EEENG5610 – Image Analysis and Applications

Formula Sheet

Histogram manipulation:

Equalisation: $g_k' = \left(\frac{2^m-1}{N^2}\right) \sum_{i=0..k} h(g_i)$

where, N^2 is image size, 2^m is #of gray levels

Stretching: $g_i' = \frac{g_i - \min(g)}{\max(g) - \min(g)}(b-a) + a$

where $[a, b]$ is the new range

Convolution:

$$g(x,y) = \sum_{u=-a}^{a} \sum_{v=-b}^{b} w(u,v) \cdot f(x+u, y+v)$$

where, $f(x,y)$ is the image; and $w(x,y)$ is the filter/mask/kernel

Image sharpening:

$$g_{sharp} = f + \gamma(f - h_{blur} * f)$$

Fourier Series:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx)$$

where, $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$,

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx$

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx$

for $n = 0, \pm 1, \pm 2, \ldots$

Edge Detection:

$$\nabla f = [G_x, G_y] = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

$$G = \sqrt{G_x^2 + G_y^2} \quad (\text{or, } G \approx |G_x| + |G_y|)$$

and $\theta_g = \tan^{-1}\left(\frac{G_y}{G_x}\right)$

Vector Gradient Formula (for edge detection)

$$g_{xx} = \left|\frac{\partial R}{\partial x}\right|^2 + \left|\frac{\partial G}{\partial x}\right|^2 + \left|\frac{\partial B}{\partial x}\right|^2$$

$$g_{yy} = \left|\frac{\partial R}{\partial y}\right|^2 + \left|\frac{\partial G}{\partial y}\right|^2 + \left|\frac{\partial B}{\partial y}\right|^2$$

$$g_{xy} = \frac{\partial R}{\partial x}\cdot\frac{\partial R}{\partial y} + \frac{\partial G}{\partial x}\cdot\frac{\partial G}{\partial y} + \frac{\partial B}{\partial x}\cdot\frac{\partial B}{\partial y}$$

$$F(\theta) = \sqrt{\frac{\left[(g_{xx} + g_{yy}) + (g_{xx} - g_{yy})\cos 2\theta + 2g_{xy}\sin 2\theta\right]}{2}}$$

where, $\theta = \frac{1}{2}\tan^{-1}\frac{2g_{xy}}{g_{xx} - g_{yy}}$

Hough Transform:

Eqn. of lines: $y = mx + c$

$\rho = x\cos\theta + y\sin\theta$

Eqn. of circle: $(x - x_0)^2 + (y - y_0)^2 = r^2$

Morphological Operations:

$$A \circ B = (A \ominus B) \oplus B$$
$$A \bullet B = (A \oplus B) \ominus B$$

Feature Extraction:

Moments:

$$m_{pq} = \sum_x \sum_y x^p y^q f(x,y)$$

$$\mu_{pq} = \sum_x \sum_y (x - \bar{x})^p (y - \bar{y})^q f(x,y)$$

where, $\bar{x} = \frac{m_{10}}{m_{00}}, \quad \bar{y} = \frac{m_{01}}{m_{00}}$

and $\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^{\gamma}}$

where $\gamma = \frac{p+q}{2} + 1, \quad p + q \geq 2$

GLCM features (Haralick's):

Contrast: $\sum_i \sum_j (i-j)^2 C_{ij}$

Uniformity (Energy): $\sum_i \sum_j C_{ij}^2$

Entropy: $-\sum_i \sum_j C_{ij} \log_2 C_{ij}$

$$= -3.32 \sum_i \sum_j C_{ij} \log C_{ij}$$

Dissimilarity: $\sum_i \sum_j |i-j| \cdot C_{ij}$

Homogeneity: $\sum_i \sum_j \frac{1}{1+(i-j)^2} C_{ij}$

Max. Prob.: $\max(C_{ij})$

Autocorrelation: $\sum_i \sum_j i \cdot j \cdot C_{ij}$

Colour Conversion:

RGB to HSI:

$$H = \begin{cases} \theta & \text{if } B \leq G \\ 360 - \theta & \text{if } B > G \end{cases} \quad \text{where,}$$

$$\theta = \cos^{-1}\left\{\frac{\frac{1}{2}[(R-G)(R-B)]}{\sqrt{(R-G)^2 + (R-B)(G-B)}}\right\}$$

$$S = 1 - \frac{3 \times \min(R,G,B)}{R+G+B}$$

$$I = (R+G+B)/3$$

HSI to RGB:

When $0° \leq H < 120°$

$B = I(1-S)$

$R = I\left[1 + \frac{S\cos H}{\cos(60° - H)}\right]$

$G = 3I - (R+B)$

When $120° \leq H < 240°$

$H = H - 120°$

$R = I(1-S)$

$G = I\left[1 + \frac{S\cos H}{\cos(60° - H)}\right]$

$B = 3I - (R+G)$

When $240° \leq H < 360°$

$H = H - 240°$

$G = I(1-S)$

$B = I\left[1 + \frac{S\cos H}{\cos(60° - H)}\right]$

$R = 3I - (B+G)$

** Symbols have the usual meaning*

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Maximum Likelihood Rule:

for any unknown X,

$$\text{If } P(C_i|X) > P(C_j|X), \quad \forall i \neq j$$
$$\text{then } X \in C_i$$

Bayes Rule/Theorem/Formula:

for any unknown X,

$$P(C_i|X) = \frac{P(X|C_i)P(C_i)}{P(X)}$$

Statistical Distributions:

Normal: $P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right]$

where, $\mu = \frac{\sum x}{N}$, and $\sigma^2 = \frac{\sum(x-\mu)^2}{N}$

Poisson: $\mathrm{Po}(\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}, \quad k = 0,1,2,\ldots$

Binomial: $\mathrm{Bi}(n,p) = \binom{n}{x} p^x (1-p)^{n-x}$

Multivariate Gaussian:

$$P(X|C_i) = \frac{1}{(2\pi)^{\frac{N}{2}}|\Sigma_i|^{\frac{1}{2}}} \exp\left[-\frac{1}{2}(X-\mu_i)^T \Sigma_i^{-1}(X-\mu_i)\right]$$

Distance metrics:

Euclidean: $\sqrt{\sum(x_i - y_i)^2}$

Manhattan (or **City-Block)**: $\sum|x_i - y_i|$

Minkowski: $\left(\sum|x_i - y_i|^q\right)^{1/q}$

Hamming: $\sum(x_i \text{ XOR } y_i)$

Mahalanobis: $(X-\mu_i)^T \Sigma_i^{-1}(X-\mu_i)$

Feature normalisation:

Min-max rule: $\tilde{x} = \frac{x_i - \min(x)}{\max(x) - \min(x)}$

Z-score : $\tilde{x} = \frac{x_i - \mu}{\sigma}$

Fisher's Linear Discriminant Rule:

$$d(X) = w^T\left(X - \frac{1}{2}(\mu_1 + \mu_2)\right)$$
where, $w = S_w^{-1}(\mu_2 - \mu_1)$

Neural Networks:

M&P Neuron/Rosenblatt's Perceptron model :

$$o_j = f_\theta(net_j), \; net_j = \sum \omega_{ij} I_i$$

where, $f_\theta(x) = \begin{cases} 1 & \text{if } x \geq \theta \\ 0 & \text{if } x < \theta \end{cases}$, $\theta = \text{threshold}$

Learning Rule: $\Delta\omega_{ij} = \eta(t_j - o_j)I_i$

$$\omega_{ij}^{t+1} = \omega_{ij}^t + \Delta\omega_{ij}$$

Multi-layer Perceptron model :

$$o_j = f(net_j), \qquad net_j = \sum \omega_{ij} x_i + b$$

where, $f(x)$ is the activation function

sigmoid: $f(x) = \frac{1}{1+e^{-x}}$

tanh: $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1}$

ReLU: $f(x) = \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$

Learning Rule:

$$E_{total} = \sum \frac{1}{2}(target - output)^2$$
$$\omega_{ij}^{+} = \omega_{ij} - \eta \frac{\partial E_{total}}{\partial \omega_{ij}}$$

Kohonen network :

$$d_j = \|\vec{\omega}_{ij} - \vec{x}\| = \sqrt{\sum\left(\omega_{ij}(t) - x_i(t)\right)^2}$$

$$\omega_{ij}(t+1) = \omega_{ij}(t) + \eta(t) N_{j^*}(t)\left[x_i - \omega_{ij}(t)\right]$$

where, $N_{j^*}(t)$ is the neighbourhood for node j^*

Classifier performance:

$$\text{Accuracy} = \frac{\#\text{ correctly classified}}{\#\text{ of test sample} - \#\text{ unclassified}}$$

$$\text{Error rate} = \frac{\#\text{ of samples misclassified}}{\#\text{ of test sample} - \#\text{ unclassified}}$$

$$\text{Rejection rate} = \frac{\#\text{ of samples unclassified}}{\#\text{ of test sample}}$$

Biometric system performance metrics:

$$\textbf{Accuracy} = \frac{\text{GA+ TR}}{\text{GA+FA+FR+TR}}$$

$$\textbf{FAR} = \frac{\text{FA}}{\text{FA+TR}}, \qquad \textbf{FRR} = \frac{FR}{\text{GA+FR}}$$

$$\textbf{Precision} = \textbf{PPV} = \frac{\text{GA}}{\text{GA+FA}}, \qquad \textbf{NPV} = \frac{TR}{\text{TR+FR}}$$

$$\textbf{F1 score} = \frac{2\text{ GA}}{2\text{ GA} + \text{FA} + \text{FR}}$$

Fingerprint matching:

$$sd(m_j', m_i) = \sqrt{(x_j' - x_i)^2 + (y_j' - y_i)^2} \leq r_0$$
$$dd(m_j', m_i) = \min(|\theta_j' - \theta_i|, 360° - |\theta_j' - \theta_i|) \leq \theta_0$$

Multimodal fusion:

Score fusion:

$$SCORE = \sum_{i=1}^{n} \omega_i s_i$$

if $SCORE > \theta \implies ACCEPT$

else $REJECT$

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