GCV Assignment2: Direct Least Squares Perspective-n-Points with Unknown Focal Length

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1 Q1: Prove the roots of n-th polynomia

$$Cx = \lambda x$$

$$(Cx - \lambda x) = 0$$

$$(C - \lambda I)x = 0$$

since $x \neq 0$, we can obtain:

$$|\lambda I - C| = 0$$

by writing down this determinant,

$$\begin{vmatrix} \lambda & -1 & \cdots & 0 \\ 0 & \lambda & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ c_0 & c_1 & \cdots & \lambda + c_{n-1} \end{vmatrix}$$

It can be transformed into a block matrix for calculation

$$\left|\begin{array}{c|ccc} \lambda & -1 & \cdots & 0 \\ \hline 0 & \lambda & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ c_0 & c_1 & \cdots & \lambda + c_{n-1} \end{array}\right| = \lambda M_{n-1} + c_0$$

$$= \lambda(\lambda M_{n-2} + c_1) + c_0$$

$$= \lambda^2 M_{n-2} + \lambda c_1 + c_0$$

$$= \lambda^3 M_{n-3} + \lambda^2 c_2 + \lambda c_1 + c_0$$

$$= \lambda^{n-1} c_{n-1} + \cdots + \lambda^2 c_2 + \lambda c_1 + c_0$$

By continuously expanding determinant M_n as above, we can prove the roots of n-th polynomial can be found as the eigenvalue values of the companion matrix C

2 Q2: Implement the DLSPnP-PE algorithm

- reprojection error: 1.480271082406342e-13
- Absolute Rotation Error: 3.1109265818524668e-15
- Relative Translation Error: 3.6529617510443754e-14
- focal error: 8.668621376273222e-15