

Chapter 8

Final comments

This final section is not for examination, but some of the topics may nevertheless be of interest to the student.

History and sociology of the modern subject

8.1 Introduction

Algebraic geometry has over the last thirty years or so enjoyed a position in math similar to that of math in the world at large, being respected and feared much more than understood. At the same time, the ‘service’ questions I am regularly asked by British colleagues or by Warwick graduate students are generally of an elementary kind: as a rule, they are either covered in this book or in [Atiyah and Macdonald]. What follows is a view of the recent development of the subject, attempting to explain this paradox. I make no pretence at objectivity.

8.2 Prehistory

Algebraic geometry developed in the 19th century from several different sources. Firstly, the geometric tradition itself: projective geometry (and descriptive geometry, of great interest to the military at the time of Napoleon), the study of curves and surfaces for their own sake, configuration geometry; then complex function theory, the view of a compact Riemann surface as an algebraic curve, and the purely algebraic reconstruction of it from its function field. On top of this, the deep analogy between algebraic curves and the ring of integers of a number field, and the need for a language in algebra and geometry for invariant theory, which played an important role in the development of abstract algebra at the start of the 20th century.

The first decades of the 20th century saw a deep division. On the one hand, the geometric tradition of studying curves and surfaces, as pursued notably by the brilliant Italian school; alongside its own quite considerable achievements, this played a substantial motivating role in the development of topology and differential geometry, but became increasingly dependent on arguments ‘by geometric intuition’ that even the *Maestri* were unable to sustain rigorously. On the other hand, the newly emerging forces of commutative algebra were laying foundations and providing techniques of