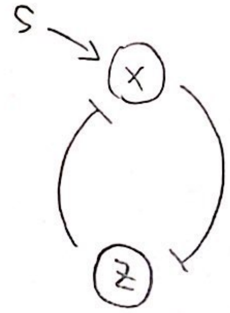
XIN DING (xd222).

Q2 (a). Bistable switch.



$$\frac{d\tilde{x}}{d\tilde{t}} = \frac{\tilde{\alpha}_x + \tilde{\beta}_x S}{1+S+(\tilde{z}/\tilde{z}_x)^{n_{zx}}} - \tilde{\delta}_x \cdot \tilde{x} \qquad \text{eqn(1)}$$

$$\frac{d\tilde{z}}{d\tilde{t}} = \frac{\tilde{\alpha}_z}{1+(\tilde{x}/\tilde{x}_z)^{n_{xz}}} - \tilde{\delta}_z \tilde{z} \qquad \text{eqn(2)}$$

(b). Non-dimensional equation.

Refer to eqn(3)-(6) in Ruben's paper, the small mistake is that $t = \tilde{t}\,\tilde{\delta}_x$ rather than $\tilde{t}\,\delta_x$.

$\tilde{x} = x \cdot \tilde{\alpha}_z / \tilde{\delta}_x$, $\tilde{t} = t/\tilde{\delta}_x$

$$\frac{d\tilde{x}}{d\tilde{t}} = \frac{d(x \cdot \frac{\tilde{\alpha}_z}{\tilde{\delta}_x})}{d(t \cdot \frac{1}{\tilde{\delta}_x})} = \tilde{\alpha}_z \frac{dx}{dt}$$

$\tilde{z} = z \cdot \tilde{\alpha}_z / \tilde{\delta}_x$. $\tilde{t} = t/\tilde{\delta}_x$

$$\frac{d\tilde{z}}{dt} = \frac{d(z \cdot \frac{\tilde{\alpha}_z}{\tilde{\delta}_x})}{d(t \cdot \frac{1}{\tilde{\delta}_x})} = \tilde{\alpha}_z \cdot \frac{dz}{dt}$$

eqn(1)

$$\frac{\tilde{\alpha}_x + \tilde{\beta}_x S}{1+S+(\tilde{z}/\tilde{z}_x)^{n_{zx}}} - \tilde{\delta}_x \cdot \tilde{x} = \frac{\alpha_x \cdot \tilde{\alpha}_z + \beta_x \tilde{\alpha}_z \cdot S}{1+S+(z/z_x)^{n_{zx}}} - \tilde{\alpha}_z x \qquad (\tilde{z}/\tilde{z}_x = z/z_x).$$

eqn(2).

$$\frac{\tilde{\alpha}_z}{1+(\tilde{x}/\tilde{x}_z)^{n_{xz}}} - \tilde{\delta}_z \cdot \tilde{z} = \frac{\tilde{\alpha}_z}{1+(x/x_z)^{n_{xz}}} - \tilde{\alpha}_z \cdot \delta_z \cdot z \qquad (\tilde{x}/\tilde{x}_z = x/x_z).$$

Overall:

$$\begin{cases} \dfrac{dx}{dt} = \dfrac{\alpha_x + \beta_x S}{1+S+(z/z_x)^{n_{zx}}} - x \\[2ex] \dfrac{dz}{dt} = \dfrac{1}{1+(x/x_z)^{n_{xz}}} - \delta_z \cdot z \end{cases}$$

Also, the $\frac{d\tilde{z}}{dt}$ equation in eqn(2) in the paper should have $\tilde{z}$ rather than z in the last segment.

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