General - Notes

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1 Hyperbolic Trig

$$sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$tanh(x) = \frac{sin(x)}{cosh(x)} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$cosech(x) = \frac{1}{sinh(x)} = \frac{2}{e^x - e^{-x}}$$

$$sech(x) = \frac{1}{cosh(x)} = \frac{2}{e^x + e^{-x}}$$

$$coth(x) = \frac{1}{tanh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\frac{d}{dx}sinh(x) = cosh(x)$$

$$\frac{d}{dx}cosh(x) = sinh(x)$$

$$\frac{d}{dx}tanh(x) = sech^2(x)$$

$$\frac{d}{dx}cosech(x) = -cosech(x).coth^2(x)$$

$$\frac{d}{dx}sech(x) = -sech(x).tanh(x)$$

$$\frac{d}{dx}coth(x) = -cosech^2(x)$$

2 Trig Identities

$$sin(\alpha + \beta) = sin(\alpha)cos(\beta) + sin(\beta)cos(\alpha)$$

$$sin(\alpha - \beta) = sin(\alpha)cos(\beta) - sin(\beta)cos(\alpha)$$

$$cos(\alpha + \beta) = cos(\alpha)cos(\beta) - sin(\alpha)sin(\beta)$$

$$cos(\alpha - \beta) = cos(\alpha)cos(\beta) + sin(\alpha)sin(\beta)$$

$$sin^2(x) + cos^2(x) = 1$$

$$tan^2(x) + 1 = sec^2(x)$$

$$1 + cot^2(x) = cosec^2(x)$$

$$
\begin{aligned}
sin(2x) &= 2.sin(x).cos(x) \\
cos(2x) &= cos^2(x) - sin^2(x) \\
&= 2cos^2(x) - 1 \\
&= 1 - 2sin^2(x) \\
sin^2(x) &= \frac{1 - cos(2x)}{2} \\
cos^2(x) &= \frac{1 + cos(2x)}{2} \\
tan^2(x) &= \frac{1 - cos(2x)}{1 + cos(2x)} \\
sin(x) + sin(y) &= 2.sin\left(\frac{x+y}{2}\right).cos\left(\frac{x-y}{2}\right) \\
sin(x) - sin(y) &= 2.sin\left(\frac{x-y}{2}\right).cos\left(\frac{x+y}{2}\right) \\
cos(x) + cos(y) &= 2.cos\left(\frac{x+y}{2}\right).cos\left(\frac{x-y}{2}\right) \\
cos(x) - cos(y) &= -2.sin\left(\frac{x+y}{2}\right).sin\left(\frac{x-y}{2}\right)
\end{aligned}
$$

3   Limits

$$
\begin{aligned}
\lim_{x \to x_0} \frac{1 - x}{1 + x} &= 1 \\
\lim_{x \to \infty} \frac{1}{x} &= 0 \\
\lim_{x \to 0} \frac{sin(x)}{x} &= 1
\end{aligned}
$$

4   Differentiation

4.1   Techniques

Sum Rule

$$(f + g)' = f' + g'$$

Product Rule

$$(fg)' = f'g + fg'$$

Quotient Rule

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

Chain Rule

$$\frac{d}{dx}f(g(x)) = f'(g(x)).g'(x)$$

4.2   Common Results

$$
\begin{aligned}
\frac{d}{dx}(x^n) &= nx^{n-1} \\
\frac{d}{dx}(e^x) &= e^x
\end{aligned}
$$

$$\frac{d}{dx}(ln(x)) \quad = \frac{1}{x}$$

$$\frac{d}{dx}(sin(x)) \quad = cos(x)$$

$$\frac{d}{dx}(cos(x)) \quad = -sin(x)$$

$$\frac{d}{dx}(tan(x)) \quad = sec^2(x)$$

$$\frac{d}{dx}(cot(x)) \quad = -cosec^2(x)$$

$$\frac{d}{dx}(sec(x)) \quad = tan(x).sec(x)$$

$$\frac{d}{dx}(cosec(x)) = -cosec(x).cot(x)$$