수치해석. hw 9.

Data-Fitting

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model : $x' = a_1x + a_2y + a_3$

$y' = a_4x + a_5y + a_6$

error : $e_x = x' - x'(x,y)$
$= x' - (a_1x + a_2y + a_3)$

$e_y = y' - y'(x,y)$
$= y' - (a_4x + a_5y + a_6)$

Sum of errors for x : $S_x(a_1, a_2, a_3) = \sum(x_i' - a_1x_i - a_2y_i - a_3)$

Derivate S_x.

$$\frac{\partial S_x}{\partial a_1} = -2\sum x_i(x_i' - (a_1x_i + a_2y_i + a_3)) = 0$$

$$\frac{\partial S_x}{\partial a_2} = -2\sum y_i(x_i' - (a_1x_i + a_2y_i + a_3)) = 0$$

$$\frac{\partial S_x}{\partial a_3} = -2\sum (x_i' - (a_1x_i + a_2y_i + a_3)) = 0$$

Organize the formula

$$a_1\sum x_i^2 + a_2\sum x_iy_i + a_3\sum x_i = \sum x_ix_i'$$

$$a_1\sum x_iy_i + a_2\sum y_i^2 + a_3\sum y_i = \sum y_ix_i'$$

$$a_1\sum x_i + a_2\sum y_i + a_3\sum 1 = \sum x_i'$$

Organize to the matrix square

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} \sum x_i^2 & \sum x_iy_i & \sum x_i \\ \sum x_iy_i & \sum y_i^2 & \sum y_i \\ \sum x_i & \sum y_i & \sum 1 \end{pmatrix}^{-1} \begin{pmatrix} \sum x_ix_i' \\ \sum y_ix_i' \\ \sum x_i' \end{pmatrix}$$

$$\begin{pmatrix} a_4 \\ a_5 \\ a_6 \end{pmatrix} = \begin{pmatrix} \sum x_i^2 & \sum x_iy_i & \sum x_i \\ \sum x_iy_i & \sum y_i^2 & \sum y_i \\ \sum x_i & \sum y_i & \sum 1 \end{pmatrix}^{-1} \begin{pmatrix} \sum x_iy_i' \\ \sum y_iy_i' \\ \sum y_i' \end{pmatrix}$$