Calculus 1 - Notes

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Contents

1   Fundamental Theorem of Calculus

Definition 1.01 - *Fundamental Theorem of Calculus*
The Fundamental Theorem of Calculus states

$$\frac{d}{dx}\int_a^x f(t)dt = f(x)$$

Definition 1.02 - *Common Sets of Numbers*
Natural Numbers, set of positive integers - $\mathbb{N} := \{1, 2, 3, ...\}$.
Whole Numbers, set of all integers - $\mathbb{Z} := \{..., -2, -1, 0, 1, 2, ...\}$.
Rational Numbers, set of fractions - $\mathbb{Q} := \left\{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{N}\right\}$.
Real Numbers, set of all rational & irrational numbers - $\mathbb{R}$.

Definition 1.03 - *Intervals*
Sets of real numbers that fulfil in given ranges.
Notation

$$[a, b] := \{x \in \mathbb{R} : a \le x \le b\}$$
$$(a, b] := \{x \in \mathbb{R} : a < x \le b\}$$
$$[a, b) := \{x \in \mathbb{R} : a \le x < b\}$$
$$(a, b) := \{x \in \mathbb{R} : a < x < b\}$$

Example
In what interval does x lie such that:

$$|3x + 4| < |2x - 1$$

Solution

$$\text{Case } 1 : x \ge \frac{1}{2}$$
$$\Rightarrow \quad 1 - 2x < 3x + 4 < 2x - 1$$
$$\Rightarrow \quad 1 - 2x < 3x + 4$$
$$\Rightarrow \quad x > \frac{-3}{5}$$
$$\text{And, } \Rightarrow \quad 3x + 4 < 2x - 1$$
$$\Rightarrow \quad x < -5$$

There are no real solutions in this range.
$$\text{Case } 2 : x < \frac{1}{2}$$
$$\Rightarrow \quad 2x - 1 < 3x + 4 < 1 - 2x$$
$$\Rightarrow \quad 2x - 1 < 3x + 4$$
$$\Rightarrow \quad -5 < x$$
$$\text{And, } \Rightarrow \quad 3x + 4 < 1 - 2x$$
$$\Rightarrow \quad 5x < -3$$
$$\Rightarrow \quad x < \frac{-3}{5}$$

$$\Rightarrow \quad -5 < x < \frac{-3}{5}, \; x \in \left(-5, \frac{-3}{5}\right)$$