# Problems Sheet 2

Statistics 2

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## Question 3

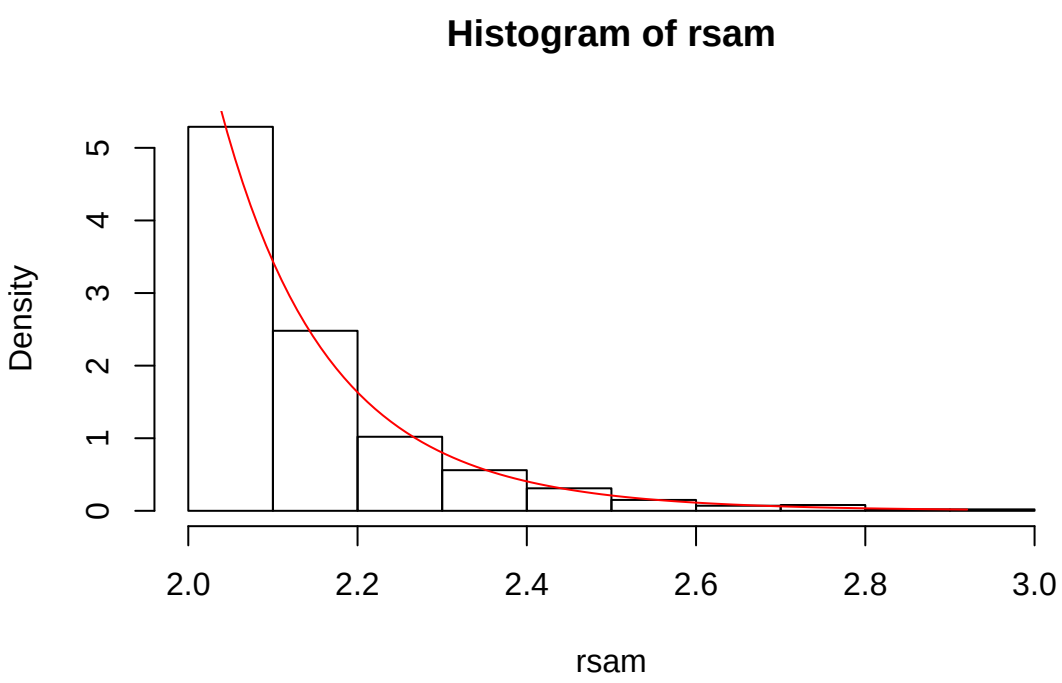
```
#f_X(x) for X~Pareto(x0,theta)
dpareto<-function(x,x0,theta) {
  stopifnot(all(x0>0), all(theta>0)) # Stop if x0 or theta are invalid
  ifelse(x<x0, 0, (theta/x)*(x0/x)^theta) # Implement indicator function to return 0 if x<x0, otherwise
}

#F_X(x) for X~Pareto(x0,theta)
ppareto<-function(x,x0,theta) {
  stopifnot(all(x0>0), all(theta>0))
  ifelse(x<x0, 0, 1-(x0/x)^theta) # Implement indicator function to return 0 if x<x0 otherwise F_X(x)
}

#F_X^{-1}(x) for X~Pareto(x0,theta)
qpareto<-function(u,x0,theta) {
  stopifnot(all(x0>0), all(theta>0), all(u>=0), all(u<=1))
  x0 * (1-u)^(-1/theta) # Return F_X^{-1}(u)
}

# Produce n random samples from X~Pareto(x0,theta)
rpareto<-function(n,x0,theta) {
  stopifnot(all(x0>0), all(theta>0), n>=0)
  u <- runif(n) # Samples Uniform[0,1] n times
  qpareto(u,x0=x0,theta=theta) # Returns values of x such F_X^{-1}(x)=u, effectively sampling X n times
}
```

### Part a)

```
x0=2; theta=15
rsam<-rpareto(1000,x0=x0,theta=theta) # Sample X~Pareto(2,15) 1000 times
```

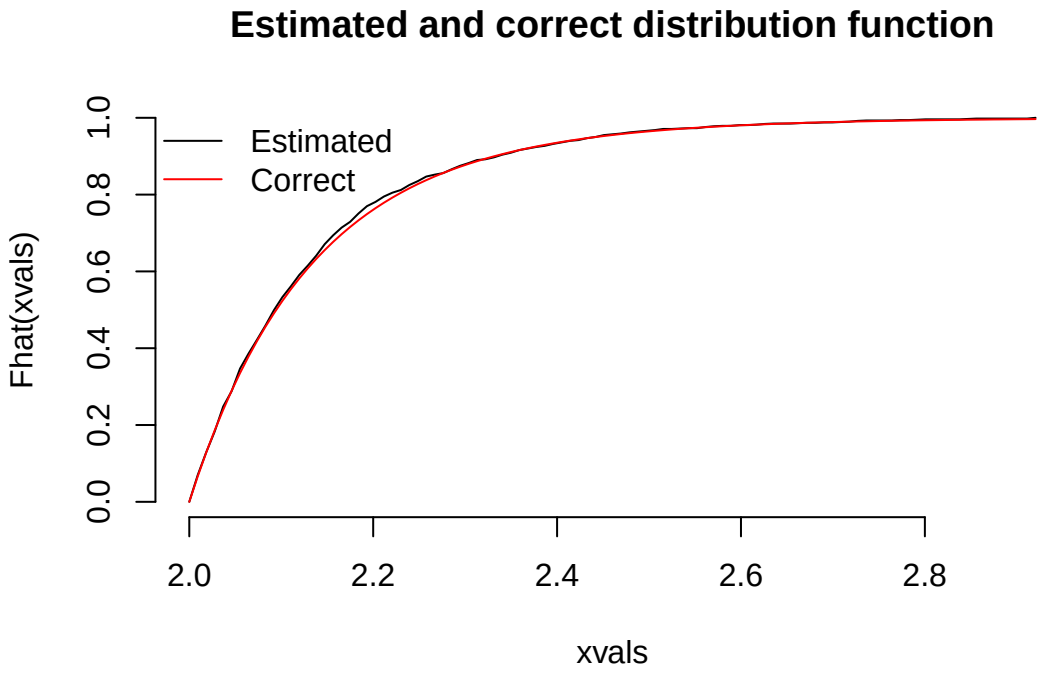
### Part b)

```
hist(rsam, freq=FALSE) # Histogram of sample
xvals<-seq(from=2,to=max(rsam),length=101) # Produce 101 linear intervals in [2,max(rsam)]
lines(xvals,dpareto(xvals,x0=x0,theta=theta),col='red') # Plot distribution of Pareto(2,15)
```



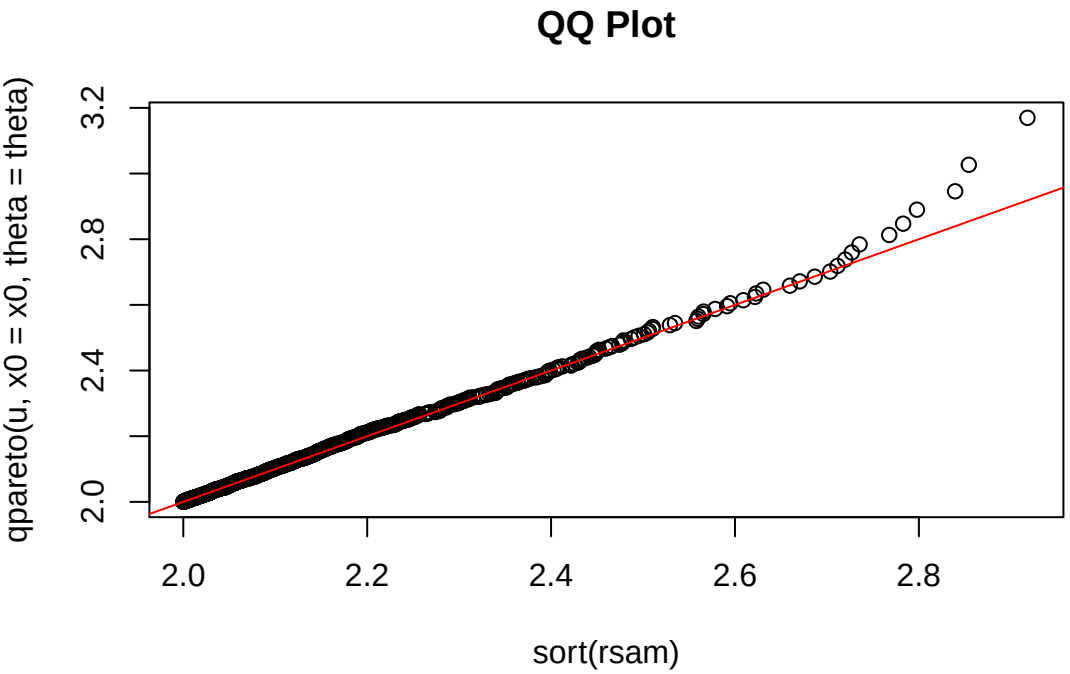

### Part c)

```
# Estimates Prob(X<=x)
Fhat<-function(x) {
  sapply(x,function(xv) mean(rsam<=xv)) # Returns propotion of sample <=x
}

plot(xvals, Fhat(xvals), type='l', main="Estimated and correct distribution function", bty='n') # Fhat
lines(xvals,ppareto(xvals,x0=x0,theta=theta),col="red") # True distribution
legend('topleft',legend=c("Estimated","Correct"),lty=1,pch=NA,col=c("black","red"),bty="n")
```




### Part d)

```
n<-length(rsam)
u<-(1:n)/n # n linear intervals of [0,1]
plot(sort(rsam),qpareto(u,x0=x0,theta=theta),main="QQ Plot") # Quantile plot
abline(a=0,b=1,col="red") # y=x
```


QQ Plot
qpareto(u, x0 = x0, theta = theta)
2.0
2.4
2.8
3.2
2.0
2.2
2.4
2.6
2.8
sort(rsam)