clear
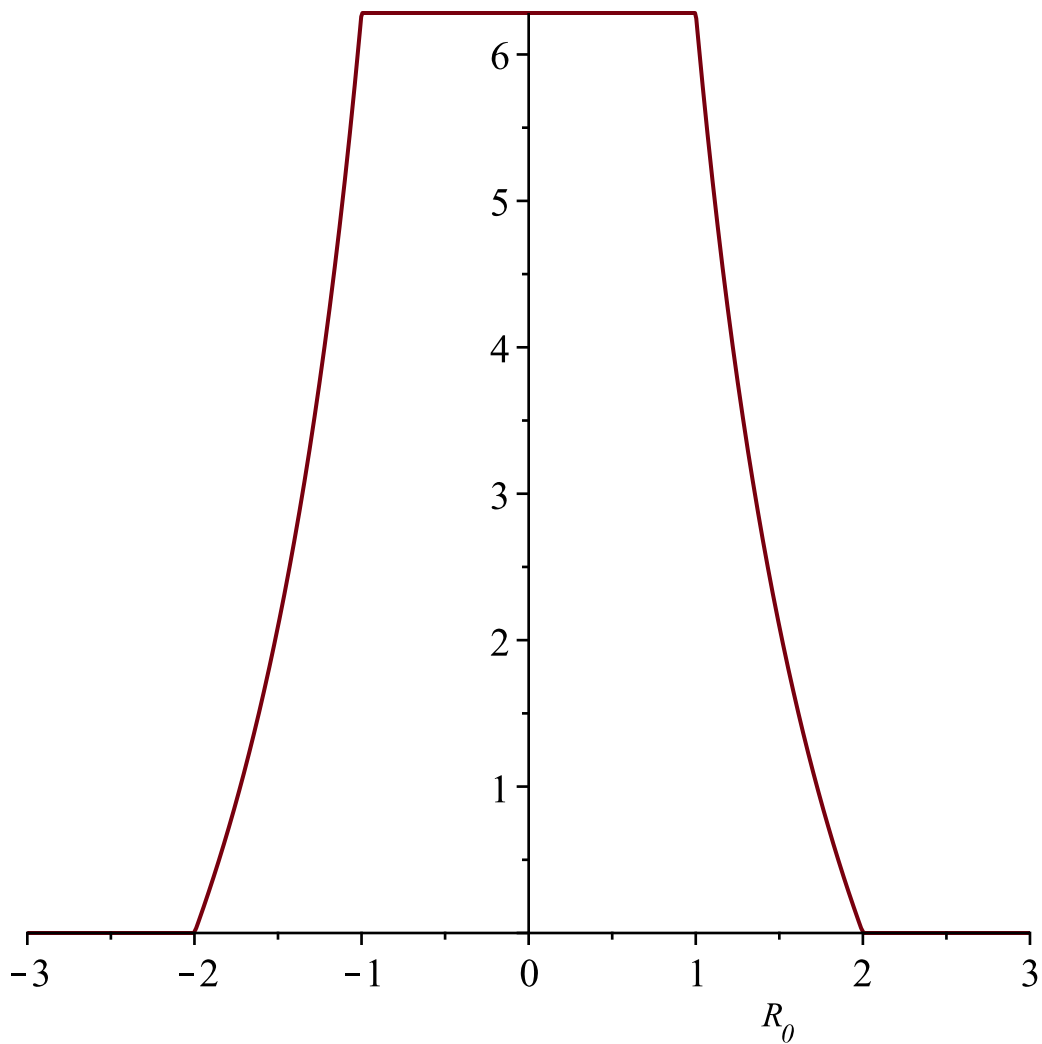
$$\varphi_R(q, R, R_0) := \int_0^{2\pi} \int_0^\pi \frac{q \cdot \sin(\theta)}{\sqrt{(R_0)^2 - 2 \cdot R_0 \cdot R \cdot \sin(\theta)\cos(\varphi) + (R)^2}}\, d\theta\, d\varphi$$

$$(q, R, R_0) \rightarrow \int_0^{2\pi} \int_0^\pi \frac{q \sin(\theta)}{\sqrt{R_0^2 - 2 R_0 R \sin(\theta)\cos(\varphi) + R^2}}\, d\theta d\varphi \tag{2}$$

$$\varphi_R(q, 1, R_0) - \varphi_R(q, 2, R_0)$$

$$\int_0^{2\pi} \int_0^\pi \frac{q \sin(\theta)}{\sqrt{R_0^2 - 2 R_0 \sin(\theta)\cos(\varphi) + 1}}\, d\theta\, d\varphi - \left(\int_0^{2\pi} \int_0^\pi \frac{q \sin(\theta)}{\sqrt{R_0^2 - 4 R_0 \sin(\theta)\cos(\varphi) + 4}}\, d\theta \right. \tag{3}$$

$$\left. d\varphi \right)$$

$$evalf\left(\varphi_R(1, 1, R_0) - \varphi_R(1, 2, R_0) \right)$$

$$\int_{0.}^{6.283185308} \int_{0.}^{3.141592654} \frac{\sin(\theta)}{\sqrt{R_0^2 - 2. R_0 \sin(\theta)\cos(\varphi) + 1.}}\, d\theta\, d\varphi - 1. \left(\int_{0.}^{6.283185308} \right. \tag{4}$$

$$\left. \int_{0.}^{3.141592654} \frac{\sin(\theta)}{\sqrt{R_0^2 - 4. R_0 \sin(\theta)\cos(\varphi) + 4.}}\, d\theta\, d\varphi \right)$$

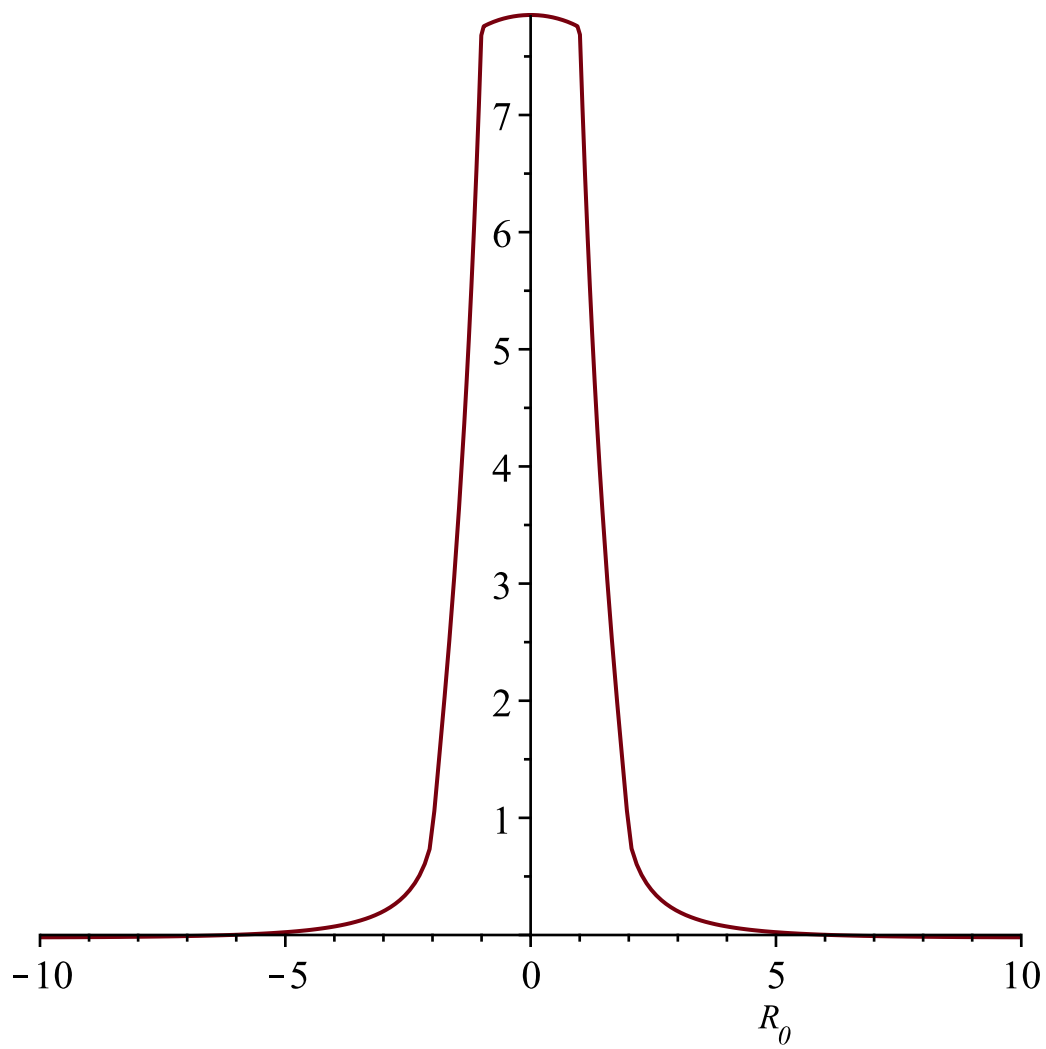$$with(plots) : plot\left(\varphi_R(1, 1, R_0) - \varphi_R(1, 2, R_0), R_0 = -3 ..3 \right)$$
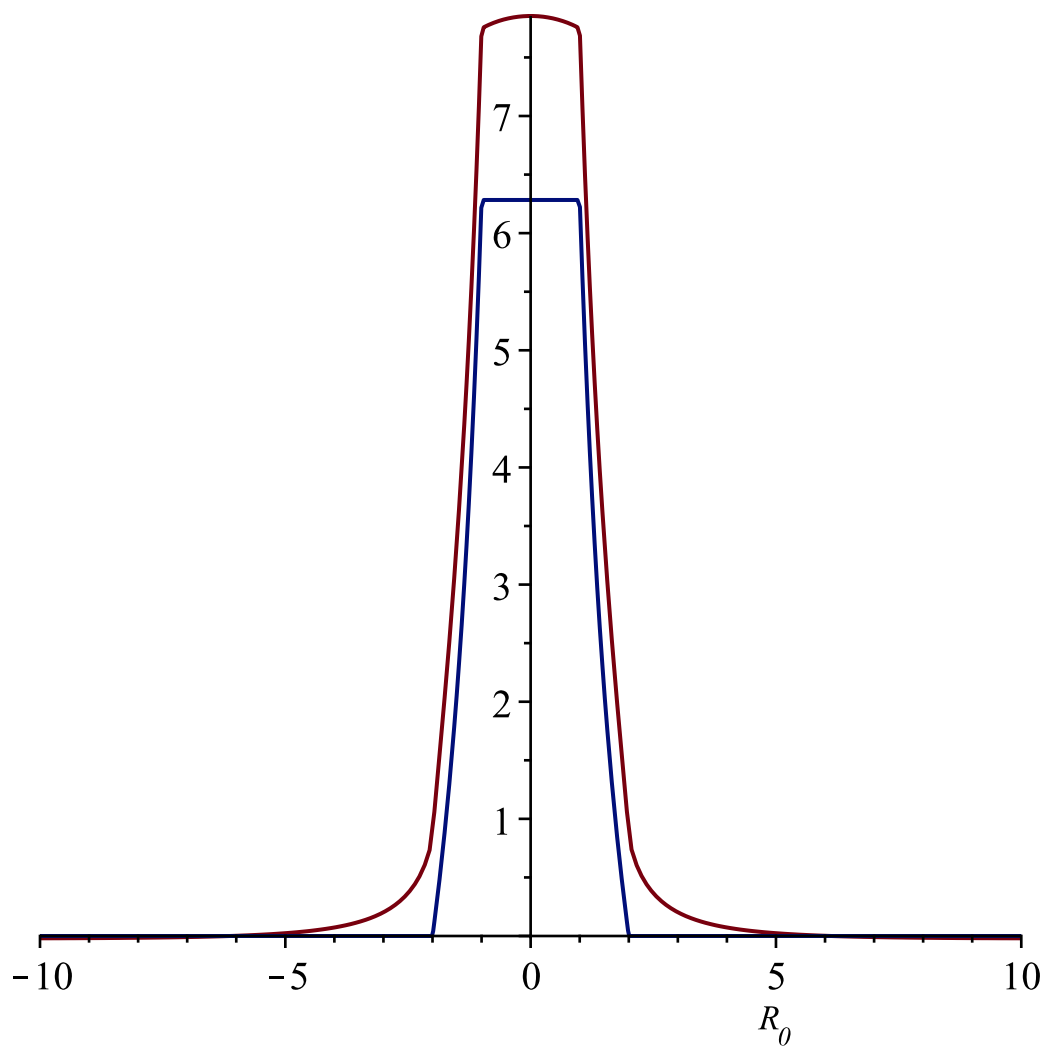
$$\varphi_{lw}(q, v, c, R, R_0) := \int_0^{2\pi}$$

$$\int_0^{\pi} \frac{q \cdot \sin(\theta)}{\sqrt{(R_0)^2 - 2 \cdot R_0 \cdot R \cdot \sin(\theta)\cos(\varphi) + (R)^2} - \frac{v}{c} \cdot (R_0 \cdot \sin(\theta)\cos(\varphi) - R)} \, d\theta \, d\varphi$$

$$(q, v, c, R, R_0) \rightarrow \int_0^{2\pi} \int_0^{\pi} \frac{q \sin(\theta)}{\sqrt{R_0^2 - 2 R_0 R \sin(\theta) \cos(\varphi) + R^2} - \frac{v(R_0 \sin(\theta)\cos(\varphi) - R)}{c}} \, d\theta \quad \textbf{(5)}$$
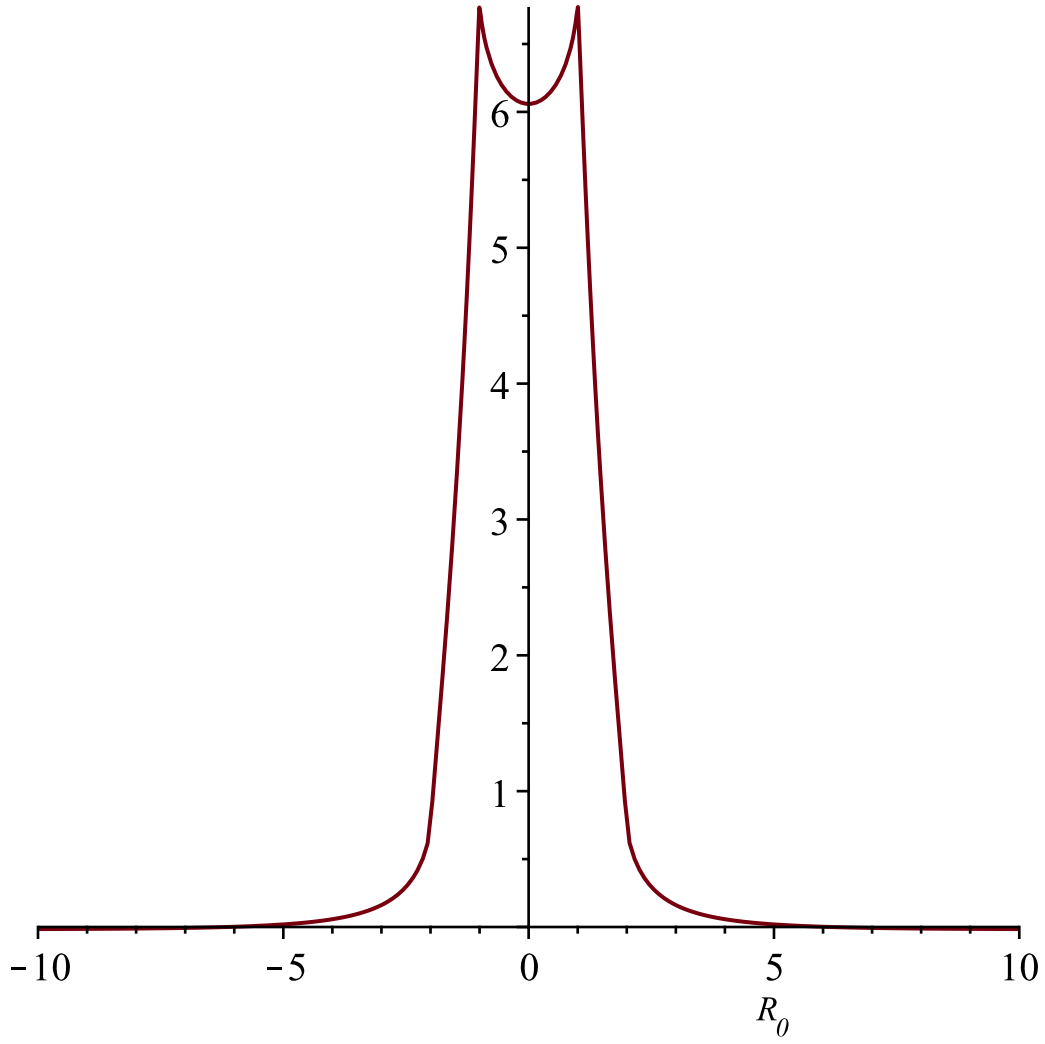
$$d\varphi$$

$with(plots): plot\left(\left[\varphi_R(1, 1, R_0) - \varphi_{lw}(1, 1, 3, 2, R_0)\right], R_0 = -10..10\right)$

$with(plots): plot\left(\left[\varphi_R(1, 1, R_0) - \varphi_{lw}(1, 1, 3, 2, R_0), \varphi_R(1, 1, R_0) - \varphi_R(1, 2, R_0)\right], R_0 = -10..10\right)$

$with(plots) : plot\left(\varphi_{lw}\left(1, 0.5, 3, 1, R_0\right) - \varphi_{lw}\left(1, 1, 3, 2, R_0\right), R_0 = -10 .. 10\right)$

$with(\, plots\,) : plot3d\big(\, \varphi_R(\,1, R, R_+\,) - \varphi_{lw}(\,1, 1, 3, R, 2\,), R = -10\,..10, R_+ = 1\,..2\,\big)$

$$E(\,q, R_+, R_-, R_0\,) := -\frac{\partial}{\partial R_0}\,\big(\, \varphi_R(\,q, R_+, R_0\,) + \varphi_R(\,-q, R_-, R_0\,)\,\big)$$

$$(\,q, R_+, R_-, R_0\,) \rightarrow -\left(\frac{\partial}{\partial R_0}\,\big(\, \varphi_R(\,q, R_+, R_0\,) + \varphi_R(\,-q, R_-, R_0\,)\,\big)\right) \tag{6}$$
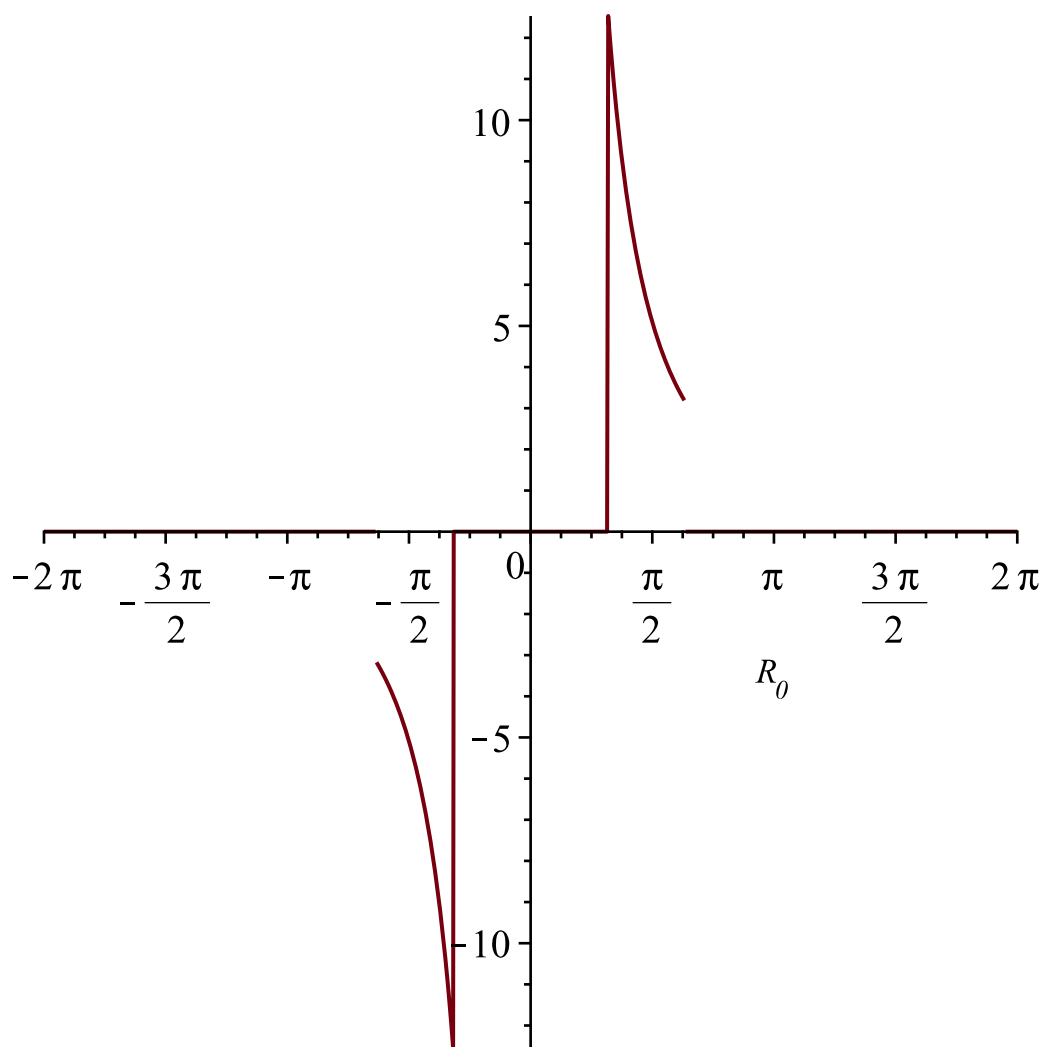
$E(\,q, R_+, R_-, R_0\,)$

$$-\left(\int_0^{2\pi}\!\!\int_0^{\pi}\left(-\frac{1}{2}\,\frac{q\,\sin(\theta)\,\big(\,2\,R_0 - 2\,R_+\sin(\theta)\cos(\varphi)\,\big)}{\big(\,R_0^{\,2} - 2\,R_0\,R_+\sin(\theta)\cos(\varphi) + R_+^{\,2}\,\big)^{3/2}}\right)\mathrm{d}\theta\,\mathrm{d}\varphi\right) - \left(\int_0^{2\pi}\right. \tag{7}$$

$$\left. \int_0^{\pi} \frac{1}{2} \frac{q \sin(\theta) \left(2 R_0 - 2 R_- \sin(\theta) \cos(\varphi)\right)}{\left(R_0^2 - 2 R_0 R_- \sin(\theta) \cos(\varphi) + R_-^2\right)^{3/2}} \, d\theta \, d\varphi \right)$$
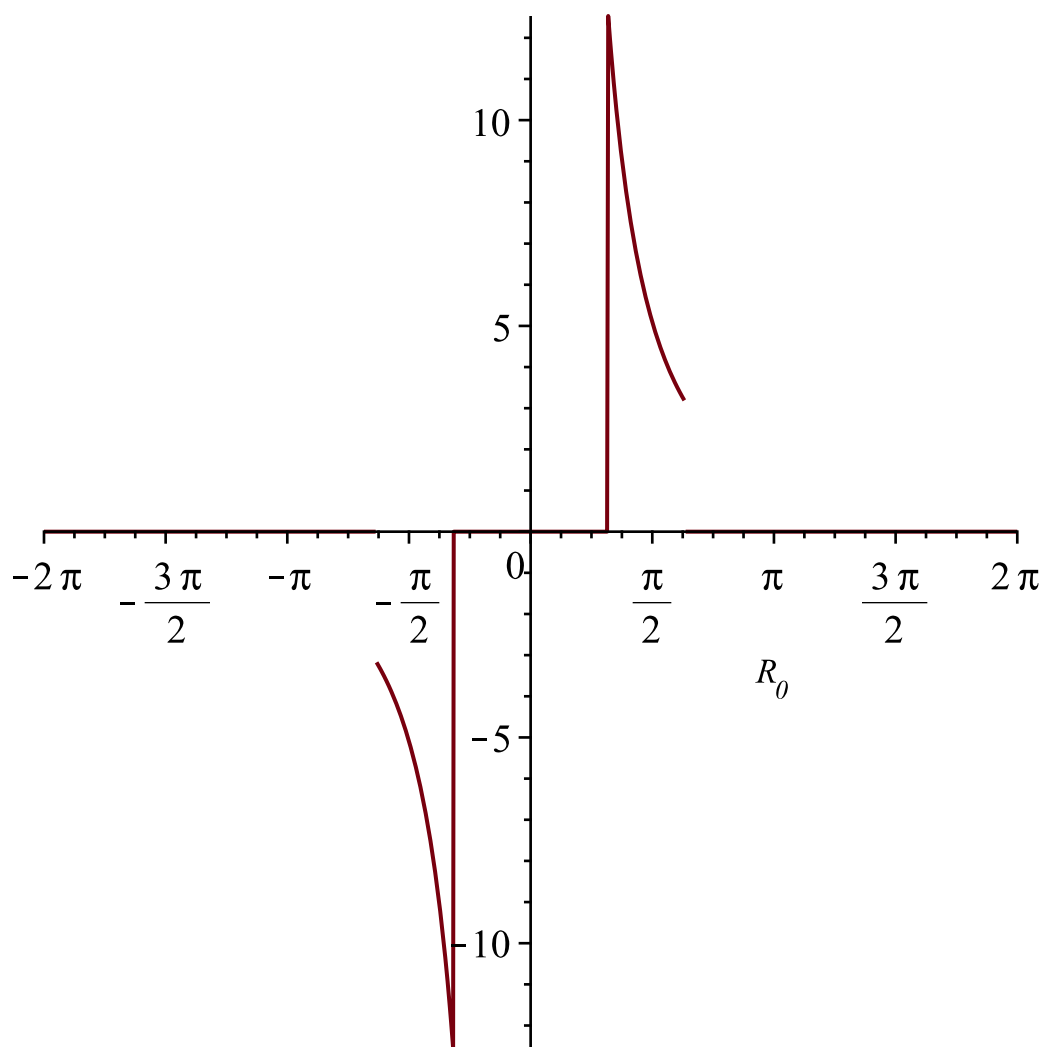
$$E_{lw}(q, v_+, v_-, c, R_+, R_-, R_0) := -\frac{\partial}{\partial R_0} \left(\varphi_{lw}(q, v_+, c, R_+, R_0) + \varphi_{lw}(-q, v_-, c, R_-, R_0)\right)$$

$$(q, v_+, v_-, c, R_+, R_-, R_0) \rightarrow -\left(\frac{\partial}{\partial R_0} \left(\varphi_{lw}(q, v_+, c, R_+, R_0) + \varphi_{lw}(-q, v_-, c, R_-, R_0)\right)\right) \tag{8}$$

$E_{lw}(q, v_+, v_-, c, R_+, R_-, R_0)$

$$-\left(\left(\int_0^{2\pi}\int_0^{\pi}\left(-\frac{q\sin(\theta)\left(\frac{1}{2}\frac{2R_0 - 2R_+\sin(\theta)\cos(\varphi)}{\sqrt{R_0^2 - 2R_0R_+\sin(\theta)\cos(\varphi) + R_+^2}} - \frac{v_+\sin(\theta)\cos(\varphi)}{c}\right)}{\left(\sqrt{R_0^2 - 2R_0R_+\sin(\theta)\cos(\varphi) + R_+^2} - \frac{v_+\left(R_0\sin(\theta)\cos(\varphi) - R_+\right)}{c}\right)^2}\right) d\theta \, d\varphi\right) - \left(\int_0^{2\pi}\right.\right. \tag{9}$$

$$\left.\left.\int_0^{\pi}\frac{q\sin(\theta)\left(\frac{1}{2}\frac{2R_0 - 2R_-\sin(\theta)\cos(\varphi)}{\sqrt{R_0^2 - 2R_0R_-\sin(\theta)\cos(\varphi) + R_-^2}} - \frac{v_-\sin(\theta)\cos(\varphi)}{c}\right)}{\left(\sqrt{R_0^2 - 2R_0R_-\sin(\theta)\cos(\varphi) + R_-^2} - \frac{v_-\left(R_0\sin(\theta)\cos(\varphi) - R_-\right)}{c}\right)^2} d\theta \, d\varphi\right)\right)$$

$with(plots) : plot\left(E(1, 1, 2, R_0), R_0\right)$

$with(plots) : plot\big(E_{lw}(1, 0, 0, 3, 1, 2, R_0), R_0\big)$

$with(plots) : plot\left(E_{lw}\left(1, 0, 1, 3, 1, 2, R_0\right), R_0\right)$

Warning,   computation interrupted

$with(plots) : plot\left(E_{lw}\left(1, 0.5, 1, 3, 1, 2, R_0\right), R_0\right)$

R_0