or she can handle and exploring outwards from there. (I actually know of a thesis on the arithmetic of cubic surfaces that was initially not considered because 'the natural context for the construction is over a general locally Noetherian ringed topos'. This is not a joke.) Many students of the time could apparently not think of any higher ambition than *Étudier les EGAs*. The study of category theory for its own sake (surely one of the most sterile of all intellectual pursuits) also dates from this time; Grothendieck himself can't necessarily be blamed for this, since his own use of categories was very successful in solving problems.

The fashion has since swung the other way. At a recent conference in France I commented on the change in attitude, and got back the sarcastic answer 'but the twisted cubic is a very good example of a prorepresentable functor'. I understand that some of the mathematicians now involved in administering French research money are individuals who suffered during this period of intellectual terrorism, and that applications for CNRS research projects are in consequence regularly dressed up to minimise their connection with algebraic geometry.

Apart from a very small number of his own students who were able to take the pace and survive, the people who got most lasting benefit from Grothendieck's ideas, and who have propagated them most usefully, were influenced at a distance: the Harvard school (through Zariski, Mumford and M. Artin), the Moscow school of Shafarevich, perhaps also the Japanese school of commutative algebraists.

8.5 The big bang

History did not end in the early 1970s, nor has algebraic geometry been less subject to swings of fashion since then. During the 1970s, although a few big schools had their own special interests (Mumford and compactification of moduli spaces, Griffiths' schools of Hodge theory and algebraic curves, Deligne and 'weights' in the cohomology of varieties, Shafarevich and K3 surfaces, Iitaka and his followers in the classification of higher dimensional varieties, and so on), it seems to me we all basically believed we were studying the same subject, and that algebraic geometry remained a monolithic block (and was in fact colonising adjacent areas of math). Perhaps the presence of just one or two experts who could handle the whole range of the subject made this possible.

By the mid-1980s, this had changed, and algebraic geometry seems at present to be split up into a dozen or more schools having quite limited interaction: curves and Abelian varieties, algebraic surfaces and Donaldson theory, 3-folds and classification in higher dimensions, K theory and algebraic cycles, intersection theory and enumerative geometry, general cohomology theories, Hodge theory, characteristic p, arithmetic algebraic geometry, singularity theory, differential equations of math physics, string theory, applications of computer algebra, etc.

Additional footnotes and highbrow comments

This section mixes elementary and advanced topics; since it is partly a 'word to the wise' for university teachers using this as a textbook, or to guide advanced students into the pitfalls of the subject, some of the material may seem obscure.

8.6 Choice of topics

The topics and examples treated in this book have been chosen partly pragmatically on the basis of small degree and ease of computation. However, they also hint at the 'classification of varieties': the material on conics applies in a sense to every rational curve, and cubic surfaces are the most essential examples of del Pezzo rational surfaces. Cubic curves with their group law are examples of Abelian varieties; the fact (2.2) that a nonsingular cubic is not rational is the very first step in classification. The intersection of two plane conics in (1.12–14) and the intersection of two quadrics of $\mathbb{P}^3_k$ referred to in Ex. 5.6 could also be fitted into a similar pattern, with the intersection of two quadrics in $\mathbb{P}^4_k$ providing another class of del Pezzo surfaces, and the family of lines on the intersection of two quadrics in $\mathbb{P}^5_k$ providing 2-dimensional Abelian varieties.

The genus of a curve, and the division into 3 cases tabulated on p. 46 is classification in a nutshell. I would have liked to include more material on the genus of a curve, in particular how to calculate it in terms of topological Euler characteristic or of intersection numbers in algebraic geometry, essential five finger exercises for young geometers. However, this would comfortably occupy a separate undergraduate lecture course, as would the complex analytic theory of elliptic curves.

8.7 Computation versus theory

Another point to make concerning the approach in these notes is that quite a lot of emphasis is given to cases that can be handled by explicit calculations. When general theory proves the existence of some construction, then doing it in terms of explicit coordinate expressions is a useful exercise that helps one to keep a grip on reality, and this is appropriate for an undergraduate textbook. This should not however be allowed to obscure the fact that the theory is really designed to handle the complicated cases, when explicit computations will often not tell us anything.

8.8 $\mathbb{R}$ versus $\mathbb{C}$

The reader with real interests may be disappointed that the treatment over $\mathbb{R}$ in §§1–2 gave way in §3 to considerations over an arbitrary field k, promptly assumed to be algebraically closed. I advise this class of reader to persevere; there are plenty of relations between real and complex geometry, including some that will come as a surprise. Asking about the real points of a real variety is a very hard question, and something of a minority interest in algebraic geometry; in any case, knowing all about its complex points will usually be an essential prerequisite. Another direct relation between geometry over $\mathbb{R}$ and $\mathbb{C}$ is that an n-dimensional nonsingular complex variety is a $2n$-dimensional real manifold – for example, algebraic surfaces are a principal source of constructions of smooth 4-manifolds.

As well as these fairly obvious relations, there are more subtle ones, for example: (a) singularities of plane curves $C \subset \mathbb{C}^2$ give rise to knots in S^3 by intersecting with the boundary of a small ball; and (b) the Penrose twistor construction views a 4-manifold (with a special kind of Riemannian metric) as the set of real valued points of a 4-dimensional complex variety that parametrises rational curves on a complex 3-dimensional variety (thus the real 4-sphere S^4 we live in can be identified as the real locus in the complex Grassmannian $\mathrm{Gr}(2,4)$ of lines in $\mathbb{P}^3_{\mathbb{C}}$).