3. (a). $\dot{m}_i = r_{x,i}\,\bar{u}_i - (\mu + \theta_{m,i})\, m_i$

$\dot{p}_i = r_{L,i}\, w_i - (\mu + \theta_{p,i})\, p_i$

At steady state $\dot{m}_i = 0$. $\dot{p}_i = 0$.

$\dot{m}_i = 0$. $\quad r_{x,i}\,\bar{u}_i = (\mu + \theta_{m,i})\, m_i^*$

$$m_i^* = \frac{r_{x,i}}{\mu + \theta_{m,i}}\,\bar{u}_i \simeq k_{x,i}\,\bar{u}_i$$

$\dot{p}_i = 0$. $\quad r_{L,i}\, w_i^* = (\mu + \theta_{p,i})\, p_i^*$

$$p_i^* = \frac{r_{L,i}}{\mu + \theta_{p,i}}\, w_i^* \simeq k_{L,i}\, w_i^*$$

w_i^* describes the translation process and is related to m_i^*

$w_i^* = w_i \cdot m_i^*$ Thus, $p_i^* \simeq k_{L,i}\, k_{x,i}\,\bar{u}_i\, w_i$.

(b). $r_{L,i} = k_{E,i}^{L}\, R_{LT} \left(\dfrac{m_i}{\tau_{L,i}\, k_{L,i} + (\tau_{L,i}+1)\, m_i} \right)$

Since $\tau_{L,i}\, k_{L,i} \gg (\tau_{L,i}+1)\, m_i$

$\rightarrow r_{L,i} = k_{E,i}^{L}\, R_{LT}\, \dfrac{m_i}{\tau_{L,i}\, k_{L,i}}$

$r_{x,i} = k_{E,i}^{x}\, R_{xT} \left(\dfrac{g_i}{\tau_{x,i}\, k_{x,i} + (\tau_{x,i}+1)\, g_i} \right)$.

To sum up:

$$P_i^* = \underbrace{\frac{k_{E,i}^{L} R_{LT}}{(\mu+\theta_{p,i})(\tau_{L,i} K_{L,i})}}_{k_{L,i}} \cdot \underbrace{\frac{k_{E,i}^{x} R_{XT}}{\mu+\theta_{m,i}}\left(\frac{g_i}{\tau_{x,i} K_{x,i} + (\tau_{x,i}+1) g_i}\right)}_{K_{x,i}} \bar{u}_i \cdot w_i$$

According to Prelim 1 Q1. $K_{x,i} = 0.575$ nmol/gDW.

$$\bar{u}_i = \frac{k_1 + k_2 f_I}{1 + k_1 + k_2 f_I} \qquad f_I = \frac{I^n}{k^n + I^n}$$

cell dry weight $= 4.3\times10^{-13}$ g $\times 0.3 = 1.29\times10^{-13}$ g. $V = 1\ \mu m^3 = 10^{-3}$ L.

$K_{L,i} = 200\ \mu M = 200\ \mu mol/L \times 10^{-3} L = 200\ nmol / (1.29\times10^{-13} g) = 1.55\times10^{15}$ nmol/gDW

Translation Rate = 14 AA/s Length = 333 AA Time = 333/14 s.
[Bio : 100233]

$k_{E,i}^{L} = 14/333 = 0.04204\ s^{-1}$ translation elongation rate

R_{LT} = 1000 ribosome/cell. ribosome conc.

$$\theta_{p,i} = \frac{\ln 2}{24h} = \frac{\ln 2}{(24\times3600)s} = 8.0225\times10^{-6}\ s^{-1}$$ degradation

$$\mu = \frac{\ln 2}{40min} = \frac{\ln 2}{(40\times60)\ s} = 2.8881\times10^{-4}\ s^{-1}$$ dilution

$$\tau_{L,i} = \frac{k_{E,i}^{L}}{k_I} = \frac{14/333\ s^{-1}}{1/1.5\ s^{-1}} = 7/111 = 0.06306$$ time constant

$k_{L,i} = 1.45\times10^{-3}$ mmol/gDW $\qquad P_i^* = 8.33\times10^{-4}\ \bar{u}_i$

Plot P_i^* curve wrt $\bar{u}_i$ in Excel [Fig. 1].

(C).

If polysome amplification constant $k_p > 1$

$$k'_{L,i} = k_p k_{L,i} > k_{L,i}$$

p_i^* will be larger, the curve will move upward and become steeper as a function of $\bar{\mu}_i$.

Plot p_i^* curve with larger k' in Excel. [Fig.2]

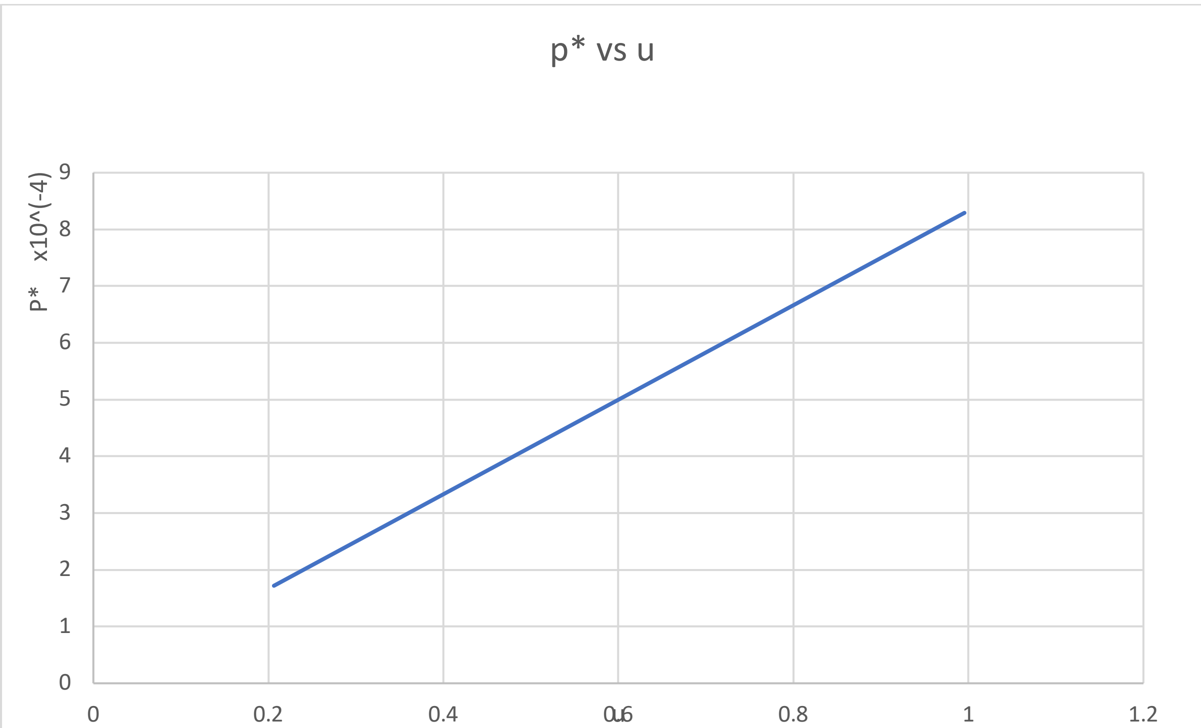

Fig.1

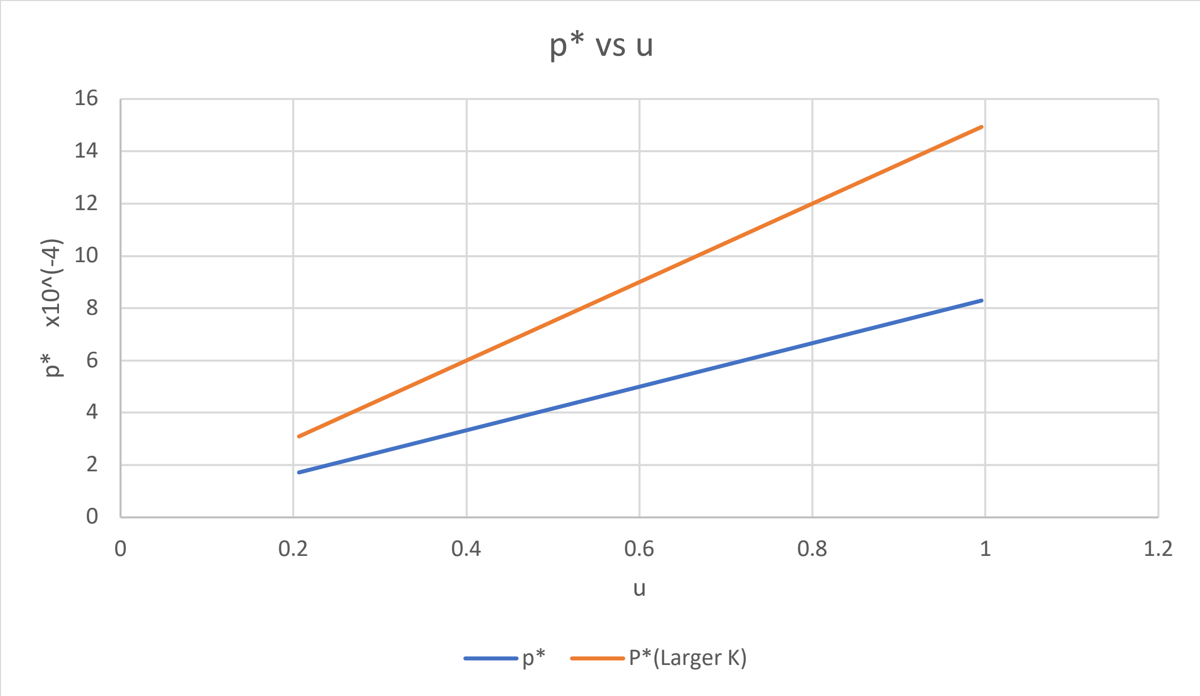

Fig.2

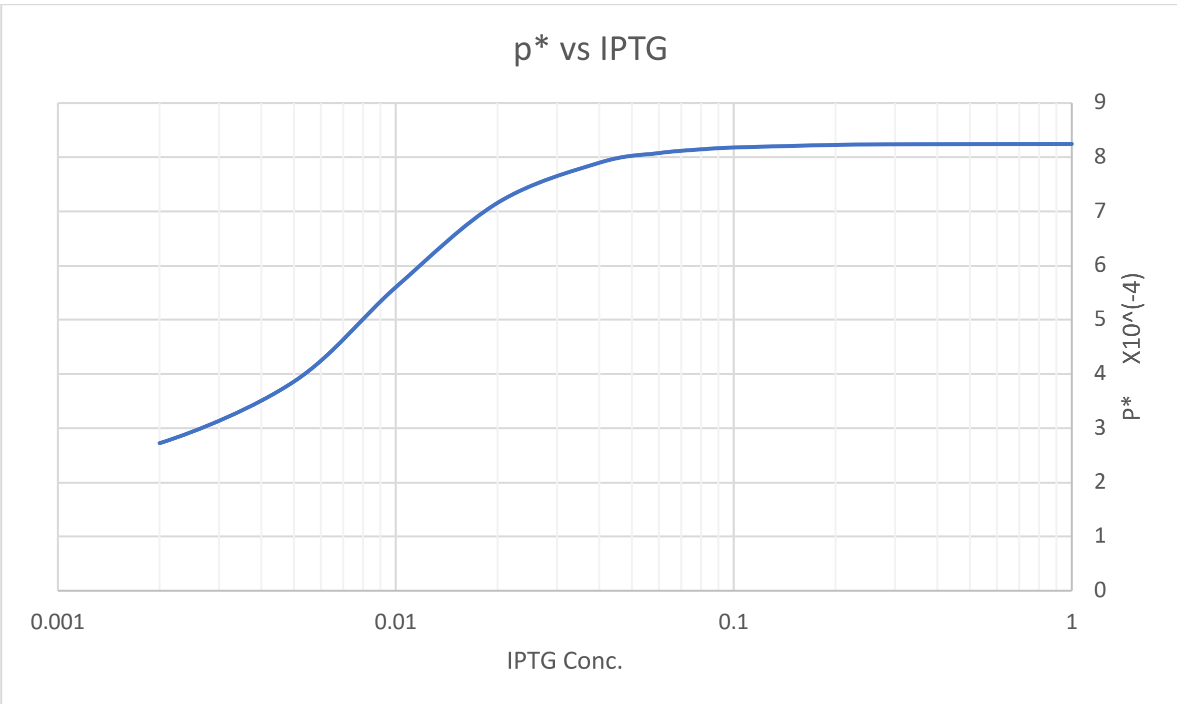

Fig.3

We can also plot the p* with IPTG concentration. Referring to the data and equation obtained from Prelim Q1, the curve is shown in Fig.3.

Additional equation:
u = (W1+W2*f[I])/(1+W1+W2*f[I])
f[I]= ([I]^n)/(KD^n+[I]^n)

KD = 0.09
n = 1.85
W1 = 0.4
W2 = 98