The behavior of weight-loss study participants in response to incentives

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Abstract

Most research examining the impact of incentive schemes on behavior has focused on simple 'main effects', showing, for example, that incentives lead to more of a particular behavior than observed in a no-incentive control group. This paper examines the impact of specific incentive schemes—the deposit contract and the lottery—in greater detail, investigating the dynamic interplay of behavior and incentives in the context of a weight-loss study. Modeling individual behavior using a state-space model, we show that past success is an important predictor of future deposit amounts and that deposit amounts influence subsequent success in dieting. We also show that neither winning nor losing the lottery have immediate transient effects on weight-loss performance.

Keywords: incentives, Bayesian inference, MCMC, dieting, state-space model, Kalman filter

1. Introduction

Volpp et al. [15] compared the efficacy of deposit contracts and lottery incentives in promoting weight loss in a group of overweight veterans. While the researchers demonstrated that both of these incentive schemes led to significantly greater weight loss than that observed in a no-incentive control group, the mechanisms that contributed to this outcome are unclear. Did deposit amounts lead to greater success or did success lead to higher deposits? Was winning or losing the lottery more motivational? To investigate these questions, we propose a model for the participants' decision making process and fit the model using Markov Chain Monte Carlo (MCMC) methods.

Incentive schemes are becoming increasingly popular mechanisms for encouraging desired behaviors, particularly in areas where people have self-control problems such as weight loss, smoking cessation, and medication adherence. Incentives are also being used to motivate students to perform better in school [1], to induce health-care professionals to wash their hands [6], and in a variety of other situations. Despite the centrality of incentives to economics, empirical investigations of the impact of incentives are still in their infancy. As a result, researchers creating incentive schemes have little to guide their design decisions other than received wisdom and seat-of-the-pants intuition.

The interest in incentives among economists, has been joined by an interest in, and an openness to, experimentation. Currently, economists are conducting a wide range of field studies examining the impact of incentives on various behaviors. By comparing different types of incentive schemes, these studies shed light on which types of incentives are effective in which situations. However, these studies are also 'throwing off' large amounts of data on individual responses to incentives that have not been adequately mined for insights. For example, a common incentive scheme involves deposit contracts in which people voluntarily deposit amounts of money that are redeemed, in some cases with a match provided by the researcher, following the successful achievement of objectives. To date, however, no research has examined either which factors influence the amounts that individuals choose to deposit or how the magnitude of such deposits subsequently affects behavior.

The design of the incentive conditions in the Volpp et al. study draws on a number of motivational effects discussed in the behavioral economics literature. Study administrators informed participants of their earnings on a daily basis via text message because immediately occurring incentives have a larger impact on behavior (see Thaler [13] and Loewenstein and Prelec [9]). Administrators also informed participants how much money they earned, or could have earned had they acheived their target weight, because regret can be motivational (see Chapman and Coups [5]). The lottery paid small amounts frequently and large amounts infrequently, because people are motivated by past rewards and the prospect of future winnings [4], and because small probabilities of large prizes are

highly motivational [10]. Deposit contracts help to motivate individuals through "loss aversion," an idea first advanced by Daniel Kahneman and Amos Tversky that utility from gains and losses is asymmetric [14].

The remainder of this paper discusses a model for the behavior of participants in the Volpp et al. weight-loss study. Section 2 summarizes the methodology used by Volpp et al. and the data collected. It also proposes a model for the participants' decision making process and discusses the assumptions underlying the model. Section 3 discusses empirical results while section 4 recommends adjustments to the study methodology and concludes.

2. Materials and methods

2.1. The Volpp et al. weight-loss study

In Volpp et al. [15], the authors randomly assigned 57 participants to one of three possible incentive conditions: lottery, deposit contract, or no-incentive control. Participants were selected based on responses to a mail survey administered to patients at a Philadelphia-area VA hospital. Initial selection criteria included Body Mass Index between thirty and forty (considered obese by the CDC) and a strong interest in losing weight. Researchers excluded candidates if they were currently undergoing treatment for drug or alcohol abuse, were addicted to medication, had had a cardiac infarction in the previous six months, or were participating in other weight-loss studies.

All participants received a set of daily weight targets that decreased by one pound per week over the sixteen week study period. Participants in the incentive conditions weighed themselves daily and reported their weight over the phone to study administrators. All participants received monthly checkups to assess their progress, administer a survey, verify reported weights, and make incentive payments, contingent on verified weight matching the most recent called-in weight. In order to reduce the dropout rate, daily weight targets were recalculated at the monthly checkup. In the case of underperformance, the slope of the target trajectory would be increased so as to smooth the necessary increase in performance over the remainder of the study. Figure 1 shows the actual weights, the resulting trajectory, and the original trajectory for a poorly performing individual.

At each monthly checkup, researchers asked the 19 participants in the deposit contract condition if they would be willing to risk from $0 to $3 per day. If a participant's daily reported weight was under the tar-
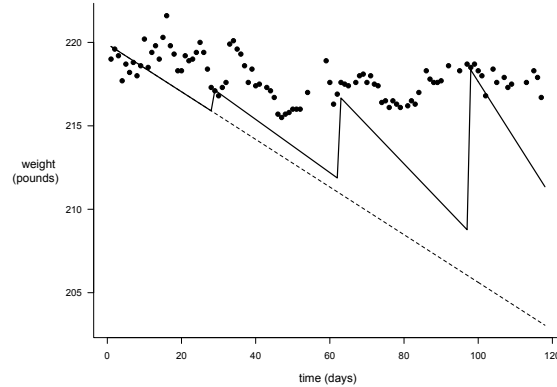

Figure 1. In the case of underperformance, the slope of the target trajectory would be increased so as to smooth the necessary increase in performance over the remainder of the study. This figure shows the actual weights (circles), the resulting adjusted trajectory (solid line), and the original trajectory (dashed line) for a poorly performing individual.

get, he would earn $3, plus twice the deposit amount for a total daily payment of up to $9. If his reported weight was over the target, he would receive nothing and lose the deposit amount. After calling in their weight for the day, participants received a text message informing them of the result. Each participant paid the entire deposit amount at the beginning of the month and received their earnings for the past month at the monthly checkup. Figure 2 shows all the deposit amounts broken down by month. Participants pick both boundary amounts ($0 or $3) and interior amounts.

Participants in the lottery condition were eligible to win a prize every day they called in an actual weight below that day's target weight. They had a one in five chance of winning $10 and a one in one hundred chance of winning $100 for an expected payout of $3 per day of enrollment. Thus, a lottery participant and a deposit contract participant who deposited no money could each expect to earn the same amount of money for the same weight-loss success.

In addition to the incentive payments, all participants received $20 for appearing at the monthly checkup. Also, incentive condition participants who lost twenty pounds or more over the course of the study received a portion of the forfeited deposits, guaranteed to be at least $50.

Researchers collected three types of data for each participant in the study. At the beginning of the study, each participant completed an intake survey. This questionnaire contained eleven demographic questions,
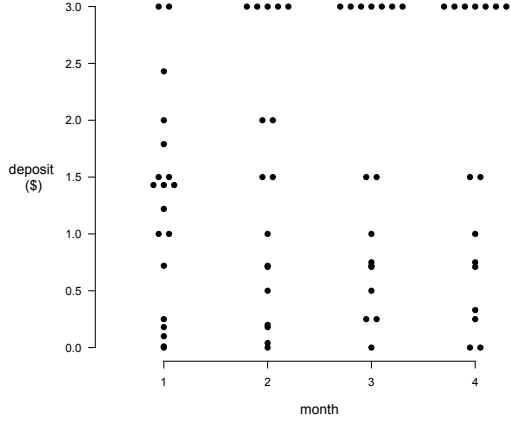
Figure 2. This figure shows deposit amounts by month for all participants over the entire study. Participants pick both boundary amounts ($0 or $3) and interior amounts.

thirty statements about diet and appearance assessed on a five-level Likert scale such as "I eat when I'm not really hungry," as well as three statements about finances like "An extra $3.00 a day could really change my life." It also asked about existing medical conditions, family medical history, past dieting experience, and physical activity. At the monthly checkup, participants adjusted deposit amounts and completed another survey containing questions similar to those on the intake survey in order to determine how and if participants had altered their diet and exercise routines in order to lose weight. Finally, study administrators recorded daily weight reports and targets.

2.2. Modeling participant decisions

According to Baumeister and Vohs [2], there are three theoretical models of human self-control or willpower. The first is similar to the standard economic model for decisions—calculate the costs and benefits of the various potential actions, and choose the one that maximizes expected benefits. This model depicts self-control as a series of calculations which are made by the individual who has some knowledge about the self and the processes with which it interacts. The second model imagines self-control as a skill which is acquired through use and understanding. The third model depicts self-control as a muscle which wears out after usage. If the muscle is strong enough to overcome temptation, then the self resists the temptation. Baumeister and Vohs argue that empirical evidence supports the third theory.

To model a participant's daily weight-loss outcome, we

imagine that the weight-loss process is a state-space model. We observe the weight and deposit amounts over time with the knowledge that both evolve as a function of willpower or self-control which we cannot observe. This unobserved "state" also evolves over time, adjusting to the incentives that have been won or lost and the likelihood of receiving future incentive payments. A flowchart demonstrating this decision making process for the deposit contract condition is shown in Figure 3. The process for the lottery condition is similar, except that there are no deposits observed, only past winnings.

For an individual in the deposit contract condition, we observe both deposit amounts and weight. This leads to the following state-space model:

$$\begin{bmatrix} d_t \\ y_t \end{bmatrix} = \begin{bmatrix} 0 \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} \psi \\ -1 \end{bmatrix} \alpha_t + \begin{bmatrix} v_t \\ \epsilon_t \end{bmatrix} \quad (1)$$

$$\alpha_{t+1} = \alpha_t + \sum_{i=1}^{k} \beta_i g_i(y, d, r) + \eta_t \quad (2)$$

where t is a daily time index, d_t is the deposit amount at time t, y_t is the weight at time t, r is the target weight and v_t, ϵ_t and η_t are mutually and serially independent, normally distributed perturbations with mean zero and variances σ_v^2, σ_ϵ^2, and σ_η^2 respectively. In this model, a change in weight from time $t-1$ to time t is taken to be equal to the underlying motivation or willpower α_t plus noise. The observed deposit amounts are taken as a constant multiple of the state, α_t. The state evolves as a nonstationary AR(1) process plus the effect of covariates and noise. The functions g_i are taken to be known functions of y and d up to time t and the target weight r at any time.

For a participant in the lottery condition, we observe only weight. This leads to the following state-space model:

$$y_t = y_{t-1} - \alpha_t + \epsilon_t \quad (3)$$

$$\alpha_{t+1} = \alpha_t + \sum_{i=1}^{k} \beta_i g_i(y, w, r) + \eta_t \quad (4)$$

where y_t is the weight at time t and ϵ_t and η_t are mutually and serially independent, normally distributed perturbations with mean zero and variances σ_ϵ^2 and σ_η^2 respectively. In this model, a change in weight from time $t-1$ to time t is taken to be equal to the underlying motivation or willpower α_t plus noise. Again, the state evolves as a nonstationary AR(1) process plus the effect of covariates and noise. The functions g_i are taken to be known functions of y and the lottery winnings w up to time t as well as the target weight r at any time.
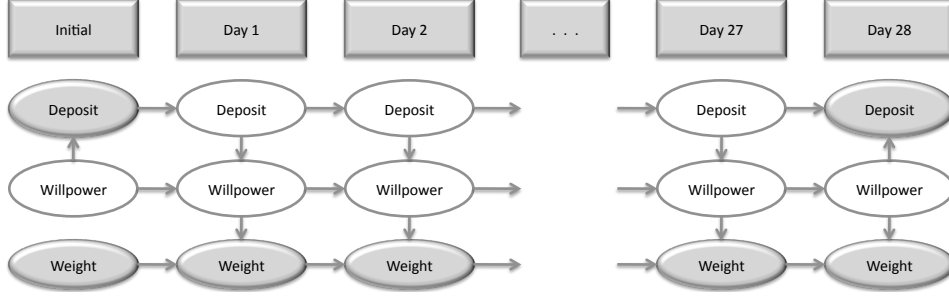
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Figure 3. Participants in the deposit contract condition made two different types of decisions during the course of the study, one type occurring on a daily basis and the other on a monthly basis. Each day they decided whether to attempt to lose weight contingent on their current weight, the weight loss target, the amount of money that they deposited, and their willpower or motivation to lose more weight. Each month, they decided how much money to deposit based on their willpower.

2.3. Combining individual-level information

The above model focuses only on inferences for individuals. The distributions for each of the parameters is calculated for individual j independent of the data or parameter distributions for all other participants in the study. However, parameter distributions for one individual should also give some information about the parameter distributions for other individuals. This leads to the formulation of a hierarchical model combining individual-level information to make inferences about deposit contract participants in general.

The previous section discussed a number of individual parameters to estimate. There are the ψ parameters which relate weight to deposit amount, the β parameters in the state equations (2) and (4), and the variance parameters in both models. In order to combine information, we specify regression models for all of the mean parameters as well as hierarchical distributions for the variance parameters. This allows not only the aggregation of individual information, but also the ability to model the mean parameters as functions of known covariates which provide some information about the relationship between individuals. The appendix contains the full model specification, including hierarchical priors and hyperpriors.

3. Analysis and results

3.1. Fitting the model

The model described in section 2.2 can be fit easily using Markov Chain Monte Carlo methods. Given ψ, β, σ_ϵ^2, σ_v^2, and σ_η^2, the unknown states α_t are determined using the Kalman [8] filter and smoother. Once the states are known, the other parameters can be estimated using conjugate updates. The remainder of this

section describes the procedure for estimating the unobserved states, the handling of missing weight data, and the choice of hyperparameters and starting values.

In order to make inferences about the coefficients of interest, we must first determine the joint distribution of the unknown states $\{\alpha_t\}_{t=1}^T$ given the observed information $\{y_t\}_{t=1}^T$ and $\{d_t\}_{t=1}^T$ for each individual. Since the observed data and the unobserved states are normally distributed in equations (1)–(4), then the conditional distribution of α_t given y_s and d_r is also normal for any $1 \leq s, r \leq T$. Therefore,

$$\alpha \mid y, d \sim \mathrm{MVN}_T(\mu, \Sigma) \qquad (5)$$

for some mean vector μ and variance covariance matrix Σ. In principle, both the mean vector and covariance matrix can be calculated given samples y and d using standard results in multivariate analysis, however, for large T, this becomes computationally intensive. An iterative solution to this problem was proposed by Rudolf Kalman [8] as a method for trajectory estimation in the Apollo space program.

The Kalman filter is a closed form recursive method for predicting the unobserved state of a Gaussian process given some observed information. The method is quite general, but the above application requires only a simplified version of the general model. The basic idea is to predict (or filter) future states iteratively using observed information up to time t for $t = 1, \ldots, T$ and then "smooth" the states from $t = T, \ldots, 1$ using all available information. This recursion produces the conditional mean, μ, and variance matrix Σ in equation (5) for the entire state vector $\alpha_1, \ldots, \alpha_T$ conditional on the observed data. Since the state errors are assumed to be Gaussian, the smoothed states will also be normally distributed and are therefore completely determined by the mean and variance. The models

specified in equations equations (1)–(4) can be written concisely as

$$x_t = Z\alpha_t + \epsilon_t, \qquad \epsilon_t \sim N(0, H)$$

$$\alpha_{t+1} = \alpha_t + \sum_{i=1}^{k} \beta_i g_i(y, w, d, r) + \eta_t, \quad \eta_t \sim N(0, Q)$$

where x_t is taken to be the column vector $\begin{bmatrix} d_t & \Delta y_t \end{bmatrix}$ in the case of the deposit contract condition or Δy_t in the case of the lottery. The recursion used for this model is shown in Figure 4.

Missing observations are easily handled using this recursion. If the entire observation vector is missing at time t, then replace step (2) with the updates $\alpha_{t+1} = \alpha_t$ and $P_{t+1} = P_t + \sigma_\eta^2$. In the case that only part of the observation vector is missing (this is often the case for the deposit participants since deposits are observed only once a month), let W be the subset of rows of the identity matrix for which observations exist. Then calculate $y^* = Wy$, $Z^* = WZ$, and $H^* = WHW'$. Replace y, Z, and H in step (2) of the Kalman filter algorithm with the corresponding starred version.

The g_i functions also require y and d. The deposit amounts are frequently missing for the observation equation, however, as covariates in the state equation, they can be considered fixed and known for the entire month. Missing weight data could be estimated from the fitted model at each step of the sampler, however, due to computational concerns, we let $y_t = y_{t-1}$ wherever y_t is missing.

The hyperparameters required to fit this model are determined by eliciting George Loewenstein. For the variance parameters, we require a guess at the mean and the 95th percentile. This is easier to do by asking about the standard deviation and then transforming to the variance. Regression coefficients have mean zero prior distributions with diffuse variance. Gelman and Rubin [7] and Brooks and Gelman [3] suggest choosing an "overdispersed" set of starting values from a posterior approximation and running multiple chains to assess convergence. This proposition is difficult in this case due to the complexity of the joint posterior. Instead, we take a sample from the prior or hyperprior distributions of each parameter. We assess convergence of a single chain using the Raftery and Lewis [11] diagnostic.

3.2. Results

We fit this model using a block Gibbs sampler run for 4000 iterations. Convergence occurs around 1000 iterations based of the convergence diagnostic of Raftery

Parameters	Value
α_0	0
P_0	1
c, p	5
c', p'	1
τ	50
τ'	1
$\mathbf{b}$ and $\mathbf{u}$	0
Σ_b and Σ_u	$100\mathbf{I}$
$\delta_b, \delta_u, \gamma_b$, and γ_u	5

Table 1. This table shows the values of the prior parameters used to fit the model. See the appendix for exact definitions.

and Lewis [11]. We take $\mathbf{M}$, the matrix of covariates in the hierarchical regression model, equal to a vector of ones to give posterior distributions of population means for the parameters of interest. Prior parameters are shown in Table 1. We are most interested in answering three questions: How does motivation affect deposits? How do deposits affect motivation? and How do various lottery outcomes affect motivation? For the deposit contract condition, we take

$$\alpha_{t+1} = \alpha_t + \beta_1(y_t - r_{t+1})_+ + \beta_2(y_t - r_{t+1})_- + \beta_3 d_t + \eta_t.$$

This means that motivation depends on past motivation, distance to tomorrow's target depending on whether the participant is currently above or below the target, and deposit amount. Figure 5 shows 50% and 95% posterior credible intervals for β_1, β_2 and β_3 for each deposit contract participant. Larger estimates correspond to decreases in motivation. Participants who are above tomorrow's target seem to experience very little increase or decrease in motivation (top panel). Participants below the target often experience severe demotivation (middle panel). The deposit amounts seem to have slight positive effects on the motivation of some individuals (bottom panel). Estimates of the ψ parameters are shown in Figure 6. These estimates show that increases in motivation lead to increases in deposit amounts.

For the lottery condition, we take

$$\alpha_{t+1} = \alpha_t + \beta_1(y_t - r_{t+1})_+ + \beta_2(y_t - r_{t+1})_- + \beta_3 I_t(\text{win } \$100) + \beta_4 I_t(\text{lose } \$100) + \beta_5 I_t(\text{win } \$10) + \beta_6 I_t(\text{lose } \$10) + \eta_t$$

where $I_t(\cdot) = 1$ if the argument occurred at time t and zero otherwise. So tomorrow's motivation depends on past motivation, distance to tomorrow's target depending on whether the participant is currently above
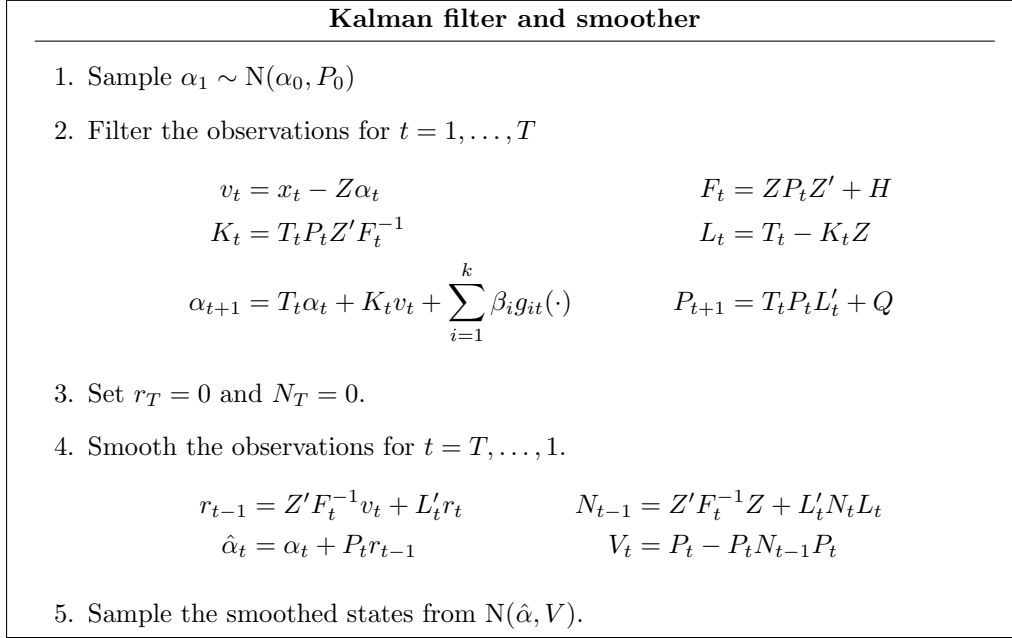

Kalman filter and smoother

1. Sample $\alpha_1 \sim \mathrm{N}(\alpha_0, P_0)$

2. Filter the observations for $t = 1, \ldots, T$

$$v_t = x_t - Z\alpha_t \qquad\qquad F_t = ZP_tZ' + H$$
$$K_t = T_tP_tZ'F_t^{-1} \qquad\qquad L_t = T_t - K_tZ$$
$$\alpha_{t+1} = T_t\alpha_t + K_tv_t + \sum_{i=1}^{k}\beta_i g_{it}(\cdot) \qquad P_{t+1} = T_tP_tL_t' + Q$$

3. Set $r_T = 0$ and $N_T = 0$.

4. Smooth the observations for $t = T, \ldots, 1$.

$$r_{t-1} = Z'F_t^{-1}v_t + L_t'r_t \qquad\qquad N_{t-1} = Z'F_t^{-1}Z + L_t'N_tL_t$$
$$\hat{\alpha}_t = \alpha_t + P_tr_{t-1} \qquad\qquad V_t = P_t - P_tN_{t-1}P_t$$

5. Sample the smoothed states from $\mathrm{N}(\hat{\alpha}, V)$.

Figure 4. This is the Kalman filter algorithm used for the simple state-space model discussed in sections 2.2 and 3.1. Except in the case of the augmented state vector in section 3.4, $T_t = 1$.

or below the target, and today's lottery outcome. Figure 7 shows 50% and 95% posterior credible intervals for β_1 to β_6 for each lottery participant. Larger estimates correspond to increases in motivation. Participants who are above tomorrows target tend to experience very little increase or decrease in motivation (top panel). Participants below the target often experience demotivation (second panel). Winning or losing the lottery has little impact on motivation (bottom four panels). Coefficients are not estimated for those individuals who never experienced the outcome—i.e. participants 9 to 19 never lost the $100 lottery and participant 10 never weighed less than tomorrow's target.

3.3. Posterior predictive simulations

Using the posterior distributions of the parameters, we can simulate new data under different conditions to answer questions about participant behavior. The general idea is that, given the model and the data, we now have information about all the parameters. Using this information, we can generate new data from the model and observe the results. This allows us to alter deposit amounts, lottery outcomes, or weight-loss success and observe the resulting behavior of the original participants. We examine behavior under four different scenarios: (1) what happens to deposits if participants lose different amounts of weight, (2) what happens to weight-loss with different deposit amounts, (3) what happens to weight-loss if the incentive struc-

ture in the lottery is altered, and (4) what happens to weight-loss if a lottery participant has a lucky streak. In each case, we simulate twenty-eight days of new data and compare the results to a baseline simulation. The charts display the median relative effect and the posterior probability that the effect exceeds the baseline.

For the first scenario, we alter the distribution of the weight shock ϵ_t such that $\epsilon_t \sim \mathrm{N}(\mu, \sigma_\epsilon^2)$. Now μ controls the amount of weight we expect the participant to lose in the coming 28 days. We take the baseline scenario to be weight-loss of four pounds (the goal for four weeks) which corresponds to $\mu = -1/7$. We also fix the initial deposit for the simulated month, since this is not available in the data. We use $1.50, the middle available deposit amount, for each individual. We compare the results to simulations in which the expected weight-loss is zero pounds, two pounds and eight pounds. Figure 8 shows the results of this simulation. Nearly every participant deposits less money when they lose less weight than in the baseline scenario. Likewise, nearly all participants deposit more money when they lose more weight than in the baseline scenario.

For the second scenario, we alter the deposits for the upcoming month and simulate new weight-loss behavior. We take the baseline scenario to be a deposit amount of $0. We compare the results to simula-
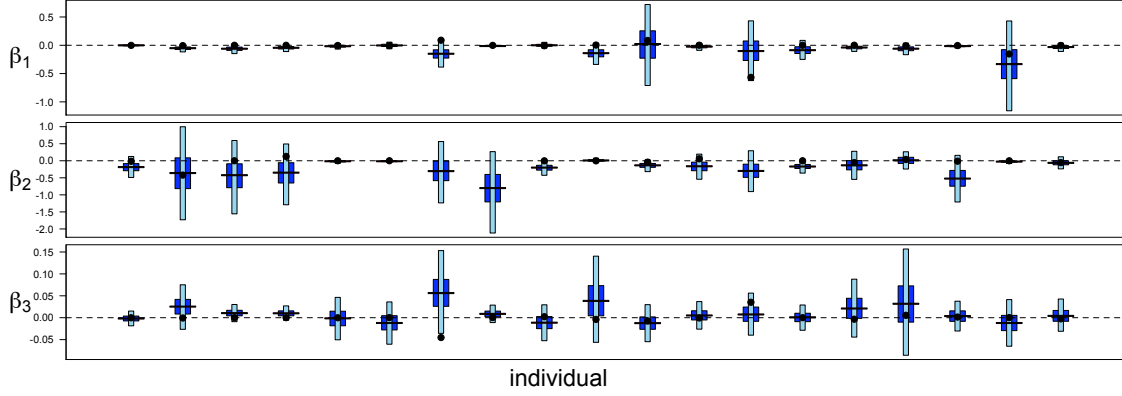
Figure 5. This figure shows 50% and 95% posterior credible intervals for β_1, β_2 and β_3 for each deposit contract participant. Larger estimates correspond to decreases in motivation. Solid circles are maximum likelihood estimates.
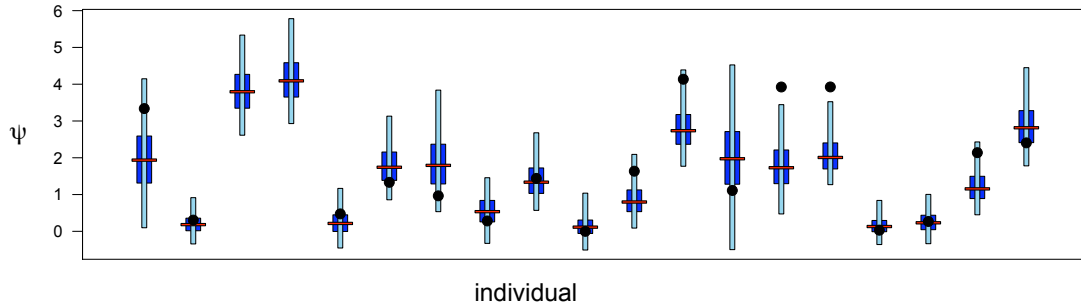

Figure 6. This figure shows 50% and 95% posterior credible intervals for ψ parameters in the deposit contract condition. Increases in motivation seem to lead to increased deposits. Solid circles are maximum likelihood estimates.

tions in which the deposit amount is $1.50, $3, and $10 (an impossible amount in the actual study). Figure 9 shows the results of this simulation. As deposit amounts increase, most individuals lose more weight with very high probability of out performing the baseline. Some individuals are unsuccessful regardless of the deposit amount. The individuals predicted to perform poorly generally did not alter their deposit amounts during the study.

The third scenario compares weight-loss outcomes under different lottery structures. It is important to get some idea of whether it is better to allocate funds to a mixture of large and small prizes (as in this study) rather than to only small, or only large, prizes. For this simulation, we take the baseline scenario to have lottery prizes as in the study: a one in five chance of winning $10 and a one in one hundred chance of winning $100. We compare the results to simulations in which the participant has a three in ten chance of winning $10 or a three in one hundred chance of winning

$100. These scenarios maintain the expected payout of $3 per day, thus comparing possible options the study may have faced given cost constraints. Figure 10 shows the results of this simulation. Most individuals seem to perform better in the scenario with only the large lottery prize, but this is not always the case. Some individuals perform better with only the small prize, while some perform best in the baseline scenario. This suggests that individual preference plays an important role in the behavioral responses of the different participants.

Finally, we want to know how the lottery participants respond to runs of good or bad luck. We alter the probabilities of the different prizes for the second week of the simulation while the rest of the simulation maintains the probabilities used in the study. Bad luck is taken to be no possibility of winning for the entire week. Two different "lucky" scenarios are tested: a three in five chance of winning the small prize with a two in seven chance of winning the large prize; and a
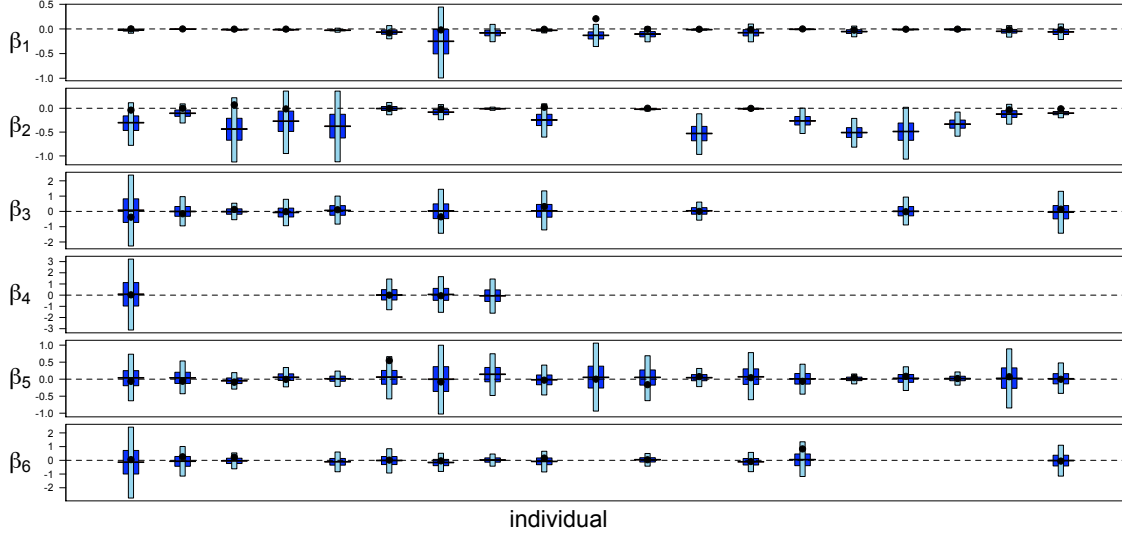
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Figure 7. This figure shows 50% and 95% posterior credible intervals for β_1 through β_6 for each lottery participant. Larger estimates correspond to increases in motivation. Solid circles are maximum likelihood estimates.

guarantee of winning the small prize every day for the week with a one in one hundred chance of winning the large prize. These probabilities represent success at random numbers, not success at weight loss, so there is no stipulation as to whether the participant actually "collected" the money in this simulation, just that the possibility existed. Figure 11 shows the results. A run of bad luck is motivational for some individuals, but is disheartening to others. Always winning the small prize has little effect, with most individuals having probabilities of exceeding the baseline between 0.4 and 0.6. Frequently winning the large prize is demotivating to some, but has little effect on others (the probability that the result will exceed baseline is not much more than 0.5 for individuals with increased weight-loss performance relative to baseline). Again, individual preferences seem to have significant roles in the responses of the participants.

3.4. Maximum likelihood methods and model checks

In order to verify our results, we fit the individual level model using maximum likelihood. Both the lottery condition and the deposit contract have around ten unknown parameters, so we reformulate the state-space model in an effort to decrease the dimensionality of the maximization problem. The formulation of the model given in equations (1)–(4) is not unique. In particular, we can augment the state vector with the β coefficients so that these will be estimated via the Kalman filter algorithm. This leaves only the ψ parameters as well

as the variance terms as the arguments to maximize over. Once the β coefficients are taken into the state vector, we must provide priors for their initial distribution which we take to be $N(0, 100)$ so as to minimize the effect of the initial conditions on the resulting estimates.

In most cases, the maximum likelihood estimates (MLEs) are similar to those obtained using Bayesian methods. Nearly all estimates fall within the credible intervals shown in Figures 5, 6, and 7. The MLEs for the variance parameters do not correspond quite so closely to their Bayesian counterparts. In particular, the MLEs for σ_η^2 are approximately zero for most individuals. This is probably due to the likelihood being relatively flat in this region: wide variations in the value of σ_η^2 will give very similar values for the likelihood. However, we expect some amount of extrinsic variation in motivation—non-zero values of σ_η^2—which is the reason for incorporating strong prior information into the Bayesian model. Posterior credible intervals and the maximum likelihood estimates are shown in Figures 12 and 13.

We use the state vectors from the maximum likelihood version of the model to examine modelling assumptions, in particular, the assumption that ϵ_t is normally distributed. Under the model, $v_t = x_t - Z\alpha_t$ is distributed normally with mean zero and variance covariance matrix F_t. Since F_t is not diagonal, the different elements of v_t are correlated. Therefore we premultiply the residual vector by the Cholesky decomposition of F_t^{-1}. We can now examine these transformed

residuals using standard tests for normality. For many individuals, a Shapiro-Wilk test [12] rejects the null hypothisis that the residuals are normally distributed with very low p-values. Q-Q plots for each individual reveal two issues that account for this result: there are often a few unexpectedly large or small standardized residuals, and there are too many standardized residuals that are approximately zero. Both of these issues are the result of data irregularities: weight reports are not always accurately recorded, the study scale was occasionally radically different from the home scale, and some individuals lied frequently. Two individuals are shown in Figure 14 along with their weight trajectories.[1] One way to account for these violations would be to assume that ϵ_t is t-distributed with two or three degrees of freedom and fit the model using importance sampling or a particle filter. We leave these options for the future. These sorts of anomalies do not alter our conclusions, rather they imply that we are unable to observe all of the signal in the data. If the model fit the data better, than the estimated states α_t would better represent the underlying motivation of the individual participants, leading to better behavioral predictions. However, since the data is clouded by these anomalies, we are unable to infer hidden behavior from the dampened signal. Better data will lead to more accurate and informed conclusions than those reported here.

4. Discussion

Using data from the Volpp et al. [15] weight-loss study, we attempt to determine the effects of incentives on weight loss and the effects of weight-loss performance on self-selected incentives. To do this, we model the evolution of the unobserved motivation of an individual as a function of past motivation, the distance to future target amounts, and the incentive payments. In section 3, we estimate the model using state-space methods with willpower, or motivation, as the unobserved state. This analysis suggests that incentives have complicated effects on day-to-day motivation after controlling for the effects of proximity to future targets. It also suggests that in the targeting framework, a current weight less than tomorrow's target is demotivating. This analysis suggests that motivation is a

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[1]There are numerous transposition errors in the data set such as an individual weighing around 186 pounds for a week and then weighing 168 for a day. Obvious errors were corrected before running the analysis, but many errors remain. For many individuals, the initial weight was as much as 4 pounds greater than the weight on the home scale leading to issues with the target weights. Finally, one individual reported exactly the target weight for two months leading to residuals of zero.

significant predictor of deposit amounts, with greater motivation leading to larger deposits.

The model that we use shows that for some individuals, incentive payments can lead to motivational effects which inturn lead to changes in predicted monthly weight-loss. The model specified in section 3 assumes that these effects are transitory, with winnings or losses effecting only the next day's motivation. However, it is possible that these effects do not occur immediately, but after some delay. Also, winning or losing the lottery may result in a persistent change in motivation rather than a transient effect as implied by this model, especially since winnings are redeemed only at the end of the month following successful performance. These theories are easily tested by choosing different $g_k(\cdot)$ functions. For example, letting the state equation for the deposit contract be

$$\alpha_{t+1} = \alpha_t + \beta_1(y_t - r_{t+1})_+ + \beta_2(y_t - r_{t+1})_-$$
$$+ \beta_3 \sum_{i=t_0}^{t} d_i I_{y_i \leq r_i} + \eta_t,$$

where t_0 is the first day of the current month gives the cumulative earnings in the current month. Now motivation depends on the accumulated winnings the participant stands to gain if he hits the target at the end of the month. Using this model specification along with the corresponding cumulative earnings model for the lottery condition gives similar results to those reported in section 3: predicted weight-loss outcomes vary widely on an individual level.

We can also extend the model to account for systematic differences between participants which may impact behavior by letting $\mathbf{M}$ be a matrix of the desired covariates from the intake or monthly surveys. We used three questions and statements from the intake survey:

- An extra \$3 a day could really change my life. (5-level Likert scale)

- I could do a lot with an extra \$100. (5-level Likert scale)

- Do you play the lottery regularly? (yes/no)

as regressors, but the results are similar to those already reported.

In order to aid in model fitting and to make more nuanced inferences, researchers should implement a few methodological changes. First, it would be helpful to have a baseline with which to compare results. A control group with daily reports rather than only monthly

weigh-ins would make it possible to separate out the effects of daily reporting from the incentive effects on motivation in a more direct manner. Second, there is almost no information in the data relating to the large lottery prize because the prize is won or lost too infrequently. Also, some individuals had the opportunity to win the prize many times during the study, while most never got the chance. One possibility would be to inform participants that there is a one in one hundred chance of winning the prize, but to actually award it deterministically so as to ensure that everyone has the opportunity to win it at least once. This would allow inferences for all individuals rather than just a handful. Finally, the deposit contracts need to be updated more frequently than monthly. Weekly updates may help, but daily updates would be ideal. In the current data sets, there is so little variation in the deposit amounts that is is difficult to gauge the behavioral effect of the deposit.

The questions in the intake and monthly surveys should be augmented to model participant behavior more effectively. It would be useful to distinguish between participants in terms of attitudes toward risk tolerance or spending habits. Questions like "Suppose you anticipate going to the grocery store for 45 minutes give or take 15 minutes. How much time would you put on your parking meter if it costs $1 per hour?" or "If it takes 10 minutes give or take 5 minutes to get to your daughter's school, how early would you leave the house in order to pick her up?" would elicit participants' relative tolerance for risk and would be more useful to distinguish between individuals on the aspects of interest. Participant responses would also aid in model validation and the construction of priors on the variance terms in both the state and observation equations.

Our analysis suggests that researchers should construct their experiments carefully in order to see desired changes in behavior in response to incentives. Target weights should be decreased for successful participants to avoid demotivation, deposit amounts should be updated frequently, and more useful survey questions would aid in the analysis. It is possible that the incentive to perform in this study is due to the idea of winning in the future or the fun of playing the game rather than the returns themselves. A control group that is observed daily could help to separate out these effects. Future research on incentives and behavior using models of the data generating process can shed light on these phenomena.

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A. Likelihood, priors and hyperpriors

A.1. Deposit Contract

Let y be the observed weights, d be the observed deposits, J be the number of individuals in the deposit contract condition, and T_j be the number of days in the study for the jth individual. Also let $\mathbf{X}$ be the matrix of covariates defined by the g_k functions and $\mathbf{M}$ be the matrix of covariates from the intake survey. For a particular individual, the likelihood for y given the states α_t and some additional parameters is given by

$$L\left(\Delta y, d \mid \alpha, \psi, \sigma_v^2, \sigma_\epsilon^2\right) = \prod_{t=1}^{T_j} \mathrm{N}\left(\begin{bmatrix} \psi \\ -1 \end{bmatrix} \alpha_t, \begin{bmatrix} \sigma_v^2 & 0 \\ 0 & \sigma_\epsilon^2 \end{bmatrix}\right)$$

$$p\left(\alpha \mid \sigma_\eta^2, \beta\right) = \prod_{t=2}^{T_j} \mathrm{N}\left(\alpha_{t-1} + (\mathbf{X}\beta)_{t-1}, \sigma_\eta^2\right)$$
$$\times \mathrm{N}(\alpha_0, P_0),$$

for each of the J individuals. This leads to the usual Kalman filtering algorithm applied to each individual. The remaining hierarchical priors and hyperpriors for combining individuals are given by

$$p\left(\beta_1, \ldots, \beta_k \mid \mathbf{b}, d^2\right) = \prod_{k=1}^{K} \prod_{j=1}^{J} \mathrm{N}\left((\mathbf{Mb}_k)_j, d_k^2\right)$$

$$p\left(\psi \mid \mathbf{u}, e^2\right) = \prod_{j=1}^{J} \mathrm{N}\left((\mathbf{Mu})_j, e^2\right)$$

$$p\left(\mathbf{b}, d^2\right) = \mathrm{N}\Gamma^{-1}\left(\mu_b, \Sigma_b, \delta_b, \gamma_b\right)$$

$$p\left(\mathbf{u}, e^2\right) = \mathrm{N}\Gamma^{-1}\left(\mu_u, \Sigma_u, \delta_u, \gamma_u\right).$$

The priors for individual variances are given by

$$p\left(\sigma_v^2\right) = \prod_{j=1}^{J} \Gamma^{-1}\left(c, c'\right)$$

$$p\left(\sigma_\epsilon^2\right) = \prod_{j=1}^{J} \Gamma^{-1}\left(p, p'\right)$$

$$p\left(\sigma_\eta^2\right) = \prod_{j=1}^{J} \Gamma^{-1}\left(\tau, \tau'\right).$$

A.2. Lottery

Similarly, let y be the observed weights, J be the number of individuals in the lottery condition, and T_j be the number of days in the study for the jth individual. Also let $\mathbf{X}$ be the matrix of covariates defined by the g_k functions and $\mathbf{M}$ be the matrix of covariates from the intake survey. For a particular individual, the likelihood for y given the states α_t and some additional parameters is given by

$$L\left(\Delta y \mid \alpha, \sigma_\epsilon^2\right) = \prod_{t=1}^{T_j} \mathrm{N}\left(-\alpha_t, \sigma_\epsilon^2\right)$$

$$p\left(\alpha \mid \sigma_\eta^2, \beta\right) = \prod_{t=2}^{T_j} \mathrm{N}\left(\alpha_{t-1} + (\mathbf{X}\beta)_{t-1}, \sigma_\eta^2\right)$$
$$\times \mathrm{N}(\alpha_0, P_0),$$

for each of the J individuals. This leads to the usual Kalman filtering algorithm applied to each individual. The remaining hierarchical priors and hyperpriors for combining individuals are given by

$$p\left(\beta_1, \ldots, \beta_k \mid \mathbf{b}, d^2\right) = \prod_{k=1}^{K} \prod_{j=1}^{J} \mathrm{N}\left((\mathbf{Mb}_k)_j, d_k^2\right)$$

$$p\left(\mathbf{b}, d^2\right) = \mathrm{N}\Gamma^{-1}\left(\mu_b, \Sigma_b, \delta_b, \gamma_b\right).$$

The priors for individual variances are given by

$$p\left(\sigma_\epsilon^2\right) = \prod_{j=1}^{J} \Gamma^{-1}\left(p, p'\right)$$

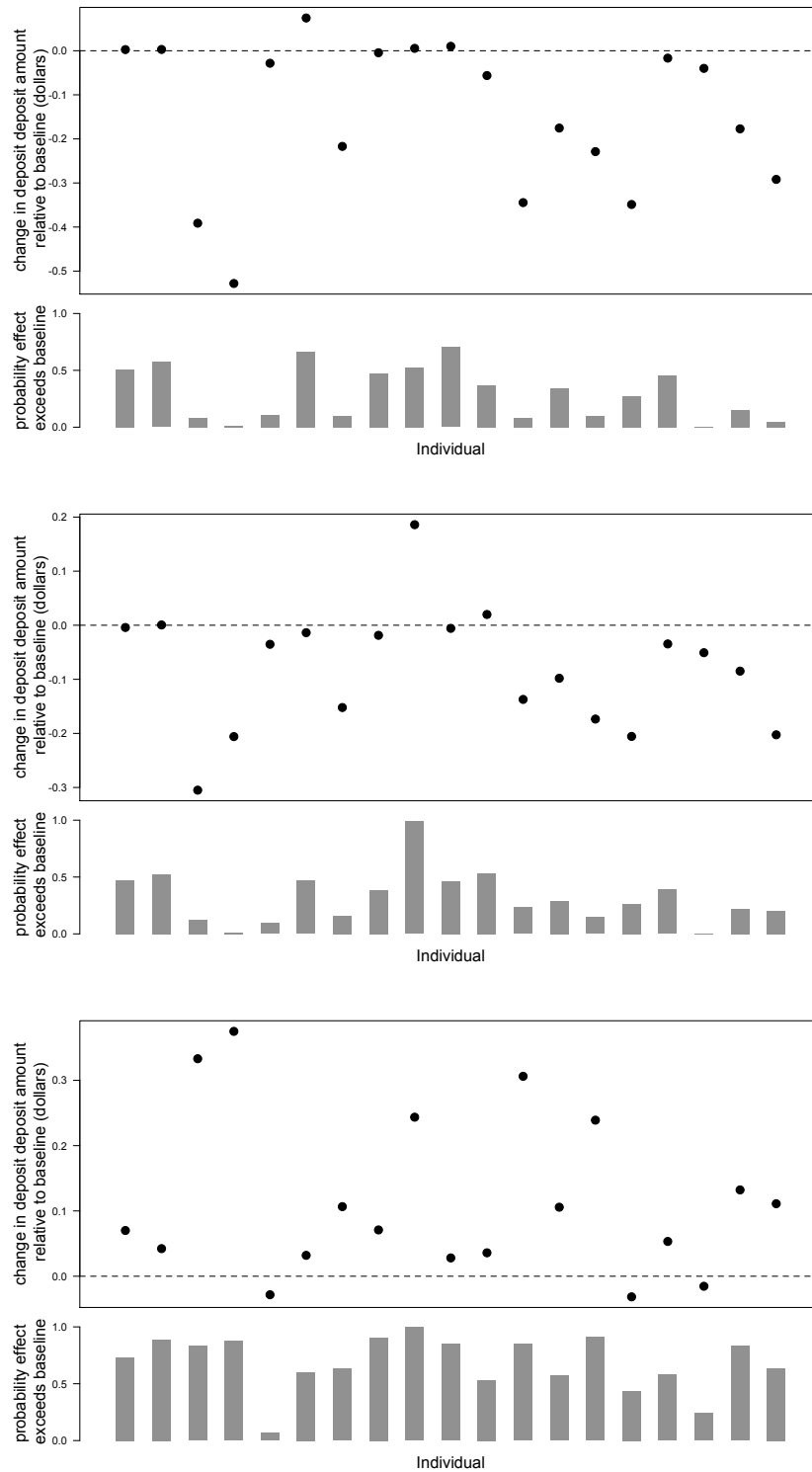$$p\left(\sigma_\eta^2\right) = \prod_{j=1}^{J} \Gamma^{-1}\left(\tau, \tau'\right)$$

Figure 8. This figure compares the predicted deposit amounts for different weight-loss successes. The baseline is taken to be twenty-eight day weight loss averaging four pounds. The top panel shows the relative change in deposit amount after zero expected weight-loss, the middle panel shows the relative ch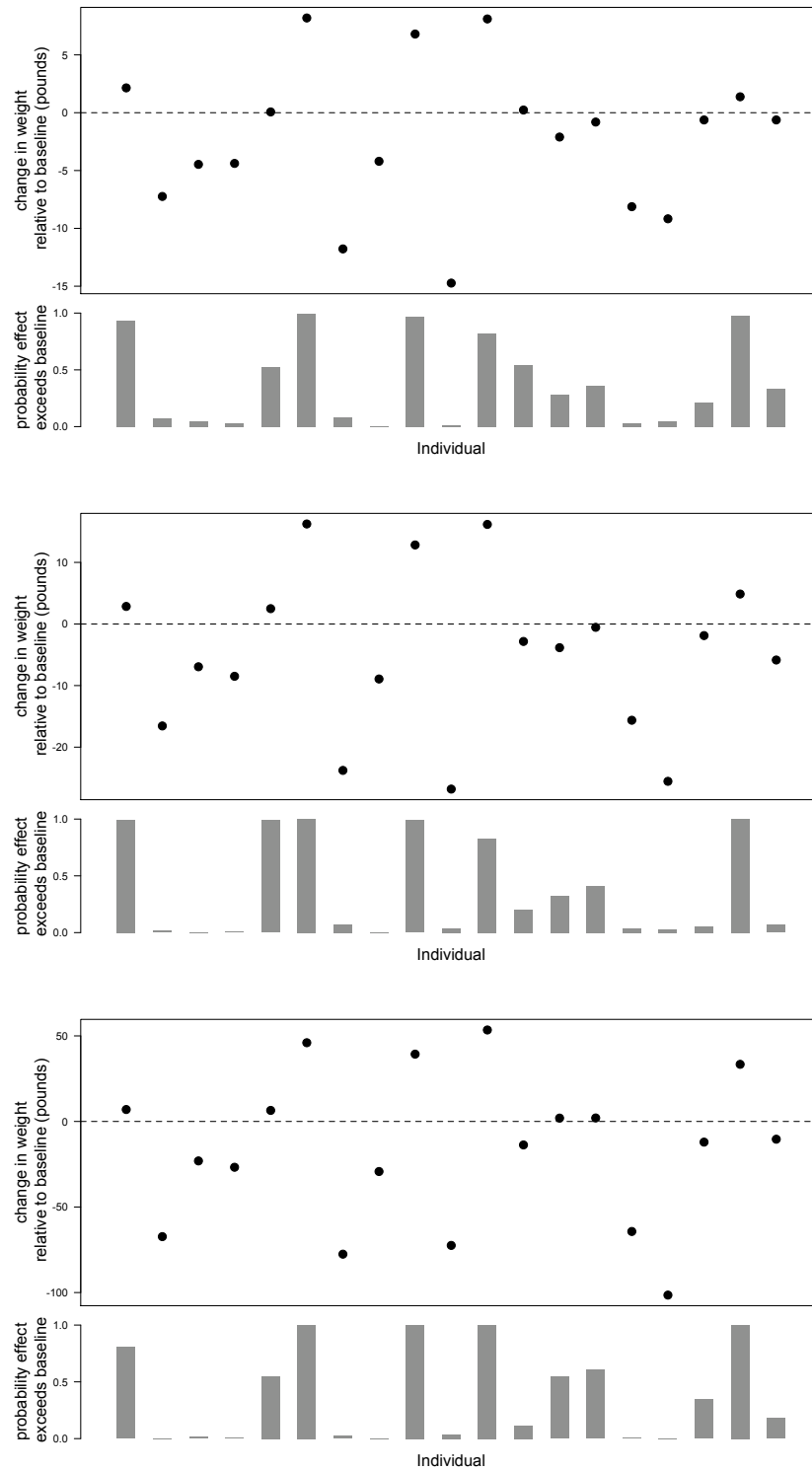ange in deposit amount after two pounds expected weight-loss and the bottom panel shows the relative change in deposit amount after eight pounds expected weight-loss.

Figure 9. This figure compares the predicted weight-loss success for different deposit amounts. The baseline is taken to be a deposit of $0. The top panel shows the relative weight-loss performance for a deposit of $1.50, the middle panel shows the relative weight-loss performance for a deposit of $3, and the bottom panel shows the relative weight-loss performance for a deposit of $10.
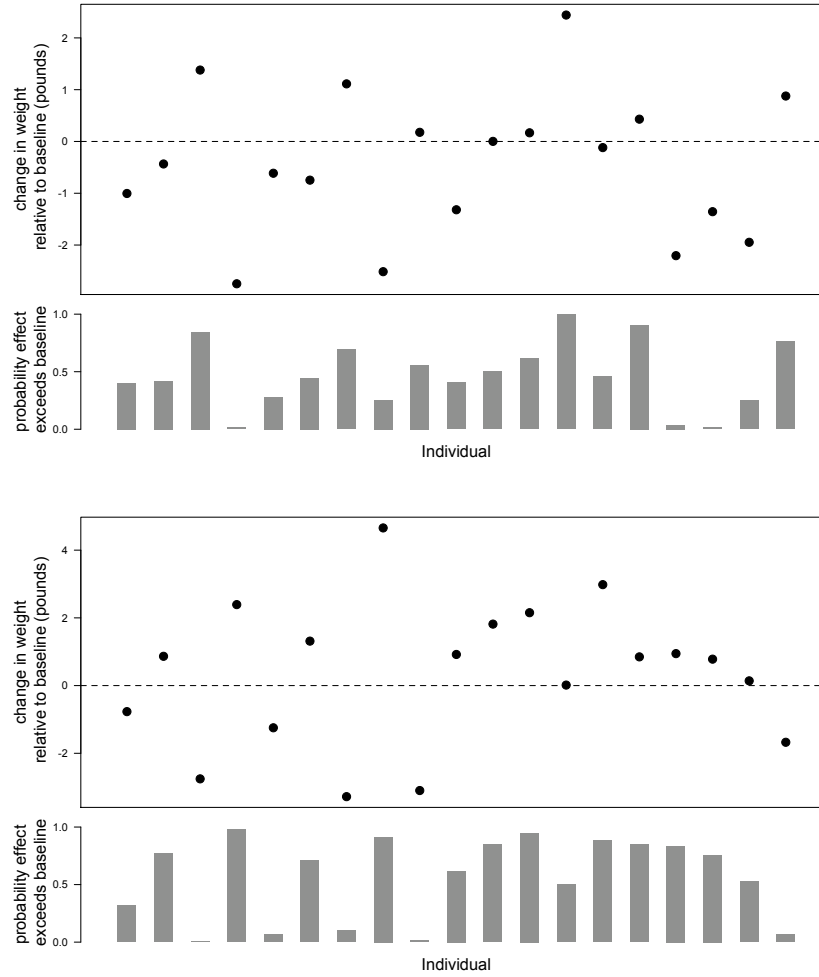
Figure 10. This figure compares the predicted weight-loss success under different lottery prize possibilities while maintaining expected costs to the study. The baseline is taken to be a a one in five chance of winning $10 and a one in one hundred chance of winning $100. The top panel shows the relative weight-loss performance for a three in ten chance of winning $10 while the bottom panel shows the relative weight-loss performance for a three in one hundred chance of winning $100.
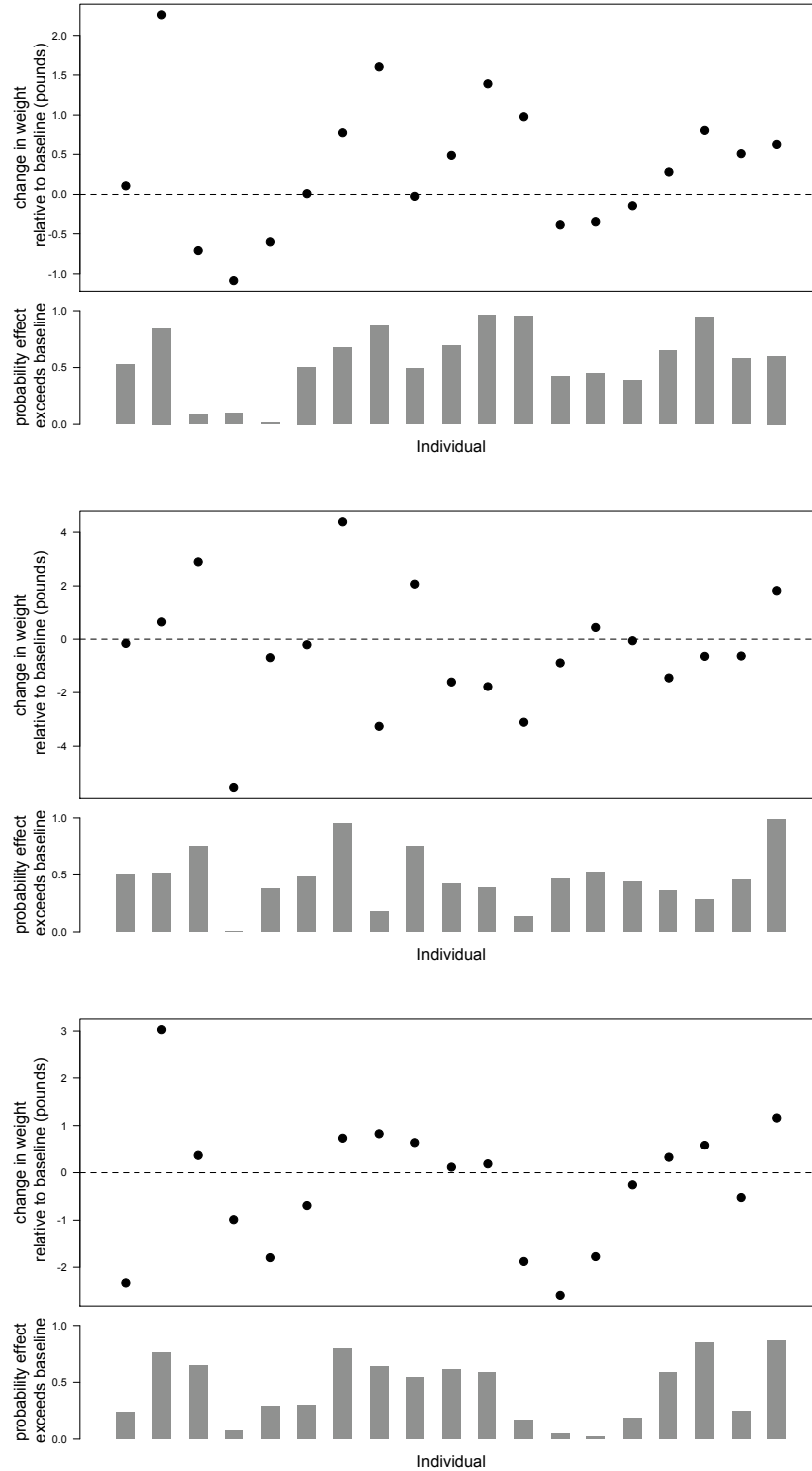
Figure 11. This figure compares the predicted weight-loss success under different scenarios of luck during the second week. The baseline is taken to be a a one in five chance of winning $10 and a one in one hundred chance of winning $100. The top panel shows the relative weight-loss performance with no possibility of winning during the second week, the middle panel shows the relative performance with a guarantee of winning the small prize every day for the week, and the bottom panel shows the relative performance for a three in five chance of winning the small prize with a two in seven chance of winning the large prize.
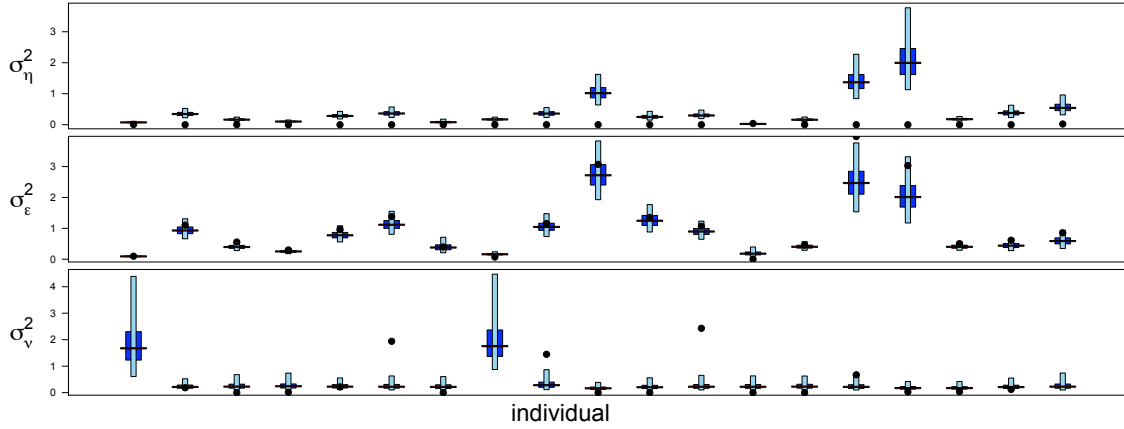
Figure 12. This figure shows posterior credible intervals for the deposit contract condition. Solid circles are the corresponding maximum likelihood estimates.
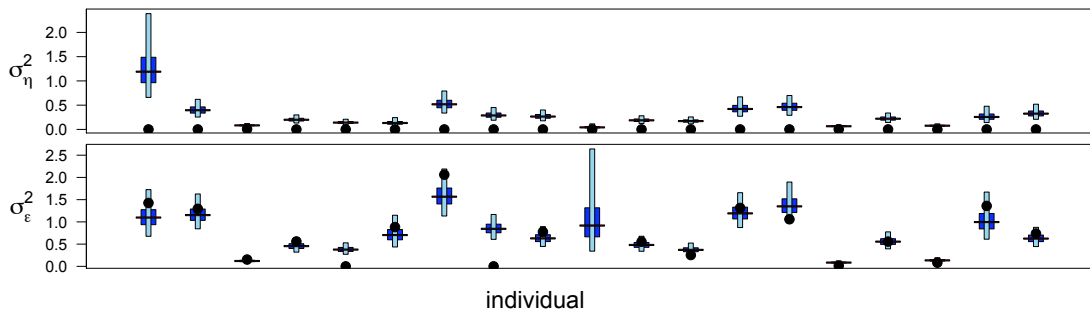

Figure 13. This figure shows posterior credible intervals for the lottery condition. Solid circles are the corresponding maximum likelihood estimates.
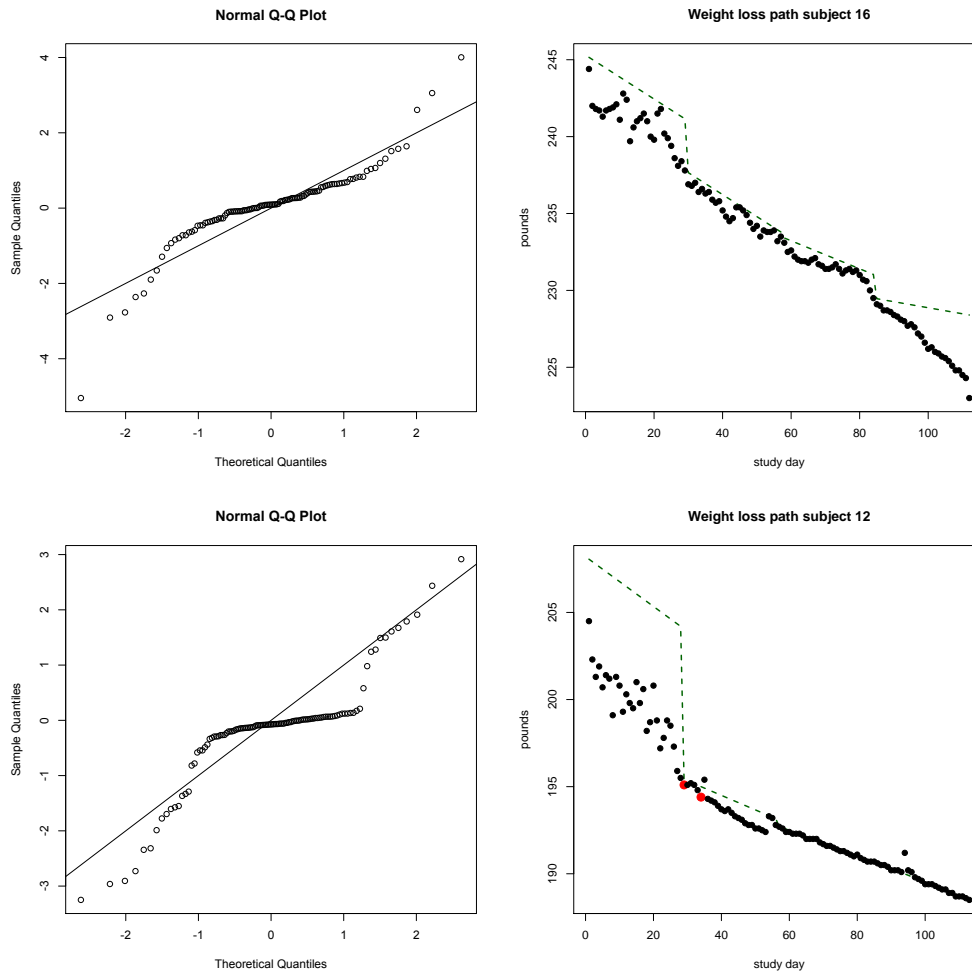
Figure 14. This figure shows Q-Q plots for a deposit contract participant (upper left plot) and a lottery participant (lower left plot) along with their reported and target weights (right side). This reveals that participant lies tend to lead to violations of modelling assumptions due to the resulting underestimates of observation weight variances.