Robots and Wage Polarization: The Effects of Robot Capital by Occupations*

Daisuke Adachi[†]

February 6, 2023

[Click here for the latest version]

Abstract

Robotics has been substituting or complementing workers in a wide range of occupations. To examine the strength of this substitutability and the distributional effects of robotization, I match unique data on imported robot prices with the occupational task information to measure the cost of using robots by occupation. The data show that a 10% reduction in costs is associated with a 1.2% reduction in wages for production and transportation occupations in the US, suggesting strong substitutability. This finding motivates the development of a model in which robots are traded and can substitute for labor with different elasticities of substitution across occupations. Using a model-implied optimal instrumental variable, I estimate a higher elasticity of substitution between robots and workers than that of general capital goods in production and transportation occupations. These estimates imply that the adoption of industrial robots significantly affects wage polarization in the US.

Keywords: Industrial Robots, Robot Prices, Elasticity of Factor Substitution, Wage Polarization

JEL Codes: J23, F16

*I thank Costas Arkolakis, Lorenzo Caliendo, Ana Cecilia Fieler, Elisa Giannone, Federico Huneeus, Shoya Ishimaru, Robert Johnson, Sam Kortum, Lei Li, Giovanni Maggi, Andreas Moxnes, Nitya Pandalai-Nayar, Shunya Noda, Yoko Okuyama, Justin Pierce, Peter Schott, Valerie Smeets, Yuta Takahashi, Kosuke Uetake, Frederic Warzynski, Ray Zhang, and participants at Aarhus, GRIPS, Hitotsubashi, Kobe, National University of Singapore, North American Summer Meeting of the Econometric Society, Nottingham, Peking, Yale, and Waseda for valuable comments. All errors are my own.

[†]Department of Economics and Business Economics, Aarhus University. E-mail: daisuke.adachi@econ.au.dk.

1 Introduction

The adoption of industrial robots has been changing factory production rapidly.[1] In the last three decades, the size of the global robot market has grown by 12% per year (IFR, 2021). Robotization has heterogeneous effects on workers across occupations, raising concern about the distributional effects of robot adoption. Policymakers have proposed various countermeasures to the potential harms of robotization, such as introducing taxation on robot adoption.[2] Motivated by these observations, emerging literature has estimated the relative effects of robot penetration on employment and the potential impact of robot taxes (e.g., Acemoglu and Restrepo 2020; Humlum 2019). However, the effects of robotization also depend on under-explored factors such as the substitutability of robots for workers in each occupation.

In this paper, I study the effect of the increased availability of robots on the wage inequality between occupations and welfare in the US. Using a new dataset on the cost of adopting Japanese robots, I show that the robot cost reduction affects the US wage and employment adversely. This suggests substitutability between robots and workers within an occupation, unlike the previous research that reveals the substitutability between occupations. Building on this fact, I develop an equilibrium model in which robots substitute labor within each occupation. I then construct a model-implied optimal instrumental variable and estimate the elasticity of substitution (EoS) between robots and workers that can be heterogeneous across occupations. Finally, based on this estimated model, I perform counterfactual exercises to study the distributional effect of robotization in the US since 1990, as well as the welfare impact of robot taxes.

[1] Throughout the paper, industrial robots (or robots) are defined as multiple-axes manipulators and are measured by the number of such manipulators, or robot arms, following a standard in the literature. A more formal definition given by ISO is provided in Appendix D.2. Such a definition implies that any automation equipment that does not have multiple axes is out of the scope of the paper, even though some of them are often called "robots" (e.g., Roomba, an autonomous home vacuum cleaner made by iRobot Corporation).

[2] The European Parliament proposed a robot tax on robot owners in 2015, although it eventually rejected the proposal (Delvaux et al. 2016). South Korea revised the corporate tax laws that downsize the "Tax Credit for Investment in Facilities for Productivity Enhancement" for enterprises investing in automation equipment (MOEF 2018).

A unique feature of my dataset is the robot price measure for each 4-digit occupation in which robots replace labor. To obtain such a dataset, I use the information about the shipment of Japanese robots, which comprises about one-third of the world's robot supply, from the Japan Robot Association (JARA). JARA's key feature is that the data are disaggregated at the level of robot application or the specified task that robots perform. I combine the JARA data with the O*NET Code Connector's match score to get an occupation-level robot price measure. Finally, I use the fixed effect regression on the unit robot price to get the robot cost shock that controls for the demand factors, which I call the Japan robot shock.

The dataset reveals two stylized facts. First, from 1990-2007, there was a sizable and heterogeneous reduction in the average cost of Japanese robots, ranging from -150% to 0% across occupations.[3] Second, there is a negative relationship between the Japan robot shock and the US wage growth, or a 1.2% decline in occupational wage growth per year associated with a 10% decrease in the cost of using Japanese robots. This finding is robust to controlling for other occupational demand shocks, such as the China trade shock, and suggests that the relative demand for labor is responsive to the robot cost reduction due to the strong substitutability of robots for labor.

However, the Japan robot shock measure may be affected by the robot quality change instead of the change in the cost of robots, and thus the reduced-form relationship does not reveal the elasticity of substitution parameters. To address this concern and derive the distributional effects of robotization, I employ an equilibrium model of robotics automation and quality changes with the following three key features. First, I incorporate the trade of robots following Armington (1969) to capture Japan's sizable robot export in my dataset. Furthermore, this large-open economy setting implies that a robot tax would affect the world price of robots, allowing a country to potentially improve the welfare by manipulating the terms-of-trade. Second, the model describes the endogenous investment in robots with a convex adjustment cost, which controls for the speed of robot

[3] I focus on this sample period and omit data after the Great Recession since the aggregate data about robots show a strikingly different trend than before, and capturing it is out of the scope of this paper.

accumulation. Therefore, compared to static models, the aggregate income implication of the robot tax is nuanced and different over the time horizon. Finally, the production function is characterized by EoS between robots and labor that varies across occupations as well as EoS between occupations. I show that this production function can be micro-founded by the task-based framework à la Acemoglu and Autor (2011), and that it yields rich predictions about the real-wage effect of robot capital accumulation.

To estimate these robot-labor EoS, I confront the identification challenge that the Japan robot shock can be correlated with the unobserved automation shock, and these shocks affect the labor market outcomes simultaneously. To overcome this challenge, I use the model solution and obtain structural residual of labor market outcomes, which controls for the effect of the automation shock. I then impose a moment condition in which this structural residual is orthogonal to the Japan robot shock. This moment condition not only provides me with consistent parameter estimates but also with an optimal instrumental variable to increase estimation precision.

Applying this estimation method, I find that the EoS between robots and workers is around 2 when estimated with a restricted constant across occupations. This estimate is higher than the typical values reported in the literature of the EoS between labor and general capital like structure and equipment, highlighting one of the main differences between robots and other capital goods. Moreover, the EoS estimates are heterogeneous when allowed to vary across occupations. Specifically, for routine occupations that perform production and material moving, the point estimates are as high as around 3, revealing the special susceptibility of workers to robots in these occupations. These estimates are identified from the strong relationship between a larger robot price drop and a lower occupational wage growth rate in these occupations. In contrast, the estimates in the other occupations are close to 1, indicating that robots and labor are neither substitutes nor complements in the other occupations. I then validate the estimated model by checking that the predicted occupational US wage changes from 1990-2007 fit well with the observed ones.

The large EoS between robots and workers in production and material moving occupations implies that the robotization in the sample period significantly decreased relative wage in these occupations. This finding indicates that the robotization shock slowed the wage growth of occupations in the middle deciles since these occupations tend to be in the middle of the occupational wage distribution in 1990. Quantitatively, it explains a 6.4% increase in the 90th-50th percentile wage ratio, a measure of wage inequality popularized by Goos and Manning (2007) and Autor, Katz, and Kearney (2008). Robotization also explains a 0.2 percentage point increase in the US real income, mostly accounted for by the rise in the producers' profit due to the accumulation of robots.

Finally, I examine the counterfactual effect of introducing a tax on robot purchases. As mentioned above, such a robot tax could potentially increase the aggregate income of a country through the change in robot prices. By contrast, the robot tax also disincentivizes the accumulation of robots in the steady state, potentially reducing aggregate income. Quantitatively, the net positive effect by terms-of-trade effect quickly disappears in 2-3 years as the effect of robot distortion starts to dominate the effect of robot price changes. As a result, the robot tax decreases the real income in the long run. Therefore, this finding provides a caution to policy measures proposed to slow down the adoption of industrial robots even when the country can strategically tap into the opportunity of terms-of-trade manipulation.

This paper contributes to the literature on the economic impacts of industrial robots by finding a sizable impact of robots on US wage polarization and a short-run positive aggregate effect of a robot tax. The closest papers to mine are Acemoglu and Restrepo (2020) and Humlum (2019). Acemoglu and Restrepo (2020) establish that the US commuting zones that experienced a greater penetration of robots in 1992-2007 saw lower growth in wages and employment.[4] To examine robot taxes, Humlum (2019) uses firm-level data on robot adoption and estimates a model that incorporates robot importers in a small-

[4]Dauth et al. (2017) and Graetz and Michaels (2018) also use the industry-level aggregate data of robot adoption to analyze its impact on labor markets.

open country, a binary decision of robot adoption, and an EoS between occupations.[5] In contrast, I use the data on the robot cost by occupation, which empirically reveals the substitutability of robots in US occupations. I also consider large open countries' trade of robots, which introduces terms-of-trade manipulation when considering robot taxes.

An increasing number of studies pay attention to occupations to learn about the potentially heterogeneous impacts of automation. While Jäger, Moll, and Lerch (2016) find no association between industrial robot adoptions and total employment at the firm level, Dinlersoz, Wolf, et al. (2018) report that the cost share of workers in the production occupation decreased after the adoption of robots within a firm. In contrast, Cheng (2018) studies the heterogeneous capital price decrease and its implication for job polarization. Jaimovich et al. (2020) construct a general equilibrium model to study the effect of automation on the labor market of routine and non-routine workers in a steady state. Compared to these papers, I provide a method of estimating the within-occupation EoS between robots and labor with the occupation-level data of robot costs and labor market outcomes, as well as incorporating the endogenous trade of robots and characterizing the transition dynamics of the effect of robot tax.

This paper is also related to the vast literature on estimating the EoS between capital and labor, as robots are one type of capital goods (to name a few, Arrow et al. 1961; Chirinko 2008; Oberfield and Raval 2014). Although the literature yields a set of estimates with a wide range, the upper limit of the range appears to be around 1.5 (Karabarbounis and Neiman 2014; Hubmer 2018). Therefore, my EoS estimates around 3 in production and material-moving occupations are significantly higher than this upper limit. In this sense, my estimates highlight one of the main differences between robots and other capital goods: the occupational workers' vulnerability to robots.

[5] There is also a growing body of studies that use the firm- and plant-level microdata to study the impact on workers in Canada (Dixon, Hong, and Wu 2019), France (Acemoglu, Lelarge, and Restrepo 2020; Bonfiglioli et al. 2020), the Netherlands (Bessen et al. 2019), Spain (Koch, Manuylov, and Smolka 2019), and the US (Dinlersoz, Wolf, et al. 2018).

2 Data and Stylized Facts

To measure the cost of using robots, I use data from the Japan Robot Association, with which I combine data from the O*NET Code Connector for matching robot application codes to labor occupation codes at the 4-digit level. I then present stylized facts about robots and workers at the occupation level that suggest strong substitutability between robots and labor to motivate the model and estimation. Throughout the paper, I set the sample period to 1992-2007 (or 1990-2007 for the labor data) and write $t_0 \equiv 1992$ and $t_1 \equiv 2007$.

2.1 Data Sources

The robot measures of my dataset are taken from the Japan Robot Association (JARA), a general incorporated association composed of Japanese robot-producing companies. As of August 2020, the association consists of 381 member companies. JARA annually surveys all these member companies about the units and monetary values of robots sold for each destination country and robot application. Robot application is defined as the specified task that robots perform, which is discussed in detail in Section 2.2 and in Appendix D.2 with examples. I use digitized JARA's annual publication of the summary cross tables starting from 1978.

Japan is a significant robot innovator, producer, and exporter. For example, as of 2017, the US had imported 5 billion dollars worth of Japanese robots, which comprises roughly one-third of the robots used in the US.[6] Therefore, the cost reduction of Japanese robots significantly affects robot adoption in the US and the world. In this paper, I use the cost drop of Japanese robots as one of the sources of robotization shocks and treat the unobserved reduction of the cost of robots from other countries as independent from the evolution of Japanese robot costs. I will clarify the source of the shocks in detail in Section 3 and discuss the plausibility of this assumption in Appendix A.3 by comparing the JARA

[6] Appendix D.5 shows the international robot flows, including Japan, the US, and the rest of the world.

data and the data from the International Federation of Robotics (IFR), a widely-used data source of robots in the world.

I also use the Occupational Information Network OnLine (O*NET) Code Connector to convert robot applications to labor occupations. The O*NET Code Connector is an online database of the definitions of occupations sponsored by the US Department of Labor, Employment, and Training Administration, and provides an occupational search service that helps workforce professionals determine relevant 4-digit level O*NET-SOC Occupation Codes. Using this service, one can search any words and get occupations that are close to the search words. Furthermore, the search algorithm provides a match score that shows the relevance of each occupation to the search term.[7] I use this match score to match robot applications and labor occupations. The set of occupations consists of all of the 324 four-digit-level occupations that exist throughout my sample period and pre-period, which is discussed in detail in Appendix D.1.

2.2 Constructing the Dataset

I describe the measurement method of robot costs, which is novel compared to the past literature that only focuses on the quantity of robots (e.g., Acemoglu and Restrepo 2020; Humlum 2019).[8]

Matching Robot Applications and Labor Occupations. Robot applications and labor occupations are close concepts, although there has not been formal concordance between application and occupation codes. On the one hand, a robot application is a task to which the robot is applied, and each task has different technological requirements for

[7]The match score is the result of the *weighted search algorithm* used by the O*NET Code Connector, which is the internal search algorithm developed and employed by O*NET since September 2005. Since then, the O*NET has continually updated the algorithm and improved the quality of the search results. Morris (2019) reports that the updated weighted search algorithm scored 95.9% based on the position and score of a best 4-digit occupation for a given query.

[8]While Graetz and Michaels (2018) provide data about robot prices from IFR, the price data is aggregated but not distinguished by occupations. By contrast, I will use the variation at the occupation level to estimate the substitutability between robots and workers.

robotics automation. On the other hand, an occupation also requires multiple types of tasks. Therefore, a heterogeneous mix of tasks in each occupation generates a difference in the ease of automation across occupations, implying the heterogeneous adoption level of robots (Manyika et al. 2017). Appendix D.2 provides further descriptions of robot applications and labor occupations using examples.

Specifically, let a denote robot application and o denote labor occupation. The JARA data measure the quantity of robots sold and total monetary transaction values for each application a. I write these as robot measures X_a^R, a generic notation that means both quantity and monetary values. The goal is to convert an application-level robot measure X_a^R to an occupation-level measure X_o^R. First, I search occupations in the O*NET Code Connector by the title of robot application a. Second, I web scrape the match score m_{oa} between a and o. Finally, I allocate X_a^R to each occupation o according to m_{oa}-weight by

$$X_o^R = \sum_a \omega_{oa} X_a^R \text{ where } \omega_{oa} \equiv \frac{m_{oa}}{\sum_{o'} m_{o'a}}. \tag{1}$$

As a result, X_o^R is the robot measure at the occupation level. Note that $\sum_o \omega_{oa} X_a^R = X_a^R$ since $\sum_o \omega_{oa} = 1$, which is a desired property of allocation that occupation-level robot values return to the application level when summed across occupations. Further details of matching are described using examples in Appendix D.7.[9]

It is worthwhile to comment on recent literature that studies the task contents of recent technological development. For example, Webb (2019) provides a natural-language-processing method to match technological advances (e.g., robots, software, and artificial intelligence) embodied in the patent title and abstract to occupations. Furthermore, Montobbio et al. (2020) extend this approach to analyzing full patent texts by applying the topic modeling method of machine learning. My matching method between robot appli-

[9]Although it is transparent to match applications and occupations in a completely automatic way instead of using a researcher's judgment, a concern about this matching method is that one has potentially erroneous matching due to noise in the text description in the occupation dictionary. In order to mitigate such a concern, I explore a manual hard-cut matching between applications and occupations, which is described in greater detail in Section D.8. The regression table confirms that my qualitative results are maintained.

cation and occupation complements these studies by matching the data of robot quantities with lower data requirements, as I only observe the title of robot applications but not such detailed descriptions as in patent texts.

Discussion on the Cost of Using Robots. A modern industrial robot is typically not stand-alone hardware (e.g., robot joints and arms) but an ecosystem that includes the hardware and control units operated by software (e.g., computers and robot-programming language). Furthermore, due to its complexity, installing robots in the production environment often requires hiring costly system integrators that offer engineering knowledge for integration. Therefore, the relevant cost of robots for adopters includes hardware, software, and integration costs.[10]

In this paper, I measure the price of robots by average price, or the total sales divided by the quantity of hardware. Thus, my measure of robot price should be interpreted as reflecting part of overall robot costs. Note that this follows the literature's convention due to the data limitation about the robot software and integration. Nevertheless, I will address this point in the model section by separately defining the observable hardware cost using my data and the unobserved components of the cost.

The Japan Robot Shock. The matching method described above provides the robot quantity $q^R_{i,o,t}$ and sales $(pq)^R_{i,o,t}$ in destination country i, occupation o, and year t. Using them, I construct the cost shocks to the users of robots in each occupation. Specifically, I start with the average price $p^R_{i,o,t} \equiv (pq)^R_{i,o,t} / q^R_{i,o,t}$.[11] Since this measure may be affected by the demand shock in country i, it is not suitable as a cost shock. To mitigate this concern, I exclude the US prices from the sample, following the ideas in the automation literature

[10] As Leigh and Kraft (2018) pointed out, the current industry and occupation classifications do not allow separating system integrators, making it difficult to estimate the cost from these classifications. In addition, relevant costs associated with the robot still remain, e.g., maintenance fees, of which we also lack quantitative evidence. Although understanding these components of the costs is of first-order importance, this paper follows the literature convention and measures robots from the market transaction of hardware.

[11] I have also computed the chain-weighted robot price index which is commonly used when measuring the capital good price. The results using this index are not qualitatively different from the main findings and are available upon request.

(Acemoglu and Restrepo 2020). Formally, I fit the fixed-effect regression

$$\ln\left(p^{R}_{i,o,t}\right) - \ln\left(p^{R}_{i,o,t_0}\right) = \psi^{D}_{i,t} + \psi^{J}_{o,t} + \epsilon_{i,o,t},\ i \neq USA \tag{2}$$

where t_0 is the initial year, $\psi^{D}_{i,t}$ is the destination-year fixed effect, $\psi^{J}_{o,t}$ is the occupation-year fixed effect, and $\epsilon_{i,o,t}$ is the residual. This regression controls for any country-year specific effect $\psi^{D}_{i,t}$, which includes country i's demand shock or trade shock between Japan and i that are constant across occupations. I use the remaining variation across occupations $\psi^{J}_{o,t}$ as a cost shock of robot adoption, and specifically define $\psi^{J}_{o} \equiv \psi^{J}_{o,t_1}$ as the "Japan robot shock."

Another issue with the average price approach is that the average price includes the component of robot quality upgrades. Namely, a rapid innovation in robotics technology could entail both a quality upgrading that makes robots perform more tasks at a greater efficiency as well as the cost saving of producing robots that perform the same task as before. The inseparability of these two components makes it hard to compare prices over time, which poses an identification threat. To work around this issue, I will use the general equilibrium model to predict the labor market effects of quality upgrading in Section 3. Other possible approaches and their limitations are discussed in Appendix D.3.

2.3 Stylized Facts

I convert the Japan robot shock data at the O*NET-SOC 4-digit occupation level to the ones at the OCC2010 occupation level to match the labor market measures from the US Census, American Community Survey (ACS), retrieved from the Integrated Public Use Microdata Series (IPUMS) USA (Ruggles et al. 2018). These labor data are standard in the literature, and their description is relegated to Appendix D.1. With all these data combined, I show stylized facts about the Japan robot shock and its relation to the labor market outcome in the US.

Fact 1: Trends of the Japan Robot Shock. Figure 1a plots the distribution (10th, 50th, and 90th percentile) of the growth rates of the price of Japanese robots in the US each year relative to the initial year. The figure shows two patterns: (i) the robot prices follow an overall decreasing trend, with a median growth rate of -17% from 1992 to 2007, or -1.1% annually, and (ii) there is significant heterogeneity in the rate of price decline across occupations. Specifically, the 10th percentile occupation experienced -34% growth (-2.8% per annum), while in the 90th percentile occupation, the price changed little in the sample period. This price drop is consistent with the trend of decreasing prices of general investment goods since 1980; Karabarbounis and Neiman (2014) report a 10% decrease per decade.

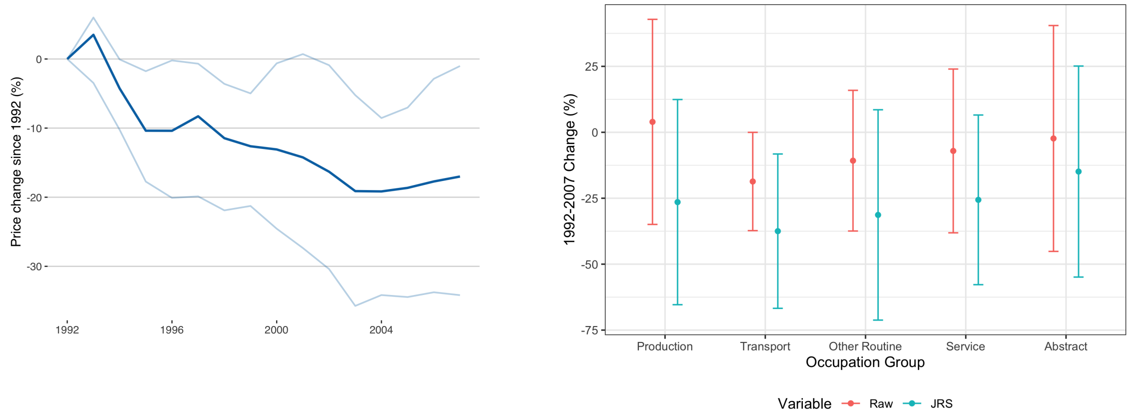
Figure 1b shows the distribution of the long-run trend (1992-2007) for each occupation group. The occupation groups are routine, service (manual), and abstract following Autor, Levy, and Murnane (2003). Routine is further divided into production, transportation, and others to reflect the rapid robot adoption in production and transportation occupations. The figure confirms a significant price variation across occupations, and that variation is observed even within occupation groups. Perhaps surprisingly, the average change of production robot prices is not as large as other robots but is slightly positive. This indicates that the robotics technology change in production occupations is not reflected by the price decline but by the quality improvement, so the unit value rises. Furthermore, the figure also shows the variation in Japan Robot Shock, or ψ^J_{i,t_1}, in equation (2). The large variation of the changes in prices by occupations persists even after controlling for the destination-year fixed effect $\psi^D_{i,t}$. It also confirms that after controlling for US demand shocks, the cost of Japanese robots is strongly decreasing, especially in the production occupation. In the following, I will use this cost variation to study the impact on the labor market and estimate the elasticity of substitution between robots and workers.

Fact 2: Effects of the Japan robot shock on US occupations. Since the labor demand may be affected by trade liberalization, notably the China shock in my sample period, I control for the occupational China shock by the method developed by Autor, Dorn, and

Figure 1: Distribution of the Cost of Robots

(a) US Robot Price by Occupations

(b) Variation in the Japan Robot Shock



Note: The left panel shows the trend of prices of robots in the US by occupations, $p^R_{USA,o,t}$. The bold and dark line shows the median price in each year, and two thin and light lines are the 10th and 90th percentile. Three-year moving averages are taken to smooth out yearly noises. The right panel shows the mean and standard deviation of long-run (1992-2007) raw price decline ("Raw") and Japan Robot Shock measured by the fixed effect ψ^C_{o,t_1} in equation (2) ("JRS"). The occupation group is routine, service (manual) and abstract, where routine is further divided into production, transportation, and other.

Hanson (2013). Namely, I compute

$$IPW_{o,t} \equiv \sum_s l_{s,o,t_0} \Delta m^C_{s,t}, \tag{3}$$

where l_{s,o,t_0} is sector-s share of employment for occupation o, and $\Delta m^C_{s,t}$ is the per-worker Chinese export growth to non-US developed countries.[12] Intuitively, an occupation receives a large trade shock if sectors that faced increased import competition from China intensively employ the corresponding occupation. With this measure of the trade shock, I run the following regression

$$\Delta \ln (Y_o) = \alpha_0 + \alpha_1 \times \left(-\psi^J_o\right) + \alpha_2 \times IPW_{o,t_1} + \boldsymbol{X}_o \cdot \boldsymbol{\alpha} + \varepsilon_o, \tag{4}$$

where Y_o is a labor market outcome by occupations such as hourly wage and employment, $\boldsymbol{X}_o$ is the vector of baseline demographic control variables which are the female

[12]Specifically, following Autor, Dorn, and Hanson (2013), I take eight countries: Australia, Denmark, Finland, Germany, Japan, New Zealand, Spain, and Switzerland. Appendix D.1 shows the distribution of occupational employment l_{s,o,t_0} for each sector.

Table 1: The effects of the Japan robot shock on US occupations

VARIABLES	(1) $\Delta \ln(w)$	(2) $\Delta \ln(w)$	(3) $\Delta \ln(L)$	(4) $\Delta \ln(L)$
Japan Robot Shock, $-\psi^J$	-0.203	-0.206	-0.449	-0.454
	(0.0412)	(0.0402)	(0.0863)	(0.0800)
Exposure to China Trade		-1.035		-1.696
		(0.566)		(1.000)
Observations	324	324	324	324
R-squared	0.273	0.284	0.102	0.107
Demographic controls	✓	✓	✓	✓

Note: The table shows the coefficients in regression (4), based on the dataset constructed from JARA, O*NET, and the US Census/ACS. Observations are 4-digit level occupations, and the sample includes all occupations that existed throughout 1970 and 2007. ψ^C stands for the Japan robot shock from equation (2) and IPW stands for the occupation-level import penetration measure (in thousand USD) in equation (3). Demographic control variables are the female share, the college-graduate share, the age distribution (shares of age 16-34, 35-49, and 50-64 among workers aged 16-64), and the foreign-born share as of 1990. All time differences, Δ, are taken with a long difference between 1990 and 2007. Bootstrapped standard errors are reported in the parentheses.

share, the college-graduate share, the age distribution, and the foreign-born share, and Δ is the long-run difference between 1990 and 2007. Since ψ_o^J is an estimated object, standard errors are bootstrapped.

Table 1 shows the result of regression (4). The first two columns take hourly wages as the outcome, while the last two columns take employment. Columns 2 and 4 are the main specifications that include both the Japan robot shock and the China shock, in which I find that the negative Japan robot shock (the reduction in the cost of Japanese robots) drives the reduced growth rate of the labor market outcomes by occupation. Quantitatively, a 10% decrease in the robot cost implies a fall in the occupational wage growth rate by 1.2%. This finding suggests the substitutability between robots and workers; when the cost of robots falls in an occupation, the relative demand for robots (resp. labor) increases (resp. decreases) in the same occupation.

Again, these findings are unique in the use of the robot cost reduction at the occupation level. By contrast, in Appendix D.9, I show additional results that complement the findings in Table 1 by taking similar approaches as in the literature, such as Acemoglu and Restrepo (2020), and confirm past findings. The Appendix also shows a number of robustness checks, such as measuring robot stocks by quantity, using quality-adjusted robot measures following the method of Khandelwal, Schott, and Wei (2013), and the pre-trend analysis showing no systematic relation between the wage growth rates in 1970-1990 and the Japan robot shock in 1992-2007. Furthermore, to address a concern that the US is a large country that affects robot prices more directly, I also confirm that the pattern is qualitatively similar in a small-open economy as well in Appendix A.2.

Although these data patterns and regressions are informative about the substitutability of robots, they do not definitively give answers to the value of the substitution parameter or the distributional and aggregate effect of robotization. Namely, the observed Japan robot shock may reflect the quality upgrading of robots, meaning the quality-adjusted robot cost reduction might be even more drastic. Furthermore, the relative effect of the Japan robot shock on the growth of occupational labor market outcomes does not reveal the real wage impact per se. I will develop and estimate a general equilibrium model in the following sections to overcome these issues.

3 Model

The basis of the model is a multi-country multi-factor Armington model. It has the following three features: (i) occupation-specific elasticities of substitution (EoS) of robots for workers, (ii) robot trade in a large open economy, and (iii) endogenous investment in robots with an adjustment cost. To emphasize these features, other standard points are relegated to Appendix B.2. The estimation and quantitative exercises are based on the full model described in Appendix B.2.

3.1 Setup

The Environment. Time is discrete and has infinite horizon $t = 0, 1, \ldots$. There are N countries, O occupations, and two types of tradable goods g, non-robot goods $g = G$ and robots $g = R$. To clarify country subscripts, I use l, i, and j, where l is a robot-exporting country, i means a non-robot good-exporting and robot-importing country, and j indicates a non-robot good-importing country, whenever I can. There is a representative household and producer in each country. The non-robot goods are differentiated by origin countries and can be consumed by households, invested to produce robots, and used as an input for robot integration. Robots are differentiated by country of origin and occupation. There are bilateral and good-specific iceberg trade costs $\tau^{g}_{ij,t}$ for each $g = G, R$. I use notation Y for the total production, Q for the quantity arrived at the destination. There is no intra-country trade cost, so $\tau^{g}_{ii,t} = 1$ for all i, g and t. Due to the iceberg cost, the bilateral price of good g that country j pays to i is $p^{g}_{ij,t} = p^{g}_{i,t}\tau^{g}_{ij,t}$. The non-robot goods (resp. robots) demand elasticity is ε (resp. ε^{R}), so that the price indices in country j are

$$P^{G}_{j,t} = \left[\sum_{i}\left(p^{G}_{ij,t}\right)^{1-\varepsilon}\right]^{1/(1-\varepsilon)} \text{ and } P^{R}_{j,o,t} = \left[\sum_{i}\left(p^{R}_{ij,o,t}\right)^{1-\varepsilon^{R}}\right]^{1/\left(1-\varepsilon^{R}\right)},$$

respectively.

There are two factors of production of non-robot goods G: labor $L_{i,o,t}$ and robot capital $K^{R}_{i,o,t}$ in each occupation o.[13] There is no international movement of factors. Producers own and accumulate robot capital. Households own the producers' share in each country. All good and factor markets are perfectly competitive. Workers are forward-looking, draw an idiosyncratic utility shock from a generalized extreme value (GEV) distribution, pay a switching cost for changing occupation, and choose the occupation o that achieves the highest expected value $V_{i,o,t}$ among O occupations (Caliendo, Dvorkin, and Parro 2019). The elasticity of occupation switch probability with respect to the expected value is ϕ. The detail of the worker's problem is discussed in Appendix B.2.

The government in each country exogenously sets the robot tax. Specifically, buyer i of robot o from country l in year t has to pay ad-valorem robot tax $u_{li,t}$ on top of the robot producer price $p^{R}_{li,o,t}$ to buy from l. The tax revenue is uniformly rebated to country i's workers.

[13] Appendix B.2 shows the model with intermediate goods and non-robot capital. The main analytical results are unchanged.

Production Functions. In country i and period t, the representative producer of non-robot good G inputs the occupation-o service $T^O_{i,o,t}$ and produces with the production function

$$Y^G_{i,t} = A^G_{i,t} \left[\sum_o \left(b_{i,o,t}\right)^{\frac{1}{\beta}} \left(T^O_{i,o,t}\right)^{\frac{\beta-1}{\beta}} \right]^{\frac{\beta}{\beta-1}}, \tag{5}$$

where $A^G_{i,t}$ is a Hicks-neutral productivity, $b_{i,o,t}$ is the cost share parameter of each occupation o, and β is the elasticity of substitution between each occupation from the production side. Parameters satisfy $b_{i,o,t} > 0$, $\sum_o b_{i,o,t} = 1$, and $\beta > 0$. Each occupation o is performed by labor $L_{i,o,t}$ and robot capital $K^R_{i,o,t}$ by the following occupation performance function:

$$T^O_{i,o,t} = \left[\left(1 - a_{o,t}\right)^{\frac{1}{\theta_O}} \left(L_{i,o,t}\right)^{\frac{\theta_O - 1}{\theta_O}} + \left(a_{o,t}\right)^{\frac{1}{\theta_O}} \left(K^R_{i,o,t}\right)^{\frac{\theta_O - 1}{\theta_O}} \right]^{\frac{\theta_O}{\theta_O - 1}}, \tag{6}$$

where $\theta_o > 0$ is the elasticity of substitution between robots and labor within occupation o that affects the changes in real wages due to adopting robots, and $a_{o,t}$ is the cost share of robot capital in tasks performed by occupation o. Equation (6) is key to understanding the automation and is discussed in detail in the next paragraph.

Robots R for occupation o are produced by investing non-robot goods $I^R_{i,o,t}$ with productivity $A^R_{i,o,t}$:[14]

$$Y^R_{i,o,t} = A^R_{i,o,t} I^R_{i,o,t}. \tag{7}$$

Note that the change in the productivity of robot production in Japan captures the Japan robot shock in my data since, combined with the perfect competition assumption, the robot price is inversely proportional to the productivity term in the competitive market.

Discussion–The Occupation Performance Function and Automation. It is worth mentioning the relationship between the occupation performance function (6) and how au-

[14]The assumption simplifies the solution of the model because occupation services, intermediate goods, and non-robot capital are used only to produce non-robot goods, but not robots. To conduct the estimation and counterfactual exercises without this simplification, one would need to observe the cost shares of intermediate goods and non-robot capital for robot producers, which is hard to measure.

tomation is treated in the literature. A standard approach in the literature, called the task-based framework, sets up a producer's allocation problem of factors (e.g., robots, labor) to a set of tasks. It then solves the allocation problem using an assumption on the efficiency structure of performing tasks for each factor. In Appendix E, I show that this task-based approach implies the unit cost function that is identical to the one derived from the occupation production function (6) with the Fréchet distribution assumption on the task-efficiency structure. Intuitively, one can regard the occupation service as the aggregate of robot capital and labor inputs after optimally allocating robots and workers to each task.

Since this task-based approach consists of the allocation of factors to tasks, the cost-share parameter $a_{o,t}$ of equation (6) has an additional interpretation of the share of the space of tasks performed by robot capital as opposed to labor. Following Acemoglu and Restrepo (2020), who defined automation as the expansion of the space of tasks that robots perform, I call the change in $a_{o,t}$ the *automation shock*. Real-world examples of the automation shock and its relationship to the models in the literature are discussed in Appendix E.1.

By contrast, the robot cost share $a_{o,t}$ also represents the quality of robots. Specifically, the quality of goods can be regarded as a non-pecuniary "attribute whose valuation is agreed upon by all consumers" (Khandelwal 2010). Since the increase in the cost-share parameter $a_{o,t}$ implies the rise in the value of the robot input among robots and labor, it can also be interpreted as quality upgrading of robots relative to labor when combined with a suitable adjustment in the TFP term.[15] In particular, equation (6) implies that in the long run (hence dropping the time subscript), the demand for robot capital is

$$K_{i,o,t}^{R} = a_{o,t} \left(\frac{c_{i,o,t}^{R}}{P_{i,o,t}^{O}} \right)^{-\theta_o} T_{i,o,t}^{O},$$

where $c_{i,o,t}^{R}$ is the user cost of robot capital formally defined in Appendix E.8, and $P_{i,o}^{O}$ is

[15] I show the exact adjustment method in Section B.1

the unit cost of performing occupation o. In this equation, a_o is the quality term as defined above.

These considerations imply that the automation shock and the quality upgrading are not distinguished in my model but have the same implication for the equilibrium. This is the implication of the Fréchet distribution assumption. It is useful to maintain this assumption since I can keep complex technology improvement in a single exogenous variable $a_{o,t}$.[16] One of the reasons for the need to impose this assumption is the lack of data on the set of tasks for each robot or the quality of robots. Relaxing this assumption using rich data on this dimension would be future work.

In this paper, I consider not only the automation shock but also the shock to the price of adopting robots. I call these two shocks as "robotization shocks" collectively. The robotization shocks are likely to be correlated at the occupation level since innovation in robot technology improves the applicability of robots and the cost efficiency of production at the same time. An example of such a correlation is provided in Appendix D.2.

To the best of my knowledge, equation (6) is the most flexible formulation of substitution between robots and labor in the literature. Specifically, I show that the industry-level unit cost function of Acemoglu and Restrepo (2020) can be obtained by $\theta_o \to 0$ for any o in Lemma E.1 in Appendix E.2. I also show that my model can imply the production structure of Humlum (2019) in Lemma E.2 in the same Appendix.

The Producer's Problem. The producer's problem is made of two tiers–static optimization about labor input in each occupation and dynamic optimization about robot investment. The static part is to choose the employment conditional on market prices and current stock of robot capital. Namely, for each i and t, conditional on the o-vector of the

[16]Note, however, that the Fréchet distribution does not solve the potential endogeneity problem of $a_{o,t}$, although it reduces the parameter dimensionality. This point will be discussed in detail in the estimation result section.

stock of robot capital $\left\{K^R_{i,o,t}\right\}_o$, producers solve

$$\pi_{i,t}\left(\left\{K^R_{i,o,t}\right\}_o\right) \equiv \max_{\left\{L_{i,o,t}\right\}_o} p^G_{i,t} Y^G_{i,t} - \sum_o w_{i,o,t} L_{i,o,t}, \tag{8}$$

where $Y^G_{i,t}$ is given by the production function (5).

The dynamic optimization is about choosing the quantity of new robots to purchase, or the size of the robot investment, given the current stock of robot capital. It is derived from the following three assumptions. First, for each i, o, and t, robot capital $K^R_{i,o,t}$ accumulates according to

$$K^R_{i,o,t+1} = (1-\delta) K^R_{i,o,t} + Q^R_{i,o,t}, \tag{9}$$

where $Q^R_{i,o,t}$ is the amount of new robot investment and δ is the depreciation rate of robots. Second, I assume that the new investment is given by a CES aggregation of robot hardware from country l, $Q^R_{li,o,t}$, and the non-robot good input $I^{int}_{i,o,t}$ that represents the input of software and integration, or

$$Q^R_{i,o,t} = \left[\sum_l \left(Q^R_{li,o,t}\right)^{\frac{\varepsilon^R-1}{\varepsilon^R}}\right]^{\frac{\varepsilon^R}{\varepsilon^R-1}\alpha^R} \left(I^{int}_{i,o,t}\right)^{1-\alpha^R} \tag{10}$$

where l denotes the origin of the newly purchased robots, and α^R is the expenditure share of robot arms in the cost of investment.[17] Third, installing robots is costly and requires a per-unit convex adjustment cost $\gamma Q^R_{i,o,t}/K^R_{i,o,t}$ measured in units of robots, where γ governs the size of the adjustment cost (e.g., Holt 1960; Cooper and Haltiwanger 2006), which reflects the complexity and sluggishness of robot adoption, as reviewed in Autor, Mindell, and Reynolds (2020) and discussed in detail in Appendix E.6.

Given these assumptions, a producer of non-robot good G in country i solves the dy-

[17] Equation (9) follows the formulation of the trade of capital goods in Anderson, Larch, and Yotov (2019) in the sense that the robots are traded because they are differentiated by origin country l. Note that equation (10) implies that the origin-differentiated investment good is aggregated at first and then added to the stock of capital following equation (9). This trick helps reduce the number of capital stock variables and is also used in Engel and Wang (2011).

namic optimization problem

$$\max_{\left\{\left\{Q^R_{li,o,t}\right\}_l, I^{int}_{i,o,t}\right\}_o} \sum_{t=0}^{\infty} \left(\frac{1}{1+\iota}\right)^t \left[\pi_{i,t}\left(\left\{K^R_{i,o,t}\right\}_o\right)\right.$$
$$\left. - \sum_o \left(\sum_l p^R_{li,o,t}\left(1+u_{li,t}\right) Q^R_{li,o,t} + P^G_{i,t} I^{int}_{i,o,t} + \gamma P^R_{i,o,t} Q^R_{i,o,t} \frac{Q^R_{i,o,t}}{K^R_{i,o,t}}\right)\right], \tag{11}$$

subject to accumulation equations (9) and (10), and given $\left\{K^R_{i,o,0}\right\}_o$. A standard Lagrangian multiplier method yields Euler equations for investment, which I derive in Appendix E.8. Note that the Lagrange multiplier $\lambda^R_{i,o,t}$ represents the equilibrium marginal value of robot capital.

Equilibrium. To close the model, the employment level must satisfy an adding-up constraint

$$\sum_o L_{i,o,t} = \overline{L}_{i,t}, \tag{12}$$

and markets for robots and non-robot goods clear. There is one numeraire good to pin down the price system. I first define a temporary equilibrium in each period and then a sequential equilibrium, which leads to the definition of a steady state. To save space, detailed expressions are relegated in Appendix E.8.

I define the bold symbols as column vectors of robot capital $\boldsymbol{K}^R_t \equiv \left[K^R_{i,o,t}\right]_{i,o}$, marginal values of robot capital $\boldsymbol{\lambda}^R_t \equiv \left[\lambda^R_{i,o,t}\right]_{i,o}$, employment $\boldsymbol{L}_t \equiv \left[L_{i,o,t}\right]_{i,o}$, workers' value functions $\boldsymbol{V}_t \equiv \left[V_{i,o,t}\right]_{i,o}$, non-robot goods prices $\boldsymbol{p}^G_t \equiv \left[p^G_{i,t}\right]_i$, robot prices $\boldsymbol{p}^R_t \equiv \left[p^R_{i,o,t}\right]_{i,o}$, wages, $\boldsymbol{w}_t \equiv \left[w_{i,o,t}\right]_{i,o}$, bilateral non-robot goods trade levels $\boldsymbol{Q}^G_t \equiv \left[Q^G_{ij,t}\right]_{i,j}$, bilateral non-robot goods trade levels $\boldsymbol{Q}^R_t \equiv \left[Q^R_{ij,o,t}\right]_{i,j,o}$, and occupation transition shares $\boldsymbol{\mu}_t \equiv \left[\mu_{i,oo',t}\right]_{i,oo'}$, where $\boldsymbol{V}_t$ and $\boldsymbol{\mu}_t$ are explained in detail in Appendix B.2. I write $\boldsymbol{S}_t \equiv \left[\boldsymbol{K}^{R\prime}_t, \boldsymbol{\lambda}^{R\prime}_t, \boldsymbol{L}'_t, \boldsymbol{V}'_t\right]'$ as state variables.

Definition 1. In each period t, given state variables $\boldsymbol{S}_t$, a *temporary equilibrium* (TE) $\boldsymbol{x}_t$ is the set of prices $\boldsymbol{p}_t \equiv \left[\boldsymbol{p}^{G\prime}_t, \boldsymbol{p}^{R\prime}_t, \boldsymbol{w}'_t\right]'$ and flow quantities $\boldsymbol{Q}_t \equiv \left[\boldsymbol{Q}^{G\prime}_t, \boldsymbol{Q}^{R\prime}_t, \boldsymbol{\mu}'_t\right]$ that satisfy: (i) given $\boldsymbol{p}_t$, workers choose occupation optimally by equation (B.6), (ii) given $\boldsymbol{p}_t$, producers maximize flow profit by equation (8) and demand robots by equation (E.21), and (iii) markets clear: Labor adds up as in equation (12), and goods markets clear with trade balances as in equations (E.29) and (E.31).

In other words, the inputs of the temporary equilibrium are all state variables, while the outputs are all remaining endogenous variables that are determined in each period. Adding the conditions about state variable transitions, sequential equilibrium determines all state variables given initial conditions as follows.

Definition 2. Given initial robot capital stocks and employment $\left[\boldsymbol{K}_0^{R'}, \boldsymbol{L}_0'\right]'$, a *sequential equilibrium* (SE) is a sequence of vectors $\boldsymbol{y}_t \equiv \left[\boldsymbol{x}_t', \boldsymbol{S}_t'\right]_t'$ that satisfies the TE conditions and employment law of motion (B.8), value function condition (B.7), capital accumulation equation (9), producer's dynamic optimization (E.25) and (E.24).

Finally, I define the steady state as a SE $\boldsymbol{y}$ that does not change over time.

3.2 Approximated Solution

Since the GE system is highly nonlinear and does not have a closed form colution due to flexible robot-labor substitution, I log-linearize the system around the initial steady state. I choose this strategy because it is well-known that the errors due to first-order approximation with respect to productivity shocks are considerably small (cf. Kleinman, Liu, and Redding 2020). Consider increases of the robot task space $a_{o,t}$ and of the productivity of the robot production $A_{i,o,t}^R$ in baseline period t_0, and combine all these changes into a column vector $\boldsymbol{\Delta}$. Write state variables $\boldsymbol{S}_t = \left[\boldsymbol{K}_t^{R'}, \boldsymbol{\lambda}_t^{R'}, \boldsymbol{L}_t', \boldsymbol{V}_t'\right]'$, and use "hat" notation to denote changes from t_0, or $\widehat{z}_t \equiv \ln(z_t) - \ln(z_{t_0})$ for any variable z_t,. I take the following three steps to solve the model.

Step 1. In given period t, I combine the vector of shocks $\boldsymbol{\Delta}$ and (given) changes in state variables $\widehat{\boldsymbol{S}}_t$ into a column vector $\widehat{\boldsymbol{A}_t} = \left[\boldsymbol{\Delta}', \widehat{\boldsymbol{S}}_t'\right]'$. Log-linearizing the TE conditions, I solve for matrices $\overline{\boldsymbol{D}^{x}}$ and $\overline{\boldsymbol{D}^{A}}$ such that the log-difference of the TE $\widehat{\boldsymbol{x}}_t$ satisfies

$$\overline{\boldsymbol{D}^{x}}\widehat{\boldsymbol{x}}_t = \overline{\boldsymbol{D}^{A}}\widehat{\boldsymbol{A}_t}. \tag{13}$$

In this equation, $\overline{\boldsymbol{D}^{x}}$ is a substitution matrix, and $\overline{\boldsymbol{D}^{A}}\widehat{\boldsymbol{A}_t}$ is a vector of partial equilibrium shifts in period t (Adao, Arkolakis, and Esposito 2019).[18]

Step 2. Log-linearizing laws of motion and Euler equations around the initial steady state, I solve for matrices $\overline{\boldsymbol{D}^{y,SS}}$ and $\overline{\boldsymbol{D}^{\Delta,SS}}$ such that $\overline{\boldsymbol{D}^{y,SS}}\widehat{\boldsymbol{y}} = \overline{\boldsymbol{D}^{\Delta,SS}}\boldsymbol{\Delta}$, where superscript SS denotes the steady state. Note that there exists a block separation $\overline{\boldsymbol{D}^{A}} = \left[\overline{\boldsymbol{D}^{A,\Delta}}|\overline{\boldsymbol{D}^{A,S}}\right]$ such that equation (13) can be written as

$$\overline{\boldsymbol{D}^{x}}\widehat{\boldsymbol{x}}_t - \overline{\boldsymbol{D}^{A,S}}\widehat{\boldsymbol{S}}_t = \overline{\boldsymbol{D}^{A,\Delta}}\boldsymbol{\Delta}. \tag{14}$$

Combined with this equation evaluated at the steady state, I have

$$\overline{\boldsymbol{E}^{y}}\widehat{\boldsymbol{y}} = \overline{\boldsymbol{E}^{\Delta}}\boldsymbol{\Delta}, \tag{15}$$

where

$$\overline{\boldsymbol{E}^{y}} \equiv \begin{bmatrix} \overline{\boldsymbol{D}^{x}} & -\overline{\boldsymbol{D}^{A,T}} \\ \multicolumn{2}{c}{\overline{\boldsymbol{D}^{y,SS}}} \end{bmatrix}, \text{ and } \overline{\boldsymbol{E}^{\Delta}} \equiv \begin{bmatrix} \overline{\boldsymbol{D}^{A,\Delta}} \\ \overline{\boldsymbol{D}^{\Delta,SS}} \end{bmatrix},$$

which implies $\widehat{\boldsymbol{y}} = \overline{\boldsymbol{E}}\boldsymbol{\Delta}$, where matrix $\overline{\boldsymbol{E}} = \left(\overline{\boldsymbol{E}^{y}}\right)^{-1}\overline{\boldsymbol{E}^{\Delta}}$ represents the first-order steady-state impact of the shock $\boldsymbol{\Delta}$. This steady-state matrix $\overline{\boldsymbol{E}}$ will be a key object in estimating the model in Section 4.

Step 3. Log-linearizing laws of motion and Euler equations around the new steady state, I solve for matrices $\overline{\boldsymbol{D}^{y,TD}_{t+1}}$ and $\overline{\boldsymbol{D}^{y,TD}_{t}}$ such that $\overline{\boldsymbol{D}^{y,TD}_{t+1}}\check{\boldsymbol{y}}_{t+1} = \overline{\boldsymbol{D}^{y,TD}_{t}}\check{\boldsymbol{y}}_t$, where the super-

[18] Since the temporary equilibrium vector $\widehat{\boldsymbol{x}}_t$ includes wages $\widehat{\boldsymbol{w}_t}$, equation (13) generalizes the general equilibrium comparative statics formulation in Adao, Arkolakis, and Esposito (2019), who consider the variant of equation (13) with $\widehat{\boldsymbol{x}}_t = \widehat{\boldsymbol{w}}_t$.

script TD stands for transition dynamics, and $\check{z}_{t+1} \equiv \ln z_{t+1} - \ln z'$ and z' is the new steady state value for any variable z. Log-linearized sequential equilibrium satisfies the following first-order difference equation

$$\overline{\boldsymbol{F}^{y}_{t+1}}\widehat{\boldsymbol{y}_{t+1}} = \overline{\boldsymbol{F}^{y}_{t}}\widehat{\boldsymbol{y}}_{t} + \overline{\boldsymbol{F}^{\Delta}_{t+1}}\boldsymbol{\Delta}. \tag{16}$$

Following the insights in Blanchard and Kahn (1980), there is a converging matrix representing the first-order transitional dynamics $\overline{\boldsymbol{F}_{t}}$ such that

$$\widehat{\boldsymbol{y}}_{t} = \overline{\boldsymbol{F}_{t}}\boldsymbol{\Delta} \text{ and } \overline{\boldsymbol{F}_{t}} \to \overline{\boldsymbol{E}}. \tag{17}$$

The matrix $\overline{\boldsymbol{F}_{t}}$ characterizes the transition dynamics after robotization shocks and is used to study the effect of policy changes in the counterfactual section. Appendix G gives the details of the derivation of these matrices.

4 Estimation

Using the Japan robot shock described in Section 2 and the general equilibrium model in Section 3, I develop an estimation method using the model-implied optimal instrumental variable (MOIV) from Adao, Arkolakis, and Esposito (2019). First, Section 4.1 provides the implementation detail of the model. I then define the MOIV estimator in Section 4.2, which gives the estimation results shown in Section 4.3. Section 4.4 discusses the performance of my estimates.

4.1 Bringing the Model to the Data

Since I observe the prices of Japanese robots and study the US labor market, I set $N = 3$ and aggregate country groups to the US (USA, country index 1), Japan (JPN, index 2), and the Rest of the World (ROW, index 3). To allow the heterogeneity of the EoS between robots and labor across occupations and maintain the estimation power at the same time, I define the occupation groups as follows. First, occupations are separated into three broad occupation groups, Abstract, Service (Manual), and Routine following Acemoglu and Autor (2011).[19] Given the trend that robots are introduced intensively in production and transportation (material-moving) occupations in the sample period, I further divide routine occupations into three sub-categories, Production (e.g., welders), Transportation (indicating transportation and material-moving, e.g., hand laborer), and Others (e.g., repairer). As a result, I obtain five occupation groups.[20] Within each group, I assume a constant EoS between robots and labor. Each occupation group is denoted by subscript g, and thus the robot-labor EoS for group g is written as θ_g.

I fix some parameters of the model at conventional values as follows. The annual discount rate is $\iota = 0.05$, and the robot depreciation rate is 10%, following Graetz and Michaels (2018).[21] I take the trade elasticity of $\varepsilon = 4$ from the large literature of trade elasticity estimation (e.g., Simonovska and Waugh 2014), and $\varepsilon^R = 1.2$ derived from applying the estimation method developed by Caliendo and Parro (2015) to the robot trade data, discussed in detail in Appendix F.1. Following Leigh and Kraft (2018), I assume $\alpha^R = 2/3$. By Cooper and Haltiwanger (2006), I set the parameter of adjustment cost at $\gamma = 0.295$. I use the estimates from Traiberman (2019) and set the dynamic occupation switching elasticity as $\phi = 0.8$. With these parametrizations, structural parameters to be estimated are $\mathbf{\Theta} \equiv \left\{\theta_g, \beta\right\}$.

Finally, since I use the first-order approximated solution, I need to measure the pre-shock steady state $\boldsymbol{y}_{t_0}$, which is an input to the solution matrix $\overline{\boldsymbol{E}}$ in equation (15). I take these data from JARA, IFR, IPUMS USA and CPS, BACI, and World Input-Output Data (WIOD). The measurement of labor market outcomes is standard and relegated to

[19] Routine occupations include occupations such as production, transportation and material moving, sales, clerical, and administrative support. Abstract occupations are professional, managerial, and technical occupations; service occupations are protective service, food preparation, cleaning, personal care, and personal services.

[20] In terms of OCC2010 codes in the US Census, Routine production occupations are in [7700, 8965], Routine transportation occupations are in [9000, 9750], Routine others are in [4700, 6130], Service occupations are in [3700, 4650], and Abstract occupations are in [10, 3540].

[21] For example, see King and Rebelo (1999) for the source of the conventional value of ι which matches the discount rate to the average real return on capital.

Appendix D.11. I set robot tax in the initial period to be zero in all countries.

In the estimation, I use the changes in US occupational wages $\widehat{\boldsymbol{w}_1}$ between 1992 and 2007 as the target variables. I use the steady-state changes from the model to match these 15-year changes in the data. Recall that the robot production function (7) implies that $\widehat{A^R_{2,o}}$ is equal to the negative cost shock to produce robots in Japan, so I measure the robot efficiency gain by

$$\widehat{A^R_{2,o}} = -\psi^J_o, \tag{18}$$

where ψ^J_o is defined in equation (2) and observed using my dataset.

4.2 Estimation Method

I begin by discussing the identification challenge of the Japan robot shock correlated with the unobserved automation shock. For this purpose, I decompose the automation shock $\widehat{a_o}$ into the component $\widehat{a_o^{\text{imp}}}$ implied from the relative demand function and unobserved error component $\widehat{a_o^{\text{err}}}$ such that $\widehat{a_o} = \widehat{a_o^{\text{imp}}} + \widehat{a_o^{\text{err}}}$ for all o. Implied component $\widehat{a_o^{\text{imp}}}$ is implicitly defined by the steady-state change of relative demand for robots and labor

$$\left(\widehat{\frac{c^R_{i,o} K^R_{i,o}}{w_{i,o} L_{i,o}}}\right) = \frac{\widehat{a_o^{\text{imp}}}}{1 - a_{o,t_0}} + \left(1 - \theta_g\right) x^R_{12} \psi^J_o + \epsilon_o, \tag{19}$$

where x^R_{12} is the import share of robots from Japan in the US, and ϵ_o is the error term that depends on the changes in wages and robot costs in the other countries. The identification challenge is that the Japan robot shock ψ^J_o does not work as an instrumental variable (IV) in equation (19) because of a potential correlation between ψ^J_o and an observed task-space expansion shock $\widehat{a_o^{\text{imp}}}$ as mentioned in Section 3.1.

To overcome this identification issue, I employ a method based on the model solution. A key observation is that conditional on $\widehat{a_o^{\text{imp}}}$, and using the solution of the wage change, the error component $\widehat{a_o^{\text{err}}}$ can be inferred from the observed endogenous variables. Specifically, from the steady-state solution matrix $\overline{\boldsymbol{E}}$, I obtain $O \times O$ sub-matrices $\overline{\boldsymbol{E}}_{\boldsymbol{w}_1,\boldsymbol{a}}$ and

$\overline{\boldsymbol{E}}_{w_1,\boldsymbol{A}_2^R}$ such that[22]

$$\widehat{\boldsymbol{w}} = \overline{\boldsymbol{E}}_{w_1,\boldsymbol{a}}\widehat{\boldsymbol{a}} + \overline{\boldsymbol{E}}_{w_1,\boldsymbol{A}_2^R}\widehat{\boldsymbol{A}_2^R}. \tag{20}$$

Using $\widehat{\boldsymbol{a}} = \widehat{\boldsymbol{a}^{\text{obs}}} + \widehat{\boldsymbol{a}^{\text{err}}}$, I derive the structural residual $\boldsymbol{\nu}_{\boldsymbol{w}} \equiv \overline{\boldsymbol{E}}_{w_1,\boldsymbol{a}}\widehat{\boldsymbol{a}^{\text{err}}} \equiv [\nu_{w,o}]_o$, which is a vector of length O generated from the linear combination of the unobserved component of the automation shocks:

$$\boldsymbol{\nu}_{\boldsymbol{w}} = \boldsymbol{\nu}_{\boldsymbol{w}}(\boldsymbol{\Theta}) = \widehat{\boldsymbol{w}} - \overline{\boldsymbol{E}}_{w_1,\boldsymbol{a}}\widehat{\boldsymbol{a}^{\text{obs}}} - \overline{\boldsymbol{E}}_{w_1,\boldsymbol{A}_2^R}\widehat{\boldsymbol{A}_2^R}.$$

I assume the following moment condition regarding this structural residual and the Japan robot shock $\boldsymbol{\psi}^J \equiv \left\{\psi_o^J\right\}_o$.

Assumption 1. *(Moment Condition)*

$$\mathbb{E}\left[\nu_{w,o}|\boldsymbol{\psi}^J\right] = 0. \tag{21}$$

Given this moment condition, it is straightforward to construct the optimal instrument and implement it with the two-step estimator (Adao, Arkolakis, and Esposito 2019). Therefore, I relegate the detailed explanation to Appendix B.3 and instead discuss the interpretation of Assumption 1 and a case in which it may not hold. Assumption 1 restricts that the structural residual $\boldsymbol{\nu}$ should not be predicted by the Japan robot shock. Note that it allows that the automation shock $\widehat{a_o}$ may correlate with the change in the robot producer productivity $\widehat{A_{2,o}^R}$. The structural residual $\nu_{w,o}$ purges out the first-order effects of all shocks, $\widehat{\boldsymbol{a}}$ and $\widehat{\boldsymbol{A}_2^R}$, on endogenous variables. I then place the assumption that the remaining variation should not be predicted by the Japan robot shock from the data. Furthermore, note that the correlation of the structural residuals with other shocks, such as trade shocks, is unlikely to break Assumption 1 as I have confirmed that controlling for such shocks does not qualitatively change the reduced-form findings in Section 2.3.

To further clarify the role of Assumption 1, consider the circumstances under which Assumption 1 breaks. One such threat is a directed technological change, in which the

[22] Appendix E.10 explains the technical reason for the choice of the steady-state matrix in equation (20).

Table 2: Parameter Estimates

	Case 1: $\theta_g = \theta$	Case 2: Free θ_g	
θ	2.05 (0.19)	Routine, Production	2.95 (0.42)
		Routine, Transportation	2.90 (0.48)
		Routine, Others	1.16 (0.32)
		Manual	1.23 (0.55)
		Abstract	0.64 (1.24)
β	0.83 (0.03)		0.73 (0.06)

Note: The estimates of the structural parameters based on the estimator in Proposition B.2. Standard errors are in parentheses. Parameter θ is the within-occupation elasticity of substitution between robots and labor. Parameter β is the elasticity of substitution between occupations. The column "Case 1: $\theta_g = \theta$" shows the result with the restriction that θ_o is constant across occupation groups. The column "Case 2: Free θ_g" shows the result with θ_g allowed to be heterogeneous across five occupation groups. Transportation indicates "Transportation and Material Moving" occupations in the Census 4-digit occupation codes (OCC2010 from 9000 to 9750). See the main text for other details.

occupational labor demand drives the changes in the cost of robots (Acemoglu and Restrepo 2018). Specifically, suppose a positive labor demand shock in occupation o induces the research and development of robots in occupation o and drives costs down in the long run instead of simply assuming my production function (7) with exogenous technological change. In this case, the structural residual ν_o does not control for this effect and is negatively correlated with Japan robot shock ψ_o^J. Another possibility that breaks Assumption 1 is the increasing returns for robot producers, which would also imply that the unobserved robot demand increase drives a reduction of robot costs. However, even if this is the case, the positive impact of Japan robot costs found in Section 2.3 shows the lower limit, and thus my qualitative results about strong substitutability are maintained.

4.3 Estimation Results

Table 2 gives the estimates of the structural parameters. The first column shows the estimation result when I restrict the EoS between robots and labor to be constant across occupation groups (Case 1). The estimate of the within-occupation EoS between robots and labor θ is around 2 and implies that robots and labor are substitutes within an occupation, and rejects the Cobb-Douglas case $\theta_g = 1$ at the conventional significance levels. The high estimate of the EoS between labor and automation capital is also found in Eden and Gaggl (2018), while their estimate is about the elasticity with respect to ICT capital. The point estimate of the EoS between occupations, β, is 0.83, implying that occupation groups are complementary. The estimate is slightly higher than Humlum's (2019) central estimate of 0.49.

The second column shows the estimation result when I allow the heterogeneity across occupation groups (Case 2). I find that the EoS for routine production occupations and routine transportation occupations is around 3, while those for other occupation groups (other occupations in routine occupations, service, and abstract occupations) are not significantly different from 1 and thus do not reject the Cobb-Douglas. Therefore, the estimates for routine production and transportation indicate the special susceptibility of workers in these occupations to robot capital. Note also that the estimate of the EoS between occupations β does not change qualitatively between Case 1 and Case 2.

As in the literature on estimating the capital-labor substitution elasticity, the source of identification of these large and heterogeneous EoS between robots and labor is the negative correlation between the Japan robot shock and the change in the labor market outcome. Intuitively, if θ_g is large, then the steady-state relative robot (resp. labor) demand responds strongly in the positive (resp. negative) direction conditional on a unit decrease in the cost of using robots. To examine this point, I run the main reduced-form regression (4) using partitioned samples by occupation groups. Table 3 shows the results. The negative correlation between occupational wage changes and the Japan robot shock $-\psi_o^J$ is seen only in the group of production occupations and transportation occupations, which is consistent with the heterogeneous elasticity of substitution between robots and labor found in Table 2. Appendix D.4 also shows the robot price variation within each occupation group to check the source of variation in these partitioned regressions.

Table 3: Wages and Robot Prices by Occupation Groups

VARIABLES	(1) $\Delta \ln(w)$	(2) $\Delta \ln(w)$	(3) $\Delta \ln(w)$	(4) $\Delta \ln(w)$	(5) $\Delta \ln(w)$
$-\psi^J$	-0.607	-0.625	-0.00285	-0.0717	-0.00742
	(0.109)	(0.0850)	(0.0435)	(0.120)	(0.0581)
Observations	55	25	109	106	29
R-squared	0.474	0.595	0.000	0.025	0.001
Occupation Group	Routine-Prod.	Routine-Tran.	Routine-Others	Abstract	Service

Note: The author's calculation based on JARA, O*NET, and US Census/ACS. The table shows regression coefficients of the main reduced-form specification (4) with separated samples according to occupation groups. Occupation group "Routine-Prod." indicates the production occupation within routine occupation, "Routine-Tran." indicates the transportation occupation within routine occupation, and "Routine-Others" indicates the other occupations within routine occupation. All regressions control for the same variables as in the full specifications of Table 1. Heteroskedasticity-robust standard errors are reported in the parentheses.

4.4 Measuring Shocks and Model Fit

I apply the observed and simulated data to the linear regression model (4) to examine the model fit and the role of the automation shock to estimate the robot-labor EoS.[23] Specifically, I consider the following two simulations. First, I hit the Japan robot shock and the implied automation shock, and I call this counterfactual wage change "targeted." In this case, the prediction of wage changes is consistent with the moment condition (21), and thus the linear regression coefficient α_1 of equation (4) is expected to be close to the one in Table 1. Second, I hit only the Japan robot shock but not the automation shock, and I call this counterfactual wage change "untargeted." In this case, the moment condition (21) is violated since the structural residual does not incorporate the unobserved automation shock, which causes a bias in the regression. The difference in estimates from the one from the targeted wage change reveals the size of this bias. Therefore, this exercise reveals how important it is to consider the automation shock in estimation. The exact method for simulating data is standard and explained in Appendix C.1.

Table 4 shows the result of these exercises. The first column shows the estimates in column (3) of Table 1 again, the second column is the estimate based on the targeted wage change, and the third column is the estimate based on the untargeted wage change. Comparing the first and second columns confirms that the targeted moments match well as expected. Furthermore, examining the third column compared to these two columns, one can see a stronger negative correlation between the simulated wage and the Japan robot shock. This is due to the positive correlation between the Japan robot shock $-\psi_o^J$ and the implied automation shock $\widehat{a_o^{\text{imp}}}$, which is consistent with the fact that robotic innovations that save costs (thus $\widehat{A_{2,o}^R} > 0$ or $-\widehat{\psi_o^J} > 0$) and that upgrade quality (thus $\widehat{a_o^{\text{imp}}} > 0$) are likely to happen at the same time.

More specifically, with the real data, the regression specification (4) contains a positive

[23] As another model validation exercise, I predict the stock of robots by occupation and find that the model predicts the actual robot accumulation dynamics well, described in detail in Appendix F.3. Furthermore, Appendix F.4 gives a detailed discussion on the Japan robot shock and the backed-out implied automation shocks.

Table 4: Model Fit: Linear Regression with Observed and Simulated Data

VARIABLES	(1) $\widehat{w}_{data}$	(2) $\widehat{w}_{\psi^J \widehat{a^{obs}}}$	(3) $\widehat{w}_{\psi^J}$
$-\psi^J$	-0.118	-0.107	-0.536
	(0.0569)	(0.0711)	(0.175)
Observations	324	324	324

Note: The author's calculation based on the dataset generated by JARA, O*NET, and the US Census. Column (1) is the coefficient of the Japan robot shock ψ^J in the reduced-form regression with IPW. Column (2) takes the US wage change predicted by GE with ψ^J as well as other shocks such as the implied automation shock $\widehat{a^{\text{imp}}}$. Column (3) takes the US wage change predicted by GE with shocks including the Japan robot shock, but counterfactually fixing the implied automation shock to be zero. Heteroskedasticity-robust standard errors in parentheses.

bias due to this positive correlation. By contrast, the untargeted wage is free from this bias since its data-generating process does not contain the automation shock but only the Japan robot shock. Thus, the linear regression coefficient α_1 is higher than the one obtained from the real data. In other words, if I had wrongfully assumed that the economy did not experience the automation shock and believed the regression coefficient in Table 1 is bias-free, I would have estimated higher EoS by ignoring the actual positive correlation between $-\psi_o^J$ and $\widehat{a_o^{\text{imp}}}$. This thought experiment reveals that it is critical to take into account the automation shock in estimating the EoS between robots and labor using the Japan robot shock, and that the large EoS in my structural estimates are robust even after taking this point into account.

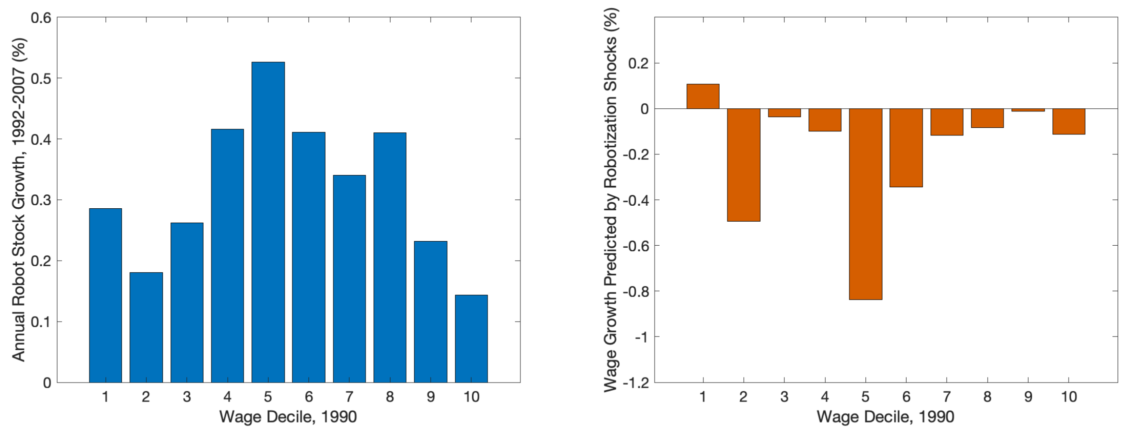
5 Counterfactual Exercises

I examine a few policy questions using the estimated model and shocks in the previous section. The first one is the question about the distributional effects of robotization. For example, Autor, Katz, and Kearney (2008) argue that the wage inequality measured by the ratio of the wages between the 90th percentile and the 50th percentile (90-50 ratio) has steadily increased since 1980.[24] I study how much such an increase can be explained

[24] Furthermore, as Heathcote, Perri, and Violante (2010) argue, wage inequality comprises a sizable part of the overall economic inequality in the US.

Figure 2: Robots, Wage Inequality, and Polarization

(a) Robot growths by Wage Decile (b) The Effect of Robots on Wages



Note: The left panel shows the average annual growth rates of the observed robot stock between 1992 and 2007 for each of ten deciles of the occupational wage distribution in 1990. The right panel shows the annualized occupational wage growth rates for each wage decile, predicted by the first-order steady-state solution of the estimated model given in equation (15).

by the increased use of industrial robots from 1990. Next, I examine the implications of counterfactual policies regarding regulating robot adoption. Due to the fear of automation, policymakers have proposed regulating industrial robots using robot taxes. The estimated model provides an answer of the short-run and long-run effects of taxing robot purchases on real wages across occupations and aggregate welfare losses.[25]

5.1 The Distributional Effects of Robot Adoption

To study the effect of robots to wage polarization, I show the pattern of robot accumulations over the occupational wage distribution revealed by my data. Figure 2a shows the average annual growth rates of observed robot stock between 1992 and 2007 for each decile of the occupational wage distribution in 1990. The figure clarifies that more robots were adopted in occupations in the middle deciles of the distribution than in the top and bottom deciles.

By contrast, Panel 2b shows the steady-state predicted wage growths per annum due

[25]In Appendix 5.2 and F.7, I also examine the effect of robot tax on occupational wages and workers' welfare and the role of trade liberalization of robots.

to the robotization shocks and the estimated model with the first-order solution given in equation (17). Consistent with the high growth rate of robot stocks in the middle of the wage distribution and the strong substitutability between robots and labor, I find that the counterfactual wage growth rate in the middle deciles of the initial wage distribution is more negative than that in the other part of the wage distribution. Quantitatively, the 90-50 ratio observed in 1990 and 2007 is, respectively, 1.588 and 1.668. On the other hand, the 90-50 ratio predicted by the initial 1990 data and the first-order solution (17) is 1.594. These numbers imply that a 6.4% increase in the 90-50 ratio can be explained by the robotization shock captured in this paper.

It is worth emphasizing that we consider two shocks in this main exercise, the automation shock $\widehat{\boldsymbol{a}}$ and the Japan robot shock $\widehat{\boldsymbol{A}}_2$. When these two shocks are distinguished in the quantitative exercise, the automation shock reduces the labor demand due to task reallocation from labor to robots, while the Japan robot shock increases the stock of robots and the marginal product of labor. Appendix C.2 clarifies this point. Furthermore, Appendix F.5 describes the counterfactual wage changes for each of the five occupation groups.

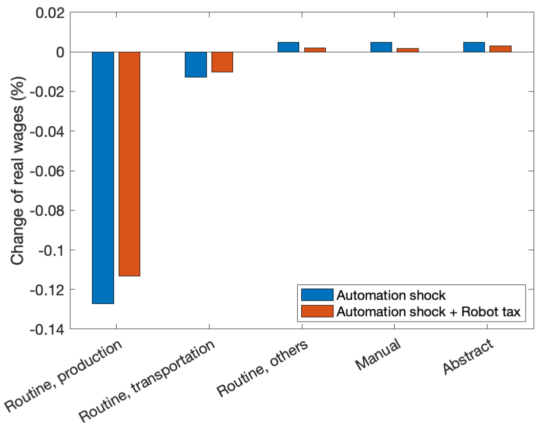
5.2 The Effect of Robot Tax on Occupations

To study the effect of counterfactually introducing a robot tax, consider an unexpected, unilateral, and permanent increase in the robot tax by 6% in the US, which I call the general tax scenario. I also consider the tax on only imported robots by 33.6%, and call it the import tax scenario, which implies the same amount of tax revenue as in the general tax scenario and makes the comparison straightforward between the two scenarios.[26] First, I examine the effect of the general robot tax on occupational inequality.

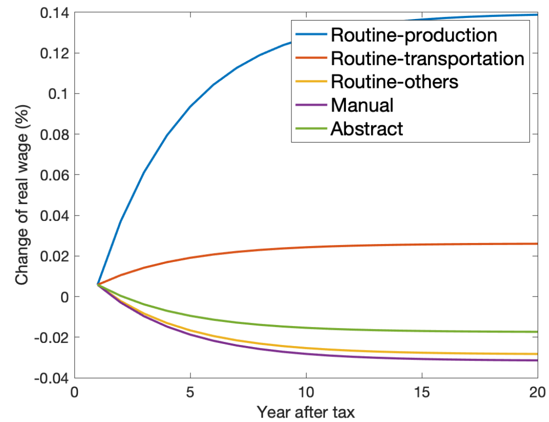
In Figure 3a, I show two scenarios of the steady-state changes in real occupational wages. In one scenario, I shock the economy only with the automation shocks. In the other scenario, I shock the economy with both the automation shocks and the robot tax. The result shows heterogeneous effects on real occupational wages of the robot tax. The

[26] The 6% rate of the general tax is more modest than the 30% rate considered in Humlum (2019) for the Danish case.

Figure 3: Effects of the Robot Tax on Real Occupational Wages



(a) Steady-state Comparison



(b) Transitional Effect of Tax

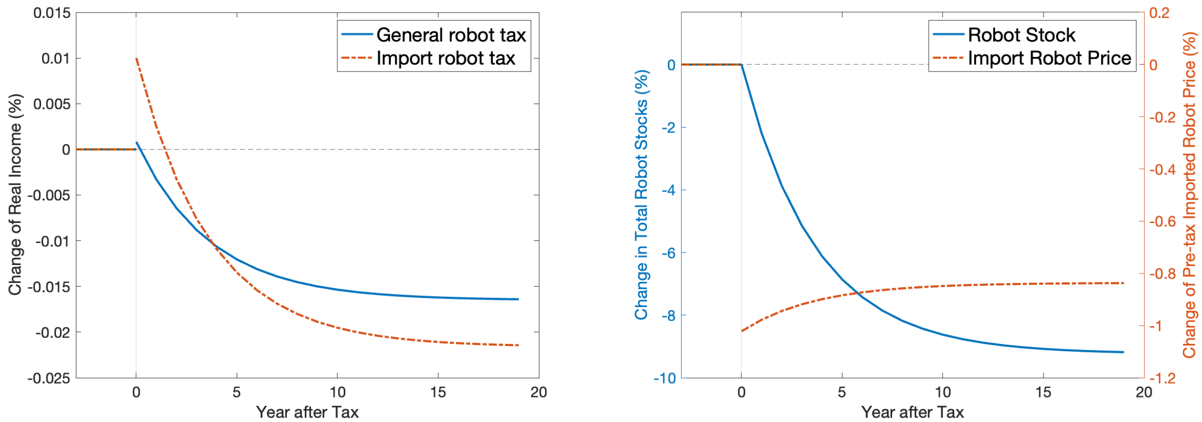
tax mitigates the negative effect of automation on routine production workers and routine transportation workers, while the tax marginally decreases the small gains that workers in the other occupations would have enjoyed. Overall, the robot tax mitigates the large heterogeneous effects of the automation shocks, which could go in negative and positive directions depending on occupation groups, and compresses the effects towards zero. Figure 3b shows the dynamics of the effects of only the robot tax. Although the steady-state effects of robot tax were heterogeneous, as shown in Figure 3a, the effect is not immediate but materializes after around 10 years, due to the sluggish adjustment in the accumulation of the robot capital stock. Overall, I find that since the robot tax slows down the adoption of robots, it rolls back the real wage effect of automation–workers in occupations that experienced significant automation shocks (e.g., production and transportation in the routine occupation groups) benefit from the tax, while the others lose.

5.3 Robot Tax and Aggregate Income

Next, I study how the two robot tax schemes affect the US real income. In Figure 4a, the solid line tracks the real-income effect of the general robot tax over a 20-year time horizon after the tax introduction. First, the magnitude of the effect is small because the

Figure 4: Effects of the Robot Tax

(a) US Real Income (b) US Robot Stock and Import Robot Price



Note: The left panel shows the counterfactual effect on the US real income of the two robot tax scenarios described in the main text over a 20-year time horizon. The right panel shows that of the import robot tax on the US total robot stocks (solid line) and the pre-tax robot price from Japan (dash-dot line) over the same time horizon.

cost of buying robots compared to the aggregate production cost is small. Second, there is a positive effect in the short run, but this effect turns negative quickly and continues to be negative in the long run.

To understand why there is a short-run positive effect on real income, it is useful to distinguish the source of national income in the model. A country's total income comprises workers' wage income, non-robot goods producers' profit, and the tax revenue rebate. Since robots are traded, and the US is a large economy that can affect the robot price produced in other countries, there is a terms-of-trade effect of robot tax in the US. Namely, the robot tax reduces the demand for robots traded in the world market and lets the equilibrium robot price go down along the supply curve. This reduction in the robot price contributes to compressing the cost of robot investment thus to increasing the firm's profit, raising the real income. This positive effect is stronger in the import robot tax scenario because the higher tax rate induces a more substantial drop in the import robot price. While this terms-of-trade manipulation is well-studied in the trade policy literature, my setting is novel since it implies the upward-sloping export supply curve from the GE. This point is discussed in detail in Appendix E.3.

The reason for the different effects on real income, in the long run, is as follows. The solid line in Figure 4b shows the dynamic impact of the import robot tax on the accumulation of robot stock. The robot tax significantly slows the accumulation of robot stocks and decreases the steady-state stock of robots by 9.7% compared to the no-tax case. The small robot stock reduces the firm profit, which contributes to low real income.[27] These results highlight the role that costly robot capital (de-)accumulation plays in the effect of the robot tax on aggregate income.

Figure 4b also shows the dynamic effect on import robot prices in the dash-dot line. In the short run, the price decreases due to the decreased demand from the US, as explained above. As the sequential equilibrium reaches the new steady state where the US stock of robots decreases, the marginal value of the robots is higher. This increased marginal value partially offsets the reduced price of robots in the short run. To further demonstrate the role of robot trade, I also consider an alternative model with no trade of robots due to prohibitively high robot costs and give the robot tax counterfactual exercise in Appendix F.6.

6 Conclusion

In this paper, I study the distributional and aggregate effects of the increased use of industrial robots, with the emphasis that robots perform specified tasks and are internationally traded. I make three contributions. First, I construct a dataset that tracks shocks to the cost of buying robots from Japan (the Japan robot shock) across occupations in which robots are adopted. Second, I develop a general equilibrium model that features the trade of robots in a large open economy and endogenous robot accumulation with an adjustment cost. Third, when estimating the model, I construct a model-implied optimal instrumental variable from the Japan robot shock in my dataset and the approximated solution of

[27] For each occupation, the counterfactual evolution of robot stocks is similar to each other in percentage and, thus, similar to the aggregate trend in percentage. This is not surprising since the robot tax is ad-valorem and uniform across occupations.

the model to identify the occupation-specific EoS between robots and labor.

The estimates of within-occupation EoS between robots and labor is heterogeneous and as high as 3 in production and material-moving occupations. These estimates are significantly larger than estimates of the EoS of capital goods and workers, with a maximum of about 1.5, revealing the special susceptibility to robot adaptation of workers in these occupations. The estimated model also implies that robots contributed to the wage polarization across occupations in the US from 1990-2007. A commonly advertised robot tax could increase the US real income in the short run but leads to a decline in the income in the long run due to the decreased steady-state robot stock. These exercises inform discussions on the regulation policies of industrial robots.

References

Acemoglu, Daron and David Autor (2011). "Skills, tasks and technologies: Implications for employment and earnings". In: *Handbook of labor economics*. Vol. 4. Elsevier, pp. 1043–1171.

Acemoglu, Daron, Claire Lelarge, and Pascual Restrepo (2020). "Competing with Robots: Firm-Level Evidence from France".

Acemoglu, Daron and Pascual Restrepo (2018). "The race between man and machine: Implications of technology for growth, factor shares, and employment". In: *American Economic Review* 108.6, pp. 1488–1542.

— (2020). "Robots and jobs: Evidence from US labor markets". In: *Journal of Political Economy* 128.6, pp. 2188–2244.

Adao, Rodrigo, Costas Arkolakis, and Federico Esposito (2019). "General Equilibrium Indirect Effects in Space: Theory and Measurement".

Anderson, James E, Mario Larch, and Yoto V Yotov (2019). "Trade and investment in the global economy: A multi-country dynamic analysis". In: *European Economic Review* 120, p. 103311.

Armington, Paul S (1969). "A theory of demand for products distinguished by place of production". In: *Staff Papers* 16.1, pp. 159–178.

Arrow, Kenneth J et al. (1961). "Capital-labor substitution and economic efficiency". In: *The review of Economics and Statistics*, pp. 225–250.

Autor, David, David Dorn, and Gordon H Hanson (2013). "The China syndrome: Local labor market effects of import competition in the United States". In: *American Economic Review* 103.6, pp. 2121–68.

Autor, David, Lawrence Katz, and Melissa Kearney (2008). "Trends in US wage inequality: Revising the revisionists". In: *The Review of economics and statistics* 90.2, pp. 300–323.

Autor, David, Frank Levy, and Richard Murnane (2003). "The skill content of recent technological change: An empirical exploration". In: *The Quarterly journal of economics* 118.4, pp. 1279–1333.

Autor, David, David Mindell, and Elisabeth B Reynolds (2020). "The Work of the Future: Building Better Jobs in an Age of Intelligent Machines". In.

Bessen, James E et al. (2019). "Automatic Reaction-What Happens to Workers at Firms that Automate?" In: *Boston Univ. School of Law, Law and Economics Research Paper*.

Blanchard, Olivier Jean and Charles M Kahn (1980). "The solution of linear difference models under rational expectations". In: *Econometrica: Journal of the Econometric Society*, pp. 1305–1311.

Bonfiglioli, Alessandra et al. (2020). "Robot Imports and Firm-Level Outcomes". In.

Caliendo, Lorenzo, Maximiliano Dvorkin, and Fernando Parro (2019). "Trade and labor market dynamics: General equilibrium analysis of the china trade shock". In: *Econometrica* 87.3, pp. 741–835.

Caliendo, Lorenzo and Fernando Parro (2015). "Estimates of the Trade and Welfare Effects of NAFTA". In: *The Review of Economic Studies* 82.1, pp. 1–44.

Cheng, Wan-Jung (2018). "Explaining Job Polariztion: The Role of Heterogeneity in Capital Intensity".

Chirinko, Robert S (2008). "σ: The long and short of it". In: *Journal of Macroeconomics* 30.2, pp. 671–686.

Connector, O*NET Code (2020). *Occupation Search*. URL: https://www.onetcodeconnector.org.

Cooper, Russell W and John C Haltiwanger (2006). "On the nature of capital adjustment costs". In: *The Review of Economic Studies* 73.3, pp. 611–633.

Dauth, Wolfgang et al. (2017). "German robots-the impact of industrial robots on workers". In.

Delvaux, Mady et al. (2016). "Draft report with recommendations to the Commission on Civil Law Rules on Robotics". In: *European Parliament Committee on Legal Affairs http://www. europarl. europa. eu/sides/getDoc. do*.

Dinlersoz, Emin, Zoltan Wolf, et al. (2018). *Automation, Labor Share, and Productivity: Plant-Level Evidence from US Manufacturing*. Tech. rep.

Dixon, Jay, Bryan Hong, and Lynn Wu (2019). "The Employment Consequences of Robots: Firm-Level Evidence". In: *Available at SSRN 3422581*.

Eden, Maya and Paul Gaggl (2018). "On the welfare implications of automation". In: *Review of Economic Dynamics* 29, pp. 15–43.

Engel, Charles and Jian Wang (2011). "International trade in durable goods: Understanding volatility, cyclicality, and elasticities". In: *Journal of International Economics* 83.1, pp. 37–52.

Goos, Maarten and Alan Manning (2007). "Lousy and lovely jobs: The rising polarization of work in Britain". In: *The review of economics and statistics* 89.1, pp. 118–133.

Graetz, Georg and Guy Michaels (2018). "Robots at work". In: *Review of Economics and Statistics* 100.5, pp. 753–768.

Heathcote, Jonathan, Fabrizio Perri, and Giovanni L. Violante (2010). "Unequal we stand: An empirical analysis of economic inequality in the United States, 1967–2006". In: *Review of Economic Dynamics* 13.1. Special issue: Cross-Sectional Facts for Macroeconomists, pp. 15–51. ISSN: 1094-2025. DOI: https://doi.org/10.1016/j.red.2009.10.010. URL: http://www.sciencedirect.com/science/article/pii/S1094202509000659.

Holt, Charles C (1960). "Planning Production, Inventories, and Work Force." In.

Hubmer, Joachim (2018). "The race between preferences and technology". In: *Unpublished paper, Yale University*.

Humlum, Anders (2019). *Robot Adoption and Labor Market Dynamics*. Tech. rep. Working Paper.

Jäger, A, C Moll, and C Lerch (2016). "Analysis of the impact of robotic systems on employment in the European Union–Update". In: *Policy Report, European Commission*.

Jaimovich, Nir et al. (2020). "The macroeconomics of automation: data, theory, and policy analysis". In.

Karabarbounis, Loukas and Brent Neiman (2014). "The global decline of the labor share". In: *The Quarterly journal of economics* 129.1, pp. 61–103.

Khandelwal, Amit (2010). "The long and short (of) quality ladders". In: *The Review of Economic Studies* 77.4, pp. 1450–1476.

Khandelwal, Amit, Peter Schott, and Shang-Jin Wei (2013). "Trade liberalization and embedded institutional reform: Evidence from Chinese exporters". In: *American Economic Review* 103.6, pp. 2169–95.

King, Robert G and Sergio T Rebelo (1999). "Resuscitating real business cycles". In: *Handbook of macroeconomics* 1, pp. 927–1007.

Kleinman, Benny, Ernest Liu, and Stephen J Redding (2020). *International friends and enemies*. Tech. rep. National Bureau of Economic Research.

Koch, Michael, Ilya Manuylov, and Marcel Smolka (2019). "Robots and firms". In.

Leigh, Nancey Green and Benjamin R Kraft (2018). "Emerging robotic regions in the United States: insights for regional economic evolution". In: *Regional Studies* 52.6, pp. 804–815.

Manyika, James et al. (2017). "Harnessing automation for a future that works". In: *McKinsey Global Institute*.

MOEF, Korea (2018). *Korean Taxation*. The Ministry of Economy and Finance, Republic of Korea.

Montobbio, Fabio et al. (2020). "Robots and the origin of their labour-saving impact". In.

Morris, Jeremiah (2019). *A Weighted O*NET Keyword Search (WWS)*. URL: https://www.onetcenter.org/dl_files/WWS.pdf.

Oberfield, Ezra and Devesh Raval (2014). *Micro data and macro technology*. Tech. rep. National Bureau of Economic Research.

Ruggles, Steven et al. (2018). "IPUMS USA: Version 8.0 [dataset]". In: *Minneapolis, MN: IPUMS* 10, p. D010.

Simonovska, Ina and Michael E Waugh (2014). "The elasticity of trade: Estimates and evidence". In: *Journal of international Economics* 92.1, pp. 34–50.

Traiberman, Sharon (Dec. 2019). "Occupations and Import Competition: Evidence from Denmark". In: *American Economic Review* 109.12, pp. 4260–4301. DOI: 10.1257/aer.20161925. URL: http://www.aeaweb.org/articles?id=10.1257/aer.20161925.

Webb, Michael (2019). "The Impact of Artificial Intelligence on the Labor Market". In: *Available at SSRN 3482150*.

Appendix

A Data Appendix

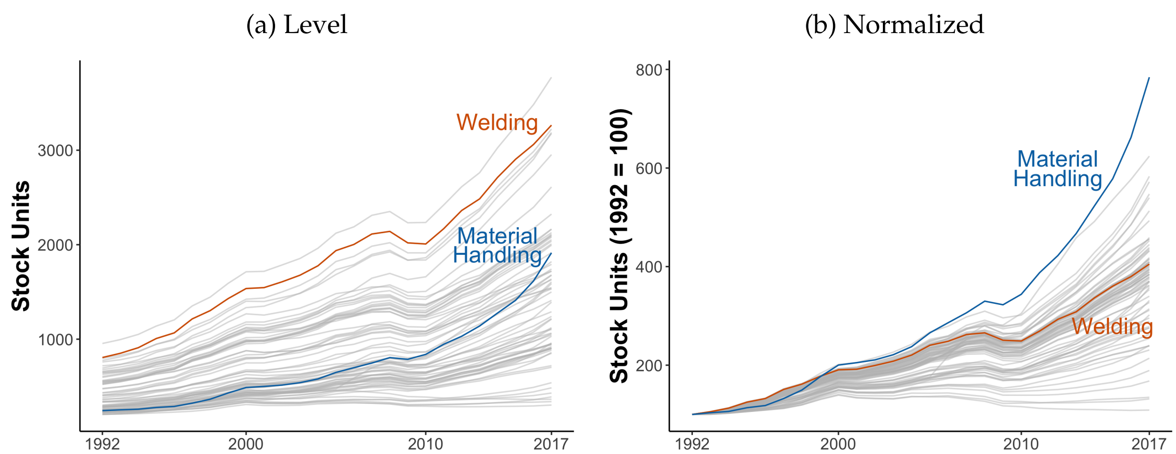
A.1 Trends of Robot Stocks and Prices

In this section, I show that each occupation experienced different trends in robot adoption. Figure A.1 shows the trend of US robot stocks at the occupation level. In the left panel, I show the trend of raw stock, which reveals the following two facts. Firstly, it shows that the overall robot stocks increased rapidly in the period, as found in the previous literature. Second, the panel also depicts that the increase occurred at different speeds across occupations. To highlight such a difference, I plot the normalized trend at 100 in the initial year in the right panel. There is significant heterogeneity in the growth rates, ranging from a factor of one to eight.

To further emphasize the different speeds, I color the following two occupations: "Welding, Soldering, and Brazing Workers" (or "Welding") and "Laborers and Freight, Stock, and Material Movers, Hand" (or "Material Handling") in these two figures. On the one hand, the stock of welding robots grew continuously throughout the period, as can be confirmed in the left panel. However, the growth rate is not outstanding but within the range of growth rates of other occupations. On the other hand, material handling was not a majority occupation as of the initial year, but it grew at the most rapid pace in the sample period. These findings indicate the difference between the automation shocks to each occupation. Some occupations were already somewhat automated by robots as of the initial year, and the automation process continued afterward (e.g., welding). There are a few occupations where robotics automation had not occurred initially, but the adoption proceeded rapidly in the sample period (e.g., material handling). This observation motivates the model that incorporates the heterogeneity across occupations in the Model section.

Next, Figure A.2 shows the trend of prices of robots in the US for each occupation. In

Figure A.1: US Robot Stocks at the Occupation Level

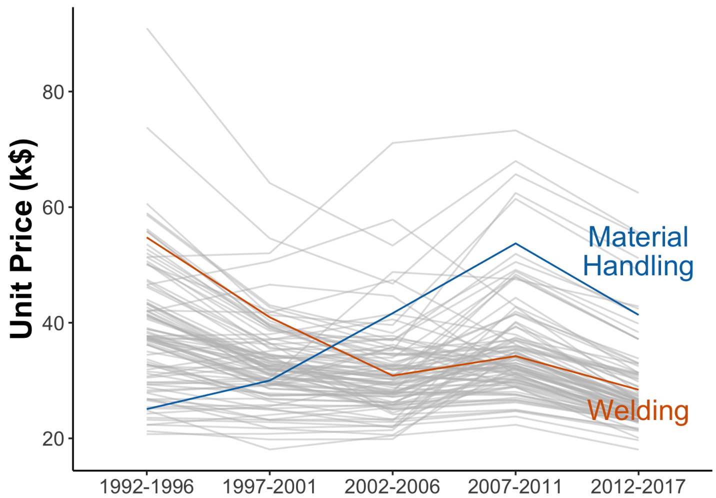


Note: The figure shows the trend of stocks of robots in the US for each occupation. The left panel shows the level, whereas the right panel shows the normalized trend at 100 in 1992. In both panels, I highlight two occupations. "Welding" corresponds to the occupation code in IPUMS USA, OCC2010 = 8140 "Welding, Soldering, and Brazing Workers." "Material Handling" corresponds to the occupation code OCC2010 = 9620 "Laborers and Freight, Stock, and Material Movers, Hand." Years are aggregated into five-year bins (with the last bin 2012-2017 being six-year one) to smooth out yearly noises.

addition to the overall decreasing trend, there is significant heterogeneity in the pattern of price falls across occupations. For instance, although welding robots saw a large drop in price during the 1990s, the price of material handling robots increased the over the sample period. These patterns are strongly correlated across countries, as indicated by the correlation coefficient of 0.968 between the US and non-US prices at the occupation-year level. Based on this finding, I use the non-US countries' prices as the Japan robot shock to the US in the Data section.

In Figure A.2, one might wonder if there is an anomaly to the overall decreasing trend in the 2007-2011 period, in which the trends of robot prices halt dropping across the board. This pattern emerges because of the method for generating these series. Namely, total sales divided by total units measure the average price. During the Great Recession period, the total units decreased more than the total sales. The relative pattern caused a temporary increase in average robot prices. In addition, after the Great Recession, both the growth of sales and units of robots accelerated. These observations suggest the structural break of the robot industry during the Great Recession, which is out of the scope of

Figure A.2: Robot Prices at the Occupation Level



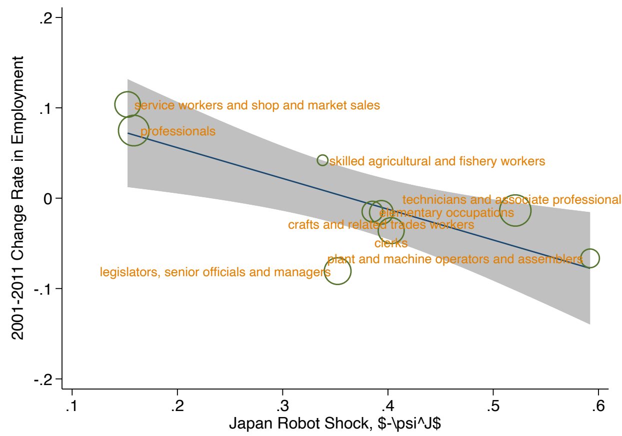
Note: The figure shows the trend of prices of robots in the US for each occupation. I highlight two occupations. "Welding" corresponds to the occupation code in IPUMS USA, OCC2010 = 8140 "Welding, Soldering, and Brazing Workers." "Material Handling" corresponds to the occupation code OCC2010 = 9620 "Laborers and Freight, Stock, and Material Movers, Hand." Years are aggregated into five-year bins (with the last bin 2012-2017 being six-year one) to smooth out yearly noises. The dollars are converted to 2000 real US dollars using CPI.

the paper.

A.2 Robustness Exercise in a Small Country

A concern of my main analysis is that the US is a large buyer of robots, and thus its demand may influence the price. To mitigate it, I conduct a robustness exercise using data from a small country that is unlikely to affect the world price of robots. Specifically, I use data from the Netherlands as a case since it is the largest exporting destination of Japanese robots in Europe, following Germany, the UK, Italy, and France, and it is a small-open economy at the same time. The data are taken from the IPUMS international and provide the ISCO 1-digit level occupation indicator in the years 2001 and 2011. I aggregate the occupational robot prices at the same level and examine the relationship between the Japan robot shock and occupational employment growth. Figure A.3 summarizes the results. Despite a significant difference in context and the level of data aggregation, I find a significant negative relationship between these two variables. Quantitatively, the

Figure A.3: The Effect of Japan Robot Shock in the Netherlands



Note: The bubble plot and fitted line between the Netherland occupational growth and the Japan robot shock are shown. The period is from 2001 to 2011. The size of the bubble reflects the initial period size of employment. The occupations are aggregated to the ISCO 1-digit level. The shade indicates the 95% confidence interval.

regression coefficient is -0.33 with the standard error of 0.08, which is comparable to the reduced-form coefficient found in the case of the US in Table 1.

A.3 The Effect of Robots from Japan and Other Countries

A potential concern for my empirical setting is the selection issue regarding the robot origin country of Japan. Specifically, robots from Japan may differ from those from other countries, so the labor market implications may also differ between them. Unfortunately, it is hard to directly compare the effects of these two different groups of robots due to the data limitation, so I will focus on the best comparable measures of robotization between Japan-sourced robots and robots from all countries, which is the quantity of robot stock. Specifically, I take the total stock of robots in the US from the IFR data. This measure does not contain the monetary value at the occupation level for all the sample periods, but it is the number of units. Note also that the IFR variable is the total number that does not specify the source country. I then convert the IFR application codes to the JARA application codes to use the allocation rule for matching the JARA application codes and the occupation codes. As a result, I obtain the robots used in the US and sourced from any country at the occupation level. I then run the following regression using the obtained robot measures and my preferred measure from the JARA:

$$\Delta Y_o = \beta^Q \Delta K_o^{R,Q} + X_o \gamma^Q + \varepsilon_o^Q, \tag{A.1}$$

where ΔY_o is either the changes in wages or employment at the occupation-o level, ΔK_o^Q is the measure of the number of robots taken either from JARA (i.e., robots from Japan) or IFR (i.e., robots from the world), and ε_o^Q is the error term. The coefficient of interest is β^Q, which gives us an insight into the correlation between the changes in labor market outcomes and the changes in robot quantity, depending on whether the robots are conditioned to be sourced from Japan. Specifically, if robots from Japan may substitute workers stronger than robots from the other countries, coefficient β^Q is expected to be larger when we use the JARA robot measure than IFR.

Table A.1 shows the regression result of equation (A.1). Columns 1-4 consider the changes in occupational wage in the outcome variable, while columns 5-8 take occupational employment. Columns 1, 2, 5, and 6 do not include the demographic control variables (female share, age distribution, college-graduate share, and foreign-born share), while columns 3, 4, 7, and 8 do. Columns 1, 3, 5, and 7 take the robots from Japan from the JARA data, while columns 2, 4, 6, and 8 take the robots from the world from the IFR data. Table A.1 reveals that both the JARA- and IFR-based robot measures capture the substitution of workers with robots. The result for the IFR data is in line with the previous findings by Acemoglu and Restrepo (2020). In contrast, comparing the size of coefficients, one can find that the coefficient is somewhat stronger for JARA robot measures than for IFR. Overall, I find some evidence that Japanese robots substitute workers stronger than other countries' robots, while all sorts of robots do seem to have some substitution effect on workers.

Table A.1: Regression Result of Labor Market Outcome on JARA and IFR Robot Stocks

VARIABLES	(1) $\Delta\ln(w)$	(2) $\Delta\ln(w)$	(3) $\Delta\ln(w)$	(4) $\Delta\ln(w)$	(5) $\Delta\ln(L)$	(6) $\Delta\ln(L)$	(7) $\Delta\ln(L)$	(8) $\Delta\ln(L)$
$\Delta\ln(K^{R,Q}_{JPN\to USA})$	-0.372		-0.271		-0.765		-0.648	
	(0.0311)		(0.0315)		(0.0903)		(0.0830)	
$\Delta\ln(K^{R,Q}_{USA})$		-0.144		-0.111		-0.311		-0.500
		(0.0161)		(0.0208)		(0.0447)		(0.0487)
Observations	324	324	324	324	324	324	324	324
R-squared	0.307	0.200	0.349	0.262	0.182	0.131	0.188	0.273
Controls			✓	✓			✓	✓

Note: Observations are 4-digit level occupations, and the regression is between 1990 and 2007 with the sample of all occupations that existed between 1970 and 2007. Columns 1-4 consider the changes in occupational wage in the outcome variable, while columns 5-8 take occupational employment. Columns 1, 2, 5, and 6 do not include the control variables such as demographic variables (female share, age distribution, college-graduate share, and foreign-born share) and China trade shock constructed in the main text, while columns 3, 4, 7, and 8 do. Columns 1, 3, 5, and 7 take the robots from Japan from JARA data, while columns 2, 4, 6, and 8 take the robots from the world from IFR data. Heteroskedasticity-robust standard errors are reported in the parenthesis.

B Theory Appendix

B.1 The Real-Wage Effect of Automation

In this section, I demonstrate the analytical result that the effect of automation on real wages depends negatively on substitution elasticity parameters θ_o and β conditional on the changes in input and trade shares. The key insight is that real wages are the relative price of labor to the consumer price index that reflects a price bundle of other factors, and the relative price changes are related to changes in the corresponding input shares and trade shares via the demand elasticities of factors and goods.

For this purpose, I modify notations in equation (6) to express the result in a concise way. Namely, define

$$A^K_{i,o,t} \equiv \left(A^G_{i,t}\right)^{\theta-1} a_{o,t},\ A^L_{i,o,t} \equiv \left(A^G_{i,t}\right)^{\theta-1} (1 - a_{o,t}). \tag{B.2}$$

Substituting these into production functions (5) and (6), I have

$$Y^G_{i,t} = \left[\sum_o \left(b_{i,o,t}\right)^{\frac{1}{\beta}} \left(\widetilde{T}^O_{i,o,t}\right)^{\frac{\beta-1}{\beta}}\right]^{\frac{\beta}{\beta-1}},$$

where

$$\widetilde{T}^O_{i,o,t} \equiv \left[\left(A^L_{i,o,t}\right)^{\frac{1}{\theta_O}} \left(L_{i,o,t}\right)^{\frac{\theta_O-1}{\theta_O}} + \left(A^K_{i,o,t}\right)^{\frac{1}{\theta_O}} \left(K^R_{i,o,t}\right)^{\frac{\theta_O-1}{\theta_O}}\right]^{\frac{\theta_O}{\theta_O-1}}.$$

Therefore, one can interpret the new terms $A^K_{i,o,t}$ and $A^L_{i,o,t}$ as the productivity shock on robots and labor, respectively.[28] Furthermore, define the labor share of producers of non-robot good G within occupation o by $\widetilde{x}^L_{i,o,t}$, occupation o's cost share among the occupation

[28] By equation (B.2), robot productivity change $\widehat{A^K_{i,o,t}}$ and automation shock $\widehat{a_{o,t}}$ satisfy that $\widehat{A^K_{i,o,t}} = \frac{\theta-1}{\alpha_{i,L}}\widehat{A^G_{i,t}} + \widehat{a_{o,t}}$. Namely, robot productivity change is the sum of total factor productivity change caused by robotics and the automation shock. I choose to use the automation shock in my main specification in equations (5) and (6) since it has a tight connection to the task-based approach, a common approach in the automation literature (e.g., Acemoglu and Restrepo 2020), as discussed in Section 3.1.

aggregate by $\widetilde{x}^{O}_{i,o,t}$, and trade share by $\widetilde{x}^{G}_{ij,t}$. Formally, these terms are defined as:

$$\widetilde{x}^{L}_{i,o,t} \equiv \frac{w_{i,o,t} L_{i,o,t}}{P^{O}_{i,o,t} T^{O}_{i,o,t}},\ \widetilde{x}^{O}_{i,o,t} \equiv \frac{P^{O}_{i,o,t} T^{O}_{i,o,t}}{p^{G}_{i,t} Q^{G}_{i,t}},\ \widetilde{x}^{G}_{ij,t} \equiv \frac{p^{G}_{i,t} Q^{G}_{ij,t}}{P^{G}_{i,t} Q^{G}_{i,t}}, \tag{B.3}$$

where $P^{O}_{i,o,t}$ is the price index of occupation o. The following proposition characterizes the real-wage changes in the steady state.

Proposition B.1. *Suppose robot productivity grows $\widehat{A^{K}_{i,o}} > 0$. For each country i and occupation o,*

$$\widehat{\left(\frac{w_{i,o}}{P^{G}_{i}}\right)} = \frac{\widehat{\widetilde{x}^{L}_{i,o}}}{1-\theta_o} + \frac{\widehat{\widetilde{x}^{O}_{i,o}}}{1-\beta} + \frac{\widehat{\widetilde{x}^{G}_{ii}}}{1-\varepsilon}. \tag{B.4}$$

Proposition B.1 clarifies how the elasticity parameters and change of shares of input and trade affect real wages at the occupation level. For example, one can observe that if $\theta_o > 1$, then (i) the larger the fall of the labor share within occupation $\widehat{\widetilde{x}^{L}_{i,o}}$, the larger the real wage gains, and (ii) pattern (i) is stronger if θ_o is small and close to 1. Therefore, conditional on other terms, the steady state changes of real occupational wages depend on the elasticity of substitution between robots and labor θ_o.

The intuition of Proposition B.1 comes from a series of revealed cost reductions, $\widehat{\widetilde{x}^{L}_{i,o}}$, $\widehat{\widetilde{x}^{O}_{i,o}}$, and $\widehat{\widetilde{x}^{G}_{ii}}$. The first term reveals the robot cost reduction relative to the labor cost. If $\theta_o > 1$, then the reduction in the price index or cost savings induced by robotization shocks dominate the drop in nominal wage, increasing the real wage. Similarly, the second term reflects the reduction of the relative cost of the occupation, and the last term represents the decrease in the production cost relative to other countries.

Proposition B.1 also extends the result of the welfare sufficient statistic in the trade literature. In particular, Arkolakis, Costinot, and Rodriguez-Clare (2012, ACR) showed that under a large class of trade models, the welfare effect of the reduction in trade costs can be characterized by the well-known ACR formula, or log-difference of the trade shares times the negative inverse of the trade elasticity. Specifically, suppose I drop robots and non-robot capital from the model and aggregate all occupations into one factor (labor).

Then, one can prove that the shocks to the productivity $\left\{A_{i,t}^{G}\right\}$ implies

$$\widehat{\left(\frac{w_i}{P_i^G}\right)} = \frac{1}{1-\varepsilon}\widehat{\tilde{x}_{ii}^G},$$

which is the ACR formula. In the next section, motivated by Proposition B.1, I estimate the model and back out the automation shock to study the impact of robotization on the occupational wage.

B.2 The Full Model

The full model used for structural estimation extends the one in the model section with intermediate goods and non-robot capital. The intermediate goods are the same goods as the non-robot goods, but are an input to the production function. The stock of non-robot capital is exogenously given in each period for each country, and producers rent non-robot capital from the rental market. The non-robot good production function is given by

$$Y_{i,t}^G = A_{i,t}^G \left\{\alpha_{i,L}\left(T_{i,t}^O\right)^{\frac{\vartheta-1}{\vartheta}} + \alpha_{i,M}\left(M_{i,t}\right)^{\frac{\vartheta-1}{\vartheta}} + \alpha_{i,K}\left(K_{i,t}\right)^{\frac{\vartheta-1}{\vartheta}}\right\}^{\frac{\vartheta}{\vartheta-1}},$$

where ϑ is the elasticity of substitution between occupation aggregates, intermediates goods, and non-robot capital, and $\alpha_{i,L}$, $\alpha_{i,M}$, and $\alpha_{i,K} \equiv 1 - \alpha_{i,L} - \alpha_{i,M}$ are cost share parameters for the occupation aggregates, intermediates, and non-robot capital, respectively. Parameters satisfy $\vartheta > 0$ and $\alpha_{i,L}, \alpha_{i,M}, \alpha_{i,K} > 0$, and in the structural estimation, I set $\vartheta = 1$ and compute each country's cost share parameters from the data. Intermediate goods are aggregated by

$$M_{i,t} = \left[\sum_l \left(M_{li,t}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{B.5}$$

where $\varepsilon > 0$ is the elasticity of substitution. Since intermediate goods are traded across countries and aggregated by equation (B.5), the elasticity parameter ε plays the role of the trade elasticity. The static decision of the producers now includes the rental amount of non-robot capital and the purchase of intermediate goods from each source country.

Workers solve a dynamic discrete choice problem to select an occupation (Traiberman 2019; Humlum 2019). Specifically, workers choose the occupations that maximize the lifetime utility based on switching costs and the draw of an idiosyncratic shock. The problem has a closed form solution when the shock follows an extreme value distribution, which is the property that the previous literature utilized (e.g., Caliendo, Dvorkin, and Parro 2019). Since I follow a similar strategy, I relegate the formal problem statement and derivation to Appendix E.7. The worker's problem can be characterized by, for each country i and period t, the transition probability $\mu_{i,oo',t}$ from occupation o in period t to occupation o' in period $t+1$, and the exponential expected value $V_{i,o,t}$ for occupation o that satisfy

$$\mu_{i,oo',t} = \frac{\left(\left(1-\chi_{i,oo',t}\right)\left(V_{i,o',t+1}\right)^{\frac{1}{1+\iota}}\right)^{\phi}}{\sum_{o''}\left(\left(1-\chi_{i,oo'',t}\right)\left(V_{i,o'',t+1}\right)^{\frac{1}{1+\iota}}\right)^{\phi}}, \tag{B.6}$$

$$V_{i,o,t} = \widetilde{\Gamma} C_{i,o,t}\left[\sum_{o'}\left(\left(1-\chi_{i,oo',t}\right)\left(V_{i,o',t+1}\right)^{\frac{1}{1+\iota}}\right)^{\phi}\right]^{\frac{1}{\phi}}, \tag{B.7}$$

respectively, where $C_{i,o,t+1}$ is the real consumption, $\chi_{i,oo',t}$ is an ad-valorem switching cost from occupation o to o', ϕ is the occupation-switch elasticity, $\widetilde{\Gamma} \equiv \Gamma\left(1-1/\phi\right)$ is a constant that depends on the Gamma function $\Gamma\left(\cdot\right)$. For each i and t, employment level satisfies the law of motion

$$L_{i,o,t+1} = \sum_{o'} \mu_{i,o'o,t} L_{i,o',t}. \tag{B.8}$$

B.3 Detail in Constructing the Instrumental Variable

Using Assumption 1, I develop a consistent and asymptotically efficient two-step estimator. Specifically, I follow the method developed by Adao, Arkolakis, and Esposito (2019), who extend the estimator of Newey and McFadden (1994) to the general equilibrium environment and define the model-implied optimal instrumental variable (MOIV). The key idea is that the optimal GMM estimator is based on the instrumental variable that depends on unknown structural parameters. Therefore, the two-step estimator solves this unknown-dependent problem and achieves desirable properties of consistency and asymptotic efficiency. As a result, I define IVs $Z_{o,n}$ where $n = 0, 1$ as follows:

$$Z_{o,n} \equiv H_{o,n}\left(\boldsymbol{\psi}^J\right) = \mathbb{E}\left[\nabla_{\boldsymbol{\Theta}} \nu_o\left(\boldsymbol{\Theta}_n\right) | \boldsymbol{\psi}^J\right] \mathbb{E}\left[\nu_o\left(\boldsymbol{\Theta}_n\right)\left(\nu_o\left(\boldsymbol{\Theta}_n\right)\right)^\top | \boldsymbol{\psi}^J\right]^{-1}. \tag{B.9}$$

Then, I achieve the following result.

Proposition B.2. *Under Assumptions 1 and E.1, the estimator $\boldsymbol{\Theta}_2$ obtained in the following procedure is consistent, asymptotically normal, and optimal:*

Step 1: With a guess $\boldsymbol{\Theta}_0$, estimate $\boldsymbol{\Theta}_1 = \boldsymbol{\Theta}_{\boldsymbol{H}_0}$ using $Z_{o,0}$ defined in equation (B.9).

Step 2: With $\boldsymbol{\Theta}_1$, estimate $\boldsymbol{\Theta}_2$ by $\boldsymbol{\Theta}_2 = \boldsymbol{\Theta}_{\boldsymbol{H}_1}$ using $Z_{o,1}$ defined in equation (B.9).

See Propositions E.1 in Appendix E.5 for discussion in further detail.

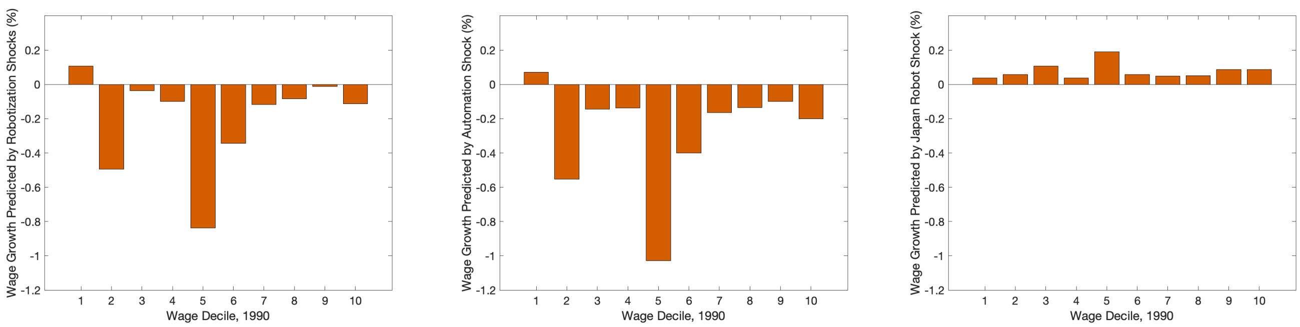
C Counterfactuals Appendix

C.1 Simulation Method

The simulation for the counterfactual analysis comprises three steps. First, I back out the observed shocks from the estimated model for each year between 1992 and 2007. Namely, I obtain the efficiency increase of Japanese robots $\widehat{A_{2,o,t}^R}$ using equation (18). With the point estimates in Table 2, the implied automation shock $\widehat{a_{o,t}^{\text{imp}}}$ using (19). To back out the efficiency shock of robots in the other countries, I assume that $\widehat{A_{i,o,t}^R} = \widehat{A_{i,t}^R}$ for $i = 1, 3$. Then by the robot trade prices $p_{ij,t}^R$ from BACI, I fit fixed effect regression $\Delta \ln\left(p_{ij,t}^R\right) = \widetilde{\psi}_{j,t}^D + \widetilde{\psi}_{i,t}^C + \widetilde{e}_{ij,t}$, and use $\widehat{A_{i,t}^R} = -\widetilde{\psi}_{i,t_1}^C$. The idea to back out the negative efficiency shock $\widetilde{\psi}_{i,t_1}^C$ is similar to the fixed-effect regression in Section 2, but without the occupational variation that is not observed in BACI data. Second, applying the backed-out shocks $\widehat{A_{i,o,t}^R}$ and $\widehat{a_{o,t}^{\text{obs}}}$ to the first-order solution of the GE in equation (17), I obtain the prediction of changes in endogenous variables to these shocks to the first-order. Finally, applying

Figure C.4: The Effect on Occupational Wages by Sources of Shocks

(a) Both Shocks | (b) Automation Shock | (c) Japan Robot Shock



Note: The left panel shows the annualized occupational wage growth rates for each wage decile, predicted by the first-order steady-state solution of the estimated model given in equation (15), for each of ten deciles of the occupational wage distribution in 1990, and is equivalent to Figure 2b. The center and right panels distinguish the effect of the automation shock (center) and the Japan robot shock (right).

the predicted changes to the initial data in $t_0 = 1992$, I obtain the predicted level of endogenous variables.

C.2 The Effect of Robotization and the Sources of Shocks

In Figure 2b, I show the effect of two robotization shocks: the automation shock $\widehat{\boldsymbol{a}}$ and the Japan robot shock $\widehat{A}_2$. Although both are relevant shocks to the robotics technology during the sample period, the result is a mixture of these two effects, making it hard to assess the contribution of each shock. To address this concern, Figure C.4 shows the decomposition of the main exercise. The left panel shows the same result as Figure 2b, while the center panel shows the predicted wage changes with only the automation shock and the right only the Japan robot shock. Notably, it is the automation shock that reduces the labor demand and, thus, the wage across many occupations. By contrast, the Japan robot shock reduces the price of robots and increases the marginal product of labor, and thus the occupational wages are increased.

Online Appendix

D Online Data Appendix

D.1 Data Sources in Detail

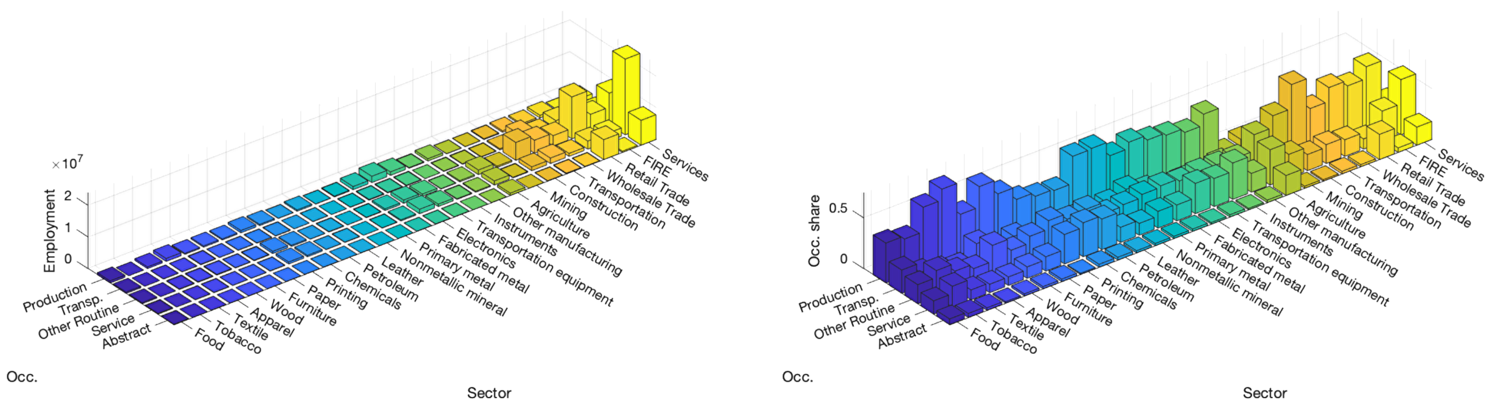
I complement data from JARA data and O*NET data by the ones from IFR, BACI, IPUMS USA and CPS. IFR is a standard data source of industrial robot adoption in several countries (e.g., Graetz and Michaels 2018; Acemoglu and Restrepo, 2020, AR thereafter), to which JARA provides the robot data of Japan. I use IFR data to show the total robot adoption in each destination country as opposed to the import from Japan. I use Federal Reserve Economic Data (FRED) to convert JARA variables denominated in JPY to USD. BACI provides disaggregated data on trade flows for more than 5000 products and 200 countries and is a standard data source of international trade (Gaulier and Zignago 2010). I use BACI data to obtain the measure of international trade of industrial robots and baseline trade shares. IPUMS USA collects and harmonizes US census microdata (Ruggles et al. 2018). I use Population Censuses (1970, 1980, 1990, and 2000) and American Community Surveys (ACS, 2006-2008 3-year sample and 2012-2016 5-year sample). I obtain occupational wages, employment, and labor cost shares from these data sources. To obtain the intermediate inputs shares, I take data from the World Input-Output Data (WIOD) in the closest year to the initial year, 1992.

I focus on consistent occupations between the 1970 Census and the 2007 ACS that cover the sample period and pre-trend analysis period to obtain consistent data across periods. Therefore, this paper focuses on the intensive-margin substitution in occupations as opposed to the extensive-margin effect of automation that creates new labor-intensive tasks and occupations (Acemoglu and Restrepo 2018). My dataset shows that 88.7 percent of workers in 2007 worked in the occupations that existed in 1990. It is an open question how to attribute the creation of new occupations to different types of automation goods like occupational robots in my case, although Autor and Salomons (2019) explore how to measure the task contents of new occupations.

I follow Autor, Dorn, and Hanson (2013) for Census/ACS data cleaning procedure. Namely, I extract the 1970, 1980, 1990, 2000 Censuses, the 2006-2008 3-year file of American Community

Figure D.1: Occupational Employment Distribution

(a) Employment size L_{s,o,t_0} (b) Employment share l_{s,o,t_0}



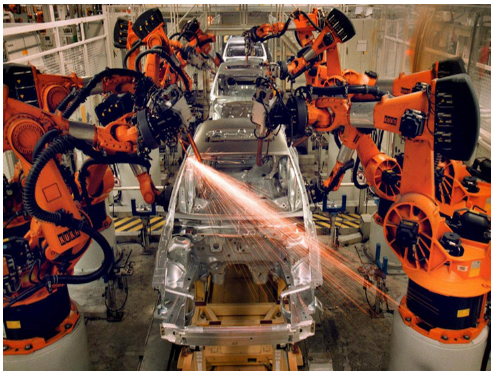
Note: The author's calculation from the 1990 US Census. The axis on the left indicates the 5 occupation groups defined in Section 4.1, and the one on the right shows sectors (roughly 4-digit for manufacturing sectors and 2-digit for the non-manufacturing). The left panel shows the size of employment, and the right one indicates the occupation share for each given sector.

Survey (ACS), and the 2012-2016 5-year file of ACS from Integrated Public Use Micro Samples. For each file, I select all workers with the OCC2010 occupation code whose age is between 16 and 64 and who is not institutionalized. I compute education share in each occupation by the share of workers with more than "any year in college," and foreign-born share by the share of workers with BPL (birthplace) variable greater than 150, or those whose birthplace is neither in the US nor in US outlying areas/territories. I compute hours worked by multiplying usual weeks worked and hours worked per week. For 1970, I use the median values in each bin of the usual weeks worked variable and assume all workers worked for 40 hours a week since the hour variable does not exist. To compute hourly wage, I first impute each state-year's top-coded values by multiplying 1.5 and divide by the hours worked. To remove outliers, I take wages below first percentile of the distribution in each year, and set the maximum wage as the top-coded earning divided by 1,500. I compute the real wage in 2000 dollars by multiplying CPI99 variable prepared by IPUMS. I use the person weight variable for aggregating all of these variables to the occupation level. Figure D.1 shows the occupational employment distribution for each sector, a variable used for creating the occupational China shock in equation (3).

To estimate the model with workers' dynamic discrete choice of occupation, I further use the bilateral occupation flow data following the idea of Caliendo, Dvorkin, and Parro (2019). Specifically, I obtain the Annual Social and Economic Supplement (ASEC) of the CPS since 1976. For each year, I select all workers with the 2010 occupation code for the current year (OCC2010) and the last year (OCC10LY) whose age is between 16 and 64 and who is not institutionalized, and treated top-coded wage income, converted nominal wage income to real one, and computed labor hours worked, education, foreign born flag variable with the same method as the one used for Census/ACS above. When computing the occupation switch probability, note that the 4-digit occupations are too disaggregated to precisely estimate with the small sample size of CPS-ASEC, as pointed out by Artuç, Chaudhuri, and McLaren (2010). Therefore, I assume that the occupations do not flow between 4-digit occupations within the 5 groups defined in Section 4.1, but do between the 5 groups. I assume that workers draw a destination 4-digit occupation occupation from the initial-year occupational employment distribution within the destination group when switching occupations. With these data and assumptions, I compute the occupation switching probability by year.

Figure D.2: Examples of Industrial Robots

(a) Spot Welding



(b) Material Handling

Sources: Autobot Systems and Automation (https://www.autobotsystems.com) and PaR Systems (https://www.par.com)

D.2 Detailed Description of Industrial Robots

Robot Definition and Examples As defined in Footnote 1, industrial robots are defined as multiple-axes manipulators. More formally, following International Organization for Standardization (ISO), I define robots as "automatically controlled, reprogrammable, multipurpose manipulator, programmable in three or more axes, which can be either fixed in place or mobile for use in industrial automation applications" (ISO 8373:2012). This section gives a detailed discussion about such industrial robots. Figure D.2 shows the pictures of examples of industrial robots that are intensively used in the production process and considered in this paper. The left panel shows spot-welding robots, while the right panel shows the material-handling robots. The spot welding robots are an example of robots in routine-production occupations, while the material-handling robots are that in routine-transportation (material-moving) occupations.

It is also worthwhile to give an example of technologies that are *not* robots according to the definition in this paper. An example of a growth in technology in the material-handling area is autonomous driving. Mehta and Levy (2020) predicts that such automation will grow strong and result in the reduction of total number of jobs in this area in eight to ten years since 2020. However, since autonomous vehicles do not operate multiple-axes, they are not treated in this paper at all. A similar observation applies for computers or artificial intelligence more generally.

Examples of Robotics Innovation In the model, I call a change in the robot task space $a_{o,t}$ as the automation shock, and that in robot producer's TFP $A^R_{l,o,t}$ as the cost shock to produce robots. In this section, I show some examples of changes of robot technology and new patents to facilitate understandings of these interpretation. An example of task space expansion is adopting *Programmed Article Transfer* (PAT, Devol 1961). PAT was machine that moves objects by a method called "teaching and playback". Teaching and playback method needs one-time teaching of how to move, after which the machine playbacks the movement repeatedly and automatically. This feature frees workers of performing repetitive tasks. PAT was intensively introduced in spot welding tasks. KHI (2018) reports that among 4,000 spot welding points, 30% were done be human previously, which PAT took over. Therefore, I interpret the adoption of PAT as the example of the expansion of the robot task space, or increase in $a_{o,t}$. Note that AR also analyze this type of technological change.

An example of cost reduction is adopting *Programmable Universal Manipulator for Assembly (PUMA).* PUMA was designed to quickly and accurately transport, handle and assemble automobile accessories. A new computer language, *Variable Assembly Language (VAL)*, made it possible because it made the teaching process less work and more sophisticated. In other words, PUMA made tasks previously done by other robots but at cheaper unit cost per unit of task.

It is also worth mentioning that introduction of a new robot brand typically contains both components of innovation (task space expansion and cost reduction). For example, PUMA also expanded task space of robots. Since VAL allowed the use of sensors and "expanded the range of applications to include assembly, inspection, palletizing, resin casting, arc welding, sealing and research" (KHI 2018).

JARA Robot Applications In addition to applications in Section D.2, the full list of robot applications avaliable in JARA data is Die casting; Forging; Resin molding; Pressing; Arc welding; Spot welding; Laser welding; Painting; Load/unload; Mechanical cutting; Polishing and deburring; Gas cutting; Laser cutting; Water jet cutting; General assembly; Inserting; Mounting; Bonding; Soldering; Sealing and gluing; Screw tightening; Picking alignment and packaging; Palletizing; Measurement/inspection/test; and Material handling.

One might wonder if robots can be classified as one of these applications since robots are characterized by versatility as opposed to older specified industrial machinery (KHI 2018). Although it is true that a robot may be reprogrammed to perform more than one task, I claim that robots are well-classified to one of the applications listed above since the layer of dexterity is different. Robots might be able to adjust a model change of the products, but are not supposed to perform different tasks across the 4-digit occupation level. To support this point, recall that "SMEs are mostly high-mix/low-volume producers. Robots are still too inflexible to be switched at a reasonable cost from one task to another" (Autor, Mindell, and Reynolds 2020). These technological bottlenecks still make it hard for producers to have such a versatile robot that can replace a wide range of workers at the 4-digit occupation level even today, all the more for the sample period of my study.

D.3 Methods for Adjusting the Robot Prices

In the paper, I use the general equilibrium model to control for the quality component of robot prices. However, there are other methods proposed in the literature of measuring the price of capital goods. In this subsecsion, I briefly describe these methods and their limitations.

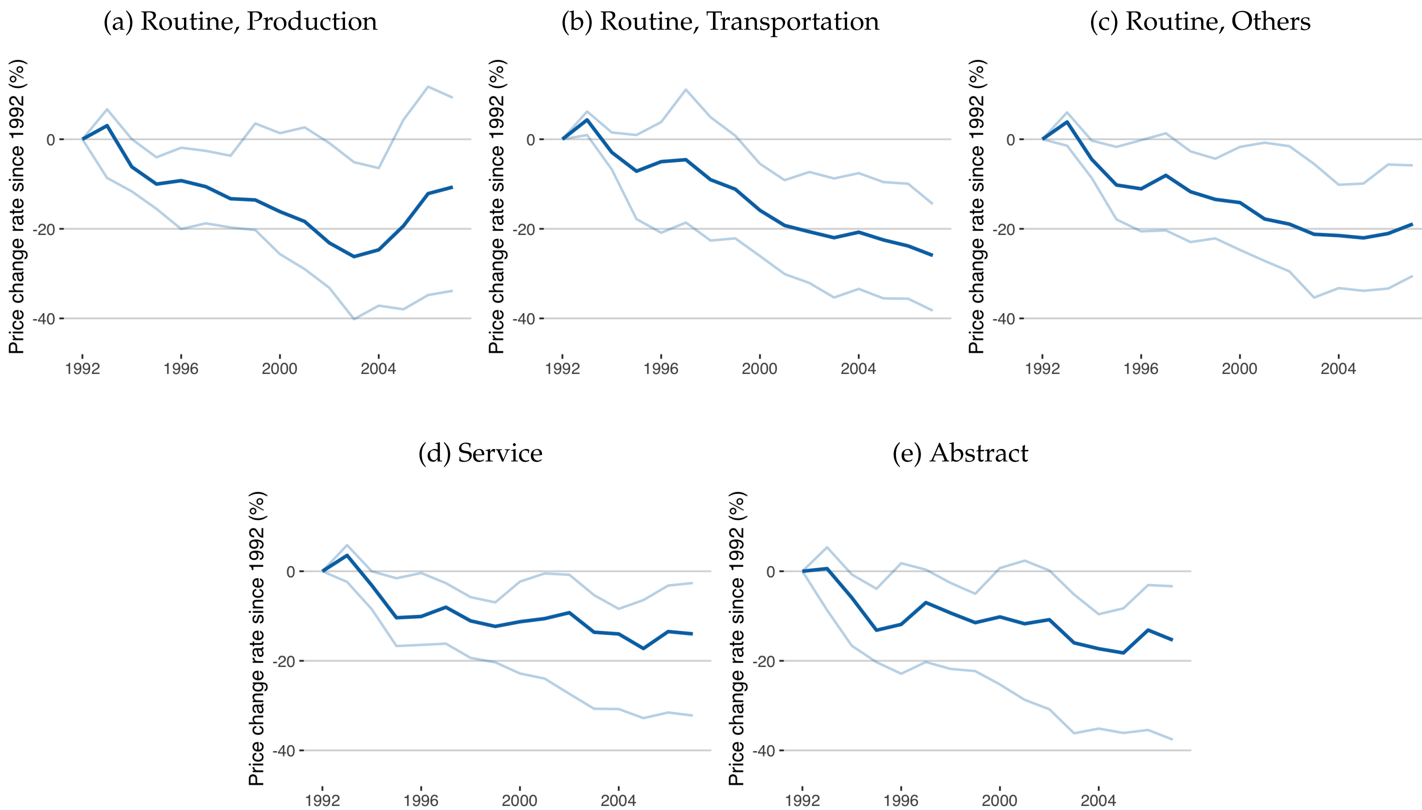
Another approach to solving this issue is to control for the quality change by the hedonic approach as in Timmer, Van Ark, et al. (2007), and in the application to digital capital in Tambe et al. (2019). The hedonic approach requires detailed information about the detailed specification of each robot. Unfortunately, it is difficult to keep track of the detailed specifications of commonly used robots as the robotics industry is rapidly changing.

Another method is a more data-driven one. Specifically, the Bank of Japan (BoJ) provides the quality-controlled price index. However, the method is not clearly declared. In fact, it is claimed to be "cost-evaluation method," in which the BoJ asks producer firms to measure the component of quality upgrading for price changes between periods. Unfortunately, I do not know the survey firms and quality components. Therefore, it is hard for me to determine better measures, and so I stick to use my raw price measure based on the representativeness of my data.

D.4 Robot Price Trends by Occupation Groups

In this section, I examine the facts discussed in Section 2.3 for each occupation group described in Section 4.1. Figure D.3 shows the plot of the trend of the robot price distribution since 1992 for each occupation group, a version of Figure 1a, disaggregated by occupation groups. The top three panels show the trends for routine occupations, namely, from the left, routine-production, routine-transportation, and routine-others. The bottom two panels show the trends for service

Figure D.3: Robot Price Trends by Occupation Groups



Note: The author's calculation based on JARA and O*NET. Panels show the trend of robot prices by occupation groups defined in the main text.

occupations and abstract occupations, from the left. All these panels show the overall decreasing trend of robot prices, and the dispersion of prices within each occupation group. Having such a dispersion is important because in Section 4 when I estimate heterogeneous EoS between robots and labor, I use the price variation within each occupation group.

D.5 Trade of Industrial Robots

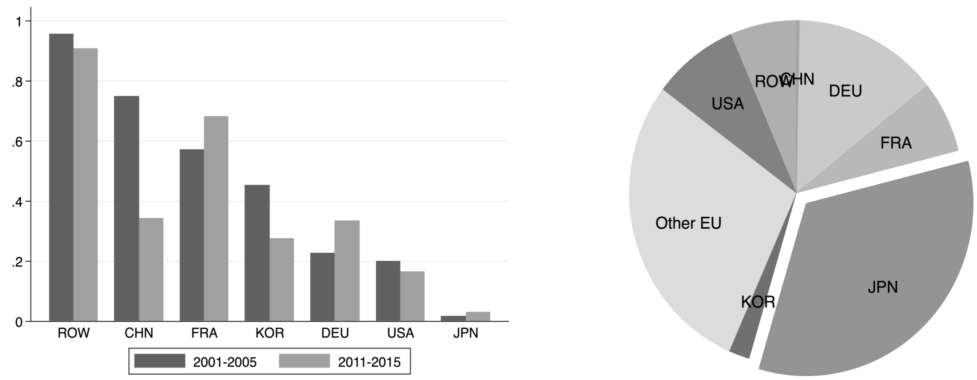
To compute the trade shares of industrial robots, I combine BACI and IFR data. In particular, I use the HS code 847950 ("Industrial Robots For Multiple Uses") to measure the robots, following Humlum (2019). I approximate the initial year value by year of 1998, when the this HS code of robots is first available. To calculate the total absorption value of robots in each country, I use the IFR data's robot units (quantities), combined with the price indices of robots occasionally released by IFR's annual reports for selected countries. These price indices do not give disaggregation by robot tasks or occupations, highlighting the value added of the JARA data. Figure D.4 the pattern of international trade of international robots. In the left panel, I compute the import-absorption ratio. To remove the noise due to yearly observations and focus on a long-run trend, I aggregate by five-year bins 2001-2005 and 2011-2015. The result indicates that many countries import robots as opposed to produce in their countries. Japan's low import ratio is outstanding, revealing that its comparative advantage in this area. It is noteworthy that China largely domesticated the production of robots over the sample period. Another way to show grasp the comparative advantage of the robot industry is to examine the share of exports as in the right panel of Figure D.4. Roughly speaking, the half of the world robot market was dominated by EU and one-third by Japan in 2001-2005. The rest 20% is shared by the rest of the world, mostly by the US and South Korea.

D.6 Robots from Japan in the US, Europe, and the Rest of the World

To internationally compare the pattern of robot adoption, I generate the growth rates of stock of robots between 1992 and 2017 at the occupation level for each group of destination countries. The groups are the US, the non-US countries, (namely, the world excluding the US and Japan), and five European countries (or "EU-5"), Denmark, Finland, France, Italy, and Sweden used in AR. To calculate the stock of robots, I employ the perpetual inventory method with depreciation rate of

Figure D.4: Trade of Industrial Robots

(a) Robot Import-Absorption Ratio (b) World Robot Export Share, 2001-2005



Note: The author's calculation from the IFR, and BACI data. The left panel show the fraction of import in the total absorption value. The import value is computed by aggregating trade values across origin country in the BACI data (HS-1996 code 847950), and the absorption value is computed by the price index and the quantity variable available for selected countries in the IFR data. The data are five-year aggregated in 2001-2005 and 2011-2015, and countries are sorted according to the import shares in 2001-2005 in the descending order. The right panel shows the export share for 2001-2005 aggregates obtained from the BACI data.

$\delta = 0.1$, following Graetz and Michaels (2018).

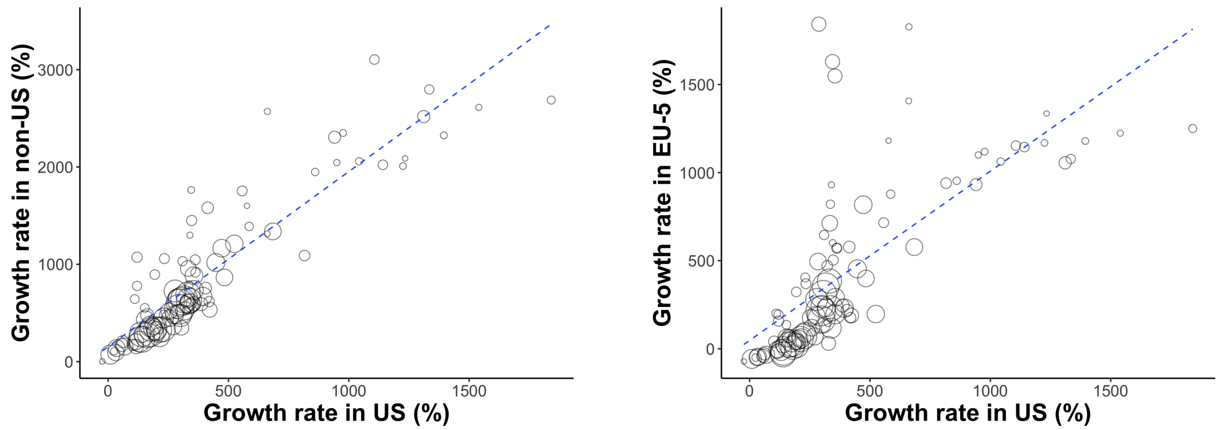
Figure D.5 shows scatterplots of the growth rates at the occupation level. The left panel shows the growth rates in the US on the horizontal axis and the ones in non-US countries on the vertical axis. The right panel shows the same measures on the horizontal axis, but the growth rates in the set of EU-5 countries on the vertical axis. These panels show that the stocks of robots at the occupation level grow (1992-2017) between the US and non-US proportionately relative to those between the US and EU-5. This finding is in contrast to AR, who find that the US aggregate robot stocks grew at a roughly similar rate as those did in EU-5. It also indicates that non-US growth patterns reflect growths of robotics technology at the occupation level available in the US. I will use these patterns as the proxy for robotics technology available in the US. In Section 3 and on, I take a further step and solve for the robot adoption quantity and values in non-US countries in general equilibrium including the US and non-US countries.

It is worth mentioning that a potential reason for the difference between my finding and AR's is the difference in data sources. In contrast to the JARA data I use, AR use IFR data that include all robot seller countries. Since EU-5 is closer to major robot producer countries other than Japan, including Germany, the robot adoption pattern across occupations may be influenced by their presence. If these close producers have a comparative advantage in producing robots for a

Figure D.5: Growth Rates of Robots at the Occupation Level

(a) Comparison between the US and non-US

(b) Comparison between the US and EU-5



Note: The author's calculation based on JARA, and O*NET. The left panel shows the correlation between occupation-level growth rates of robot stock quantities from Japan to the US and the growth rates of the quantities to the non-US countries. The right one shows the correlation between growth rates of the quantities to the US and EU-5 countries. Non-US are the aggregate of all countries excluding the US and Japan. EU-5 are the aggregate of Denmark, France, Finland, Italy, and Sweden used in Acemoglu and Restrepo (2020). Each bubble shows an occupation. The bubble size reflects the stock of robot in the US in the baseline year, 1992. See the main text for the detail of the method to create the variables.

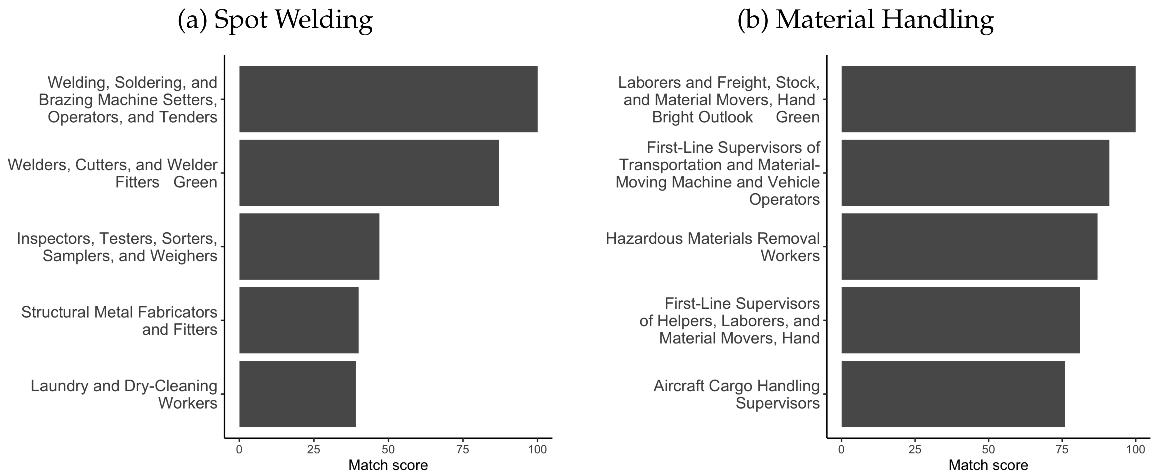
specific occupation, then EU-5 may adopt the robots for such occupations intensively from close producers. In contrast, countries out of EU-5, including the US, may not benefit the closeness to these producers. Thus they are more likely to purchase robots from far producers from EU-5, such as Japan.

D.7 Details in Application-Occupation Matching

Concrete examples of the pairs of an application and an occupation that are close are spot welding and material handling. On the one hand, spot welding is a task of welding two or more metal sheets into one by applying heat and pressure to a small area called spot. In contrast, O*NET-SOC Code 51-4121.06 has the title "Welders, Cutters, and Welder Fitters" ("Welders" below). Therefore, both spot welding robots and welders perform the welding task. On the other hand, Material handling is a short-distance movement of heavy materials. It is another major application of robots. In comparison, ONET-SOC Code 53-7062.00 has the title "Laborers and Freight, Stock, and Material Movers, Hand" ("Material Handler" below). Therefore, both material handling robots and material handlers perform the material handling task.

Figure D.6 shows examples of the O*NET match scores for spot welding and material han-

Figure D.6: Examples of Match Scores



Note: The author's calculation from the search result of O*NET Code Connector. The bars indicate match scores for the search query term "Spot Welding" in panel (a) and "Material Handling" in panel (b). Occupations codes are 2010 O*NET SOC codes. In each panel, occupations are sorted in a descending way with the relative relevance scores, and the top 5 occupations are shown. See the main text for the detail of the score.

dling. On the left panel, welding occupations are listed as relevant occupations for spot welding. In contrast, on the right panel, a material-moving laborer is a top occupation in terms of the relevance to the material-handling task, as I described above.

D.8 Hard-cut Matching of Applications and Occupations

Although it is transparent to match applications and occupations in a completely automatic way instead of using a researcher's judgment, a concern about such a matching method described in Section 2.2 is that one has potentially erroneous matching due to noise in the text description in occupation dictionary. For example, Section (D.7) reveals a case in which spot welding robots are matched to "Laundry and Dry-cleaning Workers" with a high score. This is primarily because the textual description for these workers includes "Apply bleaching powders to spots and spray them with steam to remove stains from fabrics...," which has a high matching score with the term "spot." In order to mitigate such a concern, I explore to introduce a manual hard-cut matching between applications and occupations. To be more specific, I drop all application-occupation matching with the matching score of 75 or below to exclude problematic matches while including enough data variation. I then construct the matching score following equation (1), match applications and occupations, and compute robot quantity and price variables to obtain the Japan robot shock with hard-cut matching. Table D.1 shows the result of regression specification (4). The esti-

Table D.1: Effects of the Japan robot shock on US occupations with Hard-cut Matching

VARIABLES	(1) $\Delta \ln(w)$	(2) $\Delta \ln(w)$	(3) $\Delta \ln(L)$	(4) $\Delta \ln(L)$
Japan Robot Shock, $-\psi^J$	-0.321	-0.316	-1.271	-1.266
	(0.135)	(0.107)	(0.454)	(0.555)
Exposure to China Trade		-0.743		-0.745
		(0.556)		(1.011)
Observations	324	324	324	324
R-squared	0.285	0.291	0.301	0.302
Demographic controls	✓	✓	✓	✓

Note: The table shows the coefficients in regression (4), based on the dataset constructed from JARA, O*NET, and the US Census/ACS. Observations are 4-digit level occupations, and the sample includes all occupations that existed throughout 1970 and 2007. ψ^C stands for the Japan robot shock from equation (2) and IPW stands for the occupation-level import penetration measure (in thousand USD) in equation (3). The Japan robot shock is constructed with the hard-cut matching method between applications and occupations described in Section D.8. Demographic control variables are the female share, the college-graduate share, the age distribution (shares of age 16-34, 35-49, and 50-64 among workers aged 16-64), and the foreign-born share as of 1990. All time differences, Δ, are taken with a long difference between 1990 and 2007. Bootstrapped standard errors are reported in the parentheses.

mated coefficients are somewhat larger than the ones with the preferred data matching procedure primarily because, in the hard-cut matching, erroneous matches that potentially contain noises are removed. Furthermore, the statistical significance remains in all columns.

D.9 Further Analysis about Fact 2

We provide series of evidence about the robotics substitution of existing occupations in this section. Recall that, to control for demand factors that drive the price and quantities of robots in the US, Acemoglu and Restrepo (2020) used the robot stock changes in the other countries instead of the US that show a similar trend of robot stocks as a proxy for the robot technological change and find the negative impact on the US regional labor market.[29] Following this approach, I use the changes in robot stocks and the Japan robot shocks in non-US countries (all countries except for the US and Japan), which are defined in the main text.[30]

Figure D.7 show the result of this analysis in graph, after controlling for the demographic

[29] The analysis result using US robot stocks and prices is shown in Appendix D.10 and shows a similar pattern.

[30] Note that the robot stock growths are similar between the US and the non-US countries by occupations. In contrast, the occupation-level trend in the five countries Acemoglu and Restrepo (2020) used as comparison (Denmark, Finland, France, Italy, and Sweden) is less similar to the US trend than the non-US countries. These facts are shown in Section D.6.

variables.[31] Each dot represents 4 digit-level occupations and the size of the bubble is based on the initial employment. The top two panels take the occupational wage as a labor market outcome on the y-axis, while the bottom two take the occupational employment. The left two panels take log monetary value of robot stock in non-US countries as a robot measure on the left panel, and the right two take the Japan robot shock. These results provides more direct evidence of the substitution of robots for workers who perform the same task as robots, corroborating the finding of Acemoglu and Restrepo (2020).

Finally, one might be concerned that the correlation between the US labor market outcomes and robot measures was simply an artifact of the long-run trend that has nothing to do with the robotization. To mitigate such a concern, I examine the pre-trend correlation between them. To do so, I take the 20-year difference of occupational wage since 1970 to 1990 as the outcome variable and run the main reduced-form regression (4). The result is shown in Figure D.8. I have considered wage and employment as an outcome variable, robot stock and robot prices measured in the US and non-US countries. The interaction of the choice of these variables yield eight panels in Figure D.8, while none of them shows significant relationship between (lagged) the changes in labor market outcome variables and the changes in robot measures. After these thorough analysis, I have concluded that the risk of capturing the pretrend in the main analysis is minimal.

To further check the correlation systematically, I run the following regressions and report the results in Table D.2:

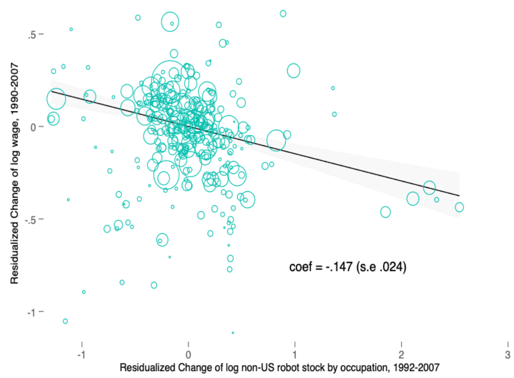
$$\Delta \ln (y_o) = a_R \Delta \ln (R_o) + (X_o)^\top \boldsymbol{a} + e_o,$$

where y_o is labor-market outcome of occupation o (wage and employment), R_o is the measure of robots (stocks and prices), X_o are the demographic control variables, e_o is the regression residual, and Δ indicates the long-difference between 1990 (1992 for $\Delta \ln (R_o)$) and 2007. The coefficient of interest is a_R. I expect negative a_R if I take robot stocks as the explanatory variable, while I expect positive a_R when I take robot price as the right-hand side variable.

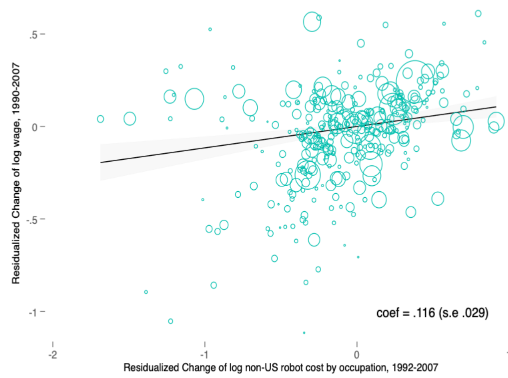
[31] A similar result can be found when I do not control for the demographic variables described above as well, as shown in Figure D.7 in Appendix D.10.

Figure D.7: Correlation between US Occupational Wage and Non-US Robot Measures (Controlled)

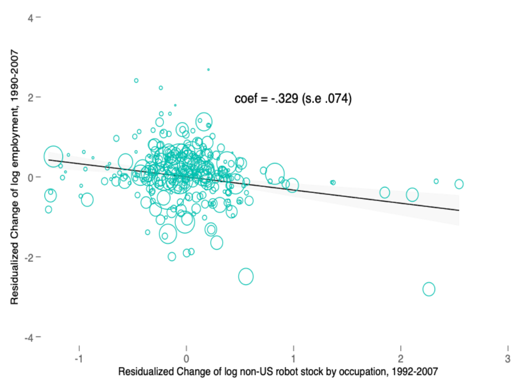
(a) Wage and Robot Values



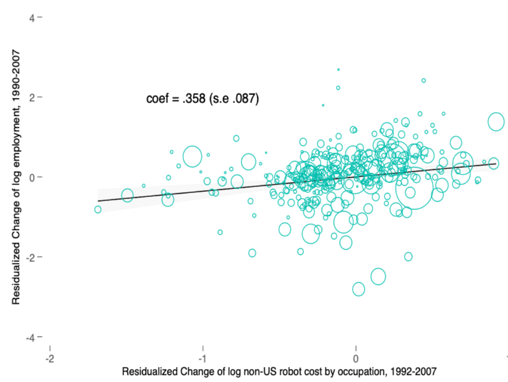
(b) Wage and Robot Prices



(c) Employment and Robot Values



(d) Employment and Robot Prices



Note: The author's calculation based on JARA, O*NET, and the US Census/ACS. The figures show the scatterplot, weighted fit line, and the 95 percent confidence interval of the changes in log robot measures and changes in log labor market outcomes. Each bubble represents a 4-digit occupation. The bubble size reflects the employment in the baseline year (1990, which is the closest Census year to the initial year that I observe the robot adoption, 1992). All variables are partialled out by control variables (the occupational female share, college share, age distribution, and foreign born share). In all panels, the number of observation is 324. In panel (a), y-axis and x-axis are log occupational wages and the change of log non-US robot stock measured by the monetary value. In panel (b), y-axis and x-axis are log occupational wages and the change of log non-US robot price. In panel (c), y-axis and x-axis are log occupational wages and the change of log non-US robot stock measured by the monetary value. In panel (d), y-axis and x-axis are log occupational wages and the change of log non-US robot prices.

D.10 Labor Market Outcomes and Various Robot Measures

Figure D.9 plots the correlation between the changes in robot measures and the changes in log labor market outcomes in the US at the occupation level, weighted by the size of occupation measured by initial the employment level. The top two panels take the occupational wage as a labor market outcome on the y-axis, while the bottom two take the occupational employment. The left two panels take log monetary value of robot stock in US as a robot measure on the left panel, and

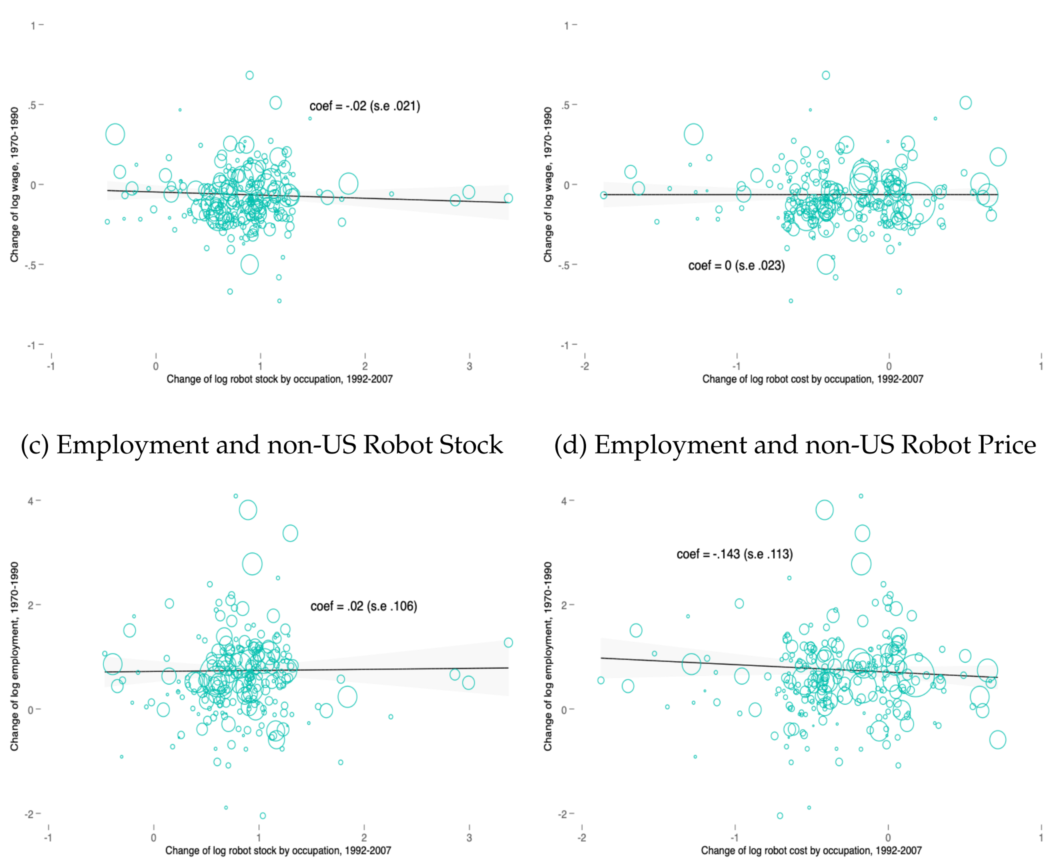
Figure D.8: Pretrend Analysis

(a) Wage and non-US Robot Stock

(b) Wage and non-US Robot Price

(c) Employment and non-US Robot Stock

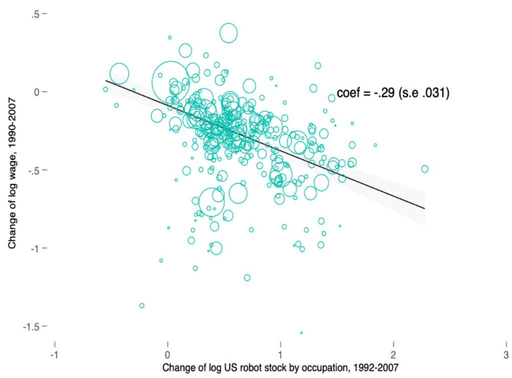
(d) Employment and non-US Robot Price



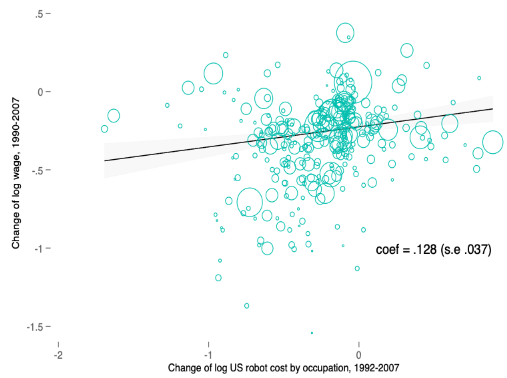
Note: The author's calculation based on JARA, O*NET, and the US Census/ACS. The figures show the scatterplot, weighted fit line, and the 95 percent confidence interval of the 1990-2007 changes in log robot measures and the lagged (1970-1990) changes in log labor market outcomes. Each bubble represents a 4-digit occupation. The bubble size reflects the employment in the baseline year (1990, which is the closest Census year to the initial year that I observe the robot adoption, 1992). In all panels, the number of observation is 324. In panel (a), y-axis and x-axis are log occupational wages and the change of log non-US robot stock. In panel (b), y-axis and x-axis are log occupational wages and the change of log non-US robot price. In panel (c), y-axis and x-axis are log occupational employment and the change of log non-US robot stock. In panel (d), y-axis and x-axis are log occupational employment and the change of log non-US robot price.

Figure D.9: Correlation between US Occupational Wage and US Robot Measures (Raw)

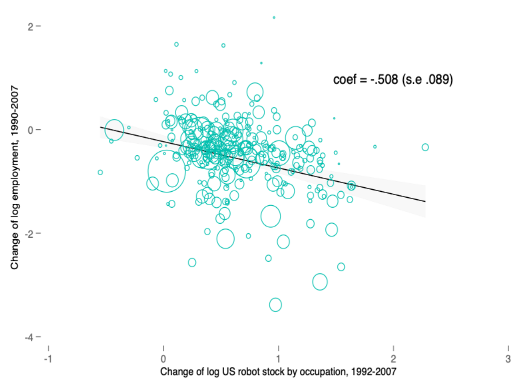
(a) Wage and Robot Stock



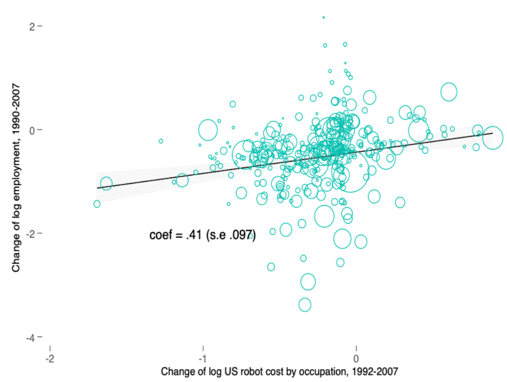
(b) Wage and Robot Price



(c) Employment and Robot Stock



(d) Employment and Robot Price



Note: The author's calculation based on JARA, O*NET, and the US Census/ACS. The figures show the scatterplot, weighted fit line, and the 95 percent confidence interval of the changes in log robot measures and changes in log labor market outcomes. Each bubble represents a 4-digit occupation. The bubble size reflects the employment in the baseline year (1990, which is the closest Census year to the initial year that I observe the robot adoption, 1992). All variables are partialled out by control variables (the occupational female share, college share, age distribution, and foreign born share). In all panels, the number of observation is 324. In panel (a), y-axis and x-axis are log occupational wages and the change of log US robot stock measured by the monetary value. In panel (b), y-axis and x-axis are log occupational wages and the change of log US robot price. In panel (c), y-axis and x-axis are log occupational wages and the change of log US robot stock measured by the monetary value. In panel (d), y-axis and x-axis are log occupational wages and the change of log US robot prices.

Table D.2: Regression Result of Labor Market Outcome on Robot Measures

VARIABLES	(1) $\Delta\ln(w)$	(2) $\Delta\ln(w)$	(3) $\Delta\ln(w)$	(4) $\Delta\ln(w)$	(5) $\Delta\ln(L)$	(6) $\Delta\ln(L)$	(7) $\Delta\ln(L)$	(8) $\Delta\ln(L)$
$\Delta\ln(K^R_{USA})$	-0.196				-0.446			
	(0.0339)				(0.0939)			
$-\Delta\ln(p^R_{USA})$		-0.171				-0.463		
		(0.0443)				(0.0792)		
$\Delta\ln(K^R_{ROW})$			-0.0798				-0.272	
			(0.0421)				(0.0914)	
Japan Robot Shock, $-\psi^J$				-0.206				-0.454
				(0.0441)				(0.0805)
Female share	0.0749	0.0627	0.0924	0.0477	0.0426	0.00942	0.102	-0.0174
	(0.0463)	(0.0501)	(0.0451)	(0.0377)	(0.119)	(0.152)	(0.146)	(0.152)
Col. grad. share	0.447	0.410	0.428	0.424	-0.104	-0.193	-0.137	-0.157
	(0.0487)	(0.0598)	(0.0550)	(0.0598)	(0.187)	(0.171)	(0.156)	(0.170)
Age 35-49 share	-0.430	-0.304	-0.303	-0.355	1.191	1.470	1.461	1.367
	(0.234)	(0.280)	(0.231)	(0.229)	(0.682)	(0.691)	(0.650)	(0.744)
Age 50-64 share	-0.00461	-0.0825	-0.103	-0.0609	-1.522	-1.675	-1.704	-1.657
	(0.275)	(0.350)	(0.277)	(0.307)	(0.621)	(0.757)	(0.766)	(0.747)
Foreign-born share	0.0254	-0.0283	0.0549	-0.0285	1.101	0.929	1.130	0.987
	(0.269)	(0.272)	(0.260)	(0.256)	(0.580)	(0.698)	(0.652)	(0.558)
Exposure to China Trade	-1.176	-1.049	-1.023	-1.035	-2.027	-1.794	-1.801	-1.696
	(0.666)	(0.539)	(0.676)	(0.432)	(0.891)	(0.938)	(0.829)	(1.131)
Observations	324	324	324	324	324	324	324	324
R-squared	0.283	0.245	0.214	0.284	0.112	0.095	0.069	0.107

Note: The author's calculation based on JARA, O*NET, and US Census/ACS. Observations are 4-digit level occupations, and the sample is all occupations that existed throughout 1970 and 2007. In each country $i \in \{USA, ROW\}$, K^R_i stands for the 2000-dollar value of the robot stock in the occupation and p^R_i stands for the average price of robot transacted in each year. All time differences (Δ) are taken with a long difference between 1990 and 2007. All demographic control variables are as of 1990. "Col. Grad. Share" stands for the college graduate share. Bootstrapped standard errors are reported in the parentheses.

the right two take the log robot prices from Japan. As expected, the correlation between the two labor market outcomes and the robot stock measure is negative, while that with the Japanese robot price is positive.

Next, I consider the role of control variables. Figure D.10 shows the results of a set of robustness checks. Each panel shows the same result as the ones corresponding to the same panel in Figure D.9, but after partialling out all variables with respect to the demographic control variables (initial-year female share, college graduates share, age 35-49 share, age 50-64 share, and foreign-born share in each occupation). The main result is robust to the control of these demographic variables.

Note that, the robot adoption in the US, as used in Figure D.9 and D.10, might suffer from the endogeneity bias due to the demand factors that drive both the labor market outcome and robot adoption. This concern is addressed in Appendix D.9. Figure D.11 shows the result without controlling for the demographic variables, which shows a qualitatively similar pattern as the main results.

Next, one may be concerned that robot quality changes over year. Specifically, if the per-unit efficiency of robots increases over year, the average unit price may understate the decrease in the price of robots. To deal with this concern, I consider the following method of quality adjustment, based on the spirit of Khandelwal, Schott, and Wei (2013). Namely, I fit the following equation with the fixed-effect regression:

$$\ln\left(X^R_{JPN\to i,o,t}\right) = -\varsigma \ln\left(p^R_{JPN\to i,o,t}\right) + a^R_{o,t} + e^R_{i,o,t},$$

from which I obtain the fixed effect $a^R_{o,t}$, which absorbs the occupation-o specific log sales component that is not explained by the prices. I then proxy the quality change by the change in such fixed effects, $\Delta a^R_{o,t} \equiv a^R_{o,t} - a^R_{o,t_0}$. The (log) quality-adjusted price is then obtained by $\ln\left(p^R_{JPN\to i,o,t}\right) - \Delta a^R_{o,t}$. Figure D.12 shows the result of correlation using quality-adjusted robot prices All the results are robust to these considerations–wage growths are negatively correlated with stock growths, and positively correlated with price growths, both across occupations.

Figure D.10: Correlation between US Occupational Wage and US Robot Measures (Controlled)

Note: The author's calculation based on JARA, O*NET, and the US Census/ACS. The figures show the scatterplot, weighted fit line, and the 95 percent confidence interval of the changes in log robot measures and changes in log labor market outcomes. Each bubble represents a 4-digit occupation. The bubble size reflects the employment in the baseline year (1990, which is the closest Census year to the initial year that I observe the robot adoption, 1992). All variables are partialled out by control variables (the occupational female share, college share, age distribution, and foreign born share). In all panels, the number of observation is 324. In panel (a), y-axis and x-axis are log occupational wages and the change of log US robot stock measured by the monetary value. In panel (b), y-axis and x-axis are log occupational wages and the change of log US robot price. In panel (c), y-axis and x-axis are log occupational wages and the change of log US robot stock measured by the monetary value. In panel (d), y-axis and x-axis are log occupational wages and the change of log US robot prices.

D.11 Data about Initial Shares

Since the log-linearized sequential equilibrium solution depends on several initial share data generated from the initial steady state, I discuss the data sources and methods for measuring these shares. I define $t_0 = 1992$ and the time frequency is annual. I consider the world that consists of three countries $\{USA, JPN, ROW\}$. Table D.3 summarizes overview of the variable notations, descriptions, and data sources.

Table D.3: List of Data Sources

Variable	Description	Source
$\widetilde{y}^G_{ij,t_0}, \widetilde{x}^G_{ij,t_0}, \widetilde{y}^R_{ij,t_0}, \widetilde{x}^R_{ij,t_0}$	Trade shares of goods and robots	BACI, IFR
$\widetilde{x}^O_{i,o,t_0}$	Occupation cost shares	IPUMS
l_{i,o,t_0}	Labor shares within occupation	JARA, IFR, IPUMS
$s^G_{i,t_0}, s^V_{i,t_0}, s^R_{i,t_0}$	Robot expenditure shares	BACI, IFR, WIOT
$\alpha_{i,M}$	Intermediate input share	WIOT

I take matrices of trade of goods and robots by BACI data. As in Humlum (2019), I measure robots by HS code 847950 ("Industrial Robots For Multiple Uses") and approximate the initial year value by year of 1998, in which the robot HS code is first available. Figure D.13 shows the trend of export and import shares of robots among the world for the US, Japan, and the Rest Of the World. The trends are fairly stable for the three regions of the world, except that the import share of the US has declined relative to the ROW.

To obtain the domestic robot absorption data, I take from IFR data the flow quantity variable and the aggregate price variable for a selected set of countries. I then multiply these to obtain USA and JPN robot adoption value. For robot prices in ROW, I take the simple average of the prices among the set of countries (France, Germany, Italy, South Korea, and the UK, as well as Japan and the US) for which the price is available in 1999, the earliest year in which the price data are available. Graetz and Michaels (2018) discuss prices of robots with the same data source. Figure D.14 shows the comparison of the US price index measure available between JARA and IFR. The JARA measures are disaggregated by 4-digit occupations. The figure shows the 10th, 50th (median), and 90th percentiles each year, as in Figure 1a. All measures are normalized at 1999, the year in which the first price measure is available in the IFR data. Overall, the JARA price trend variation tracks the overall price evolution measured by IFR reasonably well: The long-run trends from 1999 to the late 2010s are similar between the JARA median price and the IFR price index. During the 2000s, the IFR price index drops faster than the median price in the JARA data. It compares with the JARA 10th percentile price, which could be due to robotic technological changes in other countries than Japan in the corresponding period.

I construct occupation cost shares $\widetilde{x}^O_{i,o,t_0}$ and labor shares within occupation l_{i,o,t_0} as follows. To measure $\widetilde{x}^O_{i,o,t_0}$, I aggregate the total wage income of workers that primarily works in each occupation o in year 1990, the Census year closest to t_0. I then take the share of this total compensation

Table D.4: Baseline Shares by 5 Occupation Group

Occupation Group	$\widetilde{x}^O_{1,o,t_0}$	l^O_{1,o,t_0}	y^R_{2,o,t_0}	x^R_{1,o,t_0}	x^R_{2,o,t_0}	x^R_{3,o,t_0}
Routine, Production	17.58%	99.81%	64.59%	67.49%	62.45%	67.06%
Routine, Transportation	7.82%	99.93%	12.23%	11.17%	13.09%	11.04%
Routine, Others	28.78%	99.99%	10.88%	9.52%	11.68%	10.40%
Service	39.50%	99.99%	8.87%	8.58%	9.17%	8.32%
Abstract	6.32%	99.97%	3.43%	3.24%	3.60%	3.18%

Note: The author's calculation of initial-year share variables based on the US Census, IFR, and JARA. As in the main text, country 1 indicates the US, country 2 Japan, and country 3 the rest of the world. See the main text for the construction of each variable.

measure for each occupation. To measure l_{i,o,t_0}, I take the total compensation as the total labor cost and a measure of the user cost of robots for each occupation. The user cost of robots is calculated with the occupation-level robot price data available in IFR and the set of calibrated parameters in Section 4.1. Table D.4 summarizes these statistics for the aggregated 5 occupation groups in the US. One can see that the cost for production occupations and transportation occupations comprise 18% and 8% of the US economy, respectively, totaling more than one-fourth. Furthermore, the share of robot cost in all occupations is still quite low with the highest share of 0.19% in production occupations, revealing still small-scale adoption of robots from the overall US economy perspective.

To calculate the effect on total income, I also need to compute the sales share of robots by occupations $y^R_{i,o,t_0} \equiv Y^R_{i,o,t_0} / \sum_o Y^R_{i,o,t_0}$ and the absorption share $x^R_{i,o,t_0} \equiv X^R_{i,o,t_0} / \sum_o X^R_{i,o,t_0}$. To obtain y^R_{i,o,t_0}, I compute the share of robots by occupations produced in Japan $y^R_{2,o,t_0} = Y^R_{2,o,t_0} / \sum_o Y^R_{2,o,t_0}$ and assume the same distribution for other countries due to the data limitation: $y^R_{i,o,t_0} = y^R_{2,o,t_0}$ for all i. To have x^R_{i,o,t_0}, I compute the occupational robot adoption in each country by $X^R_{i,o,t_0} = P^R_{i,t_0} Q^R_{i,o,t_0}$, where Q^R_{i,o,t_0} is the occupation-level robot quantity obtained by the O*NET concordance generated in Section 2.2 applied to the IFR application classification. As mentioned above, the robot price index P^R_{i,t_0} is available for a selected set of countries. To compute the rest-of-the-world price index P^R_{3,t_0}, I take the average of all available countries weighted by the occupational robot values each year. The summary table for these variables y^R_{i,o,t_0} and x^R_{i,o,t_0} at 5 occupation groups are shown in Table D.4. All values in Table D.4 are obtained by aggregating 4-digit-level occupations, and raw and disaggregated data are available upon request.

I take a more standard measure, the intermediate input share $\alpha_{i,M}$, from World Input-Output Tables (WIOT Timmer, Dietzenbacher, et al. 2015). Finally, I combine the trade matrix generated

Table D.5: 1990 Occupation Group Switching Probability

		Routine			Service	Abstract
		Production	Transportation	Others		
Routine	Production	0.961	0.011	0.010	0.006	0.012
	Transportation	0.020	0.926	0.020	0.008	0.025
	Others	0.005	0.006	0.955	0.020	0.014
Service		0.003	0.002	0.020	0.967	0.007
Abstract		0.014	0.014	0.036	0.015	0.922

Note: The author's calculation from the CPS-ASEC 1990 data. The conditional switching probability to column occupation group conditional on being in each row occupation.

above and WIOT to construct the good and robot expenditure shares s^G_{i,t_0}, s^V_{i,t_0}, and s^R_{i,t_0}. In particular, with the robot trade matrix, I take the total sales value by summing across importers for each exporter, and total absorption value by summing across exporters for each importers. I also obtain the total good absorption by WIOT. From these total values, I compute expenditure shares. are obtained by aggregating 4-digit occupations, and the disaggregated data are available upon request.

As initial year occupation switching probabilities μ_{i,oo',t_0}, I take 1990 flow Markov transition matrix from the cleaned CPS-ASEC data created in Section D.1. Table D.5 shows this initial-year conditional switching probability. The matrix for the other years are available upon request. As for other countries than the US, although Freeman, Ganguli, and Handel (2020) has begun to develop occupational wage measures consistent across country, world-consistent occupation employment data are hard to obtain. Therefore, I assign the same flow probabilities for other countries in my estimation.

E Online Theory Appendix

E.1 Further Discussion of Model Assumptions

Capital-Skill Complementarity Occupation production function (6) also nest the one in the literature of capital-skill complementarity (Krusell et al. 2000 among others). To simplify, I focus on individual producer's production function in the steady state. Thus I drop subscripts and superscripts of country i and time period t. Suppose the set of occupations is $O \equiv \{R, U\}$ and $a_U = 0$. R

stands for the robotized occupation (e.g., spot welding) and U stands for "unrobotized" (e.g., programming). Note that since U is unrobotized $a_U = 0$. Then the unit cost of occupation aggregate (6), P^O, is

$$P^O = \left[(b_R)^{\frac{1}{\beta}} \left((1 - a_R) (w_R)^{1-\theta_R} + a_R (c_R)^{1-\theta_R} \right)^{\frac{1-\beta}{1-\theta_R}} + (b_U)^{\frac{1}{\beta}} (w_U)^{1-\beta} \right]^{\frac{1}{1-\beta}}.$$

Thus different skills R and U are substituted by robots with different substitution parameters θ_R and β, respectively. Since the literature of capital-skill complementarity studies the rising skill premium, the current model also has an ability to discuss the occupation (or skill) premium given the different level of automation across occupations.

E.2 Relationship with Other Models of Automation

The model in Section 3 is general enough to nest models of automation in the previous literature. In particular, I show how the production functions (5) and (6) imply to specifications in AR and Humlum (2019). Throughout Section E.2, I fix country i and focus on steady states and thus drop subscripts i and t since the discussion is about individual producer's production function.

E.2.1 Relationship with the model in Acemoglu and Restrepo (2020, AR)

Following AR that abstract from occupations, I drop occupations by setting $O = 1$ in this paragraph. Therefore, the EoS between occupations β plays no role, and $\theta_o = \theta$ is a unique value. AR show that the unit cost (hence the price given perfect competition) is written as

$$p^{AR} \equiv \frac{1}{\widetilde{A}} \left[(1 - \widetilde{a}) \frac{w}{A^L} + \widetilde{a} \frac{c^R}{A^R} \right]^{\alpha_L} r^{1-\alpha_L},$$

for each sector and location (See AR, Appendix A1, equation A5). In this equation, c^R is the steady state marginal cost of robot capital defined in equation (E.33) and A^L and A^R represent per-unit efficiency of labor and robots, respectively. In Lemma E.1 below, I prove that my model implies a unit cost function that is strict generalization of p^{AR} with proper modification to the shock terms and parameter configuration. I begin with the modification that allows per-unit efficiency terms in my model.

Definition E.1. For labor and robot per-unit efficiency terms $A^L > 0$ and $A^R > 0$ respectively, modified robot task space $\widetilde{a}$ and TFP term $\widetilde{A}$ are

$$\widetilde{a} \equiv \frac{a\left(A^L\right)^{\theta-1}}{a\left(A^L\right)^{\theta-1} + (1-a)\left(A^R\right)^{\theta-1}}, \tag{E.1}$$

$$\widetilde{A} \equiv \frac{A}{\left[(1-\widetilde{a})\left(A^L\right)^{\theta-1} + \widetilde{a}\left(A^R\right)^{\theta-1}\right]}. \tag{E.2}$$

Lemma E.1. *Set the number of occupations $O = 1$. In the steady state,*

$$p^G = \frac{1}{\widetilde{A}}\left[(1-\widetilde{a})\left(\frac{w}{A^L}\right)^{1-\theta} + \widetilde{a}\left(\frac{c^R}{A^R}\right)^{1-\theta}\right]^{\frac{\alpha_L}{1-\theta}}\left(p^G\right)^{\alpha_M} r^{1-\alpha_M-\alpha_L}. \tag{E.3}$$

Proof. Note that modified robot task space (E.1) and modified TFP (E.2) can be inverted to have

$$a \equiv \frac{\widetilde{a}\left(A^R\right)^{\theta-1}}{(1-\widetilde{a})\left(A^L\right)^{\theta-1} + \widetilde{a}\left(A^R\right)^{\theta-1}}, \tag{E.4}$$

$$A \equiv \left[(1-\widetilde{a})\left(A^L\right)^{\theta-1} + \widetilde{a}\left(A^R\right)^{\theta-1}\right]\widetilde{A}. \tag{E.5}$$

Cost minimization problem with the production functions (5) and (6) and perfect competition imply

$$p^G = \frac{1}{A}\left(P^O\right)^{\alpha_L} p^{\alpha_M} r^{1-\alpha_L-\alpha_M},$$

and

$$P^O = \left[(1-a)\, w^{1-\theta} + a^{1-\theta}\right]^{\frac{1}{1-\theta}},$$

where P^O is the unit cost of tasks performed by labor and robots. Substituting equations (E.4) and (E.5) and rearranging, I have

$$p^G = \frac{1}{\widetilde{A}}\left(\widetilde{P^O}\right)^{\alpha_L}\left(p^G\right)^{\alpha_M} r^{1-\alpha_L-\alpha_M},$$

where $\widetilde{P^O}$ is the cost of the tasks performed by labor and robots:

$$\widetilde{P^O} = \left[(1 - \widetilde{a}) \left(\frac{w}{A^L} \right)^{1-\theta} + a \left(\frac{c^R}{A^R} \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}.$$

□

Lemma E.1 immediately implies the following corollary that shows that the steady state modified unit cost (E.3) strictly nests the unit cost formulation of AR as a special case of Leontief occupation aggregation.

Corollary E.1. *Suppose $\alpha_M = 0$. Then as $\theta \to 0$, $p^G \to p^{AR}$.*

E.2.2 Relationship with the model in Humlum (2019)

I show that production functions (5) and (6) nest the production function used by Humlum (2019). Since the setting of Humlum (2019) does not have non-robot capital, in this section, I simplify the notation for robot capital K^R by dropping the superscript and denote as K. For each firm in each period, Humlum (2019) specifies

$$Q^D = \exp\left[\varphi_H^D + \gamma_H^D K\right] \left[\sum_o \left(\exp\left[\varphi_o^D + \gamma_o^D K\right] \right)^{\frac{1}{\beta}} (L_o)^{\frac{\beta-1}{\beta}} \right]^{\frac{\beta}{\beta-1}}, \tag{E.6}$$

where $K = \{0, 1\}$ is a binary choice, $\varphi_H^D, \gamma_H^D, \varphi_o^D$ and γ_o^D are parameters, and superscript D represents the discrete adoption problem of Humlum (2019). As normalization, suppose that

$$\sum_o \exp\left(\varphi_o^D + \gamma_o^D K\right) = 1.$$

I will start from production function (5) and (6), place restrictions, and arrive at equation (E.6). As a key observation, relative to the discrete choice of robot adoption in Humlum (2019), the continuous choice of robot *quantity* in production function (6) allows significant flexibility. In this paragraph, I assume away with intermediate inputs. This is because Humlum (2019) assumes that intermediate inputs enter in an element of CES, while production function (5) implies that intermediate inputs enter as an element of the Cobb-Douglas function.

Now, given my production functions (5) and (6), suppose producers follow the binary decision rule defined below.

Definition E.2. A binary decision rule of a producer is that producers can choose between two choices: adopting robots $K = 1$ or not $K = 0$. If they choose $K = 1$, they adopt robots at the same unit as labor $K_o = L_o \geq 0$ for all occupation o. If they choose $K = 0$, $K_o = 0$ for all o.

Note that the binary decision rule is nested in the original choice problem from $K_o^R \geq 0$ for each o. Set

$$A_o^D\left(K^R\right) \equiv \begin{cases} A_o\left(\left(1-a_o\right)^{\frac{1}{\theta}}+\left(a_o\right)^{\frac{1}{\theta}}\right)^{\frac{\theta}{\theta-1}(\beta-1)} & \text{if } K^R = L_o \\ A_o\left(1-a_o\right)^{\frac{1}{\theta-1}(\beta-1)} & \text{if } K^R = 0 \end{cases}.$$

Then I have

$$Q = \left[\sum_o \left(A_o^D\left(K_o\right)\right)^{\frac{1}{\beta}}\left(L_o\right)^{\frac{\beta-1}{\beta}}\right]^{\frac{\beta}{\beta-1}}.$$

To normalize, define

$$\widetilde{A_o^D} \equiv \left(\sum_o A_o^D\left(K_o\right)\right)^{\frac{1}{\beta-1}}$$

and

$$a_o^D\left(K_o^R\right) \equiv \frac{A_o^D\left(K_o\right)}{\sum_{o'} A_{o'}^D\left(K_{o'}\right)}.$$

Then I have

$$Q = \widetilde{A_o^D}\left[\sum_o \left(a_o^D\left(K_o\right)\right)^{\frac{1}{\beta}}\left(L_o\right)^{\frac{\beta-1}{\beta}}\right]^{\frac{\beta}{\beta-1}}. \tag{E.7}$$

Finally, let

$$A_{o,0} \equiv \left[\exp\left(\varphi_H^D + \varphi_o^D\right)\right]^{\frac{\theta_o-1}{\beta-1}}$$

and

$$A_{o,1} \equiv \left[\left(\exp\left(\varphi_H^D + \varphi_o^D + \gamma_H^D + \gamma_o^D\right)\right)^{\frac{1}{\theta_o}\frac{\theta_o-1}{\beta-1}} - \left(\exp\left(\varphi_H^D + \varphi_o^D\right)\right)^{\frac{1}{\theta_o}\frac{\theta_o-1}{\beta-1}}\right]^{\theta_o}.$$

and also let A_o and a_o satisfy

$$A_o = \left(A_{o,0} + A_{o,1}\right)^{\frac{\beta-1}{\theta_o-1}} \tag{E.8}$$

and

$$a_o = \frac{A_{o,1}}{A_{o,0} + A_{o,1}}. \tag{E.9}$$

Then one can substitute equations (E.8) and (E.9) to equation (E.7) and confirm that $Q = Q^D$. Summarizing the discussion above, I have the result that my model can be restricted to produce the production side of the model of Humlum (2019) as follows.

Lemma E.2. *Suppose that (i) producers follow the binary decision rule in Definition E.2 and that (ii) occupation productivity A_o and robot task space a_o satisfy equations (E.8) and (E.9) for each o. Then $Q = Q^D$.*

E.3 The Source of the Upward-Sloping Export Supply Curve

Compared to the trade policy literature, this paper provides the upward sloping export supply curve as a result of the general equilibrium, as opposed to the supply curve that is assumed to be upward-sloping (e.g., Broda, Limao, and Weinstein 2008). Namely, when the demand for robots in a robot exporter country decreases, the resources to produce robots in the exporter country are freed up and reallocated to produce the non-robot goods. In my case, the resources are the non-robot goods which are the input to robot production in equation (7). This increases the supply of non-robot goods in the robot-exporting country, depressing their prices. In turn, due to robot production function (7) again, this decrease in the non-robot goods price entails the decrease of the cost of producing robots. Therefore, it reduces the price of robots produced in the exporter country.

E.4 Proof of Proposition B.1

The proof takes the following four conceptual steps. First, I will write the real wage change $\left(\widehat{w_{i,o}/P_i^G}\right)$ in terms of the weighted average of relative price changes, making use of the fact that the sum of shares equals one. Second, I rewrite relative price change into layers of relative price changes with the technique of addition and subtraction. Third, I show that each layer of relative price changes is a change of relevant input or trade shares controlled by elasticity substitution. In other words, an input or trade shares reveals a layer of relative price changes. Finally, I make use of the fact that the sum of shares do not change after the shock to arrive at equation (B.4).

Cost minimization given production functions (5), (6), and (B.5) imply

$$\widehat{\left(\frac{w_{i,o}}{P_i^G}\right)} = \frac{1}{1-\alpha_{i,M}} \sum_l \widetilde{x}_{li,t_0}^G \sum_{o'} \widetilde{x}_{l,o',t_0}^O \left[\widetilde{x}_{l,o',t_0}^L \left(\widehat{w_{i,o}} - \widehat{w_{l,o'}}\right) + \left(1-\widetilde{x}_{l,o',t_0}^L\right) \left(\widehat{w_{i,o}} - \left(\frac{\widehat{A_{l,o'}^K}}{1-\theta_o} + \widehat{c_{l,o'}^R}\right)\right)\right]. \tag{E.10}$$

Note that by additions and subtractions, I can rewrite

$$\begin{aligned} \widehat{w_{i,o}} - \widehat{w_{l,o'}} = & \left(\widehat{w_{i,o}} - \widehat{P_{i,o}^O}\right) - \left(\widehat{w_{l,o'}} - \widehat{P_{l,o'}^O}\right) + \left(\widehat{P_{i,o}^O} - \widehat{P_i^O}\right) - \left(\widehat{P_{l,o'}^O} - \widehat{P_l^O}\right) \\ & + \left(\widehat{P_i^O} - \widehat{p_i^G}\right) - \left(\widehat{P_l^O} - \widehat{p_l^G}\right) + \left(\widehat{p_i^G} - \widehat{P_i^G}\right) - \left(\widehat{p_l^G} - \widehat{P_l^G}\right), \end{aligned} \tag{E.11}$$

where $\widehat{P_{i,o}^O}$, $\widehat{P_i^O}$, and $\widehat{P_i^G}$ are the price (cost) index of occupation o, occupation aggregate $T_{i,t}^O \equiv \left[\sum_o \left(T_{i,o,t}^O\right)^{(\beta-1)/\beta}\right]^{\beta/(\beta-1)}$, and consumption of non-rogot good G, and

$$\begin{aligned} \widehat{w_{i,o}} - \left(\frac{\widehat{A_{l,o'}^K}}{1-\theta} + \widehat{c_{l,o'}^R}\right) = & \left(\widehat{w_{i,o}} - \widehat{P_{i,o}^O}\right) - \left(\frac{\widehat{A_{l,o'}^K}}{1-\theta} + \widehat{c_{l,o'}^R} - \widehat{P_{l,o'}^O}\right) + \left(\widehat{P_{i,o}^O} - \widehat{P_i^O}\right) - \left(\widehat{P_{l,o'}^O} - \widehat{P_l^O}\right) \\ & + \left(\widehat{P_i^O} - \widehat{p_i^G}\right) - \left(\widehat{P_l^O} - \widehat{p_l^G}\right) + \left(\widehat{p_i^G} - \widehat{P_i^G}\right) - \left(\widehat{p_l^G} - \widehat{P_l^G}\right). \end{aligned} \tag{E.12}$$

Note that the cost minimizing input and trade shares satisfy

$$\begin{cases} \widehat{\widetilde{x}_{i,o}^L} = (1-\theta_o)\left(\widehat{w_{i,o}} - \widehat{P_{i,o}^O}\right), \widehat{1-\widetilde{x}_{i,o}^L} = \widehat{A_{i,o}^K} + (1-\theta_o)\left(\widehat{c_{i,o}^R} - \widehat{P_{i,o}^O}\right) \\ \widehat{\widetilde{x}_{i,o}^O} = (1-\beta)\left(\widehat{P_{i,o}^O} - \widehat{P_i^O}\right), \widehat{\widetilde{x}_i^T} = (1-\vartheta)\left(\widehat{P_i^O} - \widehat{p_i^G}\right), \widehat{\widetilde{x}_{li}^G} = (1-\varepsilon)\left(\widehat{p_l^G} - \widehat{P_i^G}\right) \end{cases} \tag{E.13}$$

Combined with the Cobb-Douglas assumption of production function (5), equations (E.11), (E.12), and (E.13) imply

$$\widehat{w_{i,o}} - \widehat{w_{l,o'}} = \frac{\widehat{\widetilde{x}_{i,o}^L}}{1-\theta_o} - \frac{\widehat{\widetilde{x}_{l,o'}^L}}{1-\theta_o} + \frac{\widehat{\widetilde{x}_{i,o}^O}}{1-\beta} - \frac{\widehat{\widetilde{x}_{l,o'}^O}}{1-\beta} + \frac{\widehat{\widetilde{x}_i^T}}{1-\vartheta} - \frac{\widehat{\widetilde{x}_l^T}}{1-\vartheta} + \frac{\widehat{\widetilde{x}_{ii}^G}}{1-\varepsilon} - \frac{\widehat{\widetilde{x}_{li}^G}}{1-\varepsilon}$$

$$\widehat{w_{i,o}} - \left(\frac{\widehat{A_{l,o'}^K}}{1-\theta_o} + \widehat{c_{l,o'}^R}\right) = \frac{\widehat{\widetilde{x}_{i,o}^L}}{1-\theta_o} - \frac{\widehat{\left(1-\widetilde{x}_{l,o'}^L\right)}}{1-\theta_o} + \frac{\widehat{\widetilde{x}_{i,o}^O}}{1-\beta} - \frac{\widehat{\widetilde{x}_{l,o'}^O}}{1-\beta} + \frac{\widehat{\widetilde{x}_i^T}}{1-\vartheta} - \frac{\widehat{\widetilde{x}_l^T}}{1-\vartheta} + \frac{\widehat{\widetilde{x}_{ii}^G}}{1-\varepsilon} - \frac{\widehat{\widetilde{x}_{li}^G}}{1-\varepsilon}.$$

Substituting these in equation (E.10) and using the facts that $\widetilde{x}_{i,o,t_0}^L \widehat{\widetilde{x}_{i,o}^L} + \left(1-\widetilde{x}_{i,o,t_0}^L\right)\left(\widehat{1-\widetilde{x}_{i,o}^L}\right) = 0$

for all i and o, $\sum_o \tilde{x}^O_{i,o,t_0}\widehat{\tilde{x}^O_{i,o}} = 0$, and $\sum_l \tilde{x}^G_{li,t_0}\widehat{\tilde{x}^G_{li}} = 0$ for all i, I have equation (B.4).

E.5 Details of the Two-step Estimator

I use notation $d \equiv \dim(\mathbf{\Theta})$ to denote the dimension of parameters. Assumption 1 implies that, for any d-dimensional functions $\boldsymbol{H} \equiv \{H_o\}_o$, $\mathbb{E}\left[H_o\left(\boldsymbol{\psi}^J_{t_1}\right)\nu_o\right] = 0$. The GMM estimator based on $\boldsymbol{H}$ is

$$\mathbf{\Theta}_{\boldsymbol{H}} \equiv \arg\min_{\mathbf{\Theta}} \sum_{o=1}^{O} \left[H_o\left(\boldsymbol{\psi}^J_{t_1}\right)\nu_o(\mathbf{\Theta})\right]^\top \left[H_o\left(\boldsymbol{\psi}^J_{t_1}\right)\nu_o(\mathbf{\Theta})\right], \tag{E.14}$$

which is consistent under the moment condition (21) if $\boldsymbol{H}$ satisfies the rank conditions in Newey and McFadden (1994). The exact specification of $\boldsymbol{H}$ determines the optimality, or the minimal variance, of estimator (E.14). To specify $\boldsymbol{H}$, I apply the approach that achieves the asymptotic optimality developed in Chamberlain (1987). Formally, define the instrumental variable Z_o as follows:

$$Z_o \equiv H^*_o\left(\boldsymbol{\psi}^J_{t_1}\right) \equiv \mathbb{E}\left[\nabla_{\mathbf{\Theta}}\nu_o(\mathbf{\Theta})\,|\boldsymbol{\psi}^J_{t_1}\right]\mathbb{E}\left[\nu_o(\mathbf{\Theta})\left(\nu_o(\mathbf{\Theta})\right)^\top|\boldsymbol{\psi}^J_{t_1}\right]^{-1}, \tag{E.15}$$

and assume the regularity conditions (Assumption E.1) in Appendix E.9.

Proposition E.1. *Under Assumptions 1 and E.1, $\mathbf{\Theta}_{\boldsymbol{H}^*}$ is asymptotically normal with the minimum variance among the asymptotic variances of the class of estimators in equation* (E.14)*.*

Proof. See Appendix E.9. □

To understand the optimality of the IV in equation (E.15), note that it has two components. The first term is the conditional expected gradient vector $\mathbb{E}\left[\nabla_{\mathbf{\Theta}}\nu_o(\mathbf{\Theta})\,|\boldsymbol{\psi}^J_{t_1}\right]$, which takes the gradient with respect to the structural parameter vector. Thus, it assigns large weight to occupation that changes the the predicted outcome variable sensitively to the parameters. The second term is the conditional inverse expected variance matrix $\mathbb{E}\left[\nu_o(\mathbf{\Theta})\left(\nu_o(\mathbf{\Theta})\right)^\top|\boldsymbol{\psi}^J_{t_1}\right]^{-1}$, which put large weight to occupation that has small variance of the structural residuals.

Substituting equation (E.15) to the general GMM estimator (E.14), I have an estimator $\mathbf{\Theta}_{\boldsymbol{H}^*} = \arg\min_{\mathbf{\Theta}}\left[\sum_o Z_o\nu_o(\mathbf{\Theta})\right]^\top\left[\sum_o Z_o\nu_o(\mathbf{\Theta})\right]$. Since Z_o depends on unknown parameters $\mathbf{\Theta}$, I implement the estimation by the two-step feasible method, or the model-implied optimal IV (Adao, Arkolakis, and Esposito 2019). I first estimate the first-step estimate $\mathbf{\Theta}_1$ from arbitrary initial values

$\mathbf{\Theta}_0$. Since the IV is a function of the Japan robot shock $\boldsymbol{\psi}^J_{t_1}$, $\mathbf{\Theta}_1$ is consistent by Assumption 1. However, it is not optimal. To achieve the optimality, in the second step, I obtain the optimal IV using the consistent estimator $\mathbf{\Theta}_1$. These arguments lead to the IV definition given in equation B.9 and Proposition (B.2).

E.6 Adjustment Cost of Robot Capital

Another key feature of the model is the convex adjustment cost of robot adoption. To interpret this, consider the cost of adopting new technology and integration. With the convex adjustment cost, the model predicts the staggered adoption of robots over years that I observe in the data (see Figure 4b), and implies a rich prediction about the short- and long-run effects of robotization.

First, when adopting new technology including robots, it is necessary to re-optimize the overall production process since the production process is typically optimized to employ workers. More generally, the literature of organizational dynamics studies the difficulty, not to say the impossibility, of changing strategies of a company due to complementarities (see Brynjolfsson and Milgrom 2013 for a review). Such a re-optimization incurs an additional cost of adoption in addition to the purchase of robot arms. Moreover, even within a production unit, there is a variation of this difficulty of adopting robots across production processes. In this case, the part where the adjustment is easy adopts the robots first, and vice versa. This allocation of robot adoptions over years may aggregate to make the robot stock increase slowly (Baldwin and Lin 2002). Waldman-Brown (2020) also finds that the incremental and sluggish automation is particularly well-observed in small and medium-sized firms, as they add "a machine here or there, rather than installing whole new systems that are more expensive to buy and integrate" (Autor, Mindell, and Reynolds 2020).

The second component of the adjustment cost may come from the cost of integration as discussed in Section 2.1. The marginal integration cost may increase as the massive upgrading of robotics system may require large-scale overhaul of production process, which increases the complexity and so is costly. The adjustment cost may capture the increasing marginal cost component of the integration cost. It explains an additional component of the integration cost implied by constant returns-to-scale robot aggregation in equation (10).

Another potential choice of modeling a staggered growth of robot stocks is to assume a fixed cost of robot adoption and lumpy investment. Humlum (2019) finds that many plants buy robots only once during the sample period. Since JARA data does not observe plant-level adoptions, I do not separately identify lumpy investment from the staggered growth of robot stocks in the data. To the extent that fixed cost of investment may make the policy intervention less effective (e.g., Koby and Wolf 2019), the counterfactual analysis in this paper may overestimate the effect of robot taxes since it does not take into account the fixed cost and lumpiness of investment.

E.7 Derivation of Worker's Optimality Conditions

In this section, I formalize the assumptions behind the derivation and show equations (B.6) and (B.7). One trick new to the below discussion is that I characterize the switching cost by an ad-valorem term, which makes the log-linearization simpler when solving the model.

Fix country i and period t. There is a mass $\overline{L}_{i,t}$ of workers. In the beginning of each period, worker $\omega \in \left[0, \overline{L}_{i,t}\right]$ draws a multiplicative idiosyncratic preference shock $\left\{Z_{i,o,t}\left(\omega\right)\right\}_{o}$ that follows an independent Fréchet distribution with scale parameter $A^{V}_{i,o,t}$ and shape parameter $1/\phi$. Note that one can simply extend that the idiosyncratic preference follows a correlated Fréchet distribution to allow correlated preference across occupations, as in Lind and Ramondo (2018). To keep the expression simple, I focus on the case of independent distribution. A worker ω then works in the current occupation, earns income, consumes and derives logarithmic utility, and then chooses the next period's occupation with discount rate ι. When choosing the next period occupation o', she pays an ad-valorem switching cost $\chi_{i,oo',t}$ in terms of consumption unit that depends on current occupation o. She consumes her income in each period. Thus, worker ω who currently works in occupation o_t maximizes the following objective function over the future stream of utilities by choosing occupations $\{o_s\}_{s=t+1}^{\infty}$:

$$E_t \sum_{s=t}^{\infty} \left(\frac{1}{1+\iota}\right)^{s-t} \left[\ln\left(C_{i,o_s,s}\right) + \ln\left(1-\chi_{i,o_s o_{s+1},s}\right) + \ln\left(Z_{i,o_{s+1},s}\left(\omega\right)\right)\right]$$

where $C_{i,o,s}$ is a consumption bundle when working in occupation o in period $s \geq t$, and E_t is the expectation conditional on the value of $Z_{i,o_t,t}\left(\omega\right)$. Each worker owns occupation-specific labor endowment $l_{i,o,t}$. I assume that her income is comprised of labor income $w_{i,o,t}$ and occupation-specific ad-valorem government transfer with rate $T_{i,o,t}$. Given the consumption price $P^{G}_{i,t}$, the budget constraint is

$$P_{i,t}^{G}C_{i,o,t} = w_{i,o,t}l_{i,o,t}\left(1+T_{i,o,t}\right)$$

for any worker, with $P_{i,t}^{G}$ being the price index of the non-robot good G.

By linearity of expectation,

$$\begin{aligned}
&E_t \sum_{s=t}^{\infty}\left(\frac{1}{1+\iota}\right)^{s-t}\left[\ln\left(C_{i,o_s,s}\right)+\ln\left(1-\chi_{i,o_s o_{s+1},s}\right)+\ln\left(Z_{i,o_{s+1},s}\left(\omega\right)\right)\right] \\
&= \sum_{s=t}^{\infty}\left(\frac{1}{1+\iota}\right)^{s-t}\left[E_t\ln\left(C_{i,o_s,s}\right)+E_t\ln\left(1-\chi_{i,o_s o_{s+1},s}\right)+E_t\ln\left(Z_{i,o_{s+1},s}\left(\omega\right)\right)\right].
\end{aligned}$$

By monotone transformation with exponential function,

$$\begin{aligned}
&\exp\left\{\sum_{s=t}^{\infty}\left(\frac{1}{1+\iota}\right)^{s-t}\left[E_t\ln\left(C_{i,o_s,s}\right)+E_t\ln\left(1-\chi_{i,o_s o_{s+1},s}\right)+E_t\ln\left(Z_{i,o_{s+1},s}\left(\omega\right)\right)\right].\right\} \\
&= \prod_{s=t}^{\infty}\exp\left\{\left(\frac{1}{1+\iota}\right)^{s-t}\left[E_t\ln\left(C_{i,o_s,s}\right)+E_t\ln\left(1-\chi_{i,o_s o_{s+1},s}\right)+E_t\ln\left(Z_{i,o_{s+1},s}\left(\omega\right)\right)\right]\right\}.
\end{aligned}$$

Write the value function conditional on the realization of shocks at period t as follows:

$$V_{i,o_t,t}\left(\omega\right) \equiv \max_{\{o_s\}_{s=t+1}^{\infty}}\prod_{s=t}^{\infty}\exp\left\{\left(\frac{1}{1+\iota}\right)^{s-t}\left[E_t\ln\left(C_{i,o_s,s}\right)+E_t\ln\left(1-\chi_{i,o_s o_{s+1},s}\right)+E_t\ln\left(Z_{i,o_{s+1},s}\left(\omega\right)\right)\right]\right\}.$$

I apply Bellman's principle of optimality as follows:

$$\begin{aligned}
&V_{i,o_t,t}\left(\omega\right) \\
&= \max_{\{o_s\}_{s=t+1}^{\infty}}\prod_{s=t}^{\infty}\exp\left\{\left(\frac{1}{1+\iota}\right)^{s-t}\left[E_t\ln\left(C_{i,o_s,s}\right)+E_t\ln\left(1-\chi_{i,o_s o_{s+1},s}\right)+E_t\ln\left(Z_{i,o_{s+1},s}\left(\omega\right)\right)\right]\right\} \\
&= \max_{o_{t+1}}\exp\left\{\ln\left(C_{i,o_t,t}\right)+\ln\left(1-\chi_{i,o_t o_{t+1},t}\right)+\ln\left(Z_{i,o_{t+1},t}\left(\omega\right)\right)\right\}\times \\
&\max_{\{o_s\}_{s=t+2}^{\infty}}\prod_{s=t+1}^{\infty}\exp\left\{\left(\frac{1}{1+\iota}\right)^{s-(t+1)}\left[E_{t+1}\ln\left(C_{i,o_s,s}\right)+E_{t+1}\ln\left(1-\chi_{i,o_s o_{s+1},s}\right)+E_{t+1}\ln\left(Z_{i,o_{s+1},s}\left(\omega\right)\right)\right]\right\} \\
&= \max_{o_{t+1}}\exp\left\{\ln\left(Z_{i,o_t,t}\left(\omega\right)\right)+\ln\left(C_{i,o_t,t}\right)+\ln\left(1-\chi_{i,o_t o_{t+1},t}\right)\right\}V_{i,o_{t+1},t+1},
\end{aligned}$$

where $V_{i,o_t,t}$ is the unconditional expected value function $V_{i,o_t,t} \equiv E_{t-1}V_{i,o_t,t}\left(\omega\right)$. Changing the notation from (o_t, o_{t+1}) into (o, o'), I have

$$V_{i,o,t}(\omega) = \max_{o'} C_{i,o,t}\left(1-\chi_{i,oo',t}\right) Z_{i,o',t}(\omega)\, V_{i,o',t+1}.$$

Solving the worker's maximization problem is equivalent to finding:

$$\begin{aligned}
\mu_{i,oo',t} &\equiv \Pr\left(\text{worker } \omega \text{ in } o \text{ chooses occupation } o'\right) \\
&= \Pr\left(\max_{o''} C_{i,o,t}\left(1-\chi_{i,oo'',t}\right) Z_{i,o'',t}(\omega)\, V_{i,o'',t+1} \leq C_{i,o,t}\left(1-\chi_{i,oo',t}\right) Z_{i,o',t}(\omega)\, V_{i,o',t+1}\right).
\end{aligned}$$

By the independent Fréchet assumption, I have the maximum value distribution

$$\begin{aligned}
\Pr\left(\max_{o''} C_{i,o,t}\left(1-\chi_{i,oo',t}\right) Z_{i,o',t}(\omega)\, V_{i,o',t+1} \leq v\right) &= \prod_{o'} \Pr\left(Z_{i,o',t}(\omega) \leq \frac{v}{C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}}\right) \\
&= \prod_{o''} \exp\left(\left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi} v^{-\phi}\right) \\
&= \exp\left(\sum_{o''} \left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi} v^{-\phi}\right).
\end{aligned}$$

Therefore, the conditional choice probability satisfies, again by the independent Fréchet assumption,

$$\begin{aligned}
&\mu_{i,oo',t} \\
=& \int_0^\infty \Pr\left(\max_{o''\neq o'} C_{i,o,t}\left(1-\chi_{i,oo'',t}\right) Z_{i,o',t}(\omega)\, V_{i,o'',t+1} \leq v\right) d\Pr\left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) Z_{i,o',t}(\omega)\, V_{i,o',t+1} \geq v\right) \\
=& \int_0^\infty \exp\left(\sum_{o''\neq o'} \left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi} v^{-\phi}\right) \times \\
& \qquad \left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi} \exp\left(\left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi} v^{-\phi}\right) \times \left(-\phi v^{-\phi-1}\right) dv \\
=& \frac{\left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi}}{\sum_{o''}\left(C_{i,o,t}\left(1-\chi_{i,oo'',t}\right) V_{i,o'',t+1}\right)^{\phi}} \times \\
& \qquad \int_0^\infty \exp\left(\sum_{o'''} \left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right) V_{i,o',t+1}\right)^{\phi} v^{-\phi}\right) \sum_{o''} \left(C_{i,o,t}\left(1-\chi_{i,oo'',t}\right) V_{i,o'',t+1}\right)^{\phi} \times \left(-\phi v^{-\phi-1}\right) dv.
\end{aligned}$$

The last integral term is one by integration and the definition of distribution. Therefore, I arrive at

$$\mu_{i,oo',t} = \frac{\left(C_{i,o,t}\left(1-\chi_{i,oo',t}\right)V_{i,o',t+1}\right)^{\phi}}{\sum_{o''}\left(C_{i,o,t}\left(1-\chi_{i,oo'',t}\right)V_{i,o'',t+1}\right)^{\phi}} = \frac{\left(\left(1-\chi_{i,oo',t}\right)V_{i,o',t+1}\right)^{\phi}}{\sum_{o''}\left(\left(1-\chi_{i,oo'',t}\right)V_{i,o'',t+1}\right)^{\phi}},$$

$$V_{i,o,t+1} = \widetilde{\Gamma}C_{i,o,t}\left(\sum_{o'}\left(\left(1-\chi_{i,oo',t+1}\right)V_{i,o',t+2}\right)^{\phi}\right)^{\frac{1}{\phi}}.$$

E.8 Equilibrium Characterization

To characterize the producer problem, I show the static optimization conditions and then the dynamic ones. For simplicity, I focus on the case with $\vartheta = 1$, or Cobb-Douglas in the mix of occupation aggregates, intermediates, and non-robot capital. To solve for the static problem of labor, intermediate goods, and non-robot capital, consider the FOCs of equation (8)

$$p^{G}_{i,t}\alpha_{i,L}\frac{Y^{G}_{i,t}}{T^{O}_{i,t}}\left(b_{i,o,t}\frac{T^{O}_{i,t}}{T^{O}_{i,o,t}}\right)^{\frac{1}{\beta}}\left(\left(1-a_{o,t}\right)\frac{T^{O}_{i,o,t}}{L_{i,o,t}}\right)^{\frac{1}{\theta_{O}}} = w_{i,o,t}, \tag{E.16}$$

where $T^{O}_{i,t}$ is the aggregated occupations $T^{O}_{i,t} \equiv \left[\sum_{o}\left(T^{O}_{i,o,t}\right)^{(\beta-1)/\beta}\right]^{\beta/(\beta-1)}$,

$$p^{G}_{i,t}\alpha_{i,M}\frac{Y^{G}_{i,t}}{M_{i,t}}\left(\frac{M_{i,t}}{M_{li,t}}\right)^{\frac{1}{\varepsilon}} = p^{G}_{li,t}, \tag{E.17}$$

and

$$p^{G}_{i,t}\alpha_{i,K}\frac{Y^{G}_{i,t}}{K_{i,t}} = r_{i,t}, \tag{E.18}$$

where $\alpha_{i,K} \equiv 1-\alpha_{i,L}-\alpha_{i,M}$. Note also that by the envelope theorem,

$$\frac{\partial\pi_{i,t}\left(\left\{K^{R}_{i,o,t}\right\}\right)}{\partial K^{R}_{i,o,t}} = p^{G}_{i,t}\frac{\partial Y_{i,t}}{\partial K^{R}_{i,o,t}} = p^{G}_{i,t}\left(\alpha_{L}\frac{Y^{G}_{i,t}}{T^{O}_{i,t}}\left(b_{i,o,t}\frac{T^{O}_{i,t}}{T^{O}_{i,o,t}}\right)^{\frac{1}{\beta}}\left(a_{o,t}\frac{T^{O}_{i,o,t}}{K^{R}_{i,o,t}}\right)^{\frac{1}{\theta}}\right). \tag{E.19}$$

Another static problem of producers is robot purchase. Define the "before-integration" robot aggregate $Q^{R,BI}_{i,o,t} \equiv \left[\sum_{l}\left(Q^{R}_{li,o,t}\right)^{\frac{\varepsilon^{R}-1}{\varepsilon^{R}}}\right]^{\frac{\varepsilon^{R}}{\varepsilon^{R}-1}}$ and the corresponding price index $P^{R,BI}_{i,o,t}$. By the first order condition with respect to $Q^R_{li,o,t}$ for equation (10), I have $p^R_{li,o,t} Q^R_{li,o,t} = \left(\frac{p^R_{li,o,t}}{P^{R,BI}_{i,o,t}}\right)^{1-\varepsilon^R} P^{R,BI}_{i,o,t} Q^{R,BI}_{i,o,t}$, and $P^{R,BI}_{i,o,t} Q^{R,BI}_{i,o,t} = \alpha P^R_{i,o,t} Q^R_{i,o,t}$. Thus $p^R_{li,o,t} Q^R_{li,o,t} = \alpha \left(\frac{p^R_{li,o,t}}{P^{R,BI}_{i,o,t}}\right)^{1-\varepsilon^R} P^R_{i,o,t} Q^R_{i,o,t}$. Hence

$$Q^R_{li,o,t} = \alpha \left(p^R_{li,o,t}\right)^{-\varepsilon^R} \left(P^{R,BI}_{i,o,t}\right)^{\varepsilon^R - 1} P^R_{i,o,t} Q^R_{i,o,t}.$$

Writing $P^R_{i,o,t} = \left(P^{R,BI}_{i,o,t}\right)^{\alpha^R} \left(P_{i,t}\right)^{1-\alpha^R}$, I have

$$Q^R_{li,o,t} = \alpha \left(\frac{p^R_{li,o,t}}{P^{R,BI}_{i,o,t}}\right)^{-\varepsilon^R} \left(\frac{P^{R,BI}_{i,o,t}}{P_{i,t}}\right)^{-\left(1-\alpha^R\right)} Q^R_{i,o,t}.$$

Alternatively, one can define the robot price index by $\widetilde{P}^R_{i,o,t} = \alpha^{\frac{1}{\varepsilon^R}} \left(P^{R,BI}_{i,o,t}\right)^{\frac{\varepsilon^R - \left(1-\alpha^R\right)}{\varepsilon^R}} P_{i,t}^{\frac{1-\alpha^R}{\varepsilon^R}}$ and show

$$Q^R_{li,o,t} = \left(\frac{p^R_{li,o,t}}{\widetilde{P}^R_{i,o,t}}\right)^{-\varepsilon^R} Q^R_{i,o,t}, \tag{E.20}$$

which is a standard gravity representation of robot trade.

To solve the dynamic problem, set up the (current-value) Lagrangian function for non-robot goods producers

$$\begin{aligned}\mathcal{L}_{i,t} = \sum_{t=0}^{\infty} \Bigg\{ \left(\frac{1}{1+\iota}\right)^t \Bigg[\pi_{i,t}\left(\left\{K^R_{i,o,t}\right\}_o\right) - \sum_{l,o} \left(p^R_{li,o,t}\left(1+u_{li,t}\right) Q^R_{li,o,t} + P^G_{i,t} I^{int}_{i,o,t} + \gamma P^R_{i,o,t} Q^R_{i,o,t} \frac{Q^R_{i,o,t}}{K^R_{i,o,t}} \right) \Bigg] \Bigg\}. \\ - \lambda^R_{i,o,t} \left\{ K^R_{i,o,t+1} - (1-\delta) K^R_{i,o,t} - Q^R_{i,o,t} \right\}\end{aligned}$$

Taking the FOC with respect to the hardware from country l, $Q^R_{li,o,t}$, I have

$$p^R_{li,o,t}\left(1+u_{li,t}\right) + 2\gamma P^R_{i,o,t} \left(\frac{Q^R_{i,o,t}}{K^R_{i,o,t}}\right) \frac{\partial Q^R_{i,o,t}}{\partial Q^R_{li,o,t}} = \lambda^R_{i,o,t} \frac{\partial Q^R_{i,o,t}}{\partial Q^R_{li,o,t}}. \tag{E.21}$$

Taking the FOC with respect to the integration input $I^{int}_{i,o,t}$, I have

$$P^G_{i,t} + 2\gamma P^R_{i,o,t} \left(\frac{Q^R_{i,o,t}}{K^R_{i,o,t}}\right) \frac{\partial Q^R_{i,o,t}}{\partial I^{int}_{i,o,t}} = \lambda^R_{i,o,t} \frac{\partial Q^R_{i,o,t}}{\partial I^{int}_{i,o,t}}, \tag{E.22}$$

Taking the FOC with respect to $K^R_{i,o,t+1}$, I have

$$\left(\frac{1}{1+\iota}\right)^{t+1}\left[\frac{\partial\pi_{i,t+1}\left(\left\{K^R_{i,o,t+1}\right\}_o\right)}{\partial K^R_{i,o,t+1}}+\gamma P^R_{i,o,t+1}\left(\frac{Q^R_{i,o,t+1}}{K^R_{i,o,t+1}}\right)^2+(1-\delta)\,\lambda^R_{i,o,t+1}\right]-\left(\frac{1}{1+\iota}\right)^t\lambda^R_{i,o,t}=0, \tag{E.23}$$

and the transversality condition: for any j and o,

$$\lim_{t\to\infty}e^{-\iota t}\lambda^R_{j,o,t}K^R_{j,o,t+1}=0. \tag{E.24}$$

Rearranging equation (E.23), I obtain the following Euler equation.

$$\lambda^R_{i,o,t}=\frac{1}{1+\iota}\left[(1-\delta)\,\lambda^R_{i,o,t+1}+\frac{\partial}{\partial K^R_{i,o,t+1}}\pi_{i,t+1}\left(\left\{K^R_{i,o,t+1}\right\}\right)+\gamma p^R_{i,o,t+1}\left(\frac{Q^R_{i,o,t+1}}{K^R_{i,o,t+1}}\right)^2\right]. \tag{E.25}$$

Turning to the demand for non-robot good, I will characterize bilateral intermediate good trade demand and total expenditure. Write $X^G_{j,t}$ the total purchase quantity (but not value) of good G in country j in period t. By equation (B.5), the bilateral trade demand is given by

$$p^G_{ij,t}Q^G_{ij,t}=\left(\frac{p^G_{ij,t}}{P^G_{j,t}}\right)^{1-\varepsilon}P^G_{j,t}X^G_{j,t}, \tag{E.26}$$

for any i, j, and t. In this equation, $P^G_{j,t}X^G_{j,t}$ is the total expenditures on non-robot goods. The total expenditure is the sum of final consumption $I_{j,t}$, payment to intermediate goods $\alpha_M p^G_{j,t}Y^G_{j,t}$, input to robot productions $\sum_o P^G_{j,t}I^R_{j,o,t}=\sum_{o,k}p^R_{jk,o,t}Q^R_{jk,o,t}$, and payment to robot integration $\sum_o P^G_{j,t}I^{int}_{j,o,t}=\left(1-\alpha^R\right)\sum_o P^R_{j,o,t}Q^R_{j,o,t}$. Hence

$$P^G_{j,t}X^G_{j,t}=I_{j,t}+\alpha_M p^G_{j,t}Y^G_{j,t}+\sum_{o,k}p^R_{jk,o,t}Q^R_{jk,o,t}+\left(1-\alpha^R\right)\sum_o P^R_{j,o,t}Q^R_{j,o,t}.$$

For country j and period t, by substituting into income $I_{j,t}$ the period cash flow of non-robot good producer that satisfies

$$\Pi_{j,t}\equiv\pi_{j,t}\left(\left\{K^R_{j,o,t}\right\}_o\right)-\sum_{i,o}\left(p^R_{ij,o,t}\left(1+u_{ij,t}\right)Q^R_{ij,o,t}+\sum_o P^G_{j,t}I^{int}_{j,o,t}+\gamma P^R_{j,o,t}Q^R_{j,o,t}\left(\frac{Q^R_{j,o,t}}{K^R_{j,o,t}}\right)\right)$$

and robot tax revenue $T_{j,t} = \sum_{i,o} u_{ij,t} p^R_{ij,o,t} Q^R_{ij,o,t}$, I have

$$I_{j,t} = (1-\alpha_M) \sum_k p^G_{jk,t} Q^G_{jk,t} - \left(\sum_{i,o} p^R_{ij,o,t} Q^R_{ij,o,t} + \left(1 - \alpha^R\right) \sum_o P^R_{j,o,t} Q^R_{j,o,t} \right), \tag{E.27}$$

or in terms of variables in the definition of equilibrium,

$$I_{j,t} = (1-\alpha_M) \sum_k p^G_{jk,t} Q^G_{jk,t} - \frac{1}{\alpha^R} \sum_{i,o} p^R_{ij,o,t} Q^R_{ij,o,t}.$$

Hence, the total expenditure measured in terms of the production side as opposed to income side is

$$P^G_{j,t} X^G_{j,t} = \sum_k p^G_{jk,t} Q^G_{jk,t} - \sum_{i,o} p^R_{ij,o,t} Q^R_{ij,o,t} \left(1 + \gamma \frac{Q^R_{ij,o,t}}{K^R_{j,o,t}}\right). \tag{E.28}$$

Note that this equation embeds the balanced-trade condition. By substituting equation (E.28) into equation (E.26), I have

$$p^G_{ij,t} Q^G_{ij,t} = \left(\frac{p^G_{ij,t}}{P^G_{j,t}}\right)^{1-\varepsilon^G} \left(\sum_k p^G_{jk,t} Q^G_{jk,t} + \sum_{k,o} p^R_{jk,o,t} Q^R_{jk,o,t} - \sum_{i,o} p^R_{ij,o,t} Q^R_{ij,o,t} \right). \tag{E.29}$$

The good and robot-o market-clearing conditions are given by,

$$Y^R_{i,t} = \sum_j Q^G_{ij,t} \tau^G_{ij,t}, \tag{E.30}$$

for all i and t, and

$$p^R_{i,o,t} = \frac{P^G_{i,t}}{A^R_{i,o,t}} \tag{E.31}$$

for all i, o, and t, respectively.

Conditional on state variables $\boldsymbol{S}_t = \left\{\boldsymbol{K}^R_t, \boldsymbol{\lambda}^R_t, \boldsymbol{L}_t, \boldsymbol{V}_t\right\}$, equations (B.6), (E.16), (E.21), (E.29), (E.30), and (E.31) characterize the temporary equilibrium $\left\{\boldsymbol{p}^G_t, \boldsymbol{p}^R_t, \boldsymbol{w}_t, \boldsymbol{Q}^G_t, \boldsymbol{Q}^R_t, \boldsymbol{L}_t\right\}$. In addition, conditional on initial conditions $\left\{\boldsymbol{K}^R_0, \boldsymbol{L}_0\right\}$, equations (9), (E.25), and (E.24) characterize the sequential equilibrium.

Finally, the steady state conditions are given by imposing the time-invariance condition to equations (9) and (E.25):

$$Q_{i,o}^{R} = \delta K_{i,o}^{R}, \tag{E.32}$$

$$\frac{\partial}{\partial K_{i,o}^{R}} \pi_i \left(\left\{ K_{i,o}^{R} \right\} \right) = (\iota + \delta) \lambda_{i,o}^{R} - \sum_{l} \gamma p_{li,o}^{R} \left(\frac{Q_{li,o}^{R}}{K_{i,o}^{R}} \right)^{2} \equiv c_{i,o}^{R}. \tag{E.33}$$

Note that equation (E.33) can be interpreted as the flow marginal profit of capital must be equalized to the marginal cost term. Thus I define the steady state marginal cost of robot capital $c_{i,o}^{R}$ from the right-hand side of equation (E.33). Note that if there is no adjustment cost $\gamma = 0$, the steady state Euler equation (E.33) implies

$$\frac{\partial}{\partial K_{i,o}^{R}} \pi_i \left(\left\{ K_{i,o}^{R} \right\} \right) = c_{i,o}^{R} = (\iota + \delta) \lambda_{i,o}^{R},$$

which states that the marginal profit of capital is the user cost of robots in the steady state (Hall and Jorgenson 1967).

E.9 Remaining Proofs

Proof of Proposition E.1 To prove Proposition I follow the arguments made in Sections 2 and 3 of Newey and McFadden (1994). The proof consists of four sub results in the following Lemma. Proposition E.1 can be obtained as a combination of the four results. The formal statement requires the following additional assumptions.

Assumption E.1. *(i) A function of* $\widetilde{\boldsymbol{\Theta}}$, $\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^{J}\right)\nu_o\left(\widetilde{\boldsymbol{\Theta}}\right)\right] \neq 0$ *for any* $\widetilde{\boldsymbol{\Theta}} \neq \boldsymbol{\Theta}$. *(ii)* $\underline{\theta} \leq \theta_o \leq \overline{\theta}$ *for any o,* $\underline{\beta} \leq \beta \leq \overline{\beta}$, $\underline{\gamma} \leq \gamma \leq \overline{\gamma}$, *and* $\underline{\phi} \leq \phi \leq \overline{\phi}$ *for some positive values* $\underline{\theta}, \underline{\beta}, \underline{\gamma}, \underline{\phi}, \overline{\theta}, \overline{\beta}, \overline{\gamma}, \overline{\phi}$. *(iii)* $\mathbb{E}\left[\sup_{\boldsymbol{\Theta}} \| H_o\left(\boldsymbol{\psi}_{t_1}^{J}\right)\nu_o\left(\widetilde{\boldsymbol{\Theta}}\right) \|\right] < \infty$. *(iv)* $\mathbb{E}\left[\| H_o\left(\boldsymbol{\psi}_{t_1}^{J}\right)\nu_o\left(\widetilde{\boldsymbol{\Theta}}\right) \|^{2}\right] < \infty$ *(v)* $\mathbb{E}\left[\sup_{\boldsymbol{\Theta}} \| H_o\left(\boldsymbol{\psi}_{t_1}^{J}\right)\nabla_{\widetilde{\boldsymbol{\Theta}}}\nu_o\left(\widetilde{\boldsymbol{\Theta}}\right) \|\right] < \infty$.

Lemma E.3. *Assume Assumptions 1 and E.1(i)-(iii).*

(a) The estimator of the form (E.14) *is consistent.*

Additionally, assume Assumptions E.1(iv)-(v).

(b) The estimator of the form (E.14) *is asymptotically normal.*

(c) $\sqrt{O}\left(\boldsymbol{\Theta}_{\boldsymbol{H}^*} - \boldsymbol{\Theta}\right) \to_d \mathcal{N}\left(0, \left(\boldsymbol{G}^\top \boldsymbol{\Omega}^{-1} \boldsymbol{G}\right)^{-1}\right)$*, and the asymptotic variance is the minimum of that of the estimator of the form* (E.14) *for any function* $\boldsymbol{H}$.

Proof. (a) I follow Theorems 2.6 of Newey and McFadden (1994), which implies that it suffices to show conditions (i)-(iv) of this theorem are satisfied. Assumption E.1(i) guarantees condition (i). Condition (ii) is implied by Assumption E.1(ii). Condition (iii) follows because all supply and demand functions in the model is continuous. Condition (iv) is implied by Assumption E.1(iii).

(b) I follow Theorem 3.4 of Newey and McFadden (1994), which implies that it suffices to show conditions (i)-(v) of this theorem are satisfied. Condition (i) is satisfied by Assumption E.1(i). Condition (ii) follows because all supply and demand functions in the model is continuously differentiable. Condition (iii) is implied by Assumption 1 and Assumption E.1(iv). Assumption E.1(v) implies condition (iv). Finally, the gradient vectors of the structural residual is linear independent, guaranteeing the non-singularity of the variance matrix and condition (v).

(c) By Theorem 3.4 of Newey and McFadden (1994), for an arbitrary IV-generating function $\boldsymbol{H}$, the asymptotic variance of the GMM estimator $\boldsymbol{\Theta_H}$ is

$$\left(\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right]\right)^{-1}\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nu_o\nu_o^\top\left(H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\right)^\top\right]\left(\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right]\right)^{-1},$$

where $G_o \equiv \mathbb{E}\left[\nabla_{\boldsymbol{\Theta}}\nu_o\left(\boldsymbol{\Theta}\right)|\boldsymbol{\psi}_{t_1}^J\right]$. Therefore, if $H_o\left(\boldsymbol{\psi}_{t_1}^J\right) = Z_o \equiv \mathbb{E}\left[\nabla_{\boldsymbol{\Theta}}\nu_o\left(\boldsymbol{\Theta}\right)|\boldsymbol{\psi}_{t_1}^J\right]\mathbb{E}\left[\nu_o\left(\boldsymbol{\Theta}\right)\left(\nu_o\left(\boldsymbol{\Theta}\right)\right)^\top|\boldsymbol{\psi}_{t_1}^J\right]^{-1}$, then this expression is equal to $\left(\boldsymbol{G}^\top\boldsymbol{\Omega}^{-1}\boldsymbol{G}\right)^{-1}$, where

$$\boldsymbol{G} \equiv \mathbb{E}\left[\nabla_{\boldsymbol{\Theta}}\nu_o\left(\boldsymbol{\Theta}\right)\right] \text{ and } \boldsymbol{\Omega} \equiv \mathbb{E}\left[\nu_o\left(\boldsymbol{\Theta}\right)\left(\nu_o\left(\boldsymbol{\Theta}\right)\right)^\top\right].$$

To show that this variance is minimal, I will check that

$$\begin{aligned}\boldsymbol{\Delta} \equiv &\left(\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right]\right)^{-1}\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nu_o\nu_o^\top\left(H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\right)^\top\right]\left(\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right]\right)^{-1}\\ &-\left(\boldsymbol{G}^\top\boldsymbol{\Omega}^{-1}\boldsymbol{G}\right)^{-1}\end{aligned}$$

is positive semi-definite. In fact, note that

$$\begin{aligned}\boldsymbol{\Delta} = &\left(\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right]\right)^{-1}\times\\ &\left\{\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nu_o\nu_o^\top\left(H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\right)^\top\right]-\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right]\left(\boldsymbol{G}^\top\boldsymbol{\Omega}^{-1}\boldsymbol{G}\right)^{-1}\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right]\right\}\times\\ &\left(\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right]\right)^{-1}.\end{aligned}$$

Define

$$\widetilde{\nu}_o = H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nu_o - \mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nu_o\left(\left(G_o\right)^\top\Omega_o^{-1}\nu_o\right)^{-1}\right]\mathbb{E}\left(\left(G_o\right)^\top\Omega_o^{-1}\nu_o\right)^{-1}\left(G_o\right)^\top\Omega_o^{-1}\nu_o,$$

where $\Omega_o \equiv \mathbb{E}\left[\nu_o\left(\boldsymbol{\Theta}\right)\left(\nu_o\left(\boldsymbol{\Theta}\right)\right)^\top|\boldsymbol{\psi}_{t_1}^J\right]$. Applying Theorem 5.3 of Newey and McFadden (1994), I have

$$\mathbb{E}\left[\widetilde{\nu}_o\left(\widetilde{\nu}_o\right)^\top\right] = \mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\nu_o\nu_o^\top\left(H_o\left(\boldsymbol{\psi}_{t_1}^J\right)\right)^\top\right] - \mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right]\left(\boldsymbol{G}^\top\boldsymbol{\Omega}^{-1}\boldsymbol{G}\right)^{-1}\mathbb{E}\left[H_o\left(\boldsymbol{\psi}_{t_1}^J\right)G_o\right].$$

Since $\mathbb{E}\left[\widetilde{\nu}_o\left(\widetilde{\nu}_o\right)^\top\right]$ is positive semi-definite, so is $\boldsymbol{\Delta}$, which completes the proof. □

Proof of Proposition B.2 I apply arguments in Section 6.1 of Newey and McFadden (1994). Namely, I define the joint estimator of the first-step and second-step estimator in Proposition B.2 that falls into the class of general GMM estimation, and discuss the asymptotic property using the general result about GMM estimation. In the proof, I modify the notation of the set of functions that yield optimal IV, $\boldsymbol{H}^*$, to clarify that it depends on parameters $\boldsymbol{\Theta}$ as follows:

$$H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}\right) = \mathbb{E}\left[\nabla_{\boldsymbol{\Theta}}\nu_o\left(\boldsymbol{\Theta}\right)|\boldsymbol{\psi}_{t_1}^J\right]\mathbb{E}\left[\nu_o\left(\boldsymbol{\Theta}\right)\left(\nu_o\left(\boldsymbol{\Theta}\right)\right)^\top|\boldsymbol{\psi}_{t_1}^J\right]^{-1}.$$

Define the joint estimator as follows:

$$\begin{pmatrix}\boldsymbol{\Theta}_2\\ \boldsymbol{\Theta}_1\end{pmatrix} \equiv \arg\min_{\boldsymbol{\Theta}_2,\boldsymbol{\Theta}_1}\left[\sum_o e_o\left(\boldsymbol{\Theta}_2,\boldsymbol{\Theta}_1\right)\right]^\top\left[\sum_o e_o\left(\boldsymbol{\Theta}_2,\boldsymbol{\Theta}_1\right)\right],$$

where

$$e_o\left(\boldsymbol{\Theta}_2,\boldsymbol{\Theta}_1\right) \equiv \begin{pmatrix} H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_1\right)\nu_o\left(\boldsymbol{\Theta}_2\right)\\ H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_0\right)\nu_o\left(\boldsymbol{\Theta}_1\right)\end{pmatrix}.$$

Since for any $\boldsymbol{\Theta}$, IV-generating function $H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_0\right)$ gives the consistent estimator for $\boldsymbol{\Theta}$, I have $\boldsymbol{\Theta}_1 \to \boldsymbol{\Theta}$ and $\boldsymbol{\Theta}_2 \to \boldsymbol{\Theta}$. I also have the asymptotic variance

$$\mathrm{Var}\begin{pmatrix}\boldsymbol{\Theta}_2\\ \boldsymbol{\Theta}_1\end{pmatrix} = \left[\left(\widetilde{\boldsymbol{G}}\right)^\top\widetilde{\boldsymbol{\Omega}}\widetilde{\boldsymbol{G}}\right]^{-1},$$

where

$$
\begin{aligned}
\widetilde{\boldsymbol{G}} &\equiv \mathbb{E}\left[\nabla_{(\boldsymbol{\Theta}_2,\boldsymbol{\Theta}_1)^\top} e_o\left(\boldsymbol{\Theta}_2,\boldsymbol{\Theta}_1\right)\right] \\
&= \mathbb{E}\begin{bmatrix} H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_1\right)\nabla\nu_o\left(\boldsymbol{\Theta}_2\right) & \nabla H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_1\right)\nu_o\left(\boldsymbol{\Theta}_2\right) \\ \boldsymbol{0} & H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_0\right)\nabla\nu_o\left(\boldsymbol{\Theta}_1\right) \end{bmatrix}
\end{aligned}
$$

and

$$
\begin{aligned}
&\widetilde{\boldsymbol{\Omega}} \equiv \mathbb{E}\left[e_o\left(\boldsymbol{\Theta}_2,\boldsymbol{\Theta}_1\right)\left[e_o\left(\boldsymbol{\Theta}_2,\boldsymbol{\Theta}_1\right)\right]^\top\right] \\
&= \mathbb{E}\begin{bmatrix} H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_1\right)\nu_o\left(\boldsymbol{\Theta}_2\right)\left[H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_1\right)\nu_o\left(\boldsymbol{\Theta}_2\right)\right]^\top & H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_1\right)\nu_o\left(\boldsymbol{\Theta}_2\right)\left[H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_0\right)\nu_o\left(\boldsymbol{\Theta}_1\right)\right]^\top \\ H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_0\right)\nu_o\left(\boldsymbol{\Theta}_1\right)\left[H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_1\right)\nu_o\left(\boldsymbol{\Theta}_2\right)\right]^\top & H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_0\right)\nu_o\left(\boldsymbol{\Theta}_1\right)\left[H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_0\right)\nu_o\left(\boldsymbol{\Theta}_1\right)\right]^\top \end{bmatrix}.
\end{aligned}
$$

Note that Assumption 1 implies that any function of $\boldsymbol{\psi}_{t_1}^J$ is orthogonal to ν_o, implying $\mathbb{E}\left[\nabla H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}_1\right)\nu_o\left(\boldsymbol{\Theta}_2\right)\right] = 0$. Therefore, $\widetilde{\boldsymbol{G}}$ is a block-diagonal matrix and thus the marginal asymptotic distribution of $\boldsymbol{\Theta}_2$ is normal with variance $\mathrm{Var}\left(\boldsymbol{\Theta}_2\right) = \left(\boldsymbol{G}^\top\boldsymbol{\Omega}^{-1}\boldsymbol{G}\right)^{-1}$, noting that $\boldsymbol{G} = \mathbb{E}\left[H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}\right)\nabla\nu_o\left(\boldsymbol{\Theta}\right)\right]$ and $\boldsymbol{\Omega} \equiv \mathbb{E}\left[H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}\right)\nu_o\left(\boldsymbol{\Theta}\right)\left(H_o^*\left(\boldsymbol{\psi}_{t_1}^J;\boldsymbol{\Theta}\right)\nu_o\left(\boldsymbol{\Theta}\right)\right)^\top\right]$. By Proposition E.1, this asymptotic variance is minimal among the GMM estimator (E.14).

E.10 On the Choice of the Steady-State Matrix in Equation (20)

In equation (20), I use the steady-state matrix $\overline{\boldsymbol{E}}$ instead of the transitional dynamics matrix $\overline{\boldsymbol{F}_t}$ for a computational reason. Since I have annual observation for occupational robot costs, it is potentially possible to leverage this rich variation for the structural estimation, which may permit me to estimate the EoS θ_o at a narrower occupation group level. However, the bottleneck of this approach is the computational burden to compute the dynamic solution matrix $\overline{\boldsymbol{F}_t}$. Specifically, dynamic substitution matrix $\overline{\boldsymbol{F}_{t+1}^y}$ in equation (16) is based on the conditions of Blanchard and Kahn (1980). This requires computing the eigenspace, as described in detail in Section G. This is computationally hard since I cannot rely on the sparse structure of the matrix $\overline{\boldsymbol{F}_{t+1}^y}$. In contrast, the estimation method in Proposition B.2 does not involve such computation, but only requires computing the steady-state solution matrix $\overline{\boldsymbol{E}}$. Then I only need to invert steady-state substitution matrix $\overline{\boldsymbol{E}^y}$, which is feasible given the sparse structure of $\overline{\boldsymbol{E}^y}$.

F Online Appendix for Estimation and Simulation

F.1 Robot Trade Elasticity

To estimate robot trade elasticity ε^R, I apply and extend the trilateral method of Caliendo and Parro (2015). Namely, decompose the robot trade cost $\tau^R_{li,t}$ into $\ln \tau^R_{li,t} = \ln \tau^{R,T}_{li,t} + \ln \tau^{R,D}_{li,t}$, where $\tau^{R,T}_{li,t}$ is tariff on robots taken from the UNCTAD-TRAINS database and $\tau^{R,D}_{li,t}$ is asymmetric non-tariff trade cost. The latter term is assumed to be $\ln \tau^{R,D}_{li,t} = \ln \tau^{R,D,S}_{li,t} + \ln \tau^{R,D,O}_{l,t} + \ln \tau^{R,D,D}_{i,t} + \ln \tau^{R,D,E}_{li,t}$, where $\tau^{R,D,S}_{li,t}$ captures symmetric bilateral trade costs such as distance, common border, language, and FTA belonging status and satisfies $\tau^{R,D,S}_{li,t} = \tau^{R,D,S}_{il,t}$, $\tau^{R,D,O}_{l,t}$ and $\tau^{R,D,D}_{i,t}$ are the origin and destination fixed effects such as non-tariff barriers respectively, and $\tau^{R,D,E}_{li,t}$ is the random error that is orthogonal to tariffs. From the robot gravity equation (E.20) that I derive in Section E.8, I have

$$\ln\left(\frac{X^R_{li,t}X^R_{ij,t}X^R_{jl,t}}{X^R_{lj,t}X^R_{ji,t}X^R_{il,t}}\right) = \left(1-\varepsilon^R\right)\ln\left(\frac{\tau^{R,T}_{li,t}\tau^{R,T}_{ij,t}\tau^{R,T}_{jl,t}}{\tau^{R,T}_{lj,t}\tau^{R,T}_{ji,t}\tau^{R,T}_{il,t}}\right) + e_{lij,t}, \tag{F.1}$$

where $X^R_{li,t}$ is the bilateral sales of robots from l to i in year t and $e_{lij,t} \equiv \ln \tau^{R,D,E}_{li,t} + \ln \tau^{R,D,E}_{ij,t} + \ln \tau^{R,D,E}_{jl,t} - \ln \tau^{R,D,E}_{lj,t} - \ln \tau^{R,D,E}_{ji,t} - \ln \tau^{R,D,E}_{il,t}$. The benefit of this approach is that it does not require symmetry for non-tariff trade cost $\tau^{R,D}_{li}$, but only requires the orthogonality for the asymmetric component of the trade cost. My method also extends Caliendo and Parro (2015) in using the time-series variation as well as trilateral country-level variation to complement the relatively small number of observations in robot trade data.

When implementing regression of equation (F.1), I further consider controlling for two separate sets of fixed effects. The first set is the unilateral fixed effect indicating if a country is included in the trilateral pair of countries, and the second set is the bilateral fixed effect for the twin of countries is included in the trilateral pair. These fixed effects are relevant in my setting as a few number of countries export robots, and controlling for these exporters' unobserved characteristics is critical.

Table F.1 shows the result of regression of equation (F.1). The first two columns show the result for the HS code 847950 (Industrial robots for multiple uses, the definition of robots used in Humlum 2019), and the last two columns HS code 8479 (Machines and mechanical appliances having individual functions, not specified or included elsewhere in this chapter). The first and

Table F.1: Coefficient of equation (F.1)

	(1) HS 847950	(2) HS 847950	(3) HS 8479	(4) HS 8479
Tariff	-0.272***	-0.236***	-0.146***	-0.157***
	(0.0718)	(0.0807)	(0.0127)	(0.0131)
Constant	-0.917***	-0.893***	-1.170***	-1.170***
	(0.0415)	(0.0381)	(0.00905)	(0.00853)
FEs	h-i-j-t	ht-it-jt	h-i-j-t	ht-it-jt
N	4610	4521	88520	88441
r2	0.494	0.662	0.602	0.658

Note: The author's calculation based on BACI data from 1996 to 2018 and equation (F.1). The first two columns show the result for the HS code 847950 (Industrial robots for multiple uses), while the last two columns HS code 8479 (Machines and mechanical appliances having individual functions, not specified or included elsewhere in this chapter). The first and third columns control the unilateral fixed effect, while the second and fourth the bilateral fixed effect. See the text for the detail.

third columns control for the unilateral fixed effect, and the second and fourth the bilateral fixed effect. The implied trade elasticity of robots ε^R is fairly tightly estimated and ranges between 1.13-1.34. Given these estimation results, I use $\varepsilon^R = 1.2$ in the estimation and counterfactuals.

To assess the estimation result, note that Caliendo and Parro (2015) show in Table 1 that the regression coefficient of equation (F.1) is 1.52, with the standard error of 1.81, for "Machinery n.e.c", which roughly corresponds to HS 84. Therefore, my estimate for industrial robots falls in the one-standard-deviation range of their estimate for a broader category of goods.

Note that the average trade elasticity across sectors is estimated significantly higher than these values, such as 4 in Simonovska and Waugh (2014). The low trade elasticity for robots ε^R is intuitive given robots are highly heterogeneous and hardly substitutable. This low elasticity implies small gains from robot taxes, with the robot tax incidence almost on the US (robot buyer) side rather than the robot-selling country.

F.2 Robot Tax and Workers' Welfare

To examine how the robot tax affects workers in different occupations, I define the equivalent variation (EV) implicitly as follows:

$$\sum_{t=t_0}^{\infty} \left(\frac{1}{1+\iota}\right)^t \ln\left(\left[C'_{i,o,t}\right]\right) = \sum_{t=t_0}^{\infty} \left(\frac{1}{1+\iota}\right)^t \ln\left(C_{i,o,t}\left[1+EV_{i,o}\right]\right). \tag{F.2}$$

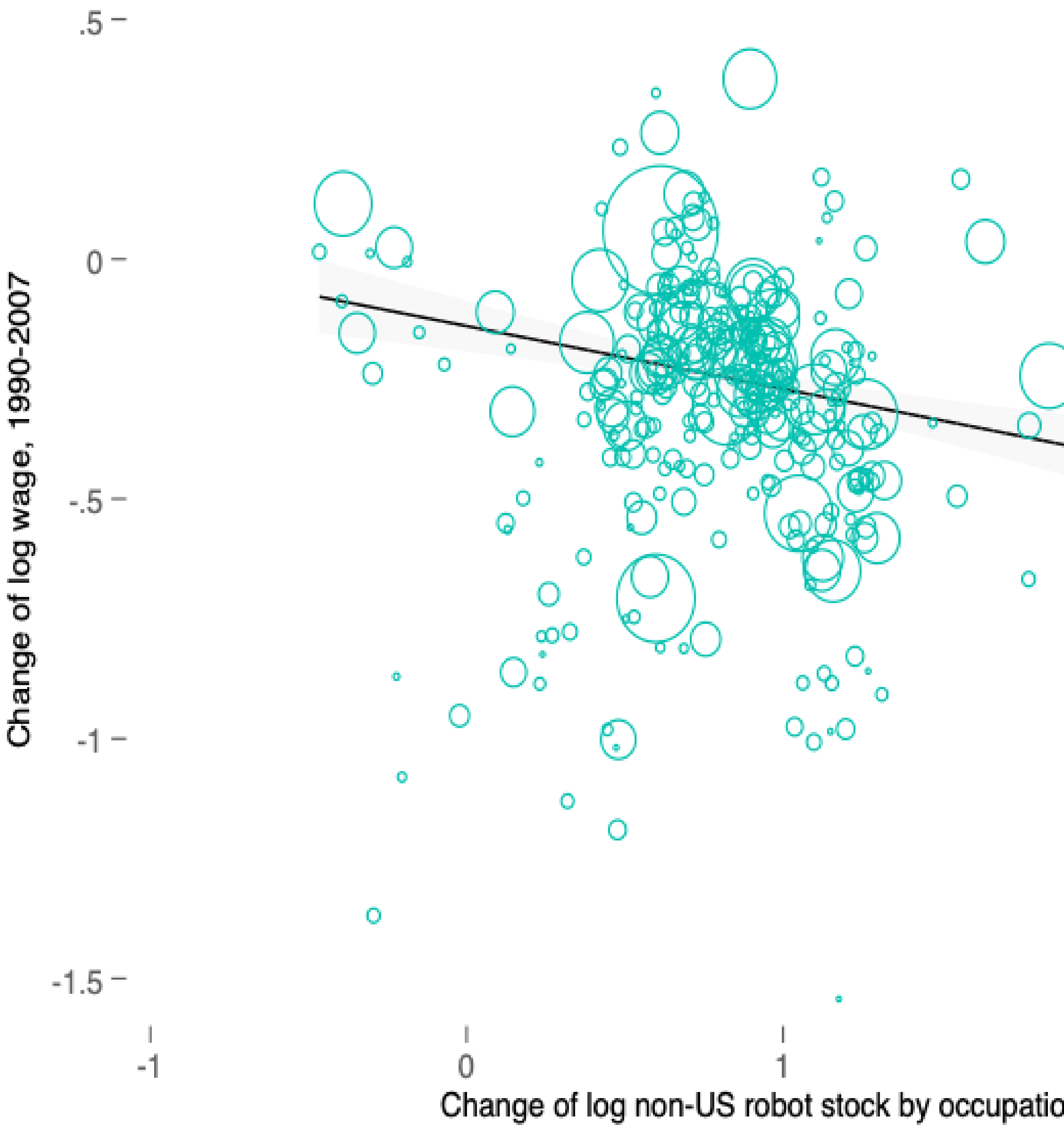
Namely, the EV is the fraction of the occupation-specific subsidy that would make the present discounted value (PDV) of the utility in the robotized and taxed equal to the PDV of the utility if the occupation-specific subsidy were exogenously given in the initial steady state. On the left-hand side, I hit the robotization shock backed out in Section 4.4. As in Section 5.3, I consider the US unilateral (not inducing a reaction in other countries), unexpected, and permanent tax on robot purchases. By this definition, the worker in occupation o prefers the robotized and taxed world if and only if the EV is positive for o.

Figure F.1a shows this occupation-specific EV as a function of the tax rate. The far-left side of the figure is the case of zero robot tax, thus a case of only the robotization shock. Consistent with the occupational wage effects (cf. Figure F.5), workers in production and transportation occupations lose significantly due to robotization. In contrast other workers are roughly indifferent between the robotized world and the non-robotized initial steady state or slightly prefer the former world. Going right through the figure, the production and transportation workers' EV improves as the robot tax reduces competing robots. The EV of production workers turns positive when the tax rate is around 6%, and that of transportation workers is positive when the rate is about 7%. However, these tax rates are too high and would make EVs in other occupations negative. In fact, in production and transportation occupations, robots do not accumulate and adversely affect labor demand in the other occupations.

To study if the reallocation policy by robot tax may work, I also compute the equivalent variation in terms of monetary value aggregated by occupation groups (total EV) and compare it with the robot tax revenue, both as a function of robot tax. Figure F.1b shows the result. One can confirm that the marginal robot tax revenue is far from enough to compensate for workers' loss that concentrates on production and transportation workers, at the initial steady state with zero robot tax rate. The robot tax revenue is negligible at this margin compared with the workers' loss due to robotization. It is true that as the robot tax rate increases, the total EV rises: When the rate is as large as 6-7%, the sum of the total EV and the robot tax revenue is positive. However, one should be cautious that my solution to the model is to the first order. Thus the approximation error may play an important role when the robot tax rate is significantly higher than the one in the initial steady state, zero. Extending my solution to the higher-order or even finding the exact solution is left for future research.

Figure D.11: Correlation between US Occupational Wage and Non-US Robot Measures (Raw)

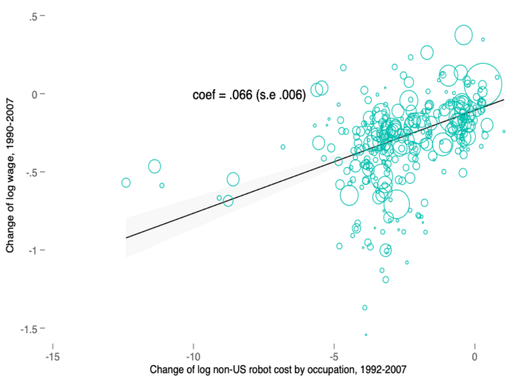
(a) Wage and Robot Values



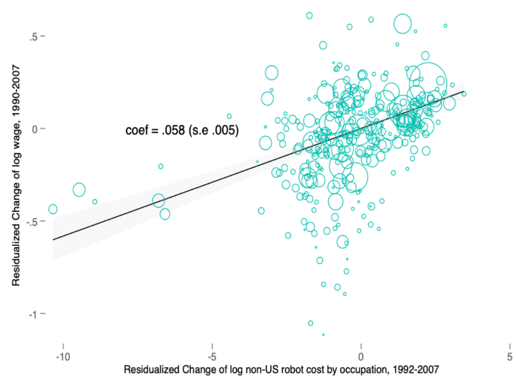
(c) Employment and Robot Values

Figure D.12: Correlation between US Occupational Wage and Non-US Quality-adjusted Robot Prices

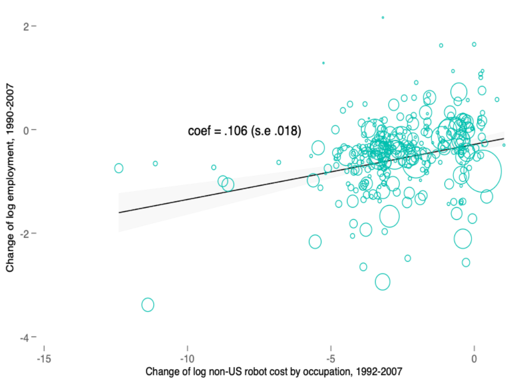
(a) Wage and Robot Price, Raw



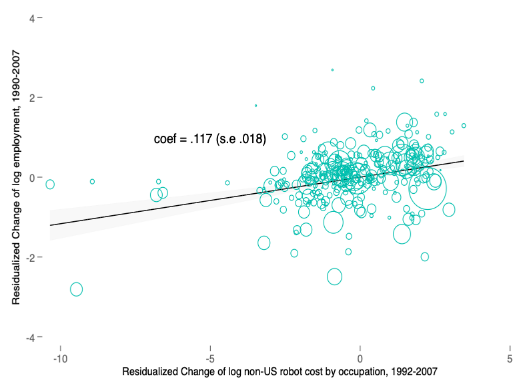
(b) Wage and Robot Price, Controlled



(c) Employment and Robot Price, Raw



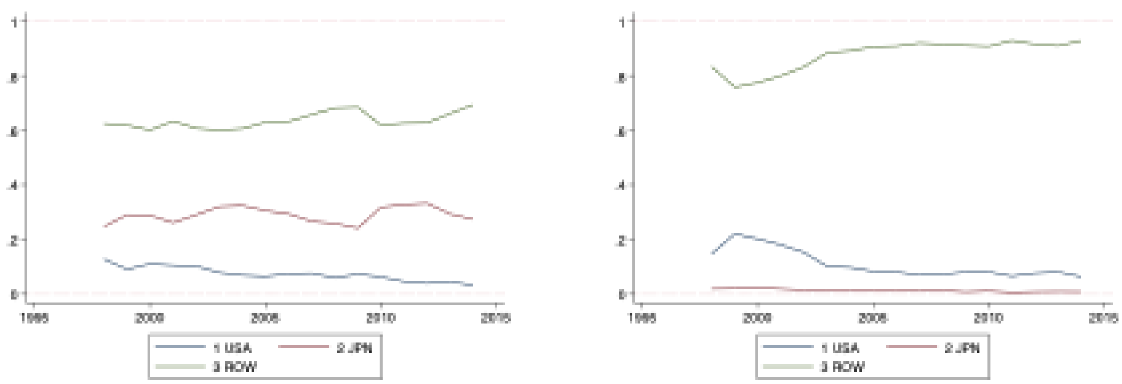
(d) Employment and Robot Price, Control



Note: The author's calculation based on JARA, O*NET, and the US Census/ACS. The figures show the scatterplot, weighted fit line, and the 95 percent confidence interval of the changes in log robot measures and changes in log labor market outcomes. Each bubble represents a 4-digit occupation. The bubble size reflects the employment in the baseline year (1990, which is the closest Census year to the initial year that I observe the robot adoption, 1992). In all panels, the number of observation is 324. In panel (a), y-axis and x-axis are log occupational wages and the change of log non-US quality-adjusted robot price measured in raw values. In panel (b), y-axis and x-axis are log occupational wages and the change of log non-US quality-adjusted robot price, both partialled out by control variables (the occupational female share, college share, age distribution, and foreign born share). In panel (c), y-axis and x-axis are log occupational wages and the change of log non-US quality-adjusted robot price in raw values. In panel (d), y-axis and x-axis are log occupational wages and the change of log non-US quality-adjusted robot prices, both partialled out by control variables.

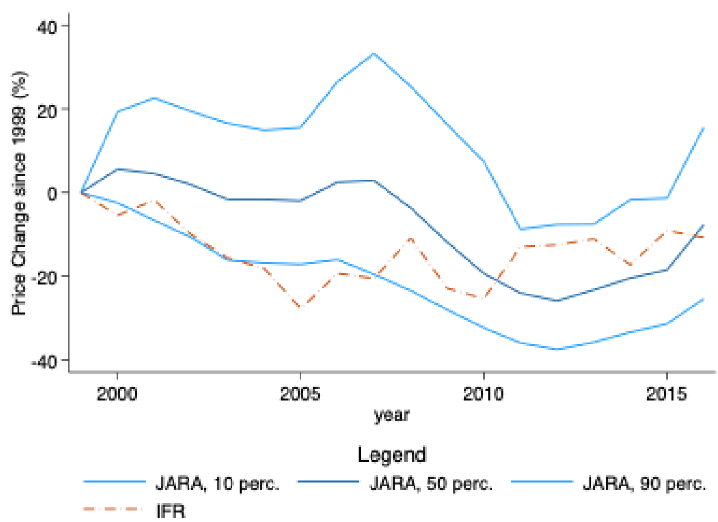
Figure D.13: Robot Trade Share Trends

(a) Exports (b) Imports



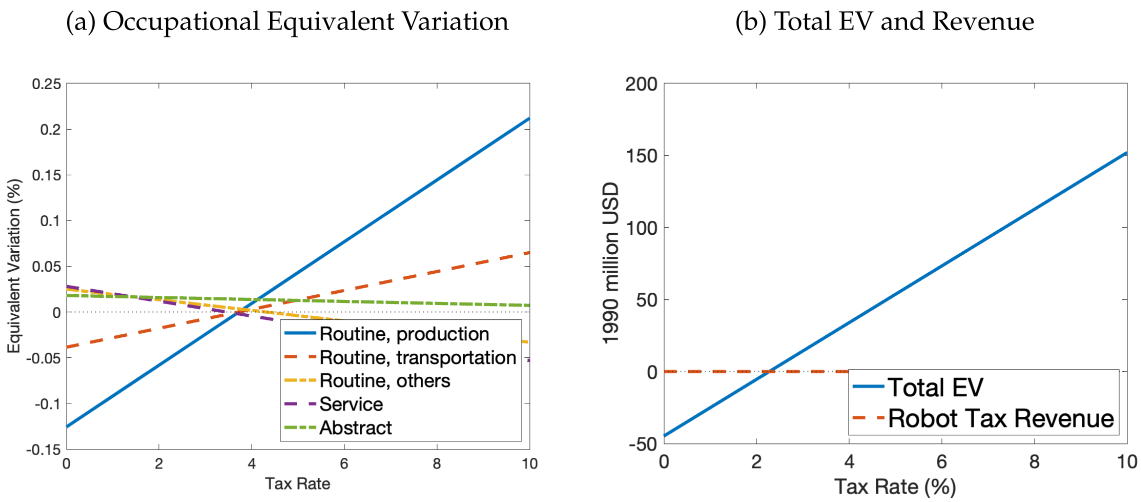
Note: The author's calculation of world trade shares based on the BACI data. Industrial robots are measured by HS code 847950 (Industrial robots for multiple uses).

Figure D.14: Comparison of US Price Indices between JARA and IFR



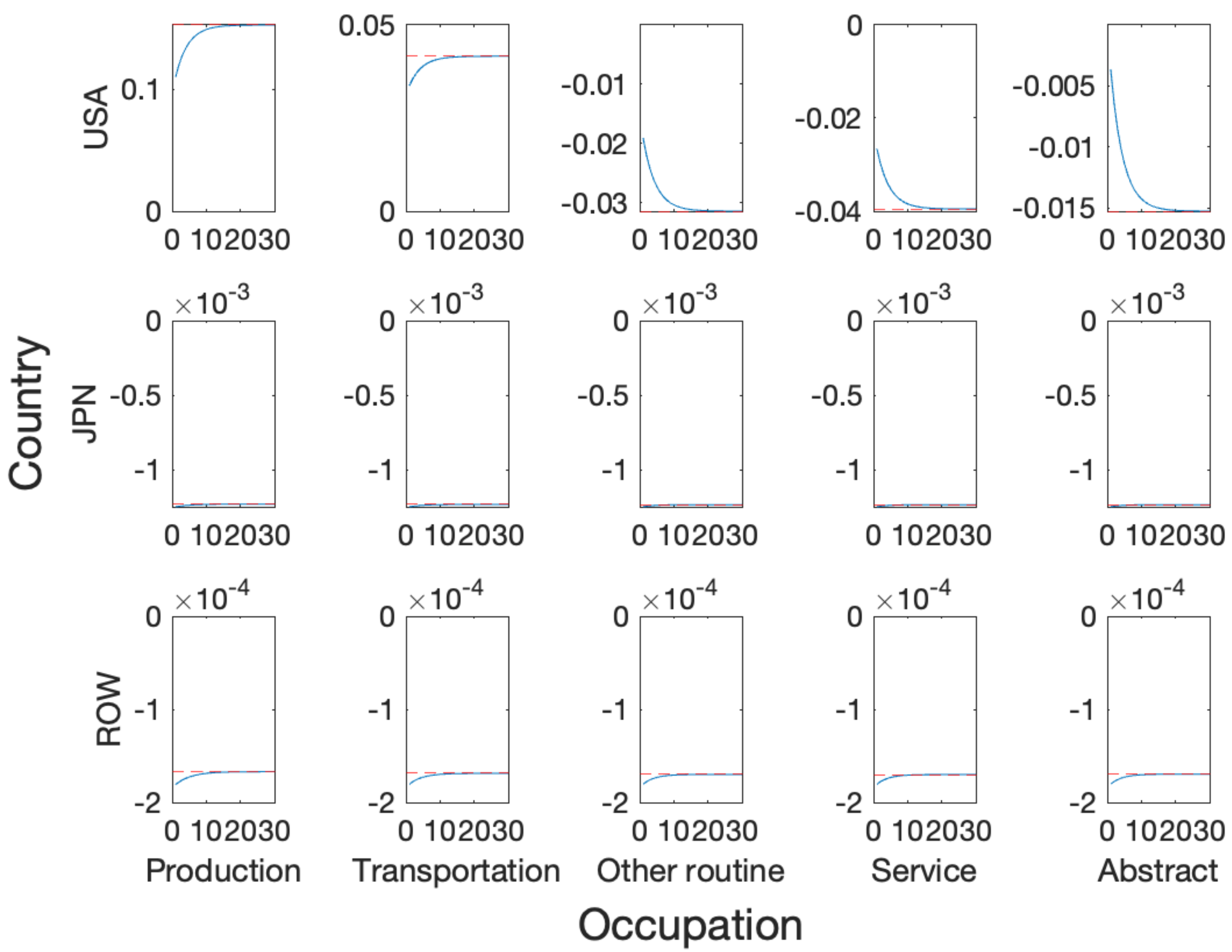
Note: The author's calculation of US robot price measures in JARA and IFR. The JARA measures are disaggregated by 4-digit occupations, and the figure shows the 10th, 50th (median), and 90th percentiles each year. All measures are normalized at 1999, the year in which the first price measure is available in the IFR data.

Figure F.1: Robot Tax and Workers' Welfare



Note: The left panel shows the US workers' equivalent variation defined in equation (F.2) as a function of the US robot tax rate. Labels "Rout., prod.", "Rout., transp.", and "Rout., others" mean routine, production; routine, transportation; and routine, others occupations, respectively. The right panel shows monetary values of equivalent variations aggregated across workers and robot tax revenue as a function of the robot tax rate, measured in 1990 million USD.

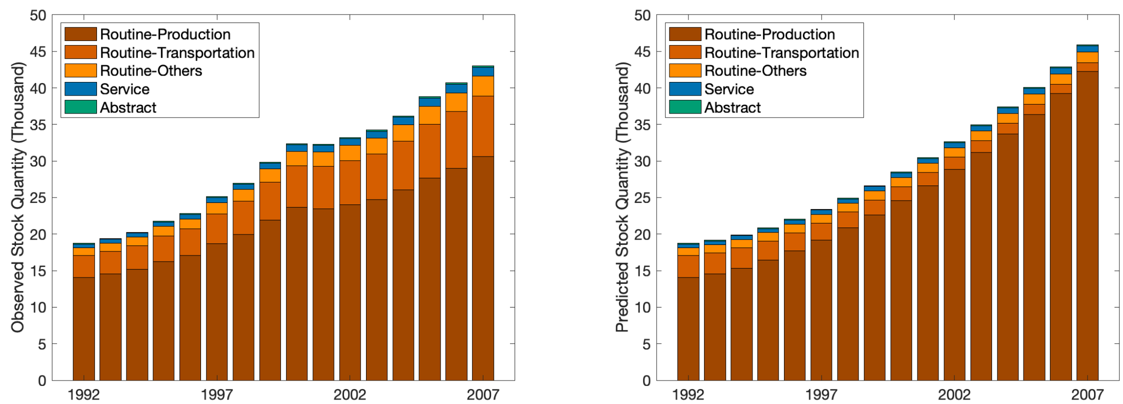
Figure F.2: US General Robot Tax and Global Occupational Value Evolution



Note: Transition dynamics of workers' occupation-specific values given the US's unexpected, unilateral, and permanent 6% general robot tax for all occupational robots at the initial steady state (period 0) are shown. Blue solid lines are the transitional dynamics, and red dashed lines are the steady-state values.

Figure F.3: Trends of Robot Stocks

(a) Data (b) Model



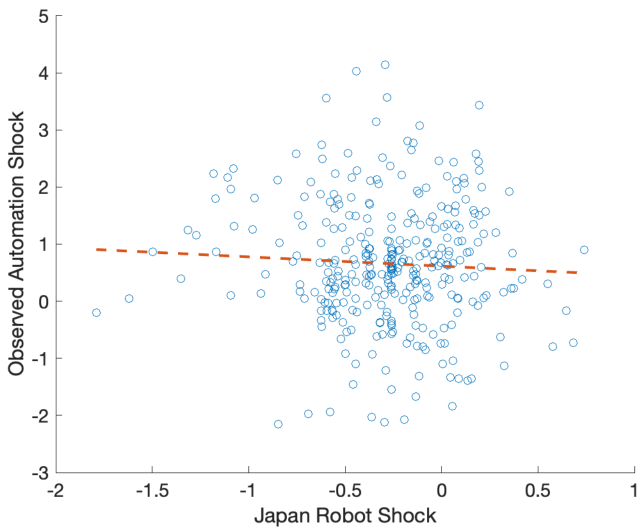
Note: Figures show the trend of the observed (left) and predicted (right) stock of robots for each occupation group measured by quantities. The predicted robot stocks are computed by shocks backed out from the estimated model and applying the first-order solution to the general equilibrium described in equation (17).

To study how the occupational effects unfold over time and if the US policy affects third countries, I study occupational value evolution given the US general robot tax. Figure F.2 shows the impact of the US's unilateral, unexpected, and permanent 6% general robot tax on the world's occupational values in the short run and the long run. In the first row, panels show the US occupational values and corroborate the finding in Figure 3 that production and transportation workers gain from the robot tax but not other workers. As can be seen from the figure, it takes roughly 10 years until the worker values reach steady states. In other countries than the US, the US robot tax effect is negative but quantitatively limited.

F.3 Actual and Predicted Robot Accumulation Dynamics

Figure F.3 shows the trends of robot stock in the US in the data and the model. Although I do not match the overall robot capital stocks, the estimated model tracks the observed pattern well between 1992 and the late 2010s, consistent with the fact that I target the changes between 1992 and 2007. Although there is over-prediction of the growth of production robots and under-prediction of the growth of transportation (material moving) robots between occupation groups, the overall predicted stock of robots matches well with the actual data.

Figure F.4: Correlation between Japan Robot Shock ψ_o^J and Automation Shock $\widehat{a_o^{obs}}$



Note: The author's calculation based on JARA, O*NET, and US Census/ACS. The x-axis shows the Japan robot shock, and is taken from the regression of equation (2). The y-axis shows the implied automation shock, and is backed out from equation (19) with the estimated parameters in Table 2. Each circle is 4-digit occupation and dashed line is the fitted line.

F.4 The Japan Robot Shock and The Implied Automation Shock

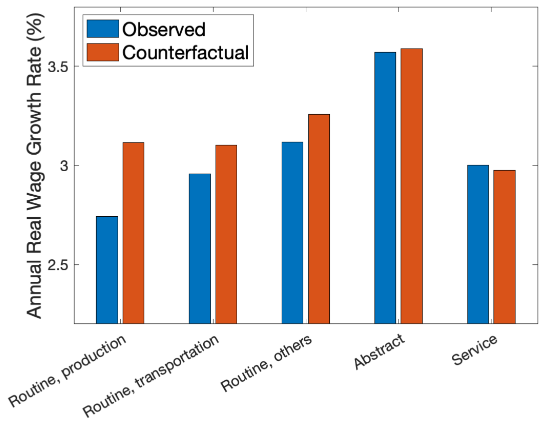
In turn, Figure F.4 shows a further detailed scatter plot between the two shocks, delivering a mild negative relationship. This negative correlation is consistent with the example of robotic innovations in Appendix D.2.

F.5 Automation and Wages for each Occupation

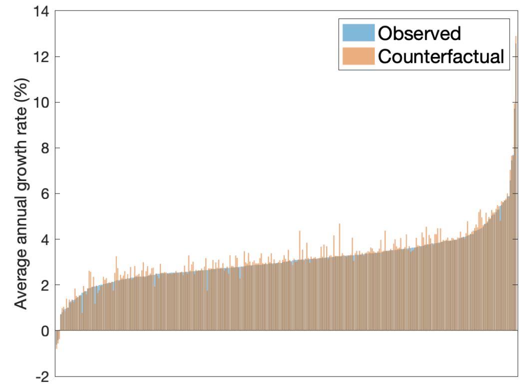
Figure F.5 shows the observed and counterfactual growth rate of real wages for each occupation, where the counterfactual change means the simulated change when there is no the automation shock. Figure F.5a shows the results aggregated at the 5 occupations groups defined in Section 4.1. I compute the counterfactual growth rate from the observed rate of the wage change, subtracted by the change predicted by the first-order steady-state solution $\overline{\boldsymbol{E}}$ and the implied automation shock $\widehat{\boldsymbol{a}^{\text{imp}}}$. The result is based on the observed high growth rates of robots in routine production and transportation (material moving) occupations, and these occupations' high EoS estimates between robots and workers. In particular, at the 5-occupation aggregate level, most of the observed differences in the real wage growth rates in the three routine occupation groups are closed absent

Figure F.5: The Steady-state Effect of Robots on Wages

(a) Occupation Groups



(b) All Occupations



the automation shock. Applying the similar exercise for all occupations in my sample, Figure F.5b shows a more granular result, where occupations are sorted by the observed changes of wages from 1990-2007.

F.6 Trade and the Effect of the Robot Tax

Figure F.6 shows the dynamic effect of the robot tax on the US real income. If the robot trade is not allowed, the robot tax does not increases the real income in any period since the terms-of-trade effect does not show up, but only the long-run capital decumulation effect does. On the other hand, once I allow the robot trade as observed in the data, the robot tax may increase the real income because it decreases the price of imported robots. The effect is concentrated in the short-run before the capital decumulation process matures. In the long run, the negative decumulation effect dominates the positive terms-of-trade effect.

F.7 Trade Liberalization of Robots

What is the effect of liberalizing the trade of robots? To approach this question and gain insights about dynamics gains from trade, I consider unexpected and permanent 20% reduction in the bilateral trade costs, following Ravikumar, Santacreu, and Sposi (2019). Figure F.7 shows the result of such a simulation for a 20-years time horizon. All country groups in the model gain

Figure F.6: Effects of the Robot Tax on the US Real Income

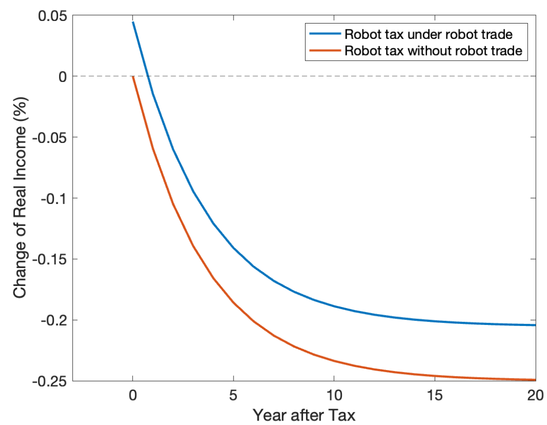
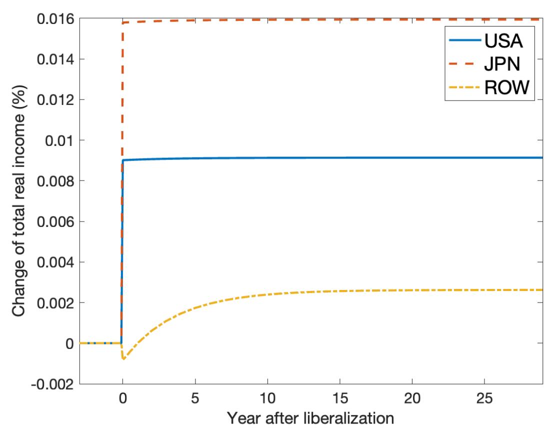


Figure F.7: The Effect of Robot Trade Cost Reduction



from the trade liberalization. The US gain materialize almost immediately after the trade cost change. A possible explanation is the combination of the following two observation. First, it takes time to accumulate robots after the trade liberalization, which makes the gains from trade liberalization sluggish. Second, by exporting robots to ROW, the US increases the revenue of robot sales immediately after the trade cost drop, improving the short-run real income gain. The real income gain is the largest for Japan, a large net robot exporter. It is noteworthy that ROW loses from the reduction in the robot trade cost, possibly due to the terms-of-trade deterioration in the short-run.

G Expressions of the GE Solution

I discuss the derivation log-linearization in equations (13), (15), and (17), so that I can bring the theory with computation. Throughout the section, relational operator $\circ$ is Hadamard product, $\oslash$ indicates Hadamard division, and $\otimes$ means Kronecker product. In this section, I use θ_o to denote the elasticity of substitution between robots and workers for each occupation

It is useful to show that the CES production structure implies the share-weighted log-change expression for both prices and quantities. Namely, I have a formula for the change in destination price index $\widehat{P^G_{j,t}} = \sum_i \widetilde{x}^G_{ij,t_0}\widehat{p^G_{ij,t}}$ and one for the change in destination expenditure $\widehat{P^G_{j,t}} + \widehat{Q^G_{j,t}} = \sum_i \widetilde{x}^G_{ij,t_0}\left(\widehat{p^G_{ij,t}} + \widehat{Q^G_{ij,t}}\right)$. These imply that

$$\widehat{Q^G_{j,t}} = \sum_i \widetilde{x}^G_{ij,t_0}\widehat{Q^G_{ij,t}},$$

or the changes of quantity aggregate $\widehat{Q^G_{j,t}}$ are also share-weighted average of changes of origin quantity $\widehat{Q^G_{ij,t}}$.

By log-linearizing equation (E.30) for any i ,

$$\begin{aligned}
&-\alpha_M\widehat{p^G_{i,t}} + \alpha_M\sum_l \widetilde{x}^G_{li,t_0}\widehat{p^G_{l,t}} + (1-\alpha_M)\sum_j \widetilde{y}^G_{ij,t_0}\widehat{Q^G_{ij,t}} - \alpha_L\sum_o \widetilde{x}^O_{i,o,t_0} l^O_{i,o,t_0}\widehat{L_{i,o,t}} \\
&= \frac{\alpha_L}{\theta_o - 1}\sum_o \frac{\widetilde{x}^O_{i,o,t_0}}{1-a_{o,t_0}}\left(-a_{o,t_0} l^O_{i,o,t_0} + (1-a_{o,t_0})\left(1 - l^O_{i,o,t_0}\right)\right)\widehat{a_{o,t}} + \alpha_L\sum_o \widetilde{x}^O_{i,o,t_0}\frac{1}{\beta-1}\widehat{b_{i,o,t}} \\
&\quad + \widehat{A^G_{i,t}} + (1-\alpha_L-\alpha_M)\widehat{K_{i,t}} - \alpha_M\sum_l \widetilde{x}^G_{li,t_0}\widehat{\tau^G_{li,t}} - (1-\alpha_M)\sum_j \widetilde{y}^G_{ij,t_0}\widehat{\tau^G_{ij,t}} + \alpha_L\sum_o \widetilde{x}^O_{i,o,t_0}\left(1 - l^O_{i,o,t_0}\right)\widehat{K^R_{i,o,t}},
\end{aligned}$$

To write a matrix notation, write

$$\overline{\boldsymbol{M}^{yG}} \equiv \begin{bmatrix} \left[\widetilde{y}^G_{11,t_0},\ldots,\widetilde{y}^G_{1N,t_0}\right] & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \left[\widetilde{y}^G_{N1,t_0},\ldots,\widetilde{y}^G_{NN,t_0}\right] \end{bmatrix}$$

a $N \times N^2$ matrix,

$$\overline{\boldsymbol{M}^{xOl}} \equiv \begin{bmatrix} \left(\widetilde{\boldsymbol{x}}_{1,\cdot,t_0} \circ \widetilde{\boldsymbol{l}}_{1,\cdot,t_0}\right)^\top & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \left(\widetilde{\boldsymbol{x}}_{N,\cdot,t_0} \circ \widetilde{\boldsymbol{l}}_{N,\cdot,t_0}\right)^\top \end{bmatrix}$$

a $N \times NO$ matrix where

$$\widetilde{\boldsymbol{x}}_{1,\cdot,t_0} \equiv \left(\widetilde{x}^O_{1,o,t_0}\right)_o \text{ and } \widetilde{\boldsymbol{l}}_{1,\cdot,t_0} \equiv \left(l^O_{1,o,t_0}\right)_o \tag{G.1}$$

are $O \times 1$ vectors, $\overline{\boldsymbol{M}^{al}}$ as a matrix with its element

$$M^{al}_{i,o} = \frac{-a_{o,t_0} l^O_{i,o,t_0} + (1 - a_{o,t_0}) \left(1 - l^O_{i,o,t_0}\right)}{1 - a_{o,t_0}},$$

and a $N \times O$ matrix,

$$\overline{\boldsymbol{M}^{xO}} \equiv \begin{bmatrix} \left[\widetilde{x}^O_{1,1,t_0}, \ldots, \widetilde{x}^O_{1,O,t_0}\right] & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \left[\widetilde{x}^O_{N,1,t_0}, \ldots, \widetilde{x}^O_{N,O,t_0}\right] \end{bmatrix},$$

a $N \times NO$ matrix,

$$\overline{\boldsymbol{M}^{xG}} \equiv \begin{bmatrix} \operatorname{diag}\left(\widetilde{x}^G_{1\cdot,t_0}\right) & \ldots & \operatorname{diag}\left(\widetilde{x}^G_{N\cdot,t_0}\right) \end{bmatrix},$$

a $N \times N^2$ matrix, and

$$\overline{\boldsymbol{M}^{xOl,2}} \equiv \begin{bmatrix} \left(\widetilde{\boldsymbol{x}}_{1,\cdot,t_0} \circ \left(\mathbf{1}_O - \widetilde{\boldsymbol{l}}_{1,\cdot,t_0}\right)\right)^\top & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \left(\widetilde{\boldsymbol{x}}_{N,\cdot,t_0} \circ \left(\mathbf{1}_O - \widetilde{\boldsymbol{l}}_{N,\cdot,t_0}\right)\right)^\top \end{bmatrix},$$

a $N \times NO$ matrix where $\widetilde{\boldsymbol{x}}_{1,\cdot,t_0}$ and $\widetilde{\boldsymbol{l}}_{1,\cdot,t_0}$ are defined in equation (G.1). Then I have

$$\begin{aligned} &- \alpha_M \left(\overline{\boldsymbol{I}} - \left(\widetilde{\boldsymbol{x}}^G_{t_0}\right)^\top\right) \widehat{\boldsymbol{p}}^G_t + (1 - \alpha_M) \overline{\boldsymbol{M}^{yG}} \widehat{\boldsymbol{Q}}^G_t - \alpha_L \overline{\boldsymbol{M}^{xOl}} \widehat{\boldsymbol{L}}_t \\ &= \frac{\alpha_L}{\theta_o - 1} \left(\widetilde{\boldsymbol{x}}^O_{t_0} \circ \overline{\boldsymbol{M}^{al}}\right) \widehat{\boldsymbol{a}}_t + \frac{\alpha_L}{\beta - 1} \overline{\boldsymbol{M}^{xO}} \widehat{\boldsymbol{b}}_t + \widehat{\boldsymbol{A}}^G_t + (1 - \alpha_L - \alpha_M) \widehat{\boldsymbol{K}}_t \\ &\quad - \left[\alpha_M \overline{\boldsymbol{M}^{xG}} + (1 - \alpha_M) \overline{\boldsymbol{M}^{yG}}\right] \widehat{\boldsymbol{\tau}}^G_t + \alpha_L \overline{\boldsymbol{M}^{xOl,2}} \widehat{\boldsymbol{K}}^R_t, \end{aligned}$$

By log-linearizing equation (E.31) for any i and o,

$$\widehat{p_{i,o,t}^R} = \widehat{P_{i,t}^G} - \widehat{A_{i,o,t}^R}$$

$$-\sum_l \widetilde{x}_{li,t_0}^G \widehat{p_{l,t}^G} + \widehat{p_{i,o,t}^R} = -\widehat{A_{i,o,t}^R} + \sum_l \widetilde{x}_{li,t_0}^G \widehat{\tau_{li,t}^G}.$$

In matrix notation, write

$$\overline{\boldsymbol{M}^{xG,2}} \equiv \begin{bmatrix} \mathbf{1}_O \left[\widetilde{x}_{11,t_0}^G, \ldots, \widetilde{x}_{N1,t_0}^G\right] \\ \vdots \\ \mathbf{1}_O \left[\widetilde{x}_{1N,t_0}^G, \ldots, \widetilde{x}_{NN,t_0}^G\right] \end{bmatrix}$$

a $NO \times N$ matrix, and

$$\overline{\boldsymbol{M}^{xG,3}} \equiv \begin{bmatrix} \widetilde{x}_{11,t_0}^G & \cdots & \widetilde{x}_{N1,t_0}^G & & & \mathbf{0} \\ & & & \ddots & & \\ & \mathbf{0} & & \widetilde{x}_{1N,t_0}^G & \cdots & \widetilde{x}_{NN,t_0}^G \end{bmatrix} \otimes \mathbf{1}_O$$

a $NO \times N^2$ matrix. Then I have

$$-\overline{\boldsymbol{M}^{xG,2}}\widehat{\boldsymbol{p}_t^G} + \widehat{\boldsymbol{p}_t^R} = -\widehat{\boldsymbol{A}_t^R} + \overline{\boldsymbol{M}^{xG,3}}\widehat{\boldsymbol{\tau}_t^G}.$$

By log-linearizing equations (B.6), (B.7), and (B.8) for any i, o, and o', I have

$$\widehat{\mu_{i,oo',t}} = \phi\left(-d\chi_{i,oo',t} + \frac{1}{1+\iota}\widehat{V_{i,o',t+1}}\right) - \sum_{o''}\mu_{i,oo'',t_0}\left(-d\chi_{i,oo'',t} + \frac{1}{1+\iota}\widehat{V_{i,o'',t+1}}\right), \tag{G.2}$$

$$\widehat{V_{i,o,t+1}} = \widehat{w_{i,o,t+1}} + dT_{i,o,t+1} - \widehat{P_{i,t+1}} + \sum_{o'}\mu_{i,oo',t_0}\left(-d\chi_{i,oo',t+1} + \frac{1}{1+\iota}\widehat{V_{i,o',t+2}}\right), \tag{G.3}$$

and

$$\widehat{L_{i,o,t+1}} = \sum_{o'}\frac{L_{i,o',t_0}}{L_{i,o,t_0}}\mu_{i,o'o,t_0}\left(\widehat{\mu_{i,o'o,t}} + \widehat{L_{i,o',t}}\right). \tag{G.4}$$

In matrix notation, by equation (G.2),

$$\widehat{\boldsymbol{\mu}_t^{\text{vec}}} = -\phi\left(\overline{\boldsymbol{I}_{NO^2}} - \overline{\boldsymbol{M}^{\mu}}\right)d\boldsymbol{\chi}_t^{\text{vec}} + \frac{\phi}{1+\iota}\left(\overline{\boldsymbol{I}_{NO^2}} - \overline{\boldsymbol{M}^{\mu}}\right)\left(\overline{\boldsymbol{I}_{NO}} \otimes \mathbf{1}_O\right)\widehat{\boldsymbol{V}_{t+1}}.$$

where

$$\overline{\boldsymbol{M}^{\mu}} \equiv \overline{\boldsymbol{M}^{\mu,3}} \otimes \mathbf{1}_O,$$

$$\overline{\boldsymbol{M}^{\mu,3}} \equiv \begin{bmatrix} \left(\boldsymbol{\mu}_{i,1\cdot,t_0}\right)^\top & & & & & & \\ & \ddots & & & & \mathbf{0} & \\ & & \left(\boldsymbol{\mu}_{i,O\cdot,t_0}\right)^\top & & & & \\ & & & \ddots & & & \\ & & & & \left(\boldsymbol{\mu}_{N,1\cdot,t_0}\right)^\top & & \\ & \mathbf{0} & & & & \ddots & \\ & & & & & & \left(\boldsymbol{\mu}_{i,O\cdot 1,t_0}\right)^\top \end{bmatrix},$$

$$d\boldsymbol{\chi}_t^{\text{vec}} \equiv \begin{bmatrix} d\boldsymbol{\chi}_{1,1\cdot,t} & \ldots & d\boldsymbol{\chi}_{1,O\cdot,t} & \ldots & d\boldsymbol{\chi}_{N,1\cdot,t} & \ldots & d\boldsymbol{\chi}_{N,O\cdot,t} \end{bmatrix}^\top,$$

and

$$\boldsymbol{\mu}_{i,o\cdot,t_0} \equiv \left(\mu_{i,oo',t_0}\right)_{o'} \text{ and } d\boldsymbol{\chi}_{1,o\cdot,t} \equiv \left(d\chi_{1,oo',t}\right)_{o'} \tag{G.5}$$

are $O \times 1$ vectors. By equation (G.3),

$$\frac{1}{1+\iota}\overline{\boldsymbol{M}^{\mu,2}}\boldsymbol{V}_{t+2}^{\vee} = \overline{\boldsymbol{M}^{x\mathrm{G},2}}\boldsymbol{p}_{t+1}^{\check{\mathrm{G}}} - \boldsymbol{w}_{t+1}^{\vee} + \boldsymbol{V}_{t+1}^{\vee}.$$

where

$$\overline{\boldsymbol{M}^{\mu,2}} \equiv \begin{bmatrix} \left(\boldsymbol{\mu}_{1,1\cdot,t_0}\right)^\top & & \\ \vdots & & \mathbf{0} \\ \left(\boldsymbol{\mu}_{1,O\cdot,t_0}\right)^\top & & \\ & \ddots & \\ & & \left(\boldsymbol{\mu}_{N,1\cdot,t_0}\right)^\top \\ \mathbf{0} & & \\ & & \left(\boldsymbol{\mu}_{N,O\cdot,t_0}\right)^\top \end{bmatrix},$$

and $\boldsymbol{\mu}_{i,o\cdot,t_0}$ is given by equation (G.5) for any i and o. By equation (G.3),

$$\boldsymbol{L}_{t+1}^{\vee} = \overline{\boldsymbol{M}^{\mu L,2}}\boldsymbol{\mu}_t^{\check{\text{vec}}} + \overline{\boldsymbol{M}^{\mu L}}\check{\boldsymbol{L}}_t$$

where $\overline{\boldsymbol{M}^{\mu L}}$ being the $NO \times NO$ matrix

$$\overline{\boldsymbol{M}^{\mu L}} = \overline{\boldsymbol{M}^{\mu,2}} \circ \left(\begin{bmatrix} \left(\boldsymbol{L}_{1,\cdot,t_0}\right)^\top & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \left(\boldsymbol{L}_{N,\cdot,t_0}\right)^\top \end{bmatrix} \otimes \mathbf{1}_O \right) \oslash \left(\begin{bmatrix} \boldsymbol{L}_{1,\cdot,t_0} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \boldsymbol{L}_{N,\cdot,t_0} \end{bmatrix} \otimes \left(\mathbf{1}_O\right)^\top \right)$$

and $\overline{\boldsymbol{M}^{\mu L,2}}$ being the $NO \times NO^2$ matrix

$$\overline{\boldsymbol{M}^{\mu L,2}} = \overline{\boldsymbol{M}^{\mu,4}} \circ \left(\begin{bmatrix} \left(\boldsymbol{L}_{1,\cdot,t_0}\right)^\top & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \left(\boldsymbol{L}_{N,\cdot,t_0}\right)^\top \end{bmatrix} \otimes \overline{\boldsymbol{I}_O} \right) \oslash$$
$$\begin{pmatrix} \left(\mathbf{1}_O\right)^\top \otimes \operatorname{diag}\left(L_{1,o,t_0}\right) & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \left(\mathbf{1}_O\right)^\top \otimes \operatorname{diag}\left(L_{N,o,t_0}\right) \end{pmatrix},$$

where

$$\overline{\boldsymbol{M}^{\mu,4}} \equiv \begin{bmatrix} \operatorname{diag}\left(\boldsymbol{\mu}_{1,1\cdot,t_0}\right) & \ldots & \operatorname{diag}\left(\boldsymbol{\mu}_{i,O\cdot,t_0}\right) & & \mathbf{0} & \\ & & & \ddots & & \\ & \mathbf{0} & & \operatorname{diag}\left(\boldsymbol{\mu}_{N,1\cdot,t_0}\right) & \ldots & \operatorname{diag}\left(\boldsymbol{\mu}_{N,O\cdot,t_0}\right) \end{bmatrix},$$

and $\boldsymbol{\mu}_{i,o\cdot,t_0}$ is given by equation (G.5) for any i and o.

By log-linearizing equation (E.29) for each i and j,

$$\widehat{Q_{ij,t}^G} = -\varepsilon^G \widehat{p_{ij,t}^G} - \left(1 - \varepsilon^G\right) \widehat{P_{j,t}^G} + \left[s_{j,t_0}^G \sum_k \widehat{p_{jk,t}^G Q_{jk,t}^G} + s_{j,t_0}^V \sum_{i,o} \widehat{p_{ij,o,t}^R Q_{ij,o,t}^R} + s_{j,t_0}^R \sum_{o,k} \widehat{p_{jk,o,t}^R Q_{jk,o,t}^R} \right]$$

where
$$s_{j,t_0}^G \equiv \frac{\sum_k p_{jk,t_0}^G Q_{jk,t_0}^G}{\sum_k p_{jk,t_0}^G Q_{jk,t_0}^G - \sum_{i,o} p_{ij,o,t_0}^R Q_{ij,o,t_0}^R + \sum_{o,k} p_{jk,o,t_0}^R Q_{jk,o,t_0}^R}$$

is the baseline share of non-robot good production in income,

$$s_{j,t_0}^R \equiv \frac{\sum_{o,k} p_{jk,o,t_0}^R Q_{jk,o,t_0}^R}{\sum_k p_{jk,t_0}^G Q_{jk,t_0}^G - \sum_{i,o} p_{ij,o,t_0}^R Q_{ij,o,t_0}^R + \sum_{o,k} p_{jk,o,t_0}^R Q_{jk,o,t_0}^R},$$

is the baseline share of robot production, and

$$s^V_{j,t_0} \equiv -\frac{\sum_{i,o} p^R_{ij,o,t_0} Q^R_{ij,o,t_0}}{\sum_k p^G_{jk,t_0} Q^G_{jk,t_0} - \sum_{i,o} p^R_{ij,o,t_0} Q^R_{ij,o,t_0} + \sum_{o,k} p^R_{jk,o,t_0} Q^R_{jk,o,t_0}},$$

is the (negative) baseline absorption share of robots. Thus

$$\begin{aligned}
&\left[\varepsilon^G \widehat{p^G_{i,t}} + \left(1-\varepsilon^G\right)\sum_l \widetilde{x}^G_{lj,t_0}\widehat{p^G_{l,t}} - s^G_{j,t_0}\widehat{p^G_{j,t}}\right] - \left[s^V_{j,t_0}\sum_{l,o}\widetilde{x}^R_{lj,o,t_0}\widetilde{x}^R_{j,o,t_0}\widehat{p^R_{l,o,t}} + s^R_{t_0}\sum_o \widetilde{y}^R_{j,o,t_0}\widehat{p^R_{j,o,t}}\right] \\
&+\left(\widehat{Q^G_{ij,t}} - s^G_{j,t_0}\sum_k \widetilde{y}^G_{jk,t_0}\widehat{Q^G_{jk,t}}\right) - \left(s^V_{j,t_0}\sum_{l,o}\widetilde{x}^R_{lj,o,t_0}\widetilde{x}^R_{j,o,t_0}\widehat{Q^R_{lj,o,t}} + s^R_{j,t_0}\sum_{k,o}\widetilde{y}^R_{jk,o,t_0}\widetilde{y}^R_{j,o,t_0}\widehat{Q^R_{jk,o,t}}\right) \\
&= -\left[\varepsilon^G\widehat{\tau^G_{ij,t}} + \left(1-\varepsilon^G\right)\sum_l \widetilde{x}^G_{lj,t_0}\widehat{\tau^G_{lj,t}} - s^G_{j,t_0}\sum_k \widetilde{y}^G_{jk,t_0}\widehat{\tau^G_{jk,t}}\right] + \left[s^V_{j,t_0}\sum_{l,o}\widetilde{x}^R_{lj,t_0}\widehat{\tau^R_{lj,t}} + s^R_{j,t_0}\sum_{k,o}\widetilde{y}^R_{jk,t_0}\widehat{\tau^R_{jk,t}}\right]
\end{aligned}$$

where

$$\widetilde{x}^R_{ij,o,t_0} \equiv \frac{p^R_{ij,o,t_0}Q^R_{ij,o,t_0}}{P^R_{j,o,t_0}Q^R_{j,o,t_0}}, \widetilde{x}^R_{j,o,t_0} \equiv \frac{P^R_{j,o,t_0}Q^R_{j,o,t_0}}{P^R_{j,t_0}Q^R_{j,t_0}}, \widetilde{x}^R_{ij,t_0} \equiv \frac{\sum_o p^R_{ij,o,t_0}Q^R_{ij,o,t_0}}{P^R_{j,t_0}Q^R_{j,t_0}},$$

$$\widetilde{y}^R_{ij,o,t_0} \equiv \frac{p^R_{ij,o,t_0}Q^R_{ij,o,t_0}}{\sum_k p^R_{ik,o,t_0}Q^R_{ik,o,t_0}}, \widetilde{y}^R_{i,o,t_0} \equiv \frac{\sum_k p^R_{ik,o,t_0}Q^R_{ik,o,t_0}}{\sum_{k,o'} p^R_{ik,o',t_0}Q^R_{ik,o',t_0}}, \widetilde{y}^R_{ij,t_0} \equiv \frac{\sum_o p^R_{ij,o,t_0}Q^R_{ij,o,t_0}}{\sum_{k,o} p^R_{ik,o,t_0}Q^R_{ik,o,t_0}}.$$

In matrix notation, define

$$\overline{\boldsymbol{M}^{xR}} \equiv \mathbf{1}_N \otimes \begin{bmatrix} \widetilde{\boldsymbol{x}}^R_{t_0} \circ \widetilde{\boldsymbol{x}}^R_{\cdot 1,\cdot,t_0} & \dots & \widetilde{\boldsymbol{x}}^R_{t_0} \circ \widetilde{\boldsymbol{x}}^R_{\cdot N,\cdot,t_0} \end{bmatrix},$$

a $N^2 \times NO$ matrix,

$$\overline{\boldsymbol{M}^{yR}} \equiv \mathbf{1}_N \otimes \begin{bmatrix} \widetilde{y}^R_{1,1} & \dots & \widetilde{y}^R_{1,O} & & & \mathbf{0} \\ & & & \ddots & & \\ & \mathbf{0} & & \widetilde{y}^R_{N,1} & \dots & \widetilde{y}^R_{N,O} \end{bmatrix},$$

a $N^2 \times NO$ matrix,

$$\overline{\boldsymbol{M}^{yG,2}} \equiv \mathbf{1}_N \otimes \overline{\boldsymbol{M}^{yG}}.$$

a $N^2 \times N^2$ matrix,

$$\overline{\boldsymbol{M}^{xR,2}} \equiv \mathbf{1}_N \otimes \begin{bmatrix} \left[\widetilde{x}^R_{1,o,t_0}\widetilde{x}^R_{11,o,t_0}\right]_o & & \mathbf{0} & \left[\widetilde{x}^R_{1,o,t_0}\widetilde{x}^R_{N1,o,t_0}\right]_o & & \mathbf{0} \\ & \ddots & & \dots & \ddots & \\ \mathbf{0} & & \left[\widetilde{x}^R_{N,o,t_0}\widetilde{x}^R_{1N,o,t_0}\right]_o & \mathbf{0} & & \left[\widetilde{x}^R_{N,o,t_0}\widetilde{x}^R_{NN,o,t_0}\right]_o \end{bmatrix}$$

a $N^2 \times N^2 O$ matrix ,

$$\overline{\boldsymbol{M}^{yR,2}} \equiv \mathbf{1}_N \otimes \begin{bmatrix} \left[\widetilde{y}^R_{1,o,t_0}\widetilde{y}^R_{11,o,t_0}\right]_o & \dots & \left[\widetilde{y}^R_{N,o,t_0}\widetilde{y}^R_{1N,o,t_0}\right]_o & & \mathbf{0} & \\ & & & \ddots & & \\ & \mathbf{0} & & \left[\widetilde{y}^R_{1,o,t_0}\widetilde{y}^R_{N1,o,t_0}\right]_o & \dots & \left[\widetilde{y}^R_{N,o,t_0}\widetilde{y}^R_{NN,o,t_0}\right]_o \end{bmatrix}$$

a $N^2 \times N^2 O$ matrix,

$$\overline{\boldsymbol{M}^{xG,4}} \equiv \mathbf{1}_N \otimes \overline{\boldsymbol{M}^{xG}}$$

a $N^2 \times N^2$ matrix,

$$\overline{\boldsymbol{M}^{xR,3}} \equiv \mathbf{1}_N \otimes \begin{bmatrix} \operatorname{diag}\left(\widetilde{x}^R_{1\cdot,t_0}\right) & \dots & \operatorname{diag}\left(\widetilde{x}^R_{N\cdot,t_0}\right) \end{bmatrix}$$

a $N^2 \times N^2$ matrix,

$$\overline{\boldsymbol{M}^{yR,3}} \equiv \mathbf{1}_N \otimes \begin{bmatrix} \left[\widetilde{y}^R_{11,t_0},\dots,\widetilde{y}^R_{1N,t_0}\right] & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \left[\widetilde{y}^R_{N1,t_0},\dots,\widetilde{y}^R_{NN,t_0}\right] \end{bmatrix}$$

a $N^2 \times N^2$ matrix, and

$$\overline{\boldsymbol{M}^{xO,2}} \equiv \mathbf{1}_N \otimes \overline{\boldsymbol{M}^{xO}},$$

a $N^2 \times NO$ matrix. Then I have

$$\left(\varepsilon^G \left[\overline{\boldsymbol{I}_N} \otimes \boldsymbol{1}_N\right] + \left(1-\varepsilon^G\right)\left[\boldsymbol{1}_N \otimes \left(\widetilde{\boldsymbol{x}}^G_{t_0}\right)^\top\right] - \operatorname{diag}\left(\boldsymbol{1}_N \otimes \boldsymbol{s}^G_{t_0}\right)\left[\boldsymbol{1}_N \otimes \overline{\boldsymbol{I}_N}\right]\right)\widehat{\boldsymbol{p}^G_t}$$
$$-\left(\operatorname{diag}\left(\boldsymbol{1}_N \otimes \boldsymbol{s}^V_{t_0}\right)\overline{\boldsymbol{M}^{xR}} + \operatorname{diag}\left(\boldsymbol{1}_N \otimes \boldsymbol{s}^R_{t_0}\right)\overline{\boldsymbol{M}^{yR}}\right)\widehat{\boldsymbol{p}^R_t}$$
$$+\left(\overline{\boldsymbol{I}_{N^2}} - \operatorname{diag}\left(\boldsymbol{1}_N \otimes \boldsymbol{s}^G_{t_0}\right)\overline{\boldsymbol{M}^{yG,2}}\right)\widehat{\boldsymbol{Q}^G_t} - \left[\operatorname{diag}\left(\boldsymbol{1}_N \otimes \boldsymbol{s}^V_{t_0}\right)\overline{\boldsymbol{M}^{xR,2}} + \operatorname{diag}\left(\boldsymbol{1}_N \otimes \boldsymbol{s}^R_{t_0}\right)\overline{\boldsymbol{M}^{yR,2}}\right]\widehat{\boldsymbol{Q}^R_t}$$
$$= -\left(\varepsilon^G + \left(1-\varepsilon^G\right)\overline{\boldsymbol{M}^{xG,4}} - \operatorname{diag}\left(\boldsymbol{1}_N \otimes \boldsymbol{s}^G_{t_0}\right)\overline{\boldsymbol{M}^{yG,2}}\right)\widehat{\boldsymbol{\tau}^G_t}$$
$$+\left(\operatorname{diag}\left(\boldsymbol{1}_N \otimes \boldsymbol{s}^V_{t_0}\right)\overline{\boldsymbol{M}^{xR,3}} + \operatorname{diag}\left(\boldsymbol{1}_N \otimes \boldsymbol{s}^R_{t_0}\right)\overline{\boldsymbol{M}^{yR,3}}\right)\widehat{\boldsymbol{\tau}^R_t}$$

By log-linearizing equation (E.21) for each i, j, and o,,

$$\left(1-\alpha^R\right)\frac{1+u_{ij,t_0}}{1+u_{ij,t_0}+2\gamma\delta}\sum_l \widetilde{x}^G_{lj,t_0}\widehat{p^G_{l,t}} + \frac{1+u_{ij,t_0}}{1+u_{ij,t_0}+2\gamma\delta}\widehat{p^R_{i,o,t}}$$
$$+\left[\frac{2\gamma\delta}{1+u_{ij,t_0}+2\gamma\delta} - \left(1-\alpha^R\right)\frac{1+u_{ij,t_0}}{1+u_{ij,t_0}+2\gamma\delta}\right]\sum_l \widetilde{x}^R_{lj,o,t_0}\widehat{p^R_{l,o,t}}$$
$$+\frac{1+u_{ij,t_0}}{1+u_{ij,t_0}+2\gamma\delta}\frac{1}{\varepsilon^R}\widehat{Q^R_{ij,o,t}} + \left[-\frac{1}{\varepsilon^R}\frac{1+u_{ij,t_0}}{1+u_{ij,t_0}+2\gamma\delta} + \frac{2\gamma\delta}{1+u_{ij,t_0}+2\gamma\delta}\right]\sum_l \widetilde{x}^R_{lj,o,t_0}\widehat{Q^R_{lj,o,t}}$$
$$= -\frac{1+u_{ij,t_0}}{1+u_{ij,t_0}+2\gamma\delta}du_{ij,t} - \left(1-\alpha^R\right)\frac{1+u_{ij,t_0}}{1+u_{ij,t_0}+2\gamma\delta}\sum_l \widetilde{x}^G_{lj,t_0}\widehat{\tau^G_{lj,t}} - \frac{1+u_{ij,t_0}}{1+u_{ij,t_0}+2\gamma\delta}\widehat{\tau^R_{ij,t}}$$
$$-\left[\frac{2\gamma\delta}{1+u_{ij,t_0}+2\gamma\delta} - \left(1-\alpha^R\right)\frac{1+u_{ij,t_0}}{1+u_{ij,t_0}+2\gamma\delta}\right]\sum_l \widetilde{x}^R_{lj,o,t_0}\widehat{\tau^R_{lj,t}} + \widehat{\lambda^R_{j,o,t}} + \frac{2\gamma\delta}{1+u_{ij,t_0}+2\gamma\delta}\widehat{K^R_{j,o,t}}.$$

In matrix notation, write a preliminary $N \times N$ matrix $\overline{\widetilde{\boldsymbol{u}_{t_0}}}$ as such that the (i,j)-element is

$$\frac{1+u_{ij,t_0}}{1+u_{ij,t_0}+2\gamma\delta}.$$

Then $\boldsymbol{1}_N\left(\boldsymbol{1}_N\right)^\top - \overline{\widetilde{\boldsymbol{u}_{t_0}}}$ is a matrix that is filled with $2\gamma\delta/\left(1+u_{ij,t_0}+2\gamma\delta\right)$ for its (i,j) element and

$$\overline{\boldsymbol{M}^u} \equiv \operatorname{diag}\left(\left[\widetilde{u_{1\cdot,t_0}}, \ldots, \widetilde{u_{N\cdot,t_0}}\right]^\top\right).$$

Using these, write

$$\overline{\boldsymbol{M}^{xG,5}} \equiv \left(\overline{\boldsymbol{M}^u} \otimes \overline{\boldsymbol{I}_O}\right)\left(\boldsymbol{1}_N \otimes \left(\widetilde{\boldsymbol{x}}^G_{t_0}\right)^\top \otimes \boldsymbol{1}_O\right)$$

a $N^2O \times N$ matrix,

$$\overline{\boldsymbol{M}^{u,2}} \equiv \begin{bmatrix} \widetilde{\boldsymbol{u}_{1\cdot,t_0}} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \widetilde{\boldsymbol{u}_{N\cdot,t_0}} \end{bmatrix} \otimes \overline{\boldsymbol{I}_O},$$

a $N^2O \times NO$ matrix where $\widetilde{\boldsymbol{u}_{i\cdot,t_0}} \equiv \left(\widetilde{u_{i\cdot,t_0}}\right)_j$ is a $N \times 1$ vector,

$$\overline{\boldsymbol{M}^{xR,4}} \equiv \left\{ \left[\left(\overline{\boldsymbol{I}_{N^2}} - \overline{\boldsymbol{M}^u} \right) - \left(1 - \alpha^R \right) \overline{\boldsymbol{M}^u} \right] \otimes \overline{\boldsymbol{I}_O} \right\} \times$$
$$\left(\mathbf{1}_N \otimes \begin{bmatrix} \operatorname{diag}\left(\left\{ \widetilde{x}^R_{11,o,t_0} \right\}_o \right) & \dots & \operatorname{diag}\left(\left\{ \widetilde{x}^R_{N1,o,t_0} \right\}_o \right) \\ \vdots & & \vdots \\ \operatorname{diag}\left(\left\{ \widetilde{x}^R_{1N,o,t_0} \right\}_o \right) & \dots & \operatorname{diag}\left(\left\{ \widetilde{x}^R_{NN,o,t_0} \right\}_o \right) \end{bmatrix} \right)$$

a $N^2O \times NO$ matrix,

$$\overline{\boldsymbol{M}^{xR,5}} \equiv \left\{ \left[-\frac{1}{\varepsilon^R} \overline{\boldsymbol{M}^u} + \left(\overline{\boldsymbol{I}_{N^2}} - \overline{\boldsymbol{M}^u} \right) \right] \otimes \overline{\boldsymbol{I}_O} \right\} \times$$
$$\left\{ \mathbf{1}_N \otimes \left[\operatorname{diag}\left(\begin{bmatrix} \widetilde{x}^R_{11,1,t_0} \\ \vdots \\ \widetilde{x}^R_{11,O,t_0} \\ \vdots \\ \widetilde{x}^R_{1N,O,t_0} \end{bmatrix} \right) \quad \dots \quad \operatorname{diag}\left(\begin{bmatrix} \widetilde{x}^R_{N1,1,t_0} \\ \vdots \\ \widetilde{x}^R_{N1,O,t_0} \\ \vdots \\ \widetilde{x}^R_{NN,O,t_0} \end{bmatrix} \right) \right] \right\}$$

a $N^2O \times N^2O$ matrix,

$$\overline{\boldsymbol{M}^{xG,6}} \equiv \left(\overline{\boldsymbol{M}^u} \otimes \overline{\boldsymbol{I}_O} \right) \left\{ \mathbf{1}_N \otimes \left[\operatorname{diag}\left(\begin{bmatrix} \widetilde{x}^G_{11,t_0} \\ \vdots \\ \widetilde{x}^G_{1N,t_0} \end{bmatrix} \right) \quad \dots \quad \operatorname{diag}\left(\begin{bmatrix} \widetilde{x}^G_{N1,t_0} \\ \vdots \\ \widetilde{x}^G_{NN,t_0} \end{bmatrix} \right) \right] \otimes \mathbf{1}_O \right\}$$

a $N^2O \times N^2$ matrix,

$$\overline{\boldsymbol{M}^{xR,6}} \equiv \left\{ \left[\left(\overline{\boldsymbol{I}_{N^2}} - \overline{\boldsymbol{M}^u} \right) - \left(1 - \alpha^R \right) \overline{\boldsymbol{M}^u} \right] \otimes \overline{\boldsymbol{I}_O} \right\} \times \left\{ \mathbf{1}_N \otimes \begin{bmatrix} \left[\widetilde{x}^R_{11,o,t_0} \right]_o & \mathbf{0} & \mathbf{0} & \cdots & \left[\widetilde{x}^R_{N1,o,t_0} \right]_o & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} & & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \left[\widetilde{x}^R_{1N,o,t_0} \right]_o & & \mathbf{0} & \mathbf{0} & \left[\widetilde{x}^R_{N3,o,t_0} \right]_o \end{bmatrix} \right\}$$

a $N^2O \times N^2$ matrix, and

$$\overline{\boldsymbol{M}^{u,3}} \equiv \begin{bmatrix} 1 - \widehat{u_{11,t_0}} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & 1 - \widehat{u_{1N,t_0}} \\ & \vdots & \\ 1 - \widehat{u_{N1,t_0}} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & 1 - \widehat{u_{NN,t_0}} \end{bmatrix} \otimes \overline{\boldsymbol{I}_O}$$

a $N^2O \times NO$ matrix. Finally, I have

$$\begin{aligned} &\left(1 - \alpha^R \right) \overline{\boldsymbol{M}^{xG,5}} \widehat{\boldsymbol{p}^G_t} + \left[\overline{\boldsymbol{M}^{u,2}} + \overline{\boldsymbol{M}^{xR,4}} \right] \widehat{\boldsymbol{p}^R_t} + \left\{ \frac{1}{\varepsilon^R} \left(\overline{\boldsymbol{M}^u} \otimes \overline{\boldsymbol{I}_O} \right) + \overline{\boldsymbol{M}^{xR,5}} \right\} \widehat{\boldsymbol{Q}^R_t} \\ &= - \left(\overline{\boldsymbol{M}^u} \otimes \mathbf{1}_O \right) d\boldsymbol{u}_t - \left(1 - \alpha^R \right) \overline{\boldsymbol{M}^{xG,6}} \widehat{\boldsymbol{\tau}^G_t} - \left[\left(\overline{\boldsymbol{M}^u} \otimes \mathbf{1}_O \right) + \overline{\boldsymbol{M}^{xR,6}} \right] \widehat{\boldsymbol{\tau}^R_t} + \left(\mathbf{1}_N \otimes \overline{\boldsymbol{I}_{NO}} \right) \widehat{\boldsymbol{\lambda}^R_t} + \overline{\boldsymbol{M}^{u,3}} \widehat{\boldsymbol{K}^R_t}. \end{aligned}$$

By log-linearizing equation (E.16) for each i and o,

$$\begin{aligned} &\widehat{p^G_{i,t}} + \sum_j \widetilde{y}^G_{ij,t_0} \widehat{Q^G_{ij,t}} - \widehat{w_{i,o,t}} + \left[-\frac{1}{\theta_o} + \left(-\frac{1}{\beta} + \frac{1}{\theta_o} \right) l^O_{i,o,t_0} \right] \widehat{L_{i,o,t}} + \left(-1 + \frac{1}{\beta} \right) \sum_{o'} \widetilde{x}^O_{i,o',t_0} l^O_{i,o',t_0} \widehat{L_{i,o',t}} \\ &= -\frac{1}{\beta} \widehat{b_{i,o,t}} + \frac{1}{\theta_o} \frac{a_{o,t_0}}{1 - a_{o,t_0}} \widehat{a_{o,t}} + \left(-\frac{1}{\beta} + \frac{1}{\theta_o} \right) \frac{1}{\theta_o - 1} \left[- \left(1 - l^O_{i,o,t_0} \right) + l^O_{i,o,t_0} \frac{a_{o,t_0}}{1 - a_{o,t_0}} \right] \widehat{a_{o,t}} \\ &\quad + \left(-1 + \frac{1}{\beta} \right) \frac{1}{\theta_o - 1} \sum_{o'} \widetilde{x}^O_{i,o',t_0} \left[- \left(1 - l^O_{i,o',t_0} \right) + l^O_{i,o',t_0} \frac{a_{o',t_0}}{1 - a_{o',t_0}} \right] \widehat{a_{o',t}} \\ &\quad - \sum_j y^G_{ij,t_0} \widehat{\tau^G_{ij,t}} - \left(-\frac{1}{\beta} + \frac{1}{\theta_o} \right) \left(1 - l^O_{i,o,t_0} \right) \widehat{K^R_{i,o,t}} - \left(-1 + \frac{1}{\beta} \right) \sum_{o'} \widetilde{x}^O_{i,o',t_0} \left(1 - l^O_{i,o',t_0} \right) \widehat{K^R_{i,o',t}}, \end{aligned}$$

In matrix notation, write

$$\overline{\boldsymbol{M}^{yG,3}} \equiv \overline{\boldsymbol{M}^{yG}} \otimes \mathbf{1}_O$$

a $NO \times N^2$ matrix,

$$\overline{\boldsymbol{M}^{xOl,3}} \equiv \overline{\boldsymbol{M}^{xOl}} \otimes \mathbf{1}_O$$

a $NO \times NO$ matrix,

$$\overline{\boldsymbol{M}^{a}} \equiv \mathbf{1}_N \otimes \operatorname{diag}\left(\frac{a_{o,t_0}}{1-a_{o,t_0}}\right)$$

a $NO \times O$ matrix,

$$\overline{\boldsymbol{M}^{al,2}} \equiv \begin{bmatrix} \operatorname{diag}\left(-\left(1-l^O_{1,o,t_0}\right)+l^O_{1,o,t_0}\frac{a_{o,t_0}}{1-a_{o,t_0}}\right) \\ \vdots \\ \operatorname{diag}\left(-\left(1-l^O_{N,o,t_0}\right)+l^O_{N,o,t_0}\frac{a_{o,t_0}}{1-a_{o,t_0}}\right) \end{bmatrix}$$

a $NO \times O$ matrix,

$$\overline{\boldsymbol{M}^{al,3}} \equiv \left(\widetilde{\boldsymbol{x}}^O_{t_0} \circ \overline{\boldsymbol{M}^{al}}\right) \otimes \mathbf{1}_O$$

a $NO \times O$ matrix,

$$\overline{\boldsymbol{M}^{xOl,4}} \equiv \overline{\boldsymbol{M}^{xOl,2}} \otimes \mathbf{1}_O,$$

a $NO \times NO$ matrix. I have

$$\begin{aligned}
&\left(\boldsymbol{I}_N \otimes \mathbf{1}_O\right)\widehat{\boldsymbol{p}^G_t} - \widehat{\boldsymbol{w}_t} + \overline{\boldsymbol{M}^{yG,3}}\widehat{\boldsymbol{Q}^G_t} + \left(-\frac{1}{\theta_o}\overline{\boldsymbol{I}_{NO}} + \left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\operatorname{diag}\left(\boldsymbol{l}^O_{t_0}\right) + \left(-1+\frac{1}{\beta}\right)\overline{\boldsymbol{M}^{xOl,3}}\right)\widehat{\boldsymbol{L}}_t \\
&= -\frac{1}{\beta}\widehat{\boldsymbol{b}}_t + \left[\frac{1}{\theta_o}\overline{\boldsymbol{M}^a} + \left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\frac{1}{\theta-1}\overline{\boldsymbol{M}^{al,2}} + \left(-1+\frac{1}{\beta}\right)\frac{1}{\theta-1}\overline{\boldsymbol{M}^{al,3}}\right]\widehat{\boldsymbol{a}}_t - \overline{\boldsymbol{M}^{yG,3}}\widehat{\boldsymbol{\tau}^G_t} \\
&\quad + \left[-\left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\operatorname{diag}\left(1-l^O_{i,o,t_0}\right) - \left(-1+\frac{1}{\beta}\right)\overline{\boldsymbol{M}^{xOl,4}}\right]\widehat{\boldsymbol{K}^R_t}.
\end{aligned}$$

Hence the log-linearized temporary equilibrium system is

$$\overline{\boldsymbol{D}^x}\widehat{\boldsymbol{x}} = \overline{\boldsymbol{D}^A}\widehat{\boldsymbol{A}}$$

where matrices $\overline{\boldsymbol{D}^x}$ and $\overline{\boldsymbol{D}^A}$ are defined as

$$\overline{\boldsymbol{D}^x} \equiv \begin{bmatrix} \overline{\boldsymbol{D}^x_{11}} & \mathbf{0} & \mathbf{0} & \overline{\boldsymbol{D}^x_{14}} & \mathbf{0} & \overline{\boldsymbol{D}^x_{16}} \\ -\overline{\boldsymbol{M}^{xG,2}} & \overline{\boldsymbol{I}_{NO}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \phi\overline{\boldsymbol{M}^{xG,2}} & \mathbf{0} & -\phi\overline{\boldsymbol{I}_{NO}} & \mathbf{0} & \mathbf{0} & \overline{\boldsymbol{M}^{l}} \\ \overline{\boldsymbol{D}^x_{41}} & \overline{\boldsymbol{D}^x_{42}} & \mathbf{0} & \overline{\boldsymbol{D}^x_{44}} & \overline{\boldsymbol{D}^x_{45}} & \mathbf{0} \\ \overline{\boldsymbol{D}^x_{51}} & \overline{\boldsymbol{D}^x_{52}} & \mathbf{0} & \mathbf{0} & \overline{\boldsymbol{D}^x_{55}} & \mathbf{0} \\ \overline{\boldsymbol{D}^x_{61}} & \mathbf{0} & -\overline{\boldsymbol{I}_{NO}} & \overline{\boldsymbol{M}^{yG,3}} & \mathbf{0} & \overline{\boldsymbol{D}^x_{66}} \end{bmatrix},$$

where

$$\overline{\boldsymbol{D}^x_{11}} \equiv -\alpha_M \left(\overline{\boldsymbol{I}_N} - \left(\widetilde{\boldsymbol{x}}^G_{t_0} \right)^\top \right), \ \overline{\boldsymbol{D}^x_{14}} \equiv (1-\alpha_M)\, \overline{\boldsymbol{M}^{yG}},\ \overline{\boldsymbol{D}^x_{16}} \equiv -\alpha_L \overline{\boldsymbol{M}^{xOl}},$$

$$\overline{\boldsymbol{D}^x_{41}} \equiv \varepsilon^G \left[\overline{\boldsymbol{I}_N} \otimes \mathbf{1}_N \right] + \left(1 - \varepsilon^G \right) \left[\mathbf{1}_N \otimes \left(\widetilde{\boldsymbol{x}}^G_{t_0} \right)^\top \right] - \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^G_{t_0} \right) \left[\mathbf{1}_N \otimes \overline{\boldsymbol{I}_N} \right],$$

$$\overline{\boldsymbol{D}^x_{42}} \equiv \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^V_{t_0} \right) \overline{\boldsymbol{M}^{xR}} + \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^R_{t_0} \right) \overline{\boldsymbol{M}^{yR}},$$

$$\overline{\boldsymbol{D}^x_{44}} \equiv \overline{\boldsymbol{I}_{N^2}} - \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^G_{t_0} \right) \overline{\boldsymbol{M}^{yG,2}},$$

$$\overline{\boldsymbol{D}^x_{45}} \equiv -\operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^V_{t_0} \right) \overline{\boldsymbol{M}^{xR,2}} - \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}^R_{t_0} \right) \overline{\boldsymbol{M}^{yR,2}},$$

$$\overline{\boldsymbol{D}^x_{51}} \equiv \left(1 - \alpha^R \right) \overline{\boldsymbol{M}^{xG,5}},\ \overline{\boldsymbol{D}^x_{52}} \equiv \overline{\boldsymbol{M}^{u,2}} + \overline{\boldsymbol{M}^{xR,4}},\ \overline{\boldsymbol{D}^x_{55}} \equiv \frac{1}{\varepsilon^R} \left(\overline{\boldsymbol{M}^{u}} \otimes \overline{\boldsymbol{I}_O} \right) + \overline{\boldsymbol{M}^{xR,5}},$$

$$\overline{\boldsymbol{D}^x_{61}} \equiv \boldsymbol{I}_N \otimes \mathbf{1}_N,\ \overline{\boldsymbol{D}^x_{66}} \equiv -\frac{1}{\theta_o} + \left(-\frac{1}{\beta} + \frac{1}{\theta_o} \right) \operatorname{diag}\left(\boldsymbol{l}^O_{t_0} \right) + \left(-1 + \frac{1}{\beta} \right) \overline{\boldsymbol{M}^{xOl,3}},$$

and

$$\overline{\boldsymbol{D}^A} \equiv \begin{bmatrix} \mathbf{0} & \overline{\boldsymbol{D}^A_{12}} & \overline{\boldsymbol{D}^A_{13}} & \overline{\boldsymbol{I}_N} & \mathbf{0} & \overline{\boldsymbol{D}^A_{16}} & \overline{\boldsymbol{D}^A_{17}} & \mathbf{0} & \alpha_L \overline{\boldsymbol{M}^{xOl,2}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\overline{\boldsymbol{I}_{NO}} & \mathbf{0} & \overline{\boldsymbol{M}^{xG}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\phi\overline{\boldsymbol{M}^{xG,3}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \overline{\boldsymbol{D}^A_{47}} & \overline{\boldsymbol{D}^A_{48}} & \mathbf{0} & \mathbf{0} \\ \overline{\boldsymbol{D}^A_{51}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \overline{\boldsymbol{D}^A_{57}} & \overline{\boldsymbol{D}^A_{58}} & \overline{\boldsymbol{M}^{u,3}} & \overline{\boldsymbol{D}^A_{5,10}} \\ \mathbf{0} & \overline{\boldsymbol{D}^A_{62}} & -\frac{1}{\beta}\overline{\boldsymbol{I}_{NO}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\overline{\boldsymbol{M}^{yG,3}} & \mathbf{0} & \overline{\boldsymbol{D}^A_{69}} & \mathbf{0} \end{bmatrix},$$

where

$$\overline{\boldsymbol{D}^A_{12}} \equiv \frac{\alpha_L}{\theta - 1} \left(\widetilde{\boldsymbol{x}}^O_{t_0} \otimes \overline{\boldsymbol{M}^{al}} \right),\ \overline{\boldsymbol{D}^A_{13}} \equiv \frac{\alpha_L}{\beta - 1} \overline{\boldsymbol{M}^{xO}},$$

$$\overline{\boldsymbol{D}_{16}^{A}} \equiv (1-\alpha_L-\alpha_M)\,\overline{\boldsymbol{I}_N},\; \overline{\boldsymbol{D}_{17}^{A}} \equiv -\left[\alpha_M \overline{\boldsymbol{M}^{xG}} + (1-\alpha_M)\,\overline{\boldsymbol{M}^{yG}}\right],$$

$$\overline{\boldsymbol{D}_{47}^{A}} \equiv -\varepsilon^G + \left(1-\varepsilon^G\right)\overline{\boldsymbol{M}^{xG,4}} + \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}_{t_0}^{G}\right)\overline{\boldsymbol{M}^{yG,2}},$$

$$\overline{\boldsymbol{D}_{48}^{A}} \equiv \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}_{t_0}^{V}\right)\overline{\boldsymbol{M}^{xR,3}} + \operatorname{diag}\left(\mathbf{1}_N \otimes \boldsymbol{s}_{t_0}^{R}\right)\overline{\boldsymbol{M}^{yR,3}},$$

$$\overline{\boldsymbol{D}_{51}^{A}} \equiv -\left(\overline{\boldsymbol{M}^{u}} \otimes \mathbf{1}_O\right),\; \overline{\boldsymbol{D}_{57}^{A}} \equiv -\left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xG,6}},$$

$$\overline{\boldsymbol{D}_{58}^{A}} \equiv -\left[\left(\overline{\boldsymbol{M}^{u}} \otimes \mathbf{1}_O\right) + \overline{\boldsymbol{M}^{xR,6}}\right],\; \overline{\boldsymbol{D}_{5,10}^{A}} \equiv \mathbf{1}_N \otimes \overline{\boldsymbol{I}_{NO}},$$

$$\overline{\boldsymbol{D}_{62}^{A}} \equiv \frac{1}{\theta}\overline{\boldsymbol{M}^{a}} + \left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\frac{1}{\theta_o-1}\overline{\boldsymbol{M}^{al,2}} + \left(-1+\frac{1}{\beta}\right)\frac{1}{\theta_o-1}\overline{\boldsymbol{M}^{al,3}},$$

and

$$\overline{\boldsymbol{D}_{69}^{A}} \equiv -\left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\operatorname{diag}\left(1-l_{i,o,t_0}^{O}\right) - \left(-1+\frac{1}{\beta}\right)\overline{\boldsymbol{M}^{xOl,4}}.$$

To normalize the price, one of the good-demand equation must be replaced with log-linearized numeraire condition $\widehat{P_{1,t}^{G}} = \sum_i x_{i1,t_0}^{G}\left(\widehat{p_{i,t}^{G}} + \widehat{\tau_{i1,t}^{G}}\right) = 0$, or

$$\overline{\boldsymbol{M}^{xG,num}}\widehat{\boldsymbol{p}_t^{G}} = -\overline{\boldsymbol{M}^{xG,num}}\widehat{\boldsymbol{\tau}_t^{G}},$$

where $\overline{\boldsymbol{M}^{xG,num}} \equiv \left[x_{11,t_0}^{G}, x_{21,t_0}^{G}, x_{31,t_0}^{G}\right]$.

To analyze the steady state conditions, first note that the steady state accumulation condition (E.32) implies $\widehat{Q_{i,o}^{R}} = \widehat{K_{i,o}^{R}}$. Using robot integration function, integration demand and unit cost formula, I have

$$\widehat{Q_{i,o}^{R}} = \sum_l x_{li,o,t_0}^{R}\widehat{Q_{li,o}^{R}} + \left(1-\alpha^R\right)\left(\sum_l \widetilde{x}_{ij,o,t_0}^{R}\widehat{p_{li,o}^{R}} - \sum_l \widetilde{x}_{li,t_0}^{G}\widehat{p_{li,t}^{G}}\right) \tag{G.6}$$

Thus the condition is

$$\begin{aligned}
&\sum_l \widetilde{x}_{li,o,t_0}^{R}\widehat{Q_{li,o}^{R}} + \left(1-\alpha^R\right)\sum_l \widetilde{x}_{li,o,t_0}^{R}\widehat{p_{l,o}^{R}} - \left(1-\alpha^R\right)\sum_l \widetilde{x}_{li,t_0}^{G}\widehat{p_{l}^{G}} - \widehat{K_{i,o}^{R}} \\
&= \left(1-\alpha^R\right)\sum_l \widetilde{x}_{li,t_0}^{G}\widehat{\tau_{li}^{G}} - \left(1-\alpha^R\right)\sum_l \widetilde{x}_{li,o,t_0}^{R}\widehat{\tau_{li}^{R}}.
\end{aligned}$$

In a matrix form, write

$$\overline{\boldsymbol{M}^{xR,7}} \equiv \begin{bmatrix} \operatorname{diag}\left(\tilde{x}^{R}_{1\cdot,\cdot,t_0}\right) & \dots & \operatorname{diag}\left(\tilde{x}^{R}_{N\cdot,\cdot,t_0}\right) \end{bmatrix}$$

a $NO \times N^2O$ matrix,

$$\overline{\boldsymbol{M}^{xR,8}} \equiv \begin{bmatrix} \operatorname{diag}\left(\tilde{x}^{R}_{11,\cdot,t_0}\right) & \dots & \operatorname{diag}\left(\tilde{x}^{R}_{N1,\cdot,t_0}\right) \\ \vdots & & \vdots \\ \operatorname{diag}\left(\tilde{x}^{R}_{1N,\cdot,t_0}\right) & \dots & \operatorname{diag}\left(\tilde{x}^{R}_{NN,\cdot,t_0}\right) \end{bmatrix}$$

a $NO \times NO$ matrix, and

$$\overline{\boldsymbol{M}^{xG,7}} \equiv \begin{bmatrix} \tilde{x}^{G}_{11,t_0} & & & \dots & \tilde{x}^{G}_{N1,t_0} & & \mathbf{0} \\ & \ddots & & & & \ddots & \\ \mathbf{0} & & \tilde{x}^{G}_{1N,t_0} & & & & \tilde{x}^{G}_{NN,t_0} \end{bmatrix} \otimes \mathbf{1}_O$$

a $NO \times N^2$ matrix.

$$\overline{\boldsymbol{M}^{xR,9}} \equiv \begin{bmatrix} \tilde{\boldsymbol{x}}^{R}_{11,\cdot,t_0} & & \mathbf{0} & & \tilde{\boldsymbol{x}}^{R}_{N1,\cdot,t_0} & & \mathbf{0} \\ & \ddots & & \dots & & \ddots & \\ \mathbf{0} & & \tilde{\boldsymbol{x}}^{R}_{1N,\cdot,t_0} & & \mathbf{0} & & \tilde{\boldsymbol{x}}^{R}_{NN,\cdot,t_0} \end{bmatrix},$$

a $NO \times N^2$ matrix, where $\tilde{\boldsymbol{x}}^{R}_{ij,\cdot,t_0} \equiv \left(\tilde{x}^{R}_{ij,o,t_0}\right)_o$ is an $O \times 1$ vector for any i and j. Then I have

$$-\left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xG,2}}\widehat{\boldsymbol{p}^{G}} + \left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xR,8}}\widehat{\boldsymbol{p}^{R}} + \overline{\boldsymbol{M}^{xR,7}}\widehat{\boldsymbol{Q}^{R}} - \widehat{\boldsymbol{K}^{R}} = \left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xG,7}}\widehat{\boldsymbol{\tau}^{G}} - \left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xR,9}}\widehat{\boldsymbol{\tau}^{R}}$$

Next, to study the steady state Euler equation (E.33) , note that by equation (E.19),

$$\frac{\partial\pi_{i,t}\left(\widehat{\left\{K_{i,o,t}^{R}\right\}}\right)}{\partial K_{i,o,t}^{R}}=\sum_{j}\widetilde{y}_{ij,t}^{G}\left(\widehat{p_{ij,t}^{G}}+\widehat{Q_{ij,t}^{G}}\right)+\left[-\frac{1}{\beta}\sum_{o'}x_{i,o',t_0}^{O}\widehat{b_{i,o',t}}+\frac{1}{\beta}\widehat{b_{i,o,t}}\right]$$
$$+\left\{\left(-1+\frac{1}{\beta}\right)\frac{1}{\theta_o-1}\sum_{o'}\frac{\widetilde{x}_{i,o',t_0}^{O}}{1-a_{o,t_0}}\left[-l_{i,o',t_0}^{O}a_{o,t_0}+\left(1-l_{i,o',t_0}^{O}\right)(1-a_{o,t_0})\right]\widehat{a_{o',t}}\right.$$
$$\left.+\left\{\left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\frac{1}{\theta_o-1}\frac{-l_{i,o,t_0}^{O}a_{o,t_0}+\left(1-l_{i,o,t_0}^{O}\right)(1-a_{o,t_0})}{1-a_{o,t_0}}+\frac{1}{\theta_o}\right\}\widehat{a_{o,t}}\right\}$$
$$+\left[\left(-1+\frac{1}{\beta}\right)\sum_{o'}x_{i,o',t_0}^{O}l_{i,o',t_0}^{O}\widehat{L_{i,o',t}}+\left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)l_{i,o,t_0}^{O}\widehat{L_{i,o,t}}\right]$$
$$+\left[\left(-1+\frac{1}{\beta}\right)\sum_{o'}\widetilde{x}_{i,o',t_0}^{O}\left(1-l_{i,o',t_0}^{O}\right)\widehat{K_{i,o',t}^{R}}+\left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\left(1-l_{i,o,t_0}^{O}\right)\widehat{K_{i,o,t}^{R}}+\left(-\frac{1}{\theta_o}\right)\widehat{K_{i,o,t}^{R}}\right].\quad\text{(G.7)}$$

Note that by the steady state accumulation condition (E.32), $Q_{i,o,t_0}^{R}/K_{i,o,t_0}^{R}=\delta$. Note also that investment function implies that, in the steady state,

$$\frac{\lambda_{j,o}^{R}}{P_{j,o}^{R}}=\left(\sum_{i}\frac{x_{ij,o}^{R}}{\left(1+u_{ij}\right)^{1-\varepsilon^{R}}}\right)^{\frac{1}{1-\varepsilon^{R}}\alpha^{R}}+2\gamma\delta.\tag{G.8}$$

To simplify the notation, set

$$\widetilde{u}_{j,o,t_0}^{SS}\equiv\frac{(\iota+\delta)\left[\left(\sum_i x_{ij,o,t_0}^{R}\left(1+u_{ij,t_0}\right)^{-\left(1-\varepsilon^{R}\right)}\right)^{\frac{1}{1-\varepsilon^{R}}\alpha^{R}}+2\gamma\delta\right]}{(\iota+\delta)\left[\left(\sum_i x_{ij,o,t_0}^{R}\left(1+u_{ij,t_0}\right)^{-\left(1-\varepsilon^{R}\right)}\right)^{\frac{1}{1-\varepsilon^{R}}\alpha^{R}}+2\gamma\delta\right]-\gamma\delta^{2}},$$

Then, by log-linearizing equation (E.33), after rearranging, I have:

$$
\begin{aligned}
&\left[\widehat{p_i^G} + 2\left(1-\alpha^R\right)\left(1-\widetilde{u}_{i,o,t_0}^{SS}\right)\sum_l \widetilde{x}_{li,t_0}^G \widehat{p_{l,t}^G}\right] - \left(1-\widetilde{u}_{i,o,t_0}^{SS}\right)\widehat{p_{i,o}^R} - 2\left(1-\alpha^R\right)\left(1-\widetilde{u}_{i,o,t_0}^{SS}\right)\sum_l \widetilde{x}_{ij,o,t_0}^R \widehat{p_{l,o}^R} \\
&+\sum_j \widetilde{y}_{ij,t_0}^G \widehat{Q_{ij}^G} - 2\left(1-\widetilde{u}_{i,o,t_0}^{SS}\right)\sum_l \widetilde{x}_{li,o,t_0}^R \widehat{Q_{li,o}^R} + \left[\left(-1+\frac{1}{\beta}\right)\sum_{o'} \widetilde{x}_{i,o',t_0}^O l_{i,o',t_0}^O \widehat{L_{i,o'}} + \left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right) l_{i,o,t_0}^O \widehat{L_{i,o}}\right] \\
&+\left[\left(-1+\frac{1}{\beta}\right)\sum_{o'} \widetilde{x}_{i,o',t_0}^O \left(1-l_{i,o',t_0}^O\right)\widehat{K_{i,o'}^R} + \left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\left(1-l_{i,o,t_0}^O\right)\widehat{K_{i,o}^R} + \left(-\frac{1}{\theta_o}\right)\widehat{K_{i,o}^R} + 2\left(1-\widetilde{u}_{i,o,t_0}^{SS}\right)\widehat{K_{i,o}^R}\right] \\
&-\widetilde{u}_{i,o,t_0}^{SS}\widehat{\lambda_{i,o}^R} \\
&= -\left(-1+\frac{1}{\beta}\right)\frac{1}{\theta_o-1}\sum_{o'}\frac{\widetilde{x}_{i,o',t_0}^O}{1-a_{o,t_0}}\left[\left(1-l_{i,o',t_0}^O\right)\left(1-a_{o',t_0}\right) - l_{i,o',t_0}^O a_{o',t_0}\right]\widehat{a_{o'}} \\
&-\left\{\left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\frac{1}{\theta_o-1}\frac{1}{1-a_{o,t_0}}\left[\left(1-l_{i,o,t_0}^O\right)\left(1-a_{o,t_0}\right) - l_{i,o,t_0}^O a_{o,t_0}\right] + \frac{1}{\theta_o}\right\}\widehat{a_o} \\
&-\left[-\frac{1}{\beta}\sum_{o'}\widetilde{x}_{i,o',t_0}^O \widehat{b_{i,o'}} + \frac{1}{\beta}\widehat{b_{i,o}}\right] + \left[-\sum_j \widetilde{y}_{ij,t_0}^G \widehat{\tau_{ij}^G} - 2\left(1-\alpha^R\right)\left(1-\widetilde{u}_{i,o,t_0}^{SS}\right)\sum_l \widetilde{x}_{li,t_0}^G \widehat{\tau_{li,t}^G}\right] \\
&+2\left(1-\alpha^R\right)\left(1-\widehat{u}_{i,o,t_0}^{SS}\right)\sum_l \widetilde{x}_{ij,o,t_0}^R \widehat{\tau_{li}^R}
\end{aligned}
$$

In matrix notation, write

$$\overline{\boldsymbol{M}^{xO,3}} \equiv \overline{\boldsymbol{M}^{xO}} \otimes \mathbf{1}_O$$

a $NO \times N^2$ matrix. Then

$$
\begin{aligned}
&\left[\left(\overline{\boldsymbol{I}_N}\otimes\mathbf{1}_O\right) + 2\left(1-\alpha^R\right)\operatorname{diag}\left(1-\widetilde{u}_{\cdot,\cdot,t_0}^{SS}\right)\overline{\boldsymbol{M}^{xG,2}}\right]\widehat{\boldsymbol{p}^G} - \operatorname{diag}\left(1-\widetilde{u}_{\cdot,\cdot,t_0}^{SS}\right)\left(\overline{\boldsymbol{I}_{NO}} - 2\left(1-\alpha^R\right)\overline{\boldsymbol{M}^{xR,8}}\right)\widehat{\boldsymbol{p}^R} \\
&+\overline{\boldsymbol{M}^{yG,3}}\widehat{\boldsymbol{Q}^G} - 2\operatorname{diag}\left(1-\widetilde{u}_{\cdot,\cdot,t_0}^{SS}\right)\overline{\boldsymbol{M}^{xR,7}}\widehat{\boldsymbol{Q}^R} + \left[\left(-1+\frac{1}{\beta}\right)\overline{\boldsymbol{M}^{xOl,3}} + \left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\operatorname{diag}\left(l_{\cdot,\cdot,t_0}^O\right)\right]\widehat{\boldsymbol{L}} \\
&+\left[\left(-1+\frac{1}{\beta}\right)\overline{\boldsymbol{M}^{xOl,4}} + \left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\operatorname{diag}\left(1-l_{\cdot,\cdot,t_0}^O\right) - \frac{1}{\theta_o}\overline{\boldsymbol{I}_{NO}} + 2\operatorname{diag}\left(1-\widetilde{u}_{\cdot,\cdot,t_0}^{SS}\right)\right]\widehat{\boldsymbol{K}^R} - \operatorname{diag}\left(\widetilde{u}_{\cdot,\cdot,t_0}^{SS}\right)\widehat{\boldsymbol{\lambda}^R} \\
&= -\left[\left(-1+\frac{1}{\beta}\right)\frac{1}{\theta_o-1}\overline{\boldsymbol{M}^{al,3}} - \left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\frac{1}{\theta_o-1}\overline{\boldsymbol{M}^{al,2}}\right]\widehat{\boldsymbol{a}} - \frac{1}{\beta}\left(\overline{\boldsymbol{I}_{NO}} - \overline{\boldsymbol{M}^{xO,3}}\right)\widehat{\boldsymbol{b}} \\
&+\left[-\overline{\boldsymbol{M}^{yG,3}} - 2\left(1-\alpha^R\right)\operatorname{diag}\left(1-\widetilde{u}_{\cdot,\cdot,t_0}^{SS}\right)\overline{\boldsymbol{M}^{xG,7}}\right]\widehat{\boldsymbol{\tau}^G} + 2\left(1-\alpha^R\right)\operatorname{diag}\left(1-\widetilde{u}_{\cdot,\cdot,t_0}^{SS}\right)\overline{\boldsymbol{M}^{xR,9}}\widehat{\boldsymbol{\tau}^R}
\end{aligned}
$$

In the steady state, I write equations (G.3) and (G.4) as

$$\overline{\boldsymbol{M}^{xG,2}}\widehat{\boldsymbol{p}^G} - \widehat{\boldsymbol{w}} + \left[\overline{\boldsymbol{I}_{NO}} - \frac{1}{1+\iota}\overline{\boldsymbol{M}^{\mu,2}}\right]\widehat{\boldsymbol{V}} = -\overline{\boldsymbol{M}^{xG,7}}\widehat{\boldsymbol{\tau}^G} + d\boldsymbol{T} - \overline{\boldsymbol{M}^{\mu,3}}d\boldsymbol{\chi}^{\text{vec}}$$

and

$$\left[\overline{\boldsymbol{I}_{NO}} - \overline{\boldsymbol{M}^{\mu L}}\right] \widehat{\boldsymbol{L}} - \overline{\boldsymbol{M}^{\mu L,2}} \widehat{\boldsymbol{\mu}^{\text{vec}}} = \boldsymbol{0}.$$

respectively.

Hence the log-linearized steady state system is

$$\overline{\boldsymbol{E}^{y}} \widehat{\boldsymbol{y}} = \overline{\boldsymbol{E}^{\Delta}} \boldsymbol{\Delta},$$

where

$$\overline{\boldsymbol{E}^{y}} \equiv \begin{bmatrix} \overline{\boldsymbol{D}^{x}} & -\overline{\boldsymbol{D}^{A,T}} \\ \multicolumn{2}{c}{\overline{\boldsymbol{D}^{y,SS}}} \end{bmatrix}, \text{ and } \overline{\boldsymbol{E}^{\Delta}} \equiv \begin{bmatrix} \overline{\boldsymbol{D}^{A,\Delta}} \\ \overline{\boldsymbol{D}^{\Delta,SS}} \end{bmatrix},$$

$\overline{\boldsymbol{D}^{A}} \equiv \begin{bmatrix} \overline{\boldsymbol{D}^{A,T}} & \overline{\boldsymbol{D}^{A,\Delta}} \end{bmatrix}$, and matrices $\overline{\boldsymbol{D}^{y,SS}}$ and $\overline{\boldsymbol{D}^{\Delta,SS}}$ are defined as

$$\overline{\boldsymbol{D}^{y,SS}} \equiv \begin{bmatrix} \overline{\boldsymbol{D}^{y,SS}_{11}} & \overline{\boldsymbol{D}^{y,SS}_{12}} & \boldsymbol{0} & \boldsymbol{0} & \overline{\boldsymbol{M}^{xR,7}} & \boldsymbol{0} & -\overline{\boldsymbol{I}_{NO}} & \boldsymbol{0} \\ \overline{\boldsymbol{D}^{y,SS}_{21}} & \overline{\boldsymbol{D}^{y,SS}_{22}} & \boldsymbol{0} & \overline{\boldsymbol{M}^{yG,3}} & \overline{\boldsymbol{D}^{y,SS}_{25}} & \overline{\boldsymbol{D}^{y,SS}_{26}} & \overline{\boldsymbol{D}^{y,SS}_{27}} & \overline{\boldsymbol{D}^{y,SS}_{28}} \end{bmatrix},$$

where

$$\overline{\boldsymbol{D}^{y,SS}_{11}} \equiv -\left(1-\alpha^{R}\right) \overline{\boldsymbol{M}^{xG,2}},$$

$$\overline{\boldsymbol{D}^{y,SS}_{12}} \equiv \left(1-\alpha^{R}\right) \overline{\boldsymbol{M}^{xR,8}},$$

$$\overline{\boldsymbol{D}^{y,SS}_{21}} \equiv \left(\overline{\boldsymbol{I}_{N}} \otimes \boldsymbol{1}_{O}\right) + 2\left(1-\alpha^{R}\right) \operatorname{diag}\left(1-\widetilde{u}^{SS}_{\cdot,\cdot,t_0}\right) \overline{\boldsymbol{M}^{xG,2}},$$

$$\overline{\boldsymbol{D}^{y,SS}_{22}} \equiv -\operatorname{diag}\left(1-\widetilde{u}^{SS}_{\cdot,\cdot,t_0}\right)\left(\overline{\boldsymbol{I}_{NO}} + 2\left(1-\alpha^{R}\right) \overline{\boldsymbol{M}^{xR,8}}\right),$$

$$\overline{\boldsymbol{D}^{y,SS}_{25}} \equiv -2\operatorname{diag}\left(1-\widetilde{u}^{SS}_{\cdot,\cdot,t_0}\right) \overline{\boldsymbol{M}^{xR,7}},$$

$$\overline{\boldsymbol{D}^{y,SS}_{26}} \equiv \left(-1+\frac{1}{\beta}\right) \overline{\boldsymbol{M}^{xOl,3}} + \left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right) \operatorname{diag}\left(l^{O}_{\cdot,\cdot,t_0}\right),$$

$$\overline{\boldsymbol{D}^{y,SS}_{27}} \equiv \left(-1+\frac{1}{\beta}\right) \overline{\boldsymbol{M}^{xOl,4}} + \left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right) \operatorname{diag}\left(1-l^{O}_{\cdot,\cdot,t_0}\right) - \frac{1}{\theta_o} \overline{\boldsymbol{I}_{NO}} + 2\operatorname{diag}\left(1-\widetilde{u}^{SS}_{\cdot,\cdot,t_0}\right),$$

$$\overline{\boldsymbol{D}^{y,SS}_{28}} \equiv -\operatorname{diag}\left(\widetilde{u}^{SS}_{\cdot,\cdot,t_0}\right),$$

and

$$\overline{\boldsymbol{D}^{\Delta,SS}} \equiv \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \overline{\boldsymbol{D}_{17}^{\Delta,SS}} & \overline{\boldsymbol{D}_{18}^{\Delta,SS}} \\ \mathbf{0} & \overline{\boldsymbol{D}_{22}^{\Delta,SS}} & \overline{\boldsymbol{D}_{23}^{\Delta,SS}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \overline{\boldsymbol{D}_{27}^{\Delta,SS}} & \overline{\boldsymbol{D}_{28}^{\Delta,SS}} \end{bmatrix},$$

where

$$\overline{\boldsymbol{D}_{17}^{\Delta,SS}} \equiv \left(1-\alpha^{R}\right)\overline{\boldsymbol{M}^{xG,7}},$$

$$\overline{\boldsymbol{D}_{18}^{\Delta,SS}} \equiv -\left(1-\alpha^{R}\right)\overline{\boldsymbol{M}^{xR,9}},$$

$$\overline{\boldsymbol{D}_{22}^{\Delta,SS}} \equiv \left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\frac{1}{\theta_o-1}\overline{\boldsymbol{M}^{al,2}} - \left(-1+\frac{1}{\beta}\right)\frac{1}{\theta_o-1}\overline{\boldsymbol{M}^{al,3}},$$

$$\overline{\boldsymbol{D}_{23}^{\Delta,SS}} \equiv -\frac{1}{\beta}\left(\overline{\boldsymbol{I}_{NO}}-\overline{\boldsymbol{M}^{xO,3}}\right),$$

$$\overline{\boldsymbol{D}_{27}^{\Delta,SS}} \equiv -\overline{\boldsymbol{M}^{yG,3}} - 2\left(1-\alpha^{R}\right)\operatorname{diag}\left(1-\widetilde{u}_{\cdot,\cdot,t_0}^{SS}\right)\overline{\boldsymbol{M}^{xG,7}},$$

and

$$\overline{\boldsymbol{D}_{28}^{\Delta,SS}} \equiv 2\left(1-\alpha^{R}\right)\operatorname{diag}\left(1-\widetilde{u}_{\cdot,\cdot,t_0}^{SS}\right)\overline{\boldsymbol{M}^{xR,9}}.$$

If $\overline{\boldsymbol{E}^{y}}$ is invertible, I have $\overline{\boldsymbol{E}} \equiv \left(\overline{\boldsymbol{E}^{y}}\right)^{-1}\overline{\boldsymbol{E}^{\Delta}}$ such that $\widehat{\boldsymbol{y}} = \overline{\boldsymbol{E}}\boldsymbol{\Delta}$. Write dimensions of $\boldsymbol{y}$ and $\boldsymbol{\Delta}$ as $n_y \equiv N + 3NO + N^2 + N^2O$ and $n_{\Delta} \equiv 3N^2 + O + 2NO + 2N$, respectively.

Finally, to study the transitional dynamics, the capital accumulation dynamics (9) implies

$$\check{K^{R}_{i,o,t+1}} = -\delta\left(1-\alpha^{R}\right)\sum_{l}\widetilde{x}_{li,t_0}^{G}\check{p}_{l,t}^{G} + \delta\left(1-\alpha^{R}\right)\sum_{l}\widetilde{x}_{li,o}^{R}\check{p}_{l,o,t}^{R} + \delta\sum_{l}\widetilde{x}_{li,o}^{R}\check{Q}_{li,o,t}^{R} + (1-\delta)\check{K}_{i,o,t}^{R}.$$

In a matrix form, write

$$\check{\boldsymbol{K}}_{t+1}^{R} = -\delta\left(1-\alpha^{R}\right)\overline{\boldsymbol{M}^{xG,2}}\check{\boldsymbol{p}}_{t}^{G} + \delta\left(1-\alpha^{R}\right)\overline{\boldsymbol{M}^{xR,8}}\check{\boldsymbol{p}}_{t}^{R} + \delta\overline{\boldsymbol{M}^{xR,7}}\check{\boldsymbol{Q}}_{t}^{R} + (1-\delta)\overline{\boldsymbol{I}_{NO}}\check{\boldsymbol{K}}_{t}^{R}.$$

Next, to study the Euler equation, define

$$\widetilde{u}_{i,o}^{TD,1} \equiv \frac{-(\iota+\delta)\left[\left(\sum_l x_{li,o}^{R}\left(1+u_{li}\right)^{-\left(1-\varepsilon^{R}\right)}\right)^{\frac{1}{1-\varepsilon^{R}}\alpha^{R}} + 2\gamma\delta\right] + \gamma\delta^{2}}{(1-\delta)\left[\left(\sum_l x_{li,o}^{R}\left(1+u_{li}\right)^{-\left(1-\varepsilon^{R}\right)}\right)^{\frac{1}{1-\varepsilon^{R}}\alpha^{R}} + 2\gamma\delta\right]}$$

and

$$\widetilde{u}_{i,o}^{TD,2} \equiv \frac{-\gamma\delta^{2}}{(1-\delta)\left[\left(\sum_{l} x_{li,o}^{R}\left(1+u_{li}\right)^{-\left(1-\varepsilon^{R}\right)}\right)^{\frac{1}{1-\varepsilon^{R}}\alpha^{R}}+2\gamma\delta\right]}.$$

Then I have

$$\begin{aligned}
&\left[-\widetilde{u}_{i,o}^{TD,1} p_{i,t+1}^{\check{G}}+2\left(1-\alpha^{R}\right)\widetilde{u}_{i,o}^{TD,2}\sum_{l}\widetilde{x}_{li}^{G} p_{l,t+1}^{\check{G}}\right]+\left[-\widetilde{u}_{i,o}^{TD,2} p_{i,o,t+1}^{\check{R}}-2\left(1-\alpha^{R}\right)\widetilde{u}_{i,o}^{TD,2}\sum_{l}\widetilde{x}_{li,o}^{R} p_{l,o,t+1}^{\check{R}}\right] \\
&-\widetilde{u}_{i,o}^{TD,1}\sum_{j}\widetilde{y}_{ij}^{G} Q_{ij,t+1}^{\check{G}}-2\widetilde{u}_{i,o}^{TD,2}\sum_{l}\widetilde{x}_{li,o}^{R} Q_{li,o,t+1}^{\check{R}}-\widetilde{u}_{i,o}^{TD,1}\left(-1+\frac{1}{\beta}\right)\sum_{o'} x_{i,o'}^{O}\left(1-l_{i,o'}^{O}\right) K_{i,o',t+1}^{\check{R}} \\
&-\widetilde{u}_{i,o}^{TD,1}\left[\left(-1+\frac{1}{\beta}\right)\sum_{o'} x_{i,o'}^{O} l_{i,o'}^{O}\check{L}_{i,o',t+1}+\left(-\frac{1}{\beta}+\frac{1}{\theta_{o}}\right) l_{i,o}^{O}\check{L}_{i,o,t+1}\right] \\
&-\left[\widetilde{u}_{i,o}^{TD,1}\left\{\left(-\frac{1}{\beta}+\frac{1}{\theta_{o}}\right)\left(1-l_{i,o}^{O}\right)+\left(-\frac{1}{\theta_{o}}\right)\right\}-2\widetilde{u}_{i,o}^{TD,2}\right] K_{i,o,t+1}^{\check{R}}+\lambda_{i,o,t+1}^{\check{R}}=\frac{1+\iota}{1-\delta}\lambda_{i,o,t}^{\check{R}}
\end{aligned}$$

In a matrix form, write

$$\overline{\boldsymbol{M}^{u,4}}=\begin{bmatrix}\widetilde{\boldsymbol{u}}_{1,\cdot}^{TD,1} & & \boldsymbol{0}\\ & \ddots & \\ \boldsymbol{0} & & \widetilde{\boldsymbol{u}}_{N,\cdot}^{TD,1}\end{bmatrix},$$

a $NO \times N$ matrix where $\widetilde{\boldsymbol{u}}_{i,\cdot}^{TD,1} \equiv \left(\widetilde{u}_{i,o}^{TD,1}\right)_{o}$ is an $O \times 1$ vector for any i. Then

$$\begin{aligned}
&\left(-\overline{\boldsymbol{M}^{u,4}}+2\left(1-\alpha^{R}\right)\operatorname{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,2}\right)\overline{\boldsymbol{M}^{xG,2}}\right)\boldsymbol{p}_{t+1}^{\check{G}}-\operatorname{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,2}\right)\left(\overline{\boldsymbol{I}_{NO}}+2\left(1-\alpha^{R}\right)\overline{\boldsymbol{M}^{xR,8}}\right)\boldsymbol{p}_{t+1}^{\check{R}} \\
&\quad-\left[\left(\overline{\boldsymbol{M}^{u,4}}\otimes\left(\mathbf{1}_{N}\right)^{\top}\right)\circ\overline{\boldsymbol{M}^{yG,3}}\right]\boldsymbol{Q}_{t+1}^{\check{G}}-2\left(\left(\mathbf{1}_{N}\right)^{\top}\otimes\operatorname{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,2}\right)\right)\circ\overline{\boldsymbol{M}^{xR,7}}\boldsymbol{Q}_{t+1}^{\check{R}} \\
&\quad+\left[-\left(-1+\frac{1}{\beta}\right)\left(\left(\overline{\boldsymbol{M}^{u,4}}\otimes\left(\mathbf{1}_{O}\right)^{\top}\right)\circ\overline{\boldsymbol{M}^{xOl,3}}\right)-\left(-\frac{1}{\beta}+\frac{1}{\theta_{o}}\right)\operatorname{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,1} l_{\cdot,\cdot}^{O}\right)\right]\check{\boldsymbol{L}}_{t+1} \\
&\quad+\left\{\left(-1+\frac{1}{\beta}\right)\left(\left(\overline{\boldsymbol{M}^{u,4}}\otimes\left(\mathbf{1}_{O}\right)^{\top}\right)\circ\overline{\boldsymbol{M}^{xOl,4}}\right)-\left(-\frac{1}{\beta}+\frac{1}{\theta_{o}}\right)\operatorname{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,1}\left(1-l_{\cdot,\cdot}^{O}\right)\right)\right. \\
&\quad\left.+\frac{1}{\theta}\operatorname{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,1}\right)+2\operatorname{diag}\left(\widetilde{u}_{\cdot,\cdot}^{TD,2}\right)\right\}\boldsymbol{K}_{t+1}^{\check{R}}+\overline{\boldsymbol{I}_{NO}}\boldsymbol{\lambda}_{t+1}^{\check{R}}=\frac{1+\iota}{1-\delta}\overline{\boldsymbol{I}_{NO}}\boldsymbol{\lambda}_{t}^{\check{R}}.
\end{aligned}$$

Hence the log-linearized transitional dynamic system is $\overline{\boldsymbol{D}_{t+1}^{y,TD}}\check{\boldsymbol{y}}_{t+1}=\overline{\boldsymbol{D}_{t}^{y,TD}}\check{\boldsymbol{y}}_{t}$, where matrices $\overline{\boldsymbol{D}_{t+1}^{y,TD}}$ and $\overline{\boldsymbol{D}_{t}^{y,TD}}$ are defined as

$$\overline{\boldsymbol{D}_{t+1}^{y,TD}}=\begin{bmatrix}\boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \overline{\boldsymbol{I}_{NO}} & \boldsymbol{0}\\ \overline{\boldsymbol{D}_{21,t+1}^{y,TD}} & \overline{\boldsymbol{D}_{22,t+1}^{y,TD}} & \boldsymbol{0} & \overline{\boldsymbol{D}_{24,t+1}^{y,TD}} & \overline{\boldsymbol{D}_{25,t+1}^{y,TD}} & \overline{\boldsymbol{D}_{26,t+1}^{y,TD}} & \overline{\boldsymbol{D}_{27,t+1}^{y,TD}} & \overline{\boldsymbol{I}_{NO}}\end{bmatrix},$$

where

$$\overline{D^{y,TD}_{21,t+1}} \equiv -\overline{M^{u,4}} + 2\left(1-\alpha^R\right)\text{diag}\left(\widetilde{u}^{TD,2}_{\cdot,\cdot}\right)\overline{M^{xG,2}},$$

$$\overline{D^{y,TD}_{22,t+1}} \equiv -\text{diag}\left(\widetilde{u}^{TD,2}_{\cdot,\cdot}\right)\left(\overline{I_{NO}} + 2\left(1-\alpha^R\right)\overline{M^{xR,8}}\right),$$

$$\overline{D^{y,TD}_{24,t+1}} \equiv -\left(\overline{M^{u,4}} \otimes (\mathbf{1}_N)^\top\right) \circ \overline{M^{yG,3}},$$

$$\overline{D^{y,TD}_{25,t+1}} \equiv -2\left((\mathbf{1}_N)^\top \otimes \text{diag}\left(\widetilde{u}^{TD,2}_{\cdot,\cdot}\right)\right) \circ \overline{M^{xR,7}},$$

$$\overline{D^{y,TD}_{26,t+1}} \equiv -\left(-1+\frac{1}{\beta}\right)\left(\left(\overline{M^{u,4}} \otimes (\mathbf{1}_O)^\top\right) \circ \overline{M^{xOl,3}}\right) - \left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\text{diag}\left(\widetilde{u}^{TD,1}_{\cdot,\cdot} l^O_{\cdot,\cdot}\right),$$

$$\begin{aligned}\overline{D^{y,TD}_{27,t+1}} \equiv &\left(-1+\frac{1}{\beta}\right)\left(\left(\overline{M^{u,4}} \otimes (\mathbf{1}_O)^\top\right) \circ \overline{M^{xOl,4}}\right) \\ &- \left(-\frac{1}{\beta}+\frac{1}{\theta_o}\right)\text{diag}\left(\widetilde{u}^{TD,1}_{\cdot,\cdot}\left(1-l^O_{\cdot,\cdot}\right)\right) + \frac{1}{\theta_o}\text{diag}\left(\widetilde{u}^{TD,1}_{\cdot,\cdot}\right) + 2\text{diag}\left(\widetilde{u}^{TD,2}_{\cdot,\cdot}\right),\end{aligned}$$

and

$$\overline{D^{y,TD}_t} = \begin{bmatrix} -\delta\left(1-\alpha^R\right)\overline{M^{xG,2}} & \delta\left(1-\alpha^R\right)\overline{M^{xR,8}} & \mathbf{0} & \mathbf{0} & \delta\overline{M^{xR,7}} & \mathbf{0} & (1-\delta)\overline{I_{NO}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \frac{1+\iota}{1-\delta}\overline{I_{NO}} \end{bmatrix}. \tag{G.9}$$

Since $\check{\boldsymbol{y}}_t = \widehat{\boldsymbol{y}}_t - \widehat{\boldsymbol{y}}$ for any $t \geq t_0$ and $\widehat{\boldsymbol{y}} = \overline{\boldsymbol{E}}\boldsymbol{\Delta}$, I have

$$\overline{D^{y,TD}_{t+1}}\left(\widehat{\boldsymbol{y}_{t+1}} - \widehat{\boldsymbol{y}}\right) = \overline{D^{y,TD}_t}\left(\widehat{\boldsymbol{y}}_t - \widehat{\boldsymbol{y}}\right)$$

$$\iff \overline{D^{y,TD}_{t+1}}\widehat{\boldsymbol{y}_{t+1}} = \overline{D^{y,TD}_t}\widehat{\boldsymbol{y}}_t - \left(\overline{D^{y,TD}_{t+1}} - \overline{D^{y,TD}_t}\right)\overline{\boldsymbol{E}}\boldsymbol{\Delta}.$$

Recall the temporary equilibrium condition $\overline{D^x}\widehat{\boldsymbol{x}}_t - \overline{D^{A,S}}\widehat{\boldsymbol{S}}_t = \overline{D^{A,\Delta}}\widehat{\boldsymbol{\Delta}}$ for any t. Thus

$$\overline{F^y_{t+1}}\widehat{\boldsymbol{y}_{t+1}} = \overline{F^y_t}\widehat{\boldsymbol{y}}_t + \overline{F^\Delta_{t+1}}\boldsymbol{\Delta},$$

where

$$\overline{F^y_{t+1}} \equiv \begin{bmatrix} \overline{D^x} & -\overline{D^{A,T}} \\ \multicolumn{2}{c}{\overline{D^{y,TD}_{t+1}}} \end{bmatrix}, \; \overline{F^y_t} \equiv \begin{bmatrix} \mathbf{0} \\ \overline{D^{y,TD}_t} \end{bmatrix}, \; \overline{F^\Delta_{t+1}} \equiv \begin{bmatrix} \overline{D^{A,\Delta}} \\ \left(\overline{D^{y,TD}_{t+1}} - \overline{D^{y,TD}_t}\right)\overline{\boldsymbol{E}} \end{bmatrix},$$

or with $\overline{\boldsymbol{F}^{y}} \equiv \left(\overline{\boldsymbol{F}^{y}_{t+1}}\right)^{-1} \overline{\boldsymbol{F}^{y}_{t}}$ and $\overline{\boldsymbol{F}^{\Delta}} \equiv \left(\overline{\boldsymbol{F}^{y}_{t+1}}\right)^{-1} \overline{\boldsymbol{F}^{\Delta}_{t+1}}$, one can write

$$\widehat{\boldsymbol{y}_{t+1}} = \overline{\boldsymbol{F}^{y}}\widehat{\boldsymbol{y}}_{t} + \overline{\boldsymbol{F}^{\Delta}}\boldsymbol{\Delta}. \tag{G.10}$$

It remains to find the initial values of the system (G.10) that converges to the steady state. To this end, I apply a standard method in Stokey and Lucas (1989). In particular, I first homogenize the system: Note that equation (G.10) can be rewritten as $\widehat{\boldsymbol{y}_{t+1}} = \overline{\boldsymbol{F}^{y}}\widehat{\boldsymbol{y}}_{t} + \left(\overline{\boldsymbol{I}} - \overline{\boldsymbol{F}^{y}}\right)\left(\overline{\boldsymbol{I}} - \overline{\boldsymbol{F}^{y}}\right)^{-1}\overline{\boldsymbol{F}^{\Delta}}\boldsymbol{\Delta}$ and thus

$$\widehat{\boldsymbol{z}_{t+1}} = \overline{\boldsymbol{F}^{y}}\widehat{\boldsymbol{z}}_{t} \tag{G.11}$$

where

$$\widehat{\boldsymbol{z}}_{t} \equiv \widehat{\boldsymbol{y}}_{t} - \left(\overline{\boldsymbol{I}} - \overline{\boldsymbol{F}^{y}}\right)^{-1}\overline{\boldsymbol{F}^{\Delta}}\boldsymbol{\Delta}. \tag{G.12}$$

The system (G.11) must not explode, or it must be that $\widehat{\boldsymbol{z}}_{t} \to \boldsymbol{0} \iff \widehat{\boldsymbol{y}}_{t} \to \left(\overline{\boldsymbol{I}} - \overline{\boldsymbol{F}^{y}}\right)^{-1}\overline{\boldsymbol{F}^{\Delta}}\boldsymbol{\Delta}$. I follow Blanchard and Kahn (1980) to find such a condition. Write Jordan decomposition of $\overline{\boldsymbol{F}^{y}}$ as $\overline{\boldsymbol{F}^{y}} = \overline{\boldsymbol{B}}^{-1}\overline{\boldsymbol{\Lambda}\boldsymbol{B}}$. Then Theorem 6.4 of Stokey and Lucas (1989) implies that it must be that out of n_y vector of $\overline{\boldsymbol{B}}\widehat{\boldsymbol{z}_{t_0}}$, n-th element must be zero if $|\lambda_n| > 1$. Since $\widehat{\boldsymbol{K}^{R}_{t_0}} = \boldsymbol{0}$, I can write

$$\widehat{\boldsymbol{z}_{t_0}} = \overline{\boldsymbol{F}^{\Delta}_{t_0}}\boldsymbol{\Delta} + \overline{\boldsymbol{F}^{\lambda}_{t_0}}\widehat{\boldsymbol{\lambda}^{R}_{t_0}},$$

where

$$\overline{\boldsymbol{F}^{\Delta}_{t_0}} \equiv \begin{bmatrix} \left(\overline{\boldsymbol{D}^{x}}\right)^{-1}\overline{\boldsymbol{D}^{A,\Delta}} \\ \boldsymbol{0}_{2NO\times n_\Delta} \end{bmatrix} - \left(\overline{\boldsymbol{I}} - \overline{\boldsymbol{F}^{y}}\right)^{-1}\overline{\boldsymbol{F}^{\Delta}} \text{ and } \overline{\boldsymbol{F}^{\lambda}_{t_0}} \equiv \begin{bmatrix} \left(\overline{\boldsymbol{D}^{x}}\right)^{-1}\overline{\boldsymbol{D}^{A,\lambda}} \\ \boldsymbol{0}_{NO\times NO} \\ \overline{\boldsymbol{I}_{NO}} \end{bmatrix}$$

and $\overline{\boldsymbol{D}^{A,\lambda}}$ is the right block matrix of $\overline{\boldsymbol{D}^{A}} \equiv \begin{bmatrix} \overline{\boldsymbol{D}^{A,K}} & \overline{\boldsymbol{D}^{A,\lambda}} \end{bmatrix}$ that corresponds to vector $\widehat{\boldsymbol{\lambda}^{R}}$. Extracting n-th row from $\overline{\boldsymbol{F}^{\Delta}_{t_0}}$ and $\overline{\boldsymbol{F}^{\lambda}_{t_0}}$ where $|\lambda_n| > 1$ and writing them as a $NO \times n_\Delta$ matrix $\overline{\boldsymbol{G}^{\Delta}_{t_0}}$ and $NO \times NO$ matrix $\overline{\boldsymbol{G}^{\lambda}_{t_0}}$, the condition of the Theorem is

$$\boldsymbol{0} = \overline{\boldsymbol{G}^{\Delta}_{t_0}}\boldsymbol{\Delta} + \overline{\boldsymbol{G}^{\lambda}_{t_0}}\widehat{\boldsymbol{\lambda}^{R}_{t_0}},$$

or $\widehat{\boldsymbol{\lambda}^{R}_{t_0}} = \overline{\boldsymbol{G}_{t_0}}\boldsymbol{\Delta}$ where $\overline{\boldsymbol{G}_{t_0}} \equiv -\left(\overline{\boldsymbol{G}^{\lambda}_{t_0}}\right)^{-1}\overline{\boldsymbol{G}^{\Delta}_{t_0}}$. Finally, tracing back to obtain the initial conditions for

$\widehat{y}_t$, it must be $\widehat{y_{t_0}} = \overline{F^y_{t_0}}\boldsymbol{\Delta}$,where

$$\overline{F^y_{t_0}} \equiv \begin{bmatrix} \left(\overline{D^x}\right)^{-1}\left(\overline{D^{A,\Delta}} + \overline{D^{A,\lambda}}\overline{G_{t_0}}\right) \\ \mathbf{0}_{NO\times n_\Delta} \\ \overline{G_{t_0}} \end{bmatrix}.$$

References

Acemoglu, Daron and Pascual Restrepo (2018). "The race between man and machine: Implications of technology for growth, factor shares, and employment". In: *American Economic Review* 108.6, pp. 1488–1542.

— (2020). "Robots and jobs: Evidence from US labor markets". In: *Journal of Political Economy* 128.6, pp. 2188–2244.

Adao, Rodrigo, Costas Arkolakis, and Federico Esposito (2019). "General Equilibrium Indirect Effects in Space: Theory and Measurement".

Arkolakis, Costas, Arnaud Costinot, and Andres Rodriguez-Clare (2012). "New trade models, same old gains?" In: *American Economic Review* 102.1, pp. 94–130.

Artuç, Erhan, Shubham Chaudhuri, and John McLaren (2010). "Trade shocks and labor adjustment: A structural empirical approach". In: *American economic review* 100.3, pp. 1008–45.

Autor, David, David Dorn, and Gordon H Hanson (2013). "The China syndrome: Local labor market effects of import competition in the United States". In: *American Economic Review* 103.6, pp. 2121–68.

Autor, David, David Mindell, and Elisabeth B Reynolds (2020). "The Work of the Future: Building Better Jobs in an Age of Intelligent Machines". In.

Autor, David and Anna Salomons (2019). "New Frontiers: The Evolving Content and Geography of New Work in the 20th Century".

Baldwin, John and Zhengxi Lin (2002). "Impediments to advanced technology adoption for Canadian manufacturers". In: *Research policy* 31.1, pp. 1–18.

Blanchard, Olivier Jean and Charles M Kahn (1980). "The solution of linear difference models under rational expectations". In: *Econometrica: Journal of the Econometric Society*, pp. 1305–1311.

Broda, Christian, Nuno Limao, and David E Weinstein (2008). “Optimal tariffs and market power: the evidence”. In: *American Economic Review* 98.5, pp. 2032–65.

Brynjolfsson, Erik and Paul Milgrom (2013). “Complementarity in organizations”. In: *The handbook of organizational economics*, pp. 11–55.

Caliendo, Lorenzo, Maximiliano Dvorkin, and Fernando Parro (2019). “Trade and labor market dynamics: General equilibrium analysis of the china trade shock”. In: *Econometrica* 87.3, pp. 741–835.

Caliendo, Lorenzo and Fernando Parro (2015). “Estimates of the Trade and Welfare Effects of NAFTA”. In: *The Review of Economic Studies* 82.1, pp. 1–44.

Chamberlain, Gary (1987). “Asymptotic efficiency in estimation with conditional moment restrictions”. In: *Journal of Econometrics* 34.3, pp. 305–334.

Devol, Jr George C (1961). *Programmed Article Transfer. US 2988237A*.

Freeman, Richard B., Ina Ganguli, and Michael J. Handel (2020). “Within Occupation Changes Dominate Changes in What Workers Do:A Shift-Share Decomposition, 2005-2015”. In: *AEA Papers and Proceedings* 110, pp. 1–7. URL: https://doi.org/10.1257/pandp.20201005.

Gaulier, Guillaume and Soledad Zignago (2010). *BACI: International Trade Database at the Product-Level. The 1994-2007 Version*. Working Papers 2010-23. CEPII. URL: http://www.cepii.fr/CEPII/fr/publications/wp/abstract.asp?NoDoc=2726.

Graetz, Georg and Guy Michaels (2018). “Robots at work”. In: *Review of Economics and Statistics* 100.5, pp. 753–768.

Hall, Robert E and Dale W Jorgenson (1967). “Tax policy and investment behavior”. In: *The American Economic Review* 57.3, pp. 391–414.

Humlum, Anders (2019). *Robot Adoption and Labor Market Dynamics*. Tech. rep. Working Paper.

Khandelwal, Amit, Peter Schott, and Shang-Jin Wei (2013). “Trade liberalization and embedded institutional reform: Evidence from Chinese exporters”. In: *American Economic Review* 103.6, pp. 2169–95.

KHI (2018). *Half a century of Kawasaki Robotics: The Kawasaki Robot Story 1968 - 2018*. Kawasaki Heavy Industry, Ltd.

Koby, Yann and Christian K Wolf (2019). *Aggregation in Heterogeneous-Firm Models: A Sufficient Statistics Approach*. Tech. rep. Tech. rep., Princeton University.

Krusell, Per et al. (2000). "Capital-skill complementarity and inequality: A macroeconomic analysis". In: *Econometrica* 68.5, pp. 1029–1053.

Lind, Nelson and Natalia Ramondo (2018). *Trade with correlation*. Tech. rep. National Bureau of Economic Research.

Mehta, Arshia and Frank Levy (2020). "Warehousing, Trucking, and Technology: The Future of Work in Logistics". In.

Newey, KW and Daniel McFadden (1994). "Large sample estimation and hypothesis". In: *Handbook of Econometrics, IV, Edited by RF Engle and DL McFadden*, pp. 2112–2245.

Ravikumar, B, Ana Maria Santacreu, and Michael Sposi (2019). "Capital accumulation and dynamic gains from trade". In: *Journal of International Economics* 119, pp. 93–110.

Ruggles, Steven et al. (2018). "IPUMS USA: Version 8.0 [dataset]". In: *Minneapolis, MN: IPUMS* 10, p. D010.

Simonovska, Ina and Michael E Waugh (2014). "The elasticity of trade: Estimates and evidence". In: *Journal of international Economics* 92.1, pp. 34–50.

Stokey, Nancy L and Robert E Lucas (1989). *Recursive methods in economic dynamics*. Harvard University Press.

Tambe, Prasanna et al. (2019). "IT, AI and the Growth of Intangible Capital". In: *Available at SSRN 3416289*.

Timmer, Marcel P, Erik Dietzenbacher, et al. (2015). "An illustrated user guide to the world input–output database: the case of global automotive production". In: *Review of International Economics* 23.3, pp. 575–605.

Timmer, Marcel P, Bart Van Ark, et al. (2007). "EU KLEMS growth and productivity accounts: an overview". In.

Traiberman, Sharon (Dec. 2019). "Occupations and Import Competition: Evidence from Denmark". In: *American Economic Review* 109.12, pp. 4260–4301. DOI: 10.1257/aer.20161925. URL: http://www.aeaweb.org/articles?id=10.1257/aer.20161925.

Waldman-Brown, Anna (Mar. 2020). "Redeployment or robocalypse? Workers and automation in Ohio manufacturing SMEs". In: *Cambridge Journal of Regions, Economy and Society* 13.1, pp. 99–115. ISSN: 1752-1378. DOI: 10.1093/cjres/rsz027. eprint: https://academic.oup.com/

cjres/article-pdf/13/1/99/33213560/rsz027.pdf. URL: https://doi.org/10.1093/cjres/rsz027.