Artificial Intelligence with Logic Programming - Notes

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1 Introduction

Definition 1.1 - *Types of AI*

1. *Weak AI* - Can solve a specific task.
2. *Strong AI* - Can solve general problems.
3. *Ultra Strong AI* - Can solve general problems & explain why/what it is doing.

2 Logic Programming

Definition 2.1 - *Logic Programming*
Logic Programming is a *declarative paradigm* where programs are concieved as a logical theory, rather than a set-by-step description of an algorithm. A Procedure call is viewed as a theorem which the truth needs to be established about. (*i.e.* executing a programming is analogous to searching for truth in a system).

Remark 2.1 - *Variables*
In *Logic Programming* a *Variable* is a variable in the mathematical sense, that is they are are placeholders that can take on any value.

Remark 2.2 - *Machine Model*
A *Machine Model* is an abstraction of the computer on which programs are executed. In *Imperative Programming* we assume a dynamic, state-based machine model where the state of the computer is given by the contents of its memory & a program statement is a transition from one statement to another. In *Logic Programming* we do not assume such a dynamic model.

2.1 Clausal Logic

Notation 2.1 - *Variables & Values*
Variables are denoted by having a capitalised first letter, whereas *values* are completly lowercase.

Definition 2.1 - *Clausal Logic*
Clausal Logic is a formalism for representing & reasoning with knowledge.

Keyword	**Description**
`S:-C.`	**If** condition `C` holds **then** statement `S` is true.
`S:-\+C.`	**If** condition `C` does **not** hold **then** statement `S` is true.
`S:-C1,C2.`	**If** conditions `C1` **and** `C2` both hold **then** statement `S` is true.
`S:-C1;C2.`	**If** at least one of `C1` **or** `C2` hold **then** statement `S` is true.
`S:-C,!.`	**Cut** the program after finding **first** `S` where `C` holds.

Definition 2.2 - *Facts & Rules*
Facts are logical formulae which are defined for explicit values **only**. *Facts* denoted unconiditional truth.

```
nearby(bond_street,oxford_circus).
```

Rules are logical formula which are defined in termms of variables (and explicit values). *Rules* denote conditional truth.

```
nearby(X,Y):-connected(X,Z,L),connected(Z,Y,L)
```

Definition 2.3 - *Query,* `?-`
A *Query* asks a question about the knowledgebase we have defined. If we just pass *values* to a *Query* then it shall simply return whether the statement is true or not. If we pass *unbound variables* as well then it shall return values for the variable which make the statement true, if any exist.

Example 2.1 - *Query*

```
1 ?-nearby(bond_street,oxford_circus)
2 ?-nearby(bond_street,X)
```

(1) will return *true* if we have defined `bond_street` to be near to `oxford_circus`.
(2) will return all the values of `X` (*i.e.* stations) which are near to `bond_street`.

Definition 2.4 - *Resolution*
In order to answer a query `?-Q1,Q2,...` find a rule `A:-B1,...,Bn` such that `A` matches with `Q1` then proceed to answer `?-B1,...,Bn,Q2,...`.
This is a *procedural interpretation* of logical formulae & is what allows *Logic* to be a programming language.

Definition 2.5 - *Functor*
*Functor*s provide a way to name a complex object composed of simpler objects & are never evaluated to determine a value.

```
reachable(X,Y,noroute):-connected(X,Y,L)
reachable(X,Y,route(Z,R)):-connected(X,Z,L),connected(Z,Y,R)
```

Querying `?-reachable(oxford_circus,tottenham_court_road,R)` will return a route `R` which connects the two stations, on a single line.
The above definition can be read as `X` is reachable from `Y` if they are connected **or** if there exists a station `Z` which is connected.

Definition 2.6 - *List Functor,* `.`
The *List Functor* takes two arguments, one on each side, and has terminator `[]`.

$$[a,b,c] \equiv \texttt{.(a,.(b,.(c,[])))}$$

Alternatively we can use a pipe to distinguish between a value and the rest of the list

`[X,Y|R]`

Remark 2.1 - *Logical Formulae*
Logical Formulae determine what is being modelled & the set of formulae that can be derived by applying inference rules.
i.e. *Logical Formulae* have both declarative (what is true) & procedural (how the truth is reached) meaning.
Changing the order of statements in a clause does not change the declarative meaning ($3 = 2 + 1 \equiv 3 = 1 + 2$) but does change the procedural meaning & thus chanes what loops we may get stuck in, what solutions are found first & how long execution takes.
N.B. The procedural meaning of *Logical Formulae* is what allows them to be used as a programming language.

2.2 SLD-Resolution

Definition 2.1 - *SLD-Resolution*
SLD-Resolution is the process `Prolog` uses to resolve a query.

1. ***S**election Rule* - Left-to-right (Always take the left branch if it is there).
   This is equivalent to reading clauses top to bottom in a `Prolog` program.

2. ***L**inear Resolution* - The shape of the proof trees obtained. ***D**efinite Clauses*

N.B. These rules vary for different languages.

Definition 2.2 - *SLD-Tree*
SLD-Trees are graphical representations of *SLD-Resolution*.
They are not the same as a proof tree as only the resolvents are shown (no input clauses or unifiers) andit contains every possible resolution step, leading to every leaf on an *SLD-Tree* being an empty clause $\square$.
The left-most leaf in an *SLD-Tree* will be returned as the first solution, if we end up in an infinite sub-tree then no solution will ever be returned.
Non-leaf nodes in *SLD-Trees* are called *choice points*. *N.B.* If a leaf is a failure then we underline it.

Proposition 2.1 - *Traversing an SLD-Tree*
`Prolog` traverses *SLD-Trees* in a *depth-first* fashion, backtracking whenever it reaches a success or a failure. This can be visualised as following the left branch of the tree whenever it is avaiable. This leads to `Prolog` being incomplete (does not always return all solutions) as once it is in an infinite sub-tree it has no way of escaping.

Remark 2.1 - *Backtracking*
Since we need to be able to *Backtrack* after finding successes or failures, we need to store all previous resolvents which have not been fully explored yet as well as a pointer to the most recent program clause that has been tried.
Due to the depth-first nature of `Prolog` this can be stored as a *stack*.

Remark 2.2 - *Order of clauses*
Due to the *Selection Rule* being left-to-right you should *put non-recursive clauses before recursive clauses* to ensure that solutions are found before infinite sub-trees.

Definition 2.3 - *Transitivity*
Transitivity is a property of predicate that states

$$X = Y \ \&\ Y = Z \implies X = Z$$

Transitivity clauses can cause infinite loops is not implemented carefully.

Remark 2.3 - *Why not use Breadth-First Search?*
Breadth-First Search requires for each branch at a given level to be tracked (rather than just one in *Depth-First Search*), this requires more memory and was deemed memory-inefficient by `Prolog` developers.

Remark 2.4 - *Queries with no solution*
Some queries do not have a solution as the *SLD-Tree* they produce is infinite, no matter how it is read & thus an answer can never be resolved.
Consider

```
brother_of(paul,peter).
brother_of(X,Y):-brother_of(Y,X).
```

The query `brother_of(peter,maria)` has no answer since its *SLD-Tree* is infinite.

Definition 2.4 - *Cut,* `!`
A *Cut* says that once it has been reached, stick to all the variable substitutions found after entering its clause. (*i.e.* Do not backtrack past it?).
Cuts have the effect of pruning the *SLD-Tree*.
Cuts which don't remove any successful branches are called *Green Cuts* and are harmless. *Cuts* which do remove successful branches are caleld *Red Cuts* and affects the procedular meaning of the process.
N.B. Cuts make the left sub-tree deterministic.

Notation 2.1 - *In SDL-Trees we put a box around subtrees which are pruned by* `!`*.*

2.3 Negation as Failure

Definition 2.1 - *Netgation as Faliure*
Negation as Failure is an interpretation of the semantics for failure.
It states if we cannot prove `q` to be true then we take `not(q)` to be true.
i.e. If we don't have enough information to state where `q` is true then we assume it to be false.
e.g. Negation as failure would resolve that *"I cannot prove God exists, therefore God does not exist."*.

Proposition 2.1 - *Altering Negation as Failure*

Proposition 2.2 - *Cut as Failure*
Suppose we have a program of the following form

```
p:-q,!,r.
p:-s.
```

We can interpret it as

```
p:-q,r.
p:-not_q,s.
```

where `not_q:=q,fail.`.

Definition 2.2 - *Negation Function,* `not()`
Let `Goal` be a literal we wish to negate

```
not(Goal):-Goal,!,fail.
not(Goal).
```

Definition 2.3 - *Call Function,* `call()`
TODO

Remark 2.1 - `not()` *&* `call()` *are called meta-predicates as they take formulas as arguments.*

Remark 2.2 - *Negation as Failure is* ***not*** *Logical Negation*
If we cannot prove predicate `q` we know that `q` is not a logical consequence of the program, but that does not mean that its negation `:-q` is a logical consequence of the program.
This is acceptable reasoning in some scenarios, but not others.

Definition 2.4 - *Logical Negation*
Logical Negation can only be expressed by indefinite clauses, as below

```
p;q,r.
p;q:-s.
s.
```

Definition 2.5 - *Conditionals, `if_then_else()`*
We give the following definition where if `S` holds then `T` is executed, else `U` is executed.

```
if_then_else(S,T,U):-S,!,T/
if_then_else(S,T,U):-U.
```

Notation 2.1 - *Implication, `->`*
We can nest `if_then_else()` to add more *elif* clauses but is clumsy notation.
Instead we simplify `if_then_else(S,T,U).` to `S->T;U.`.
This can be nested as `P->Q;(R->S;T)`. The keyword `otherwise` can be used instead of `true` as it can be more readable.

0 First Order Logic

Definition 0.1 - *The Alphabet*
A *First Order Logic* language can comprise the following types of symbols

Name	Description	Example Notation
Variables	Arbitrary objects	X Y (Capitalised)
Constants	Specified objects	oliver (Lower Case)
Functions	Object mappings	mother/1 father/1
Propositions	Unstructured assertions	p q
Predicates	Object properties & relations	happy/1 loves/2
Connectives	Connect two predicates	$\neg \wedge \vee \to$
Quantifiers	Amount of objects to consider	$\forall\exists$
Logical Constants	True & False	$\top\bot$
Punctuation	Structure of groupings	(), :
Equality	Equivalent values	=

Definition 0.2 - *Arity*
The *Arity* of a *Function* is the number of arguments it takes.
e.g. `father/1` has 1-*Arity*.

0.1 Syntax

Definition 0.1 - *Term*
A *Term* is a

1. Single constant, `c`.
2. Single variable, `X`.
3. n-arity function applied to n terms, $\texttt{f}(t_1,\dots,t_n)$ where $t_1,\dots,t_n$ are Terms.

Definition 0.2 - *Atom*
An *Atom* is a

1. Single proposition symbol, `p`.
2. n-arity predicate applied to n terms, $\texttt{r}(t_1,\dots,t_n)$ where $t_1,\dots,t_n$ are Terms.

Definition 0.3 - *Formula*
A *Formula* is

1. An atom, `a`.
2. A logical constant, `\top` or `\bot`.
3. A negation of a formula, $\neg\texttt{f}$.
4. A conjugtion of two formulae, $\texttt{f}\wedge\texttt{g}$.
5. A disjugtion of two formulae, $\texttt{f}\vee\texttt{g}$.
6. A condition of two formulae, $\texttt{f}\to\texttt{g}$.
7. A universal quantification of a formula and a variable, $\forall\texttt{X:f}$.
8. A existential quantification of a formula and a variable, $\exists\texttt{X:f}$.

Definition 0.4 - *Normal Forms*
Normal Forms are a restricted sub-languages of *First Order Languages.*
Normal Forms are used to facilitate the storage of logical formula in a computer's memory by simplifying the inference procedures required for their manipulation.
N.B. Prenex Normal Forms & *Conjunctive Normal Forms* are examples.

Definition 0.5 - *Prenex Normal Forms, PNFs*
Prenex Normal Forms only allow formulae of the form

<prefix><matrix>

where prefix is a string of quantifiers & matrix is a quantifier-free formula.
N.B. There is an algorithm for converting any *First Order Language* formula into *PNF* form (**Theorem 0.1**).

Definition 0.6 - *Conjunctive Normal Forms, CNFs*
Conjunctive Normal Forms are more restrictive that *PNFs* since they only allow formulae of the form

<prefix><matrix>

where prefix is a string of universal quantifiers & matrix is a conjuction of disjunctions of atoms, or their negations.
This conjuctions of disjunctions are known as clauses.

Remark 0.1 - *CNF Matrix*
The *Matrix* of a *CNF* is either

1. A set of sets of literals (*i.e.* atoms & their negations).
2. A set ofclauses of the form <body>→<head> where the head is a disjunction of literals (or ⊥), and the body is a conjunction of literals (or ⊤).

Theorem 0.1 - *FOL to PNF*

1. Replace implications,p→q by disjunction and negation, (¬p)∨q.
2. Push negations inside, so that each of them immediately precede a literal.
3. Move quantifiers to the front (the result is said to be *PNF*).
4. Replace existential variables by *Skolem Functors.*
5. Rewrite into *CNF* (*i.e.* A conjunction of disjunction of literals).
6. Rewrite each conjunct to a clause.

Example 0.1 - *Theorem 0.1*

```
0 (∀Y:∃:mother_of(X,Y))∧(¬∀Z:∃W:woman(Z)→mother_of(Z,W))
1 (∀Y:∃:mother_of(X,Y))∧(¬∀Z:∃W:¬woman(Z)∨mother_of(Z,W))
2 (∀Y:∃:mother_of(X,Y))∧(∃Z:∀W:woman(Z)∧¬mother_of(Z,W))
3 ∀Y:∃X:∃Z:∀W:(mother_of(X,Y)∧woman(Z)∧¬mother_of(Z,W))
4 ∀Y:∀W:(mother_of(sk1(Y),Y)∧woman(sk2(Y))∧¬mother_of(sk2(Y),W))
5 mother_of(sk1(Y),Y)∧woman(sk2(Y))∧¬mother_of(sk2(Y),W)
5 mother_of(mother(Y),Y)∧woman(childless_woman)∧¬mother_of(
      childless_woman,W)
6 ⊤ →mother_of(mother(Y),Y)
  ⊤ →woman(childless_woman)← ⊤
  mother_of(childless_woman,W)→ ⊥
```

This can be expressed in Prolog as

```
mother_of(mother(Y),Y).
woman(childless_woman).
:- mother_of(childless_woman,W).
```

0.2 Semantics

Definition 0.1 - *Assignment*
Let $\mathcal{L}$ be a *First Order Language* with domain $|\mathcal{L}|$.
An *Assignment* is a function `h()` from a variable symbol `v`$\in \mathcal{L}$ to $|\mathcal{L}|$.

Definition 0.2 - *Interpretation*
Let $\mathcal{L}$ be a *First Order Language* with domain $|\mathcal{L}|$.
An *Assignment* is a function $()^I$ from an n-arity function symbol `f/n`$\in \mathcal{L}$ to functions $|\mathcal{L}|^n \to |\mathcal{L}|$, and from n-arity predicate symbols `p/n` of $\mathcal{L}$ to relations on $|\mathcal{L}|^n$.

Definition 0.3 - *Value*
Let $\mathcal{L}$ be a *First Order Language* with domain $|\mathcal{L}|$.
The *Value* of a term, `t`, denoted $[t]^{I,h}$ is defined as

$$\begin{array}{ll} [X]^{I,h} := h(X) & \text{if } t \text{ is a variable } X. \\ [f(t_1,\dots,t_n)]^{I,h} := f^I([t_1]^{I,h},\dots,[t_n]^{I,h}) & \text{if } t \text{ is a term of the form } f(t_1,\dots,t_n). \end{array}$$

Definition 0.4 - *Satisfaction,* $\models$
The *Satisfaction* of a formula `f`, denoted $I,h\models f$ is defined as

$$\begin{array}{lll} I,h\models \top & & \\ I,h\not\models \bot & & \\ I,h\models p(t_1,\dots,t_n) & \text{iff} & p^I([t_1]^{I,h},\dots,[t_n]^{I,h}) \\ I,h\models f\wedge g & \text{iff} & I,h\models f \text{ and } I,h\models g \\ I,h\models \neg f & \text{iff} & I,h\not\models f \\ I,h\models \forall X:f & \text{iff} & I,h[X\mapsto d]\models f \text{ for every } d\in |\mathcal{L} \end{array}$$

N.B. Denotes if a formula is valid for the given interpretation, `I`, & assignment, `h`.

Definition 0.5 - *Model of Formula*
An interpreation, `M`, is a *Model* of a formula, `f`, denoted $M\models f$ iff $M,h\models f$ for all assignments `h` $\in \mathcal{L}_{\text{var}} \to |\mathcal{L}|$.

Definition 0.6 - *Model of Set of Formulae*
An interpreation, `M`, is a *Model* of a set of formulae, `F`, denoted $M\models F$ iff $M\models f$ for all $f\in F$.

Remark 0.1 - *A set of formulae is known as a Theory*

Definition 0.7 - *Entailment*
A set of formulae, `F`, *Entails* a set of formulae, `G`, denoted $F\models G$ iff every model of `F` is a model of `G`.
N.B. Here `F` is an alternative semantics to `G`.

0.3 Herbrand

Definition 0.1 - *Herbrand*
A *Herbrand Universe* is a set of all possible ground *Terms.*

A *Herbrand Base* is a set of all possible ground *Atoms* (all possible combinations of ground terms as arguments to predicates).
A *Herbrand Interpretation* is any subset of the *Herbrand Base*.
A *Herbrand Model* is a *Herbrand Interpreation* which is said to be true.

Example 0.1 - *Herbrand*
Consider a *First Order Language* which consists of two constants `peter` & `maria` and two predicates `teacher/1` & `student_of/2`.
Then the *Herbrand Universe* is

{`peter`,`maria`}

And the *Herbrand Base* is

{`teacher(peter)`,`teacher(maria)`,
`student_of(peter,peter)`,`student_of(peter,maria)`,
`student_of(maria,peter)`,`student_of(maria,maria)`}

A *Herbrand Interpretation* would be any subset of these six.
Which of these subsets is a *Herbrand Model* depends on how we define the predicates, `teacher/1` & `student_of/2`.

Definition 0.2 - *Horn Theory*
A *Horn Theory*, is a theory whose clausal form has the property that every clause has at most one postive literal.
N.B. A formula entails a *Horn Theory* if it satisfies the minimal model of the *Horn Theory*.
N.B. *Horn Theories* have a unique minimal model.