

restart : clear :

$R$

$R_0$

$$\sigma(q, r) := \frac{q}{4 \cdot \pi \cdot r^2}$$

$$(q, r) \rightarrow \frac{1}{4} \frac{q}{\pi r^2} \quad (1)$$

$$\frac{d}{d\theta} S := 2 \cdot \pi \cdot r^2 \cdot \sin(\theta)$$

$$\begin{aligned} \varphi_R(q, r, R_0) &:= \int_0^\pi \frac{2 \cdot \pi \cdot r^2 \cdot \sin(\theta) \cdot \sigma(q, r)}{\sqrt{(R_0)^2 - 2 \cdot R_0 \cdot r \cdot \cos(\theta) + (r)^2}} d\theta \\ (q, r, R_0) &\rightarrow \int_0^\pi \frac{2 \pi r^2 \sin(\theta) \sigma(q, r)}{\sqrt{R_0^2 - 2 R_0 r \cos(\theta) + r^2}} d\theta \end{aligned} \quad (2)$$

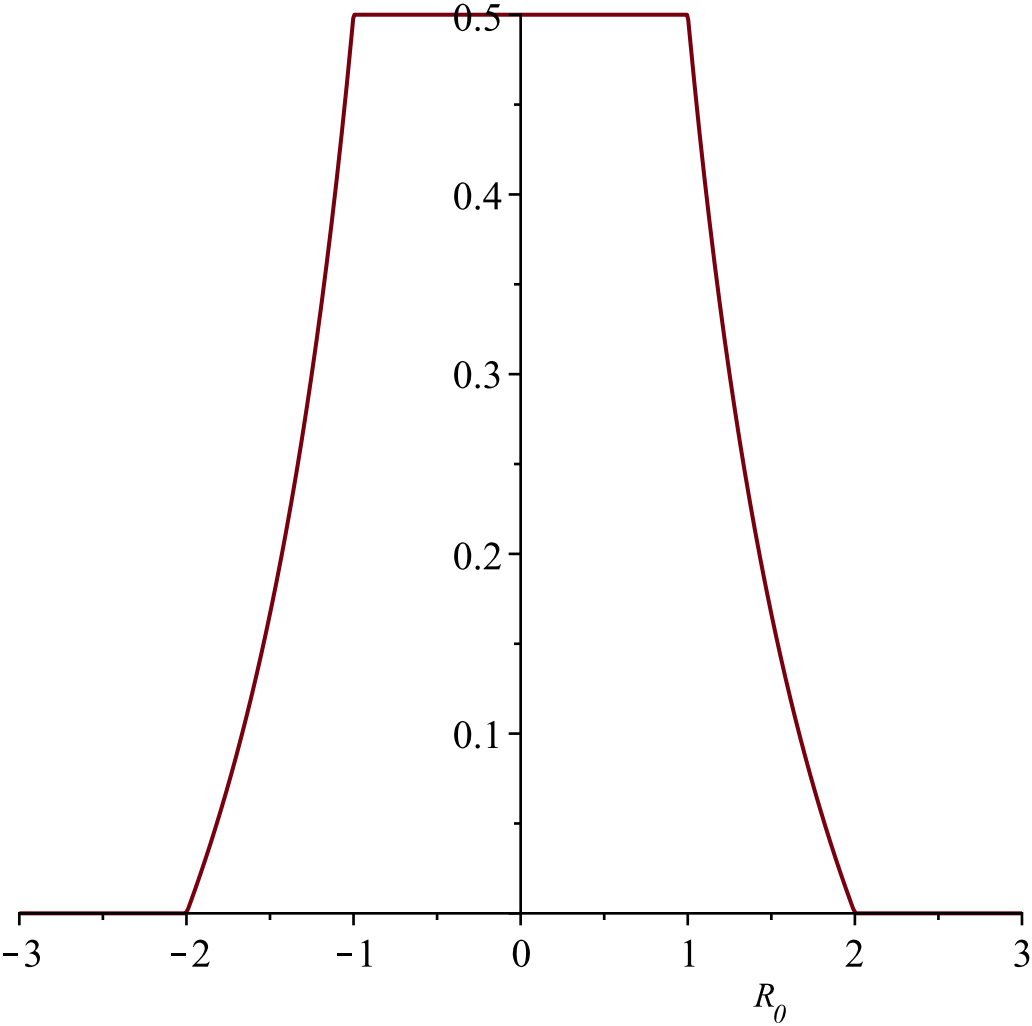
$$\begin{aligned} \varphi_R(q, r, R_0) \\ \int_0^\pi \frac{1}{2} \frac{\sin(\theta) q}{\sqrt{R_0^2 - 2 R_0 r \cos(\theta) + r^2}} d\theta \end{aligned} \quad (3)$$

$$\begin{aligned} \varphi_R(q, 1, R_0) + \varphi_R(-q, 2, R_0) \\ \int_0^\pi \frac{1}{2} \frac{\sin(\theta) q}{\sqrt{R_0^2 - 2 R_0 \cos(\theta) + 1}} d\theta + \int_0^\pi \left( -\frac{1}{2} \frac{\sin(\theta) q}{\sqrt{R_0^2 - 4 R_0 \cos(\theta) + 4}} \right) d\theta \end{aligned} \quad (4)$$

$$\begin{aligned} evalf(\varphi_R(1, 1, R_0) + \varphi_R(-1, 2, R_0)) \\ \int_{0.}^{3.141592654} \frac{0.5000000000 \sin(\theta)}{\sqrt{R_0^2 - 2. R_0 \cos(\theta) + 1.}} d\theta + \int_{0.}^{3.141592654} \left( -\frac{0.5000000000 \sin(\theta)}{\sqrt{R_0^2 - 4. R_0 \cos(\theta) + 4.}} \right) d\theta \end{aligned} \quad (5)$$

$$2 - 1$$

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with(plots):plot(φR(1, 1, Rθ) + φR(-1, 2, Rθ), Rθ=-3..3)
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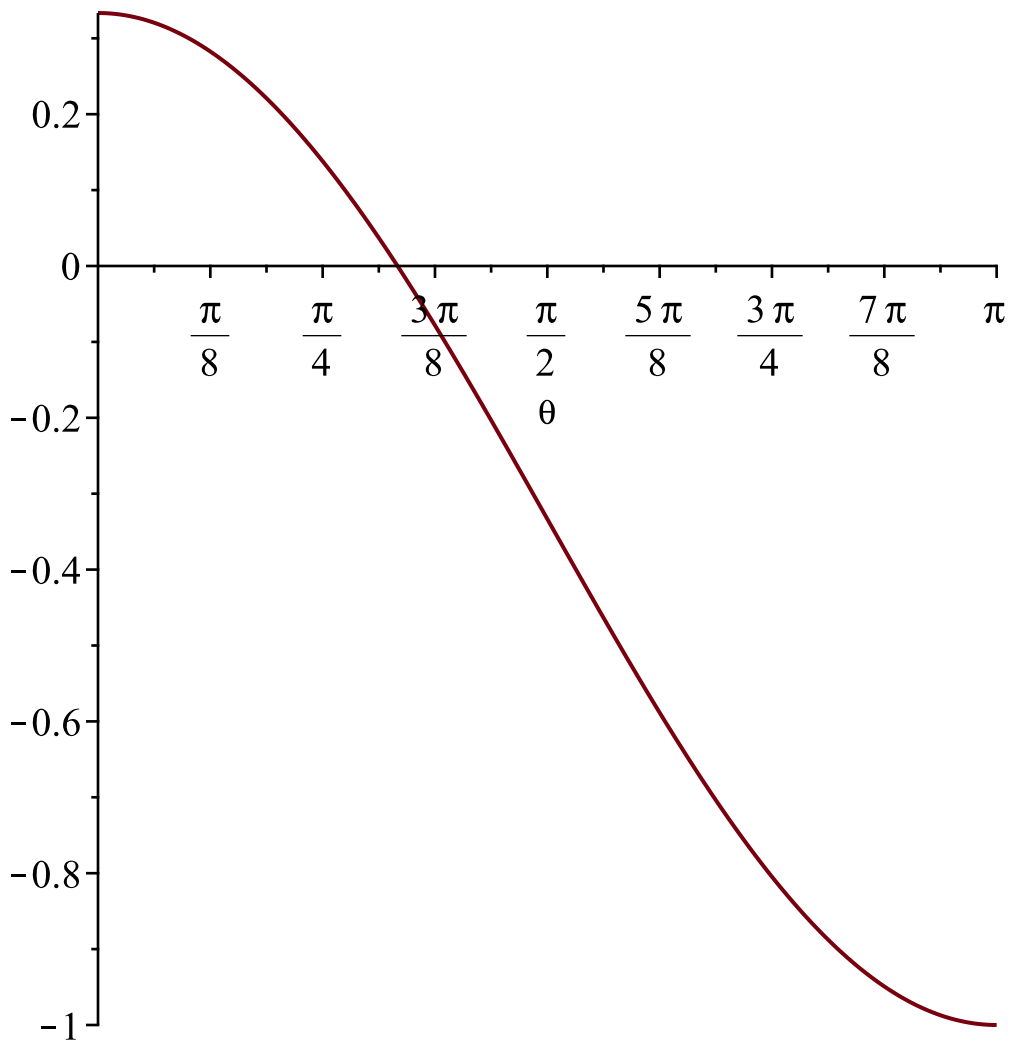


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$$vr(v, c, r, R_{\theta}, \theta) := \frac{v}{c} \cdot (R_{\theta} \cdot \cos(\theta) - r)$$

$$(v, c, r, R_{\theta}, \theta) \rightarrow \frac{v \left( R_{\theta} \cos(\theta) - r \right)}{c} \tag{6}$$

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plot(vr(1, 3, 1, 2, θ), θ=0..π)
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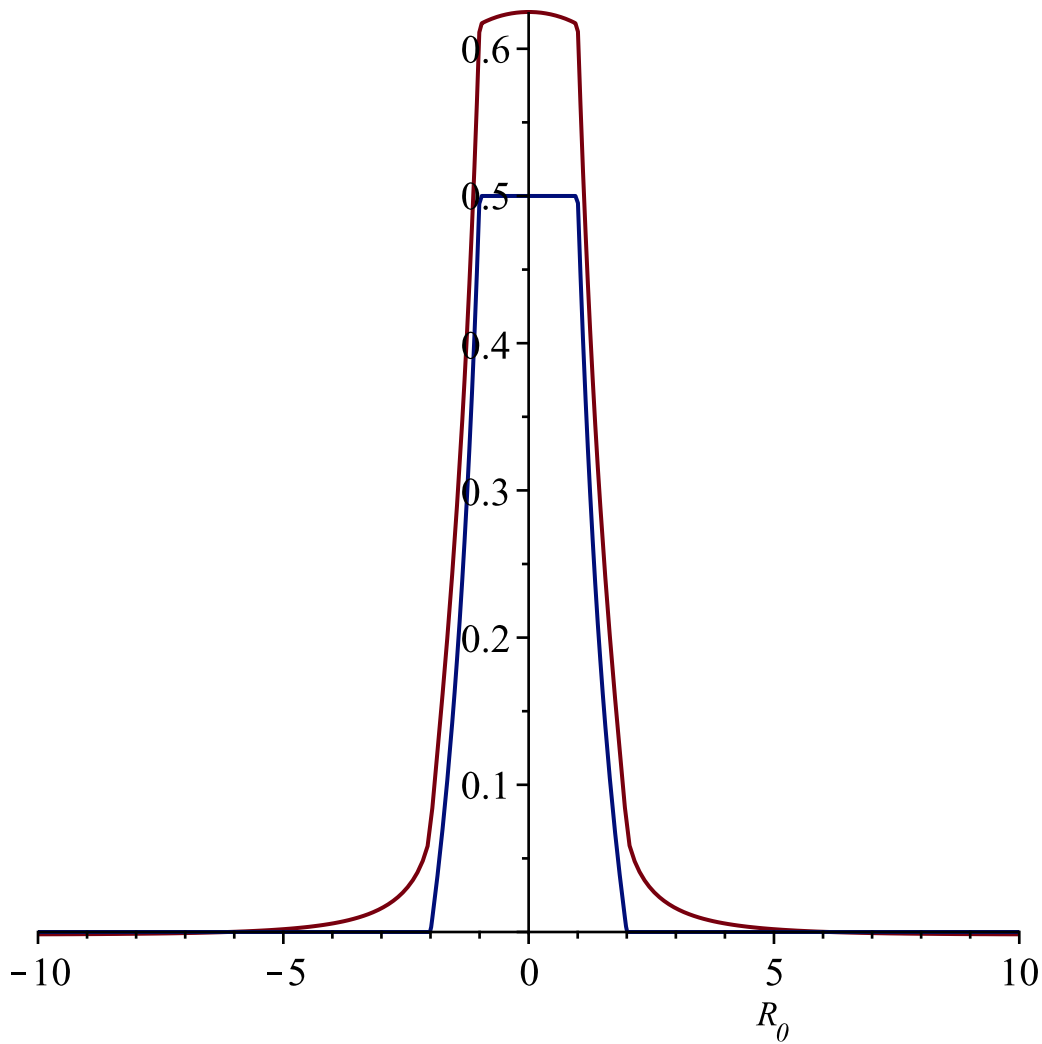
$$R_0 \; , \qquad R \qquad v \; .$$

$$\begin{aligned} \varphi_{hw}(q,v,c,r,R_0) &:= \int_0^\pi \frac{2\cdot \pi \cdot r^2 \cdot \sin(\theta) \cdot \sigma(q,r)}{\sqrt{(R_0)^2 - 2\cdot R_0 \cdot r \cdot \cos(\theta) + (r)^2} - \frac{v}{c} \cdot (R_0 \cdot \cos(\theta) - r)} \, \mathrm{d}\theta \\ (q,v,c,r,R_0) &\rightarrow \int_0^\pi \frac{2\,\pi\,r^2\sin(\theta)\,\sigma(q,r)}{\sqrt{R_0^2-2\,R_0\,r\cos(\theta)+r^2}-\frac{v\left(R_0\cos(\theta)-r\right)}{c}} \, \mathrm{d}\theta \\ \varphi_{hw}(q,v,c,r,R_0) & \end{aligned} \tag{7}$$

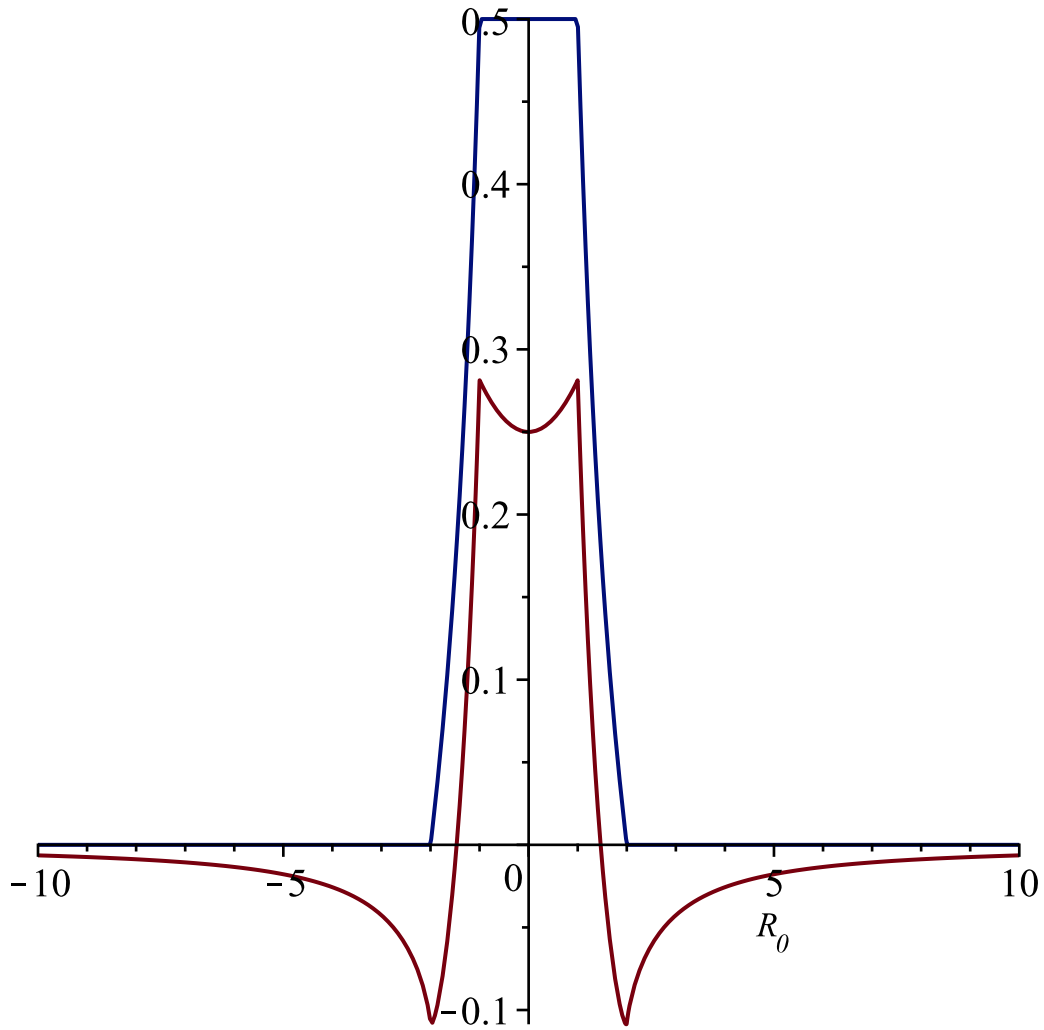
$$\int_0^{\pi} \frac{1}{2} \frac{\sin(\theta) \, q}{\sqrt{R_0^2 - 2 \, R_0 \, r \cos(\theta) + r^2} - \frac{v \, (R_0 \cos(\theta) - r)}{c}} \, \mathrm{d}\theta \tag{8}$$

$c := 3 :$ 
 $R_+ := 1 :$ 
 $R_- := 2 :$

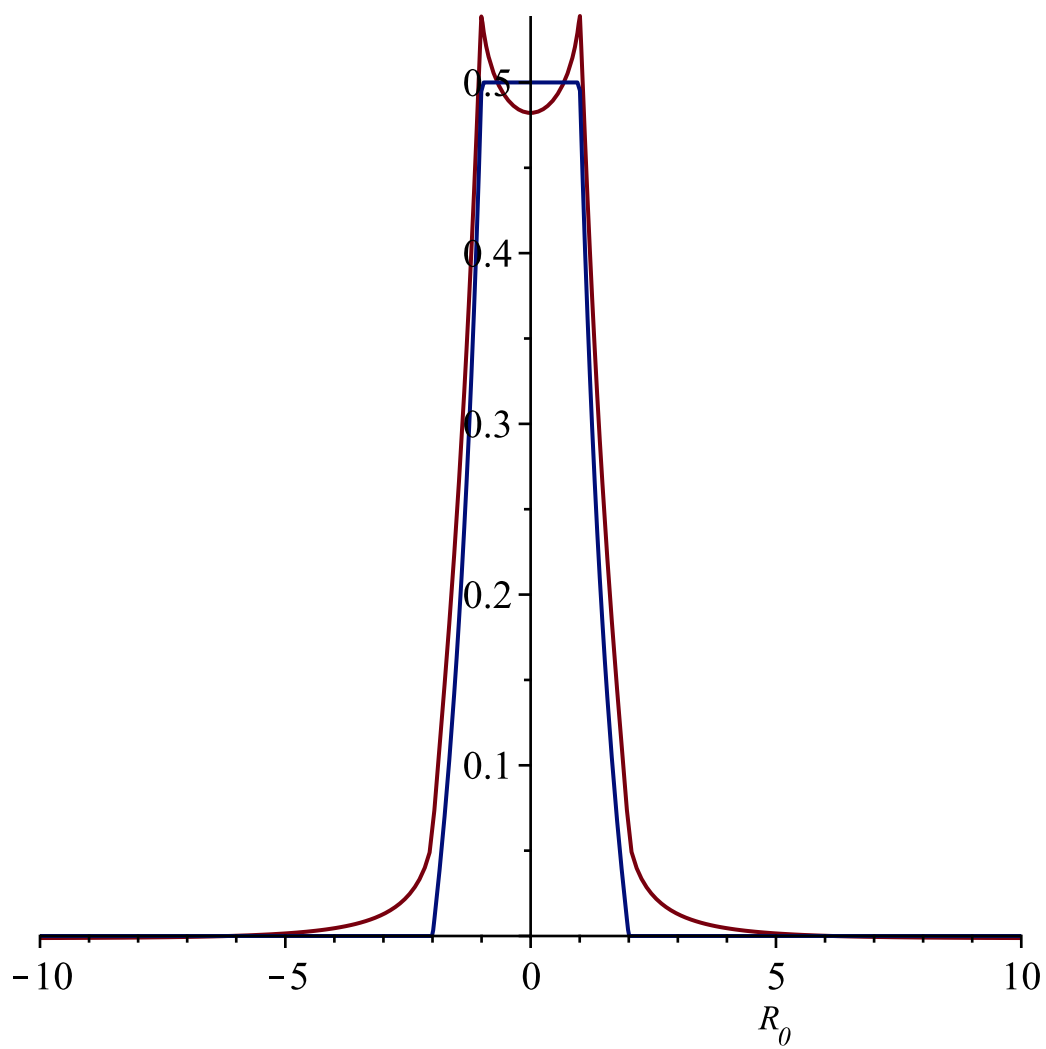
$with(plots) : plot\Big(\Big[\varphi_R(1, R_+, R_0) + \varphi_{lw}(-1, 1, c, R_-, R_0), \varphi_R(1, R_+, R_0) + \varphi_R(-1, R_-, R_0)\Big], R_0 =$   
 $-10..10)$



*with(plots):plot([ $\varphi_R(1, R_+, R_0) + \varphi_{lw}(-1, -1, c, R_-, R_0)$ ,  $\varphi_R(1, R_+, R_0) + \varphi_R(-1, R_-, R_0)$ ],  $R_0 = -10..10$ )*

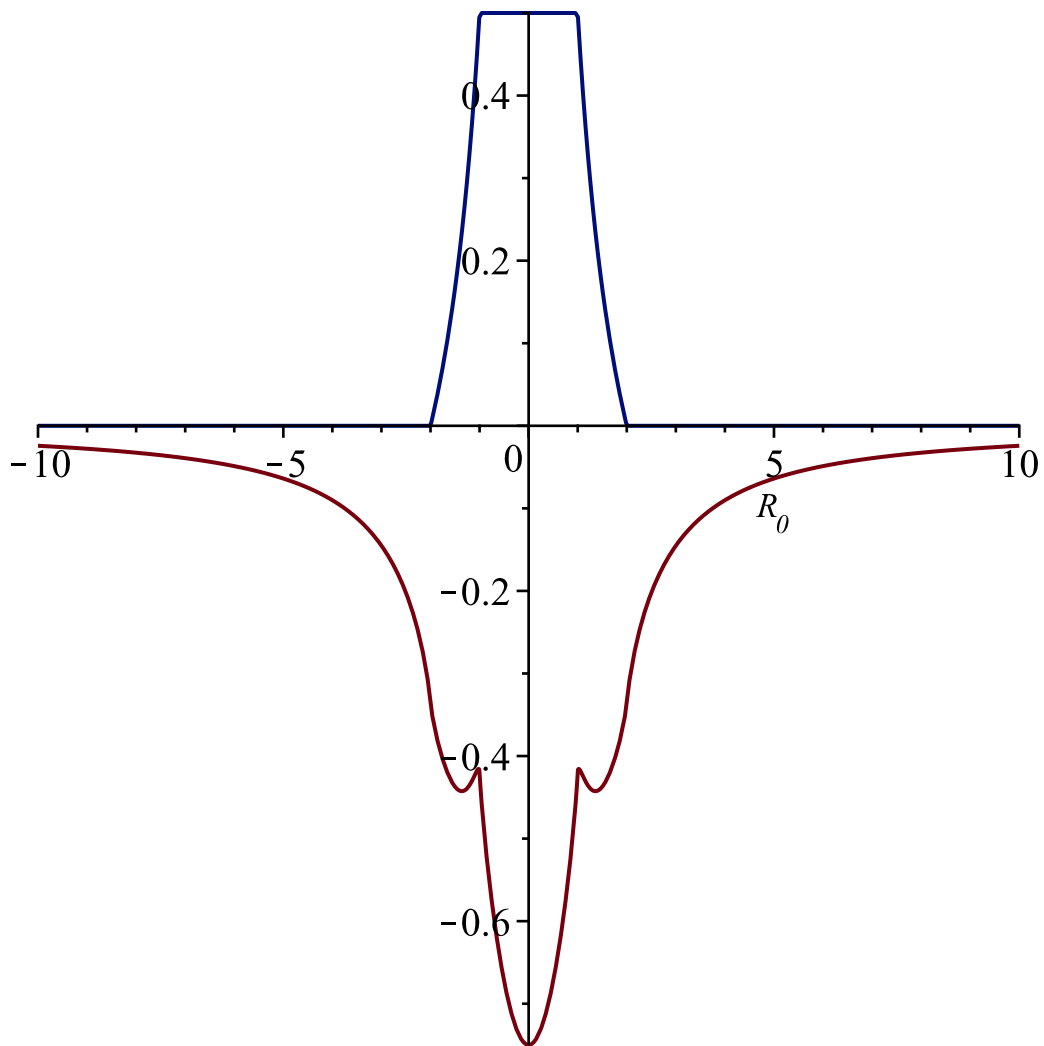


*with(plots):plot([ $\varphi_{lw}(1, 0.5, c, R_+, R_0) + \varphi_{lw}(-1, 1, c, R_-, R_0)$ ,  $\varphi_R(1, R_+, R_0) + \varphi_R(-1, R_-, R_0)$ ],  $R_0 = -10..10$ )*



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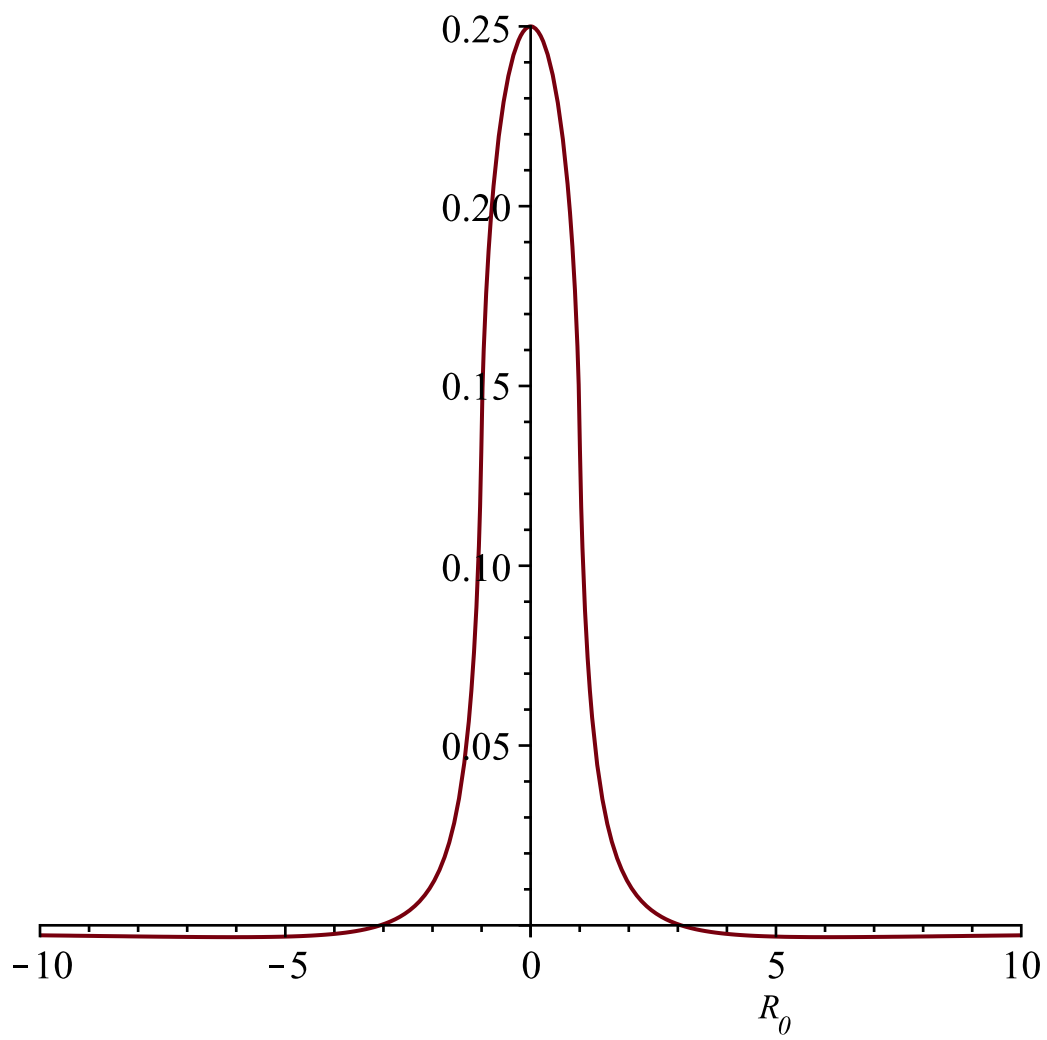
with(*plots*) : *plot*( [  $\phi_{lw}(1, 1, c, R_+, R_0) + \phi_{lw}(-1, -2, c, R_-, R_0)$ ,  $\phi_R(1, R_+, R_0) + \phi_R(-1, R_-, R_0)$  ],  
 $R_0 = -10..10$ )



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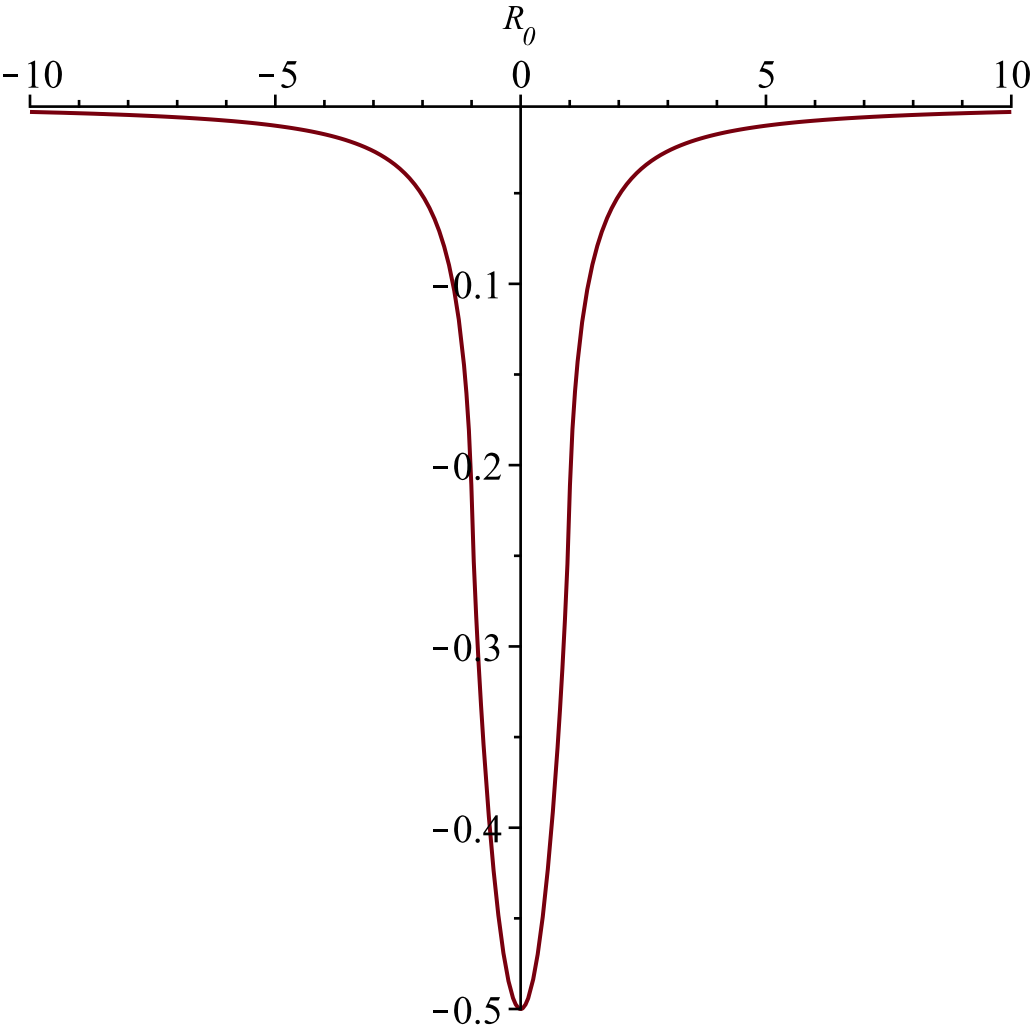
$$\begin{aligned}
 d\varphi_{lw}(q, v, c, R, R_0) &:= \varphi_{lw}(q, v, c, R, R_0) + \varphi_{lw}(-q, 0, c, R, R_0) \\
 (q, v, c, R, R_0) &\rightarrow \varphi_{lw}(q, v, c, R, R_0) + \varphi_{lw}(-q, 0, c, R, R_0)
 \end{aligned}
 \tag{9}$$

with(*plots*) : *plot*( [ *dφ<sub>lw</sub>*( -1, 1, *c*, *R<sub>+</sub>*, *R<sub>0</sub>*) ], *R<sub>0</sub>* = -10 ..10 )



*with(plots) : plot( [  $d\phi_{lw}(-1, -1, c, R_+, R_0)$  ],  $R_0 = -10..10$  )*





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$$\begin{array}{l} v_{0r}(r_0) := v \\ v_{0r}(r_0) \\ r_0 \end{array}$$

$$a_r(t, r_{\mathcal{O}} \, v_{\mathcal{O}} \, a_0) := a_0$$

$$(t, r_{\mathcal{O}} \, v_{\mathcal{O}} \, a_0) \rightarrow a_0 \tag{10}$$

$$a_r(t, r_{\mathcal{O}} \, v_{\mathcal{O}} \, a_0)$$

$$a_\theta \tag{11}$$

$$r(t, r_0, v_0, a_0) := r_0 + v_0 \cdot t + \frac{a_0 \cdot t^2}{2}$$

$$(t, r_0, v_0, a_0) \rightarrow r_0 + v_0 t + \frac{1}{2} a_0 t^2 \tag{12}$$

$$r(t, r_0, v_0, a_0) = r_0 + v_0 t + \frac{1}{2} a_0 t^2 \quad (13)$$

$$\begin{aligned} v_r(t, r_{\mathcal{O}}, v_{\mathcal{O}}, a_0) &:= v_0 + a_0 \cdot t \\ (t, r_{\mathcal{O}}, v_{\mathcal{O}}, a_0) &\rightarrow v_0 + a_0 t \end{aligned} \tag{14}$$

$$v_r(t, r_{\theta}, v_{\theta}, a_{\theta})$$

$$a_{\theta} t + v_{\theta} \tag{15}$$

$$\text{evalf}\left(\left[ \text{subs}\left(q=-1, t=0, r_0=R_-, v_0=0.2, a_0=0.1, R_0=3, \theta=0, r\left(t_{zap}, r_{\theta}, v_{\theta}, a_0\right)\right)\right]\right) \\ \left[2. + 0.2 t_{zap} + 0.05000000000 t_{zap}^2\right] \quad (16)$$

$$\begin{aligned} t_{zap}(t, r_o, v_o, a_o, R_o, \theta) &:= solve(c^2 (t - t_{zap})^2 = R_o^2 - 2 \cdot R_o \cdot r(t_{zap}, r_o, v_o, a_o) \cdot \cos(\theta) + r(t_{zap}, r_o, v_o, \\ &a_o)^2, t_{zap}) \\ (t, r_o, v_o, a_o, R_o, \theta) &\rightarrow solve(c^2 (t - t_{zap})^2 = R_o^2 - 2 R_o r(t_{zap}, r_o, v_o, a_o) \cos(\theta) + r(t_{zap}, r_o, v_o, \\ &a_o)^2, t_{zap}) \end{aligned} \quad (17)$$

$$tzap(t, r_\theta, v_\theta, a_\theta, R_\theta, \theta)$$

$$\begin{aligned} evalf & \left( subs \left( q = -1, t = 0, r_0 = R_-, v_0 = 0, a_0 = 0, R_0 = 3, \right. \right. \\ & \left. \left[ subs \left( \theta = 0, tzap \left( t, r_{0^*} v_{0^*} a_{0^*} R_{0^*} \theta \right), \right. \right. \\ & \left. \left. subs \left( \theta = \frac{\pi}{4}, tzap \left( t, r_{0^*} v_{0^*} a_{0^*} R_{0^*} \theta \right), \right. \right. \right. \\ & \left. \left. \left. subs \left( \theta = \frac{2 \cdot \pi}{4}, tzap \left( t, r_{0^*} v_{0^*} a_{0^*} R_{0^*} \theta \right), \right. \right. \right. \right) \end{aligned}$$

$$\begin{aligned}
& \text{subs}\left(\theta = \frac{3 \cdot \pi}{4}, \text{tzap}\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), \\
& \text{subs}\left(\theta = \frac{4 \cdot \pi}{4}, \text{tzap}\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), \\
& \text{subs}\left(\theta = \frac{5 \cdot \pi}{4}, \text{tzap}\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), \\
& \text{subs}\left(\theta = \frac{6 \cdot \pi}{4}, \text{tzap}\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), \\
& \text{subs}\left(\theta = \frac{7 \cdot \pi}{4}, \text{tzap}\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), \\
& \text{subs}\left(\theta = \frac{8 \cdot \pi}{4}, \text{tzap}\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), \\
& \text{plot}\left(\text{subs}\left(q=-1, t=-5, r_0=R_-, v_0=0, a_0=0, R_0=3, \text{tzap}\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), \theta=0..\pi\right), \\
& \text{plot}\left(\text{subs}\left(q=-1, t=-1, r_0=R_-, v_0=0, a_0=0, R_0=3, \text{tzap}\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), \theta=0..\pi\right), \\
& \text{plot}\left(\text{subs}\left(q=-1, t=0, r_0=R_-, v_0=0, a_0=0, R_0=3, \text{tzap}\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), \theta=0..\pi\right), \\
& \text{plot}\left(\text{subs}\left(q=-1, t=1, r_0=R_-, v_0=0, a_0=0, R_0=3, \text{tzap}\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), \theta=0..\pi\right), \\
& \text{plot}\left(\text{subs}\left(q=-1, t=2, r_0=R_-, v_0=0, a_0=0, R_0=3, \text{tzap}\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), \theta=0..\pi\right), \\
& \text{plot}\left(\text{subs}\left(q=-1, t=2, r_0=R_-, v_0=0, a_0=0, \theta=\pi, \text{tzap}\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), R_0=3..6\right), \\
& \text{plot}\left(\text{subs}\left(q=-1, t=10, r_0=R_-, v_0=0, a_0=0, \theta=\pi, \text{tzap}\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), R_0=3..6\right) \\
& \left. \right] \left. \right] \left. \right]
\end{aligned}$$

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$$\begin{aligned}
R_{\text{zap}}\left(t_{\text{zap}}, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right) &:= \sqrt{R_0^2 - 2 \cdot R_0 \cdot r(t_{\text{zap}}, r_{\theta}, v_{\theta}, a_0) \cdot \cos(\theta) + r(t_{\text{zap}}, r_{\theta}, v_{\theta}, a_0)^2} \\
&\quad \left(t_{\text{zap}}, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right) \rightarrow \sqrt{R_0^2 - 2 R_0 r(t_{\text{zap}}, r_{\theta}, v_{\theta}, a_0) \cos(\theta) + r(t_{\text{zap}}, r_{\theta}, v_{\theta}, a_0)^2}
\end{aligned} \tag{19}$$

$$R_{\text{zap}}\left(t_{\text{zap}}, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)$$

$$\sqrt{R_0^2 - 2 R_0 \left( r_0 + v_0 t_{zap} + \frac{1}{2} a_0 t_{zap}^2 \right) \cos(\theta) + \left( r_0 + v_0 t_{zap} + \frac{1}{2} a_0 t_{zap}^2 \right)^2} \quad (20)$$

**R\_\_zap := proc (t\_\_zap, r\_\_0, v\_\_0, a\_\_0, R\_\_0, theta) options  
operator, arrow; sqrt(R\_\_0^2-2\*R\_\_0\*r(t\_\_zap, r\_\_0, v\_\_0, a\_\_0)\*cos  
(theta)+r(t\_\_zap, r\_\_0, v\_\_0, a\_\_0)^2) end proc;**

$$(t_{zap}, r_0, v_0, a_0, R_0, \theta) \rightarrow \sqrt{R_0^2 - 2 R_0 r(t_{zap}, r_0, v_0, a_0) \cos(\theta) + r(t_{zap}, r_0, v_0, a_0)^2} \quad (21)$$

**evalf** $\left( \text{subs}\left( q=-1, t_{zap}=-1, r_0=R_-, v_0=0.2, a_0=0, R_0=3, \right.$

$\left[ \text{subs}\left( \theta=0, R_{zap}(t_{zap}, r_0, v_0, a_0, R_0, \theta) \right), \right.$   
 $\text{subs}\left( \theta=\frac{\pi}{4}, R_{zap}(t_{zap}, r_0, v_0, a_0, R_0, \theta) \right),$   
 $\text{subs}\left( \theta=\frac{2 \cdot \pi}{4}, R_{zap}(t_{zap}, r_0, v_0, a_0, R_0, \theta) \right),$   
 $\text{subs}\left( \theta=\frac{3 \cdot \pi}{4}, R_{zap}(t_{zap}, r_0, v_0, a_0, R_0, \theta) \right),$   
 $\text{subs}\left( \theta=\frac{4 \cdot \pi}{4}, R_{zap}(t_{zap}, r_0, v_0, a_0, R_0, \theta) \right),$   
 $\text{subs}\left( \theta=\frac{5 \cdot \pi}{4}, R_{zap}(t_{zap}, r_0, v_0, a_0, R_0, \theta) \right),$   
 $\text{subs}\left( \theta=\frac{6 \cdot \pi}{4}, R_{zap}(t_{zap}, r_0, v_0, a_0, R_0, \theta) \right),$   
 $\text{subs}\left( \theta=\frac{7 \cdot \pi}{4}, R_{zap}(t_{zap}, r_0, v_0, a_0, R_0, \theta) \right),$   
 $\text{subs}\left( \theta=\frac{8 \cdot \pi}{4}, R_{zap}(t_{zap}, r_0, v_0, a_0, R_0, \theta) \right)$   
 $\left. \right] \right)$

$$[1.200000000, 2.145517831, 3.498571137, 4.458335254, 4.800000000, 4.458335253, \\ 3.498571135, 2.145517831, 1.200000000] \quad (22)$$

$R_{zap}(t_{zap}(t, r_0, v_0, a_0, R_0, \theta), r_0, v_0, a_0, R_0, \theta);$

$$\left( R_0^2 - 2 R_0 \left( r_0 + v_0 \text{RootOf}\left( a_0^2 \_Z^4 + 4 v_0 \_Z^3 a_0 + (-4 R_0 a_0 \cos(\theta) + 4 a_0 r_0 + 4 v_0^2 \right. \right. \right. \quad (23)$$

$$\left. \left. \left. - 36 \right) \_Z^2 + (-8 R_0 v_0 \cos(\theta) + 8 r_0 v_0 + 72 t) \_Z - 8 R_0 \cos(\theta) r_0 + 4 R_0^2 + 4 r_0^2 \right. \right.$$

$$\begin{aligned}
& -36 t^2) + \frac{1}{2} a_0 \text{RootOf}(a_0^2 \_Z^4 + 4 v_0 \_Z^3 a_0 + (-4 R_0 a_0 \cos(\theta) + 4 a_0 r_0 + 4 v_0^2 \\
& -36) \_Z^2 + (-8 R_0 v_0 \cos(\theta) + 8 r_0 v_0 + 72 t) \_Z - 8 R_0 \cos(\theta) r_0 + 4 R_0^2 + 4 r_0^2 \\
& -36 t^2)^2) \cos(\theta) + \left( r_0 + v_0 \text{RootOf}(a_0^2 \_Z^4 + 4 v_0 \_Z^3 a_0 + (-4 R_0 a_0 \cos(\theta) \right. \\
& + 4 a_0 r_0 + 4 v_0^2 - 36) \_Z^2 + (-8 R_0 v_0 \cos(\theta) + 8 r_0 v_0 + 72 t) \_Z - 8 R_0 \cos(\theta) r_0 \\
& + 4 R_0^2 + 4 r_0^2 - 36 t^2) + \frac{1}{2} a_0 \text{RootOf}(a_0^2 \_Z^4 + 4 v_0 \_Z^3 a_0 + (-4 R_0 a_0 \cos(\theta) \\
& + 4 a_0 r_0 + 4 v_0^2 - 36) \_Z^2 + (-8 R_0 v_0 \cos(\theta) + 8 r_0 v_0 + 72 t) \_Z - 8 R_0 \cos(\theta) r_0 \\
& \left. + 4 R_0^2 + 4 r_0^2 - 36 t^2)^2 \right)^{1/2}
\end{aligned}$$

$$\begin{aligned}
& \text{evalf}\left(\text{subs}\left(q=-1, t=-1, r_0=R_-, v_0=0.2, a_0=0, R_0=3, \right.\right. \\
& \left.\left[\text{subs}\left(\theta=0, R_{zap}\left(tzap\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right), r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), \right.\right. \\
& \left.\text{subs}\left(\theta=\frac{\pi}{4}, R_{zap}\left(tzap\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right), r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), \right. \\
& \left.\text{subs}\left(\theta=\frac{2 \cdot \pi}{4}, R_{zap}\left(tzap\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right), r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), \right. \\
& \left.\text{subs}\left(\theta=\frac{3 \cdot \pi}{4}, R_{zap}\left(tzap\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right), r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), \right. \\
& \left.\text{subs}\left(\theta=\frac{4 \cdot \pi}{4}, R_{zap}\left(tzap\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right), r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), \right. \\
& \left.\text{subs}\left(\theta=\frac{5 \cdot \pi}{4}, R_{zap}\left(tzap\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right), r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), \right. \\
& \left.\text{subs}\left(\theta=\frac{6 \cdot \pi}{4}, R_{zap}\left(tzap\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right), r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), \right. \\
& \left.\text{subs}\left(\theta=\frac{7 \cdot \pi}{4}, R_{zap}\left(tzap\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right), r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), \right. \\
& \left.\text{subs}\left(\theta=\frac{8 \cdot \pi}{4}, R_{zap}\left(tzap\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right), r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right)\right),
\end{aligned}$$

$$\begin{aligned}
& \text{plot}\left(\text{subs}\left(q=-1, t=-5, r_0=R_-, v_0=1, a_0=0, R_0=3, R_{zap}\left(tzap\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right), r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), \theta \right. \\
& \left. =0..\pi\right), \\
& \text{plot}\left(\text{subs}\left(q=-1, t=-1, r_0=R_-, v_0=1, a_0=0, R_0=3, R_{zap}\left(tzap\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right), r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), \theta \right. \\
& \left. =0..\pi\right),
\end{aligned}$$

$$\begin{aligned}
& \text{plot}\big(\text{subs}\big(q=-1, t=0, r_0=R_-, v_0=1, a_0=0, R_0=3, R_{zap}\big(tzap\big(t, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_{\vartheta}, \theta\big), r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_{\vartheta}, \theta\big)\big), \theta \\
& \quad =0..\pi\big), \\
& \text{plot}\big(\text{subs}\big(q=-1, t=1, r_0=R_-, v_0=1, a_0=0, R_0=3, R_{zap}\big(tzap\big(t, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_{\vartheta}, \theta\big), r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_{\vartheta}, \theta\big)\big), \theta \\
& \quad =0..\pi\big), \\
& \text{plot}\big(\text{subs}\big(q=-1, t=2, r_0=R_-, v_0=1, a_0=0, R_0=3, R_{zap}\big(tzap\big(t, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_{\vartheta}, \theta\big), r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_{\vartheta}, \theta\big)\big), \theta \\
& \quad =0..\pi\big), \\
& \text{plot}\big(\text{subs}\big(q=-1, t=5, r_0=R_-, v_0=1, a_0=0, R_0=3, R_{zap}\big(tzap\big(t, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_{\vartheta}, \theta\big), r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_{\vartheta}, \theta\big)\big), \theta \\
& \quad =0..\pi\big), \\
& \text{plot}\big(\text{subs}\big(q=-1, t=10, r_0=R_-, v_0=1, a_0=0, R_0=3, R_{zap}\big(tzap\big(t, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_{\vartheta}, \theta\big), r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_{\vartheta}, \theta\big)\big), \theta \\
& \quad =0..\pi\big), \\
& \text{plot}\big(\text{subs}\big(q=-1, t=-1, r_0=R_-, v_0=1, a_0=0, \theta=0, R_{zap}\big(tzap\big(t, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_{\vartheta}, \theta\big), r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_{\vartheta}, \theta\big)\big), R_0 \\
& \quad =3..9\big), \\
& \text{plot}\big(\text{subs}\big(q=-1, t=10, r_0=R_-, v_0=1, a_0=0, \theta=\pi, R_{zap}\big(tzap\big(t, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_{\vartheta}, \theta\big), r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_{\vartheta}, \theta\big)\big), R_0 \\
& \quad =3..9\big) \\
& \bigg) \bigg) \bigg)
\end{aligned}$$

,

$$\begin{aligned}
K_{zap}\big(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_{\vartheta}, \theta\big) &:= R_{zap}\big(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_{\vartheta}, \theta\big) - \frac{v_r\big(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}\big)}{c} \cdot \big(R_0 \cdot \cos(\theta) - r\big(t_{zap}, r_{\vartheta}, \\
& \quad v_{\vartheta}, a_{\vartheta}\big)\big) \\
& \quad \big(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_{\vartheta}, \theta\big) \rightarrow R_{zap}\big(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_{\vartheta}, \theta\big) \\
& \quad - \frac{v_r\big(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}\big) \big(R_0 \cos(\theta) - r\big(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}\big)\big)}{c}
\end{aligned} \tag{25}$$

$$\begin{aligned}
& K_{zap}\big(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_{\vartheta}, \theta\big) \\
& \sqrt{R_0^2 - 2 R_0 \left(r_0 + v_0 t_{zap} + \frac{1}{2} a_0 t_{zap}^2\right) \cos(\theta) + \left(r_0 + v_0 t_{zap} + \frac{1}{2} a_0 t_{zap}^2\right)^2} - \frac{1}{3} \left(a_0 t_{zap} \right. \\
& \quad \left. + v_0\right) \left(R_0 \cos(\theta) - r_0 - v_0 t_{zap} - \frac{1}{2} a_0 t_{zap}^2\right)
\end{aligned} \tag{26}$$

$$K_{zap}\left(tzap\left(t,r_0,v_0,a_0,R_0,\theta\right),r_0,v_0,a_0,R_0,\theta\right); \quad (27)$$

$$\begin{aligned} & \left(R_0^2-2R_0\left(r_0+v_0RootOf\left(a_0^2-Z^4+4v_0-Z^3a_0+\left(-4R_0a_0\cos(\theta)+4a_0r_0+4v_0^2\right.\right.\right. \\ & -36)\_-Z^2+\left(-8R_0v_0\cos(\theta)+8r_0v_0+72t\right)\_-Z-8R_0\cos(\theta)r_0+4R_0^2+4r_0^2 \\ & -36t^2)+\frac{1}{2}a_0RootOf\left(a_0^2-Z^4+4v_0-Z^3a_0+\left(-4R_0a_0\cos(\theta)+4a_0r_0+4v_0^2\right.\right. \\ & -36)\_-Z^2+\left(-8R_0v_0\cos(\theta)+8r_0v_0+72t\right)\_-Z-8R_0\cos(\theta)r_0+4R_0^2+4r_0^2 \\ & \left.-36t^2)^2\right)\cos(\theta)+\left(r_0+v_0RootOf\left(a_0^2-Z^4+4v_0-Z^3a_0+\left(-4R_0a_0\cos(\theta)\right.\right.\right. \\ & +4a_0r_0+4v_0^2-36)\_-Z^2+\left(-8R_0v_0\cos(\theta)+8r_0v_0+72t\right)\_-Z-8R_0\cos(\theta)r_0 \\ & +4R_0^2+4r_0^2-36t^2)+\frac{1}{2}a_0RootOf\left(a_0^2-Z^4+4v_0-Z^3a_0+\left(-4R_0a_0\cos(\theta)\right.\right. \\ & +4a_0r_0+4v_0^2-36)\_-Z^2+\left(-8R_0v_0\cos(\theta)+8r_0v_0+72t\right)\_-Z-8R_0\cos(\theta)r_0 \\ & \left.+4R_0^2+4r_0^2-36t^2)^2\right)^{1/2}-\frac{1}{3}\left(v_0+a_0RootOf\left(a_0^2-Z^4+4v_0-Z^3a_0+\left(-4R_0a_0\cos(\theta)\right.\right.\right. \\ & +4a_0r_0+4v_0^2-36)\_-Z^2+\left(-8R_0v_0\cos(\theta)+8r_0v_0+72t\right)\_-Z-8R_0\cos(\theta)r_0+4R_0^2+4r_0^2-36t^2) \\ & \left.)\left(R_0\cos(\theta)-r_0-v_0RootOf\left(a_0^2-Z^4\right.\right.\right. \\ & +4v_0-Z^3a_0+\left(-4R_0a_0\cos(\theta)+4a_0r_0+4v_0^2-36)\_-Z^2+\left(-8R_0v_0\cos(\theta)+8r_0v_0\right.\right. \\ & +72t)\_-Z-8R_0\cos(\theta)r_0+4R_0^2+4r_0^2-36t^2)-\frac{1}{2}a_0RootOf\left(a_0^2-Z^4\right. \\ & +4v_0-Z^3a_0+\left(-4R_0a_0\cos(\theta)+4a_0r_0+4v_0^2-36)\_-Z^2+\left(-8R_0v_0\cos(\theta)+8r_0v_0\right.\right. \\ & \left.\left.+72t)\_-Z-8R_0\cos(\theta)r_0+4R_0^2+4r_0^2-36t^2)^2\right) \end{aligned}$$

$$\begin{aligned} evalf & \left( subs \left( q = -1, t = 1, r_0 = R_-, v_0 = 0.2, a_0 = 0, R_0 = 3, \right. \right. \\ & \left. \left[ subs \left( \theta = 0, K_{zap} \left( tzap \left( t, r_0, v_0, a_0, R_0, \theta \right), r_0, v_0, a_0, R_0, \theta \right) \right), \right. \right. \\ & \left. \left. subs \left( \theta = \frac{\pi}{4}, K_{zap} \left( tzap \left( t, r_0, v_0, a_0, R_0, \theta \right), r_0, v_0, a_0, R_0, \theta \right) \right), \right. \right. \\ & \left. \left. subs \left( \theta = \frac{2 \cdot \pi}{4}, K_{zap} \left( tzap \left( t, r_0, v_0, a_0, R_0, \theta \right), r_0, v_0, a_0, R_0, \theta \right) \right), \right. \right. \\ & \left. \left. subs \left( \theta = \frac{3 \cdot \pi}{4}, K_{zap} \left( tzap \left( t, r_0, v_0, a_0, R_0, \theta \right), r_0, v_0, a_0, R_0, \theta \right) \right) \right), \right. \end{aligned}$$

$$\begin{aligned}
& subs\left(\theta = \frac{4 \cdot \pi}{4}, K_{zap}\left(tzap\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right), r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), \\
& subs\left(\theta = \frac{5 \cdot \pi}{4}, K_{zap}\left(tzap\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right), r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), \\
& subs\left(\theta = \frac{6 \cdot \pi}{4}, K_{zap}\left(tzap\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right), r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), \\
& subs\left(\theta = \frac{7 \cdot \pi}{4}, K_{zap}\left(tzap\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right), r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), \\
& subs\left(\theta = \frac{8 \cdot \pi}{4}, K_{zap}\left(tzap\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right), r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right),
\end{aligned}$$

$$\begin{aligned}
& plot\left(subs\left(q=-1, t=-5, r_0=R_{-}, v_0=1, a_0=0, R_0=3, K_{zap}\left(tzap\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right), r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), \theta\right. \\
& \quad \left.=0..\pi\right), \\
& plot\left(subs\left(q=-1, t=-1, r_0=R_{-}, v_0=1, a_0=0, R_0=3, K_{zap}\left(tzap\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right), r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), \theta\right. \\
& \quad \left.=0..\pi\right), \\
& plot\left(subs\left(q=-1, t=0, r_0=R_{-}, v_0=1, a_0=0, R_0=3, K_{zap}\left(tzap\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right), r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), \theta\right. \\
& \quad \left.=0..\pi\right), \\
& plot\left(subs\left(q=-1, t=1, r_0=R_{-}, v_0=1, a_0=0, R_0=3, K_{zap}\left(tzap\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right), r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), \theta\right. \\
& \quad \left.=0..\pi\right), \\
& plot\left(subs\left(q=-1, t=2, r_0=R_{-}, v_0=1, a_0=0, R_0=3, K_{zap}\left(tzap\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right), r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), \theta\right. \\
& \quad \left.=0..\pi\right), \\
& plot\left(subs\left(q=-1, t=5, r_0=R_{-}, v_0=1, a_0=0, R_0=3, K_{zap}\left(tzap\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right), r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), \theta\right. \\
& \quad \left.=0..\pi\right), \\
& plot\left(subs\left(q=-1, t=10, r_0=R_{-}, v_0=1, a_0=0, R_0=3, K_{zap}\left(tzap\left(t, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right), r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right)\right), \theta\right. \\
& \quad \left.=0..\pi\right)
\end{aligned}$$

)))

$$\begin{aligned}
& t\_cur\left(t_{zap}, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right) := solve\left(c^2\left(t-t_{zap}\right)^2=R_0^2-2 \cdot R_0 \cdot r\left(t_{zap}, r_{\theta}, v_{\theta}, a_{\theta}\right) \cdot \cos (\theta)+r\left(t_{zap}, r_{\theta},\right.\right. \\
& \quad \left.\left.v_{\theta}, a_{\theta}\right)^2, t\right) \\
& \left(t_{zap}, r_{\theta}, v_{\theta}, a_{\theta}, R_{\theta}, \theta\right) \rightarrow solve\left(c^2\left(t-t_{zap}\right)^2=R_0^2-2 R_0 r\left(t_{zap}, r_{\theta}, v_{\theta}, a_{\theta}\right) \cos (\theta)+r\left(t_{zap}, r_{\theta}, v_{\theta},\right.\right. \quad (29)
\end{aligned}$$



$$a_0)^2, t)$$

$$t\_cur(t_{zap}, r_{\varnothing} v_{\varnothing} a_{\varnothing} R_{\varnothing} \theta) \\ t_{zap} \tag{30}$$

$$+ \frac{1}{6} \left( a_0^2 t_{zap}^4 - 4 R_0 \cos(\theta) a_0 t_{zap}^2 + 4 v_0 t_{zap}^3 a_0 - 8 R_0 \cos(\theta) v_0 t_{zap} \right. \\ \left. + 4 r_0 a_0 t_{zap}^2 + 4 v_0^2 t_{zap}^2 - 8 R_0 \cos(\theta) r_0 + 8 r_0 v_0 t_{zap} + 4 R_0^2 + 4 r_0^2 \right)^{1/2}, t_{zap} \\ - \frac{1}{6} \left( a_0^2 t_{zap}^4 - 4 R_0 \cos(\theta) a_0 t_{zap}^2 + 4 v_0 t_{zap}^3 a_0 - 8 R_0 \cos(\theta) v_0 t_{zap} \right. \\ \left. + 4 r_0 a_0 t_{zap}^2 + 4 v_0^2 t_{zap}^2 - 8 R_0 \cos(\theta) r_0 + 8 r_0 v_0 t_{zap} + 4 R_0^2 + 4 r_0^2 \right)^{1/2}$$

$$evalf\left(subs\left(\theta=\pi, R_0=10, t_{zap}=0, r_0=R_-, v_0=1, a_0=1, \left[t\_cur\left(t_{zap}, r_{\varnothing} v_{\varnothing} a_{\varnothing} R_{\varnothing} \theta\right), t\_cur\left(t_{zap}, r\left(t_{zap}, \right.\right.\right.\right. \\ \left.\left.\left.\left. r_{\varnothing} v_{\varnothing} a_0\right), v_{\varnothing} a_{\varnothing} R_{\varnothing} \theta\right)\right]\right)\right) \\ [4.0000000001, -4.0000000001, 4.0000000000, -4.0000000000] \tag{31}$$

$$evalf\left(subs\left(\theta=\pi, R_0=10, t_{zap}=0, r_0=R_-, v_0=1, a_0=1, t\_cur\left(t_{zap}, r_{\varnothing} v_{\varnothing} a_{\varnothing} R_{\varnothing} \theta\right)\left[1\right]\right)\right) \\ 4.0000000001 \tag{32}$$

$$r, R_0$$

$$\varphi(q, t, r_{\varnothing} v_{\varnothing} a_{\varnothing} R_0) := \int_0^{\pi} \frac{2 \cdot \pi \cdot r(t, r_{\varnothing} v_{\varnothing} a_0)^2 \cdot \sin(\theta) \cdot \sigma(q, r_0)}{K_{zap}(t_{zap}(t, r_{\varnothing} v_{\varnothing} a_{\varnothing} R_{\varnothing} \theta), r_{\varnothing} v_{\varnothing} a_{\varnothing} R_{\varnothing} \theta)} d\theta; \\ (q, t, r_{\varnothing} v_{\varnothing} a_{\varnothing} R_0) \rightarrow \int_0^{\pi} \frac{2 \pi r(t, r_{\varnothing} v_{\varnothing} a_0)^2 \sin(\theta) \sigma(q, r_0)}{K_{zap}(t_{zap}(t, r_{\varnothing} v_{\varnothing} a_{\varnothing} R_{\varnothing} \theta), r_{\varnothing} v_{\varnothing} a_{\varnothing} R_{\varnothing} \theta)} d\theta \tag{33}$$

$$evalf\left(subs\left(q=1, t=5, r_0=1, R_0=2, v_0=0, a_0=0.0, \varphi(q, t, r_{\varnothing} v_{\varnothing} a_{\varnothing} R_0)\right)\right) \\ 0.5000000000 \tag{34}$$

$$r_0$$

$$A_{R_0}(q, t, r_{\varnothing} v_{\varnothing} a_{\varnothing} R_0) := \int_0^{\pi} \frac{2 \cdot \pi \cdot r(t, r_{\varnothing} v_{\varnothing} a_0)^2 \cdot \sin(\theta) \cdot \sigma(q, r_0) \cdot (v_r(t_{zap}, r_{\varnothing} v_{\varnothing} a_0) \cdot \cos(\theta))}{K_{zap}(t_{zap}(t, r_{\varnothing} v_{\varnothing} a_{\varnothing} R_{\varnothing} \theta), r_{\varnothing} v_{\varnothing} a_{\varnothing} R_{\varnothing} \theta)} d\theta;$$

$$(q, t, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0) \rightarrow \int_0^\pi \frac{2 \pi r(t, r_{\vartheta}, v_{\vartheta}, a_{\vartheta})^2 \sin(\theta) \sigma(q, r_0) v_r(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}) \cos(\theta)}{K_{zap}(t_{zap}(t, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \theta), r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \theta)} d\theta \quad (35)$$

$$- \quad - \quad ($$

$$\begin{aligned} \cos\_alpha_{zap}(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \theta) &:= \frac{R_0 - r(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}) \cdot \cos(\theta)}{R_{zap}(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \theta)} \\ (t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \theta) &\rightarrow \frac{R_0 - r(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}) \cos(\theta)}{R_{zap}(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \theta)} \end{aligned} \quad (36)$$

$$- \quad ($$

$$\begin{aligned} aR_{zap}(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}) &:= a_r(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}) \cdot (R_0 \cdot \cos(\theta) - r(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta})) \\ (t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}) &\rightarrow a_r(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}) (R_0 \cos(\theta) - r(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta})) \end{aligned} \quad (37)$$

-

$$\begin{aligned} E\_minus\_grad\_phi_{R_0}(q, t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \theta) &:= \frac{\sigma(q, r_0)}{K_{zap}(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \theta)^2} \\ &\cdot \left( \frac{R_{zap}(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \theta) \cdot \cos\_alpha_{zap}(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \theta)}{K_{zap}(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \theta)} \left( 1 + \frac{aR_{zap}(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta})}{c^2} \right) \right. \\ &\quad \left. - \frac{v_r(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta})^2}{c^2} \right) - \frac{v_r(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}) \cdot \cos(\theta)}{c} \\ (q, t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \theta) &\rightarrow \frac{1}{K_{zap}(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \theta)^2} \left( \sigma(q, \right. \\ r_0) &\left( \frac{1}{K_{zap}(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \theta)} \left( R_{zap}(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \theta) \cos\_alpha_{zap}(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \right. \right. \\ \theta) &\left( 1 + \frac{aR_{zap}(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta})}{c^2} - \frac{v_r(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta})^2}{c^2} \right) \left. \right) - \frac{v_r(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}) \cos(\theta)}{c} \left. \right) \end{aligned} \quad (38)$$

$$\begin{aligned}
E\_minus\_grad\_varphi_{R_0}(q, t, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0) &:= \int_0^{\pi} 2 \cdot \pi \cdot r(t, r_{\vartheta}, v_{\vartheta}, a_{\vartheta})^2 \cdot \sin(\theta) \\
&\cdot E\_minus\_grad\_varphi_{R_0}(q, t_{zap}(t, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \theta), r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \theta) \, d\theta \\
(q, t, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0) &\rightarrow \int_0^{\pi} 2 \pi r(t, r_{\vartheta}, v_{\vartheta}, a_{\vartheta})^2 \sin(\theta) E\_minus\_grad\_varphi_{R_0}(q, t_{zap}(t, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, \\
&R_0, \theta), r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \theta) \, d\theta
\end{aligned} \tag{39}$$

$$\begin{aligned}
E\_minus\_I\_c\_dA\_dt_{R_0}(q, t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \theta) &:= \frac{\cos(\theta) \cdot \sigma(q, r_0)}{K_{zap}(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \theta)^2} \cdot \left( \frac{v_r(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta})}{c^2} \right. \\
&\cdot \left( \frac{R_{zap}(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \theta)}{K_{zap}(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \theta)} \cdot \left( \frac{v_r(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta})^2}{c} - \frac{a R_{zap}(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta})}{c} - c \right) + c \right) \\
&\left. - \frac{a_r(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}) \cdot R_{zap}(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \theta)}{c^2} \right) \\
(q, t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \theta) &\rightarrow \frac{1}{K_{zap}(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \theta)^2} \left( \cos(\theta) \sigma(q, r_0) \left( \frac{1}{c^2} \left( v_r(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}) \right. \right. \right. \\
&a_{\vartheta}) \left( \frac{R_{zap}(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \theta) \left( \frac{v_r(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta})^2}{c} - \frac{a R_{zap}(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta})}{c} - c \right)}{K_{zap}(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \theta)} + c \right) \right. \\
&\left. \left. - \frac{a_r(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}) R_{zap}(t_{zap}, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \theta)}{c^2} \right) \right)
\end{aligned} \tag{40}$$

$$\begin{aligned}
E\_minus\_I\_c\_dA\_dt\_integral_{R_0}(q, t, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0) &:= \int_0^{\pi} 2 \cdot \pi \cdot r(t, r_{\vartheta}, v_{\vartheta}, a_{\vartheta})^2 \cdot \sin(\theta) \\
&\cdot E\_minus\_I\_c\_dA\_dt_{R_0}(q, t_{zap}(t, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \theta), r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \theta) \, d\theta \\
(q, t, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0) &\rightarrow \int_0^{\pi} 2 \pi r(t, r_{\vartheta}, v_{\vartheta}, a_{\vartheta})^2 \sin(\theta) E\_minus\_I\_c\_dA\_dt_{R_0}(q, t_{zap}(t, r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \\
&\theta), r_{\vartheta}, v_{\vartheta}, a_{\vartheta}, R_0, \theta) \, d\theta
\end{aligned} \tag{41}$$

$$tcur := subs(t_{zap}=0, r_0=R_-, R_0=10, evalf(\max(t_{-cur}(t_{zap}, r(t_{zap}, r_0, v_0, a_0), v_0, a_0, R_0, \pi)) \max(-4.000000000, 4.000000000))) \quad (42)$$

$$tcur \quad 4.000000000 \quad (43)$$

$$ftcur(R_0) := subs(t_{zap}=0, r_0=R_-, evalf(\max(t_{-cur}(t_{zap}, r(t_{zap}, r_0, v_0, a_0), v_0, a_0, R_0, \pi)) \quad (44)$$

$$R_0 \rightarrow subs(t_{zap}=0, r_0=R_-, evalf(\max(t_{-cur}(t_{zap}, r(t_{zap}, r_0, v_0, a_0), v_0, a_0, R_0, \pi)) \quad (44)$$

$$evalf([subs(q=-1, r=R_-, v=1, c=3, R_0=3, \phi_{lw}(q, v, c, r, R_0)), subs(q=-1, t=tcur, r_0=R_-, v_0=1, a_0=0, R_0=3, \phi(q, t, r_0, v_0, a_0, R_0))]) \quad (45)$$

$$[-0.3172022481, -1.514125065]$$

$$evalf([subs(q=-1, t_{zap}=0.1, r_0=R_-, v_0=1, a_0=0, R_0=3, \theta=0, E\_minus\_grad\_ \varphi_{R_0}(q, t_{zap}, r_0, v_0, a_0, R_0, \theta))]) \quad (46)$$

$$[-0.05526213297]$$

$$evalf([subs(q=-1, t_{zap}=0.1, r_0=R_-, v_0=1, a_0=0, R_0=3, \theta=0, E\_minus\_I\_c\_dA\_dt_{R_0}(q, t_{zap}, r_0, v_0, a_0, R_0, \theta))]) \quad (47)$$

$$[0.006140236998]$$

$$evalf([subs(q=-1, t=tcur, r_0=R_-, v_0=1, a_0=0, R_0=2, \theta=1, tzap(t, r_0, v_0, a_0, R_0, \theta))]) \quad (48)$$

$$[2.676568128]$$

$$evalf([subs(q=-1, t=tcur, r_0=R_-, v_0=1, a_0=0, R_0=3, \theta=0, E\_minus\_I\_c\_dA\_dt_{R_0}(q, tzap(t, r_0, v_0, a_0, R_0, \theta), r_0, v_0, a_0, R_0, \theta))]) \quad (49)$$

$$[-0.0002456094800]$$

$$evalf([subs(q=-1, t=tcur, r_0=R_-, v_0=1, a_0=0, R_0=4, \theta=0, E\_minus\_grad\_ \varphi_{R_0}(q, tzap(t, r_0, v_0, a_0, R_0, \theta), r_0, v_0, a_0, R_0, \theta))]) \quad (50)$$

$$[0.004973591964]$$

$$\text{evalf}\left(\left[\text{subs}\left(q=-1, t=t_{\text{cur}}, r_0=R_-, v_0=1, a_0=0, R_0=4, \theta=0, E_{\text{minus\_1\_c\_dA\_dt}}(q, t, r_0, v_0, a_0, R_0, \theta)\right), r_0, v_0, a_0, R_0, \theta\right)\right] \Big) \\ [-0.0005526213299] \quad (51)$$

$$\text{evalf}\left(\left[\text{subs}\left(q=-1, t=\text{subs}(R_0=3, f_{\text{cur}}(R_0)), r_0=R_-, v_0=1, a_0=0, R_0=3, E_{\text{minus\_grad\_}\varphi\_integral_{R_0}}(q, t, r_0, v_0, a_0, R_0)\right), \text{subs}\left(q=+1, t=\text{subs}(R_0=3, f_{\text{cur}}(R_0)), r_0=R_+, v_0=0.1, a_0=0, R_0=3, E_{\text{minus\_grad\_}\varphi\_integral_{R_0}}(q, t, r_0, v_0, a_0, R_0)\right)\right] \Big) \\ [0.01662065319, 0.1512906181] \quad (52)$$

$$\text{evalf}\left(\left[\text{subs}\left(q=-1, t=\text{subs}(R_0=3, f_{\text{cur}}(R_0)), r_0=R_-, v_0=1, a_0=0, R_0=3, E_{\text{minus\_1\_c\_dA\_dt\_integral}_{R_0}}(q, t, r_0, v_0, a_0, R_0)\right), \text{subs}\left(q=+1, t=\text{subs}(R_0=3, f_{\text{cur}}(R_0)), r_0=R_+, v_0=0.1, a_0=0, R_0=3, E_{\text{minus\_1\_c\_dA\_dt\_integral}_{R_0}}(q, t, r_0, v_0, a_0, R_0)\right)\right] \Big) \\ [-0.01662065319, -0.00005605017475] \quad (53)$$

$$\text{subs}(R_0=3, f_{\text{cur}}(R_0)) \\ \max(-1.6666666666, 1.6666666666) \quad (54)$$

$$\text{subs}(R_0=5, f_{\text{cur}}(R_0)) \\ \max(-2.3333333333, 2.3333333333) \quad (55)$$

$$\text{evalf}\left(\left[\text{subs}\left(q=-1, t=\text{subs}(R_0=3, f_{\text{cur}}(R_0)), r_0=R_-, v_0=1, a_0=0, R_0=3, E_{\text{minus\_grad\_}\varphi\_integral_{R_0}}(q, t, r_0, v_0, a_0, R_0)\right), \right.\right.$$

$$\left.\text{subs}\left(q=+1, t=\text{subs}(R_0=3, f_{\text{cur}}(R_0)), r_0=R_+, v_0=0.1, a_0=0, R_0=3, E_{\text{minus\_grad\_}\varphi\_integral_{R_0}}(q, t, r_0, v_0, a_0, R_0)\right), \right. \\ \left.\text{subs}\left(q=-1, t=\text{subs}(R_0=3, f_{\text{cur}}(R_0)), r_0=R_-, v_0=1, a_0=0, R_0=3, E_{\text{minus\_1\_c\_dA\_dt\_integral}_{R_0}}(q, t, r_0, v_0, a_0, R_0)\right), \right. \\ \left.\text{subs}\left(q=1, t=\text{subs}(R_0=3, f_{\text{cur}}(R_0)), r_0=R_+, v_0=0.1, a_0=0, R_0=3, E_{\text{minus\_1\_c\_dA\_dt\_integral}_{R_0}}(q, t, r_0, v_0, a_0, R_0)\right)\right]$$

]

)

$$[0.01662065319, 0.1512906181, -0.01662065319, -0.00005605017475]$$

(56)

evalf(

[

$$\text{subs}(q=-1, t=1.1 \cdot \text{subs}(R_0=3, \text{ftcur}(R_0)), r_0=R_-, v_0=1, a_0=0, R_0=3, \\ E\_minus\_grad\_phi\_integral_{R_0}(q, t, r_0, v_0, a_0, R_0)),$$

$$\text{subs}(q=+1, t=1.1 \cdot \text{subs}(R_0=3, \text{ftcur}(R_0)), r_0=R_+, v_0=0.1, a_0=0, R_0=3, \\ E\_minus\_grad\_phi\_integral_{R_0}(q, t, r_0, v_0, a_0, R_0)),$$

$$\text{subs}(q=-1, t=1.1 \cdot \text{subs}(R_0=3, \text{ftcur}(R_0)), r_0=R_-, v_0=1, a_0=0, R_0=3, \\ E\_minus\_1\_c\_dA\_dt\_integral_{R_0}(q, t, r_0, v_0, a_0, R_0)),$$

$$\text{subs}(q=1, t=1.1 \cdot \text{subs}(R_0=3, \text{ftcur}(R_0)), r_0=R_+, v_0=0.1, a_0=0, R_0=3, \\ E\_minus\_1\_c\_dA\_dt\_integral_{R_0}(q, t, r_0, v_0, a_0, R_0))$$

]

)

$$[0.01576912403, 0.1556440828, -0.01576912403, -0.00005766304713]$$

(57)

evalf(

[

$$\text{subs}(q=-1, t=1.5 \cdot \text{subs}(R_0=3, \text{ftcur}(R_0)), r_0=R_-, v_0=1, a_0=0, R_0=3, \\ E\_minus\_grad\_phi\_integral_{R_0}(q, t, r_0, v_0, a_0, R_0)),$$

$$\text{subs}(q=+1, t=1.5 \cdot \text{subs}(R_0=3, \text{ftcur}(R_0)), r_0=R_+, v_0=0.1, a_0=0, R_0=3, \\ E\_minus\_grad\_phi\_integral_{R_0}(q, t, r_0, v_0, a_0, R_0)),$$

$$\text{subs}(q=-1, t=1.5 \cdot \text{subs}(R_0=3, \text{ftcur}(R_0)), r_0=R_-, v_0=1, a_0=0, R_0=3, \\ E\_minus\_1\_c\_dA\_dt\_integral_{R_0}(q, t, r_0, v_0, a_0, R_0)),$$

$$\text{subs}(q=1, t=1.5 \cdot \text{subs}(R_0=3, \text{ftcur}(R_0)), r_0=R_+, v_0=0.1, a_0=0, R_0=3,$$

$$E\_minus\_1\_c\_dA\_dt\_integral_{R_0}(q, t, r_0, v_0, a_0, R_0) )$$

]  
)

$$[0.01311807133, 0.1736754544, -0.01311807133, -0.00006434331287]$$

(58)

evalf(

$$[$$

$$subs(q=-1, t=1.66667, r_0=R_-, v_0=1, a_0=0, R_0=3, E\_minus\_grad\_phi\_integral_{R_0}(q, t, r_0, v_0, a_0,$$

$$R_0) ),$$

$$subs(q=+1, t=1.66667, r_0=R_+, v_0=0.1, a_0=0, R_0=3, E\_minus\_grad\_phi\_integral_{R_0}(q, t, r_0, v_0, a_0,$$

$$R_0) ),$$

$$subs(q=-1, t=1.66667, r_0=R_-, v_0=1, a_0=0, R_0=3, E\_minus\_1\_c\_dA\_dt\_integral_{R_0}(q, t, r_0, v_0, a_0,$$

$$R_0) ),$$

$$subs(q=1, t=1.66667, r_0=R_+, v_0=0.1, a_0=0, R_0=3, E\_minus\_1\_c\_dA\_dt\_integral_{R_0}(q, t, r_0, v_0, a_0,$$

$$R_0) )$$

]  
)

$$[0.01662063518, 0.1512907045, -0.01662063518, -0.00005605020679]$$

(59)

evalf(

$$[$$

$$subs(q=-1, t=2.66667, r_0=R_-, v_0=1, a_0=0, R_0=3, E\_minus\_grad\_phi\_integral_{R_0}(q, t, r_0, v_0, a_0,$$

$$R_0) ),$$

$$subs(q=+1, t=2.66667, r_0=R_+, v_0=0.1, a_0=0, R_0=3, E\_minus\_grad\_phi\_integral_{R_0}(q, t, r_0, v_0, a_0,$$

$$R_0) ),$$

$$subs(q=-1, t=2.66667, r_0=R_-, v_0=1, a_0=0, R_0=3, E\_minus\_1\_c\_dA\_dt\_integral_{R_0}(q, t, r_0, v_0, a_0,$$

$$R_0) ),$$

$$subs(q=1, t=2.66667, r_0=R_+, v_0=0.1, a_0=0, R_0=3, E\_minus\_1\_c\_dA\_dt\_integral_{R_0}(q, t, r_0, v_0, a_0,$$

$$R_0) )$$

$$\begin{aligned}
& \left. \right) \\
& \left. \right) \\
& [0.01259461396, 0.1783377694, -0.01259461396, -0.00006607060815] \quad (60)
\end{aligned}$$

$$\begin{aligned}
& evalf( \\
& [ \\
& subs(q=-1, t=2.66667, r_0=R_-, v_0=1, a_0=0, R_0=5, E\_minus\_grad\_phi\_integral_{R_0}(q, t, r_0, v_0, a_0, \\
& \quad R_0)), \\
& subs(q=+1, t=2.66667, r_0=R_+, v_0=0.1, a_0=0, R_0=5, E\_minus\_grad\_phi\_integral_{R_0}(q, t, r_0, v_0, a_0, \\
& \quad R_0)), \\
& subs(q=-1, t=2.66667, r_0=R_-, v_0=1, a_0=0, R_0=5, E\_minus\_I\_c\_dA\_dt\_integral_{R_0}(q, t, r_0, v_0, a_0, \\
& \quad R_0)), \\
& subs(q=1, t=2.66667, r_0=R_+, v_0=0.1, a_0=0, R_0=5, E\_minus\_I\_c\_dA\_dt\_integral_{R_0}(q, t, r_0, v_0, a_0, \\
& \quad R_0)) \\
& ] \\
& ) \\
& [-0.2264284024, 0.06420159698, 0.008650313562, -0.00002378541887] \quad (61)
\end{aligned}$$

$$\begin{aligned}
& plot(evalf( \\
& [ \\
& subs(q=-1, r_0=R_-, v_0=1, a_0=0, R_0=3, E\_minus\_grad\_phi\_integral_{R_0}(q, t, r_0, v_0, a_0, R_0)), \\
& subs(q=+1, r_0=R_+, v_0=0.1, a_0=0, R_0=3, E\_minus\_grad\_phi\_integral_{R_0}(q, t, r_0, v_0, a_0, R_0)), \\
& subs(q=-1, r_0=R_-, v_0=1, a_0=0, R_0=3, E\_minus\_I\_c\_dA\_dt\_integral_{R_0}(q, t, r_0, v_0, a_0, R_0)), \\
& subs(q=+1, r_0=R_+, v_0=0.1, a_0=0, R_0=3, E\_minus\_I\_c\_dA\_dt\_integral_{R_0}(q, t, r_0, v_0, a_0, R_0)) \\
& ] \\
& ), t=subs(R_0=3, fcur(R_0))..2*subs(R_0=3, fcur(R_0)))
\end{aligned}$$



$$evalf\left(\left[\begin{array}{l} subs(q=-1, t=subs(R_0=3, ficur(R_0)), r_0=R_-, v_0=0, a_0=0.1, R_0=3, \\ E\_minus\_grad\_phi\_integral_{R_0}(q, t, r_\phi, v_\phi, a_\phi, R_0)), subs(q=+1, t=subs(R_0=3, ficur(R_0)), r_0 \\ =R_+, v_0=0, a_0=0.1, R_0=3, E\_minus\_grad\_phi\_integral_{R_0}(q, t, r_\phi, v_\phi, a_\phi, R_0)) \end{array}\right]\right)$$

$$evalf\left(\left[\begin{array}{l} subs(q=-1, t=subs(R_0=3, ficur(R_0)), r_0=R_-, v_0=0, a_0=0.1, R_0=3, \\ E\_minus\_1\_c\_dA\_dt\_integral_{R_0}(q, t, r_\phi, v_\phi, a_\phi, R_0)), subs(q=+1, t=subs(R_0=3, ficur(R_0)), r_0 \\ =R_+, v_0=0, a_0=0.1, R_0=3, E\_minus\_1\_c\_dA\_dt\_integral_{R_0}(q, t, r_\phi, v_\phi, a_\phi, R_0)) \end{array}\right]\right)$$

$$\begin{aligned} & evalf\big(\big[subs(q=-1, r=R_-, v=1, c=3, R_0=3, \phi_{hw}(q, v, c, r, R_0)), subs(q=-1, t=tcu, r_0=R_-, v_0 \\ & =1, a_0=0, R_0=3, \phi(q, t, r_0, v_0, a_0, R_0))\big]\big) \end{aligned}$$

$$Digits := 5$$

$$\text{plot}\left(\text{evalf}\left(\left[\text{subs}\left(q=-1, t=t_{\text{cur}}, r_0=R_-, v_0=1, a_0=0, E_{\text{minus\_grad\_}\varphi\_integral_{R_0}}(q, t, r_0, v_0, a_0, R_0)\right), \text{subs}\left(q=+1, t=t_{\text{cur}}, r_0=R_+, v_0=0.1, a_0=0, E_{\text{minus\_grad\_}\varphi\_integral_{R_0}}(q, t, r_0, v_0, a_0, R_0)\right)\right]\right), R_0=3..10\right)$$

$$plot(evalf([subs(q=-1, t=tcur, r_0=R_-, v_0=1, a_0=0, E\_minus\_l\_c\_dA\_dt\_integral_{R_0}(q, t, r_0, v_0, a_0, R_0)), subs(q=+1, t=tcur, r_0=R_+, v_0=0.1, a_0=0, E\_minus\_l\_c\_dA\_dt\_integral_{R_0}(q, t, r_0, v_0, a_0, R_0))]), R_0=3..10)$$

C

$$with(plots) : plot \left( evalf \left( subs \left( q = 1, t = 0, r_{0e} = R_-, r_{0p} = R_+, v_{0p} = \frac{1}{2}, v_{0e} = 1, a_0 = 0, \left[ \phi_{lw}(q, v_{0p}, c, r_{0p}, R_0) + \phi_{lw}(-q, v_{0e}, c, r_{0e}, R_0), \phi(q, t, r_{0p}, v_{0p}, a_0, R_0) + \phi(-q, t, r_{0e}, v_{0e}, a_0, R_0) \right] \right) \right), R_0 = -5 .. 5, style = point, numpoints = 10 \right)$$

```

c
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with(plots):plot(evalf(subs(q=1,t=tcur,r0e=R-,r0p=R+,v0p=1,v0e=-2,a0=0,[phi_lw(q,v0p,c,
r0p,R0)+phi_lw(-q,v0e,c,r0e,R0),phi(q,t,r0p,v0p,a0,R0)+phi(-q,t,r0e,v0e,a0,R0)])),R0=-5
..5,style=point,numpoints=10)

```