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# Iterative Approximating Techniques for SDPs by Cutting-planes

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## Abstract

Solving semidefinite optimization problems (SDPs) optimally requires high demand of resources, which makes it a fundamental problem to solve it fast without losing accuracy. One critical approach is to iteratively approximate SDP with easier optimization problem such as LP / SOCP. In this paper, we review a number of these approaches of cutting methods to do inner / outer approximation of a given SDP.
**Keywords:** Semidefinite optimization, Cutting plane

## 1. Introduction

SDPs are too expensive to solve in practice because traditional interior point methods (IPMs) requires a unacceptable demand of memory. As a result, when the problem scale is large, we'll have to resort either to specialized solution techniques and algorithms that employ specific problem, or to relax the original optimization problems that leads to weaker bounds. Recently other systematic approaches to approximate SDP have been proposed which lead to less expensive optimization problems such as LP / SOCP. We'll focus on one of the most important and useful algorithms, i.e. cutting planes methods in this survey.

## 2. Related Work

Large-scale semidefinite programming (SDP) is difficult to solve due to memory limitation and a huge amount of time required. Cutting planes is one of the most general iterative approximation methods to address scalarbility for SDPs without requirement of certain problem structure. Ahmadi and Majumdar (2014, 2015) introduces Diagonally Dominant Sum of Squares (DSOS) and Scaled Diagonally Dominant Sum of Squares (SDSOS) optimization, to replace the challanging SDP problem with a sequence of linear programming (LP) and second-order cone programming (SOCP), and solve this sequence iteratively. Based on the work of Ahmadi and Majumdar (2014, 2015), Ahmadi and Hall (2016) developed a method to iteratively improve on the bounds from the DSOS and SDSOS by pursuing better bases. Inspired by Ahmadi and Majumdar (2014, 2015) and Krishnan and Mitchell (2006), Ahmadi et al. (2017) proposes an algorithm to construct inner approximations of the cone of positive semidefinite matrices via LP and SOCP by combining column generation with DSOS and SDSOS.

Except for DSOS, SDSOS and their improvements and variants, Kelley (1960) proposes the cutting-plane method for solving convex programs. However, Kelley's method performs poorly when it doesn't have a good initial approximation. Bertsimas and Cory-Wright (2020) proposes initializing Kelley's method with a SOC-representable approximation and shows that this performs better in two problems, sparse principal component analysis (PCA) and nuclear norm minimization.

## 3. Problem Formulation

We study the iterative approximation methods for the standard semidefinite optimization problems(SDP) of the form.

$$\begin{aligned} &\min_{X \in \mathbf{R}^{n\times n}} \langle C, X\rangle \\ &\text{subject to } \langle A_i, X\rangle = b_i, \forall i = 1, \cdots, m, X \geq 0 \end{aligned} \tag{1}$$

As stated before, we try to approximate the original problems by a set of simpler problems. A typical formulation is to solve the problem

$$\begin{aligned} &\min_{X \in \mathbf{R}^{n\times n}} \langle C, X\rangle \\ &\text{subject to } \langle A_i, X\rangle = b_i, \forall i = 1, \cdots, m, X \in S_k \end{aligned} \tag{2}$$

where $\{S_k\}_{k=1}^t$ is a set of sets that iteratively approximates the original semidefinite constraint. Note if problem (2) is a LP, SOCP or any simpler optimization problem rather than a SDP we can get satisfying result within an acceptable time duration.

To be specific, if $S_1 \subset S_2 \subset \cdots \subset S_t \subset S_+^n$, we say this is an inner approximation and it's an outer approximation if reversed. At each step, we add more constraints to the set $S_k$ to gradually get to our target set $S_+^n$. Adding constraints to the set is called *cutting plane* approach and the associated problem of finding violated constraints is called the *separation subproblem* (Note adding atoms to its dual are called *column generation* and the problem of finding such columns are called the *pricing subproblem*, which is actually the same as cutting plane). We'll mainly focus on these techniques that iteratively approximates the goal SDP.

## 4. Goals

We will study and implement different cutting plane methods, one of the most widely-used methods to solve semidefinite programming efficiently by iterative approximation, and test these implementations with problems on specific applications.

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