clear : *restart*

z v t

t_z ,

0 , 0 , $v \cdot t_z$

$x_a := c \cdot \left(t - t_z\right) \cdot \sin\left(\theta\right) \cdot \cos(\varphi)$

$$c\left(t - t_z\right) \sin\left(\theta\right) \cos(\varphi) \tag{1}$$

$y_a := c \cdot \left(t - t_z\right) \cdot \sin\left(\theta\right) \cdot \sin(\varphi)$

$$c\left(t - t_z\right) \sin\left(\theta\right) \sin(\varphi) \tag{2}$$

$xy_a := c \cdot \left(t - t_z\right) \cdot \sin\left(\theta\right)$

$$c\left(t - t_z\right) \sin\left(\theta\right) \tag{3}$$

( )

z $v \cdot t_z$ z

, .

.

$z_a := c \cdot \left(t - t_z\right) \cdot \cos\left(\theta\right) + v \cdot t_z$

$$c\left(t - t_z\right) \cos\left(\theta\right) + v\, t_z \tag{4}$$

(a , b , c) R

$(x-a)^2 + (y-b)^2 + (z-c)^2 - R^2 = 0$

$$(x-a)^2 + (y-b)^2 + (z-c)^2 - R^2 = 0 \tag{5}$$

$M\left(x, y, z, v, c, t, t_z\right) := (x-0)^2 + (y-0)^2 + \left(z - v \cdot t_z\right)^2 - \left(c \cdot \left(t - t_z\right)\right)^2$

$$\left(x, y, z, v, c, t, t_z\right) \rightarrow x^2 + y^2 + \left(z - v\, t_z\right)^2 - c^2 \left(t - t_z\right)^2 \tag{6}$$

$(x-0)^2 + (y-0)^2 + \left(z - v \cdot t_z\right)^2 - \left(c \cdot \left(t - t_z\right)\right)^2 = 0$

$$x^2 + y^2 + \left(-t_z\, v + z\right)^2 - c^2 \left(t - t_z\right)^2 = 0 \tag{7}$$

$simplify\left((x-0)^2 + (y-0)^2 + \left(z - v \cdot t_z\right)^2 - \left(c \cdot \left(t - t_z\right)\right)^2 = 0\right)$

$$-c^2 t^2 + 2\, c^2\, t\, t_z - c^2\, t_z^2 + t_z^2\, v^2 - 2\, t_z\, v\, z + x^2 + y^2 + z^2 = 0 \tag{8}$$

t_z $\theta = \theta_0, \varphi = \varphi_0$

$x_0 := subs\left(\theta = \theta_0, \varphi = \varphi_0, x_a\right)$

$$c\,(t - t_z)\,\sin\left(\theta_0\right)\,\cos\left(\varphi_0\right) \tag{9}$$

$y_0 := subs\left(\theta = \theta_0, \varphi = \varphi_0, y_a\right)$

$$c\,(t - t_z)\,\sin\left(\theta_0\right)\,\sin\left(\varphi_0\right) \tag{10}$$

$xy_0 := subs\left(\theta = \theta_0, \varphi = \varphi_0, xy_a\right)$

$$c\,(t - t_z)\,\sin\left(\theta_0\right) \tag{11}$$

$z_0 := subs\left(\theta = \theta_0, \varphi = \varphi_0, z_a\right)$

$$c\,(t - t_z)\,\cos\left(\theta_0\right) + v\,t_z \tag{12}$$

$\frac{\partial}{\partial x}\,M(x, y, z, v, c, t, t_z)$

$$2\,x \tag{13}$$

$\frac{\partial}{\partial y}\,M(x, y, z, v, c, t, t_z)$

$$2\,y \tag{14}$$

$\frac{\partial}{\partial z}\,M(x, y, z, v, c, t, t_z)$

$$-2\,t_z\,v + 2\,z \tag{15}$$

$\dfrac{x - x_0}{subs\left(x = x_0, \frac{\partial}{\partial x}\,M(x, y, z, v, c, t, t_z)\right)}$

$$\frac{1}{2}\,\frac{x - c\,(t - t_z)\,\sin\left(\theta_0\right)\,\cos\left(\varphi_0\right)}{c\,(t - t_z)\,\sin\left(\theta_0\right)\,\cos\left(\varphi_0\right)} \tag{16}$$

$\dfrac{y - y_0}{subs\left(y = y_0, \frac{\partial}{\partial y}\,M(x, y, z, v, c, t, t_z)\right)}$

$$\frac{1}{2}\,\frac{y - c\,(t - t_z)\,\sin\left(\theta_0\right)\,\sin\left(\varphi_0\right)}{c\,(t - t_z)\,\sin\left(\theta_0\right)\,\sin\left(\varphi_0\right)} \tag{17}$$

$\dfrac{z - z_0}{subs\left(z = z_0, \frac{\partial}{\partial z}\,M(x, y, z, v, c, t, t_z)\right)}$

$$\frac{1}{2} \frac{z-c\,(t-t_z)\cos(\theta_0)-v\,t_z}{c\,(t-t_z)\cos(\theta_0)} \tag{18}$$

$$\frac{x-x_0}{subs\left(x=x_0, \frac{\partial}{\partial x} M(x, y, z, v, c, t, t_z)\right)} = \frac{y-y_0}{subs\left(y=y_0, \frac{\partial}{\partial y} M(x, y, z, v, c, t, t_z)\right)} = \frac{z-z_0}{subs\left(z=z_0, \frac{\partial}{\partial z} M(x, y, z, v, c, t, t_z)\right)}$$

$$x\_izo\_tzap\_normal(p) := x_0 + p \cdot subs\left(x=x_0, \frac{\partial}{\partial x} M(x, y, z, v, c, t, t_z)\right)$$

$$p \rightarrow x_0 + p\ subs\left(x=x_0, \frac{\partial}{\partial x} M(x, y, z, v, c, t, t_z)\right) \tag{19}$$

$$y\_izo\_tzap\_normal(p) := y_0 + p \cdot subs\left(y=y_0, \frac{\partial}{\partial y} M(x, y, z, v, c, t, t_z)\right)$$

$$p \rightarrow y_0 + p\ subs\left(y=y_0, \frac{\partial}{\partial y} M(x, y, z, v, c, t, t_z)\right) \tag{20}$$

$$z\_izo\_tzap\_normal(p) := z_0 + p \cdot subs\left(z=z_0, \frac{\partial}{\partial z} M(x, y, z, v, c, t, t_z)\right)$$

$$p \rightarrow z_0 + p\ subs\left(z=z_0, \frac{\partial}{\partial z} M(x, y, z, v, c, t, t_z)\right) \tag{21}$$

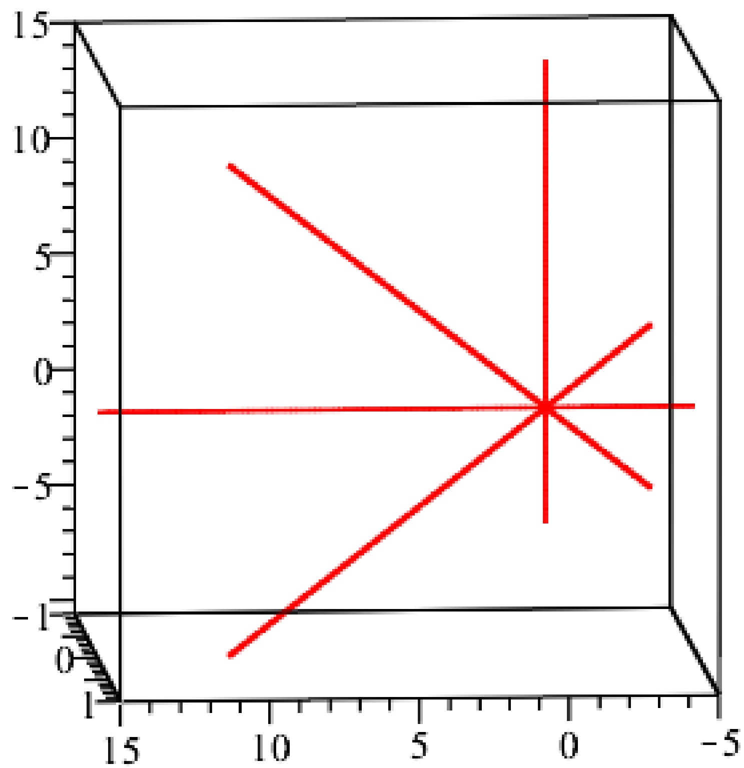
$$izo\_tzap\_normal := subs\left(c=1, t=5, v=0.5, \left[x_0 + p \cdot subs\left(x=x_0, \frac{\partial}{\partial x} M(x, y, z, v, c, t, t_z)\right), y_0 + p \cdot subs\left(y=y_0, \frac{\partial}{\partial y} M(x, y, z, v, c, t, t_z)\right), z_0 + p \cdot subs\left(z=z_0, \frac{\partial}{\partial z} M(x, y, z, v, c, t, t_z)\right)\right]\right)$$

$$\left[(5-t_z)\sin(\theta_0)\cos(\varphi_0) + 2\,p\,(5-t_z)\sin(\theta_0)\cos(\varphi_0), (5-t_z)\sin(\theta_0)\sin(\varphi_0) + 2\,p\,(5-t_z)\sin(\theta_0)\sin(\varphi_0), (5-t_z)\cos(\theta_0) + 0.5\,t_z + 2\,p\,(5-t_z)\cos(\theta_0)\right] \tag{22}$$

$with(plots):$

$$spacecurve\left(\left[subs\left(t_z=0, \theta_0=0, \varphi_0=0, izo\_tzap\_normal\right), subs\left(t_z=0, \theta_0=\frac{\pi}{4}, \varphi_0=0, izo\_tzap\_normal\right), subs\left(t_z=0, \theta_0=\frac{\pi}{2}, \varphi_0=0, izo\_tzap\_normal\right), subs\left(t_z=0, \theta_0=3 \cdot \frac{\pi}{4}, \varphi_0=0,\right.\right.\right.$$

$izo\_tzap\_normal\big)\big]$, $p=-1..1$, $thickness=1$, $numpoints=100$, $color=red\big)$



$t_z + dt_z$

$M(x, y, z, v, c, t, t_z)$

$$x^2 + y^2 + (-t_z v + z)^2 - c^2 (t - t_z)^2 \tag{23}$$

$M(x, y, z, v, c, t, t_z + dt_z)$

$$x^2 + y^2 + (z - v (t_z + dt_z))^2 - c^2 (t - t_z - dt_z)^2 \tag{24}$$

$[x\_izo\_tzap\_normal(p), y\_izo\_tzap\_normal(p), z\_izo\_tzap\_normal(p)]$

$$\big[c (t - t_z) \sin(\theta_0) \cos(\varphi_0) + 2 p c (t - t_z) \sin(\theta_0) \cos(\varphi_0), c (t - t_z) \sin(\theta_0) \sin(\varphi_0) + 2 p c (t - t_z) \sin(\theta_0) \sin(\varphi_0), c (t - t_z) \cos(\theta_0) + v t_z + 2 p c (t - t_z) \cos(\theta_0)\big] \tag{25}$$

$$t_z + dt_z$$

$$t_z$$

$M\left(x\_izo\_tzap\_normal(p), y\_izo\_tzap\_normal(p), z\_izo\_tzap\_normal(p), v, c, t, t_z + dt_z\right)$

$$\left(c\,(t-t_z)\sin(\theta_0)\cos(\varphi_0) + 2\,p\,c\,(t-t_z)\sin(\theta_0)\cos(\varphi_0)\right)^2 + \left(c\,(t-t_z)\sin(\theta_0)\sin(\varphi_0) + 2\,p\,c\,(t-t_z)\sin(\theta_0)\sin(\varphi_0)\right)^2 + \left(c\,(t-t_z)\cos(\theta_0) + v\,t_z + 2\,p\,c\,(t-t_z)\cos(\theta_0) - v\,(t_z+dt_z)\right)^2 - c^2\,(t-t_z-dt_z)^2 \tag{26}$$

$p\_t_{z2} := solve\left(M\left(x\_izo\_tzap\_normal(p), y\_izo\_tzap\_normal(p), z\_izo\_tzap\_normal(p), v, c, t, t_{z2}\right), p\right)$

$$\frac{1}{4}\,\frac{1}{c\,(t-t_z)}\Big(-2\cos(\theta_0)\,t_z\,v + 2\cos(\theta_0)\,t_{z2}\,v - 2\,c\,t + 2\,c\,t_z + 2\left(\cos(\theta_0)^2 t_z^2 v^2 - 2\cos(\theta_0)^2 t_z\,t_{z2}\,v^2 + \cos(\theta_0)^2 t_{z2}^2 v^2 + c^2 t^2 - 2\,c^2\,t\,t_{z2} + c^2 t_{z2}^2 - t_z^2 v^2 + 2\,t_z\,t_{z2}\,v^2 - t_{z2}^2 v^2\right)^{1/2}\Big), \frac{1}{4}\,\frac{1}{c\,(t-t_z)}\Big(-2\cos(\theta_0)\,t_z\,v + 2\cos(\theta_0)\,t_{z2}\,v - 2\,c\,t + 2\,c\,t_z - 2\left(\cos(\theta_0)^2 t_z^2 v^2 - 2\cos(\theta_0)^2 t_z\,t_{z2}\,v^2 + \cos(\theta_0)^2 t_{z2}^2 v^2 + c^2 t^2 - 2\,c^2\,t\,t_{z2} + c^2 t_{z2}^2 - t_z^2 v^2 + 2\,t_z\,t_{z2}\,v^2 - t_{z2}^2 v^2\right)^{1/2}\Big) \tag{27}$$

$p\_dt_z := solve\left(M\left(x\_izo\_tzap\_normal(p), y\_izo\_tzap\_normal(p), z\_izo\_tzap\_normal(p), v, c, t, t_z + dt_z\right), p\right)$

$$\frac{1}{4}\,\frac{1}{c\,(t-t_z)}\left(2\cos(\theta_0)\,dt_z\,v - 2\,c\,t + 2\,c\,t_z + 2\sqrt{\cos(\theta_0)^2 dt_z^2 v^2 + c^2 dt_z^2 - 2\,c^2 dt_z\,t + 2\,c^2 dt_z\,t_z + c^2 t^2 - 2\,c^2\,t\,t_z + c^2 t_z^2 - dt_z^2 v^2}\right), \frac{1}{4}\,\frac{1}{c\,(t-t_z)}\left(2\cos(\theta_0)\,dt_z\,v - 2\,c\,t + 2\,c\,t_z - 2\sqrt{\cos(\theta_0)^2 dt_z^2 v^2 + c^2 dt_z^2 - 2\,c^2 dt_z\,t + 2\,c^2 dt_z\,t_z + c^2 t^2 - 2\,c^2\,t\,t_z + c^2 t_z^2 - dt_z^2 v^2}\right) \tag{28}$$

$$dt_z$$

$x\_izo\_dtz(dt_z) := simplify\left(x\_izo\_tzap\_normal(p\_dt_z)\right):$

$simplify(x\_izo\_dtz(dt_z))$

$$\sin(\theta_0)\cos(\varphi_0)\left(\cos(\theta_0)\,dt_z\,v + \sqrt{\cos(\theta_0)^2 dt_z^2 v^2 + \left((t-t_z-dt_z)\,c - dt_z\,v\right)\left((t-t_z-dt_z)\,c + dt_z\,v\right)}\right) \tag{29}$$

$y\_izo\_dtz(dt_z) := simplify(y\_izo\_tzap\_normal(p\_dt_z))$:

$simplify(y\_izo\_dtz(dt_z))$

$$\sin(\theta_0)\sin(\varphi_0)\left(\cos(\theta_0)\,dt_z\,v + \sqrt{\cos(\theta_0)^2 dt_z^2 v^2 + \left((t-t_z-dt_z)\,c - dt_z\,v\right)\left((t-t_z-dt_z)\,c + dt_z\,v\right)}\right) \tag{30}$$

$z\_izo\_dtz(dt_z) := simplify(z\_izo\_tzap\_normal(p\_dt_z))$:

$simplify(z\_izo\_dtz(dt_z))$

$$\cos(\theta_0)\sqrt{\cos(\theta_0)^2 dt_z^2 v^2 + \left((t-t_z-dt_z)\,c - dt_z\,v\right)\left((t-t_z-dt_z)\,c + dt_z\,v\right)} + v\left(dt_z\cos(\theta_0)^2 + t_z\right) \tag{31}$$

$simplify\left(\sqrt{x\_izo\_dtz(dt_z)^2 + y\_izo\_dtz(dt_z)^2}\right)$

$$\left(\sin(\theta_0)^2\left(\cos(\theta_0)\,dt_z\,v + \sqrt{\cos(\theta_0)^2 dt_z^2 v^2 + \left((t-t_z-dt_z)\,c - dt_z\,v\right)\left((t-t_z-dt_z)\,c + dt_z\,v\right)}\right)^2\right)^{1/2} \tag{32}$$

$\lim_{dt_z \to 0} \frac{x\_izo\_dtz(dt_z) - x_0}{dt_z}$

$$\lim_{dt_z \to 0} \frac{1}{dt_z}\left(\sin(\theta_0)\cos(\varphi_0)\left(\cos(\theta_0)\,dt_z\,v + \sqrt{\cos(\theta_0)^2 dt_z^2 v^2 + c^2 dt_z^2 - 2c^2 dt_z\,t + 2c^2 dt_z\,t_z + c^2 t^2 - 2c^2 t\,t_z + c^2 t_z^2 - dt_z^2 v^2}\right) - c\,(t-t_z)\sin(\theta_0)\cos(\varphi_0)\right) \tag{33}$$

$\lim_{dt_z \to 0} \frac{y\_izo\_dtz(dt_z) - y_0}{dt_z}$

$$\lim_{dt_z \to 0} \frac{1}{dt_z} \left(\sin(\theta_0) \sin(\varphi_0) \left(\cos(\theta_0)\, dt_z\, v + \sqrt{\cos(\theta_0)^2\, dt_z^2\, v^2 + c^2\, dt_z^2 - 2\, c^2\, dt_z\, t + 2\, c^2\, dt_z\, t_z + c^2\, t^2 - 2\, c^2\, t\, t_z + c^2\, t_z^2 - dt_z^2\, v^2} \right) - c\, (t - t_z) \sin(\theta_0) \sin(\varphi_0) \right) \quad \textbf{(34)}$$

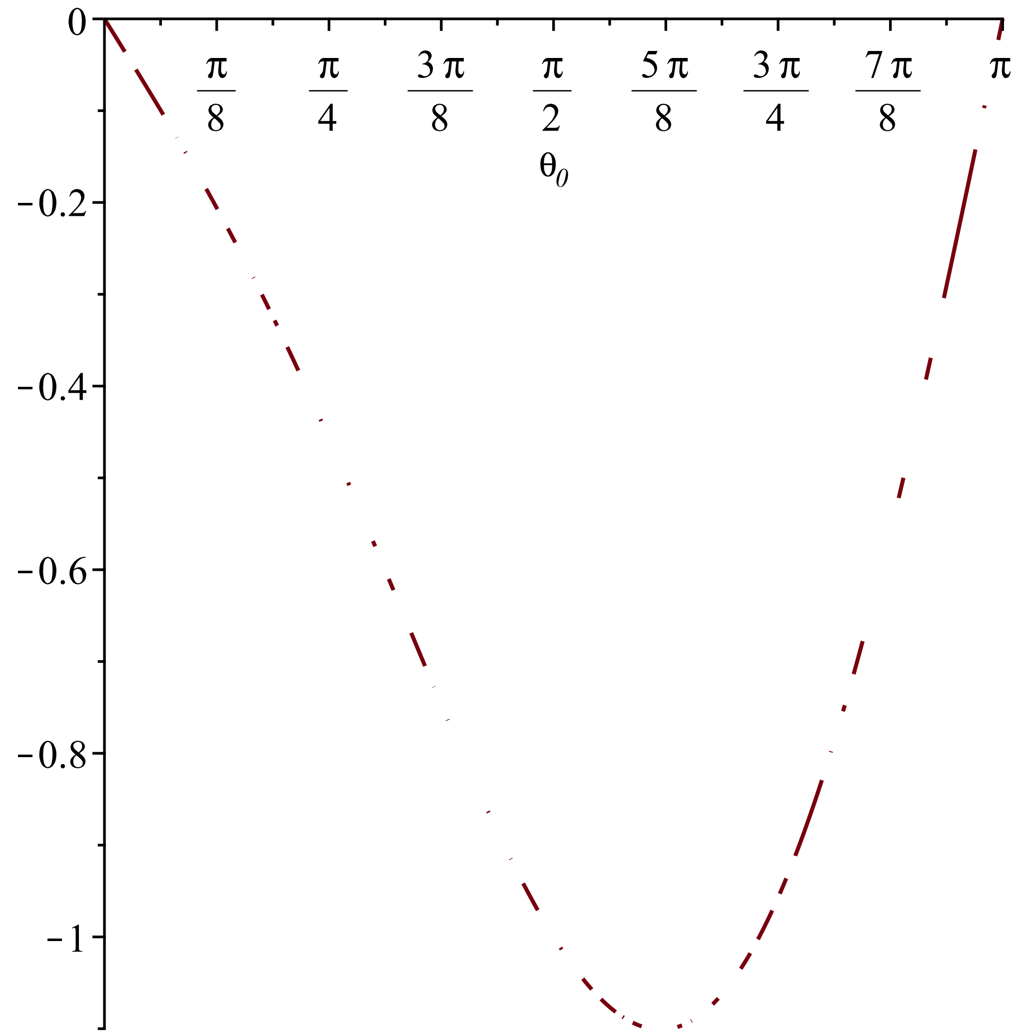
$$\lim_{dt_z \to 0} \frac{z\_izo\_dtz(dt_z) - z_0}{dt_z}$$

$$\lim_{dt_z \to 0} \frac{1}{dt_z} \left(\cos(\theta_0)^2\, dt_z\, v + \cos(\theta_0) \sqrt{\cos(\theta_0)^2\, dt_z^2\, v^2 + c^2\, dt_z^2 - 2\, c^2\, dt_z\, t + 2\, c^2\, dt_z\, t_z + c^2\, t^2 - 2\, c^2\, t\, t_z + c^2\, t_z^2 - dt_z^2\, v^2} - c\, (t - t_z) \cos(\theta_0) \right) \quad \textbf{(35)}$$

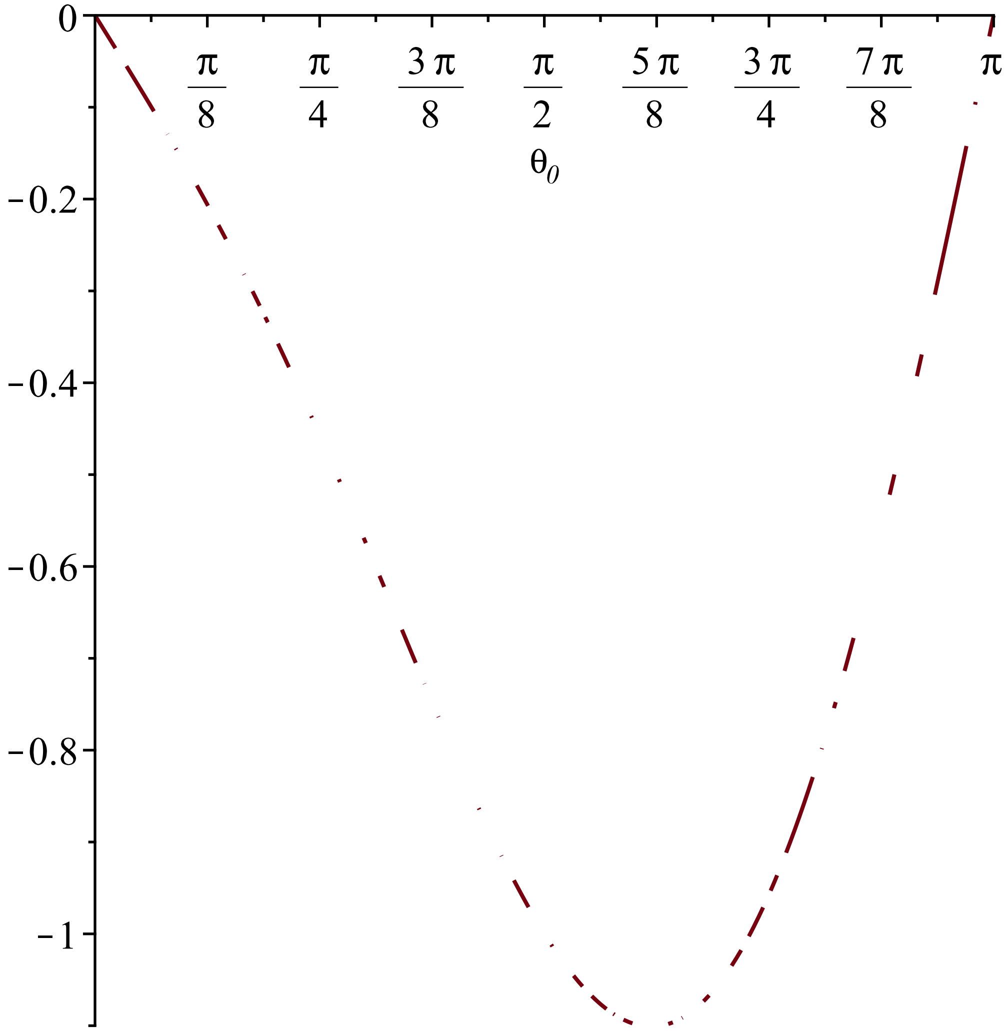
$$evalf\left(subs\left(c = 1, v = 0.5, t = 5, \varphi_0 = 0, \lim_{dt_z \to 0} \frac{x\_izo\_dtz(dt_z) - x_0}{dt_z} \right) \right)$$

$$\lim_{dt_z \to 0} \frac{1}{dt_z} \left(\sin(\theta_0) \cos(0) \left(0.5 \cos(\theta_0)\, dt_z + \sqrt{0.25 \cos(\theta_0)^2\, dt_z^2 + 0.75\, dt_z^2 - 10\, dt_z + 2\, dt_z\, t_z + 25 - 10\, t_z + t_z^2} \right) - (5 - t_z) \sin(\theta_0) \cos(0) \right) \quad \textbf{(36)}$$

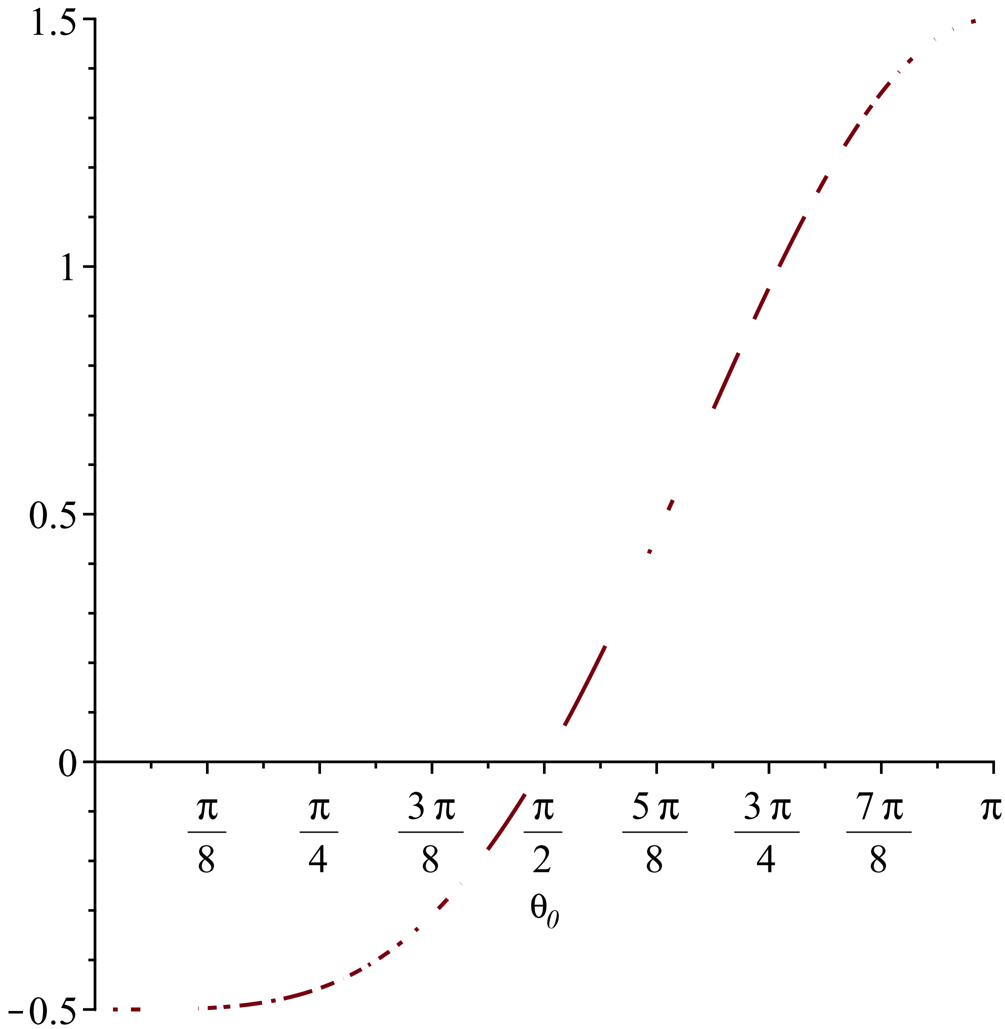
$$plot\left(subs\left(c = 1, v = 0.5, t = 5, t_z = 0, \varphi_0 = 0, \lim_{dt_z \to 0} \frac{x\_izo\_dtz(dt_z) - x_0}{dt_z} \right), \theta_0 = 0 .. \pi \right)$$



$$plot\left(subs\left(c=1,\ v=0.5,\ t=5,\ t_z=0,\ \varphi_0=\frac{\pi}{2},\ \lim_{dt_z\to 0}\frac{y\_izo\_dtz(dt_z)-y_0}{dt_z}\right),\ \theta_0=0..\pi\right)$$



$$plot\left(subs\left(c=1,\ v=0.5,\ t=5,\ t_z=0,\ \lim_{dt_z \to 0} \frac{z\_izo\_dtz(dt_z)-z_0}{dt_z}\right),\ \theta_0=0..\pi\right)$$



$xy\_izo\_dtz(dt_z) := simplify\left(\sqrt{x\_izo\_dtz(dt_z)^2 + y\_izo\_dtz(dt_z)^2}\right)$

$$dt_z \rightarrow simplify\left(\sqrt{x\_izo\_dtz(dt_z)^2 + y\_izo\_dtz(dt_z)^2}\right) \tag{37}$$

$simplify\left(\lim_{dt_z \rightarrow 0} \frac{xy\_izo\_dtz(dt_z)}{(t - t_z - dt_z)\, c}\right)$

$$\frac{\sqrt{\sin(\theta_0)^2 c^2 (t - t_z)^2}}{(t - t_z)\, c} \tag{38}$$

$$\lim_{dt_z \to 0} \frac{z\_izo\_dtz(dt_z) - v \cdot (t_z + dt_z)}{(t - t_z - dt_z)\, c}$$

$$\frac{\cos(\theta_0) \sqrt{c^2 (t - t_z)^2}}{c\, t - c\, t_z} \tag{39}$$

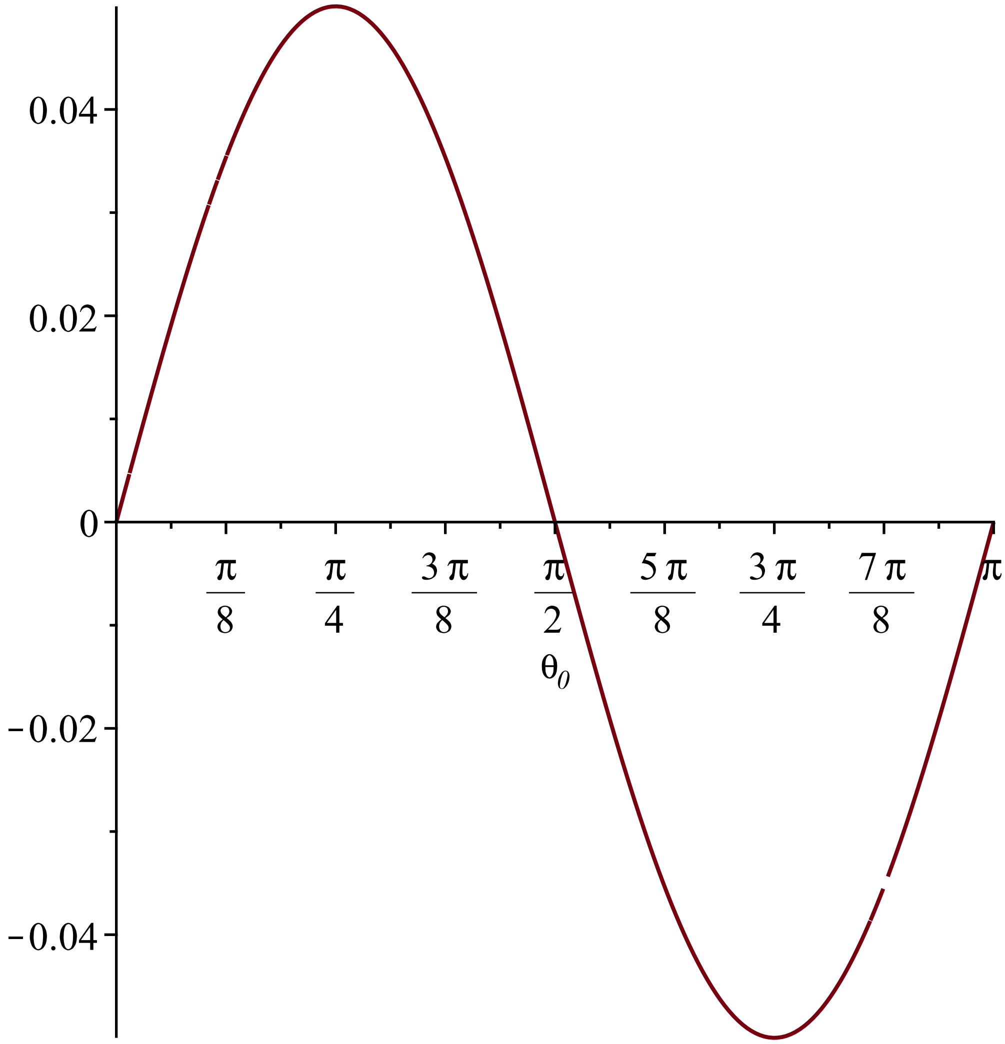
$$\lim_{dt_z \to 0} \frac{\dfrac{xy\_izo\_dtz(dt_z)}{(t - t_z - dt_z)\, c} - \sin(\theta_0)}{dt_z}$$

$$\lim_{dt_z \to 0} \frac{1}{dt_z} \left(\frac{1}{(t - t_z - dt_z)\, c} \left(\sin(\theta_0)^2 \left(\cos(\theta_0)\, dt_z\, v + \sqrt{\cos(\theta_0)^2 dt_z^2 v^2 + c^2 dt_z^2 - 2 c^2 dt_z t + 2 c^2 dt_z t_z + c^2 t^2 - 2 c^2 t\, t_z + c^2 t_z^2 - dt_z^2 v^2} \right)^2 \right)^{1/2} - \sin(\theta_0) \right) \tag{40}$$

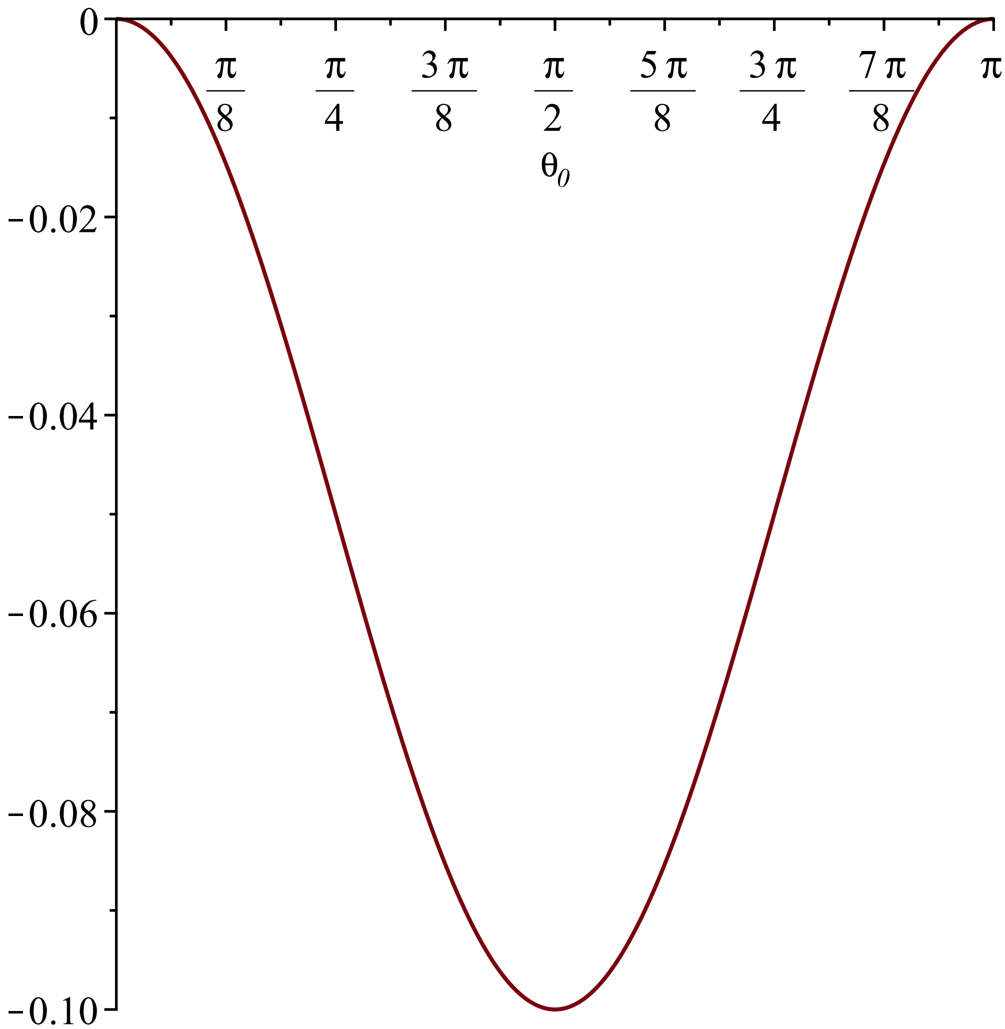
$$\lim_{dt_z \to 0} \frac{\dfrac{z\_izo\_dtz(dt_z) - v \cdot (t_z + dt_z)}{(t - t_z - dt_z)\, c} - \cos(\theta_0)}{dt_z}$$

$$\lim_{dt_z \to 0} \frac{1}{dt_z} \left(\frac{1}{(t - t_z - dt_z)\, c} \left(\cos(\theta_0)^2 dt_z\, v + \cos(\theta_0) \sqrt{\cos(\theta_0)^2 dt_z^2 v^2 + c^2 dt_z^2 - 2 c^2 dt_z t + 2 c^2 dt_z t_z + c^2 t^2 - 2 c^2 t\, t_z + c^2 t_z^2 - dt_z^2 v^2} + v\, t_z - v\, (t_z + dt_z) \right) - \cos(\theta_0) \right) \tag{41}$$

$$plot\left(subs\left(c=1,\ v=0.5,\ t=5,\ t_z=0,\ \lim_{dt_z\to 0}\frac{\dfrac{xy\_izo\_dtz(dt_z)}{(t-t_z-dt_z)\ c}-\sin(\theta_0)}{dt_z}\right),\theta_0=0\,..\pi\right)$$



$$plot\left(subs\left(c=1,\ v=0.5,\ t=5,\ t_z=0,\ \lim_{dt_z\to 0}\frac{\dfrac{z\_izo\_dtz(dt_z)-v\cdot(t_z+dt_z)}{(t-t_z-dt_z)\ c}-\cos(\theta_0)}{dt_z}\right),\theta_0=0\,..\pi\right)$$



$$sin\_theta := \sin(\theta_0) + \int_0^{tzap} subs\left(c = 1, v = 0.5, t = 5, \lim_{dt_z \to 0} \frac{\frac{xy\_izo\_dtz(dt_z)}{(t - t_z - dt_z)\,c} - \sin(\theta_0)}{dt_z}\right) \mathrm{d}t_z$$

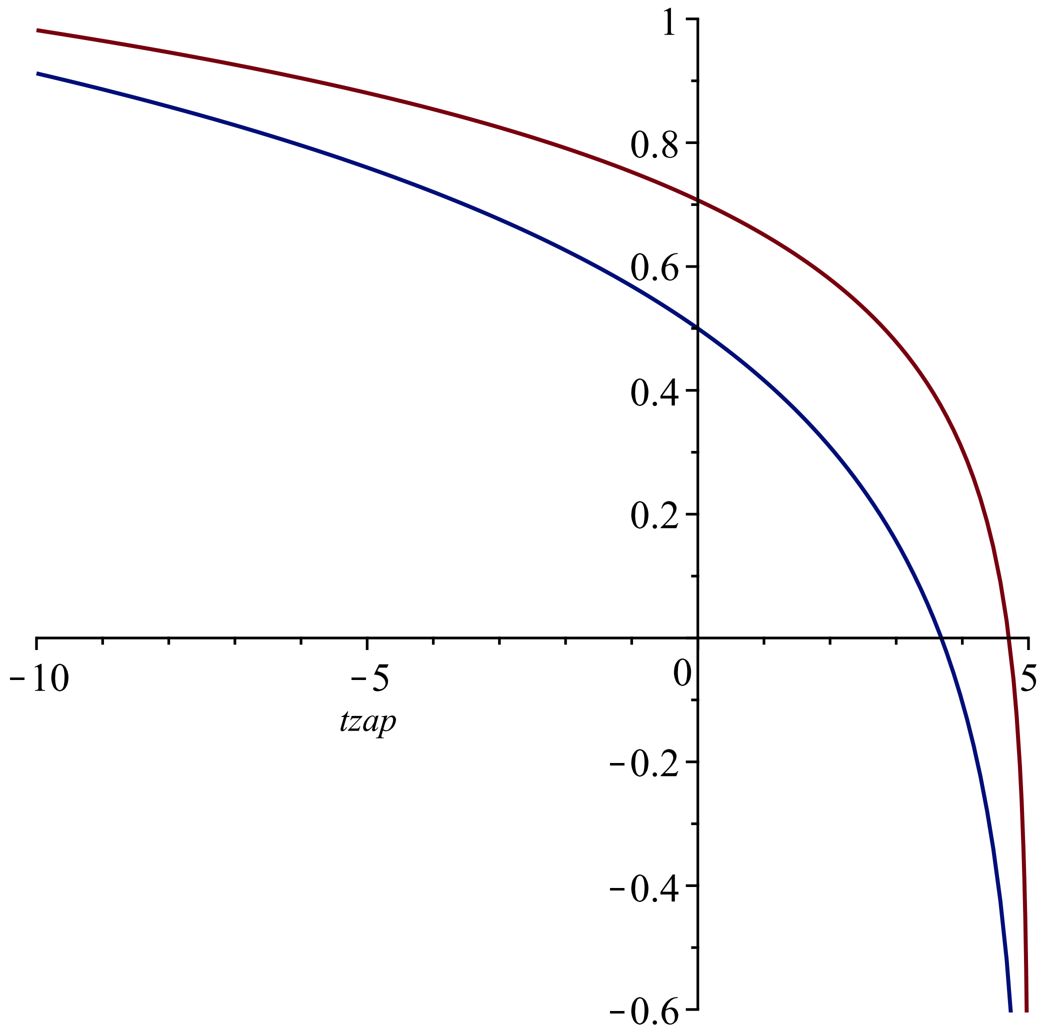
$$\sin(\theta_0) + \int_0^{tzap} \lim_{dt_z \to 0} \frac{1}{dt_z}\left(\frac{1}{5 - t_z - dt_z}\left(\sin(\theta_0)^2\left(0.5\cos(\theta_0)\,dt_z + \sqrt{0.25\cos(\theta_0)^2 dt_z^2 + 0.75\,dt_z^2 - 10\,dt_z + 2\,dt_z\,t_z + 25 - 10\,t_z + t_z^2}\right)^2\right)^{1/2} - \sin(\theta_0)\right) \tag{42}$$

$\mathrm{d}t_z$

$$cos\_theta := \cos(\theta_0) + \int_0^{tzap} subs\left(c = 1, v = 0.5, t = 5, \lim_{dt_z \to 0} \frac{\frac{z\_izo\_dtz(dt_z) - v \cdot (t_z + dt_z)}{(t - t_z - dt_z)\, c} - \cos(\theta_0)}{dt_z}\right) \mathrm{d}t_z$$

$$\cos(\theta_0) + \int_0^{tzap} \lim_{dt_z \to 0} \frac{1}{dt_z} \left(\frac{1}{5 - t_z - dt_z} \left(0.5 \cos(\theta_0)^2 dt_z + \cos(\theta_0) \sqrt{0.25 \cos(\theta_0)^2 dt_z^2 + 0.75\, dt_z^2 - 10\, dt_z + 2\, dt_z\, t_z + 25 - 10\, t_z + t_z^2} - 0.5\, dt_z\right) - \cos(\theta_0) \right) \mathrm{d}t_z \tag{43}$$

$$plot\left(\left[subs\left(\theta_0 = \frac{\pi}{4}, cos\_theta\right), subs\left(\theta_0 = \frac{\pi}{3}, cos\_theta\right)\right], tzap\right)$$



$xy_a := c\cdot\left(t-t_z\right)\cdot\sin(\theta) \quad z_a := c\cdot\left(t-t_z\right)\cdot\cos(\theta)+v\cdot t_z$

$xn := sin\_theta\cdot(t-tzap)\cdot c$

$$\left(\sin\left(\theta_0\right)+\int_0^{tzap}\lim_{dt_z\to 0}\frac{1}{dt_z}\left(\frac{1}{5-t_z-dt_z}\left(\sin\left(\theta_0\right)^2\left(0.5\cos\left(\theta_0\right)dt_z+\sqrt{0.25\cos\left(\theta_0\right)^2 dt_z^2+0.75\,dt_z^2-10\,dt_z+2\,dt_z\,t_z+25-10\,t_z+t_z^2}\right)^2\right)^{1/2}-\sin\left(\theta_0\right)\right)\mathrm{d}t_z\right)(t-tzap)\,c \tag{44}$$

$zn := cos\_theta \cdot (t - tzap) \cdot c + v \cdot tzap$

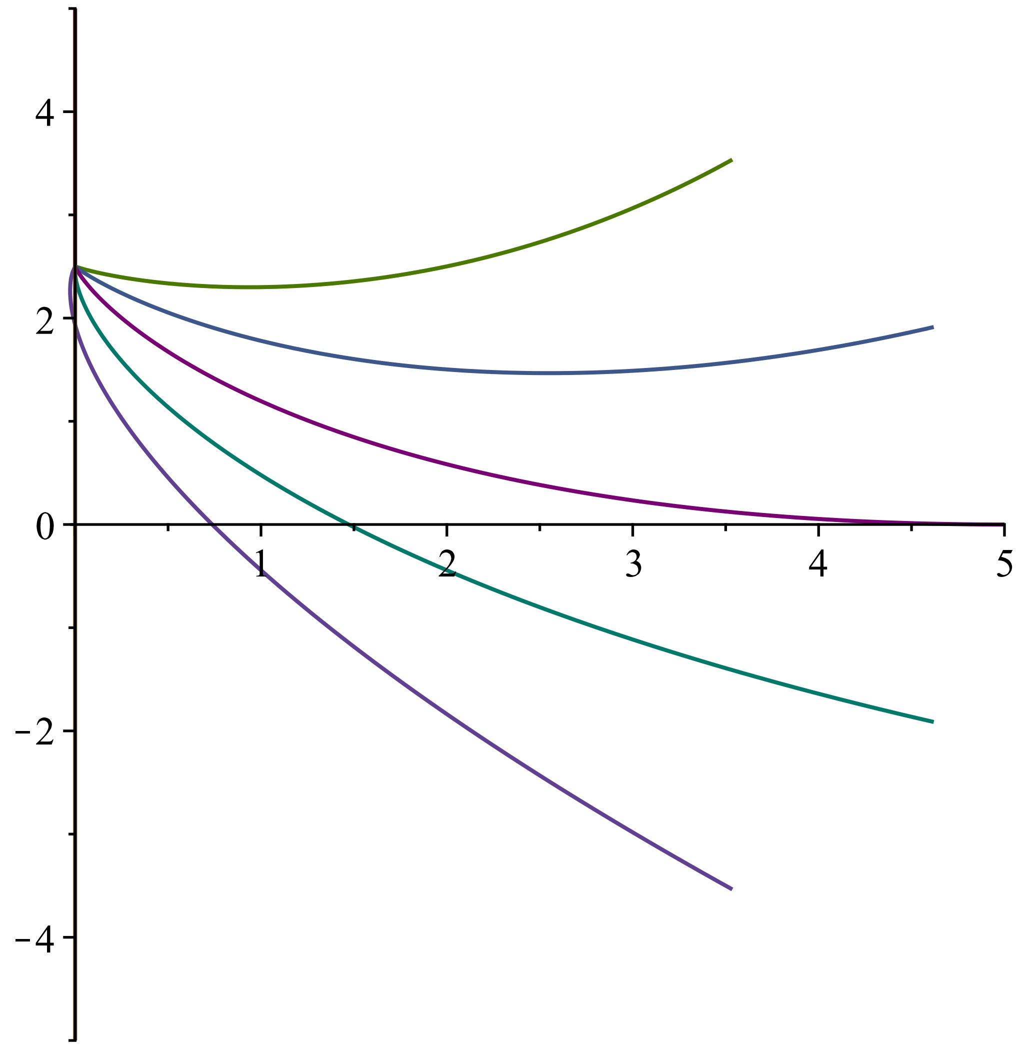
$$\left(\cos(\theta_0) + \int_0^{tzap} \lim_{dt_z \to 0} \frac{1}{dt_z} \left(\frac{1}{5 - t_z - dt_z} \left(0.5 \cos(\theta_0)^2 dt_z + \cos(\theta_0) \sqrt{0.25 \cos(\theta_0)^2 dt_z^2 + 0.75\, dt_z^2 - 10\, dt_z + 2\, dt_z t_z + 25 - 10\, t_z + t_z^2} - 0.5\, dt_z\right) - \cos(\theta_0) \right) \mathrm{d}t_z \right) (t - tzap)\, c + v\, tzap \tag{45}$$

$izo\_tzap := evalf(subs(c = 1, v = 0.5, t = 5, ([xn, zn, tzap = 0..5])))$

$$\left[\left(\sin(\theta_0) + \int_{0.}^{tzap} \lim_{dt_z \to 0} \frac{1}{dt_z} \left(\frac{1}{5 - t_z - dt_z} \left(\sin(\theta_0)^2 \left(0.5 \cos(\theta_0)\, dt_z + \sqrt{0.25 \cos(\theta_0)^2 dt_z^2 + 0.75\, dt_z^2 - 10\, dt_z + 2\, dt_z t_z + 25 - 10\, t_z + t_z^2}\right)^2\right)^{1/2} - \sin(\theta_0) \right) \mathrm{d}t_z \right) (5. - 1.\, tzap), \left(\cos(\theta_0) + \int_{0.}^{tzap} \lim_{dt_z \to 0} \frac{1}{dt_z} \left(\frac{1}{5 - t_z - dt_z} \left(0.5 \cos(\theta_0)^2 dt_z + \cos(\theta_0) \sqrt{0.25 \cos(\theta_0)^2 dt_z^2 + 0.75\, dt_z^2 - 10\, dt_z + 2\, dt_z t_z + 25 - 10\, t_z + t_z^2} - 0.5\, dt_z\right) - \cos(\theta_0) \right) \mathrm{d}t_z \right) (5. - 1.\, tzap) + 0.5\, tzap, tzap = 0. .. 5. \right] \tag{46}$$

$plot\Big(\Big[subs(\theta_0 = 0, izo\_tzap), subs\left(\theta_0 = \frac{\pi}{8}, izo\_tzap\right), subs\left(\theta_0 = \frac{2 \cdot \pi}{8}, izo\_tzap\right), subs\left(\theta_0 = \frac{3 \cdot \pi}{8}, izo\_tzap\right), subs\left(\theta_0 = \frac{\pi}{2}, izo\_tzap\right), subs\left(\theta_0 = \frac{5 \cdot \pi}{8}, izo\_tzap\right), subs\left(\theta_0 = \frac{6 \cdot \pi}{8}, izo\_tzap\right),$

$$subs\left(\theta_0 = \frac{7\cdot\pi}{8}, izo\_tzap\right), subs\left(\theta_0 = \frac{8\cdot\pi}{8}, izo\_tzap\right)\Big]\Big)$$



$$izo\_tzap2 := evalf\left(subs\left(c = 1, v = 0.5, t = 5, \left(\left[xn, zn\right]\right)\right)\right)$$

$$\left[\left(\sin\left(\theta_0\right) + \int_{0.}^{tzap} \lim_{dt_z \to 0}\right.\right. \tag{47}$$

$$\frac{1}{dt_z}\left(\frac{1}{5 - t_z - dt_z}\left(\sin\left(\theta_0\right)^2\left(0.5\cos\left(\theta_0\right) dt_z\right.\right.\right.$$

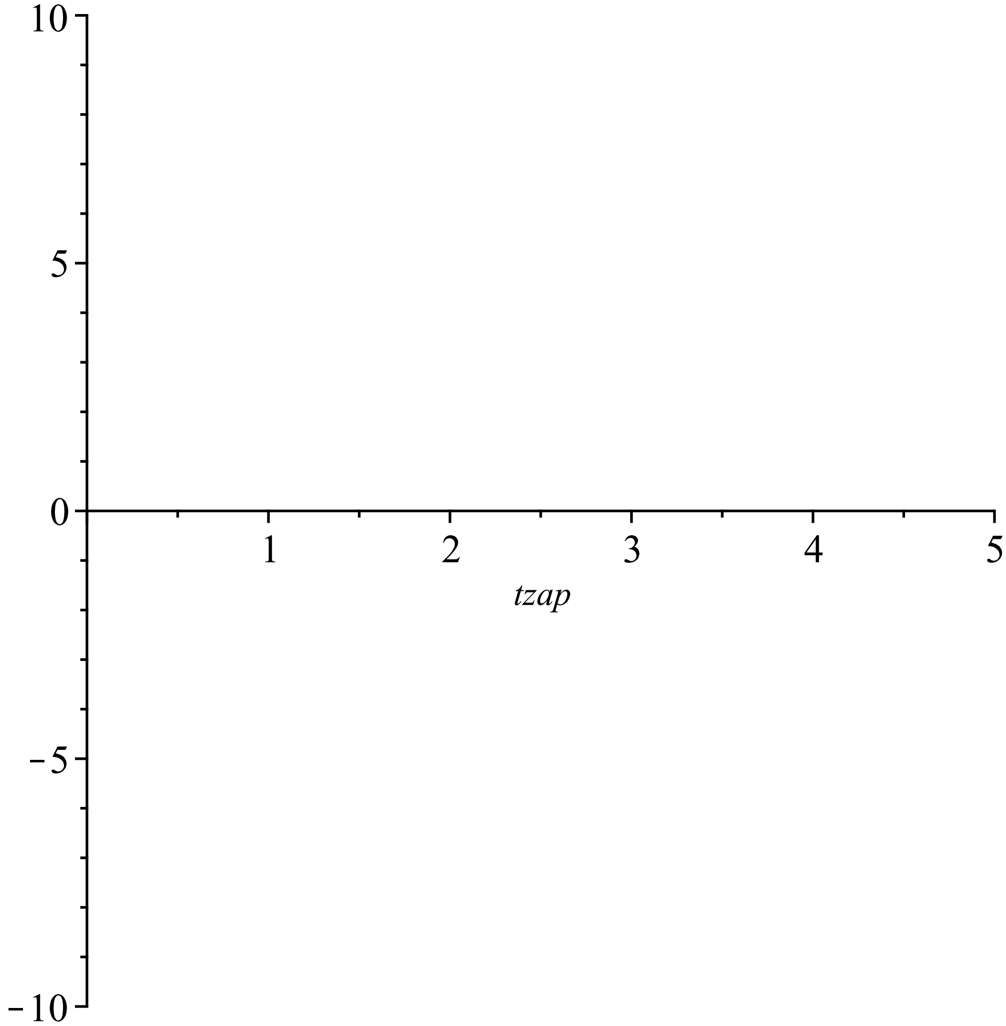
$$+\sqrt{0.25\cos(\theta_0)^2 dt_z^2+0.75\,dt_z^2-10\,dt_z+2\,dt_z t_z+25-10\,t_z+t_z^2}\Big)^2\Big)^{1/2}-\sin(\theta_0)\Big]$$

$$\mathrm{d}t_z\Big)\,(5.-1.\,tzap),\ \Big(\cos(\theta_0)+\int_{0.}^{tzap}\lim_{dt_z\to 0}\frac{1}{dt_z}\Big(\frac{1}{5-t_z-dt_z}\Big(0.5\cos(\theta_0)^2 dt_z$$

$$+\cos(\theta_0)\sqrt{0.25\cos(\theta_0)^2 dt_z^2+0.75\,dt_z^2-10\,dt_z+2\,dt_z t_z+25-10\,t_z+t_z^2}-0.5\,dt_z\Big)$$

$$-\cos(\theta_0)\Big)\,\mathrm{d}t_z\Big)\,(5.-1.\,tzap)+0.5\,tzap\Big]$$

$$plot\Big(\Big[subs(\theta_0=0, izo\_tzap2), subs\Big(\theta_0=\frac{\pi}{8}, izo\_tzap2\Big), subs\Big(\theta_0=\frac{2\cdot\pi}{8}, izo\_tzap2\Big), subs\Big(\theta_0=\frac{3\cdot\pi}{8}, izo\_tzap2\Big), subs\Big(\theta_0=\frac{\pi}{2}, izo\_tzap2\Big), subs\Big(\theta_0=\frac{5\cdot\pi}{8}, izo\_tzap2\Big), subs\Big(\theta_0=\frac{6\cdot\pi}{8}, izo\_tzap2\Big), subs\Big(\theta_0=\frac{7\cdot\pi}{8}, izo\_tzap2\Big), subs\Big(\theta_0=\frac{8\cdot\pi}{8}, izo\_tzap2\Big)\Big], tzap=0..5\Big)$$



$$plot\left(\left[evalf\left(subs\left(c=1, v=0.5, t=5, \theta_0=\frac{\pi}{3}, \varphi_0=0, xn\right)\right), evalf\left(subs\left(c=1, v=0.5, t=5, \theta_0=\frac{\pi}{3}, \varphi_0=0, zn\right)\right), tzap=0\,..5\right]\right)$$

$$int\_x := \int_0^{tzap} \lim_{dt_z \to 0} \frac{x\_izo\_dtz(dt_z) - x_0}{dt_z}\, \mathrm{d}t_z$$

$$\int_0^{tzap} \lim_{dt_z \to 0} \frac{1}{dt_z} \left(\sin(\theta_0) \cos(\varphi_0) \left(\cos(\theta_0)\, dt_z\, v\right.\right. \tag{48}$$

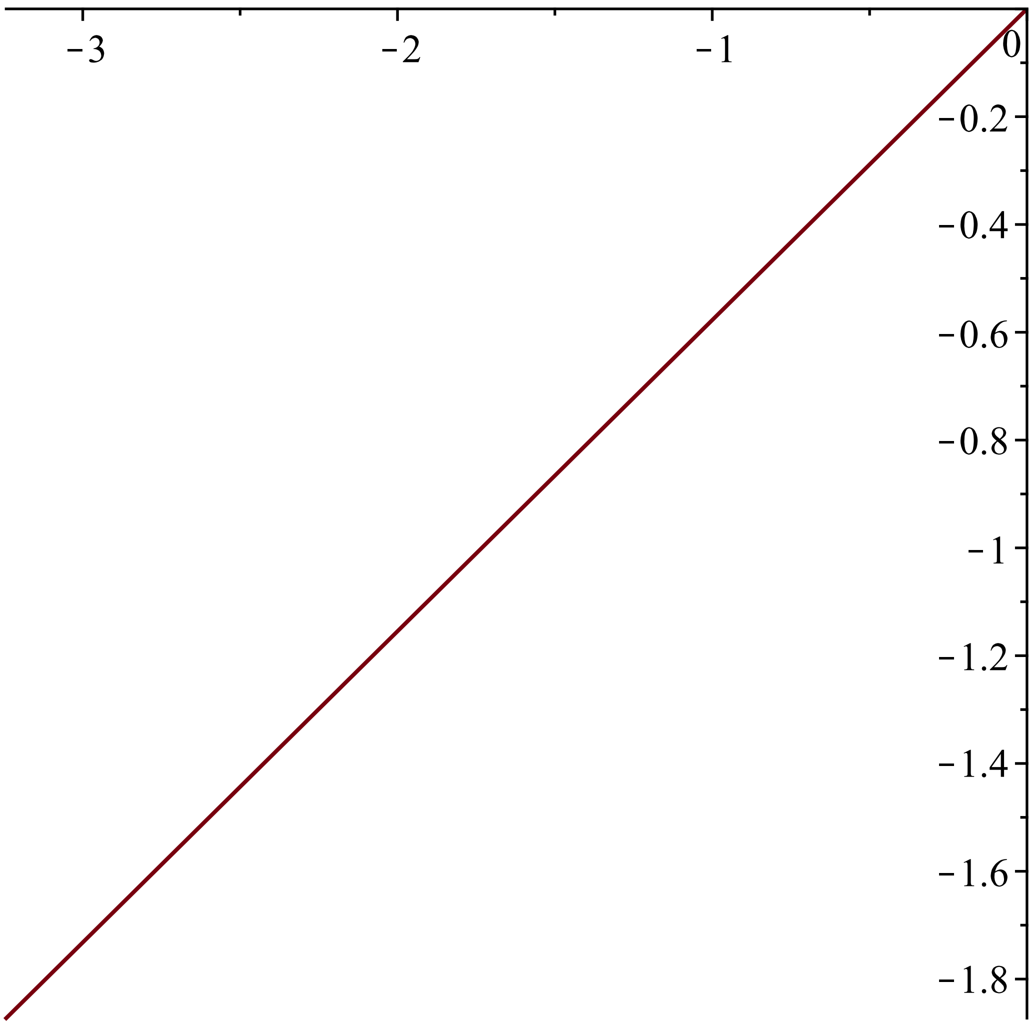
$$+\sqrt{\cos(\theta_0)^2\,dt_z^2\,v^2 + c^2\,dt_z^2 - 2\,c^2\,dt_z\,t + 2\,c^2\,dt_z\,t_z + c^2\,t^2 - 2\,c^2\,t\,t_z + c^2\,t_z^2 - dt_z^2\,v^2}\Big)$$

$$-c\,(t - t_z)\,\sin(\theta_0)\,\cos(\varphi_0)\Big)\,\mathrm{d}t_z$$

$$int\_z := \int_0^{tzap} \lim_{dt_z \to 0} \frac{z\_izo\_dtz(dt_z) - z_0}{dt_z}\,\mathrm{d}t_z$$

$$\int_0^{tzap} \lim_{dt_z \to 0} \frac{1}{dt_z}\Big(\cos(\theta_0)^2\,dt_z\,v + \cos(\theta_0)\sqrt{\cos(\theta_0)^2\,dt_z^2\,v^2 + c^2\,dt_z^2 - 2\,c^2\,dt_z\,t + 2\,c^2\,dt_z\,t_z + c^2\,t^2 - 2\,c^2\,t\,t_z + c^2\,t_z^2 - dt_z^2\,v^2} - c\,(t - t_z)\,\cos(\theta_0)\Big)\,\mathrm{d}t_z \tag{49}$$

$$plot\left(\left[evalf\left(subs\left(c=1, v=0.5, t=5, \theta_0=\frac{\pi}{3}, \varphi_0=0, int\_x\right)\right), evalf\left(subs\left(c=1, v=0.5, t=5, \theta_0=\frac{\pi}{3}, \varphi_0=0, int\_z\right)\right), tzap=0..5\right]\right)$$



$$\int_0^{t_2} evalf\left(subs\left(c=1, v=0.5, t=5, \varphi_0=0, int\_x\right)\right) \mathrm{d}t_z$$

$$\int_0^{t_2}\int_{0.}^{tzap} \lim_{dt_z \to 0} \frac{1}{dt_z} \left(\sin(\theta_0) \cos(0) \left(0.5 \cos(\theta_0)\, dt_z + \sqrt{0.25 \cos(\theta_0)^2 dt_z^2 + 0.75\, dt_z^2 - 10\, dt_z + 2\, dt_z\, t_z + 25 - 10\, t_z + t_z^2}\right) - (5 - t_z) \sin(\theta_0) \cos(0)\right) \mathrm{d}t_z\, \mathrm{d}t_z \tag{50}$$

$$\int_0^{t_2} evalf\left(subs\left(c=1, v=0.5, t=5, int\_z\right)\right) \mathrm{d}t_z$$

$$\int_0^{t_2}\int_{0.}^{tzap} \lim_{dt_z \to 0} \frac{1}{dt_z} \left(0.5 \cos(\theta_0)^2\, dt_z \tag{51}$$

$$+\cos(\theta_0)\sqrt{0.25\cos(\theta_0)^2 dt_z^2+0.75\,dt_z^2-10\,dt_z+2\,dt_z t_z+25-10\,t_z+t_z^2}-(5$$

$$-t_z)\cos(\theta_0)\Big)\,\mathrm{d}t_z\,\mathrm{d}t_z$$

$plot([subs(\,)], t_2=0..2)$

Error, invalid input: subs expects 1 or more arguments, but received 0

$$simplify\left(\frac{x\_izo\_dtz(dt_z)-x_0}{dt_z}\right)$$

$$\frac{1}{dt_z}\Big(\Big(\cos(\theta_0)\,dt_z\,v+(-t+t_z)\,c$$
$$+\sqrt{\cos(\theta_0)^2 dt_z^2 v^2+((t-t_z-dt_z)\,c+dt_z\,v)\,((t-t_z-dt_z)\,c-dt_z\,v)}\Big)$$
$$\cos(\varphi_0)\sin(\theta_0)\Big) \tag{52}$$

$$simplify\left(\frac{y\_izo\_dtz(dt_z)-y_0}{dt_z}\right)$$

$$\frac{1}{dt_z}\Big(\Big(\cos(\theta_0)\,dt_z\,v+(-t+t_z)\,c$$
$$+\sqrt{\cos(\theta_0)^2 dt_z^2 v^2+((t-t_z-dt_z)\,c+dt_z\,v)\,((t-t_z-dt_z)\,c-dt_z\,v)}\Big)$$
$$\sin(\varphi_0)\sin(\theta_0)\Big) \tag{53}$$

$$simplify\left(\frac{z\_izo\_dtz(dt_z)-z_0}{dt_z}\right)$$

$$\frac{1}{dt_z}\Big(\Big(\cos(\theta_0)\,dt_z\,v+(-t+t_z)\,c$$
$$+\sqrt{\cos(\theta_0)^2 dt_z^2 v^2+((t-t_z-dt_z)\,c+dt_z\,v)\,((t-t_z-dt_z)\,c-dt_z\,v)}\Big)\cos(\theta_0)\Big) \tag{54}$$

$$simplify\left(\frac{xy\_izo\_dtz(dt_z)-xy_0}{dt_z}\right)$$

$$\frac{1}{dt_z}\left(\left(\sin\left(\theta_0\right)^2\left(\cos\left(\theta_0\right) dt_z v\right.\right.\right.$$
$$\left.\left.+\sqrt{\cos\left(\theta_0\right)^2 dt_z^{\,2} v^2+\left(\left(t-t_z-dt_z\right) c+dt_z v\right)\left(\left(t-t_z-dt_z\right) c-dt_z v\right)}\right)^2\right)^{1/2}+\left(-t\right.$$
$$\left.\left.+t_z\right) c \sin\left(\theta_0\right)\right) \qquad (55)$$

$simplify(x\_izo\_tzap\_normal(p\_t_{z2}))$

$$\cos(\varphi_0)\left(\sqrt{v^2\left(t_z-t_{z2}\right)^2\cos\left(\theta_0\right)^2+\left(c^2-v^2\right) t_{z2}^{\,2}+\left(-2 c^2 t+2 t_z v^2\right) t_{z2}+c^2 t^2-t_z^{\,2} v^2}\right.$$
$$\left.-v\cos\left(\theta_0\right)\left(t_z-t_{z2}\right)\right)\sin\left(\theta_0\right) \qquad (56)$$

$simplify(\ y\_izo\_tzap\_normal(p\_t_{z2}))$

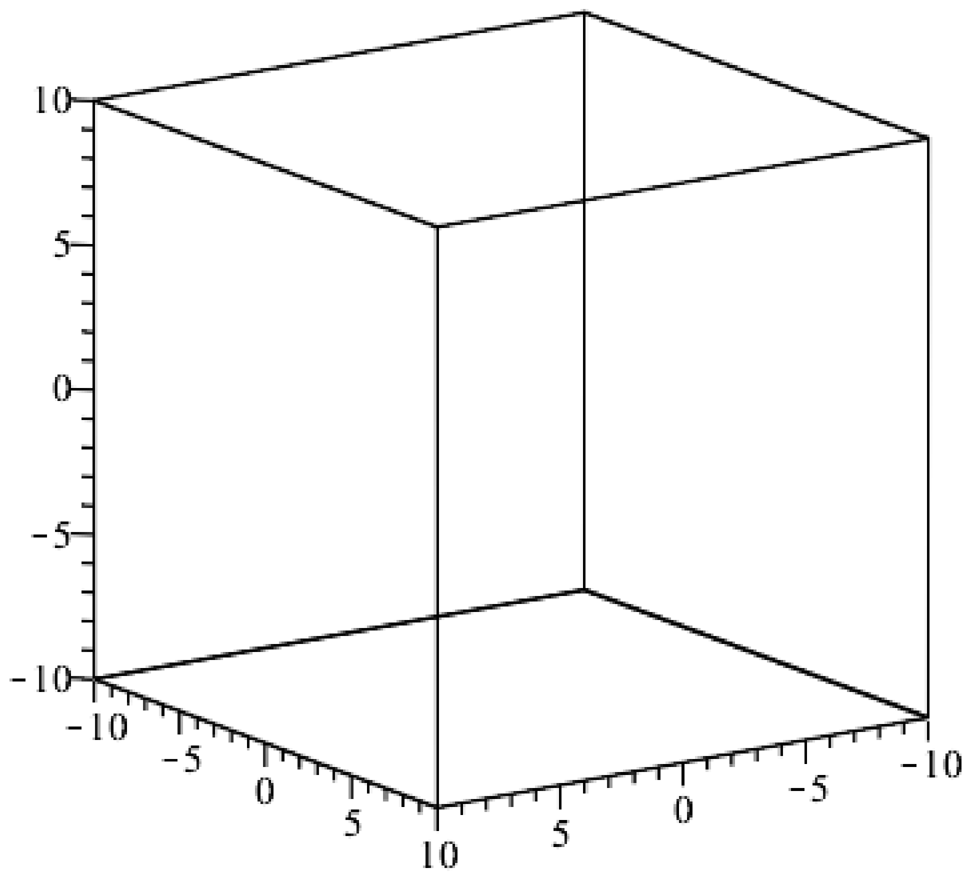
$$\sin(\varphi_0)\left(\sqrt{v^2\left(t_z-t_{z2}\right)^2\cos\left(\theta_0\right)^2+\left(c^2-v^2\right) t_{z2}^{\,2}+\left(-2 c^2 t+2 t_z v^2\right) t_{z2}+c^2 t^2-t_z^{\,2} v^2}\right.$$
$$\left.-v\cos\left(\theta_0\right)\left(t_z-t_{z2}\right)\right)\sin\left(\theta_0\right) \qquad (57)$$

$simplify(\ z\_izo\_tzap\_normal(p\_t_{z2}))$

$$\sqrt{v^2\left(t_z-t_{z2}\right)^2\cos\left(\theta_0\right)^2+\left(c^2-v^2\right) t_{z2}^{\,2}+\left(-2 c^2 t+2 t_z v^2\right) t_{z2}+c^2 t^2-t_z^{\,2} v^2}\,\cos\left(\theta_0\right)$$
$$-v\left(\left(t_z-t_{z2}\right)\cos\left(\theta_0\right)^2-t_z\right) \qquad (58)$$

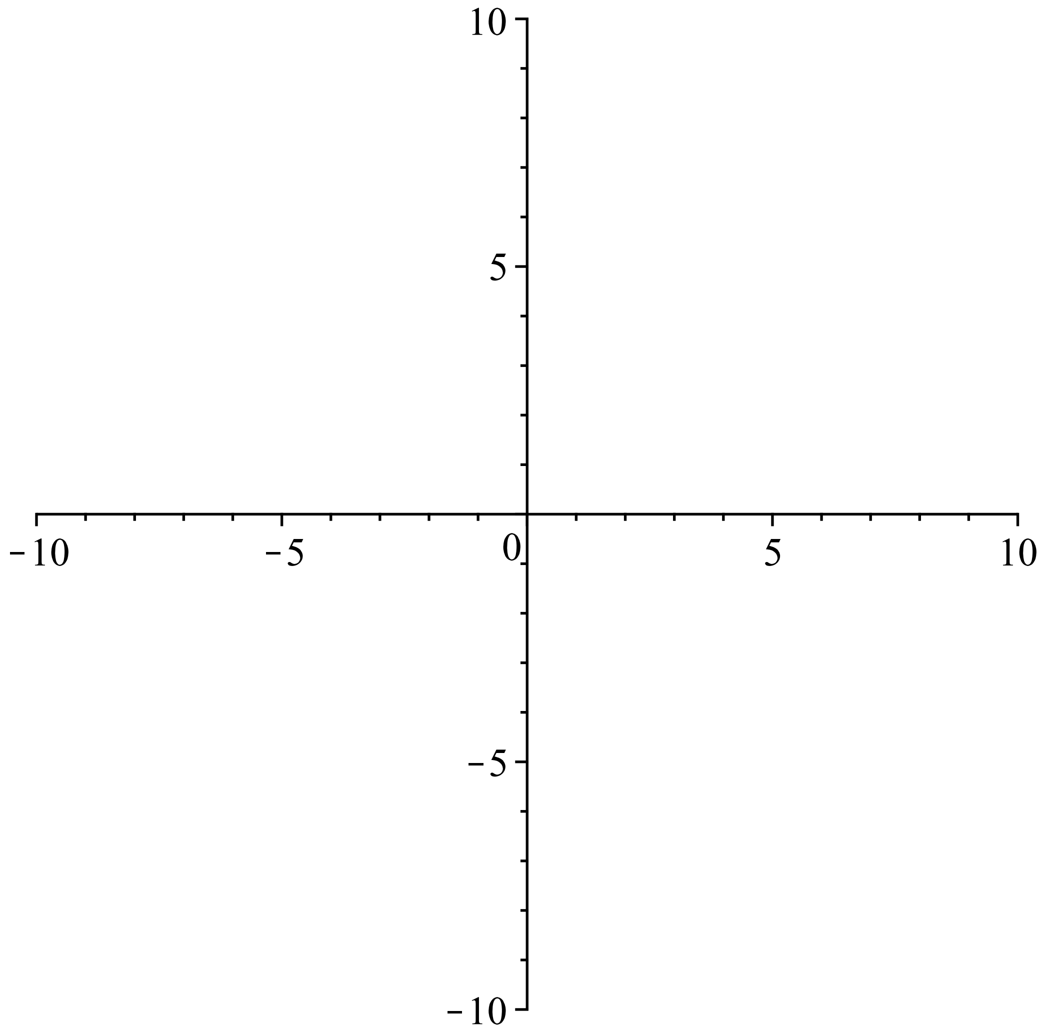
$$spacecurve\left(\left[subs\left(c=1, t=5, v=0.5, \theta=\frac{\pi}{8}, \varphi=0, \left[x_z, y_z, z_z\right]\right), subs\left(c=1, t=5, v=0.5, \theta=\frac{\pi}{10}, \varphi\right.\right.\right.$$
$$\left.\left.\left.=0, \left[x_z, y_z, z_z\right]\right)\right], t_z=0..5, thickness=1, numpoints=100, color=black\right)$$

Warning, unable to evaluate the functions to numeric values in the region; see the plotting command's help page to ensure the calling sequence is correct



$$plot\left(subs\left(c=1,\ t=5,\ v=0.5,\ \theta=\frac{\pi}{2},\ \left[xy_z,\ z_z,\ t_z=0\,..t\right]\right)\right)$$

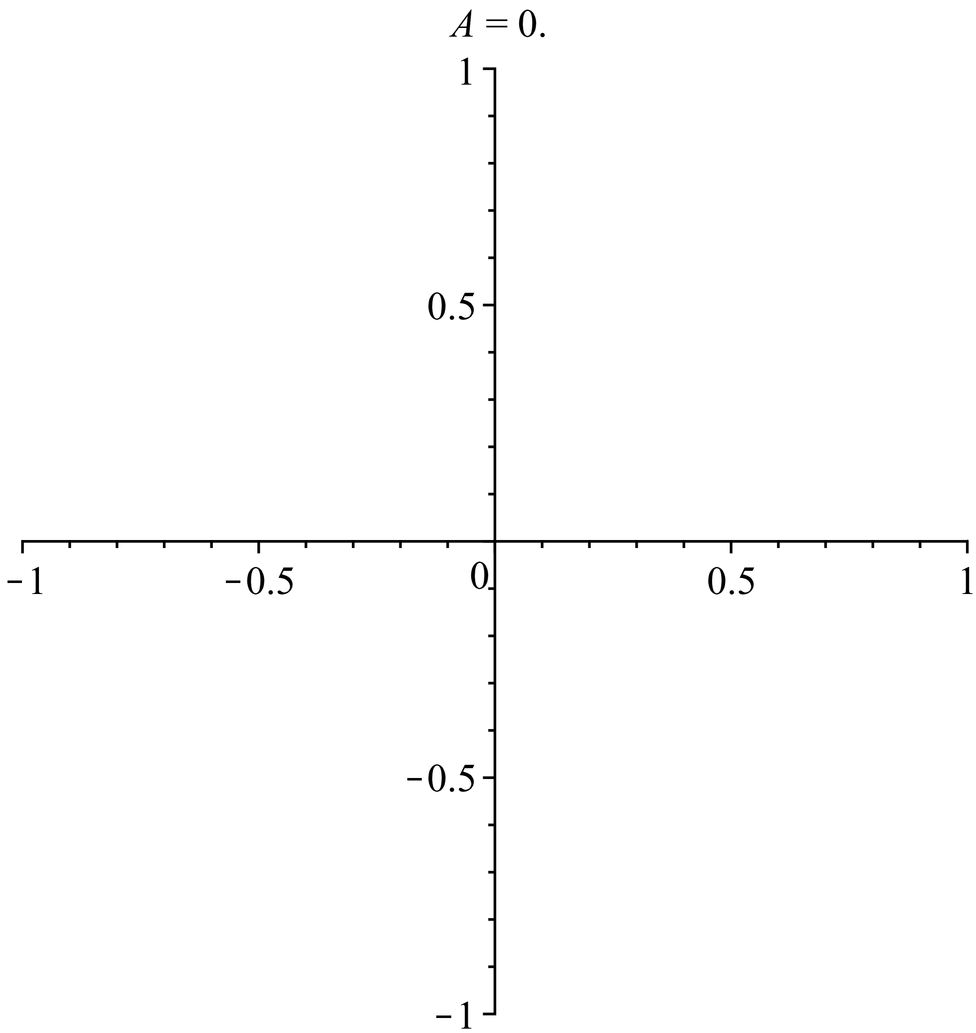
Warning, expecting only range variable t__z in expressions [xy__z, z__z] to be plotted but found names [xy__z, z__z]



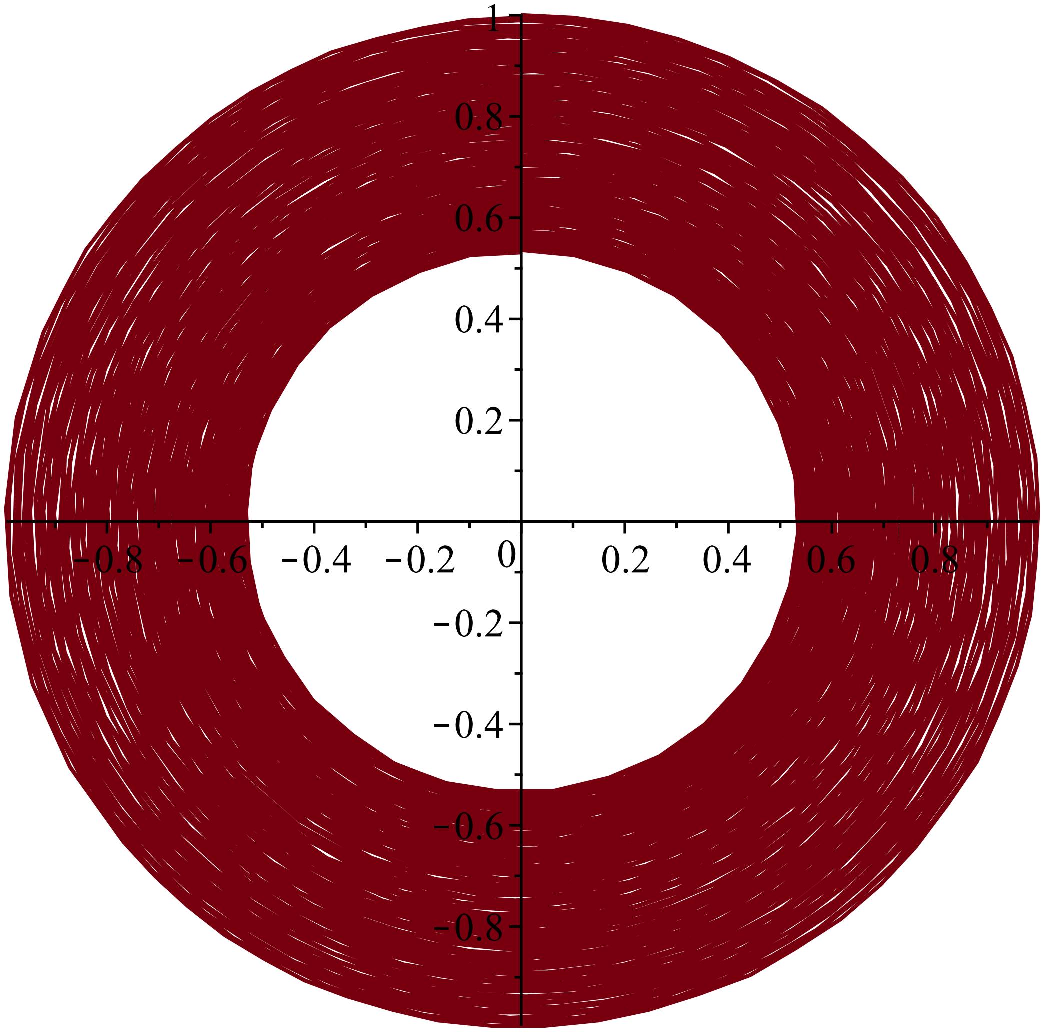
$6\,d$

$$6\,d \tag{59}$$

$animate\left(plot,\ [[\cos(t),\ \sin(t),\ t=0..A]],\ A=0..2\,\pi,\ scaling=constrained,\ frames=50\right)$

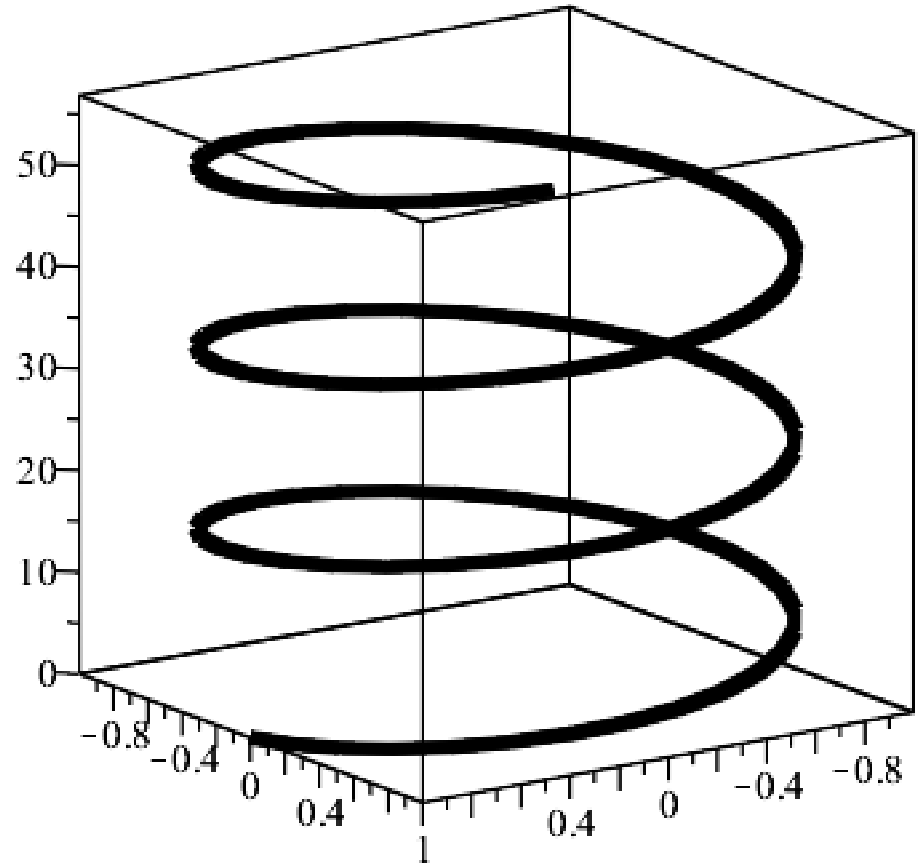


$plot([\exp(-0.001 \cdot t) \cdot \sin(t), \exp(-0.001 \cdot t) \cdot \cos(t), t = 0..200 \cdot \pi])$



$opts := thickness = 5, numpoints = 100, color = black:$

$spacecurve([\cos(t), \sin(t), (2 + \sin(1))\ t], t = 0 ..20, opts)$



$animate\left(spacecurve, [[\cos(t), \sin(t), (2+\sin(A))\ t], t=0..20, opts], A=0..20\,\pi\right)$

$A = 0.$