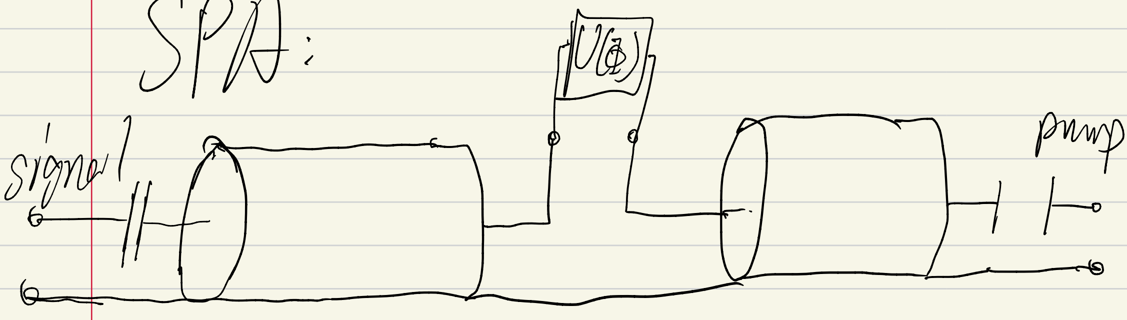
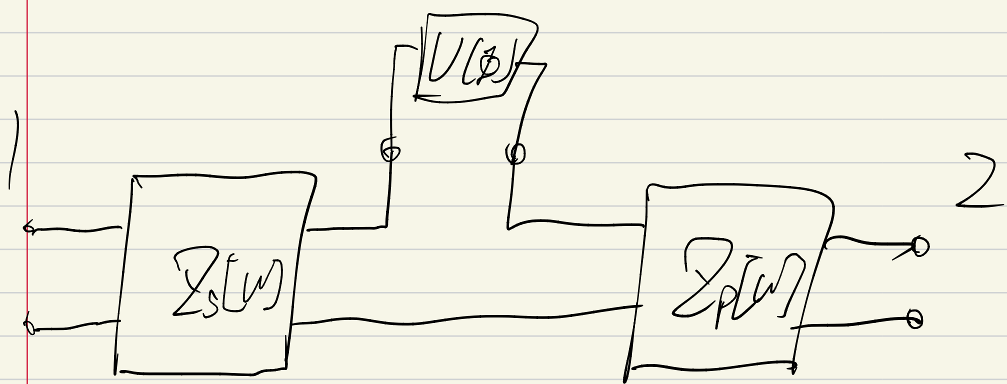
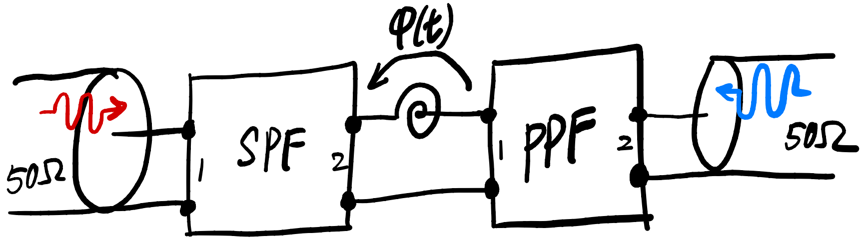
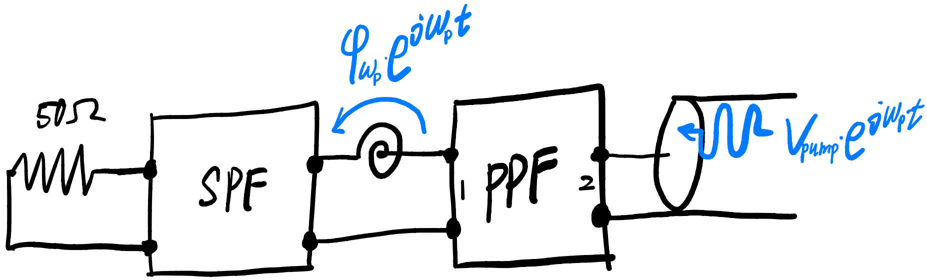
SPA:

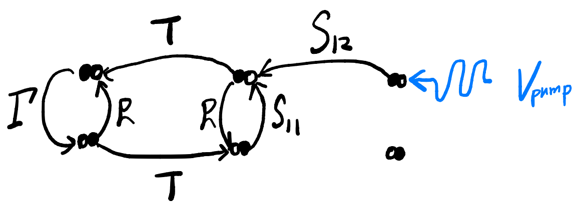


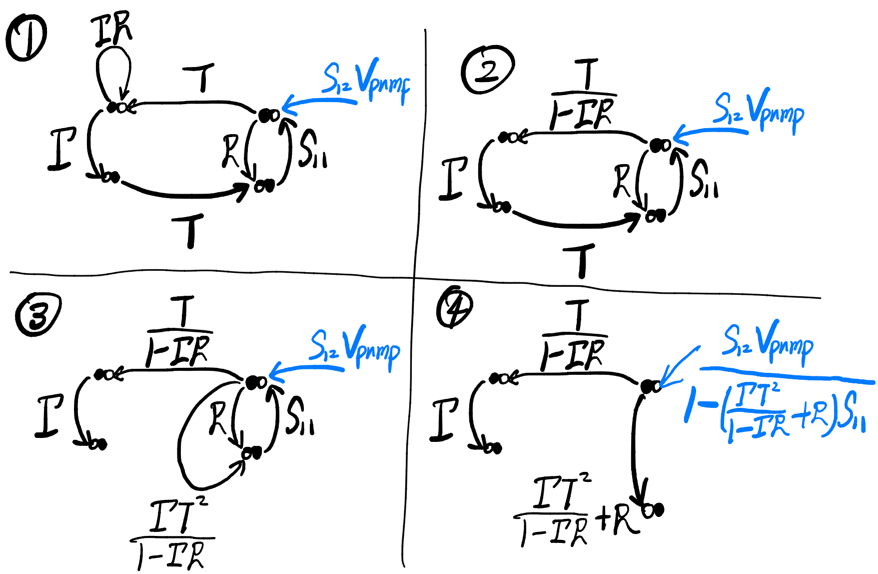


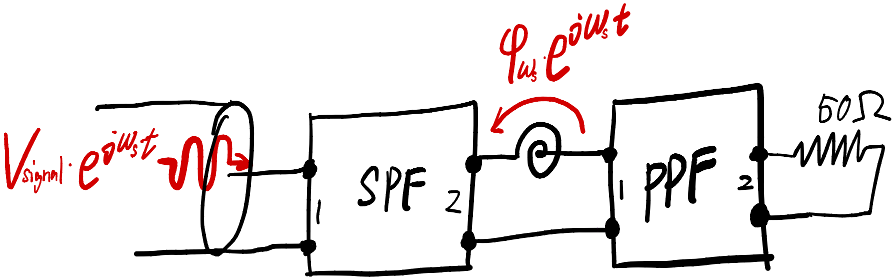
1. $S11[\omega_s]$ gain
2. $S21[\omega_s]$ small (SNR, signal leakage)
3. $S12[\omega_p]$ small (protection)

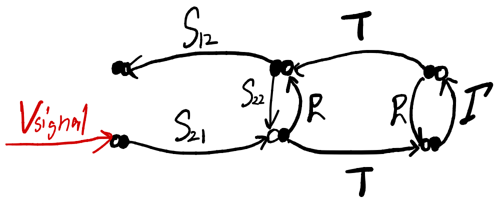

φ(t)
SPF
PPF
1
2
1
2
50Ω
50Ω

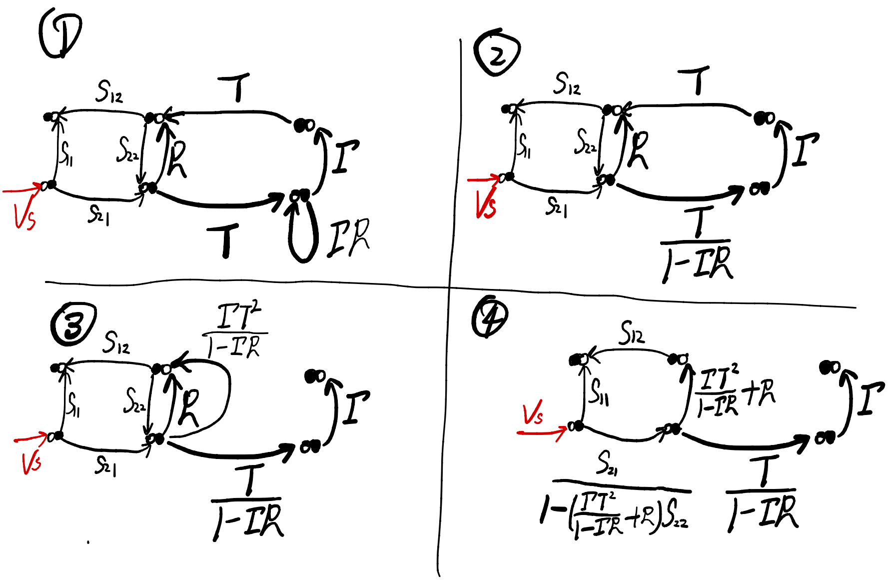

$\varphi_{\omega_p} \cdot e^{j\omega_p t}$
50Ω
SPF
PPF
1
2
1
2
$V_{pump} \cdot e^{j\omega_p t}$


T
S_{12}
Γ
R
R
S_{11}
V_{pump}
T

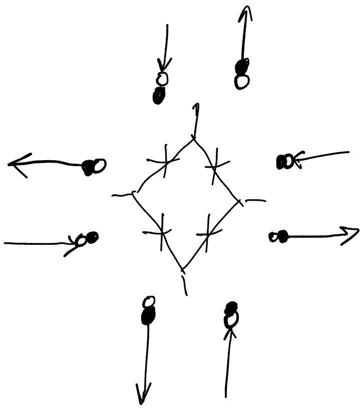

①
ΓR
T
$S_{12}V_{pump}$
Γ
R
S_{11}
T
②
$\frac{T}{1-\Gamma R}$
$S_{12}V_{pump}$
Γ
R
S_{11}
T
③
$\frac{T}{1-\Gamma R}$
$S_{12}V_{pump}$
Γ
R
S_{11}
$\frac{\Gamma T^2}{1-\Gamma R}$
④
$\frac{T}{1-\Gamma R}$
$\frac{S_{12}V_{pump}}{1-\left(\frac{\Gamma T^2}{1-\Gamma R}+R\right)S_{11}}$
Γ
$\frac{\Gamma T^2}{1-\Gamma R}+R$



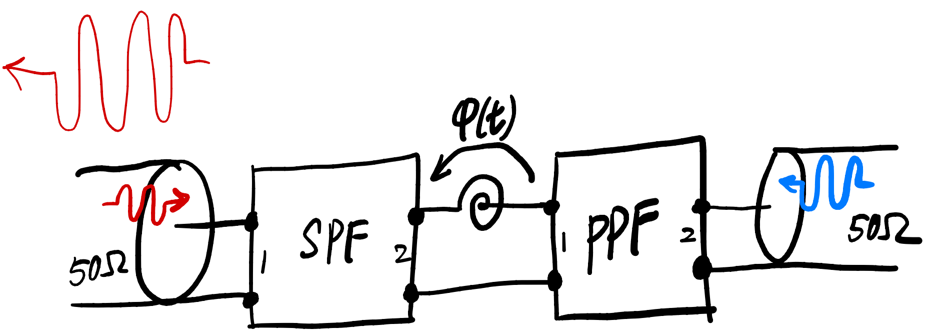


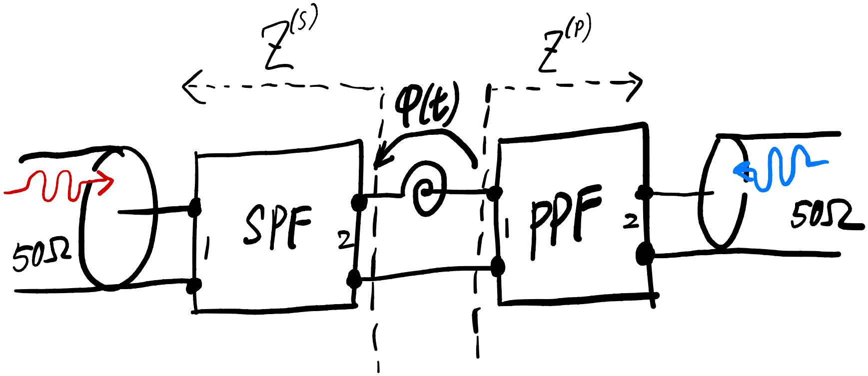


$$\frac{\Gamma T^2}{1-\Gamma R}+R=\frac{\Gamma\left(\frac{2Z_c}{Z+2Z_c}\right)^2}{1-\Gamma\frac{Z}{Z+2Z_c}}+\frac{Z}{Z+2Z_c}$$

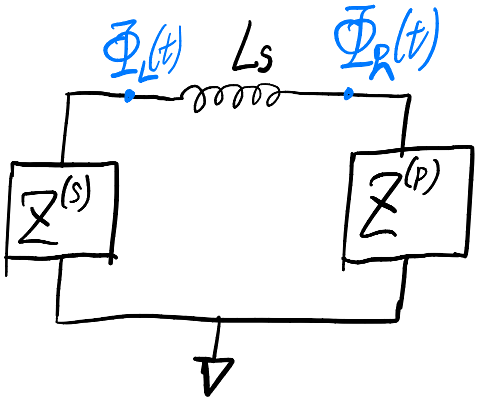


$$\begin{pmatrix} I_{\omega_s} \\ I^*_{\omega_i} \end{pmatrix} = \frac{1}{j L_s} \begin{pmatrix} \frac{1+\delta_s}{\omega_s} & \frac{-\varepsilon}{\omega_i} \\ \frac{-\varepsilon^*}{\omega_s} & \frac{1+\delta_i}{\omega_i} \end{pmatrix} \begin{pmatrix} V_{\omega_s} \\ V^*_{\omega_i} \end{pmatrix}$$

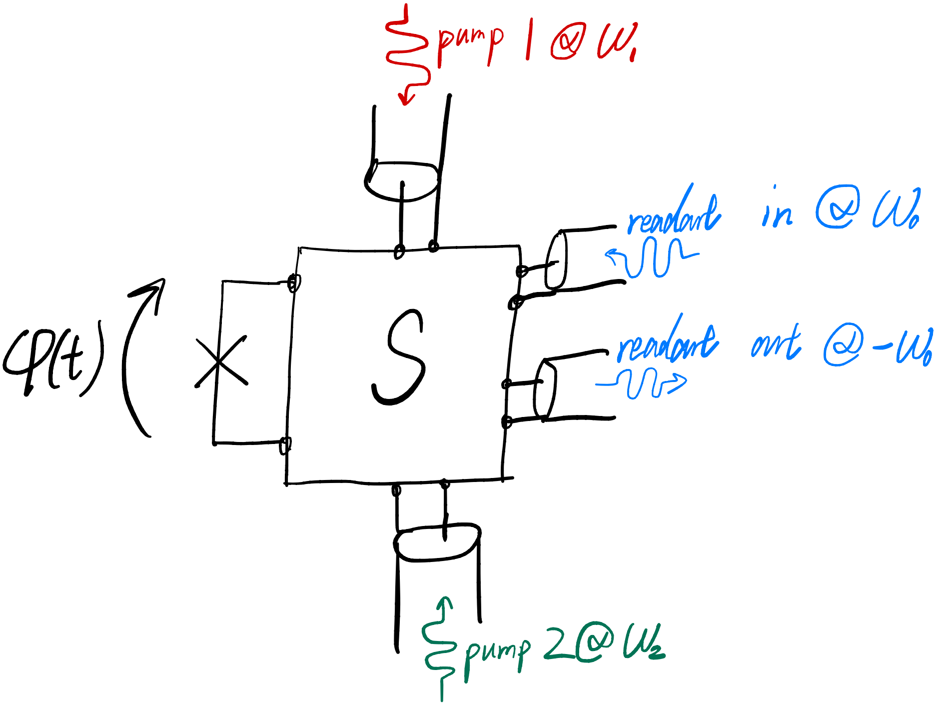


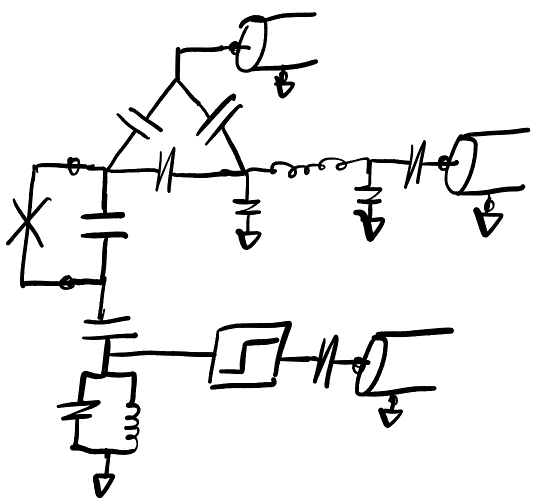


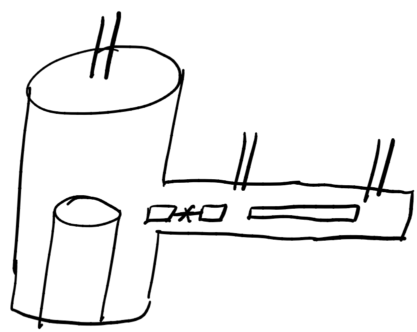
Z^(S)
Z^(P)
φ(t)
SPF
PPF
1
2
1
2
50Ω
50Ω

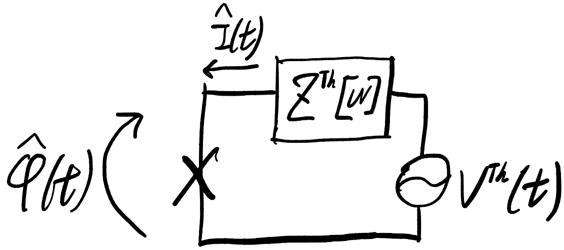


Φ_L(t)
L_s
Φ_R(t)
Z^(S)
Z^(P)


pump 1 @ ω_1
readout in @ ω_0
readout out @ $-\omega_0$
$\Phi(t)$
S
pump 2 @ ω_2







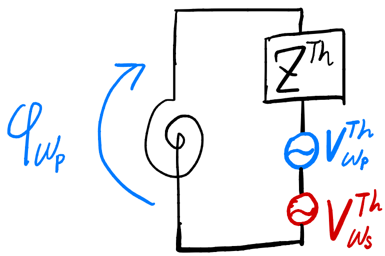
$$\mathrm{Re}\left(V_{\omega_1} e^{j\omega_1 t} + V_{\omega_2} e^{j\omega_2 t} + V_{\omega_0} e^{j\omega_0 t}\right)$$

Z^{Th} $\quad V^{Th} \quad V^{Th}$

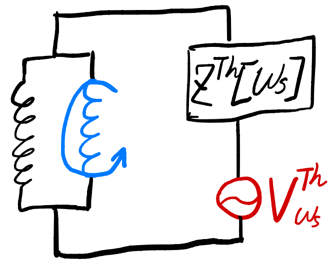
$$\hat{I} = \frac{E_J}{\phi_0}\left(c_2 \hat{\varphi} + \frac{c_3}{2}\hat{\varphi}^2 + \frac{c_4}{6}\hat{\varphi}^3 + \cdots\right)$$

$$I_{\omega_s} = \frac{E_J}{\phi_0}\left[c_2 \varphi_{\omega_s} + \frac{c_3}{2}\varphi_{\omega_p}\varphi_{\omega_i}^* + \frac{c_4}{8}\left(|\varphi_{\omega_s}|^2 + 2|\varphi_{\omega_i}|^2 + 2|\varphi_{\omega_p}|^2\right)\varphi_{\omega_s}\right]$$

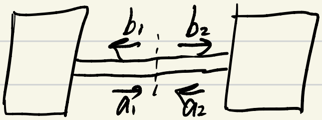
$$Y_J[\omega_s] = \frac{I_{\omega_s}}{j\omega_s \phi_0 \varphi_{\omega_s}} = \frac{1}{j\omega_s L_J} + \frac{1}{j\omega_s L_J}\cdot\frac{c_3\varphi_{\omega_p}}{2c_2}\cdot\frac{\varphi_{\omega_i}^*}{\varphi_{\omega_s}}$$



φ_{ω_p}
Z^{Th}
$V^{Th}_{\omega_p}$
$V^{Th}_{\omega_s}$



$Z^{Th}[\omega_s]$
$V^{Th}_{\omega_s}$

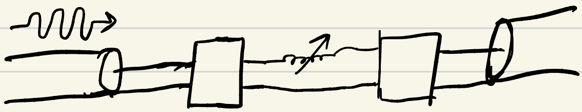


with $\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = S \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$

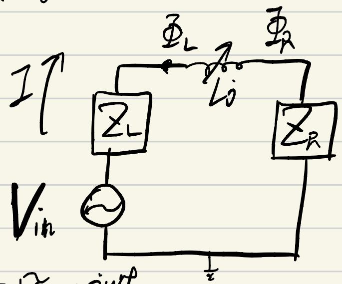
We can redefine "Transmission matrix" T_s :

$$\begin{pmatrix} b_1 \\ a_1 \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$$

$$T_s \begin{cases} T_{11} = \dfrac{-\mathrm{det}(S)}{S_{21}} \\ T_{12} = \dfrac{S_{11}}{S_{21}} \\ T_{21} = \dfrac{-S_{22}}{S_{21}} \\ T_{22} = \dfrac{1}{S_{21}} \end{cases}$$



@ $\omega \simeq \omega_s$



$= \tilde{V}_{in} \cdot e^{j\omega t}$

$$\begin{cases} j\omega\Phi_L = V_{in} - Z_L[\omega]\cdot I \\ j\omega(\Phi_L - \Phi_R) = j\omega L_j \cdot I \\ j\omega\Phi_R = Z_R[\omega]\cdot I \end{cases}$$

$$I = \tilde{I}\cdot e^{j\omega t}$$

$$\left(Z_R[\omega] + j\omega L_j + Z_L[\omega]\right)\cdot\tilde{I} = \tilde{V}_{in}$$

$$\left[j\left(\mathrm{Im}\, Z_R[\omega] + \mathrm{Im}\, Z_L[\omega] + \omega L_j\right) + \mathrm{Re}\, Z_L[\omega]\right]\cdot\tilde{I} = \tilde{V}_{in}$$