Advanced Algorithms - Notes

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0 Reference

0.1 Probability

Definition 1.1 - *Sample Space*
A *Sample Space* is the set of possible outcomes of a scenario. A *Sample Space* is not necessarily finite.
e.g. Rolling a dice $S := \{1, 2, 3, 4, 5, 6\}$.

Definition 1.2 - *Probability Measure, $\mathbb{P}$*
Probability Measure, $\mathbb{P}$, is a function from the sample space to $[0, 1]$ which fulfils $\sum_{x \in S} \mathbb{P}(x) = 1$.

$$\mathbb{P} : S \to [0, 1]$$

A *Probabiltiy Measure* must fulfil the criteria that for disjoint events $\{A_1, \dots, A_n\}$

$$\mathbb{P}\left(\bigcup_i A_i\right) = \sum_i \mathbb{P}(A_i)$$

Definition 1.3 - *Event*
An *Event* is a subset of the *Sample Space*.
The probability of an *Event*, A, happening is

$$\mathbb{P}(A) = \sum_{x \in A} \mathbb{P}(x)$$

Definition 1.4 - *Sigma Field, $\mathcal{F}$*
A *Sigma Field* is the set of possible events in a given scenario.
A *Sigma Field* must fulfil the following criteria

i) $S \in \mathcal{F}$.

ii) $\forall\ A \in \mathcal{F} \implies A^c \in \mathcal{F}$.

iii) $\forall\ A_1, \dots, A_n \in \mathcal{F} \implies \bigcup_i A_i \in \mathcal{F}$.

Definition 1.5 - *Random Variable*
A *Random Variable* is a function from the sample space, S, to the real numbers, $\mathbb{R}$.

$$X : S \to \mathbb{R}$$

The probability of a *Random Variable*, X, taking a specific value x is found by

$$\mathbb{P}(X = x) = \sum_{\{a \in S : X(a) = x\}} \mathbb{P}(a)$$

Definition 1.6 - *Indicator Random Variable*
An *Indicator Random Variable* is a *Random Variable* which only ever takes 0 or 1 and is used to indicate whether a particular event has happened (1), or not (0).

$$\mathbb{E}(I) = \mathbb{P}(I = 1)$$

Definition 1.7 - *Expected Value, $\mathbb{E}$*
The *Expected Value* of a *Random Variable* is the mean value of said *Random Variable*

$$\mathbb{E}(X) := \sum_x x\mathbb{P}(X = x)$$

Theorem 1.1 - *Linearity of Expected Value*
Let $X_1, \dots, X_n$ be random variables. Then

$$\mathbb{E}\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} \mathbb{E}(X_i)$$

Theorem 1.2 - *Markov's Inequality*
Let X be a non-negative random variable. Then

$$\mathbb{P}(X \geq a) \leq \frac{1}{a}\mathbb{E}(X) \quad \forall\ a > 0$$