Problem Chosen	2022 MCM/ICM Summary Sheet	Team Control Number
C		2221211

Quantitative investment means limitless possibilities

Summary

In order to determine an optimal investment plan, this paper proposes a quantitative inves tment model based on the time series prediction model and quantitative evaluation model to p rovide investment plan for investors.

First of all, we preprocess the original data provided by MCM to supplement the missing original data. Subsequently, ARIMA and LSTM models were used for prediction of different data. In the process of model establishment, we found that when bitcoin transaction data accu mulated to a certain amount, the prediction accuracy of LSTM model would be higher than th at of ARIMA model, while the prediction accuracy of ARIMA model of gold transaction data was always the highest. Therefore, we will use different prediction schemes for the two data.

After the end of the forecast data, we use the pre-processed data and forecast data to esta blish a quantitative model, and determine 10 first-level indicators including investment risk, P rospective benefits rate, Lookback benefit Rate,etc., among which investment risk includes c oefficient of variation within 30 days, abnormal days of fluctuation within the first 30 days,et c., four second-level indicators. We determined the weight of each index through Topsis mod el. We use these weights to determine our willingness to buy and sell Bitcoin and gold on a da ily basis, which will play an important role in our model.

We use quantified daily buy intention scores and sell intention scores to make decisions d ay by day. We compare the score difference between buy and sell to a certain threshold to det ermine whether we are buying or selling. At the same time, investment risk will affect how m uch we buy and how much we sell, which ensures that we have a certain amount of money on hand for future opportunities.

In this paper, we made a profit of $308,167.63 after 5 years of operation with the initial $ 1,000. At the same time, we use dynamic programming and random test model to verify the f easibility and optimality of the scheme. Finally, we analyze the sensitivity of the strategy to tr ansaction cost, and we find that bitcoin transaction is more sensitive to transaction cost.

Keywords: Quantitative analysis;Time series analysis;Goal programming;Trend-follow;

Memorandum

Dear investors

We are writing this memo to you about quantitative investment strategies for bitcoin and gold. We have built an adequate model to quantify how to allocate cash on hand and make decisions on the timing of adding or subtracting from a position in one product to increase your investment returns. Thus getting maximum value.

This model owes a lot to the data COMAP provided us on the last five years of bitcoin and gold price changes. The analysis of the data provided us with great inspiration before we chose to create our model.

To make good quantitative investment decisions, the first thing we need to do is to make a good analysis and forecast of the time series data of price changes. In the process of building the model, we chose ARIMA model and LSTM model. First of all, we cannot deny that the development of deep learning has gradually disrupted the field of time series data analysis, but when the data volume is small, we also cannot ignore the beauty of traditional statistical methods. For volatile data like Bitcoin, we recommend using the ARIMA model in the beginning stages, while when the data volume is large enough (we initially chose 500 days), the LSTM model can be used. This will maximize the prediction accuracy. For products like gold, where the price does not change much, we recommend using the ARIMA model all the time.

Once the forecast results are obtained, we need to quantify the daily investment risk and whether we should buy or sell. We have chosen 10 level 1 indicators including investment risk to build a quantitative model based on Topsis. The results of this model will influence our decision whether to trade and how much to trade in each day.

We use the daily willingness to buy score and willingness to sell score obtained by quantification to make decisions day by day. When the difference between the buy and sell scores is less than 0.02, we do not buy or sell, and when it is greater than 0.02, we choose the action corresponding to the larger value to execute. At the same time, the investment risk will influence the amount we buy and sell, which ensures that we have a certain amount of money on hand for future opportunities that arise. With the initial $1,000, we were able to make a profit of $308,167.63 over 5 years. At the same time we conducted simulation experiments to verify that the parameters we chose were optimal, and in more than 1,000 rounds of experiments our model turned out to be optimal. At the same time, we used five years of known data for dynamic programming to obtain results that were only $30,000 higher than our model results, which is acceptable for us without the ability to travel through time.

Finally, we also explore the transaction cost, and we find that in our model, when the transaction cost of Bitcoin increases, the total profit will change faster. Our model will be more sensitive to the transaction costs of volatile products like Bitcoin, and you will need to pay attention to the changes in the transaction costs when you actually trade.

On the model of quantitative investment. We are confident that this model will reach its full potential.

Sincerely，

MCM Team Members

Contents

1 Introduction

1.1 Problem Background

Market traders buy and sell volatile assets frequently, with a goal to maximize their total return. There is usually a commission for each purchase and sale. Two such assets are gold and bitcoin. In the face of complex investment environment, how to do quantitative investment well is the necessary condition for investors to obtain high profits. Quantitative investment refers to a trading method that issues buying and selling orders through quantitative methods and computer programs in order to obtain stable returns. With the development of statistics and machine learning, quantitative investing will also usher in a new era of development. This paper will use **time series analysis**, **Topsis**, **dynamic programming** and other methods to establish a set of quantitative bitcoin and gold trading decision model to maximize the benefits of the transaction results.

1.2 Restatement of the Problem

Given the above background information and constraints, we will address the following questions ：

- Build a mathematical model that allows you to make the best daily decision based on the price data as of that day and calculate how much value a strategy based on the model will generate as of September 10, 2021 with $1,000 of principal.
- Verify the optimality of the decision made by using the established model
- Make a sensitivity analysis of transaction commission and explain how it affects the decision outcome of the model.
- Complete a memo to communicate models, strategies and results to traders.

1.3 Our Work

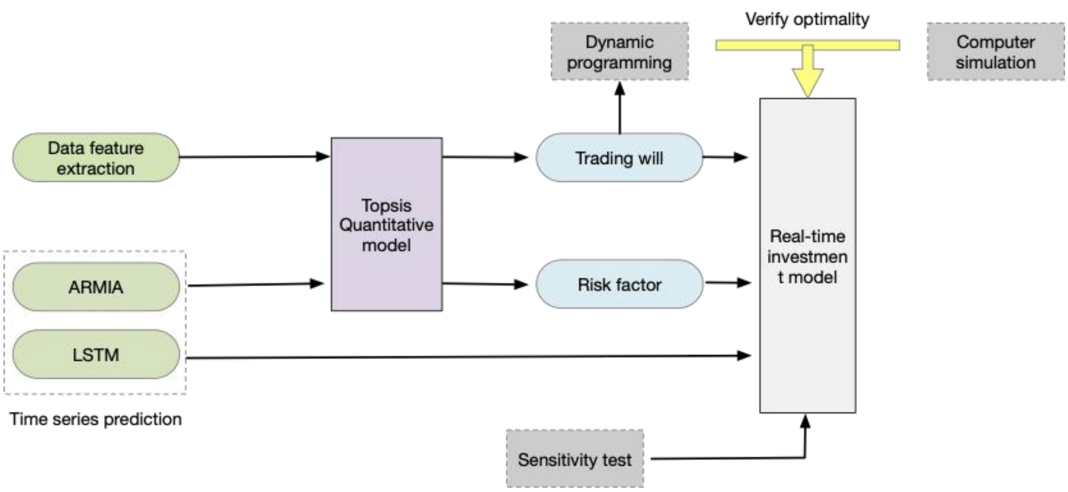

Figure 1 Model flow chart

- Gold as a traditional investment projects, investment benefit is higher in a short time, so in the first 30 days, we are only buying and selling of gold, at the beginning of 31 days, set [the cash flow, bitcoin holdings, gold holdings] for $[RM_i, B_i, G_i]$, we set up real-time investment model, by setting the threshold value, determine the daily transaction type, including buying and selling of gold and bitcoin, and then judging the number of transactions according to the risk coefficient index, and then get the investment plan of each day.

- To verify that our daily decision model is the best strategy, we test the model using two methods:

  The first investment dynamic programming model is established, the model is used to validate the optimality problem one real-time investment model, based on the results of model 2 (index quantitative model), we sort of whether the day is suitable for trading, fixed the date for the score higher for trading, in this model, we use the dynamic programming method, In the case of known specific price data every day, to the last day of the actual value of the highest as the objective function to plan, to meet only use 1000 yuan under the constraint conditions of investment, so as to get known the price of daily data planning to obtain the optimal investment scheme, the result and need daily real-time decision-making problem a model results are compared, Thus, the decision optimality of the original model is verified.

  Another validation method is to use the results of random behaviors to compare with the optimal results of model decisions. We continuously define risk levels randomly through computer simulation experiments and compare them with the risk estimates we obtained from model 2. In this model, we use the idea of cyclic iteration to constantly change the risk value and get the total profit value at this level. The results are compared with the results of the problem 1 model, which requires daily real-time decision-making, so as to verify the decision optimality of the original model.

- In order to analyze the impact of commission on decision benefit, we add the number of commission into Topsis model, in which it is reflected as a negative indicator, that is, the higher the commission, the less willing to buy or sell. Finally, we will adopt the way of ergodic solution, constantly change the commission rate, carry out many simulation experiments, and then get the conclusion.

2 Assumptions and Justifications

In order to simplify the problem and build a better mathematical model, we make reasonable assumptions and justifications.

Assumptions 1: We assume no trading at all during the first seven days of the five-year trading period given;

→**justifications 1:** As there is not enough historical transaction data in the first seven days of trading, it is impossible to evaluate the transaction risk and profit possibility if the transaction is carried out in these days, and the transaction risk in the unknown market is high. Therefore, we choose to hold the principal and wait and see in the first seven days to accumulate transaction data so that we can make the best decision later.

Assumptions 2: The difference of 0.02 between the buy and sell scores is used as the threshold for measuring whether to buy or sell gold and Bitcoin ;

→**justifications 2:** We have reviewed a large number of papers and found that the difference between the buying and selling score of 0.02 is the optimal decision threshold, which can ensure greater profits and less investment risk.

Assumptions 3: We set the buy and sell score for Saturday and Sunday of gold to 0 ;

→**justifications 3:** Because gold cannot be traded on Saturdays and Sundays, we subjectively set the score on Saturdays and Sundays to 0 when scoring the trading days of gold, so as to ensure the accuracy of decision-making.

3 Notations

The key mathematical notations used in this paper are listed in Table 1.

Table 1: Notations used in this paper

Symbol	Description	Unit
RM_i	Daily cash holding	Dollar
b_i	The market price of bitcoin	Dollar
g_i	The market price of gold	Dollar
B_i	Daily bitcoin holdings	SAT
G_i	Daily gold holdings	Ounce
BIM_i	Bitcoin buy score	
BOM_i	Bitcoin sell score	
GIM_i	Gold buy score	
GOM_i	Gold sell score	

4 Data processing and evaluation

4.1 Value prediction model

4.1.1 Data preprocessing

Sifting through the raw data, it's not hard to see that there are gaps in bitcoin and gold prices and that trading in gold stops on weekends and holidays like Christmas.

Therefore, to solve the problem of missing data, we used the weighted average method to make up the data before and after the date. For weekends and holidays, we set all data of gold trading on that day to NULL to ensure that there will be no gold trading on that day.

4.1.2 The Establishment of Prediction model

Since the birth of the stock market, the high return of the stock market has attracted researchers to try to predict the trend of stock prices. In this problem, reasonable and accurate prediction of stock price will provide important support basis for decision making. However, the stock market is a very complicated system, and its characteristics of nonlinearity, instability and complexity make it very difficult to predict the stock price. Many scholars list the daily closing prices of stocks in chronological order and construct stock time series models. ARIMA model and LSTM model are the most famous ones to predict the short-term trend of the future based on the historical stock price trend. Both ARIMA model and LSTM model can better extract the trend and periodicity contained in the original data, and then predict the future data according to the law extracted from the original data.

ARIMA (Auto regressive Integrated Moving Average model) regarded historical data as a random sequence, established a mathematical model to approximate it, and made predictions based on this model. ARIMA model mainly has five kinds of forms: AR (p), MA (q) and ARMA (p, q), ARIMA (p, d, q), ARIMA (p, d, q) x (p, d, q) s, the ARIMA (p, d, q) x (p, d, q) s. Where AR is autoregressive, MA is moving average, P and Q are autoregressive and moving average orders, P and Q are seasonal autoregressive and moving average orders, D and D are non-seasonal and seasonal difference times respectively, S is time series cycle, and its expression is:

$$\nabla^d \nabla_s^D X_t = \frac{\Theta(B)\Theta_s(B)}{\Phi(B)\Phi_s(B)} \varepsilon_t \tag{1}$$

$$\begin{gathered} \theta(B) = 1 - \theta_1 B - \cdots - \theta_q B^q \\ \phi(B) = 1 - \phi_1 B - \cdots - \phi_p B^p \\ \theta_s(B) = 1 - \theta_1 B_s - \cdots - \theta_{QS} B^{QS} \\ \phi_s(B) = 1 - \phi_1 B_s - \cdots - \phi_P B^{PS} \end{gathered} \tag{2}$$

Among them：

The modeling steps of ARIMA model are mainly divided into the following five steps :(1) the stabilization of time series; (2) Model recognition; (3) Order determination and parameter statistics of the model; (4) Model testing; (5) Prediction of the model.

With the accumulation of data, neural network and other deep learning methods have achieved good results in large-scale data emergencies. Among them, the representative methods are LSTM, CNN, etc. We will focus on LSTM. The time series model is based on cyclic time network and introduces the concepts of Input Gate, Forget Gate and Output Gate.

There are three main phases within LSTM：

1. Forget phases.： This phase is mainly about selectively forgetting the input passed in by the previous node. Simply put, you forget the unimportant and remember the important.
2. Choose the memory stage： This stage selectively "remembers" the inputs from this stage. It's basically selective memory of the input. Write down what is important and what is not.
3. Output phase.：This phase determines which outputs will be considered for the current state.

4.1.3 The Solution of Prediction model

According to known data, there are 1,265 days of time series data related to gold and 1,826 days of data related to bitcoin. (How to deal with the missing data of gold stock price on Sundays and holidays has been explained in the data pretreatment section.) According to the above data, different methods are selected for prediction in different periods. The prediction accuracy is as follows.

Prediction accuracy	ARIMA 1～500day	ARIMA 500～last day	Lstm 500～last day
Gold	73.8%	79.1%	82.7%
Bitcoin	79.3%	81.6%	79.9%

table 1 Accuracy of prediction

Therefore, we will choose to use LSTM model in the second half of gold stock price forecast to achieve better prediction accuracy, while we will always use ARIMA for bitcoin. The predicted results are shown in the figure below

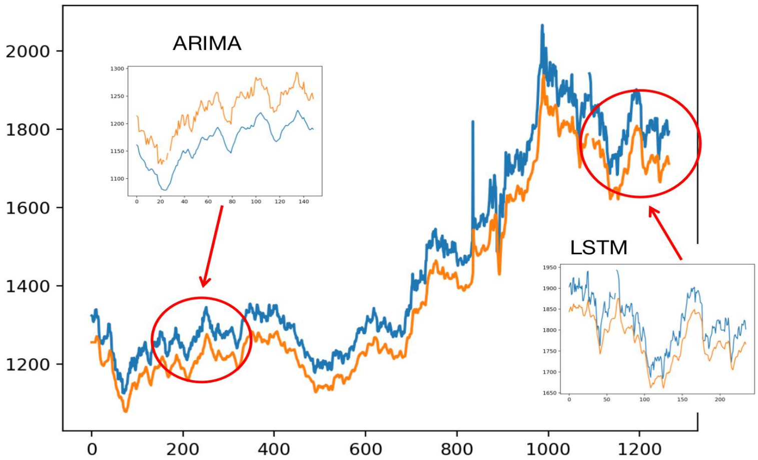

Figure 2 Bitcoin price forecast chart

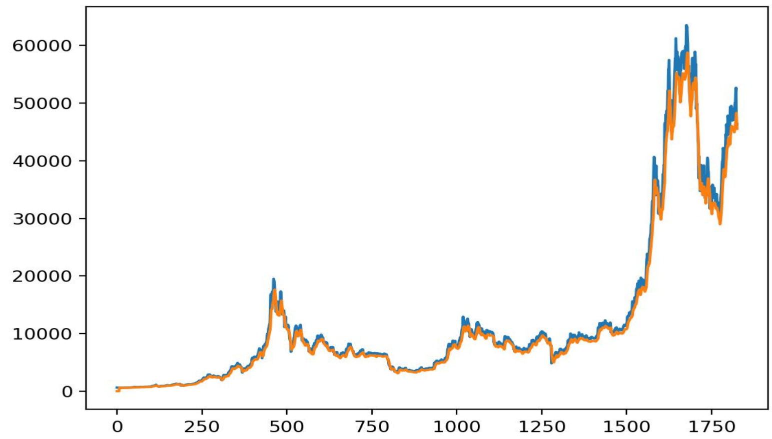

Figure 3 Gold price forecast chart

4.2 Quantitative model of evaluation indicators

4.2.1 The Establishment of Quantitative model

In the analysis of this problem, features that can influence investment buying and selling intention are mined from the historical price data of Bitcoin and Gold given in the title, so as to establish an evaluation system for whether a certain day is suitable for trading. Through entropy weight analysis of indicators, each indicator is quantified. The TOPSIS method is used to establish an evaluation system that comprehensively considers multiple indicators. Through this evaluation system, the suitable trading date is screened out.

1.Selection of characteristic variables affecting investment intention

The specific indicators that can be used to evaluate whether a certain date is suitable for trading are defined. With these indicators, an evaluation system that can be quantified and measured whether a certain date is suitable for trading is constructed. According to the analysis of data, ten first-level indicators are selected for measurement and the definitions are given as follows:

- **CurrentValue：**Represents the price of gold or Bitcoin on the day, expressed as G_{cv} and B_{cv} respectively; Obviously, this index can affect investors' investment intention on the day, and has an impact on whether to carry out transactions. Therefore, this index is included in the evaluation system.

- **Predicted value：** Represents the price of gold or Bitcoin on the day after the date, expressed as $G_{\mathrm{p}v}$ and $\mathrm{B}_{\mathrm{p}v}$ respectively. The trading price of the next day obtained through the historical price data through the prediction model will affect investors' investment intentions and have an impact on the evaluation of whether the day is suitable for trading, so this index is included in the evaluation system.

- **Prospective benefits:** Represents the difference between the predicted price of gold or Bitcoin on the next day and the price of the same day, expressed as G_{pb} and B_{pb} respectively. The expressions can be written as follows：$G_{pb} = G_{pv} - G_{cv}$、$B_{pb} = B_{pv} - B_{cv}$. Since the difference between the price of the next day and today will affect the trading intention, it is included in the evaluation system of whether it is suitable for trading.

- **Lookback benefit：** Represents the difference between the price of gold or Bitcoin on the same day and yesterday, expressed as G_{lb} and B_{lb} respectively. The expressions can be written as follows: $G_{lb} = G_{cv} - G_{yv}$、$B_{lb} = B_{cv} - B_{yv}$, where G_{yv} and B_{yv} represent the prices of gold and Bitcoin respectively yesterday. Since the difference between today's price and yesterday's price will affect the trading intention, it is included in the evaluation system of whether it is suitable for trading.

- **Prospective benefits rate：** Represents the ratio of the difference between the predicted price of the next day and the price of the current day to the price of gold or Bitcoin, respectively expressed as G_{PBR} and B_{PBR}, whose expressions can be written as follows: $G_{PBR} =$ （$G_{pv} - G_{cv}$）$/G_{cv}$、$B_{PBR} =$ （$B_{pv} - B_{cv}$）$/B_{cv}$, Since the difference between the price of the next day and today's value and the growth rate of today's value will affect the trading intention, it is included in the evaluation system of whether it is suitable for trading.

- **Lookback benefit rate：** Represents the ratio of the difference between the price of gold or Bitcoin on the same day and yesterday's price, respectively expressed as G_{lBR} and B_{lBR}, and its expressions can be written as: $G_{lBR} =$ （$G_{cv} - G_{yv}$）$/G_{yv}$、$B_{lBR} =$ （$B_{cv} - B_{yv}$）$/B_{yv}$， where G_{yv} and B_{yv} represent the prices of gold and Bitcoin respectively yesterday. Since the difference between today and yesterday's price and the growth rate of yesterday's value will affect the transaction intention, it is included in the evaluation system of whether it is suitable for transaction.

- **Maximum continuous growth days within 30 days:** This index represents the number of consecutive increasing days within 30 days before the decision making and is expressed as M_c. The data of 30 days before the decision making has important reference significance for whether to conduct trading on the decision making day. If there are many consecutive increasing days within the first 30 days, it indicates that the market situation is good, which is favorable for trading, so it can be included in the evaluation system.

- **Average price of 15 days:** This index represents the average price of 15 days before the decision date, and is represented as G_{AP} andB_{AP}. The average price level within 15 days around the decision date represents the overall price level of the date nearby. If the level is high, it indicates that the market is suitable for trading recently, so this index is included in the evaluation system.

- **BIAS：** Represents the ratio of the difference between the current day and the average price of the previous 30 days to the average price of the previous 30 days, and is expressed as G_{bias} andB_{bias}.

- **Investment risk index:** Investment risk index can be used to judge whether to carry out trading, the greater the risk, the less suitable for trading, because investment risk can not be directly obtained from the data given by the topic, it is necessary to carry out the classification of investment risk, some secondary indicators to quantify the investment risk of the primary index, the definition of the secondary indicators is as follows:

  - **Coefficient of variation:**The coefficient of variation can be used to reflect the price fluctuation in a period of time, expressed as c_{oi}, which reflects the stability of market price. The greater the coefficient of variation is, the greater the price fluctuation is and the greater the risk is

$$c_{oi} = \frac{\sigma_{oi}}{\mu_{oi}} \tag{3}$$

Where c_{oi} is the coefficient of variation in the first 30 days, σ_{oi} is the standard deviation of the price in the first 30 days, Its computation formula for $\sqrt{\sum_{w=1}^{240} \frac{(x_{owi}-\bar{x}_{ow})^2}{w-1}}$ for 30 days before the mean average price, The calculation formula is $\frac{1}{n}\sum_{w=1}^{240} x_{owi}$.

  - **Abnormal fluctuation days within the first 30 days：** If the representation of a secondary index in 30 days before the memory is normal price trend of fluctuations are larger, criterion shows that the market is not stable, there are 30 days the more the number of days, explain the market volatility, the greater the risk the greater the whether its for abnormal standard for the day and two days before and after the difference is bigger.
  - **Number of consecutive growth ranges within the first 30 days (range 2):** If there are multiple consecutive growth ranges in the first 30 days of the decision making day, if the price of the next day is higher than that of the previous day, it is recorded as a growth range, and the number of such growth ranges in the 30 days before the decision making day is recorded. The more growth ranges, the more stable the market is and the smaller the risk is.

- **Change in price over the first 30 days of the cycle:** This indicator means the difference between the price 30 days before the decision date and the price on the decision date, representing the increase of the price one month before the decision date. If there is an increase, it means that the risk is smaller.

After TOPSIS quantitative analysis, the weight of each index of investment risk is as follows:

indicators	Coefficient of variation	Continuously increase the number of sections	Abnormal number of days	Change in price
The weight(gold /Bitcoin)	0.45/0.44	0.13/0.16	0.39/0.36	0.04/0.04

table 2 Index weight

To sum up, this model summarizes the evaluation model of 10 first-level indicators on whether a certain day is suitable for trading. **CurrentValue, Predicted Value, Prospective benefits, Lookback benefit, Prospective benefits rate, Lookback benefit Rate, Maximum continuous growth days within 30 days, Average price of 15 days, BIAS, Investment risk index** and other 10 indicators were included in the evaluation system. At the same time, in order to quantify the investment risk, four secondary indicators are set, including **coefficient of variation within 30 days, abnormal days of fluctuation within the first 30 days, number of continuous growth intervals within the first 30 days, and price change within the first 30 days**. The above indicators are incorporated into the evaluation system to obtain quantitative results.

2.Select suitable trading day evaluation system construction

Using the index selected by the evaluation system to quantify the transaction characteristics of the price data, first of all, to determine the weight of each index, using the entropy weight method to eliminate the subjective weight, from the data itself to get the weight of the index, and then using TOPSIS method to quantify whether it is suitable for trading.

（1）Entropy weight method

Entropy weight method allocates weights according to the degree of change of indicators. The advantage of using entropy weight method is that it can eliminate the influence of subjective weight assignment and get the weight size from the data itself. The smaller the variation degree of indicators is, the less information is reflected and the corresponding weight should be lower.

The standard 0-1 transformation is used to normalize and forward the indicators of the evaluation system to ensure the non-negative index data, and the calculation method is：

$$z_{ij} = \frac{x_{ij} - x_j^{min}}{x_j^{max} - x_j^{min}} \tag{4}$$

Where,z_{ij} is the result after normalization, and there is no negative number;

Calculate the proportion of the ith supplier in item J index,It's the probability that we use when we calculate relative entropy

$$p_{ij} = \frac{z_{ij}}{\sum_{i=1}^{n} z_{ij}} \tag{5}$$

For the JTH indicator, its information entropy can be calculated by:

$$e_j =-\frac{1}{\ln n}\sum_{i=1}^{n} p_{ij}\ln\left(p_{ij}\right), (j = 1,2,\cdots,m) \tag{6}$$

Then the information utility value is defined as

$$d_j = 1 - e_j \tag{7}$$

The larger the information utility value d_j is, the more information it corresponds to. Finally, the entropy weight of each supply characteristic index is obtained by normalization of the information utility value:

$$\omega_j = \frac{d_j}{\sum_{j=1}^{m} d_j}, (j = 1,2,3,\cdots,m) \tag{8}$$

（2）TOPSIS method quantifies daily buying and selling feasibility

As a kind of effective multi-index evaluation method, TOPSIS method by constructing evaluation problem of positive ideal solution and negative ideal solutions, the various indicators of the optimal solution and the inferior solution of ideal solution, through the analysis of each plan and the most optimal solution and the distance of the pareto solutions, and then get more index ranking, the nearer the positive ideal solution and negative ideal solution in the farther, the solution is optimal.

Obtained by entropy weight method and normalized decision matrix $B = (bij)_{m\times n}$ construct weighted gauge matrix $C = \left(c_{ij}\right)_{m\times n}$, which is expressed as

$$c_{ij} = w_j \cdot b_{ij}，\ i = 1,2,\cdots,m; j = 1,2,\cdots,n \tag{9}$$

The optimal solution and the worst solution in each index are searched, and the positive ideal solution and the negative ideal solution are obtained by combination. The most suitable trading date is expressed as a vector：

$$C^{*} = \{c_1^*, c_2^*, \cdots c_m^*\} \tag{10}$$

Among them， $c_i^*(i = 1,2, \cdots, m)$ represents the optimal value in each indicator；
The least desirable trading day is expressed as：

$$C^{0} = \{c_1^0, c_2^0, \cdots c_m^0\} \tag{11}$$

Among them， $c_i^*(i = 1,2, \cdots, m)$ represents the worst value in each index；
Calculate the distance from each scheme to the collated solution and the negative ideal solution, and the distance from the ith trading day to the optimal trading day is：

$$s_i^* = \sqrt{\sum_{j=1}^{n} \left(c_{ij} - c_j^*\right)^2} \tag{12}$$

The distance to the least ideal trading day is：

$$s_i^0 = \sqrt{\sum_{j=1}^{n} \left(c_{ij} - c_j^0\right)^2} \tag{13}$$

Calculate the ranking index value of each scheme, i.e

$$f_i^* = \frac{S_i^0}{s_i^0 + S_i^*} \tag{14}$$

If f_i^* is closer to 1, the trading day is closer to the ideal target, and its profit may be large; otherwise, it may produce losses. The trading is sorted according to f_i^*.

4.2.2 The Solution of Evaluation index quantitative model

Using the data in the attachment, the weight distribution results of each indicator are as follows:

indicators	CurrentValue	Predicted value	Prospective benefits	Lookback benefit	Prospective benefits rate	Lookback benefit rate
The weight (Buy gold/sell gold/buy bitcoin/sell Bitcoin)	0.11/0.32/ 0.03/0.42	0.28/0.14 0.35/0.04	0.01/0.05/ 0.02/0.01	0.002/0.003 /0.004/0.005	0.001/0.03/ 0.003/0.002	0.002/0.003 /0.013/0.005
indicators	Maximum continuous growth days within 30 days	average price of 15 days	BIAS	investment risk		
The weight (Buy gold/sell gold/buy bitcoin/sell Bitcoin)	0.24/0.06/ 0.17/0.05	0.28/0.33 /0.35/0.42	0.05/0.01/ 0.03/0.01	0.05/0.05/ 0.04/0.04		

table 3 Index weight 2

Use Python, MATLAB, etc., for data processing and model solving, and get the sorting result of the most suitable date for trading (the most suitable date for trading every month)

Num	Data	Parameter values	Num	Data	Parameter values
1	8/10/20	0.946966936	6	5/10/21	0.86905376
2	11/6/20	0.945067692	7	6/11/21	0.769079069
3	1/8/21	0.921862635	8	11/12/20	0.767152575
4	9/7/20	0.910587309	9	12/5/19	0.75877059
5	3/11/21	0.903734378	10	3/5/20	0.736865051

table 4 Order results of buying gold (shown ten days before interception)

Num	Data	Parameter values	Num	Data	Parameter values
1	9/3/20	0.953048439	6	1/12/21	0.877338414
2	11/6/20	0.912886024	7	6/11/21	0.875913649
3	1/6/21	0.904374285	8	5/14/20	0.860029738

4	9/9/20	0.897149397	9	7/10/20	0.853252003
5	10/26/21	0.88377574	10	9/5/19	0.832688677

table 5 Ranking results of selling gold (shown ten days before interception)

Num	Data	Parameter values	Num	Data	Parameter values
1	4/16/21	0.9320573745	6	4/20/21	0.883100478
2	4/13/21	0.909957491	7	4/22/21	0.870656661
3	12/16/17	0.905144193	8	12/29/20	0.870029738
4	8/18/21	0.900358002	9	12/17/17	0.86903108
5	3/30/21	0.887739302	10	2/20/21	0.851860683

table 6 Ranking result of buying Bitcoin (shown ten days before interception)

Num	Data	Parameter values	Num	Data	Parameter values
1	3/17/21	0.85778660465	6	4/25/21	0.8183352243
2	3/20/21	0.8541101945	7	6/27/19	0.800972331
3	4/17/21	0.8436222603	8	1/29/18	0.80005128
4	12/27/20	0.838727702	9	9/6/17	0.80000123
5	12/20/17	0.827436879	10	3/30/19	0. 79434062

table 7 Ranking result of selling Bitcoin (shown ten days before interception)

5 Real-time investment planning problem solving

5.1 Problem analysis

As a traditional investment, gold is more profitable in a short period of time, so for the first 30 days, we only bought and sold gold, and at the beginning of the 31st day,Let 【cash flow, bitcoin holdings, gold holdings】 be 【RM_i, B_i, G_i】 ,We establish a real-time investment model and judge daily transaction types, including buying and selling of gold and bitcoin, by setting thresholds. Then we judge the number of transactions according to the risk coefficient index, and then get the investment plan of each day.

5.2 The Establishment of Real-time investment model

Let【cash flow, bitcoin holdings, gold holdings】be【RM_i, B_i, G_i】, Set the gold buy score and sell score respectively as GIM_i、GOM_i, Bitcoin has a buy score and a sell score as

BIM_i、BOM_i，Gold buy and sell score difference is BB_i，Bitcoin buy and sell score difference is GG_i,Through statistical analysis and data review, we set a bitcoin buy score and a bitcoin sell score as the threshold of 0.02 difference between the buy score and the sell score. According to this threshold, we determine when to buy and sell gold and bitcoin，Meanwhile, the average investment risks of gold and bitcoin are calculated as $\gamma_g\%$ and $\gamma_b\%$,When buying bitcoin or gold, only use $RM_{i-1} * (1-\gamma_b\%)$ or $RM_{i-1} * (1-\gamma_g\%)$ cash flow，Every time you sell bitcoin or gold， only sell $B_i * \gamma_b\%$ or $G_i * \gamma_g\%$ unit of bitcoin or gold， The model is established as follows， Among them， $\alpha_b\%$ and $\alpha_g\%$ Is the commission rate for trading bitcoin and gold。

We set a discrete integer variable Q_i，As shown below：

$$Q_i = \begin{cases} 1, BB > 0.02 \text{ and } GG > 0.02 \text{ and } BIM_i \geq GIM_i \\ 2, BB > 0.02 \text{ and } GG > 0.02 \text{ and } BIM_i \leq GIM_i \\ 3, \quad BB > 0.02 \text{ and } -0.02 \leq GG \leq 0.02 \\ 4, \quad GG > 0.02 \text{ and } -0.02 \leq BB \leq 0.02 \\ 5, \quad BB > 0.02 \text{ and } GG < -0.02 \\ 6, \quad GG > 0.02 \text{ and } BB < -0.02 \\ 7, \quad GG < -0.02 \text{ and } BB < -0.02 \\ 8, -0.02 \leq GG \leq 0.02 \text{ and } -0.02 \leq BB \leq 0.02 \end{cases} \tag{15}$$

If $Q_i = 1，3$，Buy bitcoin, not gold：

$$\begin{cases} B_i = B_{i-1} + \dfrac{RM_{i-1} * (1-\gamma_b\%)}{b_i} \\ G_i = G_{i-1} \\ RM_i = RM_{i-1} * \gamma_b\% - RM_{i-1} * (1-\gamma_b\%) * \alpha_b\% \end{cases} \tag{16}$$

If $Q_i = 2，4$，Buy gold and do not operate bitcoin：

$$\begin{cases} G_i = G_{i-1} + \dfrac{RM_{i-1} * (1-\gamma_g\%)}{g_i} \\ B_i = B_{i-1} \\ RM_i = RM_{i-1} * \gamma_g\% - RM_{i-1} * (1-\gamma_g\%) * \alpha_g\% \end{cases} \tag{17}$$

If $Q_i = 5$，Buy bitcoin, sell gold：

$$\begin{cases} G_i = (1-\gamma_g\%) * G_{i-1} \\ B_i = B_{i-1} + \dfrac{(RM_{i-1} + \gamma_g\% * G_{i-1} * g_i) * (1-\gamma_b\%)}{b_i} \\ RM_i = (RM_{i-1} + \gamma_g\% * G_{i-1} * g_i) * \gamma_b\% - \gamma_g\% * G_{i-1} * g_i * \alpha_g\% - (RM_{i-1} + \gamma_g\% * G_{i-1} * g_i) * (1-\gamma_b\%) * \alpha_b\% \end{cases} \tag{18}$$

If$Q_i = 6$，Buy gold, sell bitcoin：

$$\begin{cases} B_i = (1-\gamma_b\%) * B_{i-1} \\ G_i = G_{i-1} + \dfrac{(RM_{i-1} + \gamma_b\% * B_{i-1} * b_i) * (1-\gamma_g\%)}{b_i} \\ RM_i = (RM_{i-1} + \gamma_b\% * B_{i-1} * b_i) * \gamma_g\% - \gamma_b\% * B_{i-1} * b_i * \alpha_b\% - (RM_{i-1} + \gamma_b\% * B_{i-1} * b_i) * (1-\gamma_g\%) * \alpha_g\% \end{cases} \tag{18}$$

If$Q_i = 7$，Sell gold and bitcoin：

$$\begin{cases} B_i = (1-\gamma_b\%) * B_{i-1} \\ G_i = (1-\gamma_g\%) * G_{i-1} \\ RM_i = RM_{i-1} + \gamma_b\% * B_{i-1} * b_i * (1-\alpha_b\%) + \gamma_g\% * G_{i-1} * g_i * (1-\alpha_g\%) \end{cases} \tag{19}$$

If$Q_i = 8$，It does not operate on gold or bitcoin：

$$\begin{cases} B_i = B_{i-1} \\ G_i = G_{i-1} \\ RM_i = RM_{i-1} \end{cases} \tag{20}$$

5.3 The Solution of Real-time investment model

Python software was used to solve the problem, and the daily trading results were as follows :(partial results were intercepted)

Data(i)	12/6 /16	12/31 /16	12/20 /17	3/26 /18	6/30 /18	6/30 /19	12/20 /20	8/6 /21	9/10 /21
RM_i($)	17.08	0.01	0	14188.86	28839.40	0	125754.24	1671.36	308004.25
B_i(SAT)	3.29	3.40	3.40	0.89	2.99	7.36	1.93	6.93	0.016
G_i(ounce)	0.05	0.02	0	22.29	0.01	0	0	0	0
Total assets	2575. 34	3258.77	59125.33	51881.66	47445.67	87551.03	171828.29	284939.01	308167.63

table8 Real-time investment results

We conducted trend analysis of total assets, cash flow, bitcoin holdings and gold holdings, and the results are as follows:

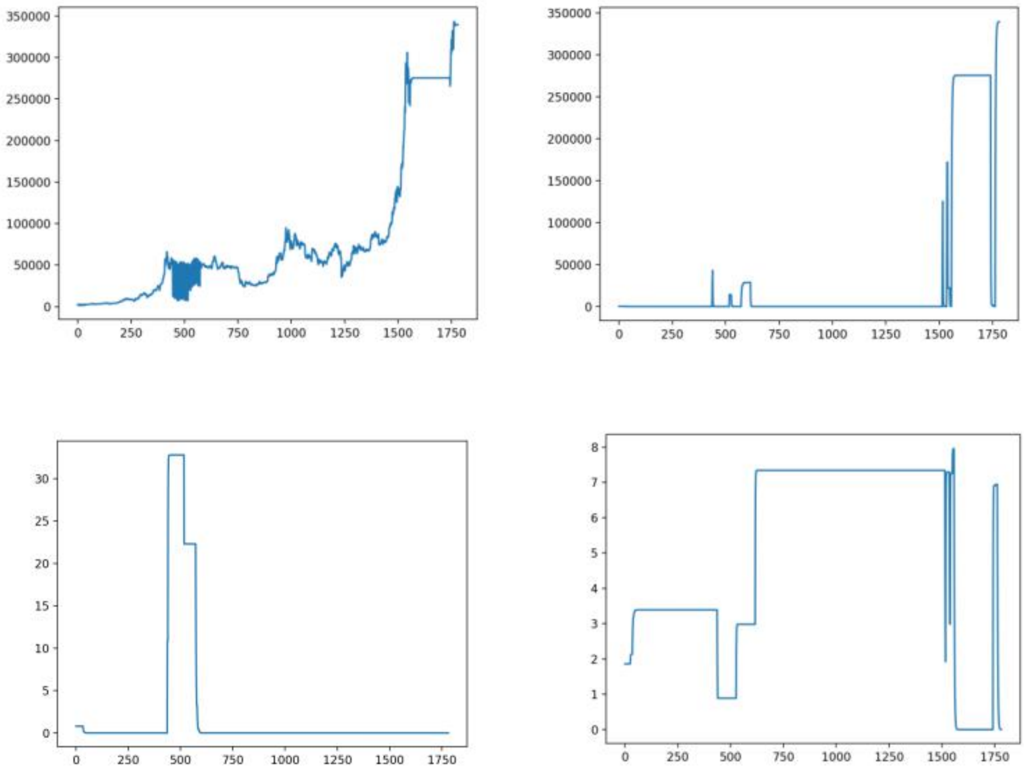

figure 4 total assets, cash flow, bitcoin holdings and gold holdings Share trend chart

6 The proof scheme of the best strategy

The first is to establish a dynamic investment planning model, which is used to verify the optimality of the real-time investment model in question 1. According to the results of model 2 (index quantitative model), we sorted whether the day was suitable for trading and determined the date with higher score suitable for trading. In this model, we use the method of dynamic programming, in the case of known specific price data every day, to the last day of the actual value of the objective function is the highest in planning, in meet using only 1000 yuan under the constraint conditions of investment, so as to get known the price of daily data planning to obtain the optimal investment scheme, This result is compared with the result of problem 1 model, which requires daily real-time decision, to verify the decision optimality of the original model.

Another validation method is to use the results of random behaviors to compare with the optimal results of model decisions. We continuously define risk levels randomly through computer simulation experiments and compare them with the risk estimates we obtained from model 2. In this model, we use the idea of cyclic iteration to constantly change the risk value and get the total profit value at this level. The result is compared with the result of problem 1, which requires daily real-time decision-making, so as to verify the decision optimality of the original model.

6.1 Problem Analysis

In order to verify that the daily decision model established by us is the optimal strategy, we use two methods to test the model, namely the dynamic investment planning model and the comparison model between the stochastic behavior results and the optimal result of model decision.

6.2 Dynamic investment planning model

6.2.1 The Establishment of dynamic investment planning model

(1)Constraints:

Daily investment operations, including buying and selling Bitcoin and gold, are represented by the following four quantities:

The daily amount of gold sold GO_i is expressed as：$GO_i = g_i * x_{1i}$

The daily amount of gold buy GI_i is expressed as：$GI_i = g_i * x_{2i}$

The daily amount of bitcion sold BO_i is expressed as：$BO_i = b_i * y_{1i}$

The daily amount of bitcion buy BI_i is expressed as：$BI_i = b_i * y_{2i}$

Where, g_i and $b_i$respectively represent the market price of gold and bitcoin on that day, x_{1i}, x_{2i}, $y_{1i}$and y_{2i}respectively represent the amount of gold sold, gold bought, bitcoin sold and bitcoin bought.

For the daily residual cash flow RM_i is expressed as

$$RM_i = RM_{i-1} + GO_i * z_{1i} + BO_i * z_{2i} - GI_i * z_{3i} - BI_i * z_{4i} - (GO_i * z_{1i} + GI_i * z_{3i}) * \alpha_g\% - (BO_i * z_{2i} + BI_i * z_{4i}) * \alpha_b \quad (21)$$

Among them,

$$z_{1i} = \begin{cases} 1, \text{ Sell gold} \\ 0, \ Unsold\ gold \end{cases}, \ z_{2i} = \begin{cases} 1, \text{ Sell Bitcoin} \\ 0, \text{ Unsold Bitcoin} \end{cases}, \ z_{3i} = \begin{cases} 1, \ Buy\ gold \\ 0, \ Not\ buy\ gold \end{cases}, \ z_{4i} = \begin{cases} 1, \text{ buy Bitcoin} \\ 0, \ not\ buy\ Bitcoin \end{cases} \quad (22)$$

For RM_i we should guarantee the RM_i>=0, at the same time, daily can't buy and sell gold at the same time, also can't buy and sell currency at the same time, namely $z_{1i}*z_{3i} = 0$, $z_{2i}*z_{4i} = 0$.

We should also ensure that the amount of gold and bitcoins we have is greater than zero per day, i.e. for the total amount of gold we have G_i and the total amount of bitcoins B_i

$$\sum_1^i (x_{2i} * z_{3i} - x_{1i} * z_{1i}) \geq 0 \qquad \sum_1^i y_{2i} * z_{4i} - y_{1i} * z_{2i}) \geq 0 \ , \quad i = 2 \cdots\cdots 679 \quad (23)$$

(1) Objective function:

According to the question, we should maximize the actual value on the last day, i.e

$$\max C \ = \ \mathrm{RM}_{679} + \sum_1^{679} (x_{2i} * z_{3i} - x_{1i} * z_{1i}) * g_i + \sum_1^{679} (y_{2i} * z_{4i} - y_{1i} * z_{2i}) * b_i \quad (24)$$

In summary, the dynamic programming model is obtained as follows:

$$\max C = \mathrm{RM}_{679} + \sum_{1}^{679} (x_{2i} * z_{3i} - x_{1i} * z_{1i}) * g_i + \sum_{1}^{679} (y_{2i} * z_{4i} - y_{1i} * z_{2i}) * b_i$$

$$\mathrm{s.t.}\begin{cases} RM_i = RM_{i-1} + GO_i * z_{1i} + BO_i * z_{2i} - GI_i * z_{3i} - BI_i * z_{4i} - (GO_i * z_{1i} + GI_i * z_{3i}) * \alpha_g\% - (BO_i * z_{2i} + BI_i * z_{4i}) * \alpha_b\% \\ z_{1i} * z_{3i} = 0,\ z_{2i} * z_{4i} = 0 \quad (z_{1i}\ ,\ z_{2i}\ , z_{3i}\ , z_{4i} = o\ or\ 1) \\ \sum_{1}^{i} (x_{2i} * z_{3i} - x_{1i} * z_{1i}) \geq 0\ , \quad i = 2\cdots\cdots 679 \\ \sum_{1}^{i} y_{2i} * z_{4i} - y_{1i} * z_{2i}) \geq 0\ , \quad i = 2\cdots\cdots 679 \end{cases}$$

6.2.2 The Solution of investment dynamic planning model

After lingo's solution, we got the maximum profit of 357,5446.36 dollars on the last day

The figure below can be obtained by comparing the data obtained with that in Question 1. As can be seen from the figure, the profit amount obtained by the model in question 1 is only slightly less than the result obtained by dynamic programming. Considering the market, subjective cognition and other influencing factors, we believe that such result is acceptable and excellent.

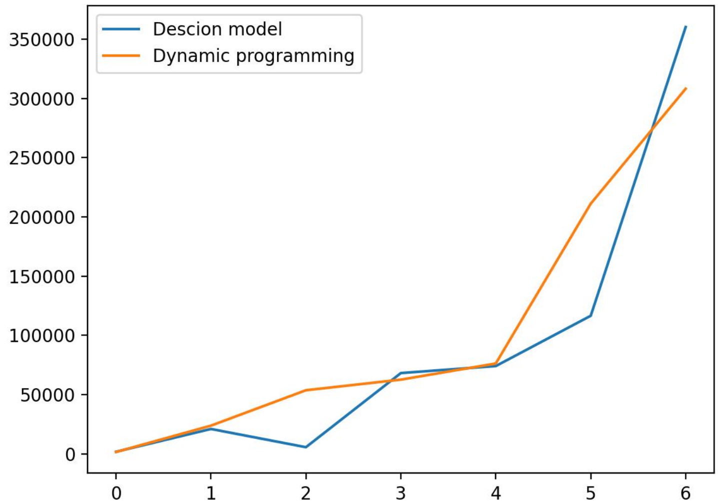

figure 5 Asset comparison chart

6.3 Stochastic test model of optimal scheme

6.3.1 The Establishment and Solution of Stochastic test model for optimal solution

This model is used to verify the optimal line of the real-time investment model established in Question 1 from the perspective of risk estimation. We continuously define the risk level randomly through computer simulation experiments and compare it with the risk estimate obtained from model 2. In this model, we use the idea of cyclic iteration to constantly change the risk value and get the total profit value at this level. The results are compared with the results of the problem 1 model, which requires daily real-time decision-making, so as to verify the decision optimality of the original model.

The Solution of model:

Based on the above, we ran a thousand iterations, randomly selecting the risk values as random floating point numbers in the range (0,1), and finally got the following result.

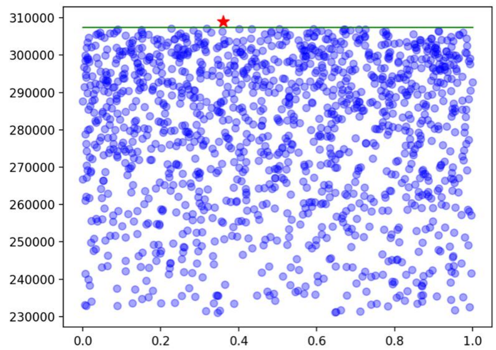

figure 6 Simulation experiment diagram

Thus, when the bitcoin risk index is 36%, the maximum return is $308167.63.

7 Sensitivity analysis of investment strategy to Transaction costs

7.1 Problem Analysis

In order to analyze the impact of transaction costs on decision-making benefits, we add Transaction costs into Topsis model, which is reflected as a negative indicator, that is, the higher transaction costs, the less willingness to buy or sell. We adopted the method of ergodic solution, constantly changing transaction costs, and conducted several simulation experiments.

7.2 Simulation of Transaction costs

As transaction costs change, the increase in transaction costs will reduce people's willingness to buy a certain stock, and conversely, as transaction costs decrease, people will have a higher desire to buy gold or bitcoin. In order to reflect the impact of transaction costs on people's subjective intentions, we added transaction costs into Topsis model. In the model, transaction costs is reflected as a negative indicator, that is, the higher the commission, the less willing to buy or sell. Finally, we will adopt the method of ergodic solution to conduct a simulation experiment to observe the change of the influence of transaction costs on decision benefit.

Transaction costs are selected for gold and bitcoin in the range (0-0.3). Through 300 rounds of simulation experiments, the following table is obtained :(partial data excerpts)

Bitcoin transaction costs	Gold transaction costs	Total revenue	Bitcoin transaction costs	Gold transaction costs	Total revenue
0.19	0.19	24370.08806	0.14	0.1	111070.6819
0.18	0.07	50602.40435	0.12	0.1	142612.4967
0.17	0.19	55911.90292	0.09	0.07	192540.5712

All the data obtained are plotted as a three-dimensional scatter chart below, with bitcoin commissions on the X-axis, gold commissions on the Y-axis, and income on the Z-axis.

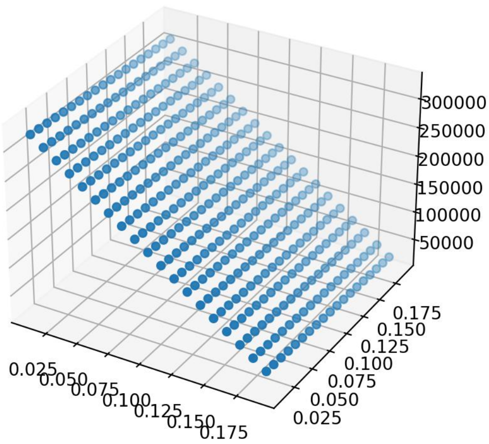

figure 7 Sensitivity analysis diagram

From this, we can see that transaction costs have a more obvious impact on Bitcoin. When the proportion of Transaction costs increases, the decrease is larger than that of gold, and it is easily affected by transaction costs.

8 Conclusion

8.1 Result

This paper successfully established a reasonable investment decision model for gold and bitcoin, found the optimal investment strategy to maximize trading benefits, and proved the o ptimality of the model through verification analysis. After five years of investment, the asset r eached $308,167.63 as of 9/10/2021, and the sensitivity of the strategy to transaction costs wa s determined through analysis.

8.2 Strengths and Weaknesses

Advantages of the model:

1. The quantitative indicators selected in the model come from the indicators with high f requency in relevant literature, which are feasible for research and can be applied to other fiel

ds, showing generalization.

2. The decision-making process of the model is divided into three modules, which are connected with each other, but can be upgraded independently for the three modules to provide continuous upgrading of the model in the later stage.

3. This paper proves the economic benefits of the model through two different experiments, which makes the model more credible.

Disadvantages of the model:

1. There is still a gap between the prediction accuracy of time series data and that of mainstream research, but a larger data set may be needed to solve this problem.

2. The use of future forecast data in the model is only limited to quantitative indicators, without the ability to perceive future trends.

3. The model does not consider people's subjective emotions and investment preferences.

9 References

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