CS 5200 Spring 2018 Analysis of Algorithms-Homework1

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1 Honor Codes

(a) Based on the syllabus uploaded into CANVAS by Dr.George. The Honor Code developed and endorsed by the Missouri S & T Student Council is at `https://stuco.mst.edu/honor-code`. Unfortunately, when I try to open this link using Google search. It turns out to be broken and it shows "Oops! That page cannot be found". Luckily, I find it other links that describe the MST Honor Code at `http://registrar.mst.edu/media/administrative/registrar/documents/academicregulations/2017-2018%20academic%20regulations_rev_9.22.17.pdf`

The Standard of Conduct violations:
1)Academic dishonesty:
a.Cheating
b.Plagiarism
c.Sabotage
2)Violation of University policies
3)Misuse in accordance with University policy of computing resources
a.Unauthorized entry into a file to use, read, or change the contents, or for any other purpose.
b.Unauthorized transfer of a file.
c.Use of computing facilities to interfere with the work of another students, faculty member, or university official.

Accordingly, proposed sanction is showed as follows:
a.Warning.
b.Probation.
c.Loss of Privileges.
d.Discretionary Sanctions.
e.University Dismissal.
f.University Suspension.
g.University Expulsion.

If someone does not obey the rules and violates these regulations, he or she might be published based on the severity of the issue. If copying other peoples work and submitting it as your own. it is the kind of plagiarism,he or she may be deal with as warning or discretionary sanctions.

(b)The URL for the ACM Code of Conduct is at `https://www.acm.org/about-acm/acm-code-of-ethics-and-professional-conduct`.

The major section of the code, sections and subsections contained are as follows:
1.General Moral Imperatives.
1.1.Contribute to society and human well-being.
1.2.Avoid harm to others.
1.3.Be honest and trustworthy.
1.4.Be fair and take action not to discriminate.
1.5.Honor property rights including copyrights and patent.
1.6.Give proper credit for intellectual property.
1.7.Respect the privacy of others.
1.8.Honor confidentiality.
2.More Specific Professional Responsibilities.
2.1. Strive to achieve the hightest quality, effectiveness and dignity in both the process and products of professional work.
2.2 Acquire and maintain professional competence.
2.3 Know and respect existing laws pertaining to professional work.
2.4 Accept and provide appropriate professional review.
2.5 Give comprehensive and thorough evaluations of computer systems and their impacts, including analysis of possible risks.
2.6 Honor contracts, agreements, and assigned responsibilities.
2.7 Improve public understanding of computing and its consequences.
2.8 Access computing and communication resources only when authorized to do so.
3.Organizational Leadership Imperatives.
3.1 Articulate social responsibilities of members of an organizational unit and encourage full acceptance of those responsibilities.
3.2 Manage personnel and resources to design and build information systems that enhance the quality of working life.
3.3 Acknowledge and support proper and authorized uses of an organization's computing and communication resources.
3.4 Ensure that users and those who will be affected by a system have their needs clearly articulated during the assessment and design of requirements; later the system must be validated to meet requirements.
3.5 Articulate and support policies that protect the dignity of users and others affected by a computing system.
3.6 Create opportunities for members of the organization to learn the principles and limitations of computer systems.
4.Compliance with the Code.
4.1 Uphold and promote the principles of this Code.
4.2 Treat violations of this code as inconsistent with membership in the ACM.

Copying other people's work and submitting it as your own, is a clear violation of such ethical principles. Plagiarism can also represent a violation of copyright law, punishable by statute. Plagiarism manifests itself in a variety of forms, including
1.Verbatim copying, near-verbatim copying, or purposely paraphrasing portions of another author's paper;
2.Copying elements of another author's paper, such as equations or illustrations that are not common knowledge, or copying or purposely paraphrasing sentences without citing the source; and
3.Verbatim copying of portions of another author's paper with citing but not clearly differentiating what text has been copied (e.g., not applying quotation marks correctly) and/or not citing the source

correctly.

ACM and the ACM Publications Board place the investigation of each claim of plagiarism at the highest priority for resolution and action.

ACM will inform the Department Chair, Dean, or supervisor of the authors of the finding of plagiarism.

1.The authors will be asked to write a formal letter of apology to the authors of the plagiarized paper, including an admission of plagiarism.

2.If the paper has appeared in press, ACM will post a Notice of Plagiarism based on the investigation on the ACM Digital Library's citation page of the plagiarizing paper and will remove access to the full text. The paper itself will be kept in the database for future research or legal purposes.

3.If the paper is under submission, the paper can be automatically rejected by the Editor-in-Chief or the Program Chair without further revisions and without any further plagiarism investigation coordinated by the ACM Director of Publications. In addition, a letter of warning will be sent by the Editor-in-Chief or the Program Chair to the authors with a copy of the ACM Policy and Procedures on Plagiarism.

(c)I have read the syllabus and it describes as follows:

"5. I want people to work on the homework individually. You can talk to each other and give help, but this help should not take the form of letting other people copy your work. It is important that you understand how to do all the problems on your own so you can do well on the exams. If you need help, please ask questions in class, on Canvas, and come during office hours"

"23. I am only interested in grading your original work. I am not interested in grading solutions to the problems that have been posted by other professors on the Web. You can lose many points if you simply copy solutions from other people or other sources."

Therefore, I have known that I will lose many points if I just simply copy solutions from other people or other sources.I cannot let other classmate copy my work, but I can talk to each other and give help.

2 Geographical related

In order to give an answer to the problem of this part. Frankly speaking, I have referred to the "WIKIPEDIA".

(a)The age of the Earth is about 4.540.05 billion years (4.54 109 years 1%). From "WIKIPEDIA", this dating is based on evidence from radiometric age-dating of meteorite material and is consistent with the radiometric ages of the oldest-known terrestrial and lunar samples.

(b)Based on the clues from Meteorites, The age of the solar system, derived from the study of meteorites (thought to be the oldest accessible material around) is near 5 billion years.

(c)The approximate age of the Milky Way as 13.6 billion years, based on this article: (`https://www.universetoday.com/9828/estimating-the-age-of-the-milky-way/`)

(d)From "WIKIPEDIA", the estimated age of the Universe is 13.7990.021 billion (109) years.

(e)From "WIKIPEDIA",Earth has about 5 billion years left. Usually, the lifespan of the earth is wholly dependent upon the aging of the sun. In about 5 to 6 billion years from now the sun is going to reach a point where its core can no longer sustain the nuclear fusion is has been and it will grow into a red giant. That means it is possible the sun could grow enough to engulf the earth itself but even if it doesn't, life will cease to exist on earth at some point during the sun's growth into a red giant.

(f)The source used and referred is at `https://blog.chron.com/sciguy/2012/11/`

and https://arxiv.org/pdf/1210.5721v1.pdf.
I have found a paper that topic is “Swansong Biospheres: Refuges for life and novel microbial biospheres on terrestrial planets near the end of their habitable lifetimes” published by Jack T. OMalley-James and Jane S.Greaves, etc. From this paper, it is easy to find that the maximum lifetime for life on Earth of 2.8 Gyr from present was found, given the presence of sheltered, high-altitude or high-latitude environments. These niches should accommodate life for about 1 Gyr beyond other surface environments. (1 Gyr=1 billion year).

The most likely sequence of events that will make the Earth uninhabitable is as follows:
1.Increased temperatures cause increased atmospheric water vapor a greenhouse gas, the presence of which further increases surface temperatures.
2.For the next billion years or so this might increase cloud cover, cooling temperatures modestly.
3.Eventually more energy from the Sun will win out, and higher temperatures lead to increased weathering of silicate rocks, drawing down more carbon from the atmosphere.
4.Carbon is normally recycled though plate tectonics; however, increasing water loss eventually halts plate tectonics. 5Once carbon dioxide levels drop below this value, higher plants will begin to die off.
6.This in turn decreases oxygen production, which, with continued consumption by biota and by oxidation of organic carbon in sedimentary rocks, leads to a steady decline in atmospheric oxygen to zero over a few million years.
7.The end of animal life would occur a few million years after the end of plant life.
8.Large endotherms (mammals, birds) would likely be the first group to become extinct due to their higher oxygen requirements.
9.Fish, amphibians, reptiles would be able to survive for longer, but eventually the oceans will evaporate.
10.The last life on Earth, in about 2.8 billion years, will be single-celled, heat-loving organisms in isolated pools of hot, salty water.

Unlike climate change, largely driven by greenhouse gases, this solar warming scenario in a couple of billion years is unstoppable. However, there is a silver lining. If the human species manages to survive its myriad other threats asteroids, increasingly sophisticated biological weapons were beginning to find a lot of planets around a lot of other worlds.

In order to extend the time horizon for this event, humans can do as follows:
1.Develop a good friendship in the world, do not develop chemical and biological weapons, and do not conduct the experiment of nuclear, such as North Korea.
2.Planting more trees for the good environment.
3.Environmental and sustainable development for humans ans animals.
4.Recuce the destruction wrought by the industrial revolution.

(g)The source is used and referred at
https://www.universetoday.com/18847/life-of-the-sun/. This lifespan began roughly 4.6 billion years ago, and will continue for about another 4.5 to 5.5 billion years, when it will deplete its supply of hydrogen, helium, and collapse into a white dwarf. But this is just the abridged version of the Suns lifespan. As always, God (or the Devil, depending on who you ask) is in the details! To break it down, the Sun is about half way through the most stable part of its life. Over the course of the past four billion years, during which time planet Earth and the entire Solar System was born, it has remained relatively unchanged. This will stay the case for another four billion years, at which point, it will have exhausted its supply of hydrogen fuel. When that happens, some pretty drastic things will take place!

(h)The source is used and referred at `https://www.cfa.harvard.edu/~dfabricant/huchra/hubble/`

The universe is now close to 14 billion years old. This age is determined by measuring the Hubble's Constant , which is a means for representing the rate at which universe is expanding.

We can't calculate the lifespan of something that is still expanding. We could calculate approximately when our star Sun will run out of fuel, it is calculated as 5 billion years from now when Sun will enter into a red giant phase. This we know because we have observed and did study on Sun for a reasonable period of time now. But with Universe, we have seen very little of what is the real picture, whatever little we have seen - we haven't been able to fully understand.

As for how universe may get destroyed at the end, these are some of the theories that claims to know the answer:
1.Big Freeze - The Big Freeze is a scenario under which continued expansion results in a universe that asymptotically approaches absolute zero temperature. It's going to be pretty cold out there.
2.Big Rip - According to this theory density of dark energy increases with time. As a result, all material objects in the universe, starting with galaxies and eventually (in a finite time) all forms, no matter how small, will disintegrate into unbound elementary particles and radiation, ripped apart.
3.Big Crunch - Just as the Big Bang started a cosmological expansion, this theory assumes that the average density of the universe is enough to stop its expansion and begin contracting. And at the end we will be left with a singular speck of dust.
Multiverse Theory - One multiverse hypothesis states that our universe is merely one Big Bang among an infinite number of simultaneously expanding Big Bangs that are spread out over endless distances.

(i)From "WIKIPEDIA", and `https://baike.baidu.com/item/\%E6\%B1\%89\%E8\%AF\%BA\%E5\%A1\%94/3468295?fr=aladdin` If we consider one year as 365 days,one year is equal to 31536000 seconds, if we consider one year as 366 days, one year is equal to 31622400 seconds, therefore, after large numbers of computation, we can find that it need 18446744073709551615 seconds. That is equal to 584.554 billion years. While the earth is not more than 4.5 billion years until now. The expected lifespan of the solar system is nearly seveal ten billion years.If 584.554 billion years pass, I truly believe anything will not exist. 584.554 billion years is multiple times than the expected lifespan of the Universe.

3 The Nine Billion Names of God

(a)Assumptions: one word per line of paper. 10 lines per paper. 0.9 billion paper will be needed to print out 9 billion words.
(b)Assumptions: if one paper is 0.01 dollars, one cabinets can contain 1000 paper. $0.01*0.9 \quad billion = 0.9 \quad billion/1000 \quad cabinets = 900,000 \quad cabinets \quad needed$
(c)it would take them about fifteen thousand years to complete the task.
(d)The time span allocated was extended to approximately 2060 A.D. What would have taken them fifteen thousand years it will be able to do in a hundred days.
(e)What the story tells us is that science, religion and peoples's belief will effect our mind. The story is great and amazing. That is will make me think about the large numbers, if we want to choose arbitrarily nine words from the 26 words. That would produce large numbers mathematically.

4 Problem 7 on page 21

The values: $f(1) = 91; f(-6) = 91; f(200) = 190; f(27) = 91;$

The experimental code is as follows:

```
def f(n):
    if n>100:
        return n-10
    else:
        return f(f(n+11))



for i in range (101):
     print i, f(i)-91
```

All outputs are zero.

5 Problem 9 on page 22

The experimental code is as follows:

```
def A(x,y):
    if x==0:
        return y+1
    elif y==0:
        return A(x-1,1)
    else:
        return A(x-1,A(x,y-1))


for i in range(4):
    for j in range(4):
        print i,j,A(i,j)
```

The output is as follows:

```
0 0 1
0 1 2
0 2 3
0 3 4
1 0 2
1 1 3
1 2 4
1 3 5
2 0 3
2 1 5
2 2 7
2 3 9
3 0 5
3 1 13
3 2 29
3 3 61
```

Calculate $A(2,2)$ by hand and list the number of calls to A that it makes as follows:

6 Problem 11 on page 23

The experimental code is as follows:

```
def extendedgcd(a,b):  # a > b > 0
    """ Extended great common divisor, returns s , t&g
       and gcd(a,b) denoted by g, so s*a + t*b = gcd(a,b)       """

    if a%b==0: return (0,1,b)
    q=[]
    while a%b != 0:
        q.append(-1*(a//b))
        (a,b)=(b,a%b)
    (a,b,gcd)=(1,q.pop(),b)
    while q:(a,b)=(b,b*q.pop()+a)
    return (a,b,gcd)

print extendedgcd(1600,300),extendedgcd(6,15),
extendedgcd(45,3000),extendedgcd(16000,64),
```

The output is as follows:

```
(1, -5, 100) (-2, 1, 3) (67, -1, 15) (0, 1, 64)
```

7 Problem 14 on page 24

The experimental code is as follows:

```
def reverse(l, first=0, last=-1):
    if first >= len(l)/2: return
    l[first], l[last] = l[last], l[first]
    reverse(l, first+1, last-1)

mylist = [101,102,106,114,1555]
print mylist
reverse(mylist)
print mylist
```

The output is as follows:

```
[101, 102, 106, 114, 1555]
[1555, 114, 106, 102, 101]
```

8 Anagram Generator

9 Monty Python skit

(a)The location of the skit is at `http://andrewchamblin.org/mitpages/MontyPython3.htm`
(b) The gathering represented is the faculty meeting in the philosophy department at the university

of Walamaloo.
(c)The organization that organize the meeting is the faculty member in the philosophy department at the university of Walamaloo.
(d)There are five people at the meeting, First Bruce, Second Bruce, third Bruce, fourth Bruce, Michael Baldwin.
(e)There is no rule six.

We assume that X denotes the number of days since the last repair or maintenance. As the problems states, the system fails with probability 1 sometime during the 30th day if the system continues for 30 days without repair or maintenance. Hence,

$$state = d + 1, d = 0, 1, ..., 30. \tag{1}$$
$$state \in (1, 31) \tag{2}$$

The actions in the Markov Decision Processes are Produce and Maintain: $Actions = \{Produce, Maintain\}$. The system will fail on the dth day since the last repair or maintenance with probability of failure of $(1 - \psi^d)$, The transition probability matrix of action "Produce" can be written as:

$$TPM_{\Pr oduce} = \begin{bmatrix} 0 & 1 & 0 & ... & 0 & ... & 0 & 0 \\ 1-\psi & 0 & \psi & 0 & 0 & ... & 0 & 0 \\ 1-\psi^2 & 0 & 0 & \psi^2 & 0 & ... & 0 & 0 \\ \vdots & & & & & & & \\ 1-\psi^d & 0 & ... & ... & 0 & \psi^d & ... & 0 \\ \vdots & & & & & & & \\ 1-\psi^{29} & 0 & ... & 0 & ... & 0 & 0 & \psi^{29} \\ 1 & 0 & ... & 0 & ... & 0 & 0 & 0 \end{bmatrix}$$

Where $d = 1, 2, ..., 29$.

Since the cost of repair is $\$450$, so reward is -450, we can formulate the transition reward matrix given the action "Produce" as:

$$TRM_{\Pr oduce} = \begin{bmatrix} 0 & 0 & 0 & ... & 0 & ... & 0 & 0 \\ -450 & 0 & 0 & 0 & 0 & ... & 0 & 0 \\ -450 & 0 & 0 & 0 & 0 & ... & 0 & 0 \\ \vdots & & & & & & & \\ -450 & 0 & ... & ... & 0 & 0 & ... & 0 \\ \vdots & & & & & & & \\ -450 & 0 & ... & 0 & ... & 0 & 0 & 0 \\ -450 & 0 & ... & 0 & ... & 0 & 0 & 0 \end{bmatrix}$$

The TPM and TRM given action "Produce" are the matrix with the size of 31×31.

From the statement that after the system is repaired or maintained, it is assumed to be as good as new. It means that that machine will go to state 0 every time after maintenance. Therefore, the transition probability matrix of action "Maintain" could be written as:

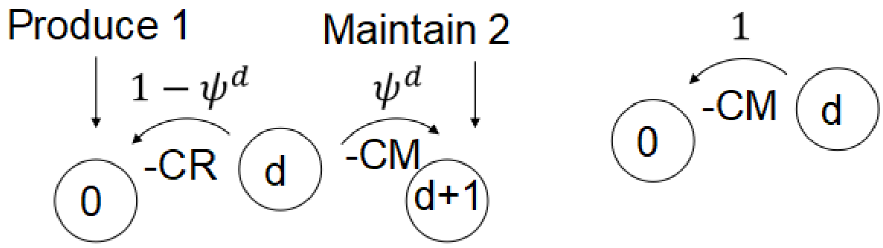

Figure 1: Roughly description about the MDP

$$TPM_{Ma\,\mathrm{int}\,ain} = \begin{bmatrix} 0 & 0 & 0 & ... & 0 & ... & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & ... & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & ... & 0 & 0 \\ \vdots & & & & & & & \\ 1 & 0 & ... & ... & 0 & 0 & ... & 0 \\ \vdots & & & & & & & \\ 1 & 0 & ... & 0 & ... & 0 & 0 & 0 \\ 1 & 0 & ... & 0 & ... & 0 & 0 & 0 \end{bmatrix}$$

The cost for maintenance is \$175, so reward is -175, similarly, we can formulate the transition reward matrix given the action "Maintain" as:

$$TRM_{Ma\,\mathrm{int}\,ain} = \begin{bmatrix} 0 & 0 & 0 & ... & 0 & ... & 0 & 0 \\ -175 & 0 & 0 & 0 & 0 & ... & 0 & 0 \\ -175 & 0 & 0 & 0 & 0 & ... & 0 & 0 \\ \vdots & & & & & & & \\ -175 & 0 & ... & ... & 0 & 0 & ... & 0 \\ \vdots & & & & & & & \\ -175 & 0 & ... & 0 & ... & 0 & 0 & 0 \\ -175 & 0 & ... & 0 & ... & 0 & 0 & 0 \end{bmatrix}$$

The mapping structure can be descriped in Figure 1.

(2) In order to identify the optimal policy for the manager, the code using relative value iteration for average reward is shown as follows:

"$main\_HW3\_Problem1\_Dai.m$" , which is the main function that shows the optimal policy and its iteration record.
"$func\_HW3\_Problem\_1\_RVI.m$" is an self-defined function for conducting relative value iteration.

If you want to know the whole process of my code, please run the the main function file, and you will find the result, simultaneously, the output will be stored into the file(diary.txt) with comand "diary on" and "diary off".

(3) Please Note that $\varepsilon = 0.01$, and the maximization number of iterations is 500 . The optimal policy for $\psi = 0.9$ is shown in Figure 2. the action "1" represents "Produce", and action "2"

State	0	1	2	3	4	5	6	7	8	9
Action	1	1	1	2	2	2	2	2	2	2
State	10	11	12	13	14	15	16	17	18	19
Action	2	2	2	2	2	2	2	2	2	2
State	20	21	22	23	24	25	26	27	28	29
Action	2	2	2	2	2	2	2	2	2	2
State	30									
Action	2									

Figure 2: Record about the states associated the actions

represents ”Maintain”.

The results show that the optimal policy for preventative maintenance of the machine is the machine should be maintained after 2 days since the last repair or maintenance.

10 Questions

The TPM associated with action a is P_a and the associated TRM is R_a.

$$P_1 = \begin{bmatrix} 0.1 & 0.9 \\ 0.8 & 0.2 \end{bmatrix}, R_1 = \begin{bmatrix} 12 & 16 \\ -7 & 13 \end{bmatrix} \qquad P_2 = \begin{bmatrix} 0.3 & 0.7 \\ 0.5 & 0.5 \end{bmatrix}, R_2 = \begin{bmatrix} 12 & -11 \\ 6 & 9 \end{bmatrix} \tag{3}$$

The state(action) trajectory is: $1(1), 2(2), 1(2), 2(2), 2(X)$. And we have known that the system starts at state 1.

$\mathbf{State 1}$. Set all the Q-factors to 0: $Q(1,1) = Q(1,2) = Q(2,1) = Q(2,2) = 0$ The set of actions allowed in state 1 is $A(1) = \{1,2\}$ and that allowed in state 2 is $A(2) = \{1,2\}$. Clearly $|A(i)| = 2, i = 1,2$. Let the step size α be defined by $\alpha = \frac{10}{20+k}$. select an action with probability $\frac{1}{|A(i)|}$. Let the selected action be 1. simulate action 1. Let the next state be 2.

$\mathbf{State 2}$.The current state (j) is 2 and the old state (i) was 1. The action (a) selected in the old state was 1. So we now have to update Q(1,1). Now:k=0, $\alpha = 0.5$;

$$r(i,a,j) = r(1,1,2) = 16 \tag{4}$$

$$\max_b Q(j,b) = \max_b Q(2,b) = \max\{Q(2,1), Q(2,2)\} = 0; \tag{5}$$

$$Q(1,1) \leftarrow (1-\alpha)Q(1,1) + \alpha[r(1,1,2) + \lambda \max_{b \in A(2)} Q(2,b)] \tag{6}$$

$$= 0.5 * 0 + 0.5 * (16 + 0.7 * 0) = 8 \tag{7}$$

Current state is 2 and the selected action is 2. Simulate action 2. Let the next state be 1.

$\mathbf{State 1(again)}$ The current state(j) is 1 and the old state (i) was 2. The action(a) selected in the old state was 2. So we now have to update Q(2,2). Now:k=1;$\alpha = 10/21$.

$$r(i,a,j) = r(2,2,1) = 6 \tag{8}$$

$$\max_b Q(1,b) = \max_b Q(1,b) = \max\{Q(1,1), Q(1,2)\} = \max\{8,0\} = 8; \tag{9}$$

$$Q(2,2) \leftarrow (1-\alpha)Q(2,2) + \alpha[r(2,2,1) + \lambda \max_{b \in A(1)} Q(1,b)] \tag{10}$$

$$= \frac{11}{21} * 0 + \frac{10}{21} * (6 + 0.7 * 8) = 5.5238 \tag{11}$$

Current state is 1 and the selected action is 2. Simulate action 2 and the next state is 2.

$\mathbf{State2(again)}$ The current state(j) is 2 and the old state(i) was 1. The action(a) selected in the old state was 2. We we now have to update Q(1,2). Now: k=2;$\alpha = 5/11$.

$$r(i,a,j) = r(1,2,2) = -11 \tag{12}$$

$$\max_b Q(2,b) = \max_b Q(2,b) = \max\{Q(2,1), Q(2,2)\} = \max\{0, 5.5238\} = 5.5238; \tag{13}$$

$$Q(1,2) \leftarrow (1-\alpha)Q(1,2) + \alpha[r(1,2,2) + \lambda \max_{b \in A(2)} Q(2,b)] \tag{14}$$

$$= \frac{6}{11} * 0 + \frac{5}{11} * (-11 + 0.7 * 5.5238) = -3.2424 \tag{15}$$

Current state is 2 and the selected action is 2. Simulate action 2 and the next state is 2.

$\mathbf{State2(a\ third\ time)}$ The current state(j) is 2 and the old state(i) was 2. The action(a) selected in the old state was 2. We we now have to update Q(2,2). Now: k=3;$\alpha = 10/23$.

$$r(i,a,j) = r(2,2,2) = 9 \tag{16}$$

$$\max_b Q(2,b) = \max_b Q(2,b) = \max\{Q(2,1), Q(2,2)\} = \max\{0, 5.5238\} = 5.5238; \tag{17}$$

$$Q(2,2) \leftarrow (1-\alpha)Q(2,2) + \alpha[r(2,2,2) + \lambda \max_{b \in A(2)} Q(2,b)] \tag{18}$$

$$= \frac{13}{23} * 5.5238 + \frac{10}{23} * (9 + 0.7 * 5.5238) = 8.7163 \tag{19}$$

Current state is 2 and the selected action is X.