Leaping Larry's Luge Launcher

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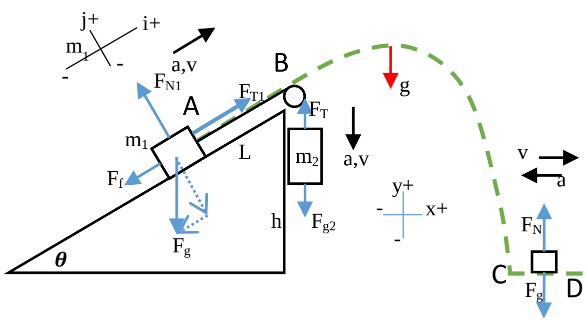
Description: Leaping Larry lies in a large luge. Launched by a lackluster pulley and ramp system, Larry is light like a lark. Laughing lyrically, Larry lands and loses all vertical velocity voraciously. He wrapped a massless stretchless rope around his massless frictionless pulley wheel. One end was connected to a barrel of rocks, and the other end he held. He started his luge as far down the ramp as the height of the ramp. The ramp and ground have friction.



Givens:

$$m_L = 31\ kg \qquad m_B = 43\ kg$$
$$\theta = 18\ deg \qquad \mu_R = .22$$
$$h = 8.5\ m \qquad x_{BD} = 54\ m$$
$$g = 9.8\frac{m}{s^2} \qquad \mu_G = ???$$

Assume:

$$F_{T1} = F_{T2} \qquad a_1 = -(a_2)$$

Strategy: I used points A through D to mark the three different stages of the luge's motion. From A to B, I created the i and j axis for going up and against the ramp, respectively. I used the sum of forces in the i and j axis for the luge, and x and y for the bucket. Setting the forces of tension equal, I got the acceleration up the ramp of the luge. Using kinematics I calculated the velocity at point B. From point B to C, it was a freefall problem. I used kinematics in the y axis to find the time until the luge hit the ground at point C, and applied that time to the x axis to get the distance between B and C. At point C, I also used the x and y components of the luge's velocity to get the speed at that point, all of which would be converted to horizontal velocity. From C to D, I could use the given distance from B to D, and the distance from B to C that I calculated previously to get the distance from C to D. Using that distance and the velocity at point C, I could use kinematics to find the acceleration needed to slow the luge down to a stop. I then used the sum of forces with the acceleration from the previous step to get the coefficient of friction that would produce such an acceleration.

Calculation:

y-axis for bucket

$$\Sigma F_{yB}: F_{T2} - F_{g2} = m_B * a_2$$
$$F_T 2 = m_B * a_2 + F_{g2}$$

j-axis for luge

$$\Sigma F_j: F_{N1} - m_L * g * Cos[\theta\ ] = m_L * a_{1j}$$
$$F_{N1} = m_L * g * Cos[\theta]$$

Friction

$$F_{f1} = \mu_R * F_{N1}$$
$$F_{f1} = \mu_R * m_L * g * Cos[\theta]$$

i-axis for luge

$$\Sigma F_i: F_{T1} - F_{gA} * Sin[\theta] - F_{f1} = m_L * a_1$$
$$F_{T1} = m_L * g * Sin[\theta] + \mu_R * m_L * g * Cos[\theta] + m_L * a_1$$

Substitute $a_1 = -a_2$

$$F_{T1} = m_L * g * Sin[\theta] + \mu_R * m_L * g * Cos[\theta] + m_L * -a_2$$

Set $F_{T1} = F_{T2}$

$$m_L * g * Sin[\theta] + \mu_R * m_L * g * Cos[\theta] + m_L * -a_2 = m_B * a_2 + F_{g2}$$

$$a_2 = \frac{gm_L\mu_R Cos[\theta] + gm_L Sin[\theta] - gm_B}{m_B + m_L}$$

$$a_2 = \frac{(9.8)(31)(.22)cos(18) + (9.8)(31)sin(18) - (9.8)(43)}{43 + 31}$$

$$\underline{a_2 = -6.56697\ m/s^2}$$

$$\underline{a_1 = 6.56697\ m/s^2}$$

Kinematics from A to B

$$EQ4)\ v_f = v_i^2 + 2ax$$

$$v_B^2 = 0 + 2a_1 h$$

$$v_B = \sqrt{2a_1 h}$$

$$v_B = \sqrt{2(6.56697)(8.5)}$$

$\underline{v_B = 7.8707\text{ m/s at 18 degrees above horizontal}}$

Componentize velocity

$$\underline{v_{BX} = v_B * \mathrm{Cos}[18] = 7.4060\ m/s}$$

$$\underline{v_{BY} = v_B * \mathrm{Sin}[18] = 2.4063\ m/s}$$

Y-dir from B to C

$$EQ3)\ y_C = \frac{1}{2} g t_{BC}^2 + v_{BY} t_{BC} + y_B$$

$$0 = \frac{1}{2}(9.8){t_{BC}}^2 + (2.4063)t_{BC} + 8.5$$

Solver) $t_{BC} = \cancel{-1.0942}, \underline{1.5853\ seconds}$

X-dir from B to C

$$x_{BC} = v_{BX} t_{BC}$$

$$x_{BC} = (7.4060)(1.5853)$$

$$\underline{x_{BC} = 11.7408\ m}$$

Y-dir velocity at C

$$EQ2)\ v_f = at + v_i$$

$$v_{CY} = g t_{BC} + v_{BY}$$

$$\underline{v_{CY} = -13.130 \frac{m}{s}}$$

x-dir velocity at C is the same as at B

$$\underline{v_{CX} = v_{BX} = 7.4060 \frac{m}{s}}$$

Speed at C, which is converted to horizontal velocity towards D

$$v_C = \sqrt{v_{CX}^2 + v_{CY}^2}$$

$$v_c = \sqrt{7.4060^2 + (-13.13)^2}$$

$$\underline{v_c = 15.074 \frac{m}{s}}$$

Acceleration from C to D

$$\mathrm{EQ4})\ v_f^2 = v_i^2 + 2a\Delta \mathrm{x}$$

$$0 = v_C^2 + 2a_C * (x_{BD} - x_{BC})$$

$$a_C = -\frac{v_C^2}{2(x_{BD} - x_{BC})}$$

$$\underline{a_c = -2.6886 \frac{m}{s^2}}$$

Net vertical forces from C to D

$$\Sigma F_Y: F_N - F_g = m a_{Cy}$$

$$F_N - mg = 0$$

$$F_N = mg$$

Frictional force from C to D

$$F_{fC} = \mu_G F_N$$

$$F_{fC} = \mu_G mg$$

Net horizontal forces from C to D

$$\Sigma F_{CX}: F_{fC} = m a_C$$

$$\mu_G mg = m a_C$$

$$\mu_G = \frac{a_C}{g}$$

$$\mu_G = \frac{-2.6886}{-9.8}$$

$$\boxed{\mu_G = .27435}$$