Dr. Jakub Wagner
Institute of Radioelectronics and Multimedia Technology

Assignment A: Estimation of lower limb joint angles

The Vicon motion-capture system enables the estimation of the three-dimensional positions of markers attached to a human body. The Plug-in Gait standard, widely used in the analysis of human gait, specifies (among other things) the placement of such markers (*cf.* Fig. 1). Each marker is assigned a 4-letter code, *e.g.* `LKNE` (corresponding to the left knee). Using the Vicon system combined with the Plug-in Gait standard, it is also possible to estimate the positions of other points – the so-called *virtual markers*[1], most of which are also assigned 4-letter codes[2] (*cf.* Fig. 2).

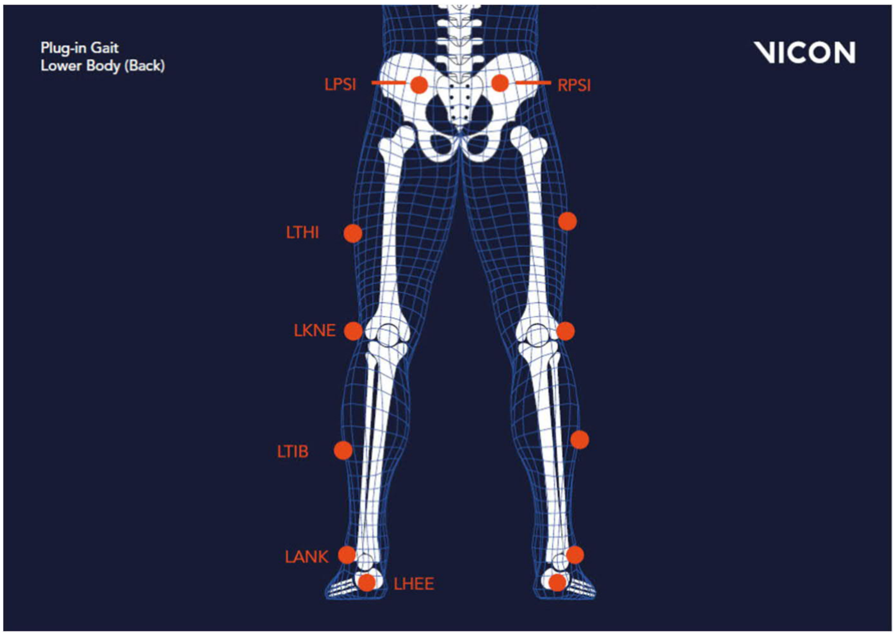

Fig. 1. Placement of selected markers in the Plug-in Gait standard [1 (p. 10)].

[1] A virtual marker is a point in the examined person's body to which no physical marker is attached, but whose position can be estimated based on the positions of the physical markers and a certain model of the human body [1 (p. 42)].

[2] The names of most virtual markers refer to the side of the body (`L` – *left* or `R` – *right*), the name of the bone (*e.g.* `FE` – *femur* or `TI` – *tibia*) and a point in the bone (*e.g.* `O` – *origin* or `P` – *proximal*); for example, the virtual marker `LFEP` corresponds to the proximal end of the left femur, which – in the Plug-in Gait standard – is identical to the centre of the left hip joint [1 (pp. 39–43)].

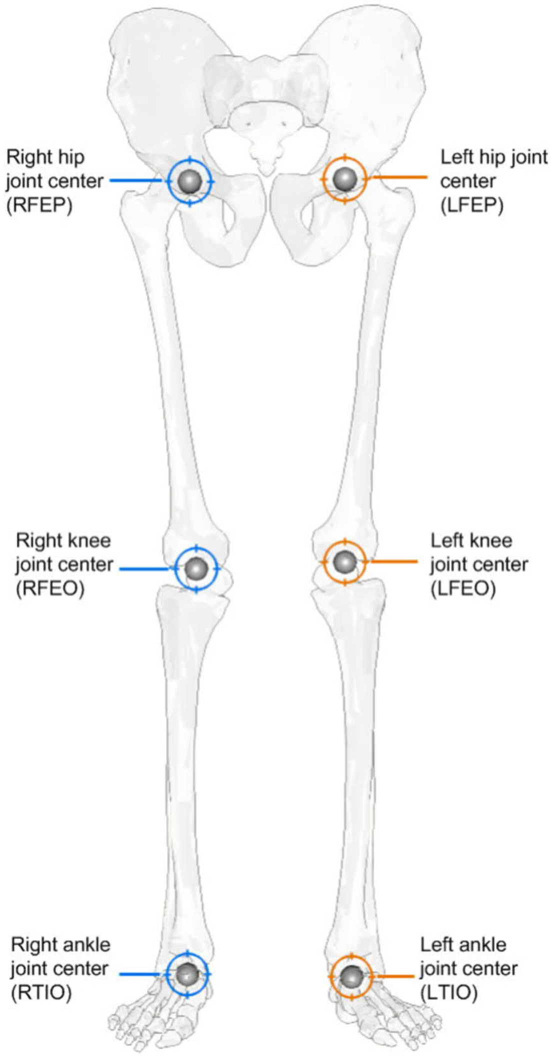

Fig. 2. Virtual markers corresponding to the centres of lower limb joints [1 (p. 43)].

Data acquired by means of the Vicon system can be used for estimating 18 angles in the lower limb joints (*cf.* Fig. 3):

– the left and right hip flexion angles, hereinafter denoted with the symbols α_{LH} and α_{RH}, respectively,
– the left and right hip adduction angles – β_{LH} and β_{RH},
– the left and right hip internal rotation angles – γ_{LH} and γ_{RH},
– the left and right knee flexion angles – α_{LK} and α_{RK},
– the left and right knee adduction angles – β_{LK} and β_{RK},
– the left and right knee internal rotation angles – γ_{LK} and γ_{RK},
– the left and right ankle dorsiflexion angles – α_{LA} and α_{RA},
– the left and right ankle adduction angles – β_{LA} and β_{RA},
– the left and right foot internal rotation angles – γ_{LA} and γ_{RA}.

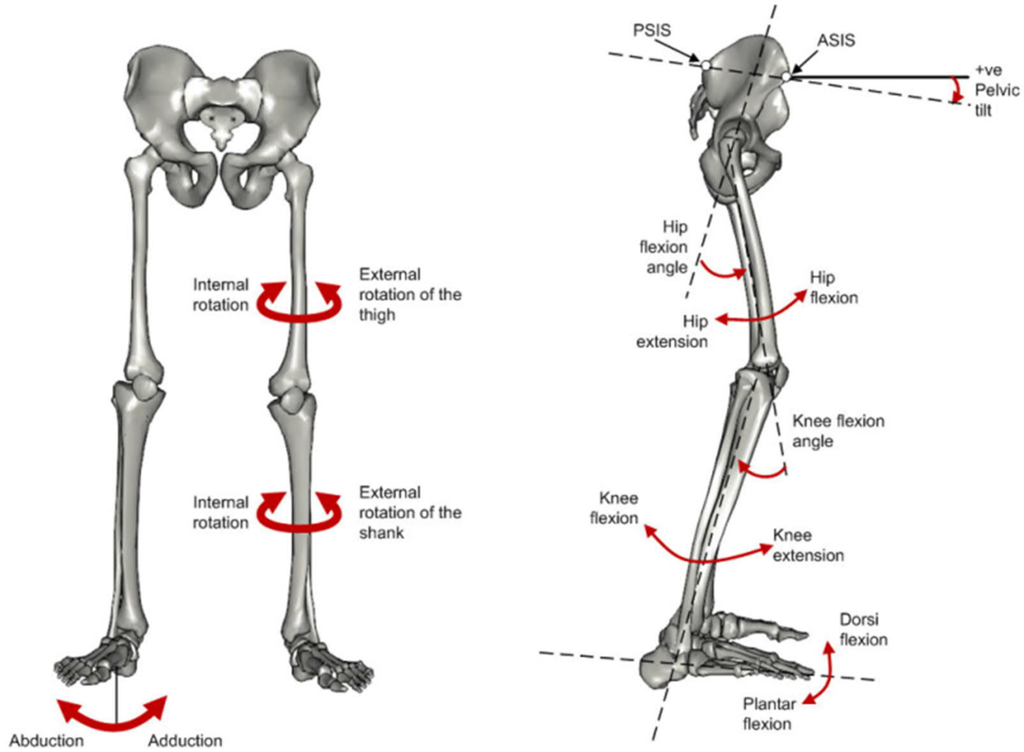

Fig. 3. Angles in lower limb joints [1 (p. 86)].

Those angles can be estimated according to the following formulae [1, 2]:

$$\alpha_{\mathrm{LH}} = -\arcsin\left(\frac{\mathbf{z}_{\mathrm{LF}}^{\mathrm{T}}\mathbf{x}_{\mathrm{PE}}}{\cos\left(\beta_{\mathrm{LH}}\right)}\right) \tag{1}$$

$$\beta_{\mathrm{LH}} = \arcsin\left(\mathbf{z}_{\mathrm{LF}}^{\mathrm{T}}\mathbf{y}_{\mathrm{PE}}\right) \tag{2}$$

$$\gamma_{\mathrm{LH}} = -\arcsin\left(\frac{\mathbf{x}_{\mathrm{LF}}^{\mathrm{T}}\mathbf{y}_{\mathrm{PE}}}{\cos\left(\beta_{\mathrm{LH}}\right)}\right) \tag{3}$$

$$\alpha_{\mathrm{RH}} = -\arcsin\left(\frac{\mathbf{z}_{\mathrm{RF}}^{\mathrm{T}}\mathbf{x}_{\mathrm{PE}}}{\cos\left(\beta_{\mathrm{RH}}\right)}\right) \tag{4}$$

$$\beta_{\mathrm{RH}} = -\arcsin\left(\mathbf{z}_{\mathrm{RF}}^{\mathrm{T}}\mathbf{y}_{\mathrm{PE}}\right) \tag{5}$$

$$\gamma_{\mathrm{RH}} = \arcsin\left(\frac{\mathbf{x}_{\mathrm{RF}}^{\mathrm{T}}\mathbf{y}_{\mathrm{PE}}}{\cos\left(\beta_{\mathrm{RH}}\right)}\right) \tag{6}$$

$$\alpha_{\mathrm{LK}} = \arcsin\left(\frac{\mathbf{z}_{\mathrm{LT}}^{\mathrm{T}}\mathbf{x}_{\mathrm{LF}}}{\cos\left(\beta_{\mathrm{LK}}\right)}\right) \tag{7}$$

$$\beta_{\mathrm{LK}} = \arcsin\left(\mathbf{z}_{\mathrm{LT}}^{\mathrm{T}}\mathbf{y}_{\mathrm{LF}}\right) \tag{8}$$

$$\gamma_{\mathrm{LK}} = -\arcsin\left(\frac{\mathbf{x}_{\mathrm{LT}}^{\mathrm{T}}\mathbf{y}_{\mathrm{LF}}}{\cos(\beta_{\mathrm{LK}})}\right) \tag{9}$$

$$\alpha_{\mathrm{RK}} = \arcsin\left(\frac{\mathbf{z}_{\mathrm{RT}}^{\mathrm{T}}\mathbf{x}_{\mathrm{RF}}}{\cos(\beta_{\mathrm{RK}})}\right) \tag{10}$$

$$\beta_{\mathrm{RK}} = -\arcsin\left(\mathbf{z}_{\mathrm{RT}}^{\mathrm{T}}\mathbf{y}_{\mathrm{RF}}\right) \tag{11}$$

$$\gamma_{\mathrm{RK}} = \arcsin\left(\frac{\mathbf{x}_{\mathrm{RT}}^{\mathrm{T}}\mathbf{y}_{\mathrm{RF}}}{\cos(\beta_{\mathrm{RK}})}\right) \tag{12}$$

$$\alpha_{\mathrm{LA}} = -\arcsin\left(\frac{\mathbf{x}_{\mathrm{LP}}^{\mathrm{T}}\mathbf{z}_{\mathrm{LT}}}{\cos(\gamma_{\mathrm{LA}})}\right) \tag{13}$$

$$\beta_{\mathrm{LA}} = -\arcsin\left(\frac{\mathbf{z}_{\mathrm{LP}}^{\mathrm{T}}\mathbf{y}_{\mathrm{LT}}}{\cos(\gamma_{\mathrm{LA}})}\right) \tag{14}$$

$$\gamma_{\mathrm{LA}} = \arcsin\left(\mathbf{x}_{\mathrm{LP}}^{\mathrm{T}}\mathbf{y}_{\mathrm{LT}}\right) \tag{15}$$

$$\alpha_{\mathrm{RA}} = -\arcsin\left(\frac{\mathbf{x}_{\mathrm{RP}}^{\mathrm{T}}\mathbf{z}_{\mathrm{RT}}}{\cos(\gamma_{\mathrm{RA}})}\right) \tag{16}$$

$$\beta_{\mathrm{RA}} = \arcsin\left(\frac{\mathbf{z}_{\mathrm{RP}}^{\mathrm{T}}\mathbf{y}_{\mathrm{RT}}}{\cos(\gamma_{\mathrm{RA}})}\right) \tag{17}$$

$$\gamma_{\mathrm{RA}} = -\arcsin\left(\mathbf{x}_{\mathrm{RP}}^{\mathrm{T}}\mathbf{y}_{\mathrm{RT}}\right) \tag{18}$$

where:

$$\mathbf{x}_{\mathrm{PE}} \equiv \frac{\mathbf{y}_{\mathrm{PE}} \times \mathbf{z}_{\mathrm{PE}}}{\left\|\mathbf{y}_{\mathrm{PE}} \times \mathbf{z}_{\mathrm{PE}}\right\|_2} \tag{19}$$

$$\mathbf{y}_{\mathrm{PE}} \equiv \frac{\mathbf{c}_{\mathrm{PELL}} - \mathbf{c}_{\mathrm{PELO}}}{\left\|\mathbf{c}_{\mathrm{PELL}} - \mathbf{c}_{\mathrm{PELO}}\right\|_2} \tag{20}$$

$$\mathbf{z}_{\mathrm{PE}} \equiv \frac{\mathbf{c}_{\mathrm{PELP}} - \mathbf{c}_{\mathrm{PELO}}}{\left\|\mathbf{c}_{\mathrm{PELP}} - \mathbf{c}_{\mathrm{PELO}}\right\|_2} \tag{21}$$

$$\mathbf{x}_{\mathrm{LF}} \equiv \frac{\mathbf{y}_{\mathrm{LF}} \times \mathbf{z}_{\mathrm{LF}}}{\left\|\mathbf{y}_{\mathrm{LF}} \times \mathbf{z}_{\mathrm{LF}}\right\|_2} \tag{22}$$

$$\mathbf{y}_{\mathrm{LF}} \equiv \frac{\mathbf{c}_{\mathrm{LKNE}} - \mathbf{c}_{\mathrm{LFEO}}}{\left\|\mathbf{c}_{\mathrm{LKNE}} - \mathbf{c}_{\mathrm{LFEO}}\right\|_2} \tag{23}$$

$$\mathbf{z}_{\mathrm{LF}} \equiv \frac{\mathbf{c}_{\mathrm{LFEP}} - \mathbf{c}_{\mathrm{LFEO}}}{\left\|\mathbf{c}_{\mathrm{LFEP}} - \mathbf{c}_{\mathrm{LFEO}}\right\|_2} \tag{24}$$

$$\mathbf{x}_{\mathrm{LF}} \equiv \frac{\mathbf{y}_{\mathrm{RF}} \times \mathbf{z}_{\mathrm{RF}}}{\left\|\mathbf{y}_{\mathrm{RF}} \times \mathbf{z}_{\mathrm{RF}}\right\|_2} \tag{25}$$

$$\mathbf{y}_{\mathrm{RF}} \equiv \frac{\mathbf{c}_{\mathrm{RFEO}} - \mathbf{c}_{\mathrm{RKNE}}}{\left\|\mathbf{c}_{\mathrm{RFEO}} - \mathbf{c}_{\mathrm{RKNE}}\right\|_2} \tag{26}$$

$$\mathbf{z}_{\mathrm{RF}} \equiv \frac{\mathbf{c}_{\mathrm{RFEP}} - \mathbf{c}_{\mathrm{RFEO}}}{\left\|\mathbf{c}_{\mathrm{RFEP}} - \mathbf{c}_{\mathrm{RFEO}}\right\|_2} \tag{27}$$

$$\mathbf{x}_{\mathrm{LT}} \equiv \frac{\mathbf{y}_{\mathrm{LT}} \times \mathbf{z}_{\mathrm{LT}}}{\left\|\mathbf{y}_{\mathrm{LT}} \times \mathbf{z}_{\mathrm{LT}}\right\|_2} \tag{28}$$

$$\mathbf{y}_{\mathrm{LT}} \equiv \frac{\mathbf{c}_{\mathrm{LANK}} - \mathbf{c}_{\mathrm{LTIO}}}{\left\|\mathbf{c}_{\mathrm{LANK}} - \mathbf{c}_{\mathrm{LTIO}}\right\|_2} \tag{29}$$

$$\mathbf{z}_{\mathrm{LT}} \equiv \frac{\mathbf{c}_{\mathrm{LFEO}} - \mathbf{c}_{\mathrm{LTIO}}}{\left\|\mathbf{c}_{\mathrm{LFEO}} - \mathbf{c}_{\mathrm{LTIO}}\right\|_2} \tag{30}$$

$$\mathbf{x}_{\mathrm{RT}} \equiv \frac{\mathbf{y}_{\mathrm{RT}} \times \mathbf{z}_{\mathrm{RT}}}{\left\|\mathbf{y}_{\mathrm{RT}} \times \mathbf{z}_{\mathrm{RT}}\right\|_2} \tag{31}$$

$$\mathbf{y}_{\mathrm{RT}} \equiv \frac{\mathbf{c}_{\mathrm{RTIO}} - \mathbf{c}_{\mathrm{RANK}}}{\left\|\mathbf{c}_{\mathrm{RTIO}} - \mathbf{c}_{\mathrm{RANK}}\right\|_2} \tag{32}$$

$$\mathbf{z}_{\mathrm{RT}} \equiv \frac{\mathbf{c}_{\mathrm{RFEO}} - \mathbf{c}_{\mathrm{RTIO}}}{\left\|\mathbf{c}_{\mathrm{RFEO}} - \mathbf{c}_{\mathrm{RTIO}}\right\|_2} \tag{33}$$

$$\mathbf{x}_{\mathrm{LP}} \equiv \frac{\mathbf{c}_{\mathrm{LFOP}} - \mathbf{c}_{\mathrm{LFOO}}}{\left\|\mathbf{c}_{\mathrm{LFOP}} - \mathbf{c}_{\mathrm{LFOO}}\right\|_2} \tag{34}$$

$$\mathbf{y}_{\mathrm{LP}} \equiv \frac{\mathbf{c}_{\mathrm{LFOL}} - \mathbf{c}_{\mathrm{LFOO}}}{\left\|\mathbf{c}_{\mathrm{LFOL}} - \mathbf{c}_{\mathrm{LFOO}}\right\|_2} \tag{35}$$

$$\mathbf{z}_{\mathrm{LP}} \equiv \frac{\mathbf{y}_{\mathrm{LP}} \times \mathbf{x}_{\mathrm{LP}}}{\left\|\mathbf{y}_{\mathrm{LP}} \times \mathbf{x}_{\mathrm{LP}}\right\|_2} \tag{36}$$

$$\mathbf{x}_{\mathrm{RP}} \equiv \frac{\mathbf{c}_{\mathrm{RFOP}} - \mathbf{c}_{\mathrm{RFOO}}}{\left\|\mathbf{c}_{\mathrm{RFOP}} - \mathbf{c}_{\mathrm{RFOO}}\right\|_2} \tag{37}$$

$$\mathbf{y}_{\mathrm{RP}} \equiv \frac{\mathbf{c}_{\mathrm{RFOL}} - \mathbf{c}_{\mathrm{RFOO}}}{\left\|\mathbf{c}_{\mathrm{RFOL}} - \mathbf{c}_{\mathrm{RFOO}}\right\|_2} \tag{38}$$

$$\mathbf{z}_{\mathrm{RP}} \equiv \frac{\mathbf{y}_{\mathrm{RP}} \times \mathbf{x}_{\mathrm{RP}}}{\left\|\mathbf{y}_{\mathrm{RP}} \times \mathbf{x}_{\mathrm{RP}}\right\|_2} \tag{39}$$

with $\|\bullet\|_2$ denoting the 2-norm, $\times$ denoting the cross product, and $\mathbf{c}_{\text{PELO}} \equiv \left[x_{\text{PELO}}\ y_{\text{PELO}}\ z_{\text{PELO}}\right]^{\text{T}} \in \mathbb{R}^3$ denoting the vector of three-dimensional coordinates of the marker named `PELO` *etc*.

References

[1] Vicon, *Plug-in Gait Reference Guide*, available online (2025/02/24): https://help.vicon.com/download/attachments/11378719/Plug-in Gait Reference Guide.pdf

[2] R. B. Davis III, S. Õunpuu, D. Tyburski, J. R. Gage, "A gait analysis data collection and reduction technique," *Human Movement Science*, vol. 10, pp. 575–587, 1991.