Multivariable Calculus - Problem Sheet 1

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My Solution 3.1

$$\mathbf{F}'(\mathbf{x}) = \underline{\begin{pmatrix} 2x_1x_2 & x_1^2 \\ \cos(x_1+x_2) & \cos(x_1+x_2) \\ x_2e^{x_1x_2} & x_1e^{x_1x_2} \end{pmatrix}}$$

My Solution 3.2

$$\begin{aligned}
\|\mathbf{v}\| &= \sqrt{5} \\
D_{\mathbf{v}}\mathbf{F}(\mathbf{x}) &= \frac{1}{\|\mathbf{v}\|}\begin{pmatrix} \frac{d}{dt}(x_1+t)^2(x_2+2t)|_{t=0} \\ \frac{d}{dt}\sin(x_1+x_2+3t)|_{t=0} \\ \frac{d}{dt}e^{(x_1+t)(x_2+2t)}|_{t=0} \end{pmatrix} \\
&= \frac{1}{\sqrt{5}}\begin{pmatrix} 2(x_1+t)(x_2+2t)+2(x_1+t)^2|_{t=0} \\ 3\cos(x_1+x_2+3t)|_{t=0} \\ (2x_1+x_2+4t)e^{(x_1+t)(x_2+2t)}|_{t=0} \end{pmatrix} \\
&= \underline{\frac{1}{\sqrt{5}}\begin{pmatrix} 2x_1x_2+2x_1^2 \\ 3\cos(x_1+x_2) \\ (2x_1+x_2)e^{x_1x_2} \end{pmatrix}}
\end{aligned}$$

My Solution 3.3

$$\begin{aligned}
D_{\mathbf{v}}\mathbf{F}(\mathbf{x}_0) &= \frac{1}{\sqrt{5}}\begin{pmatrix} 4 \\ 3\cos(2) \\ 3e \end{pmatrix} \\
\mathbf{F}'(\mathbf{x}_0)\mathbf{v} &= \begin{pmatrix} 2 & 1 \\ \cos(2) & \cos(2) \\ e & e \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} \\
&= \underline{\begin{pmatrix} 4 \\ 3\cos(2) \\ 3e \end{pmatrix}}
\end{aligned}$$

My Solution 4.1

$$\begin{array}{rrcl}
& \mathbf{F}(1,1) & = & (3,3,1) \\
\& & \mathbf{G}' & = & \begin{pmatrix} x_2 & x_1 & 0 \\ 0 & x_3 & x_2 \\ x_2x_3\cos(x_1x_2x_3) & x_1x_3\cos(x_1x_2x_3) & x_1x_2\cos(x_1x_2x_3) \end{pmatrix} \\
\implies & (\mathbf{G}'\circ\mathbf{F})(1,1) & = & \mathbf{G}'(3,3,1) \\
& & = & \begin{pmatrix} 3 & 3 & 0 \\ 0 & 1 & 3 \\ 3\cos(9) & 3\cos(9) & 9\cos(9) \end{pmatrix} \\
\text{Chain rule} & \mathbf{H}' & = & (\mathbf{G}'\circ\mathbf{F})\mathbf{F}' \\
& & = & \begin{pmatrix} 3 & 3 & 0 \\ 0 & 1 & 3 \\ 3\cos(9) & 3\cos(9) & 9\cos(9) \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 0 \end{pmatrix} \\
& & = & \underline{\begin{pmatrix} 9 & 9 \\ 5 & 1 \\ 18\cos(9) & 9\cos(9) \end{pmatrix}}
\end{array}$$

My Solution 4.2

$$\begin{array}{rrcl}
& \mathbf{H} & = & ((x_1+2x_2)(2x_1+x_2), x_1(2x_1+x_2), \sin(x_1(x_1+2x_2)(2x_1+x_2))) \\
\implies & \mathbf{H}' & = & \begin{pmatrix} (2x_1+x_2)+2(x_1+2x_2) & 2(2x_1+x_2)+(x_1+2x_2) \\ (2x_1+x_2)+2x_1 & x_1 \\ i & j \end{pmatrix} \\
\textit{where} & i & = & [(x_1+2x_2)(2x_1+x_2)+x(2x_1+x_2)+2x_1(x_1+2x_2)]\cos(x_1(x_1+2x_2)(2x_1+x_2))]] \\
\& & j & = & [2x_1(2x_1+x_2)+x_1(x_1+2x_2)]\cos(x_1(x_1+2x_2)(2x_1+x_2)) \\
\implies & \mathbf{H}'(1,1) & = & \begin{pmatrix} 3+2\times 3 & 2\times 3+3 \\ 3+2 & 1 \\ [3\times 3+1\times 3+2\times 3]\cos(1\times 3\times 3) & [2\times 3+1\times 3]\cos(1\times 3\times 3) \end{pmatrix} \\
& & = & \underline{\begin{pmatrix} 9 & 9 \\ 5 & 1 \\ 18\cos(9) & 9\cos(9) \end{pmatrix}}
\end{array}$$

My Solution 5

$$\begin{array}{rrcl}
\text{Let} & \mathbf{F}:\mathbb{R}^2\to\mathbb{R}^2 & st & \mathbf{F}=(s,t) \\
\implies & \mathbf{F}' & = & \begin{pmatrix} 3x^2 & e^y \\ -\sin(x)+y & x \end{pmatrix} \\
\implies & J_{\mathbf{F}} & = & 3x^3+e^y(\sin(x)+y) \\
\implies & J_{\mathbf{F}}(0,0) & = & 0+1(sin(0)+0) \\
& & = & \underline{0}
\end{array}$$

Since $J_{\mathbf{F}}(0,0)$ is not invertible, by the *inverse function theorem*, we <u>cannot</u> guarantee unique solutions near $(1,1)$.