$restart : with(Physics[Vectors]) : Setup(mathematicalnotation = true) :$
Лагранжиан системы двух зарядов с точностью до второго порядка (анти - Дарвиновский)

координаты зарядов

$\overrightarrow{ra} := \overrightarrow{r_a}(t) :$
$\overrightarrow{rb} := \overrightarrow{r_b}(t) :$
Вектор от заряда источника поля к пробному заряду

$R_{ba\_} := \overrightarrow{ra} - \overrightarrow{rb} :$
его длина

$R_{ba} := \| R_{ba\_} \| :$
вектор направления от заряда источника поля к пробному заряду

$n_{ba\_} := \frac{R_{ba\_}}{R_{ba}} :$
скалярный потенциал с точностью до второго порядка

$$\varphi_0 := \frac{e_b}{R_{ba}} :$$

$$\varphi_2 := \frac{e_b}{2 \cdot c^2} \cdot \frac{\partial^2}{\partial t^2} R_{ba}$$

$$\varphi_2 := \frac{1}{2\,c^2}\left(e_b\left(-\frac{\left(\left(\dot{\overrightarrow{r_a}}(t)-\dot{\overrightarrow{r_b}}(t)\right)\cdot\left(\overrightarrow{r_a}(t)-\overrightarrow{r_b}(t)\right)\right)^2}{\left\|\overrightarrow{r_a}(t)-\overrightarrow{r_b}(t)\right\|^3} + \frac{\left(\ddot{\overrightarrow{r_a}}(t)-\ddot{\overrightarrow{r_b}}(t)\right)\cdot\left(\overrightarrow{r_a}(t)-\overrightarrow{r_b}(t)\right)+\left\|\dot{\overrightarrow{r_a}}(t)-\dot{\overrightarrow{r_b}}(t)\right\|^2}{\left\|\overrightarrow{r_a}(t)-\overrightarrow{r_b}(t)\right\|}\right)\right) \quad (1)$$

$$A_{2\_} := \frac{e_b}{R_{ba}} \cdot \frac{v_{b\_}}{c}$$

$$\overrightarrow{A_2} := \frac{e_b\,\overrightarrow{v_b}}{\left\|\overrightarrow{r_a}(t)-\overrightarrow{r_b}(t)\right\|\,c} \quad (2)$$

$$\varphi_3 := -\frac{e_b}{6 \cdot c^3} \cdot \frac{\partial^3}{\partial t^3} R_{ba} :$$

$$A_{3\_} := -e_b \cdot \frac{\ddot{\overrightarrow{r_b}}(t)}{c^2} :$$

$calcF := \mathbf{proc}(L)\ \mathbf{description}$ "calc f from Lagrangian";
$L_a := subs\left(\ddot{\overrightarrow{r_a}}(t) = a_{a\_}, \ddot{\overrightarrow{r_b}}(t) = a_{b\_}, \dot{\overrightarrow{r_a}}(t) = v_{a\_}, \dot{\overrightarrow{r_b}}(t) = v_{b\_}, \overrightarrow{r_a}(t) = r_{a\_}, \overrightarrow{r_b}(t) = r_{b\_}, L\right);$

$$P\_ := \frac{\partial}{\partial v_{a\_}} L_a;$$

$$Pt\_ := subs\left(r_{a\_} = \vec{r}_a(t), r_{b\_} = \vec{r}_b(t), v_{a\_} = \dot{\vec{r}}_a(t), v_{b\_} = \dot{\vec{r}}_b(t), a_{a\_} = \ddot{\vec{r}}_a(t), a_{b\_} = \ddot{\vec{r}}_b(t), P\_\right);$$

$$F\_ := \frac{\partial}{\partial r_{a\_}} L_a;$$

$$Ft\_ := subs\left(r_{a\_} = \vec{r}_a(t), r_{b\_} = \vec{r}_b(t), v_{a\_} = \dot{\vec{r}}_a(t), v_{b\_} = \dot{\vec{r}}_b(t), a_{a\_} = \ddot{\vec{r}}_a(t), a_{b\_} = \ddot{\vec{r}}_b(t), F\_\right);$$

$$f := Ft\_ - \frac{\mathrm{d}}{\mathrm{d}t} Pt\_;$$

$$ff := subs\left(\ddot{\vec{r}}_a(t) = a_{a\_}, \ddot{\vec{r}}_b(t) = a_{b\_}, \dot{\vec{r}}_a(t) = v_{a\_}, \dot{\vec{r}}_b(t) = v_{b\_}, \vec{r}_a(t) = r_{a\_}, \vec{r}_b(t) = r_{b\_}, f\right);$$

end proc:

$$calcF\left(-v_a \cdot \varphi_0\right)$$

$$\frac{v_a e_b \vec{r}_a}{\left\|\vec{r}_a - \vec{r}_b\right\|^3} - \frac{v_a e_b \vec{r}_b}{\left\|\vec{r}_a - \vec{r}_b\right\|^3} \tag{3}$$

$$subs\left(v_{a\_} = 0, a_{a\_} = 0, calcF\left(-e_a \cdot \varphi_2\right)\right)$$

$$\frac{3 e_a e_b \left(\left(-\vec{v}_b\right) \cdot \left(\vec{r}_a - \vec{r}_b\right)\right)^2 \vec{r}_a}{2 c^2 \left\|\vec{r}_a - \vec{r}_b\right\|^5} - \frac{3 e_a e_b \left(\left(-\vec{v}_b\right) \cdot \left(\vec{r}_a - \vec{r}_b\right)\right)^2 \vec{r}_b}{2 c^2 \left\|\vec{r}_a - \vec{r}_b\right\|^5}$$
$$+ \frac{e_a e_b \left(\left(-\vec{v}_b\right) \cdot \left(\vec{r}_a - \vec{r}_b\right)\right) \vec{v}_b}{c^2 \left\|\vec{r}_a - \vec{r}_b\right\|^3} - \frac{e_a e_b \vec{r}_a \left(\left(-\vec{a}_b\right) \cdot \left(\vec{r}_a - \vec{r}_b\right)\right)}{2 c^2 \left\|\vec{r}_a - \vec{r}_b\right\|^3}$$
$$+ \frac{e_a e_b \vec{r}_b \left(\left(-\vec{a}_b\right) \cdot \left(\vec{r}_a - \vec{r}_b\right)\right)}{2 c^2 \left\|\vec{r}_a - \vec{r}_b\right\|^3} - \frac{e_a e_b \vec{r}_a \left\|-\vec{v}_b\right\|^2}{2 c^2 \left\|\vec{r}_a - \vec{r}_b\right\|^3} + \frac{e_a e_b \vec{r}_b \left\|-\vec{v}_b\right\|^2}{2 c^2 \left\|\vec{r}_a - \vec{r}_b\right\|^3} - \frac{e_a e_b \vec{a}_b}{2 c^2 \left\|\vec{r}_a - \vec{r}_b\right\|} \tag{4}$$

$$calcF\left(-e_a \cdot \varphi_2\right)$$

$$\frac{3 e_a e_b \left(\left(\vec{v}_a - \vec{v}_b\right) \cdot \left(\vec{r}_a - \vec{r}_b\right)\right)^2 \vec{r}_a}{2 c^2 \left\|\vec{r}_a - \vec{r}_b\right\|^5} - \frac{3 e_a e_b \left(\left(\vec{v}_a - \vec{v}_b\right) \cdot \left(\vec{r}_a - \vec{r}_b\right)\right)^2 \vec{r}_b}{2 c^2 \left\|\vec{r}_a - \vec{r}_b\right\|^5}$$
$$- \frac{e_a e_b \left(\left(\vec{v}_a - \vec{v}_b\right) \cdot \left(\vec{r}_a - \vec{r}_b\right)\right) \vec{v}_a}{c^2 \left\|\vec{r}_a - \vec{r}_b\right\|^3} + \frac{e_a e_b \left(\left(\vec{v}_a - \vec{v}_b\right) \cdot \left(\vec{r}_a - \vec{r}_b\right)\right) \vec{v}_b}{c^2 \left\|\vec{r}_a - \vec{r}_b\right\|^3}$$
$$- \frac{e_a e_b \vec{r}_a \left(\left(\vec{a}_a - \vec{a}_b\right) \cdot \left(\vec{r}_a - \vec{r}_b\right)\right)}{2 c^2 \left\|\vec{r}_a - \vec{r}_b\right\|^3} + \frac{e_a e_b \vec{r}_b \left(\left(\vec{a}_a - \vec{a}_b\right) \cdot \left(\vec{r}_a - \vec{r}_b\right)\right)}{2 c^2 \left\|\vec{r}_a - \vec{r}_b\right\|^3}$$
$$- \frac{e_a e_b \vec{r}_a \left\|\vec{v}_a - \vec{v}_b\right\|^2}{2 c^2 \left\|\vec{r}_a - \vec{r}_b\right\|^3} + \frac{e_a e_b \vec{r}_b \left\|\vec{v}_a - \vec{v}_b\right\|^2}{2 c^2 \left\|\vec{r}_a - \vec{r}_b\right\|^3} + \frac{e_a e_b \vec{a}_a}{2 c^2 \left\|\vec{r}_a - \vec{r}_b\right\|} - \frac{e_a e_b \vec{a}_b}{2 c^2 \left\|\vec{r}_a - \vec{r}_b\right\|} \tag{5}$$

$$calcF\left(\frac{e_a}{c}\cdot v_{a\_}\bullet A_{2\_}\right)$$

$$-\frac{e_b\,e_a\,(\vec{v}_a\bullet\vec{v}_b)\,\vec{r}_a}{\|\vec{r}_a-\vec{r}_b\|^3\,c^2}+\frac{e_b\,e_a\,(\vec{v}_a\bullet\vec{v}_b)\,\vec{r}_b}{\|\vec{r}_a-\vec{r}_b\|^3\,c^2}+\frac{e_a\,e_b\,((\vec{v}_a-\vec{v}_b)\cdot(\vec{r}_a-\vec{r}_b))\,\vec{v}_b}{c^2\,\|\vec{r}_a-\vec{r}_b\|^3}-\frac{e_a\,e_b\,\vec{a}_b}{c^2\,\|\vec{r}_a-\vec{r}_b\|} \tag{6}$$

$$calcF\left(\frac{e_a}{c}\cdot v_{a\_}\bullet A_{2\_}-e_a\cdot\varphi_2\right)$$

$$\begin{aligned}&-\frac{e_b\,e_a\,(\vec{v}_a\bullet\vec{v}_b)\,\vec{r}_a}{\|\vec{r}_a-\vec{r}_b\|^3\,c^2}+\frac{e_b\,e_a\,(\vec{v}_a\bullet\vec{v}_b)\,\vec{r}_b}{\|\vec{r}_a-\vec{r}_b\|^3\,c^2}+\frac{3\,e_a\,e_b\,((\vec{v}_a-\vec{v}_b)\cdot(\vec{r}_a-\vec{r}_b))^2\,\vec{r}_a}{2\,c^2\,\|\vec{r}_a-\vec{r}_b\|^5}\\&-\frac{3\,e_a\,e_b\,((\vec{v}_a-\vec{v}_b)\cdot(\vec{r}_a-\vec{r}_b))^2\,\vec{r}_b}{2\,c^2\,\|\vec{r}_a-\vec{r}_b\|^5}-\frac{e_a\,e_b\,((\vec{v}_a-\vec{v}_b)\cdot(\vec{r}_a-\vec{r}_b))\,\vec{v}_a}{c^2\,\|\vec{r}_a-\vec{r}_b\|^3}\\&+\frac{2\,e_a\,e_b\,((\vec{v}_a-\vec{v}_b)\cdot(\vec{r}_a-\vec{r}_b))\,\vec{v}_b}{c^2\,\|\vec{r}_a-\vec{r}_b\|^3}-\frac{e_a\,e_b\,\vec{r}_a\,((\vec{a}_a-\vec{a}_b)\cdot(\vec{r}_a-\vec{r}_b))}{2\,c^2\,\|\vec{r}_a-\vec{r}_b\|^3}\\&+\frac{e_a\,e_b\,\vec{r}_b\,((\vec{a}_a-\vec{a}_b)\cdot(\vec{r}_a-\vec{r}_b))}{2\,c^2\,\|\vec{r}_a-\vec{r}_b\|^3}-\frac{e_a\,e_b\,\vec{r}_a\,\|\vec{v}_a-\vec{v}_b\|^2}{2\,c^2\,\|\vec{r}_a-\vec{r}_b\|^3}+\frac{e_a\,e_b\,\vec{r}_b\,\|\vec{v}_a-\vec{v}_b\|^2}{2\,c^2\,\|\vec{r}_a-\vec{r}_b\|^3}\\&+\frac{e_a\,e_b\,\vec{a}_a}{2\,c^2\,\|\vec{r}_a-\vec{r}_b\|}-\frac{3\,e_a\,e_b\,\vec{a}_b}{2\,c^2\,\|\vec{r}_a-\vec{r}_b\|}\end{aligned} \tag{7}$$

$$subs\left(v_{a\_}=0,\,a_{a\_}=0,\,calcF\left(\frac{e_a}{c}\cdot v_{a\_}\bullet A_{2\_}-e_a\cdot\varphi_2\right)\right)$$

$$\begin{aligned}&-\frac{e_b\,e_a\,(0\bullet\vec{v}_b)\,\vec{r}_a}{\|\vec{r}_a-\vec{r}_b\|^3\,c^2}+\frac{e_b\,e_a\,(0\bullet\vec{v}_b)\,\vec{r}_b}{\|\vec{r}_a-\vec{r}_b\|^3\,c^2}+\frac{3\,e_a\,e_b\,((-\vec{v}_b)\cdot(\vec{r}_a-\vec{r}_b))^2\,\vec{r}_a}{2\,c^2\,\|\vec{r}_a-\vec{r}_b\|^5}\\&-\frac{3\,e_a\,e_b\,((-\vec{v}_b)\cdot(\vec{r}_a-\vec{r}_b))^2\,\vec{r}_b}{2\,c^2\,\|\vec{r}_a-\vec{r}_b\|^5}+\frac{2\,e_a\,e_b\,((-\vec{v}_b)\cdot(\vec{r}_a-\vec{r}_b))\,\vec{v}_b}{c^2\,\|\vec{r}_a-\vec{r}_b\|^3}\\&-\frac{e_a\,e_b\,\vec{r}_a\,((-\vec{a}_b)\cdot(\vec{r}_a-\vec{r}_b))}{2\,c^2\,\|\vec{r}_a-\vec{r}_b\|^3}+\frac{e_a\,e_b\,\vec{r}_b\,((-\vec{a}_b)\cdot(\vec{r}_a-\vec{r}_b))}{2\,c^2\,\|\vec{r}_a-\vec{r}_b\|^3}-\frac{e_a\,e_b\,\vec{r}_a\,\|-\vec{v}_b\|^2}{2\,c^2\,\|\vec{r}_a-\vec{r}_b\|^3}\\&+\frac{e_a\,e_b\,\vec{r}_b\,\|-\vec{v}_b\|^2}{2\,c^2\,\|\vec{r}_a-\vec{r}_b\|^3}-\frac{3\,e_a\,e_b\,\vec{a}_b}{2\,c^2\,\|\vec{r}_a-\vec{r}_b\|}\end{aligned} \tag{8}$$

$$calcF\left(-e_a \cdot \varphi_3\right)$$

$$\frac{5\, e_a\, e_b \left(\left(\vec{v}_a - \vec{v}_b\right) \cdot \left(\vec{r}_a - \vec{r}_b\right)\right)^3 \vec{r}_a}{c^3 \left\|\vec{r}_a - \vec{r}_b\right\|^7} - \frac{5\, e_a\, e_b \left(\left(\vec{v}_a - \vec{v}_b\right) \cdot \left(\vec{r}_a - \vec{r}_b\right)\right)^3 \vec{r}_b}{c^3 \left\|\vec{r}_a - \vec{r}_b\right\|^7}$$

$$- \frac{3\, e_a\, e_b \left(\left(\vec{v}_a - \vec{v}_b\right) \cdot \left(\vec{r}_a - \vec{r}_b\right)\right)^2 \vec{v}_a}{c^3 \left\|\vec{r}_a - \vec{r}_b\right\|^5} + \frac{3\, e_a\, e_b \left(\left(\vec{v}_a - \vec{v}_b\right) \cdot \left(\vec{r}_a - \vec{r}_b\right)\right)^2 \vec{v}_b}{c^3 \left\|\vec{r}_a - \vec{r}_b\right\|^5}$$

$$+ \frac{e_a\, e_b\, \vec{v}_a \left\|\vec{v}_a - \vec{v}_b\right\|^2}{c^3 \left\|\vec{r}_a - \vec{r}_b\right\|^3} - \frac{e_a\, e_b\, \vec{v}_b \left\|\vec{v}_a - \vec{v}_b\right\|^2}{c^3 \left\|\vec{r}_a - \vec{r}_b\right\|^3} + \frac{e_a\, e_b\, \vec{v}_a \left(\left(\vec{a}_a - \vec{a}_b\right) \cdot \left(\vec{r}_a - \vec{r}_b\right)\right)}{c^3 \left\|\vec{r}_a - \vec{r}_b\right\|^3}$$

$$- \frac{e_a\, e_b\, \vec{v}_b \left(\left(\vec{a}_a - \vec{a}_b\right) \cdot \left(\vec{r}_a - \vec{r}_b\right)\right)}{c^3 \left\|\vec{r}_a - \vec{r}_b\right\|^3} + \frac{e_a\, e_b \left(\left(\vec{v}_a - \vec{v}_b\right) \cdot \left(\vec{r}_a - \vec{r}_b\right)\right) \vec{a}_a}{c^3 \left\|\vec{r}_a - \vec{r}_b\right\|^3}$$

$$- \frac{e_a\, e_b \left(\left(\vec{v}_a - \vec{v}_b\right) \cdot \left(\vec{r}_a - \vec{r}_b\right)\right) \vec{a}_b}{c^3 \left\|\vec{r}_a - \vec{r}_b\right\|^3} - \frac{e_a\, e_b \left(\left(\frac{\partial}{\partial t}\, \vec{a}_a - \left(\frac{\partial}{\partial t}\, \vec{a}_b\right)\right) \cdot \left(\vec{r}_a - \vec{r}_b\right)\right) \vec{r}_b}{3\, c^3 \left\|\vec{r}_a - \vec{r}_b\right\|^3}$$

$$- \frac{e_a\, e_b \left(\left(\vec{a}_a - \vec{a}_b\right) \cdot \left(\vec{v}_a - \vec{v}_b\right)\right) \vec{r}_b}{c^3 \left\|\vec{r}_a - \vec{r}_b\right\|^3} + \frac{e_a\, e_b \left(\left(\frac{\partial}{\partial t}\, \vec{a}_a - \left(\frac{\partial}{\partial t}\, \vec{a}_b\right)\right) \cdot \left(\vec{r}_a - \vec{r}_b\right)\right) \vec{r}_a}{3\, c^3 \left\|\vec{r}_a - \vec{r}_b\right\|^3}$$

$$+ \frac{e_a\, e_b \left(\left(\vec{a}_a - \vec{a}_b\right) \cdot \left(\vec{v}_a - \vec{v}_b\right)\right) \vec{r}_a}{c^3 \left\|\vec{r}_a - \vec{r}_b\right\|^3} - \frac{e_a\, e_b \left(\frac{\partial}{\partial t}\, \vec{a}_a\right)}{3\, c^3 \left\|\vec{r}_a - \vec{r}_b\right\|} + \frac{e_a\, e_b \left(\frac{\partial}{\partial t}\, \vec{a}_b\right)}{3\, c^3 \left\|\vec{r}_a - \vec{r}_b\right\|}$$

$$+ \frac{3\, e_a\, e_b \left(\left(\vec{v}_a - \vec{v}_b\right) \cdot \left(\vec{r}_a - \vec{r}_b\right)\right) \vec{r}_b \left\|\vec{v}_a - \vec{v}_b\right\|^2}{c^3 \left\|\vec{r}_a - \vec{r}_b\right\|^5}$$

$$- \frac{3\, e_a\, e_b \left(\left(\vec{v}_a - \vec{v}_b\right) \cdot \left(\vec{r}_a - \vec{r}_b\right)\right) \vec{r}_a \left\|\vec{v}_a - \vec{v}_b\right\|^2}{c^3 \left\|\vec{r}_a - \vec{r}_b\right\|^5}$$

$$+ \frac{3\, e_a\, e_b \left(\left(\vec{v}_a - \vec{v}_b\right) \cdot \left(\vec{r}_a - \vec{r}_b\right)\right) \vec{r}_b \left(\left(\vec{a}_a - \vec{a}_b\right) \cdot \left(\vec{r}_a - \vec{r}_b\right)\right)}{c^3 \left\|\vec{r}_a - \vec{r}_b\right\|^5}$$

$$- \frac{3\, e_a\, e_b \left(\left(\vec{v}_a - \vec{v}_b\right) \cdot \left(\vec{r}_a - \vec{r}_b\right)\right) \vec{r}_a \left(\left(\vec{a}_a - \vec{a}_b\right) \cdot \left(\vec{r}_a - \vec{r}_b\right)\right)}{c^3 \left\|\vec{r}_a - \vec{r}_b\right\|^5}$$

(9)

$$calcF\left(\frac{e_a}{c} \cdot v_{a\_} \bullet A_{3\_}\right)$$

$$0$$

(10)