$$\hat{\beta} = (X^TX + \lambda I_p)^{-1}(X^TY)$$

$$= \tau^2(\tau^2 X^TX + \sigma^2 I_n)^{-1}(X^TY)$$

$$= \tau^2 X^T(\tau^2 X^TX + \sigma^2 I_n)^{-1} Y$$

$$\therefore P(\beta|Y,X) = N(\tau^2 X^T(\tau^2 X^TX + \sigma^2 I_n)^{-1} Y$$

$$\therefore E[\beta|Y,X] = \hat{\beta} = \tau^2 X^T(\tau X^TX + \sigma^2 I_n)^{-1} Y$$