Calculus 1 - Notes

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Contents

1 Before Calculus

1.1 Fundamental Theorem of Calculus

Definition 1.01 - *Fundamental Theorem of Calculus*
The Fundamental Theorem of Calculus states

$$\frac{d}{dx}\int_a^x f(t)dt = f(x)$$

Definition 1.02 - *Common Sets of Numbers*
Natural Numbers, set of positive integers - $\mathbb{N} := \{1, 2, 3, ...\}$.
Whole Numbers, set of all integers - $\mathbb{Z} := \{..., -2, -1, 0, 1, 2, ...\}$.
Rational Numbers, set of fractions - $\mathbb{Q} := \left\{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{N}\right\}$.
Real Numbers, set of all rational & irrational numbers - $\mathbb{R}$.

1.2 Intervals

Definition 1.03 - *Intervals*
Sets of real numbers that fulfil in given ranges.
Notation

$$\begin{aligned}
[a, b] &:= \{x \in \mathbb{R} : a \leq x \leq b\} \\
(a, b] &:= \{x \in \mathbb{R} : a < x \leq b\} \\
[a, b) &:= \{x \in \mathbb{R} : a \leq x < b\} \\
(a, b) &:= \{x \in \mathbb{R} : a < x < b\}
\end{aligned}$$

Example
In what interval does x lie such that:

$$|3x + 4| < |2x - 1|$$

Solution

$$\begin{aligned}
\text{Case } 1 : x \geq \frac{1}{2}& \\
&=> 1 - 2x < 3x + 4 < 2x - 1 \\
&=> 1 - 2x < 3x + 4 \\
&=> x > \frac{-3}{5} \\
\text{And,} &=> 3x + 4 < 2x - 1 \\
&=> x < -5
\end{aligned}$$

There are no real solutions in this range.

$$\begin{aligned}
\text{Case } 2 : x < \frac{1}{2}& \\
&=> 2x - 1 < 3x + 4 < 1 - 2x \\
&=> 2x - 1 < 3x + 4 \\
&=> -5 < x
\end{aligned}$$

$$\begin{aligned} \text{And, } &=> 3x+4 < 1-2x \\ &=> 5x < -3 \\ &=> x < \frac{-3}{5} \\ \\ &=> -5 < x < \frac{-3}{5},\ \underline{x \in \left(-5, \frac{-3}{5}\right)} \end{aligned}$$

Definition 1.04 - *Functions*
Functions map values between fields of numbers. The signature of a function is defined by

$$f : A \to B$$

Where f is the name of the function, A is the domain and B is the co-domain.
The *Domain* of a function is the set of numbers it can take as an input.
The *Co-Domain* is the set of numbers that the domain is mapped to.

<u>N.B.</u> - A function is valid iff it maps each value in the domain to a single value in the co-domain.

Definition 1.05 - *Maximal Domain*
The *Maximal Domain* of a function is the largest set of values which can serve as the domain of a function.

Remark 1.06 - *Types of Function*
Let $f : A \to B$
Polynomials

$$f(x) = a_0 + a_1x + ... + a_nx^n$$

Rational

$$f(x) = \frac{p(x)}{q(x)}, \quad q(x) \neq 0\ \forall\ x \in A$$

Trigonometric

$$sin(x),\ cos(x),\ tan(x) \text{ etc.}$$

2 Limits

2.1 Limits

Definition 2.01 - *Limits*
A limit is the value a function tends to, for a given x.
i.e. The value f(x) has at it gets very close to x.

Formally We say L is the limit of $f(x)$ as x tends to x_0 if

$$\forall\ \varepsilon > 0,\ \exists\ \delta > 0 \text{ st if } x \in A \text{ and } |x - x_0| < \delta => |f(x) - L| < \varepsilon$$

Notation

$$\lim_{x \to x_0} f(x) = L$$

Remark 2.02 - *Common Limits*

$$\begin{aligned}\lim_{x\to x_0}\frac{1-x}{1+x}&=1\\ \lim_{x\to\infty}\frac{1}{x}&=0\\ \lim_{x\to 0}\frac{sin(x)}{x}&=1\end{aligned}$$

Theorem 2.03 - *Operations with limits*
Let $\lim_{x\to x_0} f(x)=L_f$ and $\lim_{x\to x_0} g(x)=L_g$ Then

$$\begin{aligned}\lim_{x\to x_0}[f(x)+g(x)]&=L_f+L_g\\ \lim_{x\to x_0} f(x).g(x)&=L_f.L_g\\ \lim_{x\to x_0}\frac{f(x)}{g(x)}&=\frac{L_f}{L_g}\quad L_g\neq 0\end{aligned}$$

2.2 Exponential Function

Definition 2.04 - *Exponential Function*

$$e:=\lim_{x\to\infty}\left(1+\frac{1}{n}\right)^n\simeq 2.7182818...$$

Theorem 2.05 - *Binomial Expansion*
A techique for expanding binomial expressions

$$\begin{aligned}\left(1+\frac{x}{n}\right)^n&=\sum_{i=0}^{n}\binom{i}{n}.1^{(n-i)}.\left(\frac{x}{n}\right)^i\\ &=1+x+\frac{n-1}{2n}.x^2+...+\frac{x^n}{n^n}\end{aligned}$$