Computer Science and Engineering
Indian Institute of Information Technology - Vadodara, International Campus Diu

Predator-Prey Model Report

Team Members:

- Dhruv Bhalodia (202211011)
- Chinchkar Sneha Achyut (202211013)
- Daivik Hirpara (202211028)
- Isha Jangir (202211031)

Contents

1 Introduction

Predator-prey dynamics play a crucial role in shaping ecosystems by regulating population sizes and maintaining ecological balance. Understanding the interactions between predators and their prey is essential for predicting population fluctuations, ecosystem stability, and the overall health of natural environments. In this project, our team aims to explore predator-prey dynamics through the development and analysis of three distinct models: a grid-based model, an analytical model based on Lotka-Volterra equations, and a graphical model for visual representation. By employing these models, we seek to gain insights into the factors influencing predator-prey relationships and their implications for ecosystem management and conservation.

1.1 Example

Deep in the redwood forests of California, dusky-footed wood rats provide up to 80% of the diet for the spotted owl, the main predator of the wood rat. Example 1 uses a linear dynamical system to model the physical system of the owls and the rats. (Admittedly, the model is unrealistic in several respects, but it can provide a starting point for the study of more complicated nonlinear models used by environmental scientists.)

Denote the owl and wood rat populations at time k by $x_k = [O_k\ R_k]$ where k is the time in months (k in x_k, O_k, R_k is in subscript), O_k is the number of owls in the region studied, and R_k is the number of rats (measured in thousands). Suppose

$$O_{k+1} = 0.5(O_k) + 0.4(R_k), \quad R_{k+1} = -p \cdot O_k + 1.1(R_k) \tag{1}$$

where p is a positive parameter. The term $(0.5)O_k$ in the first equation represents that without wood rats for food, only half of the owls will survive each month. Similarly, the term $(1.1)R_k$ in the second equation indicates that without owls as predators, the rat population will grow by 10% per month. If rats are plentiful, the $(0.4)R_k$ will tend to make the owl population rise, Additionally, the term $-p \cdot O_k$ represents the deaths of rats due to predation by owls, where $1000p$ is the average number of rats eaten by one owl in one month.) Determine the evolution of this system when the predation parameter p is 0.104 .

SOLUTION: SOLUTION When $p = 0.104$, the eigenvalues of the coefficient matrix A for the equations in (3) turn out to be $\lambda_1 = 1.02$ and $\lambda_2 = 0.58$. Corresponding eigenvectors are

$\mathrm{v}_1 = [10\ 13], v_2 = [5\ 1]$

Aninitial x_0 can be written as $x_0 = c_1 v_1 + c_2 v_2$. Then, for $k >= 0$,

$$x_k = c_1(1.02)^k v_1 + c_2(0.58)^k v_2 = c_1(1.02)^k[10\ 13] + c_2(0.58)^k[5\ 1]$$

As $k \to \infty$, λ_2 rapidly approaches zero. Assume $c_i > 0$. Then, for all sufficiently large k, x is approximately the same as $c_1(1.02)^k v_1$, and we write

$$x_k \approx c_1(1.02)^k[10\ 13] \tag{2}$$

The approximation in (4) improves as k increases, and so for large k,

$$x_{k+1} \approx c_1(1.02)^{k+1}[10\ 13] = (1.02)c_1(1.02)^k[10\ 13] \approx 1.02x_k \tag{3}$$

The approximation in (3) says that eventually both entries of x (the numbers of owls and rats) grow by a factor of almost 1.02 each month, a 2% monthly growth rate. By (2), x is approximately a multiple of $(10, 13)$, so the entries in x are nearly in the same ratio as 10 to 13. That is, for every 10 owls there are about 13 thousand rats.

Example 1 illustrates two general facts about a dynamical system $x_{k+1} = Ax$ in which A is $n \times n$, its eigenvalues satisfy $|\lambda_1| \geq 1$ and $1 > |\lambda_i|$ for $i = 2, \ldots, n$, and v_1 is an eigenvector corresponding to λ_1. If x_0 is given with $c_1 \neq 0$, then for all sufficiently large k,

$$x_{k+1} \approx \lambda_1^k x_0 \tag{4}$$

$$x_k \approx c_1 \lambda_1^k v_1 \tag{5}$$

The approximations in (4) and (5) can be made as close as desired by taking k sufficiently large. By (4), the x eventually grow almost by a factor of λ_1 each time, so λ_1 determines the eventual growth rate of the system. Also, by (5), the ratio of any two entries in x (for large k) is nearly the same as the ratio of the corresponding entries in v_1.

2 Objective

The primary objective of our project is to investigate predator-prey dynamics using different modeling approaches and techniques. Specifically, our goals include:

- Developing a grid-based simulation model to simulate the interactions between prey (fish) and predators (sharks) within a two-dimensional grid environment.
- Constructing an analytical model based on Lotka-Volterra equations to describe the population dynamics of prey and predators over time.
- Creating a graphical model to visually represent the trajectory of prey and predator populations and explore the phase space of the system.
- Analyzing the outcomes of each model to gain insights into predator-prey dynamics, including population fluctuations, equilibrium points, and the effects of various ecological factors.

3 Methodology

3.1 Grid-based Model

Function initialize_grid:

```
Initialize a 2D grid with empty cells
```

Function populate_grid:

```
Calculate initial populations of fish and sharks based on predefined ratio
Randomly place fish and sharks on the grid
```

Function update_grid:

```
For each cell in the grid:
    Count fish and shark neighbors
    Apply toroidal boundary conditions
    Update cell based on predator-prey interactions and rules
    Breed new fish and sharks based on predefined time intervals
```

Function print_grid:

```
Print the current state of the grid
```

Main:

```
Initialize the grid
Populate the grid with initial fish and shark populations
Simulate predator-prey interactions for a predefined number of time steps
Print the state of the grid at each time step
```

3.2 Analytical Model

Function dx_dt:

```
Calculate the rate of change of prey population over time
Return the result
```

Function dy_dt:

```
Calculate the rate of change of predator population over time
Return the result
```

Main:

```
Initialize parameters and initial populations
Calculate equilibrium points
Print equilibrium points
Calculate and display phase trajectory using numerical integration
```

3.3 Graphical Model

Function dx_dt:

```
Calculate the rate of change of prey population over time
Return the result
```

Function dy_dt:

```
Calculate the rate of change of predator population over time
Return the result
```

Main:

```
Initialize parameters and initial populations
Initialize gnuplot
Plot trajectory and direction field using numerical integration
Close gnuplot
```

4 Observations and result

4.1 Numerical Observation

```
Enter initial value for x: 2
Enter initial value for y: 6
Enter value for alpha: 2
Enter value for beta: 1
Enter value for delta: 1
Enter value for gamma: 3
Equilibrium points:
x_eq = 3.00
y_eq = 2.00

Phase trajectory:
(x, y) = (2.00, 6.00)
(1.92, 5.94)
(1.85, 5.87)
(1.78, 5.80)
(1.71, 5.73)
(1.65, 5.66)
(1.59, 5.58)
(1.54, 5.50)
(1.48, 5.42)
(1.44, 5.33)
(1.39, 5.25)
(1.35, 5.17)
(1.30, 5.08)
(1.27, 4.99)
(1.23, 4.91)
(1.19, 4.82)
(1.16, 4.73)
(1.13, 4.64)
(1.10, 4.56)
(1.07, 4.47)
(1.05, 4.39)
(1.02, 4.30)
```

Figure 1: Numerical observation of predator-prey dynamics

4.2 Results

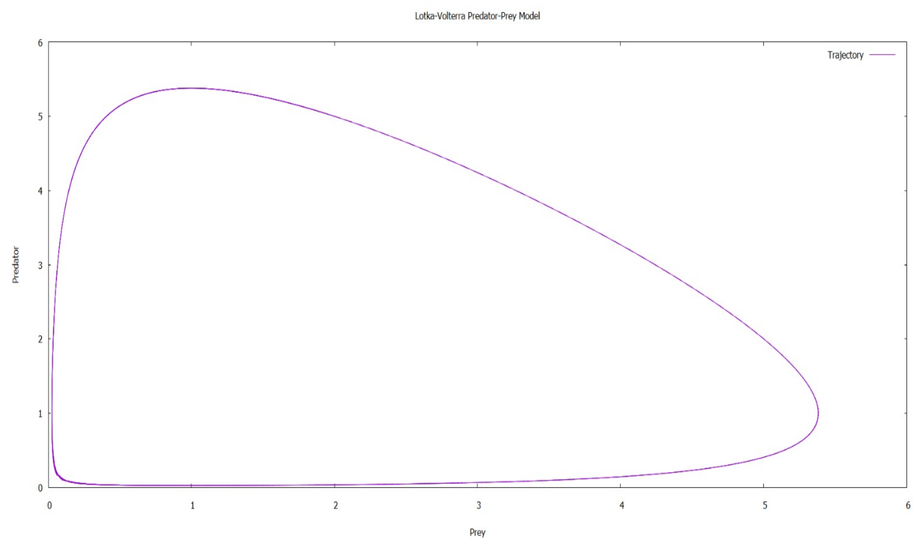

Figure 2: Result 1: Graphical Representation y-axis:Predator x-axis:Prey

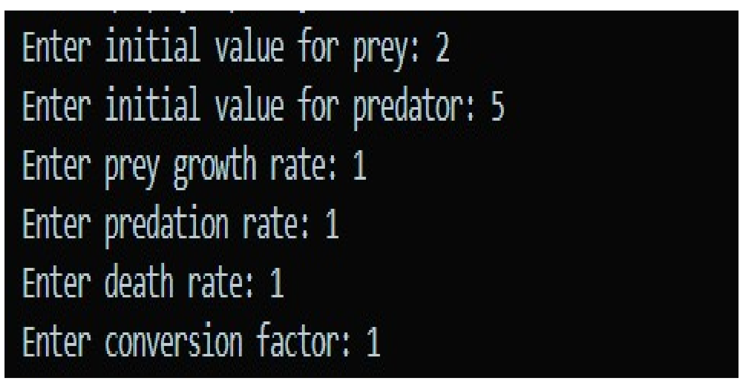

Figure 3: Result 1: Input

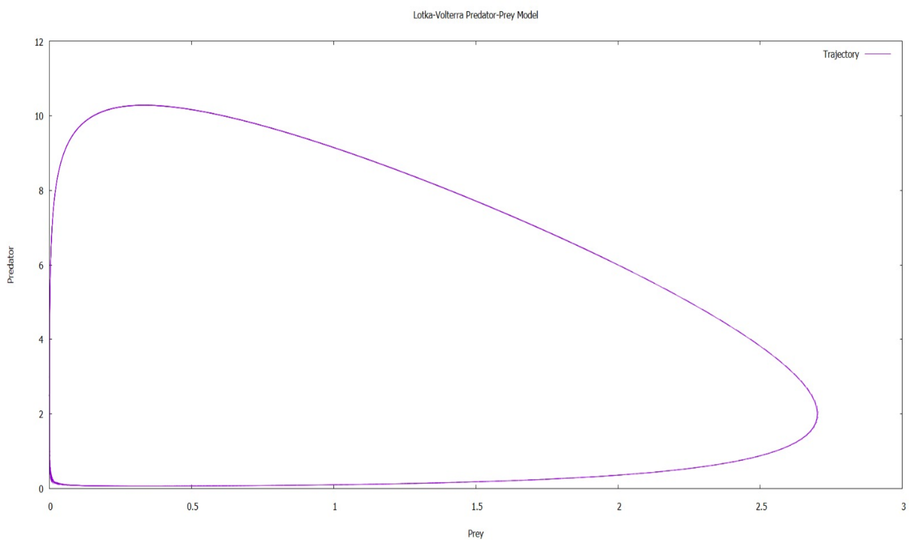

Figure 4: Result 2: Graphical Representation y-axis:Predator x-axis:Prey

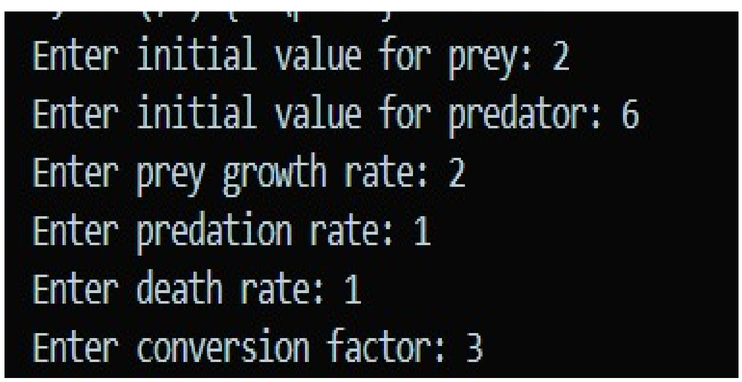

Figure 5: Result 2: Input

5 Conclusion

In summary, our exploration of the predator-prey model has provided valuable insights into the dynamics of ecological systems. Through the development and analysis of grid-based, analytical, and graphical models, we've gained a deeper understanding of how predator and prey populations interact over time. By simulating these interactions and examining equilibrium points, we've uncovered key factors influencing population fluctuations and ecosystem stability. Our findings underscore the importance of studying predator-prey dynamics for ecosystem management and conservation efforts. Moving forward, this knowledge can inform strategies for maintaining ecological balance and preserving biodiversity in natural environments.

6 References

Lay, D., McDonald, J., Lay, S. (2023). Linear Algebra and Its Applications.(Pearson) (5th ed.).