline{\boldsymbol{E}}$ and the implied automation shock $\widehat{\boldsymbol{a}^{\text{imp}}}$. The result is based on the observed high growth rates of robots in routine production and transportation (material moving) occupations, and these occupations' high EoS estimates between robots and workers. In particular, at the 5-occupation aggregate level, most of the observed differences in the real wage growth rates in the three routine occupation groups are closed absent the automation shock. Applying the similar exercise for all occupations in my

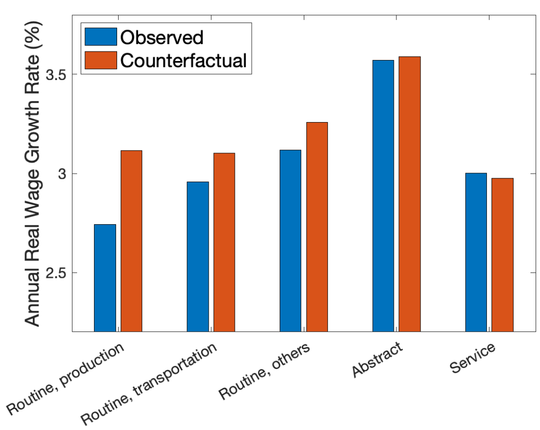
Figure H.2: Correlation between Japan Robot Shock ψ_o^J and Automation Shock $\widehat{a_o^{obs}}$



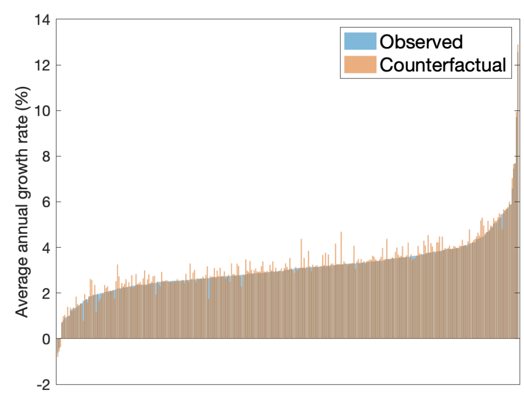
Note: The author's calculation based on JARA, O*NET, and US Census/ACS. The x-axis shows the Japan robot shock, and is taken from the regression of equation (1). The y-axis shows the implied automation shock, and is backed out from equation (21) with the estimated parameters in Table 2. Each circle is 4-digit occupation and dashed line is the fitted line.

Figure H.3: The Steady-state Effect of Robots on Wages

(a) Occupation Groups



(b) All Occupations



sample, Figure H.3b shows a more granular result, where occupations are sorted by the observed changes of wages from 1990-2007.

H.5 Robot Tax and Workers' Welfare

To examine how the robot tax affects workers in different occupations, I define the equivalent variation (EV) implicitly as follows:

$$\sum_{t=t_0}^{\infty}\left(\frac{1}{1+\iota}\right)^t \ln\left(\left[C'_{i,o,t}\right]\right)=\sum_{t=t_0}^{\infty}\left(\frac{1}{1+\iota}\right)^t \ln\left(C_{i,o,t}\left[1+EV_{i,o}\right]\right). \tag{H.2}$$

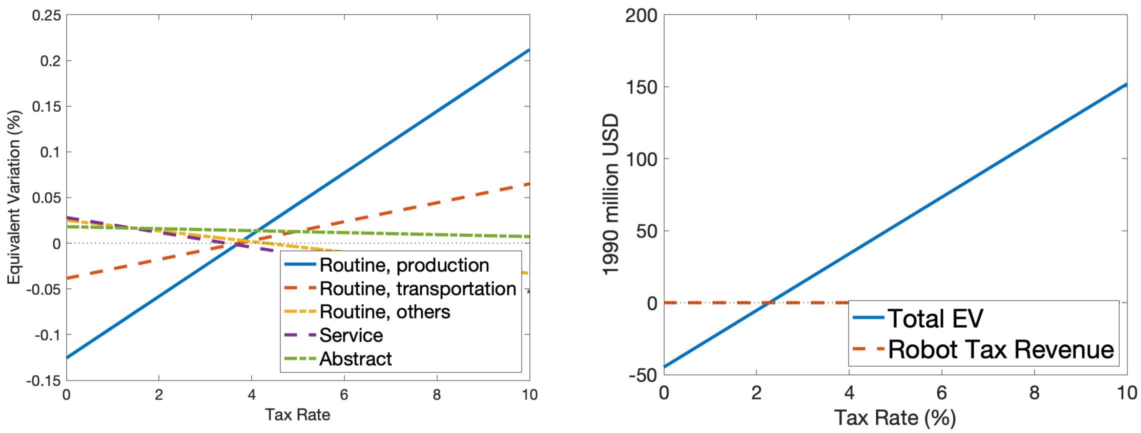
Namely, the EV is the fraction of the occupation-specific subsidy that would make the present discounted value (PDV) of the utility in the robotized and taxed equal to the PDV of the utility if the occupation-specific subsidy were exogenously given in the initial equilibrium. On the left-hand side, I hit the robotization shock backed out in Section 4.4. As in Section 5.2, I consider the US unilateral (not inducing a reaction in other countries), unexpected, and permanent tax on robot purchases. By this definition, the worker in occupation o prefers the robotized and taxed world if and only if the EV is positive for o.

Figure H.4a shows this occupation-specific EV as a function of the tax rate. The far-left side of the figure is the case of zero robot tax, thus a case of only the robotization shock. Consistent with the occupational wage effects (cf. Figure H.3), workers in production and transportation occupations lose significantly due to robotization. In contrast other workers are roughly indifferent between the robotized world and the non-robotized initial equilibrium or slightly prefer the former world. Going right through the figure, the production and transportation workers' EV improves as the robot tax reduces competing robots. The EV of production workers turns positive when the tax rate is around 6%, and that of transportation workers is positive when the rate is about 7%. However, these tax rates are too high and would make EVs in other occupations negative. In fact, in production and transportation occupations, robots do not accumulate and adversely affect labor demand in the other occupations.

To study if the reallocation policy by robot tax may work, I also compute the equivalent variation in terms of monetary value aggregated by occupation groups (total EV) and compare it with the robot tax revenue, both as a function of robot tax. Figure H.4b shows the result. One can confirm that the marginal robot tax revenue is far from enough to compensate for workers' loss that concentrates on production and transportation workers, at the initial equilibrium with zero robot tax rate. The robot tax revenue is negligible at this margin compared with the workers' loss due

Figure H.4: Robot Tax and Workers' Welfare

(a) Occupational Equivalent Variation (b) Total EV and Revenue



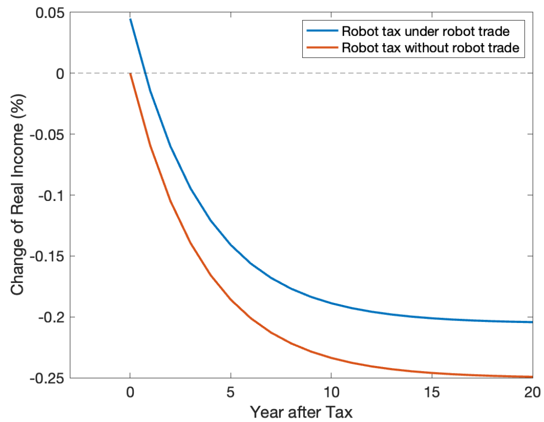
Note: The left panel shows the US workers' equivalent variation defined in equation (H.2) as a function of the US robot tax rate. Labels "Rout., prod.", "Rout., transp.", and "Rout., others" mean routine, production; routine, transportation; and routine, others occupations, respectively. The right panel shows monetary values of equivalent variations aggregated across workers and robot tax revenue as a function of the robot tax rate, measured in 1990 million USD.

to robotization. It is true that as the robot tax rate increases, the total EV rises: When the rate is as large as 6-7%, the sum of the total EV and the robot tax revenue is positive. However, one should be cautious that my solution to the model is to the first order. Thus the approximation error may play an important role when the robot tax rate is significantly higher than the one in the initial equilibrium, zero. Extending my solution to the higher-order or even finding the exact solution is left for future research.

H.6 Trade and the Effect of the Robot Tax

Figure H.5 shows the dynamic effect of the robot tax on the US real income. If the robot trade is not allowed, the robot tax does not increases the real income in any period since the terms-of-trade effect does not show up, but only the long-run capital decumulation effect does. On the other hand, once I allow the robot trade as observed in the data, the robot tax may increase the real income because it decreases the price of imported robots. The effect is concentrated in the short-run before the capital decumulation process matures. In the long run, the negative decumulation effect dominates the positive terms-of-trade effect.

Figure H.5: Effects of the Robot Tax on the US Real Income



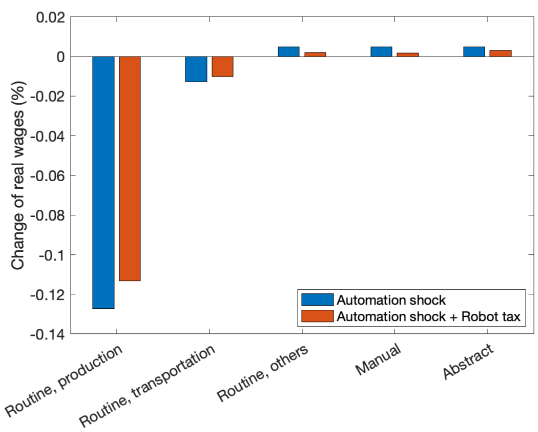
H.7 The Effect of Robot Tax on Occupations

In Figure H.6a, I show two scenarios of the steady-state changes in occupational real wages. On the one hand, I shock the economy only with the automation shocks. On the other hand, I shock the economy with both the automation shocks and the robot tax. The result shows heterogenous effects on occupational real wages of the robot tax. The tax mitigates the negative effect of automation on routine production workers and routine transportation workers, while the tax marginally decreases the small gains that workers in the other occupations would have enjoyed. Overall, the robot tax mitigates the large heterogeneous effects of the automation shocks, that could go negative and positive directions depending on occupation groups, and compresses the effects towards zero. Figure H.6b shows the dynamics of the effects of robot tax, net of the effects of automation shocks. Although the steady-state effects of robot tax were heterogeneous as shown in Figure H.6a, the effect is not immediate but materializes after around 10 years, due to the sluggish adjustment in the accumulation of the robot capital stock. Overall, I find that since the robot tax slows down the adoption of robots, it rolls back the real wage effect of automation–workers in occupations that experienced significant automation shocks (e.g., production and transportation in the routine occupation groups) benefit from the tax, while the others lose.

To study how the occupational effects unfold over time and if the US policy affects third countries, I study occupational value evolution given the US general robot tax. Figure H.7 shows the impact of the US's unilateral, unexpected, and permanent 6% general robot tax on the world's occupational values in the short run and the long run. In the first row, panels show the US oc-

Figure H.6: Effects of the Robot Tax on Occupational Real Wages

(a) Steady-state Comparison



(b) Transitional Effect of Tax

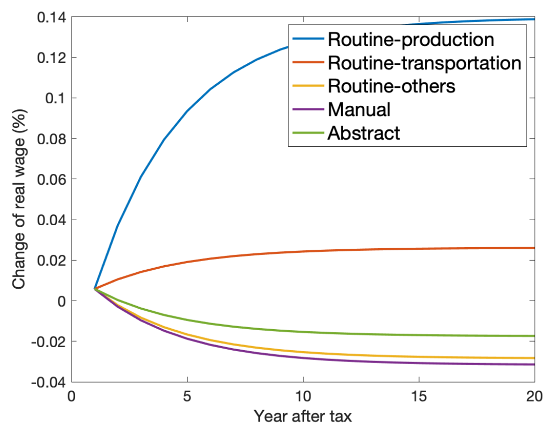
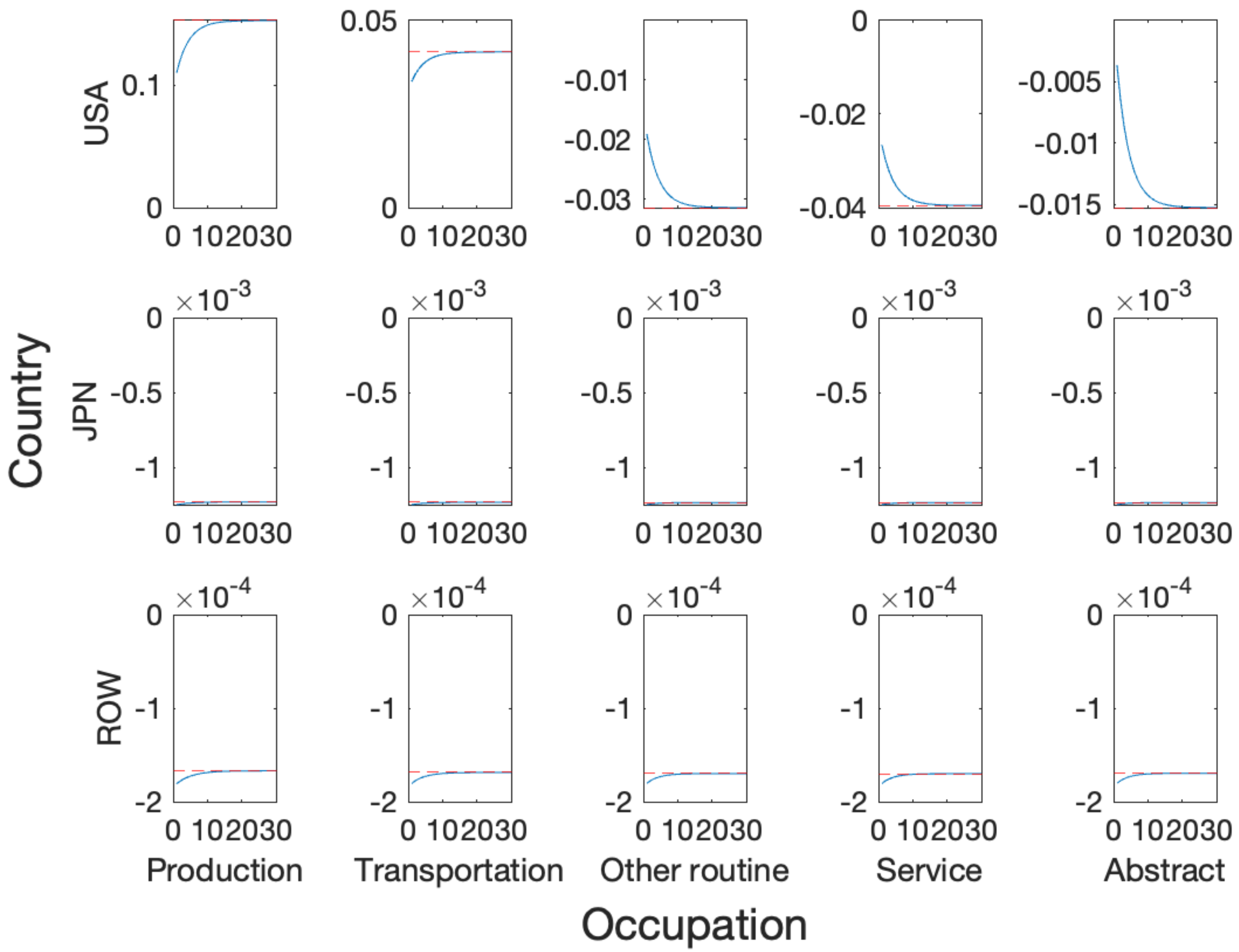
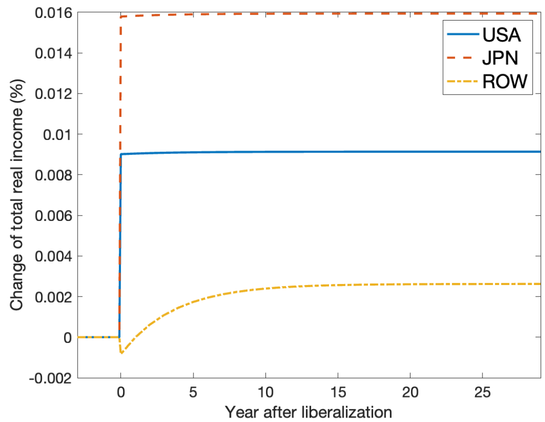


Figure H.7: US General Robot Tax and Global Occupational Value Evolution



Note: Transition dynamics of workers' occupation-specific values given the US's unexpected, unilateral, and permanent 6% general robot tax for all occupational robots at the initial steady-state (period 0) are shown. Blue solid lines are the transitional dynamics, and red dashed lines are the steady-state values.

Figure H.8: The Effect of Robot Trade Cost Reduction



cupational values and corroborate the finding in Figure H.6 that production and transportation workers gain from the robot tax but not other workers. As can be seen from the figure, it takes roughly 10 years until the worker values reach steady states. In other countries than the US, the US robot tax effect is negative but quantitatively limited.

H.8 Trade Liberalization of Robots

What is the effect of liberalizing the trade of robots? To approach this question and gain insights about dynamics gains from trade, I consider unexpected and permanent 20% reduction in the bilateral trade costs, following Ravikumar, Santacreu, and Sposi (2019). Figure H.8 shows the result of such a simulation for a 20-years time horizon. All country groups in the model gain from the trade liberalization. The US gain materialize almost immediately after the trade cost change. A possible explanation is the combination of the following two observation. First, it takes time to accumulate robots after the trade liberalization, which makes the gains from trade liberalization sluggish. Second, by exporting robots to ROW, the US increases the revenue of robot sales immediately after the trade cost drop, improving the short-run real income gain. The real income gain is the largest for Japan, a large net robot exporter. It is noteworthy that ROW loses from the reduction in the robot trade cost, possibly due to the terms-of-trade deterioration in the short-run.

I Detail of the GE Solution

I discuss the derivation log-linearization in equations (12), (14), and (16), so that I can bring the theory with computation. Throughout the section, relational operator $\circ$ is Hadamard product, $\oslash$ indicates Hadamard division, and $\otimes$ means Kronecker product. In this section, I use θ_o to denote the elasticity of substitution between robots and workers for each occupation

It is useful to show that the CES production structure implies the share-weighted log-change expression for both prices and quantities. Namely, I have a formula for the change in destination price index $\widehat{P_{j,t}^G} = \sum_i \widetilde{x}_{ij,t_0}^G \widehat{p_{ij,t}^G}$ and one for the change in destination expenditure $\widehat{P_{j,t}^G} + \widehat{Q_{j,t}^G} = \sum_i \widetilde{x}_{ij,t_0}^G \left(\widehat{p_{ij,t}^G} + \widehat{Q_{ij,t}^G}\right)$. These imply that

$$\widehat{Q_{j,t}^G} = \sum_i \widetilde{x}_{ij,t_0}^G \widehat{Q_{ij,t}^G},$$

or the changes of quantity aggregate $\widehat{Q_{j,t}^G}$ are also share-weighted average of changes of origin quantity $\widehat{Q_{ij,t}^G}$.

By log-linearizing equation (G.24) for any i ,

$$\begin{aligned}
&-\alpha_M \widehat{p_{i,t}^G} + \alpha_M \sum_l \widetilde{x}_{li,t_0}^G \widehat{p_{l,t}^G} + (1-\alpha_M) \sum_j \widetilde{y}_{ij,t_0}^G \widehat{Q_{ij,t}^G} - \alpha_L \sum_o \widetilde{x}_{i,o,t_0}^O l_{i,o,t_0}^O \widehat{L_{i,o,t}} \\
&= \frac{\alpha_L}{\theta_o - 1} \sum_o \frac{\widetilde{x}_{i,o,t_0}^O}{1-a_{o,t_0}} \left(-a_{o,t_0} l_{i,o,t_0}^O + (1-a_{o,t_0}) \left(1 - l_{i,o,t_0}^O\right)\right) \widehat{a_{o,t}} + \alpha_L \sum_o \widetilde{x}_{i,o,t_0}^O \frac{1}{\beta - 1} \widehat{b_{i,o,t}} \\
&\quad + \widehat{A_{i,t}^G} + (1 - \alpha_L - \alpha_M) \widehat{K_{i,t}} - \alpha_M \sum_l \widetilde{x}_{li,t_0}^G \widehat{\tau_{li,t}^G} - (1-\alpha_M) \sum_j \widetilde{y}_{ij,t_0}^G \widehat{\tau_{ij,t}^G} + \alpha_L \sum_o \widetilde{x}_{i,o,t_0}^O \left(1 - l_{i,o,t_0}^O\right) \widehat{K_{i,o,t}^R},
\end{aligned}$$

To write a matrix notation, write

$$\overline{\boldsymbol{M}^{yG}} \equiv \begin{bmatrix} \left[\widetilde{y}_{11,t_0}^G, \ldots, \widetilde{y}_{1N,t_0}^G\right] & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \left[\widetilde{y}_{N1,t_0}^G, \ldots, \widetilde{y}_{NN,t_0}^G\right] \end{bmatrix}$$

a $N \times N^2$ matrix,

$$\overline{\boldsymbol{M}^{xOl}} \equiv \begin{bmatrix} \left(\widetilde{\boldsymbol{x}}_{1,\cdot,t_0} \circ \widetilde{\boldsymbol{l}}_{1,\cdot,t_0}\right)^\top & & \boldsymbol{0} \\ & \ddots & \\ \boldsymbol{0} & & \left(\widetilde{\boldsymbol{x}}_{N,\cdot,t_0} \circ \widetilde{\boldsymbol{l}}_{N,\cdot,t_0}\right)^\top \end{bmatrix}$$

a $N \times NO$ matrix where

$$\widetilde{\boldsymbol{x}}_{1,\cdot,t_0} \equiv \left(\widetilde{x}^O_{1,o,t_0}\right)_o \text{ and } \widetilde{\boldsymbol{l}}_{1,\cdot,t_0} \equiv \left(l^O_{1,o,t_0}\right)_o \tag{I.1}$$

are $O \times 1$ vectors, $\overline{\boldsymbol{M}^{al}}$ as a matrix with its element

$$M^{al}_{i,o} = \frac{-a_{o,t_0} l^O_{i,o,t_0} + (1 - a_{o,t_0})\left(1 - l^O_{i,o,t_0}\right)}{1 - a_{o,t_0}},$$

and a $N \times O$ matrix,

$$\overline{\boldsymbol{M}^{xO}} \equiv \begin{bmatrix} \left[\widetilde{x}^O_{1,1,t_0}, \ldots, \widetilde{x}^O_{1,O,t_0}\right] & & \boldsymbol{0} \\ & \ddots & \\ \boldsymbol{0} & & \left[\widetilde{x}^O_{N,1,t_0}, \ldots, \widetilde{x}^O_{N,O,t_0}\right] \end{bmatrix},$$

a $N \times NO$ matrix,

$$\overline{\boldsymbol{M}^{xG}} \equiv \left[\begin{array}{ccc} \operatorname{diag}\left(\widetilde{x}^G_{1\cdot,t_0}\right) & \ldots & \operatorname{diag}\left(\widetilde{x}^G_{N\cdot,t_0}\right) \end{array} \right],$$

a $N \times N^2$ matrix, and

$$\overline{\boldsymbol{M}^{xOl,2}} \equiv \begin{bmatrix} \left(\widetilde{\boldsymbol{x}}_{1,\cdot,t_0} \circ \left(\boldsymbol{1}_O - \widetilde{\boldsymbol{l}}_{1,\cdot,t_0}\right)\right)^\top & & \boldsymbol{0} \\ & \ddots & \\ \boldsymbol{0} & & \left(\widetilde{\boldsymbol{x}}_{N,\cdot,t_0} \circ \left(\boldsymbol{1}_O - \widetilde{\boldsymbol{l}}_{N,\cdot,t_0}\right)\right)^\top \end{bmatrix},$$

a $N \times NO$ matrix where $\widetilde{\boldsymbol{x}}_{1,\cdot,t_0}$ and $\widetilde{\boldsymbol{l}}_{1,\cdot,t_0}$ are defined in equation (I.1). Then I have

$$\begin{aligned} & -\alpha_M \left(\overline{\boldsymbol{I}} - \left(\overline{\widetilde{\boldsymbol{x}}^G_{t_0}}\right)^\top\right) \widehat{\boldsymbol{p}^G_t} + (1 - \alpha_M)\, \overline{\boldsymbol{M}^{yG}}\widehat{\boldsymbol{Q}^G_t} - \alpha_L \overline{\boldsymbol{M}^{xOl}} \widehat{\boldsymbol{L}_t} \\ &= \frac{\alpha_L}{\theta_o - 1}\left(\widetilde{\boldsymbol{x}}^O_{t_0} \circ \overline{\boldsymbol{M}^{al}}\right) \widehat{\boldsymbol{a}}_t + \frac{\alpha_L}{\beta - 1} \overline{\boldsymbol{M}^{xO}} \widehat{\boldsymbol{b}}_t + \widehat{\boldsymbol{A}^G_t} + (1 - \alpha_L - \alpha_M)\, \widehat{\boldsymbol{K}_t} \\ &\quad - \left[\alpha_M \overline{\boldsymbol{M}^{xG}} + (1 - \alpha_M)\, \overline{\boldsymbol{M}^{yG}}\right] \widehat{\boldsymbol{\tau}^G_t} + \alpha_L \overline{\boldsymbol{M}^{xOl,2}} \widehat{\boldsymbol{K}^R_t}, \end{aligned}$$

By log-linearizing equation (G.25) for any i and o,

$$\widehat{p^R_{i,o,t}} = \widehat{P^G_{i,t}} - \widehat{A^R_{i,o,t}}$$

$$-\sum_l \widetilde{x}^G_{li,t_0} \widehat{p^G_{l,t}} + \widehat{p^R_{i,o,t}} = -\widehat{A^R_{i,o,t}} + \sum_l \widetilde{x}^G_{li,t_0} \widehat{\tau^G_{li,t}}.$$

In matrix notation, write

$$\overline{\boldsymbol{M}^{xG,2}} \equiv \begin{bmatrix} \mathbf{1}_O \left[\widetilde{x}^G_{11,t_0}, \ldots, \widetilde{x}^G_{N1,t_0}\right] \\ \vdots \\ \mathbf{1}_O \left[\widetilde{x}^G_{1N,t_0}, \ldots, \widetilde{x}^G_{NN,t_0}\right] \end{bmatrix}$$

a $NO \times N$ matrix, and

$$\overline{\boldsymbol{M}^{xG,3}} \equiv \begin{bmatrix} \widetilde{x}^G_{11,t_0} & \ldots & \widetilde{x}^G_{N1,t_0} & & & & \mathbf{0} & \\ & & & \ddots & & & & \\ & \mathbf{0} & & & & \widetilde{x}^G_{1N,t_0} & \ldots & \widetilde{x}^G_{NN,t_0} \end{bmatrix} \otimes \mathbf{1}_O$$

a $NO \times N^2$ matrix. Then I have

$$-\overline{\boldsymbol{M}^{xG,2}} \widehat{\boldsymbol{p}^G_t} + \widehat{\boldsymbol{p}^R_t} = -\widehat{\boldsymbol{A}^R_t} + \overline{\boldsymbol{M}^{xG,3}} \widehat{\boldsymbol{\tau}^G_t}.$$

By log-linearizing equations (B.2), (B.3), and (B.4) for any i, o, and o', I have

$$\widehat{\mu_{i,oo',t}} = \phi \left(-d\chi_{i,oo',t} + \frac{1}{1+\iota} \widehat{V_{i,o',t+1}} \right) - \sum_{o''} \mu_{i,oo'',t_0} \left(-d\chi_{i,oo'',t} + \frac{1}{1+\iota} \widehat{V_{i,o'',t+1}} \right), \tag{I.2}$$

$$\widehat{V_{i,o,t+1}} = \widehat{w_{i,o,t+1}} + dT_{i,o,t+1} - \widehat{P_{i,t+1}} + \sum_{o'} \mu_{i,oo',t_0} \left(-d\chi_{i,oo',t+1} + \frac{1}{1+\iota} \widehat{V_{i,o',t+2}} \right), \tag{I.3}$$

and

$$\widehat{L_{i,o,t+1}} = \sum_{o'} \frac{L_{i,o',t_0}}{L_{i,o,t_0}} \mu_{i,o'o,t_0} \left(\widehat{\mu_{i,o'o,t}} + \widehat{L_{i,o',t}} \right). \tag{I.4}$$

In matrix notation, by equation (I.2),

$$\widehat{\boldsymbol{\mu}^{\text{vec}}_t} = -\phi \left(\overline{\boldsymbol{I}_{NO^2}} - \overline{\boldsymbol{M}^{\mu}} \right) d\boldsymbol{\chi}^{\text{vec}}_t + \frac{\phi}{1+\iota} \left(\overline{\boldsymbol{I}_{NO^2}} - \overline{\boldsymbol{M}^{\mu}} \right) \left(\overline{\boldsymbol{I}_{NO}} \otimes \mathbf{1}_O \right) \widehat{\boldsymbol{V}_{t+1}}.$$

where

$$\overline{\boldsymbol{M}^{\mu}} \equiv \overline{\boldsymbol{M}^{\mu,3}} \otimes \mathbf{1}_O,$$

$$\overline{\boldsymbol{M}^{\mu,3}} \equiv \begin{bmatrix} \left(\boldsymbol{\mu}_{i,1\cdot,t_0}\right)^\top & & & & & & \\ & \ddots & & & & \mathbf{0} & \\ & & \left(\boldsymbol{\mu}_{i,O\cdot,t_0}\right)^\top & & & & \\ & & & \ddots & & & \\ & & & & \left(\boldsymbol{\mu}_{N,1\cdot,t_0}\right)^\top & & \\ & \mathbf{0} & & & & \ddots & \\ & & & & & & \left(\boldsymbol{\mu}_{i,O\cdot 1,t_0}\right)^\top \end{bmatrix},$$

$$d\boldsymbol{\chi}_t^{\text{vec}} \equiv \begin{bmatrix} d\boldsymbol{\chi}_{1,1\cdot,t} & \dots & d\boldsymbol{\chi}_{1,O\cdot,t} & \dots & d\boldsymbol{\chi}_{N,1\cdot,t} & \dots & d\boldsymbol{\chi}_{N,O\cdot,t} \end{bmatrix}^\top,$$

and

$$\boldsymbol{\mu}_{i,o\cdot,t_0} \equiv \left(\mu_{i,oo',t_0}\right)_{o'} \text{ and } d\boldsymbol{\chi}_{1,o\cdot,t} \equiv \left(d\chi_{1,oo',t}\right)_{o'} \tag{I.5}$$

are $O \times 1$ vectors. By equation (I.3),

$$\frac{1}{1+\iota}\overline{\boldsymbol{M}^{\mu,2}}\check{\boldsymbol{V}}_{t+2} = \overline{\boldsymbol{M}^{xG,2}}\boldsymbol{p}_{t+1}^{\check{G}} - \check{\boldsymbol{w}}_{t+1} + \check{\boldsymbol{V}}_{t+1}.$$

where

$$\overline{\boldsymbol{M}^{\mu,2}} \equiv \begin{bmatrix} \left(\boldsymbol{\mu}_{1,1\cdot,t_0}\right)^\top & & \\ \vdots & & \mathbf{0} \\ \left(\boldsymbol{\mu}_{1,O\cdot,t_0}\right)^\top & & \\ & \ddots & \\ & & \left(\boldsymbol{\mu}_{N,1\cdot,t_0}\right)^\top \\ \mathbf{0} & & \\ & & \left(\boldsymbol{\mu}_{N,O\cdot,t_0}\right)^\top \end{bmatrix},$$

and $\boldsymbol{\mu}_{i,o\cdot,t_0}$ is given by equation (I.5) for any i and o. By equation (I.3),

$$\check{\boldsymbol{L}}_{t+1} = \overline{\boldsymbol{M}^{\mu L,2}}\check{\boldsymbol{\mu}}_t^{\text{vec}} + \overline{\boldsymbol{M}^{\mu L}}\check{\boldsymbol{L}}_t$$

where $\overline{\boldsymbol{M}^{\mu L}}$ being the $NO \times NO$ matrix

$$\overline{\boldsymbol{M}^{\mu L}} = \overline{\boldsymbol{M}^{\mu,2}} \circ \left(\begin{bmatrix} \left(\boldsymbol{L}_{1,\cdot,t_0}\right)^\top & & \boldsymbol{0} \\ & \ddots & \\ \boldsymbol{0} & & \left(\boldsymbol{L}_{N,\cdot,t_0}\right)^\top \end{bmatrix} \otimes \boldsymbol{1}_O \right) \oslash \left(\begin{bmatrix} \boldsymbol{L}_{1,\cdot,t_0} & & \boldsymbol{0} \\ & \ddots & \\ \boldsymbol{0} & & \boldsymbol{L}_{N,\cdot,t_0} \end{bmatrix} \otimes \left(\boldsymbol{1}_O\right)^\top \right)$$

and $\overline{\boldsymbol{M}^{\mu L,2}}$ being the $NO \times NO^2$ matrix

$$\overline{\boldsymbol{M}^{\mu L,2}} = \overline{\boldsymbol{M}^{\mu,4}} \circ \left(\begin{bmatrix} \left(\boldsymbol{L}_{1,\cdot,t_0}\right)^\top & & \boldsymbol{0} \\ & \ddots & \\ \boldsymbol{0} & & \left(\boldsymbol{L}_{N,\cdot,t_0}\right)^\top \end{bmatrix} \otimes \overline{\boldsymbol{I}_O} \right) \oslash$$
$$\begin{pmatrix} \left(\boldsymbol{1}_O\right)^\top \otimes \operatorname{diag}\left(L_{1,o,t_0}\right) & & \boldsymbol{0} \\ & \ddots & \\ \boldsymbol{0} & & \left(\boldsymbol{1}_O\right)^\top \otimes \operatorname{diag}\left(L_{N,o,t_0}\right) \end{pmatrix},$$

where

$$\overline{\boldsymbol{M}^{\mu,4}} \equiv \begin{bmatrix} \operatorname{diag}\left(\boldsymbol{\mu}_{1,1\cdot,t_0}\right) & \ldots & \operatorname{diag}\left(\boldsymbol{\mu}_{i,O\cdot,t_0}\right) & & \boldsymbol{0} & \\ & & & \ddots & & \\ & \boldsymbol{0} & & \operatorname{diag}\left(\boldsymbol{\mu}_{N,1\cdot,t_0}\right) & \ldots & \operatorname{diag}\left(\boldsymbol{\mu}_{N,O\cdot,t_0}\right) \end{bmatrix},$$

and $\boldsymbol{\mu}_{i,o\cdot,t_0}$ is given by equation (I.5) for any i and o.

By log-linearizing equation (G.23) for each i and j,

$$\widehat{Q^G_{ij,t}} = -\varepsilon^G \widehat{p^G_{ij,t}} - \left(1 - \varepsilon^G\right) \widehat{P^G_{j,t}} + \left[s^G_{j,t_0} \sum_k \widehat{p^G_{jk,t} Q^G_{jk,t}} + s^V_{j,t_0} \sum_{i,o} \widehat{p^R_{ij,o,t} Q^R_{ij,o,t}} + s^R_{j,t_0} \sum_{o,k} \widehat{p^R_{jk,o,t} Q^R_{jk,o,t}} \right]$$

where

$$s^G_{j,t_0} \equiv \frac{\sum_k p^G_{jk,t_0} Q^G_{jk,t_0}}{\sum_k p^G_{jk,t_0} Q^G_{jk,t_0} - \sum_{i,o} p^R_{ij,o,t_0} Q^R_{ij,o,t_0} + \sum_{o,k} p^R_{jk,o,t_0} Q^R_{jk,o,t_0}}$$

is the baseline share of non-robot good production in income,

$$s^R_{j,t_0} \equiv \frac{\sum_{o,k} p^R_{jk,o,t_0} Q^R_{jk,o,t_0}}{\sum_k p^G_{jk,t_0} Q^G_{jk,t_0} - \sum_{i,o} p^R_{ij,o,t_0} Q^R_{ij,o,t_0} + \sum_{o,k} p^R_{jk,o,t_0} Q^R_{jk,o,t_0}},$$

is the baseline share of robot production, and

$$s^V_{j,t_0} \equiv -\frac{\sum_{i,o} p^R_{ij,o,t_0} Q^R_{ij,o,t_0}}{\sum_k p^G_{jk,t_0} Q^G_{jk,t_0} - \sum_{i,o} p^R_{ij,o,t_0} Q^R_{ij,o,t_0} + \sum_{o,k} p^R_{jk,o,t_0} Q^R_{jk,o,t_0}},$$

is the (negative) baseline absorption share of robots. Thus

$$\begin{aligned}
&\left[\varepsilon^G \widehat{p^G_{i,t}} + \left(1-\varepsilon^G\right)\sum_l \widetilde{x}^G_{lj,t_0}\widehat{p^G_{l,t}} - s^G_{j,t_0}\widehat{p^G_{j,t}}\right] - \left[s^V_{j,t_0}\sum_{l,o}\widetilde{x}^R_{lj,o,t_0}\widetilde{x}^R_{j,o,t_0}\widehat{p^R_{l,o,t}} + s^R_{t_0}\sum_o \widetilde{y}^R_{j,o,t_0}\widehat{p^R_{j,o,t}}\right] \\
&+\left(\widehat{Q^G_{ij,t}} - s^G_{j,t_0}\sum_k \widetilde{y}^G_{jk,t_0}\widehat{Q^G_{jk,t}}\right) - \left(s^V_{j,t_0}\sum_{l,o}\widetilde{x}^R_{lj,o,t_0}\widetilde{x}^R_{j,o,t_0}\widehat{Q^R_{lj,o,t}} + s^R_{j,t_0}\sum_{k,o}\widetilde{y}^R_{jk,o,t_0}\widetilde{y}^R_{j,o,t_0}\widehat{Q^R_{jk,o,t}}\right) \\
&= -\left[\varepsilon^G \widehat{\tau^G_{ij,t}} + \left(1-\varepsilon^G\right)\sum_l \widetilde{x}^G_{lj,t_0}\widehat{\tau^G_{lj,t}} - s^G_{j,t_0}\sum_k \widetilde{y}^G_{jk,t_0}\widehat{\tau^G_{jk,t}}\right] + \left[s^V_{j,t_0}\sum_{l,o}\widetilde{x}^R_{lj,t_0}\widehat{\tau^R_{lj,t}} + s^R_{j,t_0}\sum_{k,o}\widetilde{y}^R_{jk,t_0}\widehat{\tau^R_{jk,t}}\right]
\end{aligned}$$

where

$$\widetilde{x}^R_{ij,o,t_0} \equiv \frac{p^R_{ij,o,t_0} Q^R_{ij,o,t_0}}{P^R_{j,o,t_0} Q^R_{j,o,t_0}}, \widetilde{x}^R_{j,o,t_0} \equiv \frac{P^R_{j,o,t_0} Q^R_{j,o,t_0}}{P^R_{j,t_0} Q^R_{j,t_0}}, \widetilde{x}^R_{ij,t_0} \equiv \frac{\sum_o p^R_{ij,o,t_0} Q^R_{ij,o,t_0}}{P^R_{j,t_0} Q^R_{j,t_0}},$$

$$\widetilde{y}^R_{ij,o,t_0} \equiv \frac{p^R_{ij,o,t_0} Q^R_{ij,o,t_0}}{\sum_k p^R_{ik,o,t_0} Q^R_{ik,o,t_0}}, \widetilde{y}^R_{i,o,t_0} \equiv \frac{\sum_k p^R_{ik,o,t_0} Q^R_{ik,o,t_0}}{\sum_{k,o'} p^R_{ik,o',t_0} Q^R_{ik,o',t_0}}, \widetilde{y}^R_{ij,t_0} \equiv \frac{\sum_o p^R_{ij,o,t_0} Q^R_{ij,o,t_0}}{\sum_{k,o} p^R_{ik,o,t_0} Q^R_{ik,o,t_0}}.$$

In matrix notation, define

$$\overline{\boldsymbol{M}^{xR}} \equiv \mathbf{1}_N \otimes \begin{bmatrix} \widetilde{\boldsymbol{x}}^R_{t_0} \circ \widetilde{\boldsymbol{x}}^R_{\cdot 1,\cdot,t_0} & \dots & \widetilde{\boldsymbol{x}}^R_{t_0} \circ \widetilde{\boldsymbol{x}}^R_{\cdot N,\cdot,t_0} \end{bmatrix},$$

a $N^2 \times NO$ matrix,

$$\overline{\boldsymbol{M}^{yR}} \equiv \mathbf{1}_N \otimes \begin{bmatrix} \widetilde{y}^R_{1,1} & \dots & \widetilde{y}^R_{1,O} & & & \mathbf{0} \\ & & & \ddots & & \\ & \mathbf{0} & & \widetilde{y}^R_{N,1} & \dots & \widetilde{y}^R_{N,O} \end{bmatrix},$$

a $N^2 \times NO$ matrix,

$$\overline{\boldsymbol{M}^{yG,2}} \equiv \mathbf{1}_N \otimes \overline{\boldsymbol{M}^{yG}}.$$

a $N^2 \times N^2$ matrix,

$$\overline{\boldsymbol{M}^{xR,2}} \equiv \mathbf{1}_N \otimes \begin{bmatrix} \left[\widetilde{x}^R_{1,o,t_0}\widetilde{x}^R_{11,o,t_0}\right]_o & & \mathbf{0} & & \left[\widetilde{x}^R_{1,o,t_0}\widetilde{x}^R_{N1,o,t_0}\right]_o & & \mathbf{0} \\ & \ddots & & \dots & & \ddots & \\ \mathbf{0} & & \left[\widetilde{x}^R_{N,o,t_0}\widetilde{x}^R_{1N,o,t_0}\right]_o & & \mathbf{0} & & \left[\widetilde{x}^R_{N,o,t_0}\widetilde{x}^R_{NN,o,t_0}\right]_o \end{bmatrix}$$

a $N^2 \times N^2O$ matrix ,

$$\overline{\boldsymbol{M}^{yR,2}} \equiv \mathbf{1}_N \otimes \begin{bmatrix} \left[\widetilde{y}^R_{1,o,t_0}\widetilde{y}^R_{11,o,t_0}\right]_o & \dots & \left[\widetilde{y}^R_{N,o,t_0}\widetilde{y}^R_{1N,o,t_0}\right]_o & & & \mathbf{0} & \\ & & & \ddots & & & \\ & \mathbf{0} & & & \left[\widetilde{y}^R_{1,o,t_0}\widetilde{y}^R_{N1,o,t_0}\right]_o & \dots & \left[\widetilde{y}^R_{N,o,t_0}\widetilde{y}^R_{NN,o,t_0}\right]_o \end{bmatrix}$$

a $N^2 \times N^2O$ matrix,

$$\overline{\boldsymbol{M}^{xG,4}} \equiv \mathbf{1}_N \otimes \overline{\boldsymbol{M}^{xG}}$$

a $N^2 \times N^2$ matrix,

$$\overline{\boldsymbol{M}^{xR,3}} \equiv \mathbf{1}_N \otimes \begin{bmatrix} \operatorname{diag}\left(\widetilde{x}^R_{1\cdot,t_0}\right) & \dots & \operatorname{diag}\left(\widetilde{x}^R_{N\cdot,t_0}\right) \end{bmatrix}$$

a $N^2 \times N^2$ matrix,

$$\overline{\boldsymbol{M}^{yR,3}} \equiv \mathbf{1}_N \otimes \begin{bmatrix} \left[\widetilde{y}^R_{11,t_0},\dots,\widetilde{y}^R_{1N,t_0}\right] & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \left[\widetilde{y}^R_{N1,t_0},\dots,\widetilde{y}^R_{NN,t_0}\right] \end{bmatrix}$$

a $N^2 \times N^2$ matrix, and

$$\overline{\boldsymbol{M}^{xO,2}} \equiv \mathbf{1}_N \otimes \overline{\boldsymbol{M}^{xO}},$$

a $N^2 \times NO$ matrix. Then I have

$$\begin{aligned}
&\left(\varepsilon^G \left[\overline{\boldsymbol{I}_N} \otimes \mathbf{1}_N\right] + \left(1-\varepsilon^G\right)\left[\mathbf{1}_N \otimes \left(\widetilde{\boldsymbol{x}}^G_{t_0}\right)^\top\right] - \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^G_{t_0}\right)\left[\mathbf{1}_N \otimes \overline{\boldsymbol{I}_N}\right]\right)\widehat{\boldsymbol{p}^G_t} \\
&-\left(\operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^V_{t_0}\right)\overline{\boldsymbol{M}^{xR}} + \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^R_{t_0}\right)\overline{\boldsymbol{M}^{yR}}\right)\widehat{\boldsymbol{p}^R_t} \\
&+\left(\overline{\boldsymbol{I}_{N^2}} - \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^G_{t_0}\right)\overline{\boldsymbol{M}^{yG,2}}\right)\widehat{\boldsymbol{Q}^G_t} - \left[\operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^V_{t_0}\right)\overline{\boldsymbol{M}^{xR,2}} + \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^R_{t_0}\right)\overline{\boldsymbol{M}^{yR,2}}\right]\widehat{\boldsymbol{Q}^R_t} \\
&= -\left(\varepsilon^G + \left(1-\varepsilon^G\right)\overline{\boldsymbol{M}^{xG,4}} - \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^G_{t_0}\right)\overline{\boldsymbol{M}^{yG,2}}\right)\widehat{\boldsymbol{\tau}^G_t} \\
&\quad + \left(\operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^V_{t_0}\right)\overline{\boldsymbol{M}^{xR,3}} + \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^R_{t_0}\right)\overline{\boldsymbol{M}^{yR,3}}\right)\widehat{\boldsymbol{\tau}^R_t}
\end{aligned}$$

By log-linearizing equation (G.15) for each i, j, and o,,

$$\begin{aligned}
&\left(1-\alpha^R\right)\frac{1+u_{ij,t_0}}{1+u_{ij,t_0}+2\gamma\delta}\sum_l \widetilde{x}^G_{lj,t_0}\widehat{p^G_{l,t}} + \frac{1+u_{ij,t_0}}{1+u_{ij,t_0}+2\gamma\delta}\widehat{p^R_{i,o,t}} \\
&+\left[\frac{2\gamma\delta}{1+u_{ij,t_0}+2\gamma\delta} - \left(1-\alpha^R\right)\frac{1+u_{ij,t_0}}{1+u_{ij,t_0}+2\gamma\delta}\right]\sum_l \widetilde{x}^R_{lj,o,t_0}\widehat{p^R_{l,o,t}} \\
&+\frac{1+u_{ij,t_0}}{1+u_{ij,t_0}+2\gamma\delta}\frac{1}{\varepsilon^R}\widehat{Q^R_{ij,o,t}} + \left[-\frac{1}{\varepsilon^R}\frac{1+u_{ij,t_0}}{1+u_{ij,t_0}+2\gamma\delta} + \frac{2\gamma\delta}{1+u_{ij,t_0}+2\gamma\delta}\right]\sum_l \widetilde{x}^R_{lj,o,t_0}\widehat{Q^R_{lj,o,t}} \\
&= -\frac{1+u_{ij,t_0}}{1+u_{ij,t_0}+2\gamma\delta}du_{ij,t} - \left(1-\alpha^R\right)\frac{1+u_{ij,t_0}}{1+u_{ij,t_0}+2\gamma\delta}\sum_l \widetilde{x}^G_{lj,t_0}\widehat{\tau^G_{lj,t}} - \frac{1+u_{ij,t_0}}{1+u_{ij,t_0}+2\gamma\delta}\widehat{\tau^R_{ij,t}} \\
&-\left[\frac{2\gamma\delta}{1+u_{ij,t_0}+2\gamma\delta} - \left(1-\alpha^R\right)\frac{1+u_{ij,t_0}}{1+u_{ij,t_0}+2\gamma\delta}\right]\sum_l \widetilde{x}^R_{lj,o,t_0}\widehat{\tau^R_{lj,t}} + \widehat{\lambda^R_{j,o,t}} + \frac{2\gamma\delta}{1+u_{ij,t_0}+2\gamma\delta}\widehat{K^R_{j,o,t}}.
\end{aligned}$$

In matrix notation, write a preliminary $N \times N$ matrix $\overline{\widetilde{\boldsymbol{u}_{t_0}}}$ as such that the (i,j)-element is

$$\frac{1+u_{ij,t_0}}{1+u_{ij,t_0}+2\gamma\delta}.$$

Then $\mathbf{1}_N\left(\mathbf{1}_N\right)^\top - \overline{\widetilde{\boldsymbol{u}_{t_0}}}$ is a matrix that is filled with $2\gamma\delta/\left(1+u_{ij,t_0}+2\gamma\delta\right)$ for its (i,j) element and

$$\overline{\boldsymbol{M}^u} \equiv \operatorname{diag}\left(\left[\widetilde{u_{1\cdot,t_0}},\ldots,\widetilde{u_{N\cdot,t_0}}\right]^\top\right).$$

Using these, write

$$\overline{\boldsymbol{M}^{xG,5}} \equiv \left(\overline{\boldsymbol{M}^u} \otimes \overline{\boldsymbol{I}_O}\right)\left(\mathbf{1}_N \otimes \left(\widetilde{\boldsymbol{x}}^G_{t_0}\right)^\top \otimes \mathbf{1}_O\right)$$

a $N^2O \times N$ matrix,

$$\overline{\boldsymbol{M}^{u,2}} \equiv \begin{bmatrix} \widetilde{\boldsymbol{u}_{1\cdot,t_0}} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \widetilde{\boldsymbol{u}_{N\cdot,t_0}} \end{bmatrix} \otimes \overline{\boldsymbol{I}_O},$$

a $N^2O \times NO$ matrix where $\widetilde{\boldsymbol{u}_{i\cdot,t_0}} \equiv \left(\widetilde{u_{i\cdot,t_0}}\right)_j$ is a $N \times 1$ vector,

$$\overline{\boldsymbol{M}^{xR,4}} \equiv \left\{ \left[\left(\overline{\boldsymbol{I}_{N^2}} - \overline{\boldsymbol{M}^u} \right) - \left(1 - \alpha^R \right) \overline{\boldsymbol{M}^u} \right] \otimes \overline{\boldsymbol{I}_O} \right\} \times \left(\mathbf{1}_N \otimes \begin{bmatrix} \operatorname{diag}\left(\left\{ \widetilde{x}^R_{11,o,t_0} \right\}_o \right) & \dots & \operatorname{diag}\left(\left\{ \widetilde{x}^R_{N1,o,t_0} \right\}_o \right) \\ \vdots & & \vdots \\ \operatorname{diag}\left(\left\{ \widetilde{x}^R_{1N,o,t_0} \right\}_o \right) & \dots & \operatorname{diag}\left(\left\{ \widetilde{x}^R_{NN,o,t_0} \right\}_o \right) \end{bmatrix} \right)$$

a $N^2O \times NO$ matrix,

$$\overline{\boldsymbol{M}^{xR,5}} \equiv \left\{ \left[-\frac{1}{\varepsilon^R} \overline{\boldsymbol{M}^u} + \left(\overline{\boldsymbol{I}_{N^2}} - \overline{\boldsymbol{M}^u} \right) \right] \otimes \overline{\boldsymbol{I}_O} \right\} \times \left\{ \mathbf{1}_N \otimes \begin{bmatrix} \operatorname{diag}\left(\begin{bmatrix} \widetilde{x}^R_{11,1,t_0} \\ \vdots \\ \widetilde{x}^R_{11,O,t_0} \\ \vdots \\ \widetilde{x}^R_{1N,O,t_0} \end{bmatrix} \right) & \dots & \operatorname{diag}\left(\begin{bmatrix} \widetilde{x}^R_{N1,1,t_0} \\ \vdots \\ \widetilde{x}^R_{N1,O,t_0} \\ \vdots \\ \widetilde{x}^R_{NN,O,t_0} \end{bmatrix} \right) \end{bmatrix} \right\}$$

a $N^2O \times N^2O$ matrix,

$$\overline{\boldsymbol{M}^{xG,6}} \equiv \left(\overline{\boldsymbol{M}^u} \otimes \overline{\boldsymbol{I}_O} \right) \left\{ \mathbf{1}_N \otimes \begin{bmatrix} \operatorname{diag}\left(\begin{bmatrix} \widetilde{x}^G_{11,t_0} \\ \vdots \\ \widetilde{x}^G_{1N,t_0} \end{bmatrix} \right) & \dots & \operatorname{diag}\left(\begin{bmatrix} \widetilde{x}^G_{N1,t_0} \\ \vdots \\ \widetilde{x}^G_{NN,t_0} \end{bmatrix} \right) \end{bmatrix} \otimes \mathbf{1}_O \right\}$$

a $N^2 O \times N^2$ matrix,

$$\overline{\boldsymbol{M}^{xR,6}} \equiv \left\{ \left[\left(\overline{\boldsymbol{I}_{N^2}} - \overline{\boldsymbol{M}^u} \right) - \left(1 - \alpha^R \right) \overline{\boldsymbol{M}^u} \right] \otimes \overline{\boldsymbol{I}_O} \right\}$$
$$\times \left\{ \boldsymbol{1}_N \otimes \begin{bmatrix} \left[\widetilde{x}^R_{11,o,t_0} \right]_o & \boldsymbol{0} & \boldsymbol{0} & \dots & \left[\widetilde{x}^R_{N1,o,t_0} \right]_o & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \ddots & \boldsymbol{0} & & \boldsymbol{0} & \ddots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \left[\widetilde{x}^R_{1N,o,t_0} \right]_o & & \boldsymbol{0} & \boldsymbol{0} & \left[\widetilde{x}^R_{N3,o,t_0} \right]_o \end{bmatrix} \right\}$$

a $N^2 O \times N^2$ matrix, and

$$\overline{\boldsymbol{M}^{u,3}} \equiv \begin{bmatrix} 1 - \widehat{u_{11,t_0}} & & \boldsymbol{0} \\ & \ddots & \\ \boldsymbol{0} & & 1 - \widehat{u_{1N,t_0}} \\ & \vdots & \\ 1 - \widehat{u_{N1,t_0}} & & \boldsymbol{0} \\ & \ddots & \\ \boldsymbol{0} & & 1 - \widehat{u_{NN,t_0}} \end{bmatrix} \otimes \overline{\boldsymbol{I}_O}$$

a $N^2 O \times NO$ matrix. Finally, I have

$$\left(1 - \alpha^R\right) \overline{\boldsymbol{M}^{xG,5}} \widehat{\boldsymbol{p}^G_t} + \left[\overline{\boldsymbol{M}^{u,2}} + \overline{\boldsymbol{M}^{xR,4}} \right] \widehat{\boldsymbol{p}^R_t} + \left\{ \frac{1}{\varepsilon^R} \left(\overline{\boldsymbol{M}^u} \otimes \overline{\boldsymbol{I}_O} \right) + \overline{\boldsymbol{M}^{xR,5}} \right\} \widehat{\boldsymbol{Q}^R_t}$$
$$= - \left(\overline{\boldsymbol{M}^u} \otimes \boldsymbol{1}_O \right) d\boldsymbol{u}_t - \left(1 - \alpha^R \right) \overline{\boldsymbol{M}^{xG,6}} \widehat{\boldsymbol{\tau}^G_t} - \left[\left(\overline{\boldsymbol{M}^u} \otimes \boldsymbol{1}_O \right) + \overline{\boldsymbol{M}^{xR,6}} \right] \widehat{\boldsymbol{\tau}^R_t} + \left(\boldsymbol{1}_N \otimes \overline{\boldsymbol{I}_{NO}} \right) \widehat{\boldsymbol{\lambda}^R_t} + \overline{\boldsymbol{M}^{u,3}} \widehat{\boldsymbol{K}^R_t}.$$

By log-linearizing equation (G.10) for each i and o,

$$\widehat{p^G_{i,t}} + \sum_j \widetilde{y}^G_{ij,t_0} \widehat{Q^G_{ij,t}} - \widehat{w_{i,o,t}} + \left[-\frac{1}{\theta_o} + \left(-\frac{1}{\beta} + \frac{1}{\theta_o} \right) l^O_{i,o,t_0} \right] \widehat{L_{i,o,t}} + \left(-1 + \frac{1}{\beta} \right) \sum_{o'} \widetilde{x}^O_{i,o',t_0} l^O_{i,o',t_0} \widehat{L_{i,o',t}}$$
$$= -\frac{1}{\beta} \widehat{b_{i,o,t}} + \frac{1}{\theta_o} \frac{a_{o,t_0}}{1 - a_{o,t_0}} \widehat{a_{o,t}} + \left(-\frac{1}{\beta} + \frac{1}{\theta_o} \right) \frac{1}{\theta_o - 1} \left[- \left(1 - l^O_{i,o,t_0} \right) + l^O_{i,o,t_0} \frac{a_{o,t_0}}{1 - a_{o,t_0}} \right] \widehat{a_{o,t}}$$
$$+ \left(-1 + \frac{1}{\beta} \right) \frac{1}{\theta_o - 1} \sum_{o'} \widetilde{x}^O_{i,o',t_0} \left[- \left(1 - l^O_{i,o',t_0} \right) + l^O_{i,o',t_0} \frac{a_{o',t_0}}{1 - a_{o',t_0}} \right] \widehat{a_{o',t}}$$
$$- \sum_j y^G_{ij,t_0} \widehat{\tau^G_{ij,t}} - \left(-\frac{1}{\beta} + \frac{1}{\theta_o} \right) \left(1 - l^O_{i,o,t_0} \right) \widehat{K^R_{i,o,t}} - \left(-1 + \frac{1}{\beta} \right) \sum_{o'} \widetilde{x}^O_{i,o',t_0} \left(1 - l^O_{i,o',t_0} \right) \widehat{K^R_{i,o',t}},$$

In matrix notation, write

$$\overline{\boldsymbol{M}^{yG,3}} \equiv \overline{\boldsymbol{M}^{yG}} \otimes \mathbf{1}_O$$

a $NO \times N^2$ matrix,

$$\overline{\boldsymbol{M}^{xOl,3}} \equiv \overline{\boldsymbol{M}^{xOl}} \otimes \mathbf{1}_O$$

a $NO \times NO$ matrix,

$$\overline{\boldsymbol{M}^{a}} \equiv \mathbf{1}_N \otimes \mathrm{diag}\left(\frac{a_{o,t_0}}{1-a_{o,t_0}}\right)$$

a $NO \times O$ matrix,

$$\overline{\boldsymbol{M}^{al,2}} \equiv \begin{bmatrix} \mathrm{diag}\left(-\left(1-l^O_{1,o,t_0}\right)+l^O_{1,o,t_0}\frac{a_{o,t_0}}{1-a_{o,t_0}}\right) \\ \vdots \\ \mathrm{diag}\left(-\left(1-l^O_{N,o,t_0}\right)+l^O_{N,o,t_0}\frac{a_{o,t_0}}{1-a_{o,t_0}}\right) \end{bmatrix}$$

a $NO \times O$ matrix,

$$\overline{\boldsymbol{M}^{al,3}} \equiv \left(\widetilde{\boldsymbol{x}}^O_{t_0} \circ \overline{\boldsymbol{M}^{al}}\right) \otimes \mathbf{1}_O$$

a $NO \times O$ matrix,

$$\overline{\boldsymbol{M}^{xOl,4}} \equiv \overline{\boldsymbol{M}^{xOl,2}} \otimes \mathbf{1}_O,$$

a $NO \times NO$ matrix. I have

$$\begin{aligned}
&(\boldsymbol{I}_N \otimes \mathbf{1}_O)\,\widehat{\boldsymbol{p}^G_t} - \widehat{\boldsymbol{w}_t} + \overline{\boldsymbol{M}^{yG,3}}\widehat{\boldsymbol{Q}^G_t} + \left(-\frac{1}{\theta_o}\overline{\boldsymbol{I}_{NO}} + \left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\mathrm{diag}\left(\boldsymbol{l}^O_{t_0}\right) + \left(-1+\frac{1}{\beta}\right)\overline{\boldsymbol{M}^{xOl,3}}\right)\widehat{\boldsymbol{L}}_t \\
&= -\frac{1}{\beta}\widehat{\boldsymbol{b}}_t + \left[\frac{1}{\theta_o}\overline{\boldsymbol{M}^{a}} + \left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\frac{1}{\theta-1}\overline{\boldsymbol{M}^{al,2}} + \left(-1+\frac{1}{\beta}\right)\frac{1}{\theta-1}\overline{\boldsymbol{M}^{al,3}}\right]\widehat{\boldsymbol{a}}_t - \overline{\boldsymbol{M}^{yG,3}}\widehat{\boldsymbol{\tau}^G_t} \\
&\quad + \left[-\left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\mathrm{diag}\left(1-l^O_{i,o,t_0}\right) - \left(-1+\frac{1}{\beta}\right)\overline{\boldsymbol{M}^{xOl,4}}\right]\widehat{\boldsymbol{K}^R_t}.
\end{aligned}$$

Hence the log-linearized temporary equilibrium system is

$$\overline{\boldsymbol{D}^x}\widehat{\boldsymbol{x}} = \overline{\boldsymbol{D}^A}\widehat{\boldsymbol{A}}$$

where matrices $\overline{\boldsymbol{D}^x}$ and $\overline{\boldsymbol{D}^A}$ are defined as

$$\overline{\boldsymbol{D}^x} \equiv \begin{bmatrix} \overline{\boldsymbol{D}^x_{11}} & \mathbf{0} & \mathbf{0} & \overline{\boldsymbol{D}^x_{14}} & \mathbf{0} & \overline{\boldsymbol{D}^x_{16}} \\ -\overline{\boldsymbol{M}^{xG,2}} & \overline{\boldsymbol{I}_{NO}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \phi\overline{\boldsymbol{M}^{xG,2}} & \mathbf{0} & -\phi\overline{\boldsymbol{I}_{NO}} & \mathbf{0} & \mathbf{0} & \overline{\boldsymbol{M}^{l}} \\ \overline{\boldsymbol{D}^x_{41}} & \overline{\boldsymbol{D}^x_{42}} & \mathbf{0} & \overline{\boldsymbol{D}^x_{44}} & \overline{\boldsymbol{D}^x_{45}} & \mathbf{0} \\ \overline{\boldsymbol{D}^x_{51}} & \overline{\boldsymbol{D}^x_{52}} & \mathbf{0} & \mathbf{0} & \overline{\boldsymbol{D}^x_{55}} & \mathbf{0} \\ \overline{\boldsymbol{D}^x_{61}} & \mathbf{0} & -\overline{\boldsymbol{I}_{NO}} & \overline{\boldsymbol{M}^{yG,3}} & \mathbf{0} & \overline{\boldsymbol{D}^x_{66}} \end{bmatrix},$$

where

$$\overline{\boldsymbol{D}^x_{11}} \equiv -\alpha_M \left(\overline{\boldsymbol{I}_N} - \left(\widetilde{\boldsymbol{x}}^G_{t_0} \right)^\top \right), \overline{\boldsymbol{D}^x_{14}} \equiv (1-\alpha_M)\,\overline{\boldsymbol{M}^{yG}}, \overline{\boldsymbol{D}^x_{16}} \equiv -\alpha_L \overline{\boldsymbol{M}^{xOl}},$$

$$\overline{\boldsymbol{D}^x_{41}} \equiv \varepsilon^G \left[\overline{\boldsymbol{I}_N} \otimes \mathbf{1}_N \right] + \left(1-\varepsilon^G \right) \left[\mathbf{1}_N \otimes \left(\widetilde{\boldsymbol{x}}^G_{t_0} \right)^\top \right] - \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^G_{t_0} \right) \left[\mathbf{1}_N \otimes \overline{\boldsymbol{I}_N} \right],$$

$$\overline{\boldsymbol{D}^x_{42}} \equiv \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^V_{t_0} \right) \overline{\boldsymbol{M}^{xR}} + \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^R_{t_0} \right) \overline{\boldsymbol{M}^{yR}},$$

$$\overline{\boldsymbol{D}^x_{44}} \equiv \overline{\boldsymbol{I}_{N^2}} - \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^G_{t_0} \right) \overline{\boldsymbol{M}^{yG,2}},$$

$$\overline{\boldsymbol{D}^x_{45}} \equiv -\operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^V_{t_0} \right) \overline{\boldsymbol{M}^{xR,2}} - \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^R_{t_0} \right) \overline{\boldsymbol{M}^{yR,2}},$$

$$\overline{\boldsymbol{D}^x_{51}} \equiv \left(1-\alpha^R \right) \overline{\boldsymbol{M}^{xG,5}}, \overline{\boldsymbol{D}^x_{52}} \equiv \overline{\boldsymbol{M}^{u,2}} + \overline{\boldsymbol{M}^{xR,4}}, \overline{\boldsymbol{D}^x_{55}} \equiv \frac{1}{\varepsilon^R} \left(\overline{\boldsymbol{M}^{u}} \otimes \overline{\boldsymbol{I}_O} \right) + \overline{\boldsymbol{M}^{xR,5}},$$

$$\overline{\boldsymbol{D}^x_{61}} \equiv \boldsymbol{I}_N \otimes \mathbf{1}_N, \overline{\boldsymbol{D}^x_{66}} \equiv -\frac{1}{\theta_o} + \left(-\frac{1}{\beta} + \frac{1}{\theta_o} \right) \operatorname{diag}\left(\boldsymbol{l}^O_{t_0} \right) + \left(-1 + \frac{1}{\beta} \right) \overline{\boldsymbol{M}^{xOl,3}},$$

and

$$\overline{\boldsymbol{D}^A} \equiv \begin{bmatrix} \mathbf{0} & \overline{\boldsymbol{D}^A_{12}} & \overline{\boldsymbol{D}^A_{13}} & \overline{\boldsymbol{I}_N} & \mathbf{0} & \overline{\boldsymbol{D}^A_{16}} & \overline{\boldsymbol{D}^A_{17}} & \mathbf{0} & \alpha_L\overline{\boldsymbol{M}^{xOl,2}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\overline{\boldsymbol{I}_{NO}} & \mathbf{0} & \overline{\boldsymbol{M}^{xG}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\phi\overline{\boldsymbol{M}^{xG,3}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \overline{\boldsymbol{D}^A_{47}} & \overline{\boldsymbol{D}^A_{48}} & \mathbf{0} & \mathbf{0} \\ \overline{\boldsymbol{D}^A_{51}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \overline{\boldsymbol{D}^A_{57}} & \overline{\boldsymbol{D}^A_{58}} & \overline{\boldsymbol{M}^{u,3}} & \overline{\boldsymbol{D}^A_{5,10}} \\ \mathbf{0} & \overline{\boldsymbol{D}^A_{62}} & -\frac{1}{\beta}\overline{\boldsymbol{I}_{NO}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\overline{\boldsymbol{M}^{yG,3}} & \mathbf{0} & \overline{\boldsymbol{D}^A_{69}} & \mathbf{0} \end{bmatrix},$$

where

$$\overline{\boldsymbol{D}^A_{12}} \equiv \frac{\alpha_L}{\theta-1} \left(\widetilde{\boldsymbol{x}}^O_{t_0} \otimes \overline{\boldsymbol{M}^{al}} \right), \overline{\boldsymbol{D}^A_{13}} \equiv \frac{\alpha_L}{\beta-1} \overline{\boldsymbol{M}^{xO}},$$

$$\overline{\boldsymbol{D}_{16}^{A}} \equiv (1-\alpha_L-\alpha_M)\,\overline{\boldsymbol{I}_N},\ \overline{\boldsymbol{D}_{17}^{A}} \equiv -\left[\alpha_M\overline{\boldsymbol{M}^{xG}}+(1-\alpha_M)\,\overline{\boldsymbol{M}^{yG}}\right],$$

$$\overline{\boldsymbol{D}_{47}^{A}} \equiv -\varepsilon^{G}+\left(1-\varepsilon^{G}\right)\overline{\boldsymbol{M}^{xG,4}}+\mathrm{diag}\left(\mathbf{1}_N\otimes \boldsymbol{s}_{t_0}^{G}\right)\overline{\boldsymbol{M}^{yG,2}},$$

$$\overline{\boldsymbol{D}_{48}^{A}} \equiv \mathrm{diag}\left(\mathbf{1}_N\otimes \boldsymbol{s}_{t_0}^{V}\right)\overline{\boldsymbol{M}^{xR,3}}+\mathrm{diag}\left(\mathbf{1}_N\otimes \boldsymbol{s}_{t_0}^{R}\right)\overline{\boldsymbol{M}^{yR,3}},$$

$$\overline{\boldsymbol{D}_{51}^{A}} \equiv -\left(\overline{\boldsymbol{M}^{u}}\otimes \mathbf{1}_O\right),\ \overline{\boldsymbol{D}_{57}^{A}} \equiv -\left(1-\alpha^{R}\right)\overline{\boldsymbol{M}^{xG,6}},$$

$$\overline{\boldsymbol{D}_{58}^{A}} \equiv -\left[\left(\overline{\boldsymbol{M}^{u}}\otimes \mathbf{1}_O\right)+\overline{\boldsymbol{M}^{xR,6}}\right],\ \overline{\boldsymbol{D}_{5,10}^{A}} \equiv \mathbf{1}_N\otimes\overline{\boldsymbol{I}_{NO}},$$

$$\overline{\boldsymbol{D}_{62}^{A}} \equiv \frac{1}{\theta}\overline{\boldsymbol{M}^{a}}+\left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\frac{1}{\theta_o-1}\overline{\boldsymbol{M}^{al,2}}+\left(-1+\frac{1}{\beta}\right)\frac{1}{\theta_o-1}\overline{\boldsymbol{M}^{al,3}},$$

and

$$\overline{\boldsymbol{D}_{69}^{A}} \equiv -\left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\mathrm{diag}\left(1-l_{i,o,t_0}^{O}\right)-\left(-1+\frac{1}{\beta}\right)\overline{\boldsymbol{M}^{xOl,4}}.$$

To normalize the price, one of the good-demand equation must be replaced with log-linearized numeraire condition $\widehat{P_{1,t}^{G}}=\sum_i x_{i1,t_0}^{G}\left(\widehat{p_{i,t}^{G}}+\widehat{\tau_{i1,t}^{G}}\right)=0$, or

$$\overline{\boldsymbol{M}^{xG,num}}\widehat{\boldsymbol{p}_t^{G}}=-\overline{\boldsymbol{M}^{xG,num}}\widehat{\boldsymbol{\tau}_t^{G}},$$

where $\overline{\boldsymbol{M}^{xG,num}} \equiv \left[x_{11,t_0}^{G},x_{21,t_0}^{G},x_{31,t_0}^{G}\right]$.

To analyze the steady state conditions, first note that the steady state accumulation condition (G.26) implies $\widehat{Q_{i,o}^{R}}=\widehat{K_{i,o}^{R}}$. Using robot integration function, integration demand and unit cost formula, I have

$$\widehat{Q_{i,o}^{R}}=\sum_l x_{li,o,t_0}^{R}\widehat{Q_{li,o}^{R}}+\left(1-\alpha^{R}\right)\left(\sum_l \widetilde{x}_{ij,o,t_0}^{R}\widehat{p_{li,o}^{R}}-\sum_l \widetilde{x}_{li,t_0}^{G}\widehat{p_{li,t}^{G}}\right) \tag{I.6}$$

Thus the condition is

$$\begin{aligned}&\sum_l \widetilde{x}_{li,o,t_0}^{R}\widehat{Q_{li,o}^{R}}+\left(1-\alpha^{R}\right)\sum_l \widetilde{x}_{li,o,t_0}^{R}\widehat{p_{l,o}^{R}}-\left(1-\alpha^{R}\right)\sum_l \widetilde{x}_{li,t_0}^{G}\widehat{p_l^{G}}-\widehat{K_{i,o}^{R}}\\&=\left(1-\alpha^{R}\right)\sum_l \widetilde{x}_{li,t_0}^{G}\widehat{\tau_{li}^{G}}-\left(1-\alpha^{R}\right)\sum_l \widetilde{x}_{li,o,t_0}^{R}\widehat{\tau_{li}^{R}}.\end{aligned}$$

In a matrix form, write

$$\overline{\boldsymbol{M}^{xR,7}} \equiv \begin{bmatrix} \operatorname{diag}\left(\widetilde{x}^{R}_{1\cdot,\cdot,t_0}\right) & \dots & \operatorname{diag}\left(\widetilde{x}^{R}_{N\cdot,\cdot,t_0}\right) \end{bmatrix}$$

a $NO \times N^2O$ matrix,

$$\overline{\boldsymbol{M}^{xR,8}} \equiv \begin{bmatrix} \operatorname{diag}\left(\widetilde{x}^{R}_{11,\cdot,t_0}\right) & \dots & \operatorname{diag}\left(\widetilde{x}^{R}_{N1,\cdot,t_0}\right) \\ \vdots & & \vdots \\ \operatorname{diag}\left(\widetilde{x}^{R}_{1N,\cdot,t_0}\right) & \dots & \operatorname{diag}\left(\widetilde{x}^{R}_{NN,\cdot,t_0}\right) \end{bmatrix}$$

a $NO \times NO$ matrix, and

$$\overline{\boldsymbol{M}^{xG,7}} \equiv \begin{bmatrix} \widetilde{x}^{G}_{11,t_0} & & & \dots & \widetilde{x}^{G}_{N1,t_0} & & \mathbf{0} \\ & \ddots & & & & \ddots & \\ \mathbf{0} & & \widetilde{x}^{G}_{1N,t_0} & & & & \widetilde{x}^{G}_{NN,t_0} \end{bmatrix} \otimes \mathbf{1}_O$$

a $NO \times N^2$ matrix.

$$\overline{\boldsymbol{M}^{xR,9}} \equiv \begin{bmatrix} \widetilde{\boldsymbol{x}}^{R}_{11,\cdot,t_0} & & \mathbf{0} & & \widetilde{\boldsymbol{x}}^{R}_{N1,\cdot,t_0} & & \mathbf{0} \\ & \ddots & & \dots & & \ddots & \\ \mathbf{0} & & \widetilde{\boldsymbol{x}}^{R}_{1N,\cdot,t_0} & & \mathbf{0} & & \widetilde{\boldsymbol{x}}^{R}_{NN,\cdot,t_0} \end{bmatrix},$$

a $NO \times N^2$ matrix, where $\widetilde{\boldsymbol{x}}^{R}_{ij,\cdot,t_0} \equiv \left(\widetilde{x}^{R}_{ij,o,t_0}\right)_o$ is an $O \times 1$ vector for any i and j. Then I have

$$-\left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xG,2}}\widehat{\boldsymbol{p}^G} + \left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xR,8}}\widehat{\boldsymbol{p}^R} + \overline{\boldsymbol{M}^{xR,7}}\widehat{\boldsymbol{Q}^R} - \widehat{\boldsymbol{K}^R} = \left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xG,7}}\widehat{\boldsymbol{\tau}^G} - \left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xR,9}}\widehat{\boldsymbol{\tau}^R}$$

Next, to study the steady state Euler equation (G.27) , note that by equation (G.13),

$$
\begin{aligned}
\frac{\partial \pi_{i,t}\left(\widehat{\left\{K^R_{i,o,t}\right\}}\right)}{\partial K^R_{i,o,t}} &= \sum_j \widetilde{y}^G_{ij,t}\left(\widehat{p^G_{ij,t}}+\widehat{Q^G_{ij,t}}\right)+\left[-\frac{1}{\beta}\sum_{o'} x^O_{i,o',t_0}\widehat{b_{i,o',t}}+\frac{1}{\beta}\widehat{b_{i,o,t}}\right] \\
&+\left\{\left(-1+\frac{1}{\beta}\right)\frac{1}{\theta_o-1}\sum_{o'}\frac{\widetilde{x}^O_{i,o',t_0}}{1-a_{o,t_0}}\left[-l^O_{i,o',t_0}a_{o,t_0}+\left(1-l^O_{i,o',t_0}\right)\left(1-a_{o,t_0}\right)\right]\widehat{a_{o',t}}\right. \\
&+\left.\left\{\left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\frac{1}{\theta_o-1}\frac{-l^O_{i,o,t_0}a_{o,t_0}+\left(1-l^O_{i,o,t_0}\right)\left(1-a_{o,t_0}\right)}{1-a_{o,t_0}}+\frac{1}{\theta_o}\right\}\widehat{a_{o,t}}\right\} \\
&+\left[\left(-1+\frac{1}{\beta}\right)\sum_{o'} x^O_{i,o',t_0}l^O_{i,o',t_0}\widehat{L_{i,o',t}}+\left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)l^O_{i,o,t_0}\widehat{L_{i,o,t}}\right] \\
&+\left[\left(-1+\frac{1}{\beta}\right)\sum_{o'}\widetilde{x}^O_{i,o',t_0}\left(1-l^O_{i,o',t_0}\right)\widehat{K^R_{i,o',t}}+\left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\left(1-l^O_{i,o,t_0}\right)\widehat{K^R_{i,o,t}}+\left(-\frac{1}{\theta_o}\right)\widehat{K^R_{i,o,t}}\right].
\end{aligned}
\tag{I.7}
$$

Note that by the steady state accumulation condition (G.26), $Q^R_{i,o,t_0}/K^R_{i,o,t_0} = \delta$. Note also that investment function implies that, in the steady state,

$$
\frac{\lambda^R_{j,o}}{P^R_{j,o}} = \left(\sum_i \frac{x^R_{ij,o}}{\left(1+u_{ij}\right)^{1-\varepsilon^R}}\right)^{\frac{1}{1-\varepsilon^R}\alpha^R} + 2\gamma\delta. \tag{I.8}
$$

To simplify the notation, set

$$
\widetilde{u}^{SS}_{j,o,t_0} \equiv \frac{(\iota+\delta)\left[\left(\sum_i x^R_{ij,o,t_0}\left(1+u_{ij,t_0}\right)^{-\left(1-\varepsilon^R\right)}\right)^{\frac{1}{1-\varepsilon^R}\alpha^R}+2\gamma\delta\right]}{(\iota+\delta)\left[\left(\sum_i x^R_{ij,o,t_0}\left(1+u_{ij,t_0}\right)^{-\left(1-\varepsilon^R\right)}\right)^{\frac{1}{1-\varepsilon^R}\alpha^R}+2\gamma\delta\right]-\gamma\delta^2},
$$

Then, by log-linearizing equation (G.27), after rearranging, I have:

$$
\begin{aligned}
&\left[\widehat{p_i^G} + 2\left(1-\alpha^R\right)\left(1-\widetilde{u}^{SS}_{i,o,t_0}\right)\sum_l \widetilde{x}^G_{li,t_0}\widehat{p^G_{l,t}}\right] - \left(1-\widetilde{u}^{SS}_{i,o,t_0}\right)\widehat{p^R_{i,o}} - 2\left(1-\alpha^R\right)\left(1-\widetilde{u}^{SS}_{i,o,t_0}\right)\sum_l \widetilde{x}^R_{ij,o,t_0}\widehat{p^R_{l,o}} \\
&+\sum_j \widetilde{y}^G_{ij,t_0}\widehat{Q^G_{ij}} - 2\left(1-\widetilde{u}^{SS}_{i,o,t_0}\right)\sum_l \widetilde{x}^R_{li,o,t_0}\widehat{Q^R_{li,o}} + \left[\left(-1+\frac{1}{\beta}\right)\sum_{o'}\widetilde{x}^O_{i,o',t_0} l^O_{i,o',t_0}\widehat{L_{i,o'}} + \left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right) l^O_{i,o,t_0}\widehat{L_{i,o}}\right] \\
&+\left[\left(-1+\frac{1}{\beta}\right)\sum_{o'}\widetilde{x}^O_{i,o',t_0}\left(1-l^O_{i,o',t_0}\right)\widehat{K^R_{i,o'}} + \left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\left(1-l^O_{i,o,t_0}\right)\widehat{K^R_{i,o}} + \left(-\frac{1}{\theta_o}\right)\widehat{K^R_{i,o}} + 2\left(1-\widetilde{u}^{SS}_{i,o,t_0}\right)\widehat{K^R_{i,o}}\right] \\
&-\widetilde{u}^{SS}_{i,o,t_0}\widehat{\lambda^R_{i,o}} \\
&= -\left(-1+\frac{1}{\beta}\right)\frac{1}{\theta_o-1}\sum_{o'}\frac{\widetilde{x}^O_{i,o',t_0}}{1-a_{o,t_0}}\left[\left(1-l^O_{i,o',t_0}\right)\left(1-a_{o',t_0}\right) - l^O_{i,o',t_0}a_{o',t_0}\right]\widehat{a_{o'}} \\
&-\left\{\left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\frac{1}{\theta_o-1}\frac{1}{1-a_{o,t_0}}\left[\left(1-l^O_{i,o,t_0}\right)\left(1-a_{o,t_0}\right) - l^O_{i,o,t_0}a_{o,t_0}\right] + \frac{1}{\theta_o}\right\}\widehat{a_o} \\
&-\left[-\frac{1}{\beta}\sum_{o'}\widetilde{x}^O_{i,o',t_0}\widehat{b_{i,o'}} + \frac{1}{\beta}\widehat{b_{i,o}}\right] + \left[-\sum_j \widetilde{y}^G_{ij,t_0}\widehat{\tau^G_{ij}} - 2\left(1-\alpha^R\right)\left(1-\widetilde{u}^{SS}_{i,o,t_0}\right)\sum_l \widetilde{x}^G_{li,t_0}\widehat{\tau^G_{li,t}}\right] \\
&+2\left(1-\alpha^R\right)\left(1-\widetilde{u}^{SS}_{i,o,t_0}\right)\sum_l \widetilde{x}^R_{ij,o,t_0}\widehat{\tau^R_{li}}
\end{aligned}
$$

In matrix notation, write

$$
\overline{\boldsymbol{M}^{xO,3}} \equiv \overline{\boldsymbol{M}^{xO}} \otimes \mathbf{1}_O
$$

a $NO \times N^2$ matrix. Then

$$
\begin{aligned}
&\left[\left(\overline{\boldsymbol{I}_N}\otimes \mathbf{1}_O\right) + 2\left(1-\alpha^R\right)\operatorname{diag}\left(1-\widetilde{u}^{SS}_{\cdot,\cdot,t_0}\right)\overline{\boldsymbol{M}^{xG,2}}\right]\widehat{\boldsymbol{p}^G} - \operatorname{diag}\left(1-\widetilde{u}^{SS}_{\cdot,\cdot,t_0}\right)\left(\overline{\boldsymbol{I}_{NO}} - 2\left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xR,8}}\right)\widehat{\boldsymbol{p}^R} \\
&+\overline{\boldsymbol{M}^{yG,3}}\widehat{\boldsymbol{Q}^G} - 2\operatorname{diag}\left(1-\widetilde{u}^{SS}_{\cdot,\cdot,t_0}\right)\overline{\boldsymbol{M}^{xR,7}}\widehat{\boldsymbol{Q}^R} + \left[\left(-1+\frac{1}{\beta}\right)\overline{\boldsymbol{M}^{xOl,3}} + \left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\operatorname{diag}\left(l^O_{\cdot,\cdot,t_0}\right)\right]\widehat{\boldsymbol{L}} \\
&+\left[\left(-1+\frac{1}{\beta}\right)\overline{\boldsymbol{M}^{xOl,4}} + \left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\operatorname{diag}\left(1-l^O_{\cdot,\cdot,t_0}\right) - \frac{1}{\theta_o}\overline{\boldsymbol{I}_{NO}} + 2\operatorname{diag}\left(1-\widetilde{u}^{SS}_{\cdot,\cdot,t_0}\right)\right]\widehat{\boldsymbol{K}^R} - \operatorname{diag}\left(\widetilde{u}^{SS}_{\cdot,\cdot,t_0}\right)\widehat{\boldsymbol{\lambda}^R} \\
&= -\left[\left(-1+\frac{1}{\beta}\right)\frac{1}{\theta_o-1}\overline{\boldsymbol{M}^{al,3}} - \left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\frac{1}{\theta_o-1}\overline{\boldsymbol{M}^{al,2}}\right]\widehat{\boldsymbol{a}} - \frac{1}{\beta}\left(\overline{\boldsymbol{I}_{NO}} - \overline{\boldsymbol{M}^{xO,3}}\right)\widehat{\boldsymbol{b}} \\
&+\left[-\overline{\boldsymbol{M}^{yG,3}} - 2\left(1-\alpha^R\right)\operatorname{diag}\left(1-\widetilde{u}^{SS}_{\cdot,\cdot,t_0}\right)\overline{\boldsymbol{M}^{xG,7}}\right]\widehat{\boldsymbol{\tau}^G} + 2\left(1-\alpha^R\right)\operatorname{diag}\left(1-\widetilde{u}^{SS}_{\cdot,\cdot,t_0}\right)\overline{\boldsymbol{M}^{xR,9}}\widehat{\boldsymbol{\tau}^R}
\end{aligned}
$$

In the steady state, I write equations (I.3) and (I.4) as

$$
\overline{\boldsymbol{M}^{xG,2}}\widehat{\boldsymbol{p}^G} - \widehat{\boldsymbol{w}} + \left[\overline{\boldsymbol{I}_{NO}} - \frac{1}{1+\iota}\overline{\boldsymbol{M}^{\mu,2}}\right]\widehat{\boldsymbol{V}} = -\overline{\boldsymbol{M}^{xG,7}}\widehat{\boldsymbol{\tau}^G} + d\boldsymbol{T} - \overline{\boldsymbol{M}^{\mu,3}}d\boldsymbol{\chi}^{\text{vec}}
$$

and

$$\left[\overline{\boldsymbol{I}_{NO}} - \overline{\boldsymbol{M}^{\mu L}}\right] \widehat{\boldsymbol{L}} - \overline{\boldsymbol{M}^{\mu L,2}} \widehat{\boldsymbol{\mu}^{\text{vec}}} = \boldsymbol{0}.$$

respectively.

Hence the log-linearized steady state system is

$$\overline{\boldsymbol{E}^{y}} \widehat{\boldsymbol{y}} = \overline{\boldsymbol{E}^{\Delta}} \boldsymbol{\Delta},$$

where

$$\overline{\boldsymbol{E}^{y}} \equiv \begin{bmatrix} \overline{\boldsymbol{D}^{x}} & -\overline{\boldsymbol{D}^{A,T}} \\ \multicolumn{2}{c}{\overline{\boldsymbol{D}^{y,SS}}} \end{bmatrix}, \text{ and } \overline{\boldsymbol{E}^{\Delta}} \equiv \begin{bmatrix} \overline{\boldsymbol{D}^{A,\Delta}} \\ \overline{\boldsymbol{D}^{\Delta,SS}} \end{bmatrix},$$

$\overline{\boldsymbol{D}^{A}} \equiv \begin{bmatrix} \overline{\boldsymbol{D}^{A,T}} & \overline{\boldsymbol{D}^{A,\Delta}} \end{bmatrix}$, and matrices $\overline{\boldsymbol{D}^{y,SS}}$ and $\overline{\boldsymbol{D}^{\Delta,SS}}$ are defined as

$$\overline{\boldsymbol{D}^{y,SS}} \equiv \begin{bmatrix} \overline{\boldsymbol{D}_{11}^{y,SS}} & \overline{\boldsymbol{D}_{12}^{y,SS}} & \boldsymbol{0} & \boldsymbol{0} & \overline{\boldsymbol{M}^{xR,7}} & \boldsymbol{0} & -\overline{\boldsymbol{I}_{NO}} & \boldsymbol{0} \\ \overline{\boldsymbol{D}_{21}^{y,SS}} & \overline{\boldsymbol{D}_{22}^{y,SS}} & \boldsymbol{0} & \overline{\boldsymbol{M}^{yG,3}} & \overline{\boldsymbol{D}_{25}^{y,SS}} & \overline{\boldsymbol{D}_{26}^{y,SS}} & \overline{\boldsymbol{D}_{27}^{y,SS}} & \overline{\boldsymbol{D}_{28}^{y,SS}} \end{bmatrix},$$

where

$$\overline{\boldsymbol{D}_{11}^{y,SS}} \equiv -\left(1-\alpha^{R}\right) \overline{\boldsymbol{M}^{xG,2}},$$

$$\overline{\boldsymbol{D}_{12}^{y,SS}} \equiv \left(1-\alpha^{R}\right) \overline{\boldsymbol{M}^{xR,8}},$$

$$\overline{\boldsymbol{D}_{21}^{y,SS}} \equiv \left(\overline{\boldsymbol{I}_{N}} \otimes \boldsymbol{1}_{O}\right) + 2\left(1-\alpha^{R}\right) \operatorname{diag}\left(1-\widetilde{u}_{\cdot,\cdot,t_0}^{SS}\right) \overline{\boldsymbol{M}^{xG,2}},$$

$$\overline{\boldsymbol{D}_{22}^{y,SS}} \equiv -\operatorname{diag}\left(1-\widetilde{u}_{\cdot,\cdot,t_0}^{SS}\right) \left(\overline{\boldsymbol{I}_{NO}} + 2\left(1-\alpha^{R}\right) \overline{\boldsymbol{M}^{xR,8}}\right),$$

$$\overline{\boldsymbol{D}_{25}^{y,SS}} \equiv -2\operatorname{diag}\left(1-\widetilde{u}_{\cdot,\cdot,t_0}^{SS}\right) \overline{\boldsymbol{M}^{xR,7}},$$

$$\overline{\boldsymbol{D}_{26}^{y,SS}} \equiv \left(-1+\frac{1}{\beta}\right) \overline{\boldsymbol{M}^{xOl,3}} + \left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right) \operatorname{diag}\left(l_{\cdot,\cdot,t_0}^{O}\right),$$

$$\overline{\boldsymbol{D}_{27}^{y,SS}} \equiv \left(-1+\frac{1}{\beta}\right) \overline{\boldsymbol{M}^{xOl,4}} + \left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right) \operatorname{diag}\left(1-l_{\cdot,\cdot,t_0}^{O}\right) - \frac{1}{\theta_o} \overline{\boldsymbol{I}_{NO}} + 2\operatorname{diag}\left(1-\widetilde{u}_{\cdot,\cdot,t_0}^{SS}\right),$$

$$\overline{\boldsymbol{D}_{28}^{y,SS}} \equiv -\operatorname{diag}\left(\widetilde{u}_{\cdot,\cdot,t_0}^{SS}\right),$$

and

$$\overline{\boldsymbol{D}^{\Delta,SS}} \equiv \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \overline{\boldsymbol{D}_{17}^{\Delta,SS}} & \overline{\boldsymbol{D}_{18}^{\Delta,SS}} \\ \mathbf{0} & \overline{\boldsymbol{D}_{22}^{\Delta,SS}} & \overline{\boldsymbol{D}_{23}^{\Delta,SS}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \overline{\boldsymbol{D}_{27}^{\Delta,SS}} & \overline{\boldsymbol{D}_{28}^{\Delta,SS}} \end{bmatrix},$$

where

$$\overline{\boldsymbol{D}_{17}^{\Delta,SS}} \equiv \left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xG,7}},$$

$$\overline{\boldsymbol{D}_{18}^{\Delta,SS}} \equiv -\left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xR,9}},$$

$$\overline{\boldsymbol{D}_{22}^{\Delta,SS}} \equiv \left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\frac{1}{\theta_o-1}\overline{\boldsymbol{M}^{al,2}} - \left(-1+\frac{1}{\beta}\right)\frac{1}{\theta_o-1}\overline{\boldsymbol{M}^{al,3}},$$

$$\overline{\boldsymbol{D}_{23}^{\Delta,SS}} \equiv -\frac{1}{\beta}\left(\overline{\boldsymbol{I}_{NO}}-\overline{\boldsymbol{M}^{xO,3}}\right),$$

$$\overline{\boldsymbol{D}_{27}^{\Delta,SS}} \equiv -\overline{\boldsymbol{M}^{yG,3}} - 2\left(1-\alpha^R\right)\operatorname{diag}\left(1-\widetilde{u}_{\cdot,\cdot,t_0}^{SS}\right)\overline{\boldsymbol{M}^{xG,7}},$$

and

$$\overline{\boldsymbol{D}_{28}^{\Delta,SS}} \equiv 2\left(1-\alpha^R\right)\operatorname{diag}\left(1-\widetilde{u}_{\cdot,\cdot,t_0}^{SS}\right)\overline{\boldsymbol{M}^{xR,9}}.$$

If $\overline{\boldsymbol{E}^y}$ is invertible, I have $\overline{\boldsymbol{E}} \equiv \left(\overline{\boldsymbol{E}^y}\right)^{-1}\overline{\boldsymbol{E}^\Delta}$ such that $\widehat{\boldsymbol{y}} = \overline{\boldsymbol{E}}\boldsymbol{\Delta}$. Write dimensions of $\boldsymbol{y}$ and $\boldsymbol{\Delta}$ as $n_y \equiv N+3NO+N^2+N^2O$ and $n_\Delta \equiv 3N^2+O+2NO+2N$, respectively.

Finally, to study the transitional dynamics, the capital accumulation dynamics (8) implies

$$\check{K_{i,o,t+1}^{R}} = -\delta\left(1-\alpha^R\right)\sum_l \widetilde{x}_{li,t_0}^{G}\check{p_{l,t}^{G}} + \delta\left(1-\alpha^R\right)\sum_l \widetilde{x}_{li,o}^{R}\check{p_{l,o,t}^{R}} + \delta\sum_l \widetilde{x}_{li,o}^{R}\check{Q_{li,o,t}^{R}} + (1-\delta)\check{K_{i,o,t}^{R}}.$$

In a matrix form, write

$$\check{\boldsymbol{K}_{t+1}^{R}} = -\delta\left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xG,2}}\check{\boldsymbol{p}_t^G} + \delta\left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xR,8}}\check{\boldsymbol{p}_t^R} + \delta\overline{\boldsymbol{M}^{xR,7}}\check{\boldsymbol{Q}_t^R} + (1-\delta)\overline{\boldsymbol{I}_{NO}}\check{\boldsymbol{K}_t^R}.$$

Next, to study the Euler equation, define

$$\widetilde{u}_{i,o}^{TD,1} \equiv \frac{-(\iota+\delta)\left[\left(\sum_l x_{li,o}^R\left(1+u_{li}\right)^{-\left(1-\varepsilon^R\right)}\right)^{\frac{1}{1-\varepsilon^R}\alpha^R}+2\gamma\delta\right]+\gamma\delta^2}{(1-\delta)\left[\left(\sum_l x_{li,o}^R\left(1+u_{li}\right)^{-\left(1-\varepsilon^R\right)}\right)^{\frac{1}{1-\varepsilon^R}\alpha^R}+2\gamma\delta\right]}$$

and

$$\widetilde{u}_{i,o}^{TD,2} \equiv \frac{-\gamma\delta^2}{(1-\delta)\left[\left(\sum_l x_{li,o}^R\left(1+u_{li}\right)^{-(1-\varepsilon^R)}\right)^{\frac{1}{1-\varepsilon^R}\alpha^R}+2\gamma\delta\right]}.$$

Then I have

$$\begin{aligned}
&\left[-\widetilde{u}_{i,o}^{TD,1} p_{i,t+1}^{\check{G}} + 2\left(1-\alpha^R\right)\widetilde{u}_{i,o}^{TD,2}\sum_l \widetilde{x}_{li}^G p_{l,t+1}^{\check{G}}\right] + \left[-\widetilde{u}_{i,o}^{TD,2} p_{i,o,t+1}^{\check{R}} - 2\left(1-\alpha^R\right)\widetilde{u}_{i,o}^{TD,2}\sum_l \widetilde{x}_{li,o}^R p_{l,o,t+1}^{\check{R}}\right]\\
&-\widetilde{u}_{i,o}^{TD,1}\sum_j \widetilde{y}_{ij}^G Q_{ij,t+1}^{\check{G}} - 2\widetilde{u}_{i,o}^{TD,2}\sum_l \widetilde{x}_{li,o}^R Q_{li,o,t+1}^{\check{R}} - \widetilde{u}_{i,o}^{TD,1}\left(-1+\frac{1}{\beta}\right)\sum_{o'} x_{i,o'}^O\left(1-l_{i,o'}^O\right) K_{i,o',t+1}^{\check{R}}\\
&-\widetilde{u}_{i,o}^{TD,1}\left[\left(-1+\frac{1}{\beta}\right)\sum_{o'} x_{i,o'}^O l_{i,o'}^O L_{i,o',t+1}^{\check{}} + \left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right) l_{i,o}^O L_{i,o,t+1}^{\check{}}\right]\\
&-\left[\widetilde{u}_{i,o}^{TD,1}\left\{\left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\left(1-l_{i,o}^O\right)+\left(-\frac{1}{\theta_o}\right)\right\} - 2\widetilde{u}_{i,o}^{TD,2}\right] K_{i,o,t+1}^{\check{R}} + \lambda_{i,o,t+1}^{\check{R}} = \frac{1+\iota}{1-\delta}\lambda_{i,o,t}^{\check{R}}
\end{aligned}$$

In a matrix form, write

$$\overline{\boldsymbol{M}^{u,4}} = \begin{bmatrix} \widetilde{\boldsymbol{u}}_{1,\cdot}^{TD,1} & & \boldsymbol{0} \\ & \ddots & \\ \boldsymbol{0} & & \widetilde{\boldsymbol{u}}_{N,\cdot}^{TD,1} \end{bmatrix},$$

a $NO \times N$ matrix where $\widetilde{\boldsymbol{u}}_{i,\cdot}^{TD,1} \equiv \left(\widetilde{u}_{i,o}^{TD,1}\right)_o$ is an $O \times 1$ vector for any i. Then

$$\begin{aligned}
&\left(-\overline{\boldsymbol{M}^{u,4}} + 2\left(1-\alpha^R\right)\operatorname{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,2}\right)\overline{\boldsymbol{M}^{xG,2}}\right)\boldsymbol{p}_{t+1}^{\check{G}} - \operatorname{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,2}\right)\left(\overline{\boldsymbol{I}_{NO}} + 2\left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xR,8}}\right)\boldsymbol{p}_{t+1}^{\check{R}}\\
&\quad - \left[\left(\overline{\boldsymbol{M}^{u,4}}\otimes\left(\boldsymbol{1}_N\right)^\top\right)\circ\overline{\boldsymbol{M}^{yG,3}}\right]\boldsymbol{Q}_{t+1}^{\check{G}} - 2\left(\left(\boldsymbol{1}_N\right)^\top\otimes\operatorname{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,2}\right)\right)\circ\overline{\boldsymbol{M}^{xR,7}}\boldsymbol{Q}_{t+1}^{\check{R}}\\
&\quad + \left[-\left(-1+\frac{1}{\beta}\right)\left(\left(\overline{\boldsymbol{M}^{u,4}}\otimes\left(\boldsymbol{1}_O\right)^\top\right)\circ\overline{\boldsymbol{M}^{xOl,3}}\right) - \left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\operatorname{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,1} l_{\cdot,\cdot}^O\right)\right]\boldsymbol{L}_{t+1}^{\check{}}\\
&\quad + \left\{\left(-1+\frac{1}{\beta}\right)\left(\left(\overline{\boldsymbol{M}^{u,4}}\otimes\left(\boldsymbol{1}_O\right)^\top\right)\circ\overline{\boldsymbol{M}^{xOl,4}}\right) - \left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\operatorname{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,1}\left(1-l_{\cdot,\cdot}^O\right)\right)\right.\\
&\quad \left. + \frac{1}{\theta}\operatorname{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,1}\right) + 2\operatorname{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,2}\right)\right\}\boldsymbol{K}_{t+1}^{\check{R}} + \overline{\boldsymbol{I}_{NO}}\boldsymbol{\lambda}_{t+1}^{\check{R}} = \frac{1+\iota}{1-\delta}\overline{\boldsymbol{I}_{NO}}\boldsymbol{\lambda}_t^{\check{R}}.
\end{aligned}$$

Hence the log-linearized transitional dynamic system is $\overline{\boldsymbol{D}_{t+1}^{y,TD}}\check{\boldsymbol{y}}_{t+1} = \overline{\boldsymbol{D}_t^{y,TD}}\check{\boldsymbol{y}}_t$, where matrices $\overline{\boldsymbol{D}_{t+1}^{y,TD}}$ and $\overline{\boldsymbol{D}_t^{y,TD}}$ are defined as

$$\overline{\boldsymbol{D}_{t+1}^{y,TD}} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \overline{\boldsymbol{I}_{NO}} & \boldsymbol{0} \\ \overline{\boldsymbol{D}_{21,t+1}^{y,TD}} & \overline{\boldsymbol{D}_{22,t+1}^{y,TD}} & \boldsymbol{0} & \overline{\boldsymbol{D}_{24,t+1}^{y,TD}} & \overline{\boldsymbol{D}_{25,t+1}^{y,TD}} & \overline{\boldsymbol{D}_{26,t+1}^{y,TD}} & \overline{\boldsymbol{D}_{27,t+1}^{y,TD}} & \overline{\boldsymbol{I}_{NO}} \end{bmatrix},$$

where

$$\overline{\boldsymbol{D}_{21,t+1}^{y,TD}} \equiv -\overline{\boldsymbol{M}^{u,4}} + 2\left(1-\alpha^{R}\right)\operatorname{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,2}\right)\overline{\boldsymbol{M}^{xG,2}},$$

$$\overline{\boldsymbol{D}_{22,t+1}^{y,TD}} \equiv -\operatorname{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,2}\right)\left(\overline{\boldsymbol{I}_{NO}} + 2\left(1-\alpha^{R}\right)\overline{\boldsymbol{M}^{xR,8}}\right),$$

$$\overline{\boldsymbol{D}_{24,t+1}^{y,TD}} \equiv -\left(\overline{\boldsymbol{M}^{u,4}} \otimes (\mathbf{1}_N)^{\top}\right) \circ \overline{\boldsymbol{M}^{yG,3}},$$

$$\overline{\boldsymbol{D}_{25,t+1}^{y,TD}} \equiv -2\left((\mathbf{1}_N)^{\top} \otimes \operatorname{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,2}\right)\right) \circ \overline{\boldsymbol{M}^{xR,7}},$$

$$\overline{\boldsymbol{D}_{26,t+1}^{y,TD}} \equiv -\left(-1+\frac{1}{\beta}\right)\left(\left(\overline{\boldsymbol{M}^{u,4}} \otimes (\mathbf{1}_O)^{\top}\right) \circ \overline{\boldsymbol{M}^{xOl,3}}\right) - \left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\operatorname{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,1} l_{\cdot,\cdot}^{O}\right),$$

$$\begin{aligned}\overline{\boldsymbol{D}_{27,t+1}^{y,TD}} \equiv & \left(-1+\frac{1}{\beta}\right)\left(\left(\overline{\boldsymbol{M}^{u,4}} \otimes (\mathbf{1}_O)^{\top}\right) \circ \overline{\boldsymbol{M}^{xOl,4}}\right) \\ & - \left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\operatorname{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,1}\left(1-l_{\cdot,\cdot}^{O}\right)\right) + \frac{1}{\theta_o}\operatorname{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,1}\right) + 2\operatorname{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,2}\right),\end{aligned}$$

and

$$\overline{\boldsymbol{D}_{t}^{y,TD}} = \begin{bmatrix} -\delta\left(1-\alpha^{R}\right)\overline{\boldsymbol{M}^{xG,2}} & \delta\left(1-\alpha^{R}\right)\overline{\boldsymbol{M}^{xR,8}} & \mathbf{0} & \mathbf{0} & \delta\overline{\boldsymbol{M}^{xR,7}} & \mathbf{0} & (1-\delta)\overline{\boldsymbol{I}_{NO}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \frac{1+\iota}{1-\delta}\overline{\boldsymbol{I}_{NO}} \end{bmatrix}. \tag{I.9}$$

Since $\check{\boldsymbol{y}}_t = \widehat{\boldsymbol{y}}_t - \widehat{\boldsymbol{y}}$ for any $t \geq t_0$ and $\widehat{\boldsymbol{y}} = \overline{\boldsymbol{E}}\boldsymbol{\Delta}$, I have

$$\overline{\boldsymbol{D}_{t+1}^{y,TD}}\left(\widehat{\boldsymbol{y}_{t+1}} - \widehat{\boldsymbol{y}}\right) = \overline{\boldsymbol{D}_{t}^{y,TD}}\left(\widehat{\boldsymbol{y}}_t - \widehat{\boldsymbol{y}}\right)$$

$$\iff \overline{\boldsymbol{D}_{t+1}^{y,TD}}\widehat{\boldsymbol{y}_{t+1}} = \overline{\boldsymbol{D}_{t}^{y,TD}}\widehat{\boldsymbol{y}}_t - \left(\overline{\boldsymbol{D}_{t+1}^{y,TD}} - \overline{\boldsymbol{D}_{t}^{y,TD}}\right)\overline{\boldsymbol{E}}\boldsymbol{\Delta}.$$

Recall the temporary equilibrium condition $\overline{\boldsymbol{D}^{x}}\widehat{\boldsymbol{x}}_t - \overline{\boldsymbol{D}^{A,S}}\widehat{\boldsymbol{S}}_t = \overline{\boldsymbol{D}^{A,\Delta}}\widehat{\boldsymbol{\Delta}}$ for any t. Thus

$$\overline{\boldsymbol{F}_{t+1}^{y}}\widehat{\boldsymbol{y}_{t+1}} = \overline{\boldsymbol{F}_{t}^{y}}\widehat{\boldsymbol{y}}_t + \overline{\boldsymbol{F}_{t+1}^{\Delta}}\boldsymbol{\Delta},$$

where

$$\overline{\boldsymbol{F}_{t+1}^{y}} \equiv \begin{bmatrix} \overline{\boldsymbol{D}^{x}} & -\overline{\boldsymbol{D}^{A,T}} \\ \multicolumn{2}{c}{\overline{\boldsymbol{D}_{t+1}^{y,TD}}} \end{bmatrix}, \overline{\boldsymbol{F}_{t}^{y}} \equiv \begin{bmatrix} \mathbf{0} \\ \overline{\boldsymbol{D}_{t}^{y,TD}} \end{bmatrix}, \overline{\boldsymbol{F}_{t+1}^{\Delta}} \equiv \begin{bmatrix} \overline{\boldsymbol{D}^{A,\Delta}} \\ \left(\overline{\boldsymbol{D}_{t+1}^{y,TD}} - \overline{\boldsymbol{D}_{t}^{y,TD}}\right)\overline{\boldsymbol{E}} \end{bmatrix},$$

or with $\overline{\boldsymbol{F}^y} \equiv \left(\overline{\boldsymbol{F}_{t+1}^y}\right)^{-1} \overline{\boldsymbol{F}_t^y}$ and $\overline{\boldsymbol{F}^\Delta} \equiv \left(\overline{\boldsymbol{F}_{t+1}^y}\right)^{-1} \overline{\boldsymbol{F}_{t+1}^\Delta}$, one can write

$$\widehat{\boldsymbol{y}_{t+1}} = \overline{\boldsymbol{F}^y}\widehat{\boldsymbol{y}}_t + \overline{\boldsymbol{F}^\Delta}\boldsymbol{\Delta}. \tag{I.10}$$

It remains to find the initial values of the system (I.10) that converges to the steady state. To this end, I apply a standard method in Stokey and Lucas (1989). In particular, I first homogenize the system: Note that equation (I.10) can be rewritten as $\widehat{\boldsymbol{y}_{t+1}} = \overline{\boldsymbol{F}^y}\widehat{\boldsymbol{y}}_t + \left(\overline{\boldsymbol{I}} - \overline{\boldsymbol{F}^y}\right)\left(\overline{\boldsymbol{I}} - \overline{\boldsymbol{F}^y}\right)^{-1}\overline{\boldsymbol{F}^\Delta}\boldsymbol{\Delta}$ and thus

$$\widehat{\boldsymbol{z}_{t+1}} = \overline{\boldsymbol{F}^y}\widehat{\boldsymbol{z}}_t \tag{I.11}$$

where

$$\widehat{\boldsymbol{z}}_t \equiv \widehat{\boldsymbol{y}}_t - \left(\overline{\boldsymbol{I}} - \overline{\boldsymbol{F}^y}\right)^{-1}\overline{\boldsymbol{F}^\Delta}\boldsymbol{\Delta}. \tag{I.12}$$

The system (I.11) must not explode, or it must be that $\widehat{\boldsymbol{z}}_t \to \boldsymbol{0} \iff \widehat{\boldsymbol{y}}_t \to \left(\overline{\boldsymbol{I}} - \overline{\boldsymbol{F}^y}\right)^{-1}\overline{\boldsymbol{F}^\Delta}\boldsymbol{\Delta}$. I follow Blanchard and Kahn (1980) to find such a condition. Write Jordan decomposition of $\overline{\boldsymbol{F}^y}$ as $\overline{\boldsymbol{F}^y} = \overline{\boldsymbol{B}}^{-1}\overline{\boldsymbol{\Lambda}\boldsymbol{B}}$. Then Theorem 6.4 of Stokey and Lucas (1989) implies that it must be that out of n_y vector of $\overline{\boldsymbol{B}}\widehat{\boldsymbol{z}_{t_0}}$, n-th element must be zero if $|\lambda_n| > 1$. Since $\widehat{\boldsymbol{K}_{t_0}^R} = \boldsymbol{0}$, I can write

$$\widehat{\boldsymbol{z}_{t_0}} = \overline{\boldsymbol{F}_{t_0}^\Delta}\boldsymbol{\Delta} + \overline{\boldsymbol{F}_{t_0}^\lambda}\widehat{\boldsymbol{\lambda}_{t_0}^R},$$

where

$$\overline{\boldsymbol{F}_{t_0}^\Delta} \equiv \begin{bmatrix} \left(\overline{\boldsymbol{D}^x}\right)^{-1}\overline{\boldsymbol{D}^{A,\Delta}} \\ \boldsymbol{0}_{2NO\times n_\Delta} \end{bmatrix} - \left(\overline{\boldsymbol{I}} - \overline{\boldsymbol{F}^y}\right)^{-1}\overline{\boldsymbol{F}^\Delta} \text{ and } \overline{\boldsymbol{F}_{t_0}^\lambda} \equiv \begin{bmatrix} \left(\overline{\boldsymbol{D}^x}\right)^{-1}\overline{\boldsymbol{D}^{A,\lambda}} \\ \boldsymbol{0}_{NO\times NO} \\ \overline{\boldsymbol{I}_{NO}} \end{bmatrix}$$

and $\overline{\boldsymbol{D}^{A,\lambda}}$ is the right block matrix of $\overline{\boldsymbol{D}^A} \equiv \begin{bmatrix} \overline{\boldsymbol{D}^{A,K}} & \overline{\boldsymbol{D}^{A,\lambda}} \end{bmatrix}$ that corresponds to vector $\widehat{\boldsymbol{\lambda}^R}$. Extracting n-th row from $\overline{\boldsymbol{F}_{t_0}^\Delta}$ and $\overline{\boldsymbol{F}_{t_0}^\lambda}$ where $|\lambda_n| > 1$ and writing them as a $NO \times n_\Delta$ matrix $\overline{\boldsymbol{G}_{t_0}^\Delta}$ and $NO \times NO$ matrix $\overline{\boldsymbol{G}_{t_0}^\lambda}$, the condition of the Theorem is

$$\boldsymbol{0} = \overline{\boldsymbol{G}_{t_0}^\Delta}\boldsymbol{\Delta} + \overline{\boldsymbol{G}_{t_0}^\lambda}\widehat{\boldsymbol{\lambda}_{t_0}^R},$$

or $\widehat{\boldsymbol{\lambda}_{t_0}^R} = \overline{\boldsymbol{G}_{t_0}}\boldsymbol{\Delta}$ where $\overline{\boldsymbol{G}_{t_0}} \equiv -\left(\overline{\boldsymbol{G}_{t_0}^\lambda}\right)^{-1}\overline{\boldsymbol{G}_{t_0}^\Delta}$. Finally, tracing back to obtain the initial conditions for

$\widehat{y}_t$, it must be $\widehat{y_{t_0}} = \overline{F^y_{t_0}}\boldsymbol{\Delta}$,where

$$\overline{F^y_{t_0}} \equiv \begin{bmatrix} \left(\overline{D^x}\right)^{-1}\left(\overline{D^{A,\Delta}} + \overline{D^{A,\lambda}}\overline{G_{t_0}}\right) \\ \mathbf{0}_{NO\times n_\Delta} \\ \overline{G_{t_0}} \end{bmatrix}.$$

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