

a.

Verifying is P: Given a subset of vertices of the size of $n/2$, we can check if its a clique in $O(n^2)$. So the problem is in NP.

The problem can be reduced to a clique problem. If the problem is true, $n/2$ -clique problem is true. If the problem is false, $n/2$ -clique is false or it is true and there is a node connected to all the nodes in the clique of size $n/2$. So the problem is at least as hard as clique problem. As clique is NP-complete, the problem is NP-hard.

So the problem is NP-complete.

f.

Verifying is P: Given a subset, we can check if the sum is 0 in $O(n)$. So the problem is in NP.

If the problem is true, let the size of subset be k . Then the Subset Sum problem with the set adjusted by adding a same integer i to all the elements to make them positive, and the target sum set to be ki is true, with a solution of size k .

If the problem is false, The the problem above says false or says true but without a solution of size k .

So the problem is at least as hard as Subset Sum problem. As Subset Sum problem is NP-complete, the problem is NP-hard.

So the problem is NP complete.

e.

Verifying is P: Given a collection, we can check if it covers U in $O(mk)$. So the problem is in NP.

The problem can be reduced to a vertex cover problem. Assume that there's a Vertex Cover problem with $G=(V,E)$. Make E the ground set U . For each vertex, create a subset S_v with all the edge connected to it in it. Let k be the demanded size of vertex cover. And we've generated the required problem from a Vertex Cover problem. As Vertex Cover is NP-complete, the problem is NP-hard.

So the problem is NP-complete.

i.

Verifying a given coloring requires $O(|V| |E|)$. So the problem is NP.

Create the complement graph G' . In graph G' , if there exists a vertex cover V_c of size k in the Vertex Cover instance, we can color the vertices in V_c with one color. Remove all the vertices colored and do it again. Then we can get the 3-coloring scheme if exists.

Correctness: An edge in G' represents not-an-edge in G . So Vertex Cover in G' is a maximum set of independent vertices in G .

As Vertex Cover is NP-complete, 3-coloring is NP-hard.

So 3-coloring is NP-complete.

