Homework 1 Part 2

Xiongming Dai

Homework 1, Part 2

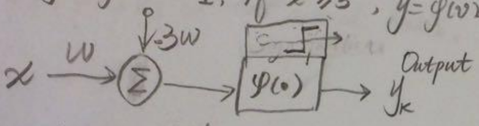
1. Based on the definition of Sign function.

$$\varphi(v) = \begin{cases} 1 & \text{if } v \geq 0 \\ -1 & \text{if } v < 0 \end{cases}$$

a. The transfer function is Symmetic Hard-Limit Transfer Function

b. $wx+b=v$. $3w+b=0$, so the bias b equals to $-3w$.
$b=-3w$, this bias is highly related to the input weight

c. $y=\varphi(v)=\varphi(wx-3w)$, w denotes the weight of the network
$y=\varphi(v)=1$, if $x \geq 3$, $y=\varphi(v)=-1$, if $x<3$.



Input signal: x.
Weight: w.
bias: $b=-3w$.
Summing junction: $v=wx-3w$.
Activation function: $\varphi(\cdot)$: Symmetric Hard-Limit Transfer Function
Output signal: $y_k = \text{Sign}(wx-3w)$

Matlab code:

```
p=[1;2;3;4];
w=ones(1,6);
b=-3;
for i=1:4
    n=w(i)*p(i)+b
    a(i)=hardlims(n);
end
plot(p,a);
```

1.

2. Because the output of 0.75 is required. so. we can consider Logistic function $\varphi(v) = \frac{1}{1+\exp(-av)}$ as the activation function. a is the slope parameter.

$v = wx^T + b = [1,-2]\begin{bmatrix}-3\\5\end{bmatrix} + b = -13+b$.

a. Yes, I suppose so, the combination of bias and activation function can be written as: $\varphi(v) = \frac{1}{1+\exp[-a(-13+b)]} = \frac{1}{1+\exp(13a-ab)}$. (a is the slope parameter)

b. if bias $b=0$. $\varphi(v) = \frac{1}{1+\exp(13a)}$, $(a>0)$, so $\varphi(v) < \frac{1}{2}$, which can not make the output equal to 0.75, so, it cannot be done with a bias of zero.

c. the appropriate activation function is logistic function

d. $W = [w_1, w_2]$, $v = wx^T + b = [w_1, w_2]\begin{bmatrix}-3\\5\end{bmatrix} + b = -3w_1 + 5w_2 + b$

$\varphi(v) = \frac{1}{1+\exp(-av)} = 0.75$. from a,b,c, we can find a bias:

$\varphi(v) = \frac{1}{1+\exp(13-b)} = 0.75\ (a=1) = \frac{1}{1+\exp(-v)}$, $-3w_1 + 5w_2 = -13$ ①

if we meet the condition of ①, we also can realize the same job, w can be $[3, -0.8]$.

2.

3.

3. Threshold function or Heaviside function can be selected.

$$\varphi(v) = \begin{cases} 1 & \text{if } v \geq 0 \\ 0 & \text{if } v < 0 \end{cases}$$

$v = wx + b$ $\quad$ $w + b < 0$ from $x = 1$, $\varphi(v) = 0$.

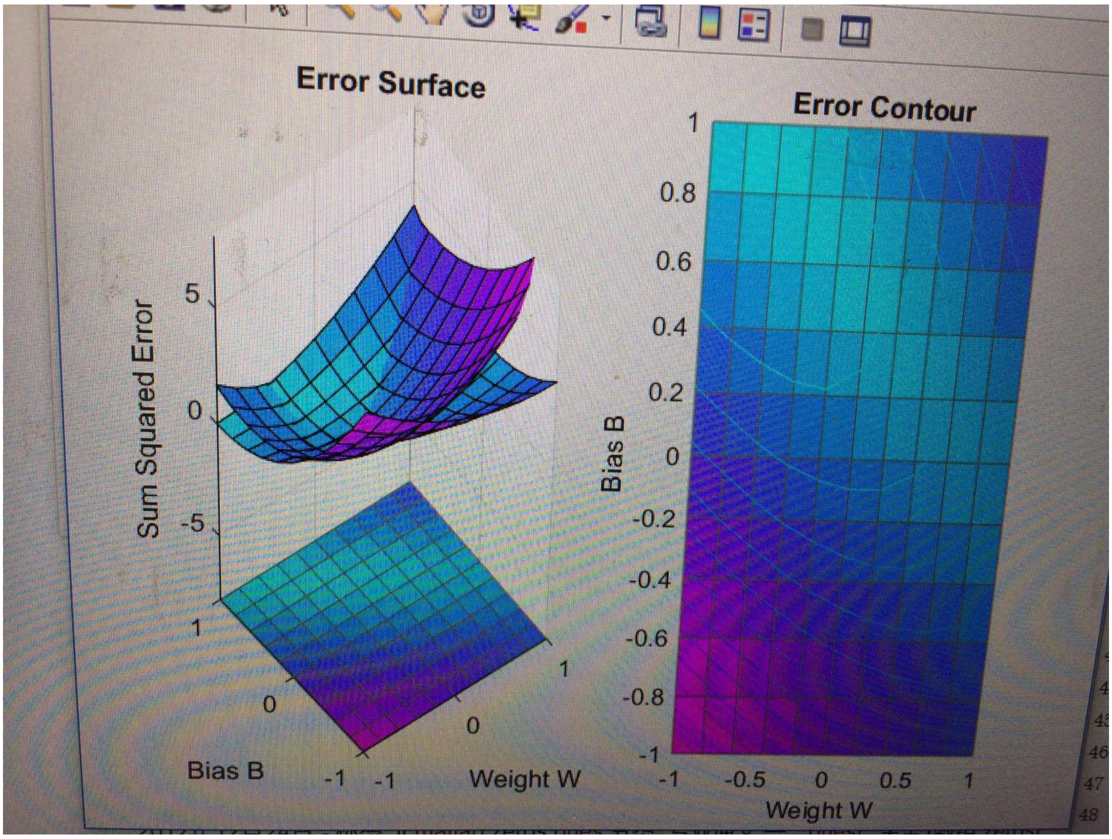
$b \geq 0$ from $x = 0$, $\varphi(v) = 1$ ①

so $w < -b$ ② so $w < -b \leq 0$ ③

We can choose any arbitrary value of w and b when ③ is satisfied. ~~We~~ Such as $w = -5$, $b = 4$. the transfic function used is "Threshold function"

```
X = [1.0 0];
T = [0 1.0];
w = -1:0.2:1;   b= -1:0.2:1;
ES = errsurf(X, T, w, b, 'purelin');
plotes(w, b, ES);
```

script



Error Surface
Sum Squared Error
5
0
-5
1
0
Bias B
-1
-1
0
1
Weight W
Error Contour
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
Bias B
-1
-0.5
0
0.5
1
Weight W

4.

4. i: $\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}1\\2\\1\end{bmatrix}$, the rank of the matrix $M = \begin{bmatrix}1&1&1\\2&0&2\\3&1&1\end{bmatrix}$

$Rank(M) = 3$, so, vectors of i are independent, the dimension of the vector space spanned by the set is 3.

ii. $\cos(2t) = \cos^2 t - \sin^2 t$, so the set of vectors is not independent

the dimension of the vector space spanned by the set is 2.

iii: $a(1+t) + b(1-t) = 0$

$a+b+(a-b)t = 0$ so. $1+t, 1-t$ are independent.

$\left.\begin{array}{l} a+b=0 \\ a-b=0 \end{array}\right\} \Rightarrow a=b=0$ the dimention is 2.

iv. Matrix of the set is $M = \begin{bmatrix}1&1&3\\2&0&4\\2&0&4\\1&1&3\end{bmatrix}$, the rank of M is

$Rank(M) = 2 < 3$. so. they are not independent.

the dimension is 2.

```
%Problem 4
M_i=[1 1 1;2 0 2;3 1 1];
r=rank(M_i)
M_iv=[1 1 3;2 0 4;2 0 4;1 1 3];
r=rank(M_iv)
```

5. ① $M=[y_1,y_2,y_3]=\begin{bmatrix}1&1&1\\0&1&1\\0&0&1\end{bmatrix}$, the rank of M. Rank(M)=3.

so, the dimension spanned by three vectors is 3. they are independent

② Gram-Schmidt orthogonalization:

$$\beta_1=y_1=\begin{bmatrix}1\\0\\0\end{bmatrix},\ \eta_1=\begin{bmatrix}1\\0\\0\end{bmatrix}$$

$$\beta_2=y_2-\langle y_2,\eta_1\rangle\eta_1=\begin{bmatrix}1\\1\\0\end{bmatrix}-(y_2^T\eta_1)\cdot\eta_1=\begin{bmatrix}1\\1\\0\end{bmatrix}-(1+0)\cdot\begin{bmatrix}1\\0\\0\end{bmatrix}=\begin{bmatrix}0\\1\\0\end{bmatrix},\ \eta_2=\begin{bmatrix}0\\1\\0\end{bmatrix}$$

$$\beta_3=y_3-\langle y_3,\eta_1\rangle\eta_1-\langle y_3,\eta_2\rangle\eta_2=\begin{bmatrix}1\\1\\1\end{bmatrix}-\begin{bmatrix}1\\0\\0\end{bmatrix}-\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}0\\0\\1\end{bmatrix}$$

β_1,β_2,β_3 is the normal basis.

5.

Matlab code:

```
function mat = Schmidt(v)
if(size(v,2)==1)
    v=v';
    mat = eye(size(v',1));
    [a b ] =max(v);
    mat(1,:)=v;
    if(b~=1)
    mat(b,:) = [1 zeros(1,size(mat,1)-1)];
    end
else
    mat = v;
end
for i=2:length(v)
    mat(i-1,:)=mat(i-1,:)/sqrt(mat(i-1,:)*mat(i-1,:)');
    b = mat(1:i-1,:)';
    P = eye(size(b,1))-b*(b'*b)^-1*b';
    mat(i,:) = mat(i,:)*P;
```

```
    end
mat(end,:)=mat(end,:)/sqrt(mat(end,:)*mat(end,:)');
end
```

After running:

```
>> M=[1 1 1;0 1 1;0 0 1];
Schmidt(M')
[100.0%]
ans =

     1     0     0
     0     1     0
     0     0     1
```