General - Notes

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1 Hyperbolic Trig

$$
\begin{aligned}
sinh(x) &= \frac{e^x - e^{-x}}{2} \\
cosh(x) &= \frac{e^x + e^{-x}}{2} \\
tanh(x) &= \frac{sin(x)}{cosh(x)} &&= \frac{e^x + e^{-x}}{e^x - e^{-x}} \\
cosech(x) &= \frac{1}{sinh(x)} &&= \frac{2}{e^x - e^{-x}} \\
sech(x) &= \frac{1}{cosh(x)} &&= \frac{2}{e^x + e^{-x}} \\
coth(x) &= \frac{1}{tanh(x)} &&= \frac{e^x - e^{-x}}{e^x + e^{-x}} \\
\frac{d}{dx}sinh(x) &= cosh(x) \\
\frac{d}{dx}cosh(x) &= sinh(x) \\
\frac{d}{dx}tanh(x) &= sech^2(x) \\
\frac{d}{dx}cosech(x) &= -cosech(x).coth^2(x) \\
\frac{d}{dx}sech(x) &= -sech(x).tanh(x) \\
\frac{d}{dx}coth(x) &= -cosech^2(x)
\end{aligned}
$$

2 Trig Identities

$$
\begin{aligned}
sin(\alpha + \beta) &= sin(\alpha)cos(\beta) + sin(\beta)cos(\alpha) \\
sin(\alpha - \beta) &= sin(\alpha)cos(\beta) - sin(\beta)cos(\alpha) \\
cos(\alpha + \beta) &= cos(\alpha)cos(\beta) - sin(\alpha)sin(\beta) \\
cos(\alpha - \beta) &= cos(\alpha)cos(\beta) + sin(\alpha)sin(\beta) \\
sin^2(x) + cos^2(x) &= 1 \\
tan^2(x) + 1 &= sec^2(x) \\
1 + cot^2(x) &= cosec^2(x)
\end{aligned}
$$

$$\begin{aligned}
sin(2x) &= 2.sin(x).cos(x) \\
cos(2x) &= cos^2(x) - sin^2(x) \\
&= 2cos^2(x) - 1 \\
&= 1 - 2sin^2(x) \\
sin^2(x) &= \frac{1 - cos(2x)}{2} \\
cos^2(x) &= \frac{1 + cos(2x)}{2} \\
tan^2(x) &= \frac{1 - cos(2x)}{1 + cos(2x)} \\
sin(x) + sin(y) &= 2.sin\left(\frac{x+y}{2}\right).cos\left(\frac{x-y}{2}\right) \\
sin(x) - sin(y) &= 2.sin\left(\frac{x-y}{2}\right).cos\left(\frac{x+y}{2}\right) \\
cos(x) + cos(y) &= 2.cos\left(\frac{x+y}{2}\right).cos\left(\frac{x-y}{2}\right) \\
cos(x) - cos(y) &= -2.sin\left(\frac{x+y}{2}\right).sin\left(\frac{x-y}{2}\right)
\end{aligned}$$

3 Limits

$$\begin{aligned}
\lim_{x \to x_0} \frac{1-x}{1+x} &= 1 \\
\lim_{x \to \infty} \frac{1}{x} &= 0 \\
\lim_{x \to 0} \frac{sin(x)}{x} &= 1
\end{aligned}$$

4 Differentiation

4.1 Common Results

$$\begin{aligned}
\frac{d}{dx}(x^n) &= nx^{n-1} \\
\frac{d}{dx}(e^x) &= e^x \\
\frac{d}{dx}(ln(x)) &= \frac{1}{x} \\
\frac{d}{dx}(sin(x)) &= cos(x) \\
\frac{d}{dx}(cos(x)) &= -sin(x) \\
\frac{d}{dx}(tan(x)) &= sec^2(x) \\
\frac{d}{dx}(cot(x)) &= -cosec^2(x) \\
\frac{d}{dx}(sec(x)) &= tan(x).sec(x) \\
\frac{d}{dx}(cosec(x)) &= -cosec(x).cot(x)
\end{aligned}$$

4.2 Techniques

Sum Rule

$$(f+g)' = f' + g'$$

Product Rule

$$(fg)' = f'g + fg'$$

Quotient Rule

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

Chain Rule

$$\frac{d}{dx}f(g(x)) = f'(g(x)).g'(x)$$

5 Integration

5.1 Common Results

Let F be a primitive of f.

$$\begin{aligned} f(x) = x^n &\quad => F(x) = \frac{x^{n+1}}{n+1} \\ f(x) = sin(x) &=> F(x) = -cos(x) \\ f(x) = cos(x) &=> F(x) = sin(x) \\ f(x) = e^x = &\quad => F(x) = e^x \end{aligned}$$

5.2 Integration by Substition

Let $f : [a,b] \to \mathbb{R}$ be continuous. Let u be a differentiable bijection with a co-domain of $[a,b]$.
Let $\alpha, \beta \in \mathbb{R}$ such that $u(\alpha) = a,\ u(\beta) = b$.
Then

$$\int_a^b f(u)du = \int_\alpha^\beta (f \circ u)(x).u'(x)dx$$

5.3 Integration by Parts

Let u, v be differentiable functions with the same domain. By the *Product Rule*

$$u(x).v(x) = \int \left[u(x).v'(x) + u'(x).v(x)\right] dx$$

So

$$\int u(x).v'(x)dx = u(x).v(x) - \int v(x).u'(x)dx$$