clear

$$clear \tag{1}$$

$$\varphi_R(q, R, R_0) := \int_0^{2\pi}\int_0^{\pi} \frac{q\cdot\sin(\theta)}{\sqrt{(R_0)^2 - 2\cdot R_0\cdot R\cdot \sin(\theta)\cos(\varphi) + (R)^2}}\, d\theta\, d\varphi$$

$$(q, R, R_0) \to \int_0^{2\pi}\int_0^{\pi} \frac{q\sin(\theta)}{\sqrt{R_0^2 - 2R_0 R\sin(\theta)\cos(\varphi) + R^2}}\, d\theta d\varphi \tag{2}$$

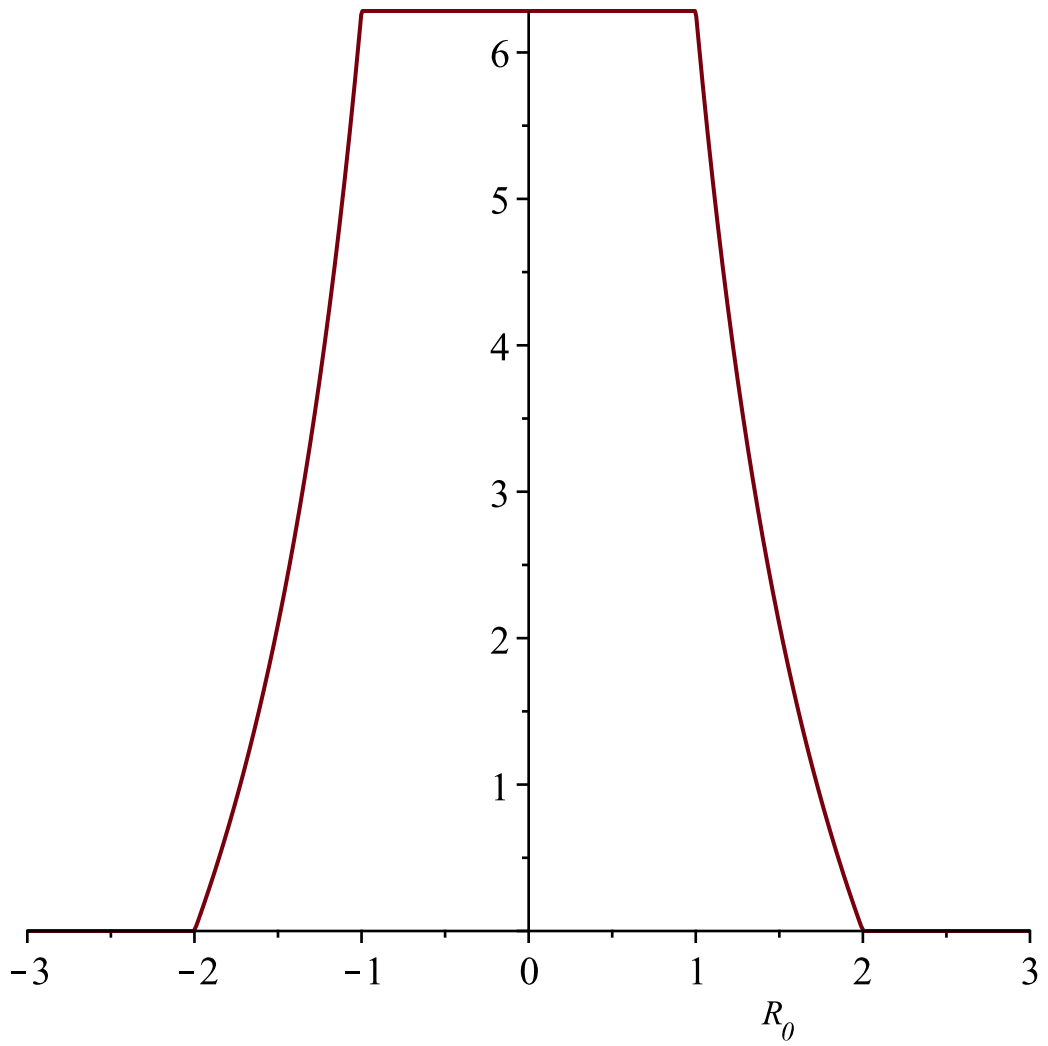
$$\varphi_R(q, 1, R_0) - \varphi_R(q, 2, R_0)$$

$$\int_0^{2\pi}\int_0^{\pi} \frac{q\sin(\theta)}{\sqrt{R_0^2 - 2R_0\sin(\theta)\cos(\varphi) + 1}}\, d\theta\, d\varphi - \left(\int_0^{2\pi}\int_0^{\pi} \frac{q\sin(\theta)}{\sqrt{R_0^2 - 4R_0\sin(\theta)\cos(\varphi) + 4}}\, d\theta\, d\varphi\right) \tag{3}$$

$$evalf\left(\varphi_R(1, 1, R_0) - \varphi_R(1, 2, R_0)\right)$$

$$\int_{0.}^{6.283185308}\int_{0.}^{3.141592654} \frac{\sin(\theta)}{\sqrt{R_0^2 - 2.R_0\sin(\theta)\cos(\varphi) + 1.}}\, d\theta\, d\varphi - 1.\left(\int_{0.}^{6.283185308}\int_{0.}^{3.141592654} \frac{\sin(\theta)}{\sqrt{R_0^2 - 4.R_0\sin(\theta)\cos(\varphi) + 4.}}\, d\theta\, d\varphi\right) \tag{4}$$

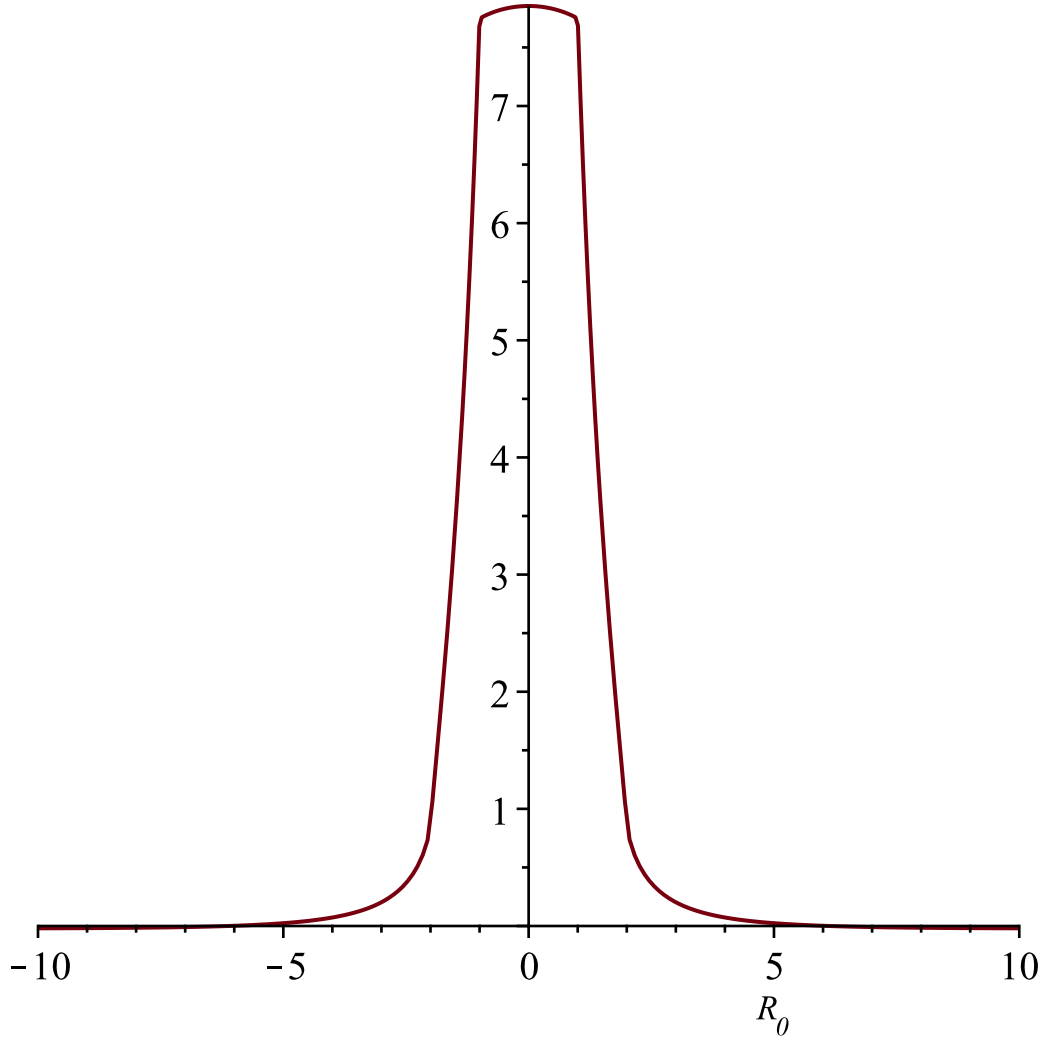
$$with(plots) : plot\left(\varphi_R(1, 1, R_0) - \varphi_R(1, 2, R_0), R_0 = -3..3\right)$$



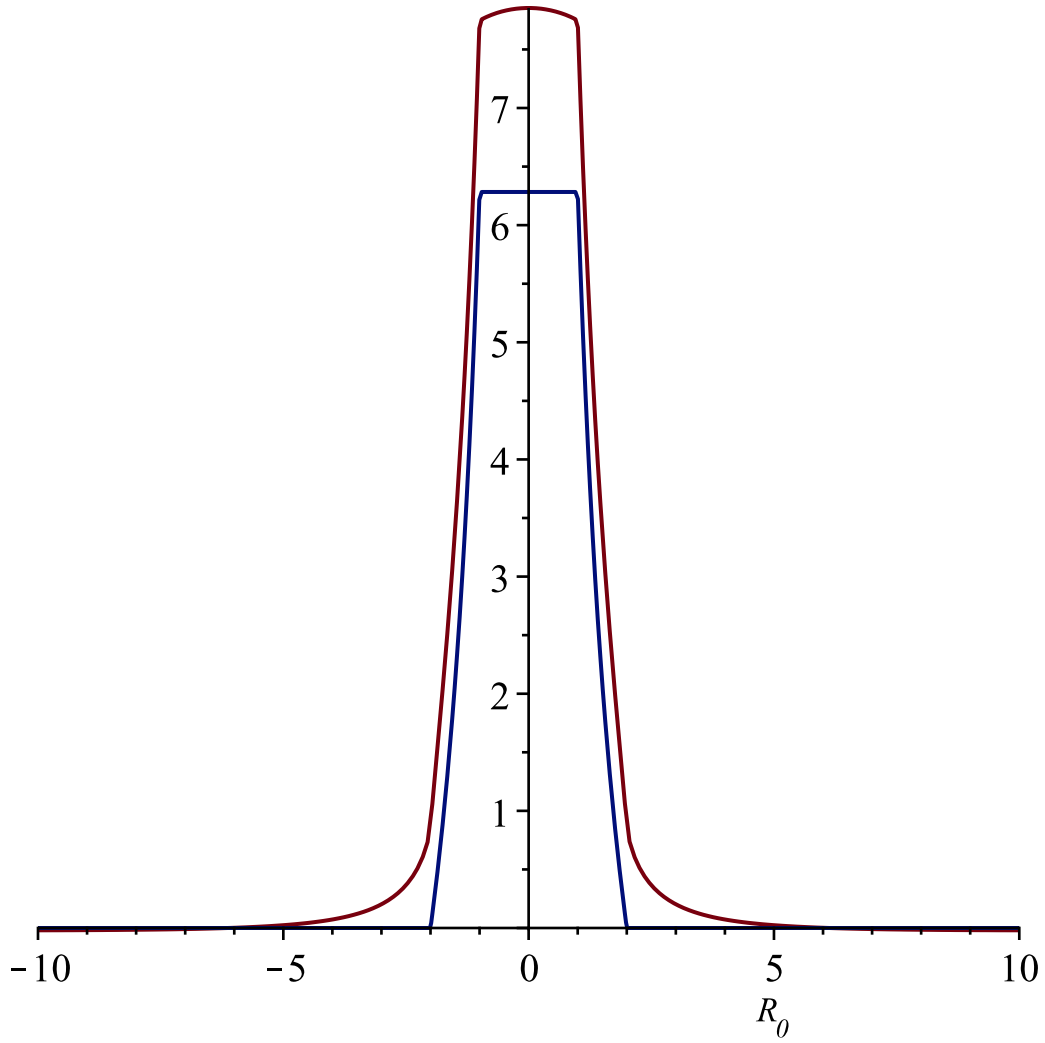
$$\varphi_{lw}(q, v, c, R, R_0) := \int_0^{2\pi} \int_0^{\pi} \frac{q \cdot \sin(\theta)}{\sqrt{(R_0)^2 - 2 \cdot R_0 \cdot R \cdot \sin(\theta) \cos(\varphi) + (R)^2} - \frac{v}{c} \cdot (R_0 \cdot \sin(\theta) \cos(\varphi) - R)} \, d\theta \, d\varphi$$

$$(q, v, c, R, R_0) \rightarrow \int_0^{2\pi} \int_0^{\pi} \frac{q \sin(\theta)}{\sqrt{R_0^2 - 2 R_0 R \sin(\theta) \cos(\varphi) + R^2} - \frac{v (R_0 \sin(\theta) \cos(\varphi) - R)}{c}} \, d\theta \, d\varphi \quad \textbf{(5)}$$

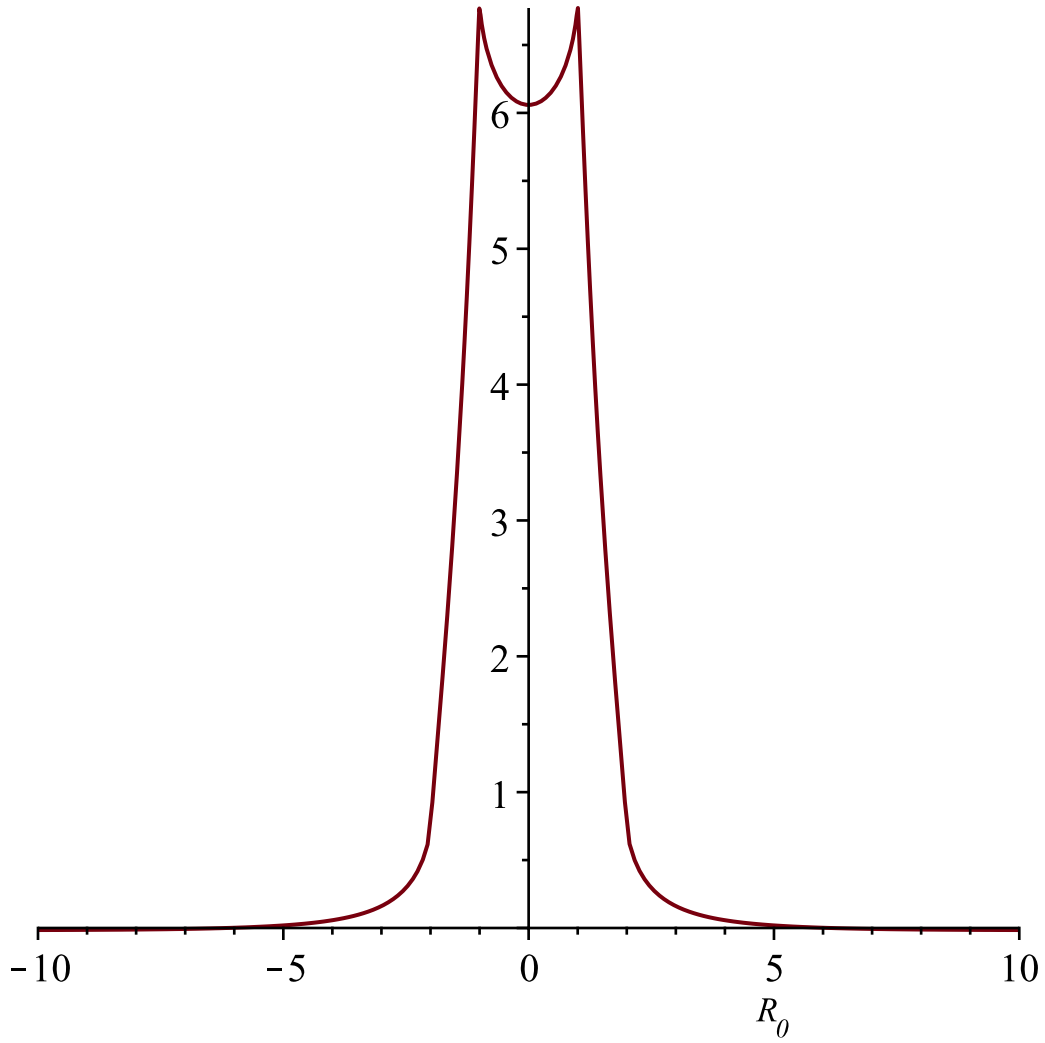
$$with(plots) : plot\left(\left[\varphi_R(1, 1, R_0) - \varphi_{lw}(1, 1, 3, 2, R_0) \right], R_0 = -10 .. 10 \right)$$



$$with(plots) : plot\left(\left[\varphi_R(1, 1, R_0) - \varphi_{lw}(1, 1, 3, 2, R_0), \varphi_R(1, 1, R_0) - \varphi_R(1, 2, R_0) \right], R_0 = -10 .. 10 \right)$$



$with(plots) : plot\left(\varphi_{lw}(1, 0.5, 3, 1, R_0) - \varphi_{lw}(1, 1, 3, 2, R_0), R_0 = -10..10\right)$



$$with(plots) : plot3d\left(\varphi_R(1, R, R_+) - \varphi_{lw}(1, 1, 3, R, 2), R = -10..10, R_+ = 1..2\right)$$

$$E(q, R_+, R_-, R_0) := -\frac{\partial}{\partial R_0}\left(\varphi_R(q, R_+, R_0) + \varphi_R(-q, R_-, R_0)\right)$$

$$(q, R_+, R_-, R_0) \to -\left(\frac{\partial}{\partial R_0}\left(\varphi_R(q, R_+, R_0) + \varphi_R(-q, R_-, R_0)\right)\right) \quad \textbf{(6)}$$

$$E(q, R_+, R_-, R_0)$$

$$-\left(\int_0^{2\pi}\int_0^{\pi}\left(-\frac{1}{2}\,\frac{q\sin(\theta)\left(2R_0 - 2R_+\sin(\theta)\cos(\varphi)\right)}{\left({R_0}^2 - 2R_0R_+\sin(\theta)\cos(\varphi) + {R_+}^2\right)^{3/2}}\right)d\theta\,d\varphi\right) - \left(\int_0^{2\pi} \quad \textbf{(7)}$$

$$\int_0^{\pi} \frac{1}{2} \frac{q \sin(\theta) \left(2 R_0 - 2 R_- \sin(\theta) \cos(\varphi)\right)}{\left(R_0^2 - 2 R_0 R_- \sin(\theta) \cos(\varphi) + R_-^2\right)^{3/2}} \, d\theta \, d\varphi \Bigg)$$

$$E_{lw}(q, v_+, v_-, c, R_+, R_-, R_0) := -\frac{\partial}{\partial R_0} \left(\varphi_{lw}(q, v_+, c, R_+, R_0) + \varphi_{lw}(-q, v_-, c, R_-, R_0)\right)$$

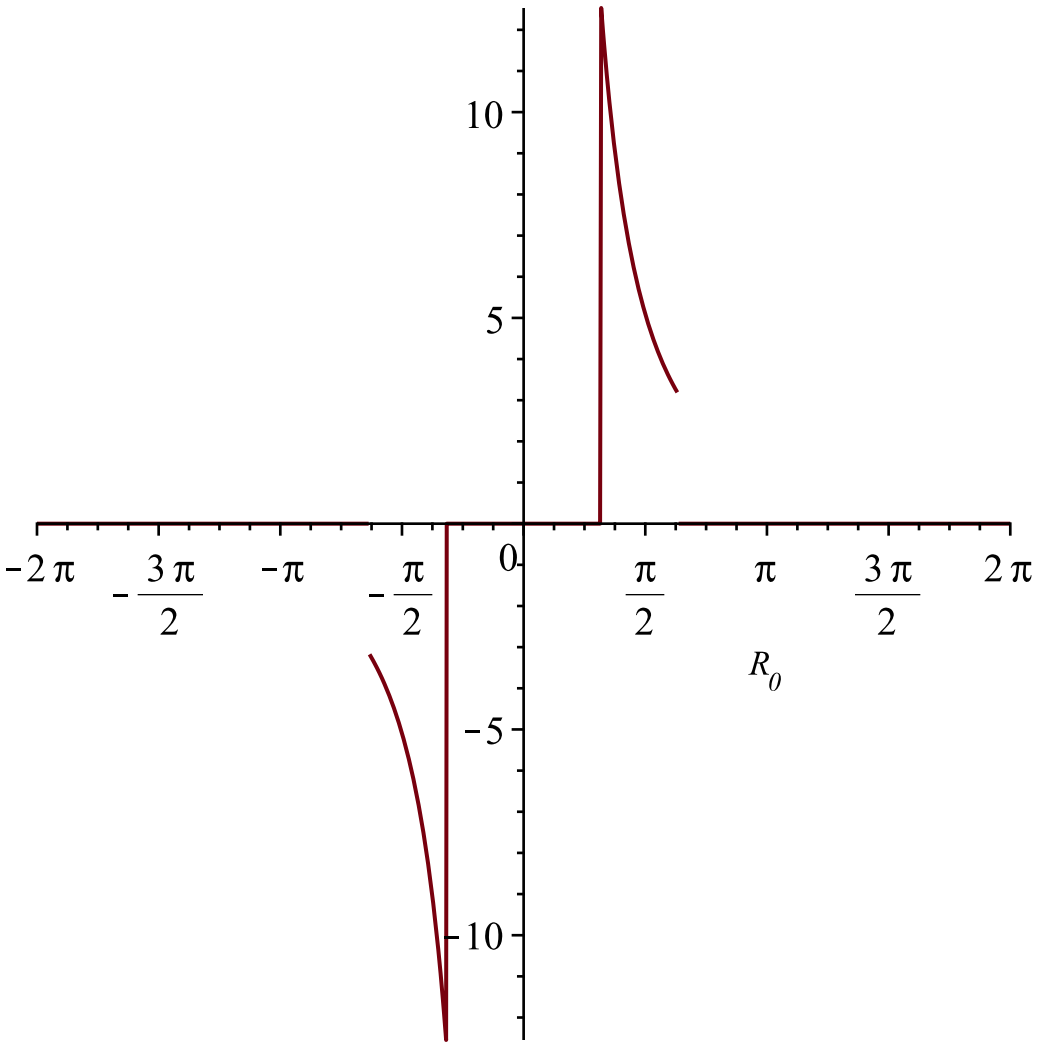
$$(q, v_+, v_-, c, R_+, R_-, R_0) \to -\left(\frac{\partial}{\partial R_0} \left(\varphi_{lw}(q, v_+, c, R_+, R_0) + \varphi_{lw}(-q, v_-, c, R_-, R_0)\right)\right) \tag{8}$$

$$E_{lw}(q, v_+, v_-, c, R_+, R_-, R_0)$$

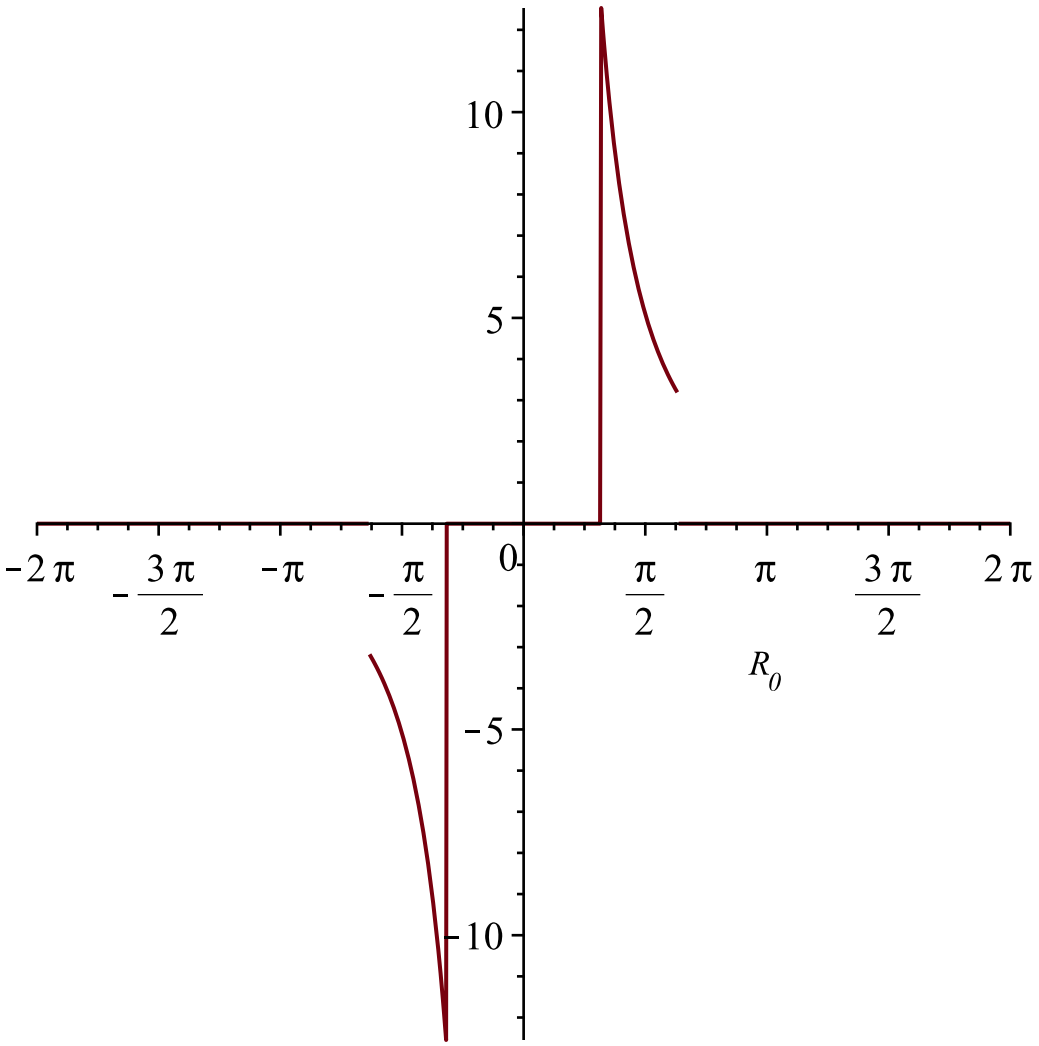
$$-\left(\int_0^{2\pi}\int_0^{\pi} \left(-\frac{q \sin(\theta) \left(\frac{1}{2} \frac{2 R_0 - 2 R_+ \sin(\theta) \cos(\varphi)}{\sqrt{R_0^2 - 2 R_0 R_+ \sin(\theta) \cos(\varphi) + R_+^2}} - \frac{v_+ \sin(\theta) \cos(\varphi)}{c}\right)}{\left(\sqrt{R_0^2 - 2 R_0 R_+ \sin(\theta) \cos(\varphi) + R_+^2} - \frac{v_+ \left(R_0 \sin(\theta) \cos(\varphi) - R_+\right)}{c}\right)^2}\right) d\theta \, d\varphi\right) - \left(\int_0^{2\pi}\right.$$

$$\left.\int_0^{\pi} \frac{q \sin(\theta) \left(\frac{1}{2} \frac{2 R_0 - 2 R_- \sin(\theta) \cos(\varphi)}{\sqrt{R_0^2 - 2 R_0 R_- \sin(\theta) \cos(\varphi) + R_-^2}} - \frac{v_- \sin(\theta) \cos(\varphi)}{c}\right)}{\left(\sqrt{R_0^2 - 2 R_0 R_- \sin(\theta) \cos(\varphi) + R_-^2} - \frac{v_- \left(R_0 \sin(\theta) \cos(\varphi) - R_-\right)}{c}\right)^2} d\theta \, d\varphi\right) \tag{9}$$

$$with(plots) : plot\left(E(1, 1, 2, R_0), R_0\right)$$



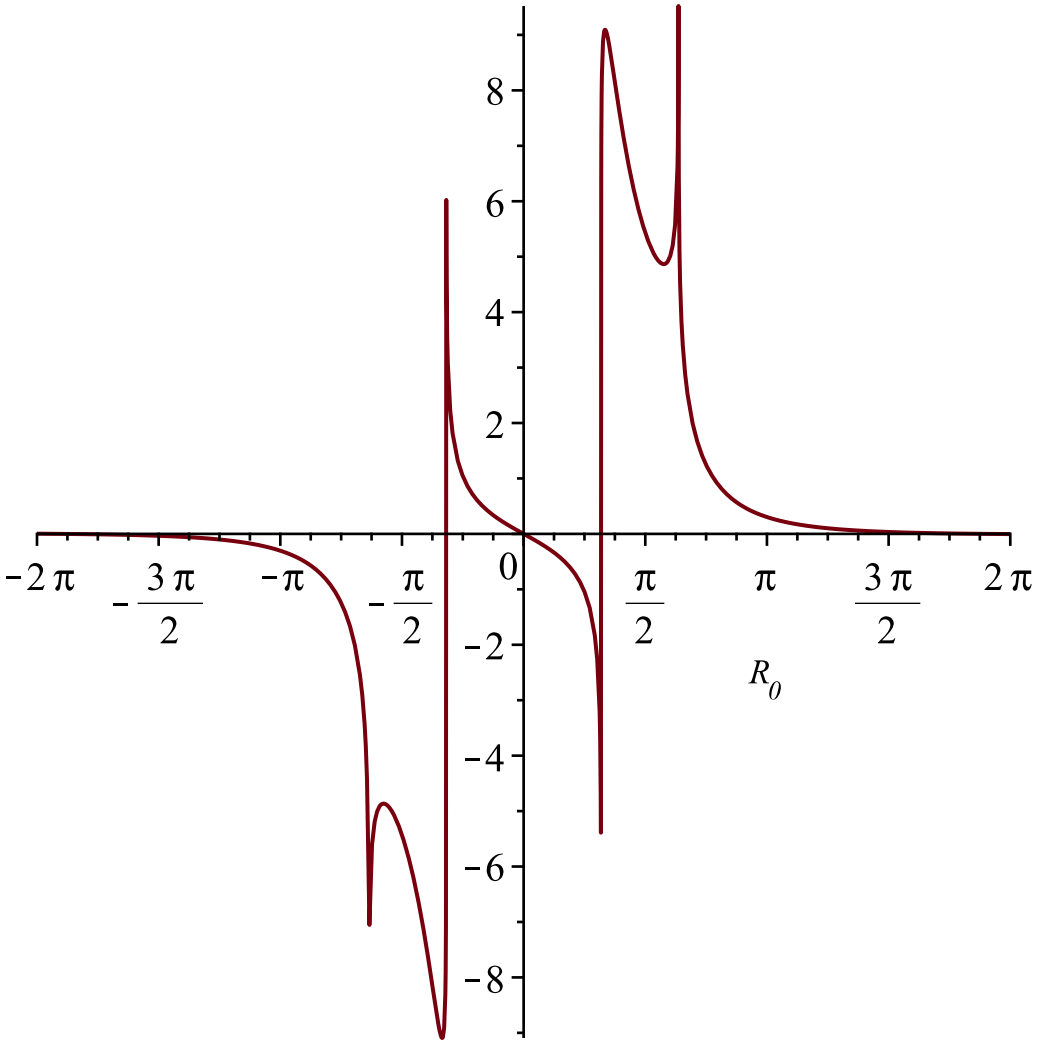
$with(plots) : plot\left(E_{lw}\left(1, 0, 0, 3, 1, 2, R_0\right), R_0\right)$



$with(plots) : plot(E_{lw}(1, 0, 1, 3, 1, 2, R_0), R_0)$

Warning, computation interrupted

$with(plots) : plot(E_{lw}(1, 0.5, 1, 3, 1, 2, R_0), R_0)$


8
6
4
2
0
-2
-4
-6
-8
-2 π
-3 π/2
-π
-π/2
π/2
π
3 π/2
2 π
R0