

# GCV Assignment5: Non-rigid Structure from Motion

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January 2024

**Q1: Prove  $\text{rank}(AB) \leq \min\{\text{rank}(A), \text{rank}(B)\}$ , for any matrices  $A$  and  $B$ .**

Show that  $\text{rank}(AB) \leq \min\{\text{rank}(A), \text{rank}(B)\}$ . It can be proved as follows:

Each column of  $AB$  is a combination of the columns of  $A$ , which implies that  $\mathcal{R}(AB) \subseteq \mathcal{R}(A)$ . Hence,  $\dim(\mathcal{R}(AB)) \leq \dim(\mathcal{R}(A))$ , or equivalently,  $\text{rank}(AB) \leq \text{rank}(A)$ . Each row of  $AB$  is a combination of the rows of  $B \rightarrow \text{rowspace}(AB) \subseteq \text{rowspace}(B)$ , but the dimension of rowspace = dimension of column space = rank, so that  $\text{rank}(AB) \leq \text{rank}(B)$ . Therefore,  $\text{rank}(AB) \leq \min\{\text{rank}(A), \text{rank}(B)\}$

**Q2: Show the average reconstruction error per frame and over all the frame. Give your analysis of the results, for example, performance, error distribution and visualization.**

The mean error is 0.014074723559774682. The error distribution is shown as Fig1.

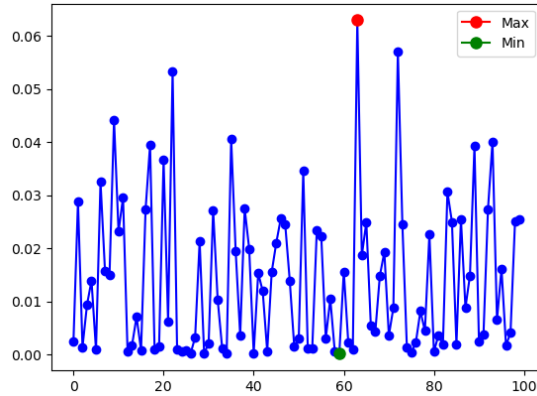


Figure 1: The error distribution.

The performance of best and worst are shown as below. We can see in the best figure, all points are well aligned. However, the worst case is in a

mess. The performance of it is not stable the standard deviation of error is 0.00020545619171280497.

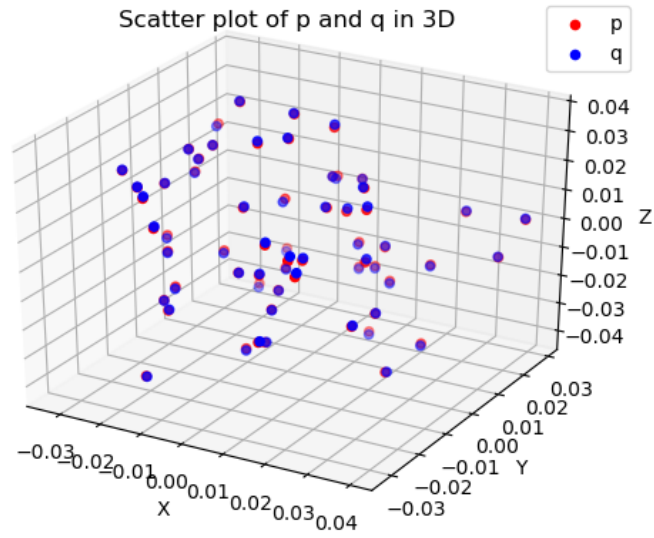


Figure 2: The best case.

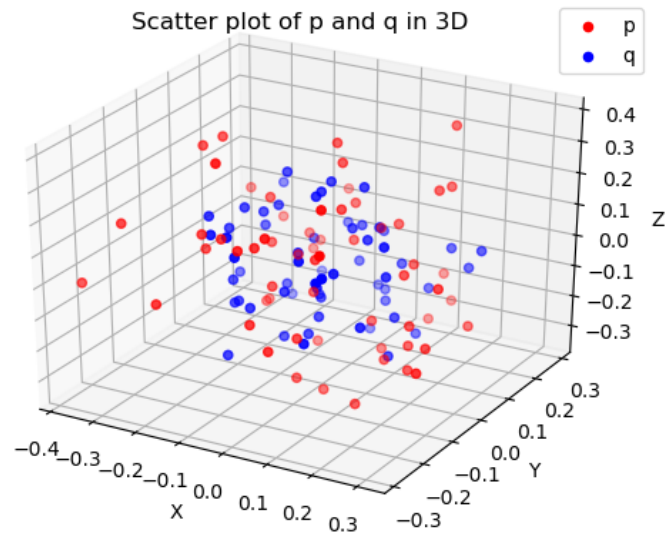


Figure 3: The worst case.