Machine Learning 6.867 - Pset 2

October 16, 2015

1 Logistic Regression

1.1 Implementation

We implemented L_2-regularized logistic regression using gradient descent.

1.2 Testing in data with $\lambda = 0$

We test the logistic regression

Table 1: Estimated logistic regrecoefficients

Data	w_0	w_1	w_2
stdev1	-66.3378	95.2461	101.1527
stdev2	-0.0466	0.7636	1.1148
stdev4	-0.0093	0.2363	0.2034
nonsep	0.0006	-0.0247	-0.0237

2 Support Vector Machine

Support Vector Machines are a popular classification method to construct linear or nonlinear decision boundaries by solving a convex optimization problem. There are two common forms of the optimization problem considered for SVM, which we refer to as the primal and dual. In this paper, we only consider the dual form, because it is computationally more tractable for many problems, and this method has the ability to generalize to different choices of kernel. The dual form of SVM for a general kernel function $k : \mathcal{X} \times \mathcal{X} \to \mathbb{R}$ is as follows:

$$\begin{aligned} \max_{\alpha \in \mathbb{R}^n} \quad & \sum_{i=1}^{n} \alpha_i - \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i \alpha_j y^{(i)} y^{(j)} k(x^{(i)}, x^{(j)}) \\ \text{s.t.} \quad & 0 \leq \alpha_i \leq C, \qquad\qquad i = 1, \ldots, n, \\ & \sum_{i=1}^{n} \alpha_i y^{(i)} = 0. \end{aligned} \tag{1}$$

2.1 Implementation

First, we implemented the dual form of the SVM with a linear kernel, where k is the usual dot product $k(x, z) = \langle x, z \rangle$ for all $x, z \in \mathcal{X}$. In MATLAB, we created a function with inputs: data $X \in \mathbb{R}^{n \times p}$, labels $Y \in \{-1, 1\}$, and cost parameter $C \in \mathbb{R}^+$. Within the function, we use the quadratic solver `quadprog` to solve the SVM dual problem (1) with these parameters to find the optimal α's. Since `quadprog` requires that the problem fit into a certain functional form, we reformulate the problem (1) as follows:

$$
\begin{aligned}
-\min_{\alpha\in\mathbb{R}^n} \quad & \tfrac{1}{2}\alpha^T H\alpha - \sum_{i=1}^{n}\alpha_i \\
\text{s.t.} \quad & 0 \le \alpha_i \le C, \qquad\qquad i = 1,\dots,n, \\
& \sum_{i=1}^{n}\alpha_i y^{(i)} = 0,
\end{aligned} \tag{2}
$$

where: $H \in \mathbb{R}^{n\times n}$ is a matrix with $(i,j)^{th}$ entry $H_{ij} = y^{(i)}y^{(j)}k(x^{(i)}, x^{(j)})$. Given the optimal solution $\alpha \in \mathbb{R}^n$ for the SVM problem with a linear kernel, the chosen linear decision boundary $\theta^T x + \theta_0 = 0$ is given by:

$$\theta = \sum_{i=1}^{n}\alpha_i y^{(i)}x^{(i)} \tag{3}$$

$$\theta_0 = \frac{1}{\mathcal{M}}\Big(\sum_{j\in\mathcal{M}}\Big(y^{(j)} - \sum_{i\in\mathcal{S}}\alpha_i y^{(i)}(x^{(j)})^T x^{(i)}\Big)\Big) \tag{4}$$

The output of our linear SVM function is $[\theta, \theta_0]$. We tested our function on the 2D example $X = \{(1,2),(2,2),(0,0),(-2,3)\}$, $Y = \{1,1,-1,-1\}$. For this problem, the objective function generated for problem (2) is:

$$\frac{1}{2}\alpha^T H\alpha - \sum_{i=1}^{4}\alpha_i, \tag{5}$$

where:

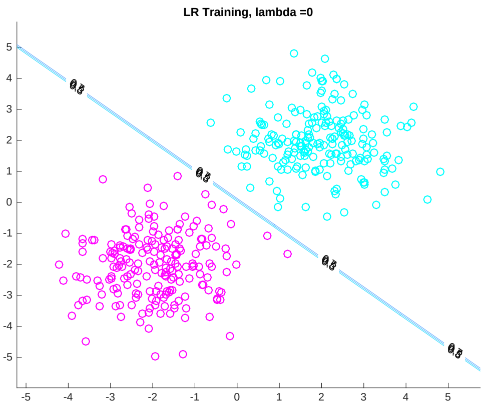
$$H = \begin{bmatrix} 5 & 6 & 0 & -4 \\ 6 & 8 & 0 & -2 \\ 0 & 0 & 0 & 0 \\ -4 & -2 & 0 & 13 \end{bmatrix}$$

The constraints are:

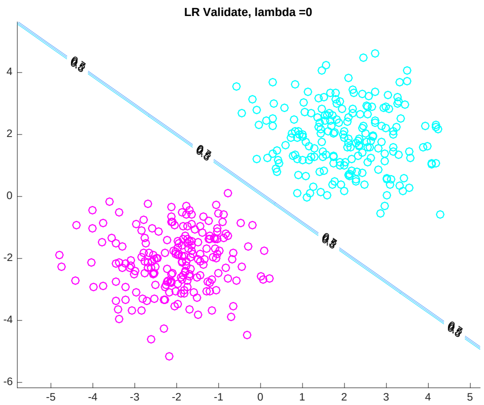
$$0 \le \alpha_i \le C, \quad i = 1,\dots,4, \tag{6}$$

$$\alpha_1 + \alpha_2 - \alpha_3 - \alpha_4 = 0. \tag{7}$$

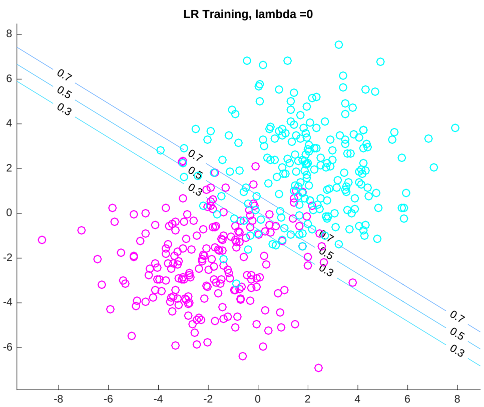
3 Titanic Data



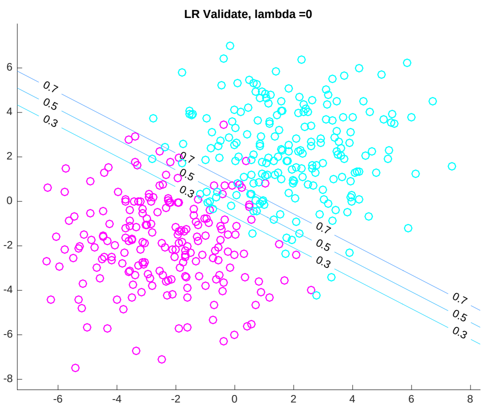
(a) Data with $\sigma = 1$, training



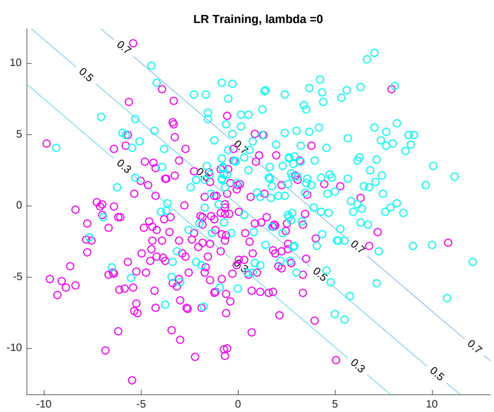
(b) Data with $\sigma = 1$, validation



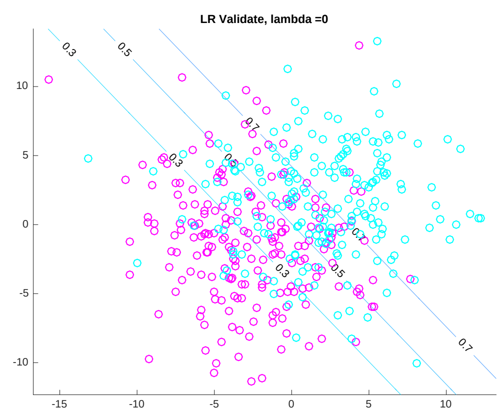
(c) CData with $\sigma = 2$, training



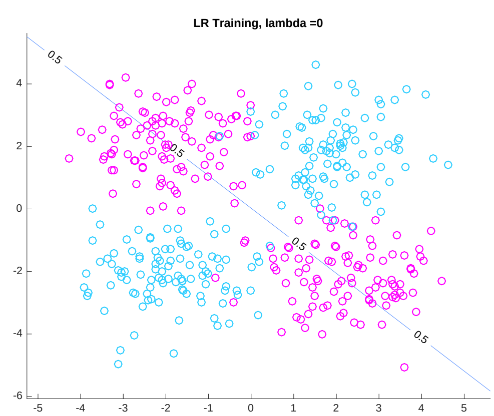
(d) CData with $\sigma = 2$, validation



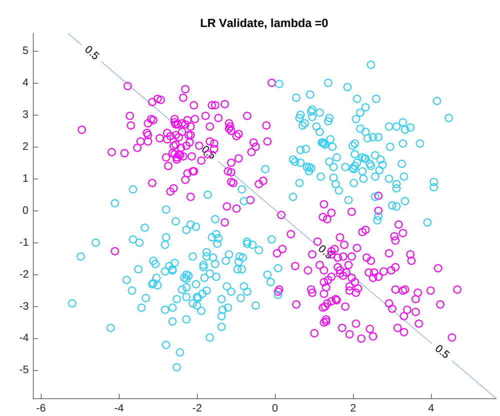
(e) Data with $\sigma = 4$, training



(f) Data with $\sigma = 4$, validation



(g) Non-seperable data, training



(h) Non-seperable data, validation