proof. An example of the difference between the two approaches was the argument between Chow and van der Waerden, who established rigorously the existence of an algebraic variety parametrising space curves of given degree and genus, and Severi, who had been making creative use of such parameter spaces all his working life, and who in his declining years bitterly resented the intrusion of algebraists (nonItalians at that!) into his field, and most especially the implicit suggestion that the work of his own school lacked rigour.

8.3 Rigour, the first wave

Following the introduction of abstract algebra by Hilbert and Emmy Noether, rigorous foundations for algebraic geometry were laid in the 1920s and 1930s by van der Waerden, Zariski and Weil (van der Waerden's contribution is often suppressed, apparently because a number of mathematicians of the immediate postwar period, including some of the leading algebraic geometers, considered him a Nazi collaborator).

A central plank of their program was to make algebraic geometry work over an arbitrary field. In this connection, a key foundational difficulty is that you can't just define a variety to be a point set: if you start life with a variety $V \subset \mathbb{A}^n_k$ over a given field k then V is not just a subset of k^n; you must also allow K-valued points of V for field extensions $k \subset K$ (see (8.13, c) for a discussion). This is one reason for the notation $\mathbb{A}^n_k$, to mean the k-valued points of a variety $\mathbb{A}^n$ that one would like to think of as existing independently of the specified field k.

The necessity of allowing the ground field to change throughout the argument added enormously to the technical and conceptual difficulties (to say nothing of the notation). However, by around 1950, Weil's system of foundations was accepted as the norm, to the extent that traditional geometers (such as Hodge and Pedoe) felt compelled to base their books on it, much to the detriment, I believe, of their readability.

8.4 The Grothendieck era

From around 1955 to 1970, algebraic geometry was dominated by Paris mathematicians, first Serre then more especially Grothendieck and his school. It is important not to underestimate the influence of Grothendieck's approach, especially now that it has to some extent gone out of fashion. This was a period in which tremendous conceptual and technical advances were made, and thanks to the systematic use of the notion of scheme (more general than a variety, see (8.12–14) below), algebraic geometry was able to absorb practically all the advances made in topology, homological algebra, number theory, etc., and even to play a dominant role in their development. Grothendieck himself retired from the scene around 1970 in his early forties, which must be counted a tragic waste (he initially left the IHES in a protest over military funding of science). As a practising algebraic geometer, one is keenly aware of the large blocks of powerful machinery developed during this period, many of which still remain to be written up in an approachable way.

On the other hand, the Grothendieck personality cult had serious side effects: many people who had devoted a large part of their lives to mastering Weil foundations suffered rejection and humiliation, and to my knowledge only one or two have adapted to the new language; a whole generation of students (mainly French) got themselves brainwashed into the foolish belief that a problem that can't be dressed up in high powered abstract formalism is unworthy of study, and were thus excluded from the mathematician's natural development of starting with a small problem he or she can handle and exploring outwards from there. (I actually know of a thesis on the arithmetic of cubic surfaces that was initially not considered because 'the natural context for the construction is over a general locally Noetherian ringed topos'. This is not a joke.) Many students of the time could apparently not think of any higher ambition than *Étudier les EGAs*. The study of category theory for its own sake (surely one of the most sterile of all intellectual pursuits) also dates from this time; Grothendieck himself can't necessarily be blamed for this, since his own use of categories was very successful in solving problems.

The fashion has since swung the other way. At a recent conference in France I commented on the change in attitude, and got back the sarcastic answer 'but the twisted cubic is a very good example of a prorepresentable functor'. I understand that some of the mathematicians now involved in administering French research money are individuals who suffered during this period of intellectual terrorism, and that applications for CNRS research projects are in consequence regularly dressed up to minimise their connection with algebraic geometry.

Apart from a very small number of his own students who were able to take the pace and survive, the people who got most lasting benefit from Grothendieck's ideas, and who have propagated them most usefully, were influenced at a distance: the Harvard school (through Zariski, Mumford and M. Artin), the Moscow school of Shafarevich, perhaps also the Japanese school of commutative algebraists.

8.5 The big bang

History did not end in the early 1970s, nor has algebraic geometry been less subject to swings of fashion since then. During the 1970s, although a few big schools had their own special interests (Mumford and compactification of moduli spaces, Griffiths' schools of Hodge theory and algebraic curves, Deligne and 'weights' in the cohomology of varieties, Shafarevich and K3 surfaces, Iitaka and his followers in the classification of higher dimensional varieties, and so on), it seems to me we all basically believed we were studying the same subject, and that algebraic geometry remained a monolithic block (and was in fact colonising adjacent areas of math). Perhaps the presence of just one or two experts who could handle the whole range of the subject made this possible.

By the mid-1980s, this had changed, and algebraic geometry seems at present to be split up into a dozen or more schools having quite limited interaction: curves and Abelian varieties, algebraic surfaces and Donaldson theory, 3-folds and classification in higher dimensions, K theory and algebraic cycles, intersection theory and enumerative geometry, general cohomology theories, Hodge theory, characteristic p, arithmetic algebraic geometry, singularity theory, differential equations of math physics, string theory, applications of computer algebra, etc.

Additional footnotes and highbrow comments

This section mixes elementary and advanced topics; since it is partly a 'word to the wise' for university teachers using this as a textbook, or to guide advanced students into the pitfalls of the subject, some of the material may seem obscure.