Does Gauss' theorem apply to the Lienard Wichert potential?

Let us check by direct calculation the applicability of Gauss's theorem in integral form to the LV potential. For this purpose, let us calculate the integral of the normal component of the electric field in the spherical coordinate system.

$restart : clear :$

Start of countdown of the delayed time tau
$T_0 := 0 :$
Charge acceleration value
$A := 0.1 :$

Initial velocity of the charge at the delayed moment of time T_0
$V := 0 :$

Initial coordinate of the charge at the delayed moment of time T_0
$Z := 0 :$

Radius of the spherical surface of integration
$R := 1 :$

Let's set the equation of charge motion depending on the delayed torque tau
Let the charge initially resting at the moment of time τ_0 gains acceleration in the direction of axis z
Thus the equation of charge motion can be expressed as a piecewise continuous function
$a_q(a_0, \mathrm{tau}, \tau_0) := piecewise(\mathrm{tau} > \tau_0, a_0, 0) :$
$a_q(a_0, \mathrm{tau}, \tau_0)$

$$\begin{cases} a_0 & \tau_0 < \tau \\ 0 & otherwise \end{cases} \tag{1}$$

$z_q(z_0, v_0, a_0, \mathrm{tau}, \tau_0) := piecewise\left(\mathrm{tau} > \tau_0, z_0 + v_0 \cdot (\mathrm{tau} - \tau_0) + a_0 \cdot \frac{(\mathrm{tau} - \tau_0)^2}{2}, z_0 + v_0 \cdot (\mathrm{tau} - \tau_0)\right) :$
$z_q(z_0, v_0, a_0, \mathrm{tau}, \tau_0)$

$$\begin{cases} z_0 + v_0 (\tau - \tau_0) + \frac{1}{2} a_0 (\tau - \tau_0)^2 & \tau_0 < \tau \\ z_0 + v_0 (\tau - \tau_0) & otherwise \end{cases} \tag{2}$$

$v_q(v_0, a_0, \mathrm{tau}, \tau_0) := piecewise(\mathrm{tau} > \tau_0, v_0 + a_0 \cdot (\mathrm{tau} - \tau_0), v_0) :$
$v_q(v_0, a_0, \mathrm{tau}, \tau_0)$

$$\begin{cases} v_0 + a_0 (\tau - \tau_0) & \tau_0 < \tau \\ v_0 & \mathit{otherwise} \end{cases} \tag{3}$$

Let us write the equation for calculating the delayed momentum tau for calculating the electric field at time t on a spherical surface of integration of radius R_0

$Eq(z_0, v_0, a_0, \text{tau}, \tau_0, t, \text{theta}) := c^2 \cdot (t - \text{tau})^2 = R_0^2 + (z_q(z_0, v_0, a_0, \text{tau}, \tau_0))^2 - 2 \cdot R_0 \cdot z_q(z_0, v_0, a_0, \text{tau}, \tau_0) \cdot \cos(\text{theta}) :$

Substitute the given numerical values into this equation

$Eqs(\text{tau}, t, \text{theta}) := subs(z_0 = Z, v_0 = V, a_0 = A, c = 1, R_0 = R, \tau_0 = \mathrm{T}_0, Eq(z_0, v_0, a_0, \text{tau}, \tau_0, t, \text{theta})) :$

Solving this equation with respect to tau, we obtain the function of dependence of the lagging moment on the current moment of time and theta angle of the spherical coordinate system. Here it is important to point out the interval

$ftau(T, \text{Theta}) := fsolve(subs(t = T, \text{theta} = \text{Theta}, Eqs(\text{tau}, t, \text{theta})), \text{tau} = -\infty..T) :$

$ftau(0, 0)$

$$-1.000000000 \tag{4}$$

$dftau(T, \text{Theta}) := T - ftau(T, \text{Theta}) :$

The following line is the key step to removing your "z is in the equation and is not solved for" error.

$Eqsu := unapply(Eqs(\text{tau}, t, \text{theta}), t, \text{theta}) :$

Construct the function inversion that occurs in the outer integrand. RootOf allows some analysis of the inverse; whereas fsolve has more flexibility numerically

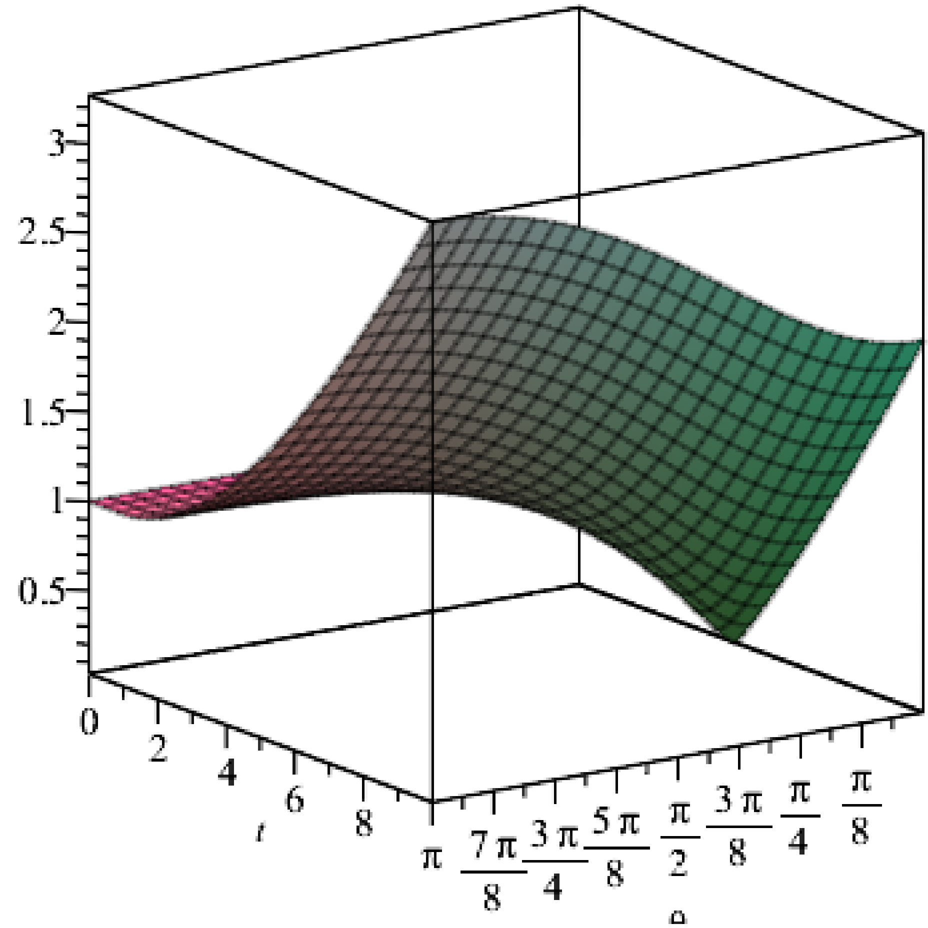
$rtau(t, \text{theta}) := RootOf(Eqsu(t, \text{theta}), \text{tau}, -\infty..t) :$

$rtau(T, \text{Theta})$

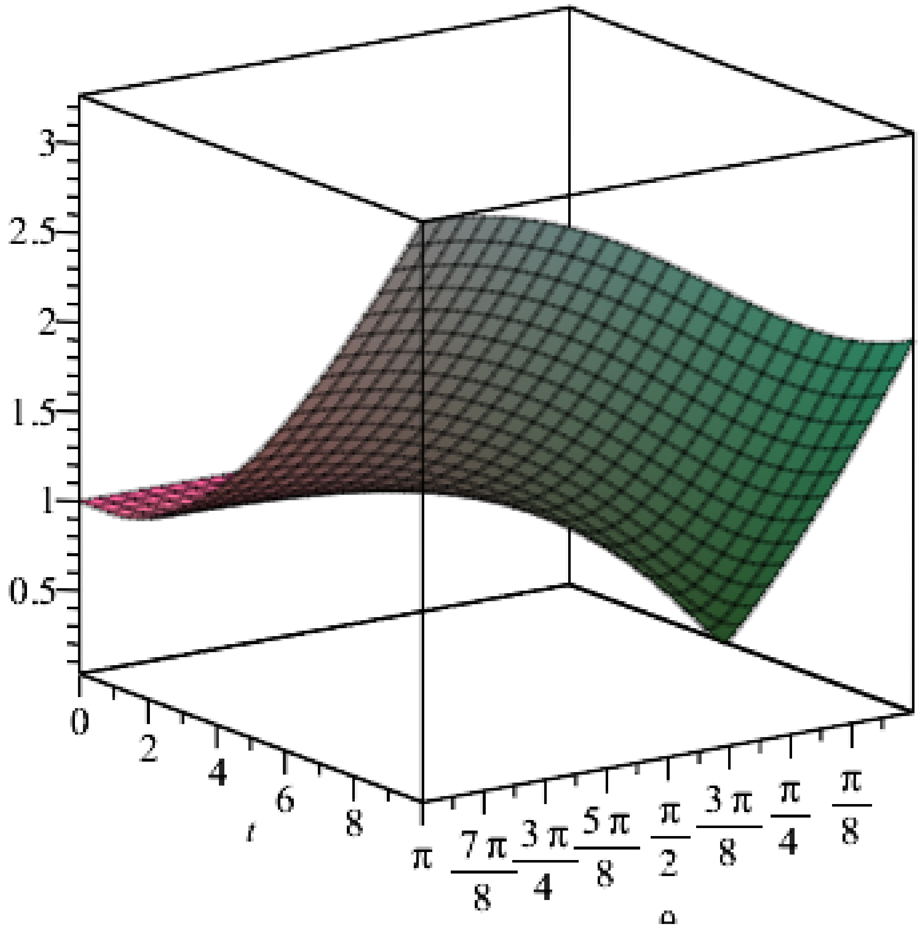
$$RootOf\left(-\left(\begin{cases} 0.05000000000\ \_Z^2 & 0 < \_Z \\ 0 & \mathit{otherwise} \end{cases}\right)^2 + 2\left(\begin{cases} 0.05000000000\ \_Z^2 & 0 < \_Z \\ 0 & \mathit{otherwise} \end{cases}\right)\cos(\Theta) + T^2 - 2\,T\,\_Z + \_Z^2 - 1, -\infty..T\right) \tag{5}$$

$drtau(T, \text{Theta}) := T - rtau(T, \text{Theta}) :$

$plot3d('dftau(t, \text{theta})', \text{theta} = 0..\text{Pi}, t = 0..10)$



plot3d(*drtau*(*t*, theta), theta = 0 ..Pi, *t* = 0 ..10)



Lienard-Wichert radius in the spherical coordinate system

$$K\left(z_0, v_0, a_0, \text{tau}, \tau_0, t, \text{theta}\right) := c\cdot(t-\text{tau}) - \frac{v_q\left(v_0, a_0, \text{tau}, \tau_0\right)}{c}\cdot\left(R_0\cdot\cos(\text{theta}) - z_q\left(z_0, v_0, a_0, \text{tau}, \tau_0\right)\right):$$

Lienard-Wichert radius with substituted numerical values

$Ks(\text{tau}, t, \text{theta}) := subs\left(z_0 = Z, v_0 = V, a_0 = A, c = 1, R_0 = R, \tau_0 = \mathrm{T}_0, K\left(z_0, v_0, a_0, \text{tau}, \tau_0, t, \text{theta}\right)\right):$

$Ks(\text{tau}, t, \text{theta})$

$$t - \tau - \left(\begin{cases} 0.1\,\tau & 0 < \tau \\ 0 & otherwise \end{cases}\right)\left(\cos(\theta) - \left(\begin{cases} 0.05000000000\,\tau^2 & 0 < \tau \\ 0 & otherwise \end{cases}\right)\right) \tag{6}$$

Besides, we substitute the lagging moment function into the formula of the LV radius

$Rlwf(T, \text{Theta}) := Ks(ftau(T, \text{Theta}), T, \text{Theta}):$

$Rlwr(T, \text{Theta}) := Ks(rtau(T, \text{Theta}), T, \text{Theta}):$

$eval(Rlwf(0, 0))$

$$1.000000000 \tag{7}$$

$evalf(Rlwr(0, 0))$

$$1.000000000 \tag{8}$$

$eval(Rlwf(1, 0))$

$$1. \tag{9}$$

$evalf(Rlwr(1, 0))$

$$1. \tag{10}$$

$eval(Rlwf(2, 0))$

$$0.8445824720 \tag{11}$$

$evalf(Rlwr(2, 0))$

$$0.8445824720 \tag{12}$$

$eval(Rlwf(2, \mathrm{Pi}))$

$$1.145341380 \tag{13}$$

$evalf(Rlwr(2, \mathrm{Pi}))$

$$1.145341380 \tag{14}$$

$evalf(Rlwr(0, 0)) + evalf(Rlwr(0, \mathrm{Pi}))$

$$2.000000000 \tag{15}$$

$evalf(Rlwr(1, 0)) + evalf(Rlwr(1, \mathrm{Pi}))$

$$2. \tag{16}$$

$evalf(Rlwr(2, 0)) + evalf(Rlwr(2, \mathrm{Pi}))$

$$1.989923852 \tag{17}$$

$evalf(Rlwr(3, 0)) + evalf(Rlwr(3, \mathrm{Pi}))$

$$1.959630751 \tag{18}$$

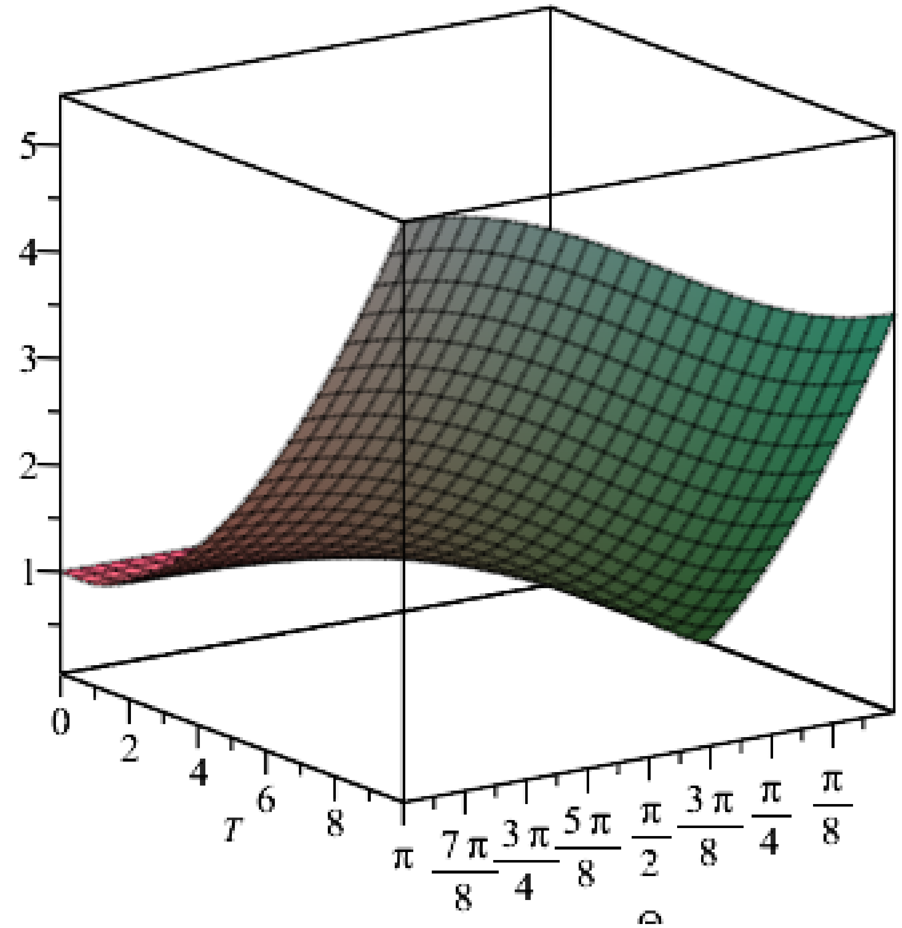
$evalf(Rlwr(4, 0)) + evalf(Rlwr(4, \mathrm{Pi}))$

$$1.914021705 \tag{19}$$

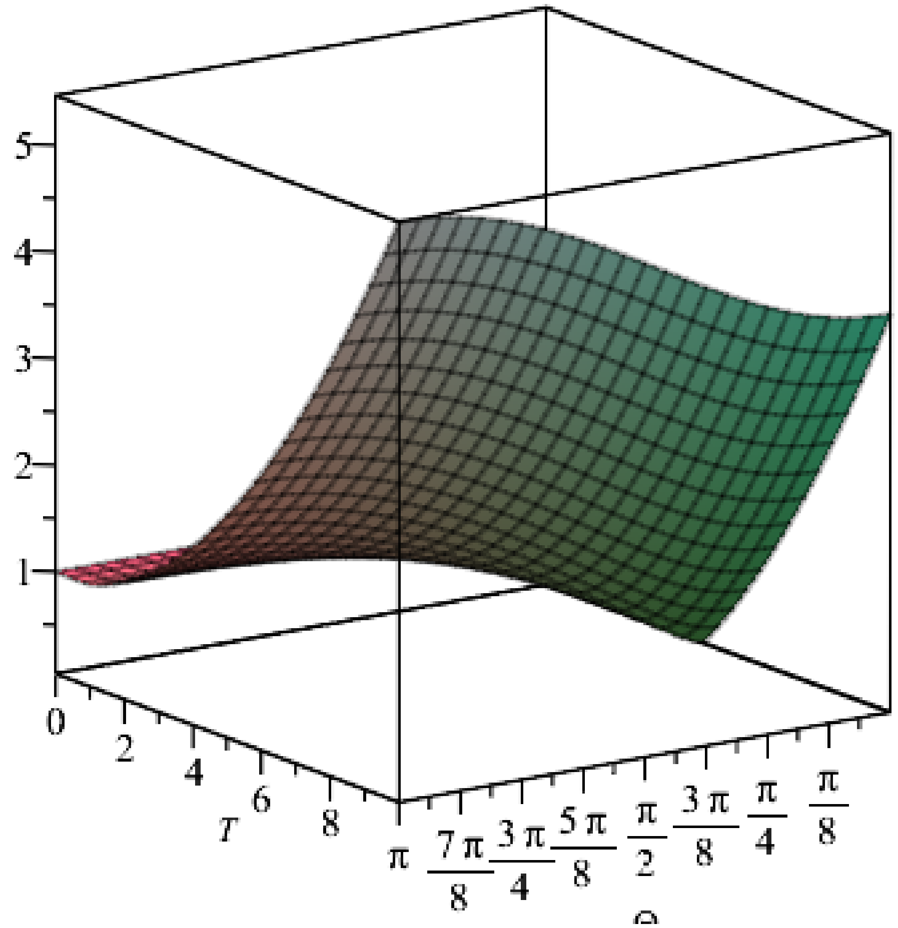
$evalf(Rlwr(5, 0)) + evalf(Rlwr(5, \mathrm{Pi}))$

$$2.373207260 \tag{20}$$

$plot3d('Rlwf(T, \mathrm{Theta})', \mathrm{Theta} = 0\,..\mathrm{Pi}, T = 0\,..10)$



$plot3d('Rlwr(T, \mathrm{Theta})', \mathrm{Theta} = 0..\mathrm{Pi}, T = 0..10)$



Теперь нормальная компонента поля

$$E_n = e\frac{R_0 - z_q(t')\cos(\theta) - (t-t')v_q(t')\cos(\theta)}{K^3}\left(1 + a_q(t')\frac{\left(R_0\cos(\theta) - z_q(t')\right)}{c^2} - \frac{v_q(t')^2}{c^2}\right) - \frac{e}{K^2}(t-t')\frac{a_q(t')}{c}\cos(\theta)$$

$$Enf(T, \text{Theta}) := subs\Big(z_0 = Z, v_0 = V, a_0 = A, c = 1, R_0 = R, \tau_0 = \mathrm{T}_0, \Big(\Big(R_0 - z_q\big(z_0, v_0, a_0, ftau(T, \text{Theta}), \text{tau}_0\big) * \cos(\text{Theta}) - (T - ftau(T, \text{Theta})) * v_q\big(v_0, a_0, ftau(T, \text{Theta}), \text{tau}_0\big) * \cos(\text{Theta})\Big) * \Big(1 + a_q\big(a_0, ftau(T, \text{Theta}), \tau_0\big) * \big(R_0 * \cos(\text{Theta}) - z_q\big(z_0, v_0, a_0, ftau(T, \text{Theta}), \text{tau}_0\big)\big) / c\text{^}2 - v_q\big(v_0, a_0, ftau(T, \text{Theta}), \text{tau}_0\big)\text{^}2 / c\text{^}2\Big) / (Rlwf(T, \text{Theta}))\text{^}3 - (T - ftau(T, \text{Theta})) * \Big(a_q\big(a_0, ftau(T, \text{Theta}), \tau_0\big) / c\Big) * \cos(\text{Theta}) / Rlwf(T, \text{Theta})\text{^}2\Big)\Big)$$

$$(T, \Theta) \rightarrow subs\left(z_0 = Z, v_0 = V, a_0 = A, c = 1, R_0 = R, \tau_0 = \mathrm{T}_0, \frac{1}{Rlwf(T, \Theta)^3}\left(\left(R_0 - z_q\left(z_0, v_0, a_0,\right.\right.\right.\right. \quad \textbf{(21)}$$

$$ftau(T,\Theta),\tau_0\big)\cos(\Theta) - \big(T - ftau(T,\Theta)\big)\, v_q\big(v_0, a_0, ftau(T,\Theta), \tau_0\big)\cos(\Theta)\Big)\Bigg(1 + \frac{a_q\big(a_0, ftau(T,\Theta),\tau_0\big)\big(R_0\cos(\Theta) - z_q(z_0, v_0, a_0, ftau(T,\Theta),\tau_0)\big)}{c^2} - \frac{v_q\big(v_0, a_0, ftau(T,\Theta),\tau_0\big)^2}{c^2}\Bigg)\Bigg) - \frac{\big(T - ftau(T,\Theta)\big)\, a_q\big(a_0, ftau(T,\Theta),\tau_0\big)\cos(\Theta)}{c\, Rlwf(T,\Theta)^2}\Bigg)$$

$Enr(T, \text{Theta}) := subs\big(z_0 = Z, v_0 = V, a_0 = A, c = 1, R_0 = R, \tau_0 = \mathrm{T}_0, \big(\big(R_0 - z_q\big(z_0, v_0, a_0, rtau(T, \text{Theta}), \text{tau}_0\big) * \cos(\text{Theta}) - (T - rtau(T, \text{Theta})) * v_q\big(v_0, a_0, rtau(T, \text{Theta}), \text{tau}_0\big) * \cos(\text{Theta})\big) * \big(1 + a_q\big(a_0, rtau(T, \text{Theta}), \tau_0\big) * \big(R_0 * \cos(\text{Theta}) - z_q\big(z_0, v_0, a_0, rtau(T, \text{Theta}), \text{tau}_0\big)\big) / c\hat{}2 - v_q\big(v_0, a_0, rtau(T, \text{Theta}), \text{tau}_0\big)\hat{}2 / c\hat{}2\big) / (Rlwr(T, \text{Theta}))\hat{}3 - (T - rtau(T, \text{Theta})) * \big(a_q\big(a_0, rtau(T, \text{Theta}), \tau_0\big) / c\big) * \cos(\text{Theta}) / Rlwr(T, \text{Theta})\hat{}2\big)\big)$

$$(T,\Theta) \rightarrow subs\Bigg(z_0 = Z, v_0 = V, a_0 = A, c = 1, R_0 = R, \tau_0 = \mathrm{T}_0, \frac{1}{Rlwr(T,\Theta)^3}\Bigg(\Big(R_0 - z_q\big(z_0, v_0, a_0, rtau(T,\Theta),\tau_0\big)\cos(\Theta) - \big(T - rtau(T,\Theta)\big)\, v_q\big(v_0, a_0, rtau(T,\Theta),\tau_0\big)\cos(\Theta)\Big)\Bigg(1 + \frac{a_q\big(a_0, rtau(T,\Theta),\tau_0\big)\big(R_0\cos(\Theta) - z_q(z_0, v_0, a_0, rtau(T,\Theta),\tau_0)\big)}{c^2} - \frac{v_q\big(v_0, a_0, rtau(T,\Theta),\tau_0\big)^2}{c^2}\Bigg)\Bigg) - \frac{\big(T - rtau(T,\Theta)\big)\, a_q\big(a_0, rtau(T,\Theta),\tau_0\big)\cos(\Theta)}{c\, Rlwr(T,\Theta)^2}\Bigg) \quad \textbf{(22)}$$

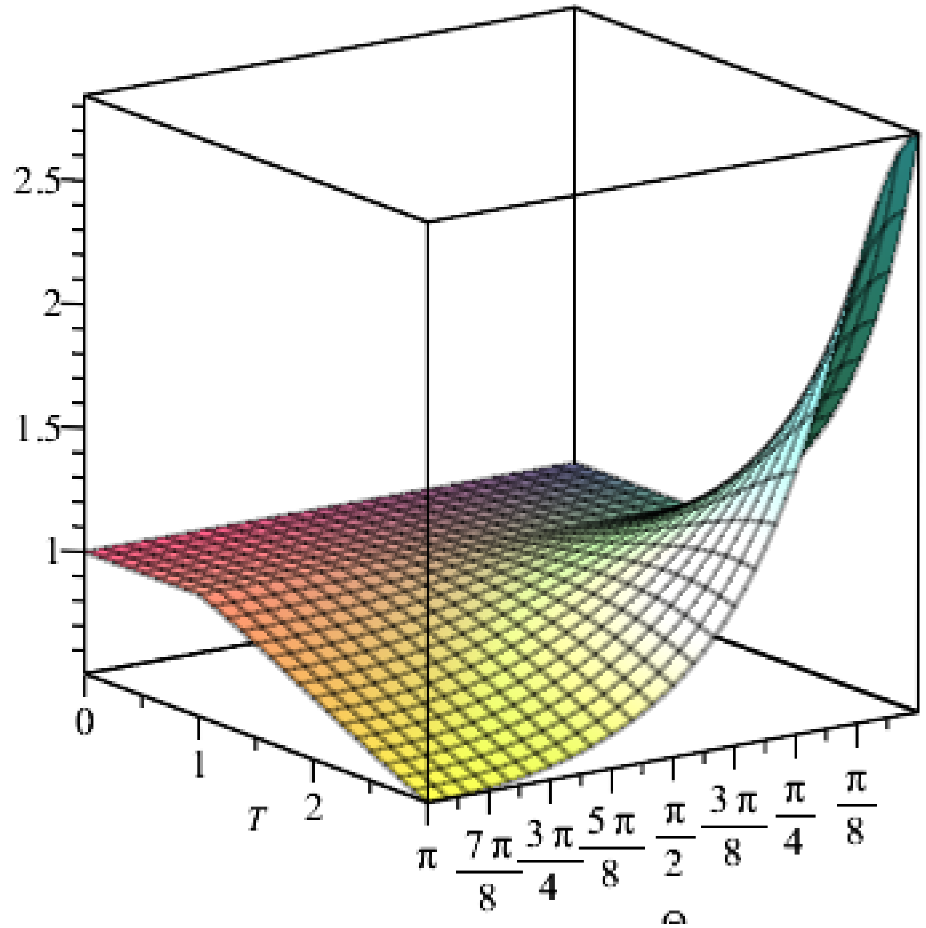
$evalf(Enf(2, 0))$

$$1.386271243 \quad \textbf{(23)}$$

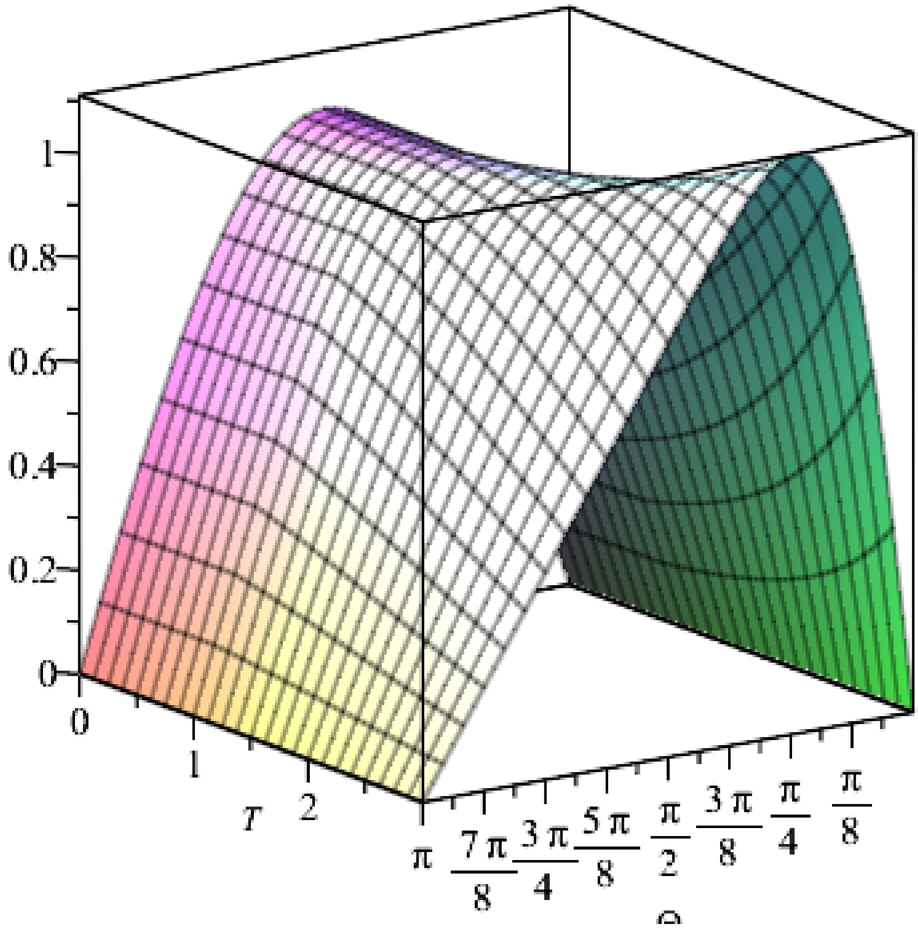
$evalf(Enr(2, 0))$

$$1.386271243 \quad \textbf{(24)}$$

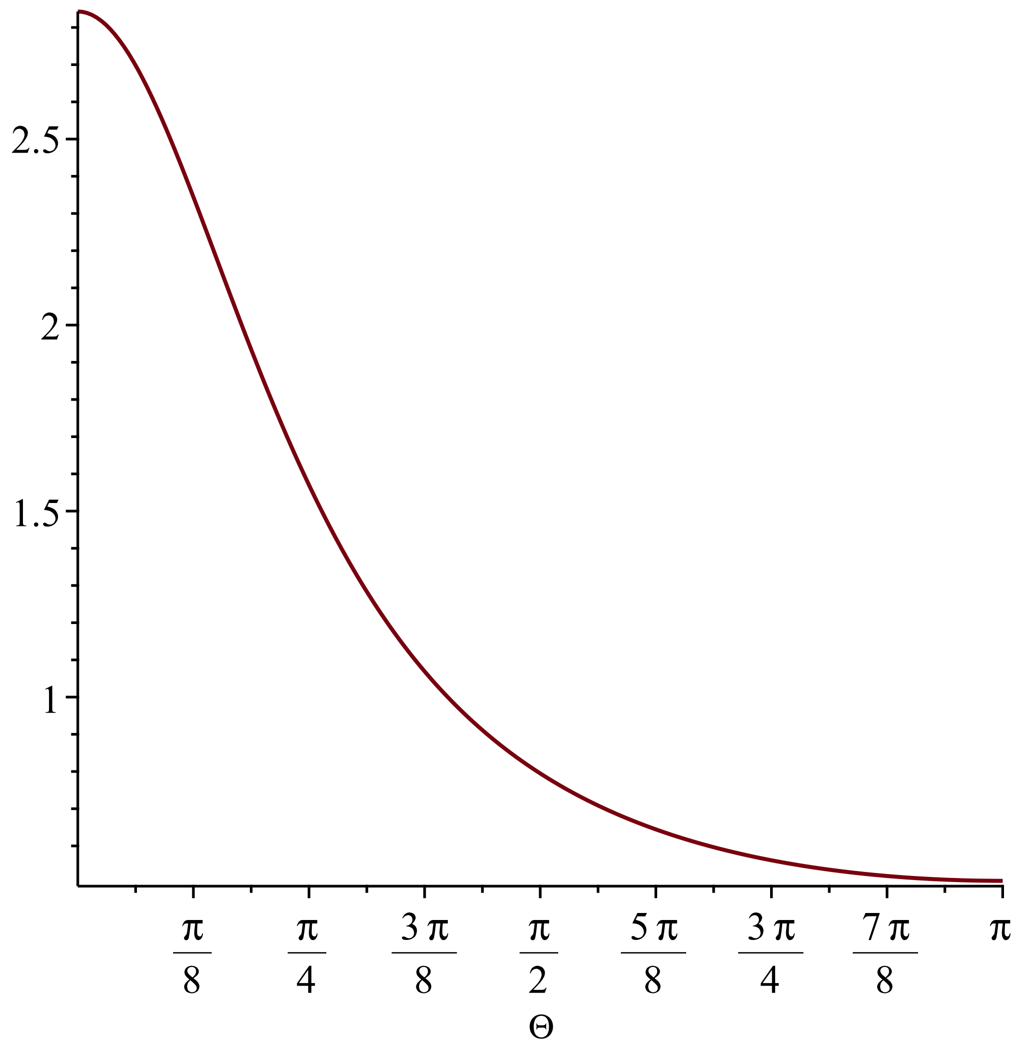
$plot3d('Enf(T, \text{Theta})', \text{Theta} = 0 ..\text{Pi}, T = 0 ..3)$



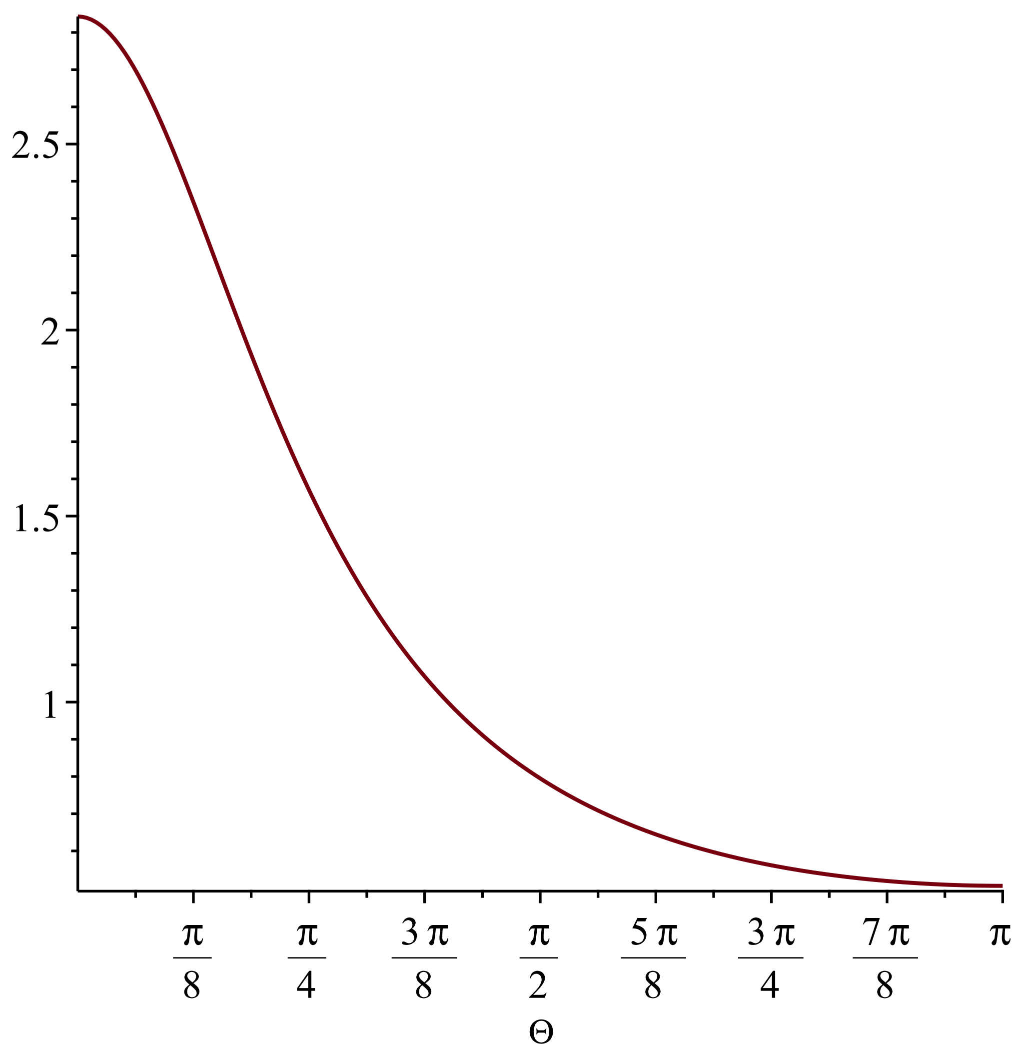
plot3d('*Enf*(*T*, Theta) · sin(Theta)', Theta = 0 ..Pi, *T* = 0 ..3)



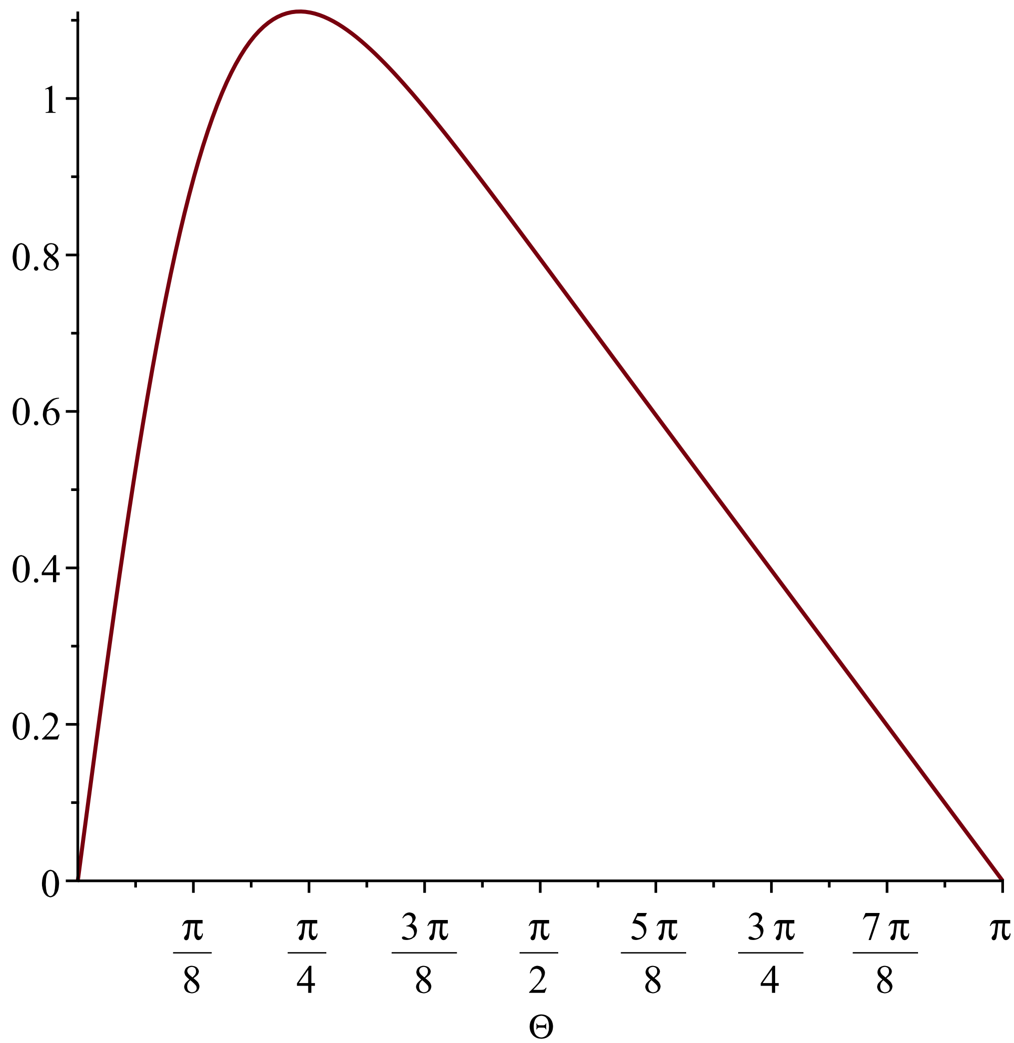
plot('*Enf*(3, Theta)', Theta = 0 ..Pi)



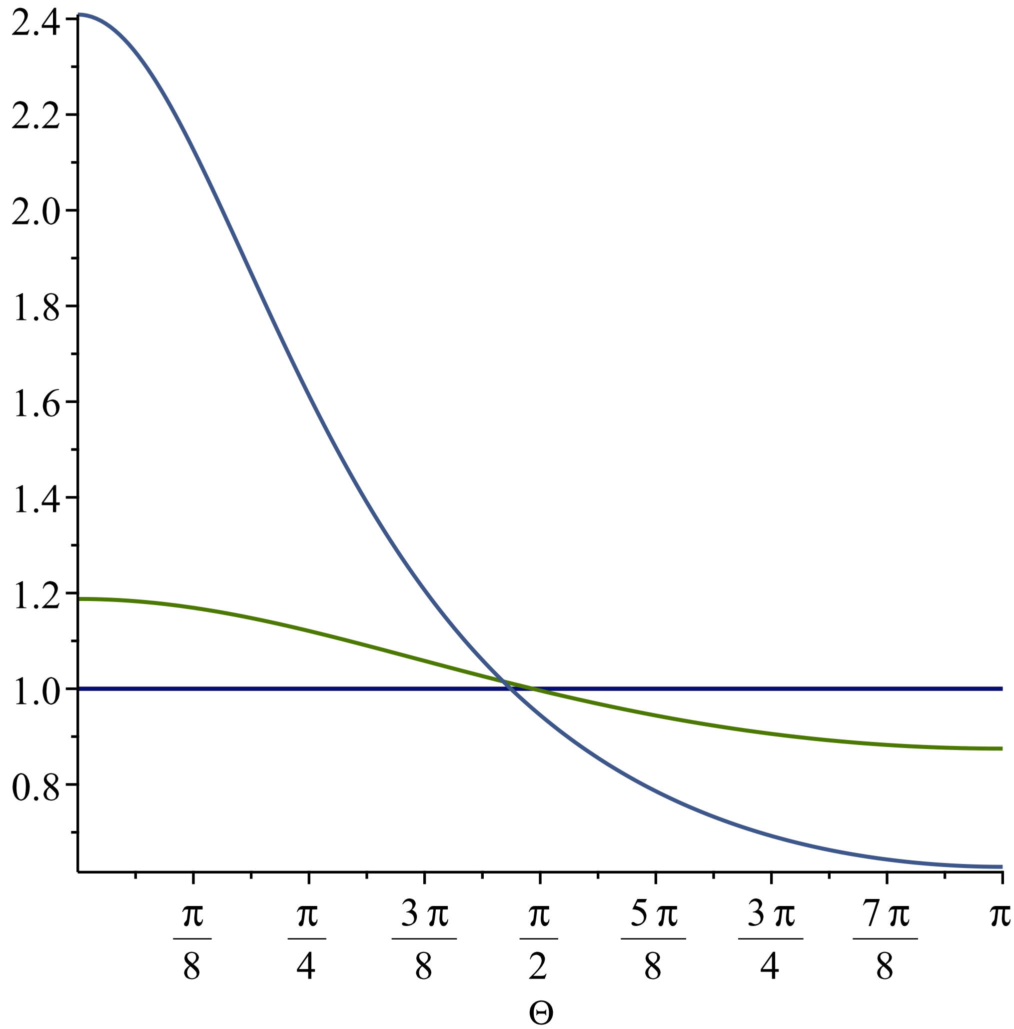
plot('*Enr*(3, Theta)', Theta = 0 ..Pi)



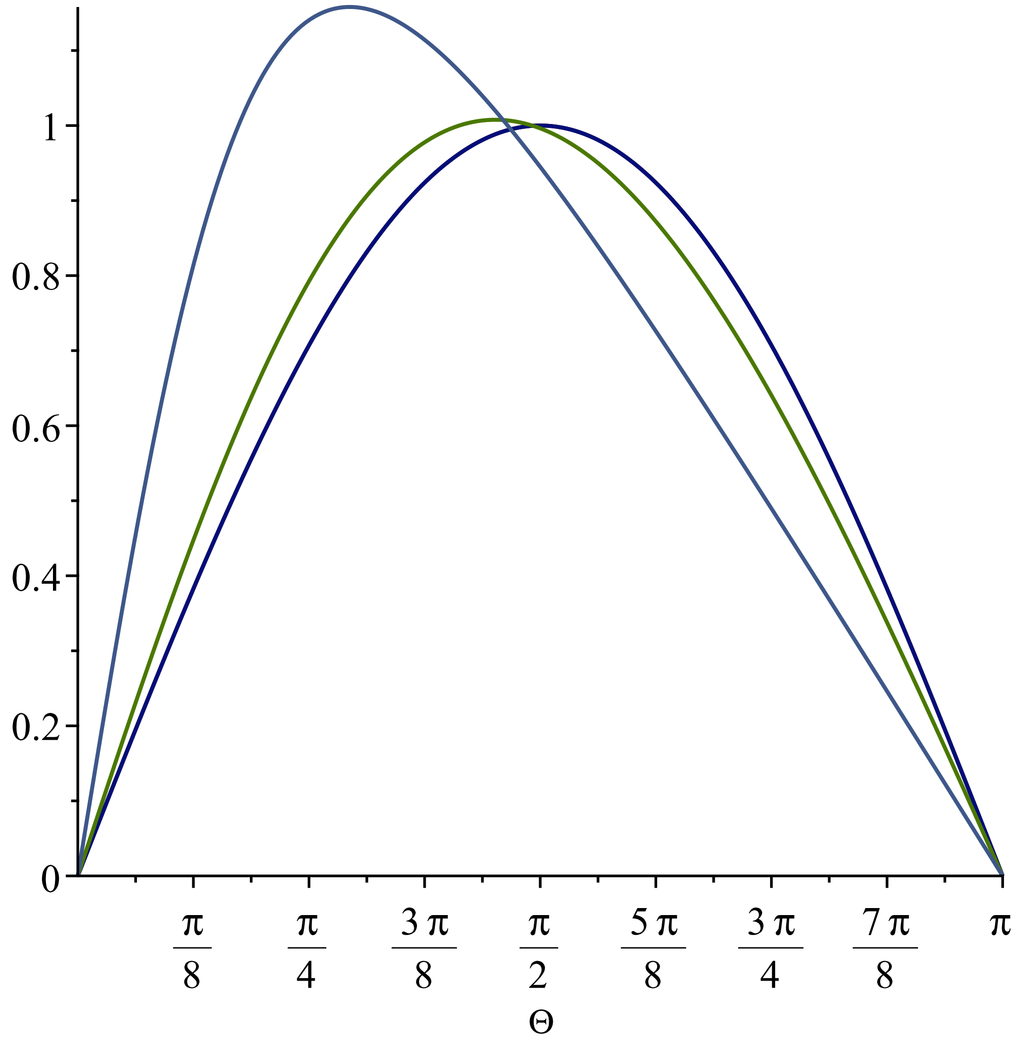
plot('*Enf*(3, Theta) · sin(Theta)', Theta = 0 ..Pi)



$plot([seq](eval(Enr(T, \text{Theta}), T=t), t=0..3), \text{Theta}=0..\text{Pi})$



plot([*seq*](*eval*(*Enr*(*T*, Theta) ·sin(Theta), *T* = *t*), *t* = 0 ..3), Theta = 0 ..Pi)



$k(T) := evalf(Int(Enr(T, \text{Theta}) \cdot (R\verb|^|2) \cdot \sin(\text{Theta}), \text{ Theta} = 0 .. \text{Pi}))$

$$T \rightarrow evalf\left(\int_0^{\pi} Enr(T, \Theta)\, R^2 \sin(\Theta)\, \mathrm{d}\Theta\right) \tag{25}$$

$evalf(k(0))$

$$2.000000000 \tag{26}$$

$evalf(k(1))$

$$2.000000000 \tag{27}$$

$evalf(k(2))$

$$2.015336698 \tag{28}$$

$evalf(k(3))$

$$2.189712498 \tag{29}$$

$evalf(k(4))$

$$3.594958824 \tag{30}$$

$evalf(k(5))$

$$6.429266743 \tag{31}$$

$plot(k(T), T = 0..4)$