This argument seems to me to be unsuitable for an undergraduate course for two reasons: statement (i) assumes results about the dimension of fibres, which however intuitively acceptable (especially to students in the last week of a course) are hard to do rigorously; whereas (ii) is the theorem that a projective variety is complete, that again requires proof (by elimination theory, compactness, or a full-scale treatment of the valuative criterion for properness).

To the best of my knowledge, my proof in (7.2) is new; the knowledgeable reader will of course see its relation to the other traditional argument by vector bundles: the Grassmannian $\mathrm{Gr}(2,4)$ has a tautological rank 2 vector bundle E (consisting of linear forms on the lines of $\mathbb{P}^3$); restricting the equation f of a cubic surface to every line $\ell \subset \mathbb{P}^3$ defines a section $s(f) \in S^3E$ of the 3rd symmetric power of E. Finally, every section of S^3E must have a zero, either by ampleness of E or by a Chern class argument (that also gives the magic number 27).

Substitute for preface

8.16 Acknowledgements and name dropping

It would be futile to try to list all the mathematicians who have contributed to my education. I owe a great debt to both my formal supervisors Pierre Deligne and Peter Swinnerton-Dyer (before he became a successful politician and media personality); I probably learned most from the books of David Mumford, and my understanding (such as it is) of the Grothendieck legacy derives largely from Mumford and Deligne. My view of the world, both as a mathematician and as a human being, has been strongly influenced by Andrei Tyurin.

My approach to what an undergraduate algebraic geometry course should be is partly based on a course designed around 1970 by Peter Swinnerton-Dyer for the Cambridge tripos, and taught in subsequent years by him and Barry Tennison; my book is in some ways a direct descendant of this, and some of the exercises have been taken over verbatim from Tennison's example sheets. However, I have benefitted enormously from the freedom allowed under the Warwick course structure, especially the philosophy of teaching (explicitly stated by Christopher Zeeman) that research experience must serve as one's main guideline in deciding how and what to teach.

The 'winking torus' appearing in (2.14–) comes to me from Jim Eells, who informs me he learnt it from H. Hopf (and that it probably goes back to an older German tradition of mathematical art work). I must thank Caroline Series, Frans Oort, Paul Cohn, John Jones, Ulf Persson, David Fowler, an anonymous referee and David Tranah from C.U.P. for helpful comments on the preprint version of this book, and apologise if on occasions I have either not been fully able to accommodate their suggestions, or preferred my own counsel.

I am grateful to Martina Jaeger for a number of corrections to the first printing, and to Isao Wakabayashi for a detailed reading, which uncovered many inaccuracies. I thank especially R.J. Chapman and Bill Bruce for pointing out the most serious error of the first printing (I avoided mention of the Hessian at the start of (7.2) by appealing to a false statement left as an exercise).

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