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Comment on the 4/3 problem in the electromagnetic mass and the Boyer-Rohrlich controversy

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The Boyer-Rohrlich controversy seems to indicate that there are yet obscure points in the old problem of the electromagnetic mass. Here we try to clarify the physical content of both points of view, which indeed are part of two different traditions to approach the physics of classical elementary particles.

I. INTRODUCTION

For nearly a century physicists have pondered the electron and its self-field, as well as the interaction that may exist between them. This ill-understood interaction has given rise to some problems such as the additional inertia associated with a charged body in uniform motion,[1] the structure of the electron,[2–5] and the correct definition of electromagnetic field momentum[6–8] recently put forward in the Boyer-Rohrlich controversy.[9,10] This last problem also involves questions about the stability of the electron, the nature of the electron mass (totally electromagnetic or partly nonelectromagnetic?), and infinite self-energy. The clarification of these problems is worthwhile since most of them appear again in quantum electrodynamics. In the present Comment we try to give a physical interpretation of both Boyer's and Rohrlich's points of view, tracing their differences to their completely different conceptions of what a relativistic theory and the electron structure should be. This is then an effort to clarify the different approaches used to tackle the problem at hand, and for better understanding we set it in its historical perspective.

The point at issue is the following: In the four-tensor formalism in space-time it is shown that the energy and momentum of a mass point constitute a four-vector. However, the energy of the electric field of a distribution of charge at rest and the field momentum, as defined by Abraham,[3] of the field convected by the charged body in uniform motion are not covariantly related. For Boyer[9] and others[11] this is natural, since the electromagnetic field and the charge distribution are only a part of an isolated static system, the other part being the nonelectromagnetic force necessary to glue the charge distribution. This can be called the two-field approach.[12] On the other hand, for Rohrlich[10] and others[8] this situation must be mended by a redefinition of the convected field momentum. In the old "electromagnetic world view" the problem was seen as a discrepancy between the momentum-defined electromagnetic mass, the coefficient of $\mathbf{v}$ after an integration of $\mathbf{g}=(1/c^2)\mathbf{S}$, $\mathbf{S}$ being the Poynting vector $(c/4\pi)(\mathbf{E}\times\mathbf{B})$, and the rest-energy-defined mass U_0/c^2, taking as U_0 the electrostatic energy of the charge distribution at rest.

II. PROPOSED SOLUTIONS

The first explanation of the discrepancy between the momentum-defined electromagnetic mass and the one defined by energy was put forth by Poincaré. Having in mind that[5] "the laws of physical phenomena must be the same for a stationary observer as for an observer carried along in a uniform motion of translation," Poincaré tried to find appropriate cohesive forces that, together with the electromagnetic forces, would comply with a covariant principle of least action. Poincaré interpreted these supplementary forces as an internal cohesive pressure, namely,

$$p_0^S=-\frac{e^2}{8\pi a^4}\,, \tag{1}$$

with a being the radius of the electron, thus obtaining an additional energy

$$p_0^S V=-\frac{U_0}{3}\,, \tag{2}$$

where V is the volume of the electron in the rest frame.

This explanation was further clarified by von Laue,[11] who developed a relativistic continuum mechanics and introduced a second-rank four-tensor $T^{\mu\nu}$ to represent the state of energy, momentum, and stress of a deformable body or a field. By demanding that the electron should be in equilibrium in every inertial frame and in order to close the system,

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von Laue took the following condition:

$$\partial_\nu (T^{\mu\nu}_{\rm em} + T^{\mu\nu}_{\rm nonem}) = 0 , \tag{3}$$

where $T^{\mu\nu}_{\rm em}$ is the tensor of the electromagnetic field and $T^{\mu\nu}_{\rm nonem}$ the tensor of the nonelectromagnetic part, which includes the Poincaré stresses and maybe other fields. Using the Gauss theorem for space-time, it is easy to show[12] that for such a closed system the integral of the total stress-energy tensor over a spacelike hypersurface τ is independent of τ. Therefore, *total* energy-momentum as measured in the rest system at $t_0 = \text{const}$ is covariantly related to the *total* energy-momentum as measured in any other inertial frame at $t = \text{const}$.

However, if compared at $t_0 = \text{const}$ and $t = \text{const}$, the electromagnetic and mechanical parts do not behave covariantly by themselves. Boyer's point is precisely that this is to be expected, since the electron in the rest system is in a state of equilibrium and if seen from another reference system it is in a *different* state of equilibrium related to the former by a net transfer of energy and momentum from the mechanical stabilizing forces to the electromagnetic field, such transfer satisfying the principle of energy momentum conservation. Indeed, Boyer has shown,[13] by analyzing open multiparticle systems from different inertial frames, that this transfer of energy and momentum is a consequence of the relativity of simultaneity; hence such transfer is a relativistic kinematical effect.

Thus, from the point of view subscribed to by Boyer, the classical electron may be modeled in the rest system as a spherical shell of charge, stabilized by an attractive pressure [Eq. (1)] that gives rise to an additional nonelectromagnetic energy $\frac{1}{3}U_0$, giving thus a total energy $\frac{4}{3}U_0$. In a reference system where the electron moves with constant velocity $\mathbf{v}$, there is a flux of energy through a two-surface perpendicular to $\mathbf{v}$, and in this case the total energy passing through the surface in unit time is $\frac{4}{3}U_0$, thus producing a momentum equal to

$$\mathbf{p} = \tfrac{4}{3}\gamma \frac{U_0}{c^2}\mathbf{v} , \tag{4}$$

in agreement with the Lorentz transformation. In this way we have an energy-momentum four-vector $(\frac{4}{3}U_0, \mathbf{0})$ in the rest system, which after a Lorentz transformation produces the four-vector $(\frac{4}{3}\gamma U_0, \frac{4}{3}\gamma(U_0/c^2)\mathbf{v})$.

Usually this viewpoint has been rejected on the following grounds:

(1) Electrodynamics being a covariant theory, one would expect the purely electromagnetic energy and momentum to behave covariantly.

(2) Until now, the origin and nature of the Poincaré stresses have been unknown.

(3) It is strange for a noncovariant electromagnetic energy and momentum to be compensated by a noncovariant contribution from the Poincaré stresses in such a way as to produce a total energy-momentum that *is* covariant.

These objections have led some other physicists to look for a *definition* of a purely electromagnetic field momentum for the Coulomb field that is covariant. Rohrlich[6,7] has achieved this aim by defining the four-vector

$$p^\mu = \int T^{\mu\nu}_{\rm em} v_\nu d^3\tau , \tag{5}$$

where $T^{\mu\nu}_{\rm em}$ is the usual electromagnetic stress-energy tensor. Since this tensor *is not* divergence-free for the Coulomb field due to the presence of the sources, that is,

$$\partial_\nu T^{\mu\nu}_{\rm em} = -\frac{1}{c}F^{\mu\nu}j_\nu , \tag{6}$$

the volume integral is not independent of the hypersurface of integration. For this reason Rohrlich chooses to integrate over the *particular* hypersurface

$$d\tau_\nu = v_\nu d^3\tau , \tag{7}$$

$d^3\tau$ being the invariant volume dV' of the rest system and v_ν is the four-velocity; that is, the volume element is the Lorentz-transformed rest element volume: "Physically (this condition) expresses the rigidity of the charged particle in the relativistic sense."[7]

This definition leads to the expressions

$$\begin{aligned} p^0_{\rm em} &= \int dV'\gamma(U - \mathbf{v}\cdot\mathbf{S}) = \gamma m_{\rm em}c^2 , \\ p^k_{\rm em} &= \int dV'\gamma(S^k + v_l T^{lk}_{\rm em}) = \gamma m_{\rm em}v^k , \end{aligned} \tag{8}$$

with

$$m_{\rm em} = \frac{U_0}{c^2} , \tag{9}$$

where the factor of $\frac{4}{3}$ does not appear.

III. DISCUSSION

Here we try to give a more physical interpretation to the previous points of view. First, we note a difference in what is to be understood as a relativistic theory. For Poincaré,[4,5] Einstein,[14] von Laue,[15] and others, the content of the principle of relativity is that the *general laws of physics* must be covariant. "The relativity theory refers to patterns not to events."[16] So we must not take the trouble to work out covariant definitions of physical quantities unless we are trying to combine them in what we believe to be general laws. This is the reason why von Laue puts emphasis on the general conservation law [Eq. (3)] rather than on a *covariant* definition of a purely electromagnetic energy-momentum *four-vector*.

On the other hand, for Rohrlich[17] and others,[8] *physical observables* must have definite transformation properties. That is, for a physical quantity to be relativistically meaningful, it must be related among different inertial observers via Lorentz transformations. Faithful to this view, Rohrlich has constructed a covariant definition of the electromagnetic field momentum associated with the convected Coulomb field of the electron.

Let us now see what the differences are with respect to the physical meaning of both definitions of electromagnetic field momentum.

The conceptions of Poincaré, von Laue, and Boyer go as follows.

(1) Relativity theory is a field theory that presupposes classical electromagnetism. Therefore, every force field is transformed as the electromagnetic field is.[18] It is precisely this assumption that renders the nature of the electron mass ambiguous, since then there is no way to distinguish

between the behavior of different kinds of mass.

(2) The electron cannot be a purely electromagnetic object. Earnshaw's theorem compels us to introduce cohesive forces that stabilize the distribution of the finite quantity of charge that constitutes the electron.

(3) Because of the postulated equilibrium in every inertial frame, for the electromagnetic and cohesive forces the volume integral of every total stress must be zero (von Laue's theorem). A necessary condition for this to be the case is that Eq. (3) is satisfied with

$$T^{\mu\nu}_{\rm tot} = T^{\mu\nu}_{\rm em} + T^{\mu\nu}_{\rm coh} \quad . \tag{10}$$

In the rest system and for a spherical shell of charge, the stress-energy tensor of the cohesive force takes the form

$$T^{\mu\nu}_{\rm coh} = \begin{bmatrix} \rho & 0 & 0 & 0 \\ 0 & p^c_0 & 0 & 0 \\ 0 & 0 & p^c_0 & 0 \\ 0 & 0 & 0 & p^c_0 \end{bmatrix} , \tag{11}$$

where ρ is the energy density and p^c_0 is an internal cohesive pressure, the Poincaré pressure given by Eq. (1). In other words, the nonelectromagnetic subsystem can be modeled by a perfect fluid. In fact, Bialynicki-Birula[19] has explored the model of a perfect charged fluid with some detail and his conclusions fall in line with the two-field approach.

Applying a Lorentz transformation to the total tensor[11] we find[18]

$$U_{\rm em} = \gamma \left[U'_0 - p^c_0 V' \frac{v^2}{c^2} \right]$$
$$\mathbf{G}_{\rm em} = \gamma (U'_0 - p^c_0 V') \frac{\mathbf{v}}{c^2} , \tag{12}$$

and

$$U_{\rm coh} = \gamma \left[U'_{\rm coh} + p^c_0 V' \frac{v^2}{c^2} \right] ,$$
$$\mathbf{G}_{\rm coh} = \gamma (U'_{\rm coh} + p^c_0 V') \frac{\mathbf{v}}{c^2} , \tag{13}$$

but we have already seen [Eq. (3)] that for the shell model of the electron $p^c_0 V' = -\frac{1}{3} U_0$.

Now, if we define the field momentum to be completely electromagnetic, as in the old electromagnetic world view, i.e., $G_{\rm coh} = 0$, then obviously $U'_{\rm coh} = -p^c_0 V' = \frac{1}{3} U_0$. Therefore, the total quantities will be

$$U_{\rm tot} = \gamma (U'_{0_{\rm em}} + U'_{\rm coh}) = \gamma (U_0 + \tfrac{1}{3} U_0) = \gamma \tfrac{4}{3} U_0 ,$$
$$\mathbf{G}_{\rm tot} = \gamma (U'_{0_{\rm em}} + U'_{\rm coh}) \frac{\mathbf{v}}{c} = \gamma \frac{4}{3} \frac{U_0}{c^2} \mathbf{v} \quad . \tag{14}$$

Again we note a covariant energy-momentum with a rest mass $\frac{4}{3} m^0_{\rm em}$ because of the inclusion of $\frac{1}{3} m^0_{\rm em}$, which corresponds to the energy of the cohesive stresses.

Rohrlich's outlook, however, is founded on the following assumptions.

(1) The rest energy of the electron is only the electrostatic energy, $U_0 = m^0_{\rm em} c^2$. The energy associated with the Poincaré stresses is normalized to zero. Maybe it is convenient to point out that in his book Rohrlich, as Fermi once did,[20] does not take into account the cohesive stresses, concluding that "we seem to have a relativistic but unstable electron."

(2) This rest energy is Lorentz transformed to any other inertial frame, so that the field momentum is the only one that corresponds to the flux of this energy, and therefore the associated mass is simply $m^0_{\rm em}$. As a justification of this treatment, Rohrlich uses the fact that the stress-energy tensor of the field is divergence-free *outside* a sphere that surrounds the charge, while the interior of the sphere is normalized to zero. This is akin to Dirac's[21] phenomenological approach, in which he treats the electron as a point charge. Therefore, Rohrlich's approach eliminates the analysis of any interaction between the electromagnetic and nonelectromagnetic subsystems and simply postulates its stability, as corresponds to a theory of a point electron.

As another justification of this treatment, Rohrlich has invoked the possible relation between classical and quantum electrodynamics, since the latter also postulates a point electron, as a phenomenological theory should. However, any constructive classical theory of the electron would have to be explored through the methods of Poincaré, von Laue, and Boyer. It must also be noted that we can get a factor of 1 in the total mass, i.e., taking into account the mechanical cohesive stresses. We simply define the cohesive mechanical rest energy as zero (i.e., $U'_{\rm coh} = 0$) and from Eq. (13) we get

$$U_{\rm coh} = \gamma \frac{v^2}{c^2} p^c_0 V' = -\tfrac{1}{3} \gamma \frac{v^2}{c^2} m^0_{\rm em} c^2 ,$$
$$\mathbf{G}_{\rm coh} = \gamma \frac{p^c_0 V'}{c^2} \mathbf{v} = -\tfrac{1}{3} \gamma m^0_{\rm em} \mathbf{v} , \tag{15}$$

then, instead of Eq. (14) we have

$$U_{\rm tot} = \gamma \left[1 + \frac{1}{3} \frac{v^2}{c^2} \right] m^0_{\rm em} c^2 - \tfrac{1}{3} \gamma m^0_{\rm em} \frac{v^2}{c^2} = \gamma m^0_{\rm em} c^2 ,$$
$$\mathbf{G}_{\rm tot} = \gamma \tfrac{4}{3} m^0_{\rm em} \mathbf{v} - \tfrac{1}{3} \gamma m^0_{\rm em} \mathbf{v} = \gamma m^0_{\rm em} \mathbf{v} \quad . \tag{16}$$

But of course here $U_{\rm nonem}$, $G_{\rm nonem}$, $U_{\rm em}$, $G_{\rm em}$ *do not* constitute four-vectors by themselves.

Schwinger[22] has also derived in a rigorous and transparent way two energy-momentum four-vectors: one corresponding to a $\frac{4}{3} m^0_{\rm em}$ and another to a $m^0_{\rm em}$. He does this by requiring that $-(1/c) F^{\mu\nu} j_\nu$ be divergenceless; that is, he construes two conserved tensors that correspond to stable charge distributions. Schwinger calls these masses "electromagnetic," but there must necessarily be forces of constriction of nonelectromagnetic origin that produce the required four-currents. These masses are electromagnetic in the sense that $\frac{4}{3} m^0_{\rm em}$ is obtained provided that all the momentum is of electromagnetic origin, while $m^0_{\rm em}$ results if all the rest energy is electrostatic. Again, this is consistent with von Laue's analysis [Eqs. (14) and (16)].

IV. CONCLUSIONS

As we can see, from the perspective of the traditional view, there is no trouble with the famous $\frac{4}{3}$ factor. The electron is considered in this view as an extended charged body, in equilibrium *in every* inertial frame, subject to the laws of electrodynamics *and* the laws of relativistic continuum mechanics through the general conservation law [Eq.

(3)]. Then it is natural that the purely electromagnetic part of the classical electron does not behave covariantly and there is nothing strange in that a noncovariant electromagnetic field energy-momentum is compensated by a noncovariant mechanical contribution: Again it is a consequence of the conservation law.

On the other hand, the new point of view treats the electron as a point particle, and the approach is rather geometrical and phenomenological. This may be the correct approach when we want to compare the classical theory with quantum electrodynamics, where the electron is also considered as a point particle. But the relation between classical and quantum electrodynamics is rather an open problem.

Of course the problem of the nature of the Poincaré stresses remains, and since progress here seems very difficult, work on classical theories in this line has been abandoned. By now the phenomenological approach, with its renormalization program, has produced important results in quantum electrodynamics, but not much so in classical electrodynamics.

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$$(dx)^2+(dy)^2+(dz)^2$$

is substituted by the relativistic invariant

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