Calculus 1 - Problem Sheet 1

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a)

$$x = 1$$

1 The Real Line; Intervals And Inequalities

1) Solve the following inequalities:

a) $-3 < 4 - x < 8$

$$\begin{aligned} & -3 < 4 - x < 8 \\ \Rightarrow \quad & -7 < -x < 4 \\ \Rightarrow \quad & 7 > x > -4 \\ \Rightarrow \quad & \underline{x = (-4, 7)} \end{aligned}$$

b) $\frac{x}{x+5} < 1, \ x \neq 0$

$$\begin{aligned} x > 0 \quad \Rightarrow \quad & 3x - 1 < 4x \\ \Rightarrow \quad & -1 < x \\ \Rightarrow \quad & x = (0, \infty) \\ x < 0 \quad \Rightarrow \quad & 3x - 1 > 4x \\ \Rightarrow \quad & -1 > x \\ \Rightarrow \quad & x = (-\infty, -1) \\ \Rightarrow \quad & \underline{x = (-\infty, -1) \cup (0, \infty)} \end{aligned}$$

c) $\frac{x}{x+5} < 1, \ x \neq -5$

$$\begin{aligned} x > -5 \quad \Rightarrow \quad & x < x + 5 \\ \Rightarrow \quad & 0 < 5 \\ \text{AlwaysTrue} \quad \Rightarrow \quad & x = (-5, \infty) \\ x < -5 \quad \Rightarrow \quad & x = x + 5 \\ \Rightarrow \quad & 0 > 5 \\ \text{Always false} \quad \Rightarrow \quad & \text{no more real solutions} \\ \Rightarrow \quad & \underline{x = (5, \infty)} \end{aligned}$$

d) $\dfrac{2x-1}{3x+2} > 3, \ x \neq \dfrac{-2}{3}$

$$
\begin{aligned}
x > \frac{-2}{3} \quad &=> \quad 2x - 1 < 9x + 6 \\
&=> \quad -7 < 7x \\
&=> \quad -1 < x \\
&=> \quad -1 < x < \frac{-2}{3} \\
&=> \quad \underline{x = \left(-1, \frac{-2}{3}\right)}
\end{aligned}
$$

e) $\left|\frac{3x-1}{x}\right| > 4$

$$
\begin{aligned}
\text{Let} \frac{3x-1}{x} = 0 \quad , \quad & x \neq 0 \\
=> \quad & 3x - 1 = 0 \\
=> \quad & x = \frac{1}{3}
\end{aligned}
$$

$$
\begin{aligned}
x > \frac{1}{3} \quad &=> \quad \frac{3x-1}{x} > 4 \\
&=> \quad 3x - 1 > 4x \\
&=> \quad -1 > x \text{ No solutions}
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{3} > x > 0 \quad &=> \quad -\left(\frac{3x-1}{x}\right) > 4 \\
&=> \quad 1 - 3x > 4x \\
&=> \quad 1 > 7x \\
&=> \quad \frac{1}{7} > x \\
&=> \quad x = (-1, 0) \\
&=> \quad \underline{x = \left(-1, \frac{1}{7}\right)}
\end{aligned}
$$

f) $\left|\frac{x}{x+5}\right| \leq 2$

$$
\begin{aligned}
\text{Let} \frac{x}{x+5} = 0 \quad , \quad & x \neq -5 \\
=> \quad & x = 0
\end{aligned}
$$

$$
\begin{aligned}
x > 0 \quad &=> \quad \frac{x}{x+5} \leq 2 \\
&=> \quad x \leq 2x + 10 \\
&=> \quad -10 \leq x \\
&=> \quad x > 0, x = (0, \infty]
\end{aligned}
$$

$$
\begin{aligned}
-5 < x < 0 \quad &=> \quad \frac{-x}{x+5} \leq 2 \\
&=> \quad -x \leq 2x + 10 \\
&=> \quad -10 \leq 3x
\end{aligned}
$$

$$
\begin{aligned}
&=> \quad x = \left[\frac{-10}{3}, 0\right) \\
x < -5 \quad &=> \quad \frac{x}{x+5} \leq 2 \\
&=> \quad x \geq 2x + 1 - \\
&=> \quad x = [-\infty, 0) \\
x \quad &= \quad \underline{(-\infty, -10] \cup \left[\frac{-10}{3}, \infty\right)}
\end{aligned}
$$

2) Solve the following:

a) $|3x - 4| = 3$

$$
\begin{aligned}
& \quad 3x - 4 = 3 \\
=> \quad & 3x = 7 \\
=> \quad & \underline{x = \frac{7}{3}} \\
\text{And,} \quad & -(3x - 4) = 3 \\
=> \quad & 4 - 3x = 3 \\
=> \quad & 1 = 3x \\
=> \quad & \underline{x = \frac{1}{3}}
\end{aligned}
$$

b) $|x + 5| = 2$

$$
\begin{aligned}
& \quad x + 5 = 2 \\
=> \quad & \underline{x = -3} \\
\text{And,} \quad & -(x + 5) = 2 \\
=> \quad & -x = 7 \\
=> \quad & \underline{x = 7}
\end{aligned}
$$

c) $|2x - 3| = |3x - 2|$

$$
\begin{aligned}
& \quad 2x - 3 = 3x - 2 \\
=> \quad & \underline{x = -1} \\
\text{And,} \quad & -(2x - 3) = 3x - 2 \\
=> \quad & 3 - 2x = 3x - 2 \\
=> \quad & 5x = 5 \\
=> \quad & \underline{x = 1}
\end{aligned}
$$

d) $|x + 2| = |x + 3|$

$$
\begin{aligned}
& \quad x + 2 = x + 3 \\
=> \quad & 2 = 3 \text{ No real solutions} \\
\text{And,} \quad & -(x + 2) = (x + 3) \\
=> \quad & -2 = 2x + 3 \\
=> \quad & \underline{x = \frac{-5}{2}}
\end{aligned}
$$

e) $|x+2| = 4x-1$

$$\begin{aligned} & x+2 = 4x-1 \\ \Rightarrow\ & 3 = 3x \\ \Rightarrow\ & \underline{x = 1} \\ \text{And,}\quad & -(x+2) = 4x-1 \\ \Rightarrow\ & -1 = 5x \\ \Rightarrow\ & \underline{x = \frac{-1}{5}} \end{aligned}$$

f) $|x-1| < |3x+1$

$$\begin{aligned} \text{Let}\quad & x-1 = 3x+1 \\ \Rightarrow\ & -2 = 2x \\ \Rightarrow\ & x = -1 \\ \text{And,}\quad & -(x-1) = 3x \\ \Rightarrow\ & 1-x = 3x+1 \\ \Rightarrow\ & x = 0 \\ \Rightarrow\ & \underline{x = (-\infty, -1) \cup (0, \infty)} \end{aligned}$$

g) $|3x+4| < |2x-1|$

$$\begin{aligned} \text{Let}\quad & 3x+4 = 2x-1 \\ \Rightarrow\ & x = -5 \\ \text{And,}\quad & -(3x+4) = 2x-1 \\ \Rightarrow\ & -3 = 5x \\ \Rightarrow\ & x = \frac{-3}{5} \\ \Rightarrow\ & \underline{x = (-5, \frac{-3}{5})} \end{aligned}$$

2 Polynomials and Roots

3) Solve the following:

a) $|x^2-2| < 1$

$$\begin{aligned} & x^2-2 < 1 \\ \Rightarrow\ & x^2 < 3 \\ \Rightarrow\ & -\sqrt{3} < x\sqrt{3} \\ \text{And,}\quad & -(x^2-2) < 1 \\ \Rightarrow\ & x^2-2 > -1 \\ \Rightarrow\ & x < -1, x > 1 \\ \Rightarrow\ & \underline{x = (-\sqrt{3}, -1) \cup (1, \sqrt{3})} \end{aligned}$$

b) $|x^2-2| < 3$

$$
\begin{aligned}
& x^2 - 2 < 3 \\
\Rightarrow\ & x^2 < 5 \\
\Rightarrow\ & x = (-\sqrt{5}, \sqrt{5}) \\
\text{And,}\quad & -(x^2 - 2) < 3 \\
\Rightarrow\ & x^2 - 2 > -3 \\
\Rightarrow\ & x^2 > -1 \text{ Norealsolutions} \\
\Rightarrow\ & \underline{x = (-\sqrt{5}, \sqrt{5})}
\end{aligned}
$$