Chapter 6

Tangent space and nonsingularity, dimension

6.1 Nonsingular points of a hypersurface

Suppose $f \in k[X_1, \dots, X_n]$ is irreducible, $f \notin k$, and set $V = V(f) \subset \mathbb{A}^n$; let $P = (a_1, \dots, a_n) \in V$, and ℓ be a line through P. Since $P \in V$, obviously P is a root of $f|\ell$.

Question: When is P a multiple root of $f|\ell$?

Answer: If and only if ℓ is contained in the affine linear subspace

$$T_P V : \Big(\sum_i \frac{\partial f}{\partial X_i}(P) \cdot (X_i - a_i) = 0 \Big) \subset \mathbb{A}^n,$$

called the *tangent space* to V at P.

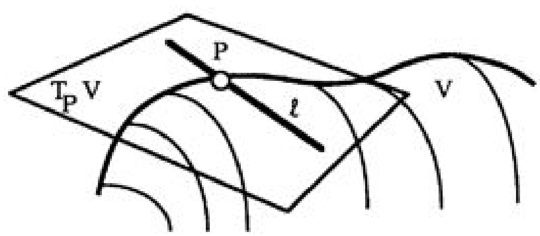

Figure 6.1: Tangent space

To prove this, parametrise ℓ as

$$\ell : X_i = a_i + b_i T,$$