

restart : clear :

« , $v_x v_y v_z$.

$$P(x_a, y_a, z_a) \cdot t = 0 - , \quad x_0, y_0, z_0$$

t $x_t = x_0 + v_x \cdot t$ $y_t = y_0 + v_y \cdot t$ $z_t = z_0 + v_z \cdot t$.

положение с учетом запаздывания, т.е. положение в момент $t' = t - r'/c$ (5.1), где r' – расстояние

t'

$$x_{t_{zap}} := x_0 + v_x \cdot t_{zap} : y_{t_{zap}} := y_0 + v_y \cdot t_{zap} : z_{t_{zap}} := z_0 + v_z \cdot t_{zap} : , \quad r_{zap}(t_{zap})$$

$$:= \sqrt{(x_a - x_{t_{zap}})^2 + (y_a - y_{t_{zap}})^2 + (z_a - z_{t_{zap}})^2} \cdot r_{zap}(t_{zap}) (5.2)$$

$$t_{zap} \rightarrow \sqrt{(x_a - x_{t_{zap}})^2 + (y_a - y_{t_{zap}})^2 + (z_a - z_{t_{zap}})^2}$$

$$\sqrt{(-t_{zap} v_x - x_0 + x_a)^2 + (-t_{zap} v_y - y_0 + y_a)^2 + (-t_{zap} v_z - z_0 + z_a)^2} \quad (1)$$

Чтобы найти r' или t' , это уравнение надо сопоставить с (5.1). Исключим сперва r' , решив (5.1) относительно r' , и подставив в (5.2). Возведя затем обе части в квадрат, получим

$$c^2 (t_a - t_{zap})^2 = (-t_{zap} v_x - x_0 + x_a)^2 + (-t_{zap} v_y - y_0 + y_a)^2 + (-t_{zap} v_z - z_0 + z_a)^2$$

$$t_{zap} := solve(c^2 (t_a - t_{zap})^2 = (-t_{zap} v_x - x_0 + x_a)^2 + (-t_{zap} v_y - y_0 + y_a)^2 + (-t_{zap} v_z - z_0 + z_a)^2, t_{zap})$$

$$\frac{1}{c^2 - v_x^2 - v_y^2 - v_z^2} \left(c^2 t_a + v_x x_0 - v_x x_a + v_y y_0 - v_y y_a + v_z z_0 - v_z z_a \right. \quad (2)$$

$$+ (c^2 t_a^2 v_x^2 + c^2 t_a^2 v_y^2 + c^2 t_a^2 v_z^2 + 2 c^2 t_a v_x x_0 - 2 c^2 t_a v_x x_a + 2 c^2 t_a v_y y_0$$

$$- 2 c^2 t_a v_y y_a + 2 c^2 t_a v_z z_0 - 2 c^2 t_a v_z z_a + c^2 x_0^2 - 2 c^2 x_0 x_a + c^2 x_a^2 + c^2 y_0^2 - 2 c^2 y_0 y_a$$

$$+ c^2 y_a^2 + c^2 z_0^2 - 2 c^2 z_0 z_a + c^2 z_a^2 - v_x^2 y_0^2 + 2 v_x^2 y_0 y_a - v_x^2 y_a^2 - v_x^2 z_0^2 + 2 v_x^2 z_0 z_a$$

$$- v_x^2 z_a^2 + 2 v_x v_y x_0 y_0 - 2 v_x v_y x_0 y_a - 2 v_x v_y x_a y_0 + 2 v_x v_y x_a y_a + 2 v_x v_z x_0 z_0 - 2 v_x v_z x_0 z_a$$

$$- 2 v_x v_z x_a z_0 + 2 v_x v_z x_a z_a - v_y^2 x_0^2 + 2 v_y^2 x_0 x_a - v_y^2 x_a^2 - v_y^2 z_0^2 + 2 v_y^2 z_0 z_a - v_y^2 z_a^2$$

$$+ 2 v_y v_z y_0 z_0 - 2 v_y v_z y_0 z_a - 2 v_y v_z y_a z_0 + 2 v_y v_z y_a z_a - v_z^2 x_0^2 + 2 v_z^2 x_0 x_a - v_z^2 x_a^2$$

$$- v_z^2 y_0^2 + 2 v_z^2 y_0 y_a - v_z^2 y_a^2)^{1/2} \Big), - \frac{1}{c^2 - v_x^2 - v_y^2 - v_z^2} \left(-c^2 t_a - v_x x_0 + v_x x_a - v_y y_0 \right.$$

$$+ v_y y_a - v_z z_0 + v_z z_a$$

$$+ (c^2 t_a^2 v_x^2 + c^2 t_a^2 v_y^2 + c^2 t_a^2 v_z^2 + 2 c^2 t_a v_x x_0 - 2 c^2 t_a v_x x_a + 2 c^2 t_a v_y y_0$$

$$- 2 c^2 t_a v_y y_a + 2 c^2 t_a v_z z_0 - 2 c^2 t_a v_z z_a + c^2 x_0^2 - 2 c^2 x_0 x_a + c^2 x_a^2 + c^2 y_0^2 - 2 c^2 y_0 y_a$$

$$\begin{aligned}
& + c^2 y_a^2 + c^2 z_0^2 - 2 c^2 z_0 z_a + c^2 z_a^2 - v_x^2 y_0^2 + 2 v_x^2 y_0 y_a - v_x^2 y_a^2 - v_x^2 z_0^2 + 2 v_x^2 z_0 z_a \\
& - v_x^2 z_a^2 + 2 v_x v_y x_0 y_0 - 2 v_x v_y x_0 y_a - 2 v_x v_y x_a y_0 + 2 v_x v_y x_a y_a + 2 v_x v_z x_0 z_0 - 2 v_x v_z x_0 z_a \\
& - 2 v_x v_z x_a z_0 + 2 v_x v_z x_a z_a - v_y^2 x_0^2 + 2 v_y^2 x_0 x_a - v_y^2 x_a^2 - v_y^2 z_0^2 + 2 v_y^2 z_0 z_a - v_y^2 z_a^2 \\
& + 2 v_y v_z y_0 z_0 - 2 v_y v_z y_0 z_a - 2 v_y v_z y_a z_0 + 2 v_y v_z y_a z_a - v_z^2 x_0^2 + 2 v_z^2 x_0 x_a - v_z^2 x_a^2 \\
& - v_z^2 y_0^2 + 2 v_z^2 y_0 y_a - v_z^2 y_a^2 \Big)^{1/2} \Big)
\end{aligned}$$

$$\begin{aligned}
c^2 (t_a - t_{zap})^2 &= \left(\underset{t_{zap}}{-t_{zap} v_x - x_0 + x_a} \right)^2 + \left(-t_{zap} v_y - y_0 + y_a \right)^2 + \left(-t_{zap} v_z - z_0 + z_a \right)^2 \\
&, \quad ? \\
&,
\end{aligned}$$

$$t_{zap1} := \text{simplify}(t_{zap}[1])$$

$$\begin{aligned}
& \frac{1}{c^2 - v_x^2 - v_y^2 - v_z^2} \Big(\left((t_a^2 v_x^2 + 2 t_a (x_0 - x_a) v_x + t_a^2 v_y^2 + 2 t_a (y_0 - y_a) v_y + t_a^2 v_z^2 + 2 t_a (z_0 \right. \\
& - z_a) v_z + x_0^2 - 2 x_0 x_a + x_a^2 + y_0^2 - 2 y_0 y_a + y_a^2 + (z_0 - z_a)^2 \Big) c^2 + \left(-y_0^2 + 2 y_0 y_a - y_a^2 \right. \\
& - (z_0 - z_a)^2 \Big) v_x^2 + 2 (x_0 - x_a) ((y_0 - y_a) v_y + v_z (z_0 - z_a)) v_x + \left(-x_0^2 + 2 x_0 x_a - x_a^2 \right. \\
& - (z_0 - z_a)^2 \Big) v_y^2 + 2 v_z (y_0 - y_a) (z_0 - z_a) v_y - \left(x_0^2 - 2 x_0 x_a + x_a^2 + (y_0 - y_a)^2 \right) v_z^2 \Big)^{1/2} \\
& + (x_0 - x_a) v_x + (y_0 - y_a) v_y + c^2 t_a + v_z (z_0 - z_a) \Big)
\end{aligned} \tag{3}$$

$$t_{zap2} := \text{simplify}(t_{zap}[2])$$

$$\begin{aligned}
& \frac{1}{c^2 - v_x^2 - v_y^2 - v_z^2} \Big(\\
& - \left((t_a^2 v_x^2 + 2 t_a (x_0 - x_a) v_x + t_a^2 v_y^2 + 2 t_a (y_0 - y_a) v_y + t_a^2 v_z^2 + 2 t_a (z_0 - z_a) v_z \right. \\
& + x_0^2 - 2 x_0 x_a + x_a^2 + y_0^2 - 2 y_0 y_a + y_a^2 + (z_0 - z_a)^2 \Big) c^2 + \left(-y_0^2 + 2 y_0 y_a - y_a^2 - (z_0 \right. \\
& - z_a)^2 \Big) v_x^2 + 2 (x_0 - x_a) ((y_0 - y_a) v_y + v_z (z_0 - z_a)) v_x + \left(-x_0^2 + 2 x_0 x_a - x_a^2 - (z_0 \right. \\
& - z_a)^2 \Big) v_y^2 + 2 v_z (y_0 - y_a) (z_0 - z_a) v_y - \left(x_0^2 - 2 x_0 x_a + x_a^2 + (y_0 - y_a)^2 \right) v_z^2 \Big)^{1/2} + (x_0 \\
& - x_a) v_x + (y_0 - y_a) v_y + c^2 t_a + v_z (z_0 - z_a) \Big)
\end{aligned} \tag{4}$$

$$r_{zap_x} := x_a - x_{t_{zap}} : r_{zap_y} := y_a - y_{t_{zap}} : r_{zap_z} := z_a - z_{t_{zap}} :$$

$$vr_{zap} := v_x \cdot r_{zap_x} + v_y \cdot r_{zap_y} + v_z \cdot r_{zap_z} \\ v_x (-t_{zap} v_x - x_0 + x_a) + v_y (-t_{zap} v_y - y_0 + y_a) + v_z (-t_{zap} v_z - z_0 + z_a) \quad (5)$$

$$vr_{zap1} := subs(t_{zap} = t_{zap1}, vr_{zap}) : simplify(vr_{zap1})$$

$$\frac{1}{c^2 - v_x^2 - v_y^2 - v_z^2} \left((-v_x^2 - v_y^2 - v_z^2) \right. \\ \left((t_a^2 v_x^2 + 2 t_a (x_0 - x_a) v_x + t_a^2 v_y^2 + 2 t_a (y_0 - y_a) v_y + t_a^2 v_z^2 + 2 t_a (z_0 - z_a) v_z + x_0^2 - 2 x_0 x_a + x_a^2 + y_0^2 - 2 y_0 y_a + y_a^2 + (z_0 - z_a)^2) c^2 + (-y_0^2 + 2 y_0 y_a - y_a^2 - (z_0 - z_a)^2) v_x^2 + 2 (x_0 - x_a) ((y_0 - y_a) v_y + v_z (z_0 - z_a)) v_x + (-x_0^2 + 2 x_0 x_a - x_a^2 - (z_0 - z_a)^2) v_y^2 + 2 v_z (y_0 - y_a) (z_0 - z_a) v_y - (x_0^2 - 2 x_0 x_a + x_a^2 + (y_0 - y_a)^2) v_z^2 \right)^{1/2} - c^2 (t_a v_x^2 + (x_0 - x_a) v_x + t_a v_y^2 + (y_0 - y_a) v_y + v_z (t_a v_z + z_0 - z_a)) \left. \right) \quad (6)$$

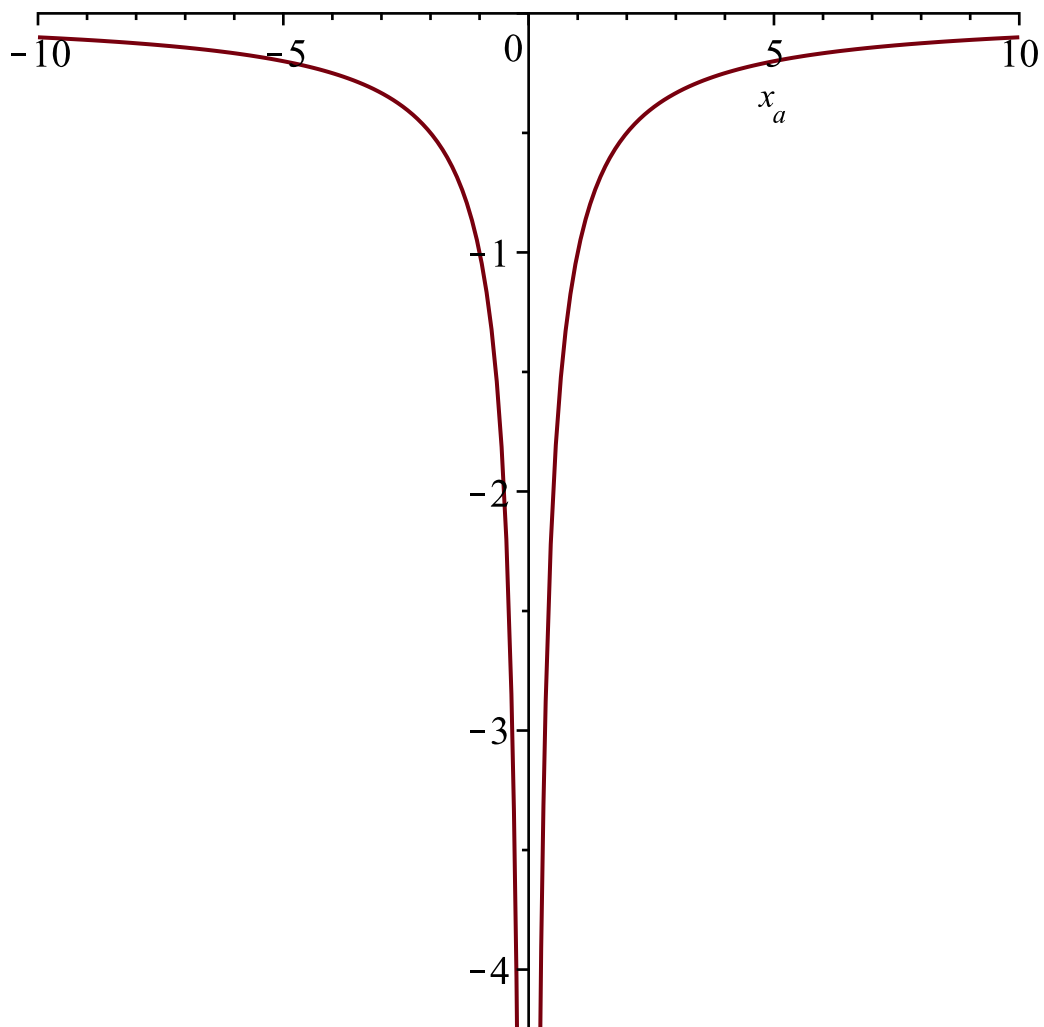
$$vr_{zap2} := subs(t_{zap} = t_{zap2}, vr_{zap}) : simplify(vr_{zap2})$$

$$\frac{1}{c^2 - v_x^2 - v_y^2 - v_z^2} \left((v_x^2 + v_y^2 + v_z^2) \right. \\ \left((t_a^2 v_x^2 + 2 t_a (x_0 - x_a) v_x + t_a^2 v_y^2 + 2 t_a (y_0 - y_a) v_y + t_a^2 v_z^2 + 2 t_a (z_0 - z_a) v_z + x_0^2 - 2 x_0 x_a + x_a^2 + y_0^2 - 2 y_0 y_a + y_a^2 + (z_0 - z_a)^2) c^2 + (-y_0^2 + 2 y_0 y_a - y_a^2 - (z_0 - z_a)^2) v_x^2 + 2 (x_0 - x_a) ((y_0 - y_a) v_y + v_z (z_0 - z_a)) v_x + (-x_0^2 + 2 x_0 x_a - x_a^2 - (z_0 - z_a)^2) v_y^2 + 2 v_z (y_0 - y_a) (z_0 - z_a) v_y - (x_0^2 - 2 x_0 x_a + x_a^2 + (y_0 - y_a)^2) v_z^2 \right)^{1/2} - c^2 (t_a v_x^2 + (x_0 - x_a) v_x + t_a v_y^2 + (y_0 - y_a) v_y + v_z (t_a v_z + z_0 - z_a)) \left. \right) \quad (7)$$

$$\begin{aligned}
r_{zap1} &:= c \cdot (t_a - t_{zap1}) : \text{simplify}(r_{zap1}) : K_1 := \text{simplify}\left(r_{zap1} - \frac{vr_{zap1}}{c}\right) \\
&- \frac{1}{c} \left((t_a^2 v_x^2 + 2 t_a (x_0 - x_a) v_x + t_a^2 v_y^2 + 2 t_a (y_0 - y_a) v_y + t_a^2 v_z^2 + 2 t_a (z_0 - z_a) v_z + x_0^2 \right. \\
&\quad - 2 x_0 x_a + x_a^2 + y_0^2 - 2 y_0 y_a + y_a^2 + (z_0 - z_a)^2) c^2 + (-y_0^2 + 2 y_0 y_a - y_a^2 - (z_0 \\
&\quad - z_a)^2) v_x^2 + 2 (x_0 - x_a) ((y_0 - y_a) v_y + v_z (z_0 - z_a)) v_x + (-x_0^2 + 2 x_0 x_a - x_a^2 - (z_0 \\
&\quad - z_a)^2) v_y^2 + 2 v_z (y_0 - y_a) (z_0 - z_a) v_y - (x_0^2 - 2 x_0 x_a + x_a^2 + (y_0 - y_a)^2) v_z^2 \Big)^{1/2}
\end{aligned} \tag{8}$$

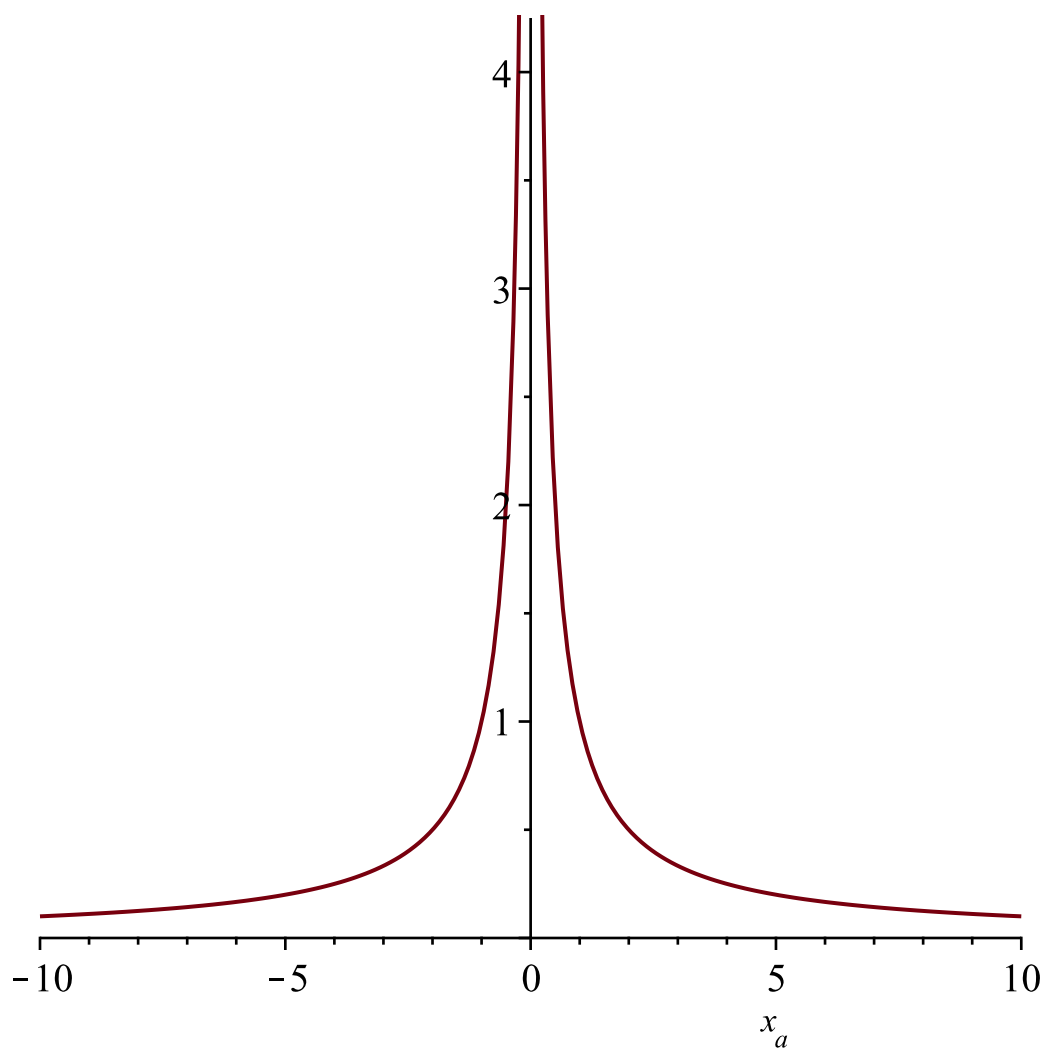
$$\begin{aligned}
r_{zap2} &:= c \cdot (t_a - t_{zap2}) : \text{simplify}(r_{zap2}) : K_2 := \text{simplify}\left(r_{zap2} - \frac{vr_{zap2}}{c}\right) \\
&\frac{1}{c} \left((t_a^2 v_x^2 + 2 t_a (x_0 - x_a) v_x + t_a^2 v_y^2 + 2 t_a (y_0 - y_a) v_y + t_a^2 v_z^2 + 2 t_a (z_0 - z_a) v_z + x_0^2 \right. \\
&\quad - 2 x_0 x_a + x_a^2 + y_0^2 - 2 y_0 y_a + y_a^2 + (z_0 - z_a)^2) c^2 + (-y_0^2 + 2 y_0 y_a - y_a^2 - (z_0 \\
&\quad - z_a)^2) v_x^2 + 2 (x_0 - x_a) ((y_0 - y_a) v_y + v_z (z_0 - z_a)) v_x + (-x_0^2 + 2 x_0 x_a - x_a^2 - (z_0 \\
&\quad - z_a)^2) v_y^2 + 2 v_z (y_0 - y_a) (z_0 - z_a) v_y - (x_0^2 - 2 x_0 x_a + x_a^2 + (y_0 - y_a)^2) v_z^2 \Big)^{1/2}
\end{aligned} \tag{9}$$

$$\begin{aligned}
&\text{simplify}\left(\text{subs}\left(v_y=0, v_z=0, x_0=0, y_0=0, z_0=0, \frac{1}{K_l}\right)\right); \text{plot}\left(\text{simplify}\left(\text{subs}\left(v_y=0, v_z=0, x_0=0, y_0\right.\right.\right. \\
&\quad \left.\left.\left.=0, z_0=0, y_a=0, z_a=0, c=3, v_x=2, t_a=0, \frac{1}{K_l}\right)\right), x_a\right) \\
&\quad - \frac{c}{\sqrt{(t_a^2 v_x^2 - 2 t_a v_x x_a + x_a^2 + y_a^2 + z_a^2) c^2 - v_x^2 (y_a^2 + z_a^2)}}
\end{aligned}$$

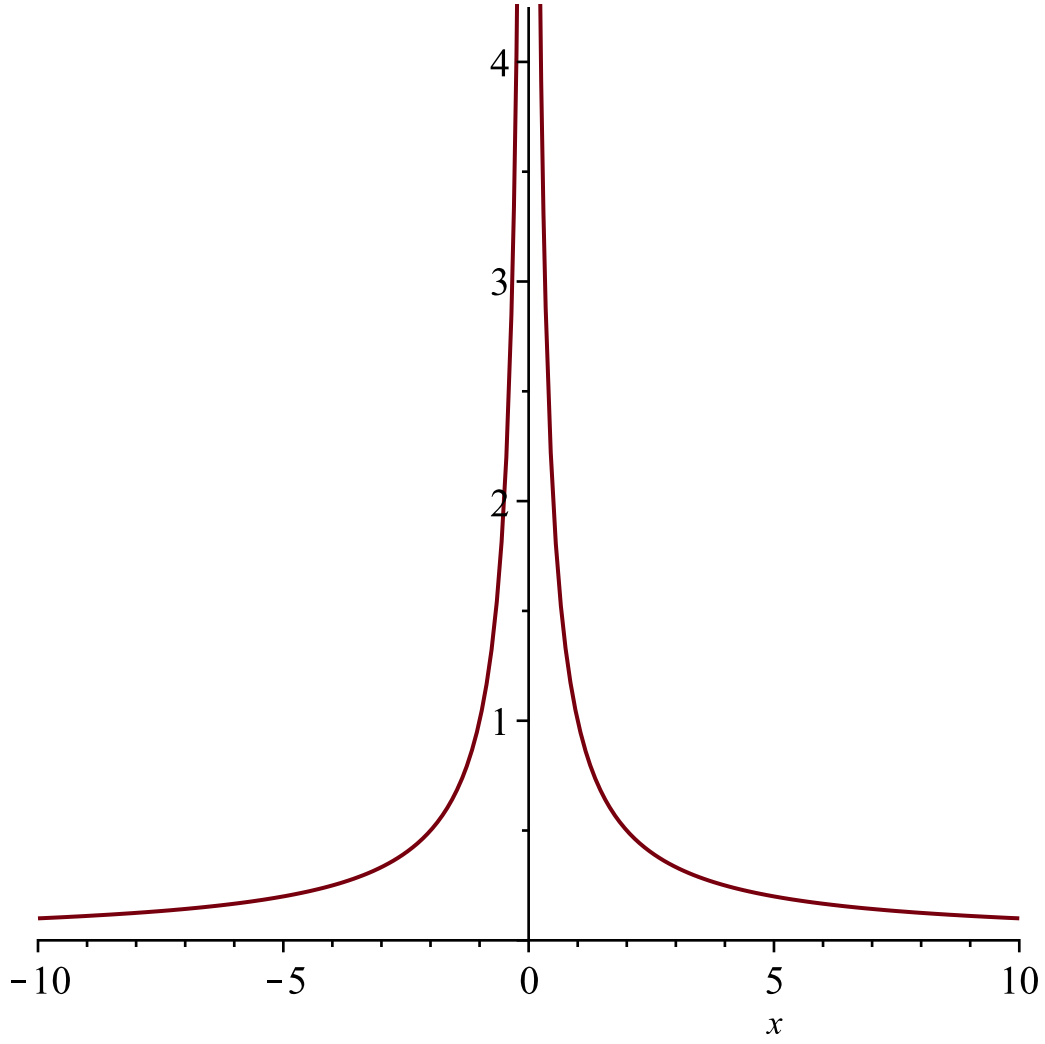


$$\text{simplify}\left(\text{subs}\left(v_y=0, v_z=0, x_0=0, y_0=0, z_0=0, \frac{1}{K_2}\right)\right); \text{plot}\left(\text{simplify}\left(\text{subs}\left(v_y=0, v_z=0, x_0=0, y_0=0, z_0=0, y_a=0, z_a=0, c=3, v_x=2, t_a=0, \frac{1}{K_2}\right)\right), x_a\right)$$

$$\frac{c}{\sqrt{\left(t_a^2 v_x^2 - 2 t_a v_x x_a + x_a^2 + y_a^2 + z_a^2\right) c^2 - v_x^2 \left(y_a^2 + z_a^2\right)}}$$



$$K := \left(\sqrt{(x - v \cdot t)^2 + \left(1 - \frac{v^2}{c^2}\right) \cdot (y^2 + z^2)} \right) : plot \left(simplify \left(subs \left(z = 0, v = 2, c = 3, y = 0, t = 0, \right. \right. \right. \\ \left. \left. \left. \frac{1}{K} \right) \right) \right)$$



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$$E\varphi_x := -\frac{\partial}{\partial x_a} \frac{1}{K_2}$$

$$\frac{1}{2} \left(c \left((-2 t_a v_x - 2 x_0 + 2 x_a) c^2 - 2 \left((y_0 - y_a) v_y + v_z (z_0 - z_a) \right) v_x + (2 x_0 - 2 x_a) v_y^2 - (\right. \right. \quad (10)$$

$$\left. -2 x_0 + 2 x_a) v_z^2 \right) \Big/ \left((t_a^2 v_x^2 + 2 t_a (x_0 - x_a) v_x + t_a^2 v_y^2 + 2 t_a (y_0 - y_a) v_y + t_a^2 v_z^2 \right.$$

$$+ 2 t_a (z_0 - z_a) v_z + x_0^2 - 2 x_0 x_a + x_a^2 + y_0^2 - 2 y_0 y_a + y_a^2 + (z_0 - z_a)^2 \Big) c^2 + (-y_0^2$$

$$+ 2 y_0 y_a - y_a^2 - (z_0 - z_a)^2) v_x^2 + 2 (x_0 - x_a) \left((y_0 - y_a) v_y + v_z (z_0 - z_a) \right) v_x + (-x_0^2$$

$$+ 2 x_0 x_a - x_a^2 - (z_0 - z_a)^2) v_y^2 + 2 v_z (y_0 - y_a) (z_0 - z_a) v_y - (x_0^2 - 2 x_0 x_a + x_a^2$$

$$+ (y_0 - y_a)^2) v_z^2)^{3/2}$$

$$E\varphi_y := -\frac{\partial}{\partial y_a} \frac{1}{K_2}$$

$$\begin{aligned} & \frac{1}{2} \left(c \left((-2 t_a v_y - 2 y_0 + 2 y_a) c^2 + (2 y_0 - 2 y_a) v_x^2 - 2 (x_0 - x_a) v_y v_x - 2 v_z (z_0 - z_a) v_y - \right. \right. \\ & \left. \left. - 2 y_0 + 2 y_a) v_z^2 \right) \right) / \left((t_a^2 v_x^2 + 2 t_a (x_0 - x_a) v_x + t_a^2 v_y^2 + 2 t_a (y_0 - y_a) v_y + t_a^2 v_z^2 \right. \\ & \left. + 2 t_a (z_0 - z_a) v_z + x_0^2 - 2 x_0 x_a + x_a^2 + y_0^2 - 2 y_0 y_a + y_a^2 + (z_0 - z_a)^2) c^2 + (-y_0^2 \right. \\ & \left. + 2 y_0 y_a - y_a^2 - (z_0 - z_a)^2) v_x^2 + 2 (x_0 - x_a) ((y_0 - y_a) v_y + v_z (z_0 - z_a)) v_x + (-x_0^2 \right. \\ & \left. + 2 x_0 x_a - x_a^2 - (z_0 - z_a)^2) v_y^2 + 2 v_z (y_0 - y_a) (z_0 - z_a) v_y - (x_0^2 - 2 x_0 x_a + x_a^2 \right. \\ & \left. + (y_0 - y_a)^2) v_z^2 \right)^{3/2} \end{aligned} \quad (11)$$

$$E\varphi_z := -\frac{\partial}{\partial z_a} \frac{1}{K_2}$$

$$\begin{aligned} & \frac{1}{2} \left(c \left((-2 t_a v_z - 2 z_0 + 2 z_a) c^2 + (2 z_0 - 2 z_a) v_x^2 - 2 (x_0 - x_a) v_z v_x + (2 z_0 - 2 z_a) v_y^2 \right. \right. \\ & \left. \left. - 2 v_z (y_0 - y_a) v_y \right) \right) / \left((t_a^2 v_x^2 + 2 t_a (x_0 - x_a) v_x + t_a^2 v_y^2 + 2 t_a (y_0 - y_a) v_y + t_a^2 v_z^2 \right. \\ & \left. + 2 t_a (z_0 - z_a) v_z + x_0^2 - 2 x_0 x_a + x_a^2 + y_0^2 - 2 y_0 y_a + y_a^2 + (z_0 - z_a)^2) c^2 + (-y_0^2 \right. \\ & \left. + 2 y_0 y_a - y_a^2 - (z_0 - z_a)^2) v_x^2 + 2 (x_0 - x_a) ((y_0 - y_a) v_y + v_z (z_0 - z_a)) v_x + (-x_0^2 \right. \\ & \left. + 2 x_0 x_a - x_a^2 - (z_0 - z_a)^2) v_y^2 + 2 v_z (y_0 - y_a) (z_0 - z_a) v_y - (x_0^2 - 2 x_0 x_a + x_a^2 \right. \\ & \left. + (y_0 - y_a)^2) v_z^2 \right)^{3/2} \end{aligned} \quad (12)$$

$$\text{simplify}(\text{subs}(v_y=0, v_z=0, x_0=0, y_0=0, z_0=0, E\varphi_x))$$

$$- \frac{c^3 (t_a v_x - x_a)}{\left((t_a^2 v_x^2 - 2 t_a v_x x_a + x_a^2 + y_a^2 + z_a^2) c^2 - v_x^2 (y_a^2 + z_a^2) \right)^{3/2}} \quad (13)$$

$$\text{simplify}(\text{subs}(v_y=0, v_z=0, x_0=0, y_0=0, z_0=0, E\varphi_y))$$

$$\frac{c y_a (c^2 - v_x^2)}{\left((t_a^2 v_x^2 - 2 t_a v_x x_a + x_a^2 + y_a^2 + z_a^2) c^2 - v_x^2 (y_a^2 + z_a^2) \right)^{3/2}} \quad (14)$$

$$\text{simplify}(\text{subs}(v_y=0, v_z=0, x_0=0, y_0=0, z_0=0, E\varphi_z))$$

$$\frac{c z_a (c^2 - v_x^2)}{\left((t_a^2 v_x^2 - 2 t_a v_x x_a + x_a^2 + y_a^2 + z_a^2) c^2 - v_x^2 (y_a^2 + z_a^2) \right)^{3/2}} \quad (15)$$

-

$$A_x := \frac{v_x}{c \cdot K_2} \quad (16)$$

$$\begin{aligned} & \left((t_a^2 v_x^2 + 2 t_a (x_0 - x_a) v_x + t_a^2 v_y^2 + 2 t_a (y_0 - y_a) v_y + t_a^2 v_z^2 + 2 t_a (z_0 - z_a) v_z \right. \\ & \quad \left. + x_0^2 - 2 x_0 x_a + x_a^2 + y_0^2 - 2 y_0 y_a + y_a^2 + (z_0 - z_a)^2 \right) c^2 + \left(-y_0^2 + 2 y_0 y_a - y_a^2 - (z_0 \right. \\ & \quad \left. - z_a)^2 \right) v_x^2 + 2 (x_0 - x_a) \left((y_0 - y_a) v_y + v_z (z_0 - z_a) \right) v_x + \left(-x_0^2 + 2 x_0 x_a - x_a^2 - (z_0 \right. \\ & \quad \left. - z_a)^2 \right) v_y^2 + 2 v_z (y_0 - y_a) (z_0 - z_a) v_y - \left(x_0^2 - 2 x_0 x_a + x_a^2 + (y_0 - y_a)^2 \right) v_z^2 \Big)^{1/2} \end{aligned}$$

$$A_y := \frac{v_y}{c \cdot K_2} \quad (17)$$

$$\begin{aligned} & \left((t_a^2 v_x^2 + 2 t_a (x_0 - x_a) v_x + t_a^2 v_y^2 + 2 t_a (y_0 - y_a) v_y + t_a^2 v_z^2 + 2 t_a (z_0 - z_a) v_z \right. \\ & \quad \left. + x_0^2 - 2 x_0 x_a + x_a^2 + y_0^2 - 2 y_0 y_a + y_a^2 + (z_0 - z_a)^2 \right) c^2 + \left(-y_0^2 + 2 y_0 y_a - y_a^2 - (z_0 \right. \\ & \quad \left. - z_a)^2 \right) v_x^2 + 2 (x_0 - x_a) \left((y_0 - y_a) v_y + v_z (z_0 - z_a) \right) v_x + \left(-x_0^2 + 2 x_0 x_a - x_a^2 - (z_0 \right. \\ & \quad \left. - z_a)^2 \right) v_y^2 + 2 v_z (y_0 - y_a) (z_0 - z_a) v_y - \left(x_0^2 - 2 x_0 x_a + x_a^2 + (y_0 - y_a)^2 \right) v_z^2 \Big)^{1/2} \end{aligned}$$

$$A_z := \frac{v_z}{c \cdot K_2} \quad (18)$$

$$\begin{aligned} & \left((t_a^2 v_x^2 + 2 t_a (x_0 - x_a) v_x + t_a^2 v_y^2 + 2 t_a (y_0 - y_a) v_y + t_a^2 v_z^2 + 2 t_a (z_0 - z_a) v_z \right. \\ & \quad \left. + x_0^2 - 2 x_0 x_a + x_a^2 + y_0^2 - 2 y_0 y_a + y_a^2 + (z_0 - z_a)^2 \right) c^2 + \left(-y_0^2 + 2 y_0 y_a - y_a^2 - (z_0 \right. \\ & \quad \left. - z_a)^2 \right) v_x^2 + 2 (x_0 - x_a) \left((y_0 - y_a) v_y + v_z (z_0 - z_a) \right) v_x + \left(-x_0^2 + 2 x_0 x_a - x_a^2 - (z_0 \right. \\ & \quad \left. - z_a)^2 \right) v_y^2 + 2 v_z (y_0 - y_a) (z_0 - z_a) v_y - \left(x_0^2 - 2 x_0 x_a + x_a^2 + (y_0 - y_a)^2 \right) v_z^2 \Big)^{1/2} \end{aligned}$$

$$EA_x := -\frac{1}{c} \cdot \frac{\partial}{\partial t_a} A_x$$

$$\frac{1}{2} \left(c v_x \left(2 t_a v_x^2 + 2 (x_0 - x_a) v_x + 2 t_a v_y^2 + 2 (y_0 - y_a) v_y + 2 t_a v_z^2 + 2 v_z (z_0 - z_a) \right) \right) / \quad (19)$$

$$\left(\left(t_a^2 v_x^2 + 2 t_a (x_0 - x_a) v_x + t_a^2 v_y^2 + 2 t_a (y_0 - y_a) v_y + t_a^2 v_z^2 + 2 t_a (z_0 - z_a) v_z + x_0^2 - 2 x_0 x_a + x_a^2 + y_0^2 - 2 y_0 y_a + y_a^2 + (z_0 - z_a)^2 \right) c^2 + \left(-y_0^2 + 2 y_0 y_a - y_a^2 - (z_0 - z_a)^2 \right) v_x^2 + 2 (x_0 - x_a) \left((y_0 - y_a) v_y + v_z (z_0 - z_a) \right) v_x + \left(-x_0^2 + 2 x_0 x_a - x_a^2 - (z_0 - z_a)^2 \right) v_y^2 + 2 v_z (y_0 - y_a) (z_0 - z_a) v_y - \left(x_0^2 - 2 x_0 x_a + x_a^2 + (y_0 - y_a)^2 \right) v_z^2 \right)^{3/2}$$

$$EA_y := -\frac{1}{c} \cdot \frac{\partial}{\partial t_a} A_y$$

$$\frac{1}{2} \left(c v_y \left(2 t_a v_x^2 + 2 (x_0 - x_a) v_x + 2 t_a v_y^2 + 2 (y_0 - y_a) v_y + 2 t_a v_z^2 + 2 v_z (z_0 - z_a) \right) \right) / \quad (20)$$

$$\left(\left(t_a^2 v_x^2 + 2 t_a (x_0 - x_a) v_x + t_a^2 v_y^2 + 2 t_a (y_0 - y_a) v_y + t_a^2 v_z^2 + 2 t_a (z_0 - z_a) v_z + x_0^2 - 2 x_0 x_a + x_a^2 + y_0^2 - 2 y_0 y_a + y_a^2 + (z_0 - z_a)^2 \right) c^2 + \left(-y_0^2 + 2 y_0 y_a - y_a^2 - (z_0 - z_a)^2 \right) v_x^2 + 2 (x_0 - x_a) \left((y_0 - y_a) v_y + v_z (z_0 - z_a) \right) v_x + \left(-x_0^2 + 2 x_0 x_a - x_a^2 - (z_0 - z_a)^2 \right) v_y^2 + 2 v_z (y_0 - y_a) (z_0 - z_a) v_y - \left(x_0^2 - 2 x_0 x_a + x_a^2 + (y_0 - y_a)^2 \right) v_z^2 \right)^{3/2}$$

$$EA_z := -\frac{1}{c} \cdot \frac{\partial}{\partial t_a} A_z$$

$$\frac{1}{2} \left(c v_z \left(2 t_a v_x^2 + 2 (x_0 - x_a) v_x + 2 t_a v_y^2 + 2 (y_0 - y_a) v_y + 2 t_a v_z^2 + 2 v_z (z_0 - z_a) \right) \right) / \quad (21)$$

$$\left(\left(t_a^2 v_x^2 + 2 t_a (x_0 - x_a) v_x + t_a^2 v_y^2 + 2 t_a (y_0 - y_a) v_y + t_a^2 v_z^2 + 2 t_a (z_0 - z_a) v_z + x_0^2 - 2 x_0 x_a + x_a^2 + y_0^2 - 2 y_0 y_a + y_a^2 + (z_0 - z_a)^2 \right) c^2 + \left(-y_0^2 + 2 y_0 y_a - y_a^2 - (z_0 - z_a)^2 \right) v_x^2 + 2 (x_0 - x_a) \left((y_0 - y_a) v_y + v_z (z_0 - z_a) \right) v_x + \left(-x_0^2 + 2 x_0 x_a - x_a^2 - (z_0 - z_a)^2 \right) v_y^2 + 2 v_z (y_0 - y_a) (z_0 - z_a) v_y - \left(x_0^2 - 2 x_0 x_a + x_a^2 + (y_0 - y_a)^2 \right) v_z^2 \right)^{3/2}$$

-

$$E_x := E\varphi_x + EA_x$$

$$\frac{1}{2} \left(c \left(\left(-2 t_a v_x - 2 x_0 + 2 x_a \right) c^2 - 2 \left((y_0 - y_a) v_y + v_z (z_0 - z_a) \right) v_x + \left(2 x_0 - 2 x_a \right) v_y^2 - \left(-2 x_0 + 2 x_a \right) v_z^2 \right) \right) / \quad (22)$$

$$\left(\left(t_a^2 v_x^2 + 2 t_a (x_0 - x_a) v_x + t_a^2 v_y^2 + 2 t_a (y_0 - y_a) v_y + t_a^2 v_z^2 + 2 t_a (z_0 - z_a) v_z + x_0^2 - 2 x_0 x_a + x_a^2 + y_0^2 - 2 y_0 y_a + y_a^2 + (z_0 - z_a)^2 \right) c^2 + \left(-y_0^2 + 2 y_0 y_a - y_a^2 - (z_0 - z_a)^2 \right) v_x^2 + 2 (x_0 - x_a) \left((y_0 - y_a) v_y + v_z (z_0 - z_a) \right) v_x + \left(-x_0^2 + 2 x_0 x_a - x_a^2 - (z_0 - z_a)^2 \right) v_y^2 + 2 v_z (y_0 - y_a) (z_0 - z_a) v_y - \left(x_0^2 - 2 x_0 x_a + x_a^2 + (y_0 - y_a)^2 \right) v_z^2 \right)^{3/2}$$

$$\begin{aligned}
& + 2 x_0 x_a - x_a^2 - (z_0 - z_a)^2 v_y^2 + 2 v_z (y_0 - y_a) (z_0 - z_a) v_y - (x_0^2 - 2 x_0 x_a + x_a^2 \\
& + (y_0 - y_a)^2 v_z^2)^{3/2} + \frac{1}{2} \left(c v_x (2 t_a v_x^2 + 2 (x_0 - x_a) v_x + 2 t_a v_y^2 + 2 (y_0 - y_a) v_y \right. \\
& + 2 t_a v_z^2 + 2 v_z (z_0 - z_a) \left. \right) \Bigg/ \left((t_a^2 v_x^2 + 2 t_a (x_0 - x_a) v_x + t_a^2 v_y^2 + 2 t_a (y_0 - y_a) v_y \right. \\
& + t_a^2 v_z^2 + 2 t_a (z_0 - z_a) v_z + x_0^2 - 2 x_0 x_a + x_a^2 + y_0^2 - 2 y_0 y_a + y_a^2 + (z_0 - z_a)^2 \Bigg) c^2 + \left(\right. \\
& - y_0^2 + 2 y_0 y_a - y_a^2 - (z_0 - z_a)^2 v_x^2 + 2 (x_0 - x_a) ((y_0 - y_a) v_y + v_z (z_0 - z_a)) v_x + (-x_0^2 \\
& + 2 x_0 x_a - x_a^2 - (z_0 - z_a)^2 v_y^2 + 2 v_z (y_0 - y_a) (z_0 - z_a) v_y - (x_0^2 - 2 x_0 x_a + x_a^2 \\
& + (y_0 - y_a)^2 v_z^2 \Bigg)^{3/2}
\end{aligned}$$

$$E_y := E\varphi_y + EA_y$$

$$\begin{aligned}
& \frac{1}{2} \left(c \left((-2 t_a v_y - 2 y_0 + 2 y_a) c^2 + (2 y_0 - 2 y_a) v_x^2 - 2 (x_0 - x_a) v_y v_x - 2 v_z (z_0 - z_a) v_y - \right. \right. \quad (23) \\
& \left. - 2 y_0 + 2 y_a) v_z^2 \right) \Bigg/ \left((t_a^2 v_x^2 + 2 t_a (x_0 - x_a) v_x + t_a^2 v_y^2 + 2 t_a (y_0 - y_a) v_y + t_a^2 v_z^2 \right. \\
& + 2 t_a (z_0 - z_a) v_z + x_0^2 - 2 x_0 x_a + x_a^2 + y_0^2 - 2 y_0 y_a + y_a^2 + (z_0 - z_a)^2 \Bigg) c^2 + \left(-y_0^2 \right. \\
& + 2 y_0 y_a - y_a^2 - (z_0 - z_a)^2 v_x^2 + 2 (x_0 - x_a) ((y_0 - y_a) v_y + v_z (z_0 - z_a)) v_x + (-x_0^2 \\
& + 2 x_0 x_a - x_a^2 - (z_0 - z_a)^2 v_y^2 + 2 v_z (y_0 - y_a) (z_0 - z_a) v_y - (x_0^2 - 2 x_0 x_a + x_a^2 \\
& + (y_0 - y_a)^2 v_z^2 \Bigg)^{3/2} + \frac{1}{2} \left(c v_y (2 t_a v_x^2 + 2 (x_0 - x_a) v_x + 2 t_a v_y^2 + 2 (y_0 - y_a) v_y \right. \\
& + 2 t_a v_z^2 + 2 v_z (z_0 - z_a) \left. \right) \Bigg/ \left((t_a^2 v_x^2 + 2 t_a (x_0 - x_a) v_x + t_a^2 v_y^2 + 2 t_a (y_0 - y_a) v_y \right. \\
& + t_a^2 v_z^2 + 2 t_a (z_0 - z_a) v_z + x_0^2 - 2 x_0 x_a + x_a^2 + y_0^2 - 2 y_0 y_a + y_a^2 + (z_0 - z_a)^2 \Bigg) c^2 + \left(\right. \\
& - y_0^2 + 2 y_0 y_a - y_a^2 - (z_0 - z_a)^2 v_x^2 + 2 (x_0 - x_a) ((y_0 - y_a) v_y + v_z (z_0 - z_a)) v_x + (-x_0^2 \\
& + 2 x_0 x_a - x_a^2 - (z_0 - z_a)^2 v_y^2 + 2 v_z (y_0 - y_a) (z_0 - z_a) v_y - (x_0^2 - 2 x_0 x_a + x_a^2 \\
& + (y_0 - y_a)^2 v_z^2 \Bigg)^{3/2}
\end{aligned}$$

$$E_z := E\varphi_z + EA_z$$

$$\begin{aligned}
& \frac{1}{2} \left(c \left((-2 t_a v_z - 2 z_0 + 2 z_a) c^2 + (2 z_0 - 2 z_a) v_x^2 - 2 (x_0 - x_a) v_z v_x + (2 z_0 - 2 z_a) v_y^2 \right. \right. \quad (24) \\
& \left. - 2 v_z (y_0 - y_a) v_y \right) \Bigg/ \left((t_a^2 v_x^2 + 2 t_a (x_0 - x_a) v_x + t_a^2 v_y^2 + 2 t_a (y_0 - y_a) v_y + t_a^2 v_z^2 \right. \\
& + 2 t_a (z_0 - z_a) v_z + x_0^2 - 2 x_0 x_a + x_a^2 + y_0^2 - 2 y_0 y_a + y_a^2 + (z_0 - z_a)^2 \Bigg) c^2 + \left(-y_0^2 \right. \\
& + 2 y_0 y_a - y_a^2 - (z_0 - z_a)^2 v_x^2 + 2 (x_0 - x_a) ((y_0 - y_a) v_y + v_z (z_0 - z_a)) v_x + (-x_0^2 \\
& + 2 x_0 x_a - x_a^2 - (z_0 - z_a)^2 v_y^2 + 2 v_z (y_0 - y_a) (z_0 - z_a) v_y - (x_0^2 - 2 x_0 x_a + x_a^2 \\
& + (y_0 - y_a)^2 v_z^2 \Bigg)^{3/2} + \frac{1}{2} \left(c v_z (2 t_a v_x^2 + 2 (x_0 - x_a) v_x + 2 t_a v_y^2 + 2 (y_0 - y_a) v_y \right. \\
& + 2 t_a v_z^2 + 2 v_z (z_0 - z_a) \left. \right) \Bigg/ \left((t_a^2 v_x^2 + 2 t_a (x_0 - x_a) v_x + t_a^2 v_y^2 + 2 t_a (y_0 - y_a) v_y \right.
\end{aligned}$$

$$\begin{aligned}
& + t_a^2 v_z^2 + 2 t_a (z_0 - z_a) v_z + x_0^2 - 2 x_0 x_a + x_a^2 + y_0^2 - 2 y_0 y_a + y_a^2 + (z_0 - z_a)^2 \Big) c^2 + \Big(\\
& - y_0^2 + 2 y_0 y_a - y_a^2 - (z_0 - z_a)^2 \Big) v_x^2 + 2 (x_0 - x_a) \Big((y_0 - y_a) v_y + v_z (z_0 - z_a) \Big) v_x + \Big(-x_0^2 \\
& + 2 x_0 x_a - x_a^2 - (z_0 - z_a)^2 \Big) v_y^2 + 2 v_z (y_0 - y_a) (z_0 - z_a) v_y - \Big(x_0^2 - 2 x_0 x_a + x_a^2 \\
& + (y_0 - y_a)^2 \Big) v_z^2 \Big)^{3/2}
\end{aligned}$$

simplify(E_x)

$$\begin{aligned}
& - \Big(c \Big(c^2 - v_x^2 - v_y^2 - v_z^2 \Big) (t_a v_x + x_0 - x_a) \Big) / \Big(\Big(t_a^2 v_x^2 + 2 t_a (x_0 - x_a) v_x + t_a^2 v_y^2 + 2 t_a (y_0 \\
& - y_a) v_y + t_a^2 v_z^2 + 2 t_a (z_0 - z_a) v_z + x_0^2 - 2 x_0 x_a + x_a^2 + y_0^2 - 2 y_0 y_a + y_a^2 + (z_0 - z_a)^2 \Big) \\
& c^2 + \Big(-y_0^2 + 2 y_0 y_a - y_a^2 - (z_0 - z_a)^2 \Big) v_x^2 + 2 (x_0 - x_a) \Big((y_0 - y_a) v_y + v_z (z_0 - z_a) \Big) v_x \\
& + \Big(-x_0^2 + 2 x_0 x_a - x_a^2 - (z_0 - z_a)^2 \Big) v_y^2 + 2 v_z (y_0 - y_a) (z_0 - z_a) v_y - \Big(x_0^2 - 2 x_0 x_a + x_a^2 \\
& + (y_0 - y_a)^2 \Big) v_z^2 \Big)^{3/2}
\end{aligned} \tag{25}$$

simplify(E_y)

$$\begin{aligned}
& - \Big(c \Big(c^2 - v_x^2 - v_y^2 - v_z^2 \Big) (t_a v_y + y_0 - y_a) \Big) / \Big(\Big(t_a^2 v_x^2 + 2 t_a (x_0 - x_a) v_x + t_a^2 v_y^2 + 2 t_a (y_0 \\
& - y_a) v_y + t_a^2 v_z^2 + 2 t_a (z_0 - z_a) v_z + x_0^2 - 2 x_0 x_a + x_a^2 + y_0^2 - 2 y_0 y_a + y_a^2 + (z_0 - z_a)^2 \Big) \\
& c^2 + \Big(-y_0^2 + 2 y_0 y_a - y_a^2 - (z_0 - z_a)^2 \Big) v_x^2 + 2 (x_0 - x_a) \Big((y_0 - y_a) v_y + v_z (z_0 - z_a) \Big) v_x \\
& + \Big(-x_0^2 + 2 x_0 x_a - x_a^2 - (z_0 - z_a)^2 \Big) v_y^2 + 2 v_z (y_0 - y_a) (z_0 - z_a) v_y - \Big(x_0^2 - 2 x_0 x_a + x_a^2 \\
& + (y_0 - y_a)^2 \Big) v_z^2 \Big)^{3/2}
\end{aligned} \tag{26}$$

simplify(E_z)

$$\begin{aligned}
& - \Big(c \Big(c^2 - v_x^2 - v_y^2 - v_z^2 \Big) (t_a v_z + z_0 - z_a) \Big) / \Big(\Big(t_a^2 v_x^2 + 2 t_a (x_0 - x_a) v_x + t_a^2 v_y^2 + 2 t_a (y_0 \\
& - y_a) v_y + t_a^2 v_z^2 + 2 t_a (z_0 - z_a) v_z + x_0^2 - 2 x_0 x_a + x_a^2 + y_0^2 - 2 y_0 y_a + y_a^2 + (z_0 - z_a)^2 \Big) \\
& c^2 + \Big(-y_0^2 + 2 y_0 y_a - y_a^2 - (z_0 - z_a)^2 \Big) v_x^2 + 2 (x_0 - x_a) \Big((y_0 - y_a) v_y + v_z (z_0 - z_a) \Big) v_x \\
& + \Big(-x_0^2 + 2 x_0 x_a - x_a^2 - (z_0 - z_a)^2 \Big) v_y^2 + 2 v_z (y_0 - y_a) (z_0 - z_a) v_y - \Big(x_0^2 - 2 x_0 x_a + x_a^2 \\
& + (y_0 - y_a)^2 \Big) v_z^2 \Big)^{3/2}
\end{aligned} \tag{27}$$

6 . 3

simplify(*subs*($v_y=0, v_z=0, x_0=0, y_0=0, z_0=0, z_a=0, E_x$))

$$\begin{aligned}
& - \frac{c (t_a v_x - x_a) (c^2 - v_x^2)}{\Big((t_a^2 v_x^2 - 2 t_a v_x x_a + x_a^2 + y_a^2) c^2 - v_x^2 y_a^2 \Big)^{3/2}}
\end{aligned} \tag{28}$$

$$\text{simplify}(\text{subs}(v_y=0, v_z=0, x_0=0, y_0=0, z_0=0, z_a=0, E_y))$$

$$\frac{c y_a (c^2 - v_x^2)}{\left((t_a^2 v_x^2 - 2 t_a v_x x_a + x_a^2 + y_a^2) c^2 - v_x^2 y_a^2 \right)^{3/2}} \quad (29)$$

$$\text{simplify}(\text{subs}(v_y=0, v_z=0, x_0=0, y_0=0, z_0=0, z_a=0, E_z))$$

$$0 \quad (30)$$

" "

$$E_{side}(X0, t_a) := \int_{-X0}^{X0} (\text{subs}(v_y=0, v_z=0, y_0=0, z_0=0, x_a=0, z_a=0, y_a=1, v_x=1, c=3, E_x) - \text{subs}(v_y$$

$$=0, v_z=0, y_0=0, z_0=0, x_a=0, z_a=0, y_a=1, v_x=0, c=3, E_x)) dx_0$$

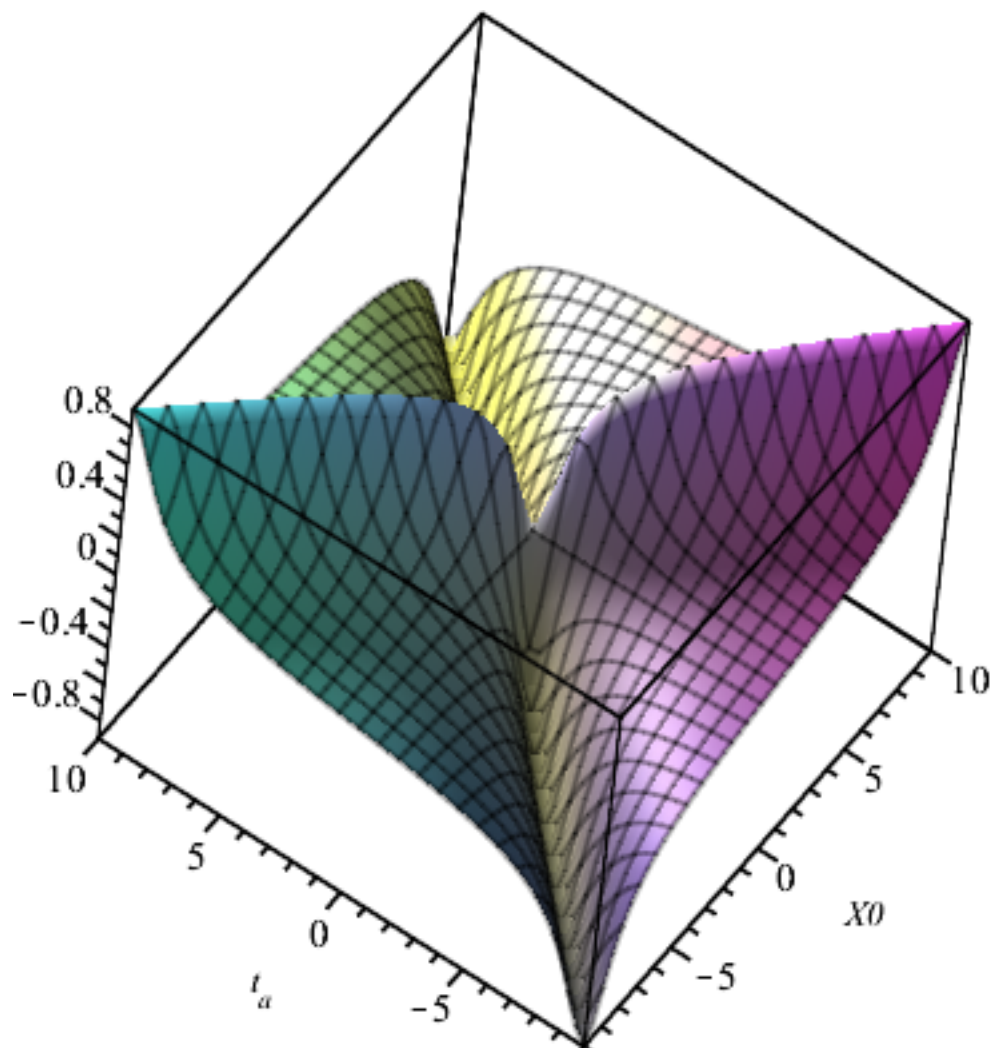
$$(X0, t_a) \rightarrow \int_{-X0}^{X0} (\text{subs}(v_y=0, v_z=0, y_0=0, z_0=0, x_a=0, z_a=0, y_a=1, v_x=1, c=3, E_x) - \text{subs}(v_y \quad (31)$$

$$=0, v_z=0, y_0=0, z_0=0, x_a=0, z_a=0, y_a=1, v_x=0, c=3, E_x)) dx_0$$

$$E_{side}(X0, t_a)$$

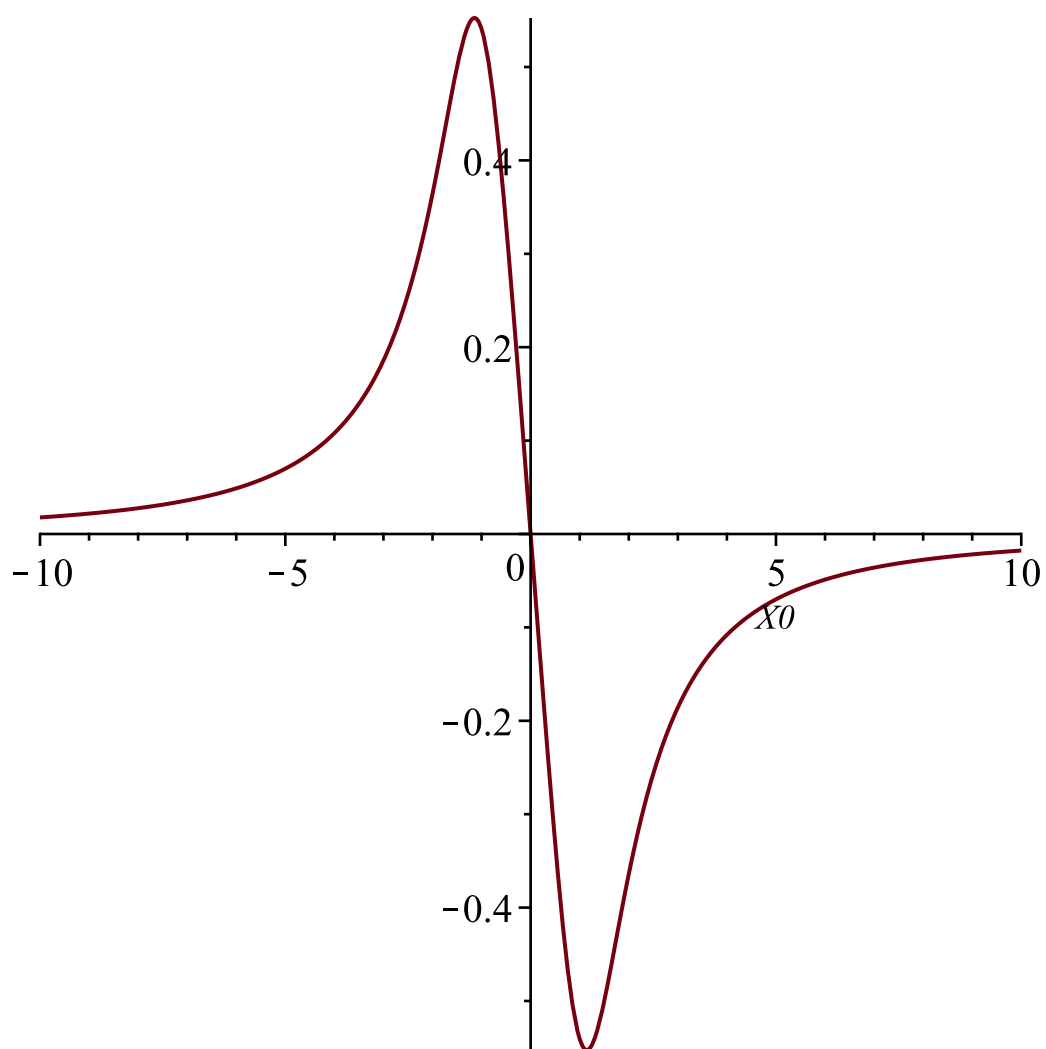
$$\frac{8}{3} \frac{-\sqrt{9 X0^2 + 18 X0 t_a + 9 t_a^2 + 8} + \sqrt{9 X0^2 - 18 X0 t_a + 9 t_a^2 + 8}}{\sqrt{9 X0^2 - 18 X0 t_a + 9 t_a^2 + 8} \sqrt{9 X0^2 + 18 X0 t_a + 9 t_a^2 + 8}} \quad (32)$$

$$\text{plot3d}(E_{side}(X0, t_a))$$



- " "

$plot(subs(t_a=1, E_{side}(X_0, t_a)))$



,
 $plot(subs(X0=1, E_{side}(X0, t_a)), t_a=-10..10)$

