restart : *clear* :

R $\qquad\qquad\qquad\qquad$ R_0

$$R_0 \tag{1}$$

$\sigma(q, r) := \dfrac{q}{4\cdot\pi\cdot r^2}$

$$(q, r) \rightarrow \frac{1}{4}\,\frac{q}{\pi\, r^2} \tag{2}$$

$\dfrac{\mathrm{d}}{\mathrm{d}\,\theta} S := 2\cdot\pi\cdot r^2\cdot\sin(\theta)$

$\varphi_R(q, r, R_0) := \displaystyle\int_0^{\pi} \frac{2\cdot\pi\cdot r^2\cdot\sin(\theta)\cdot\sigma(q, r)}{\sqrt{(R_0)^2 - 2\cdot R_0\cdot r\cdot\cos(\theta) + (r)^2}}\,\mathrm{d}\theta$

$$(q, r, R_0) \rightarrow \int_0^{\pi} \frac{2\,\pi\, r^2 \sin(\theta)\,\sigma(q, r)}{\sqrt{R_0^{\,2} - 2\,R_0\, r\cos(\theta) + r^2}}\,\mathrm{d}\theta \tag{3}$$

$\varphi_R(q, r, R_0)$

$$\int_0^{\pi} \frac{1}{2}\,\frac{\sin(\theta)\, q}{\sqrt{R_0^{\,2} - 2\,R_0\, r\cos(\theta) + r^2}}\,\mathrm{d}\theta \tag{4}$$

$\varphi_R(q, 1, R_0) + \varphi_R(-q, 2, R_0)$

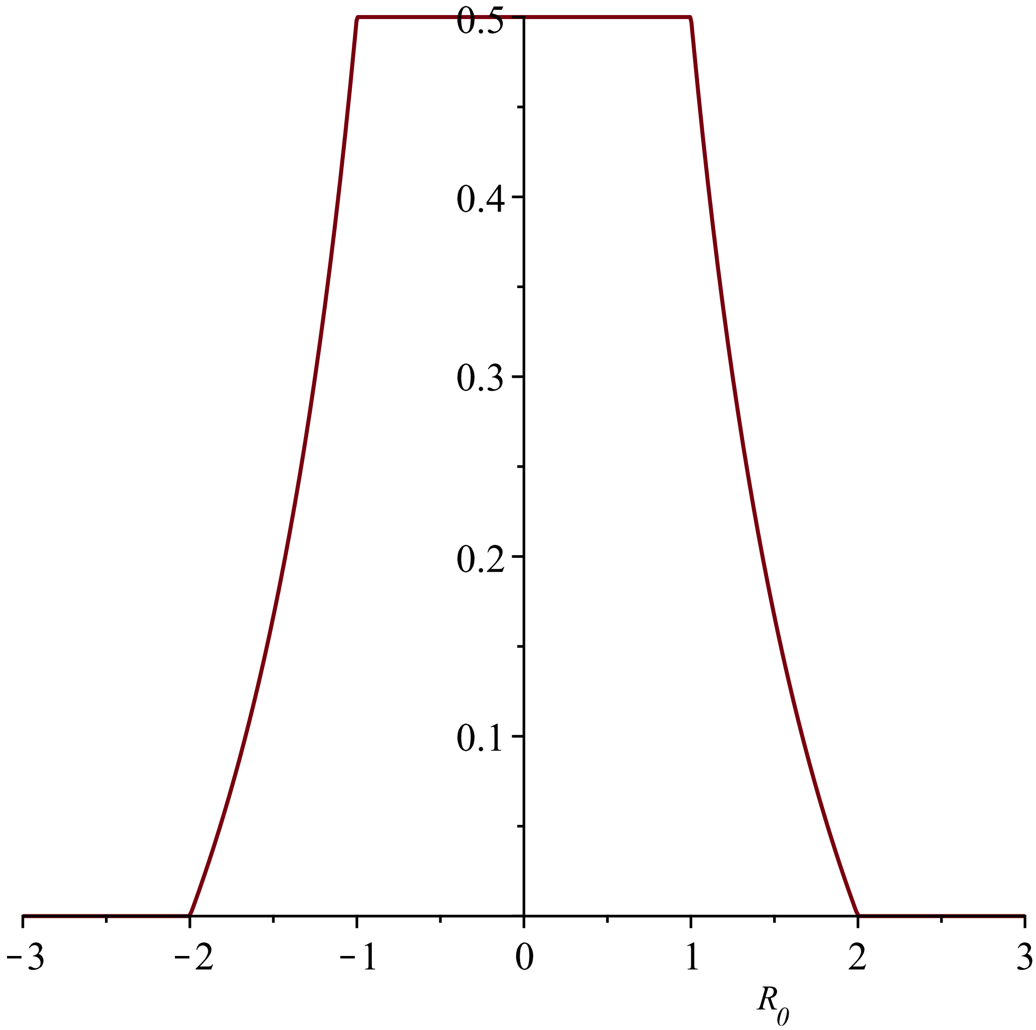
$$\int_0^{\pi} \frac{1}{2}\,\frac{\sin(\theta)\, q}{\sqrt{R_0^{\,2} - 2\,R_0\cos(\theta) + 1}}\,\mathrm{d}\theta + \int_0^{\pi}\left(-\frac{1}{2}\,\frac{\sin(\theta)\, q}{\sqrt{R_0^{\,2} - 4\,R_0\cos(\theta) + 4}}\right)\mathrm{d}\theta \tag{5}$$

$evalf\left(\varphi_R(1, 1, R_0) + \varphi_R(-1, 2, R_0)\right)$

$$\int_{0.}^{3.141592654} \frac{0.5000000000 \sin(\theta)}{\sqrt{R_0^2 - 2.\, R_0 \cos(\theta) + 1.}} \, d\theta + \int_{0.}^{3.141592654} \left(- \frac{0.5000000000 \sin(\theta)}{\sqrt{R_0^2 - 4.\, R_0 \cos(\theta) + 4.}} \right) d\theta \qquad \textbf{(6)}$$

2 - 1

$with(plots) : plot\left(\varphi_R(1, 1, R_0) + \varphi_R(-1, 2, R_0), R_0 = -3 .. 3\right)$



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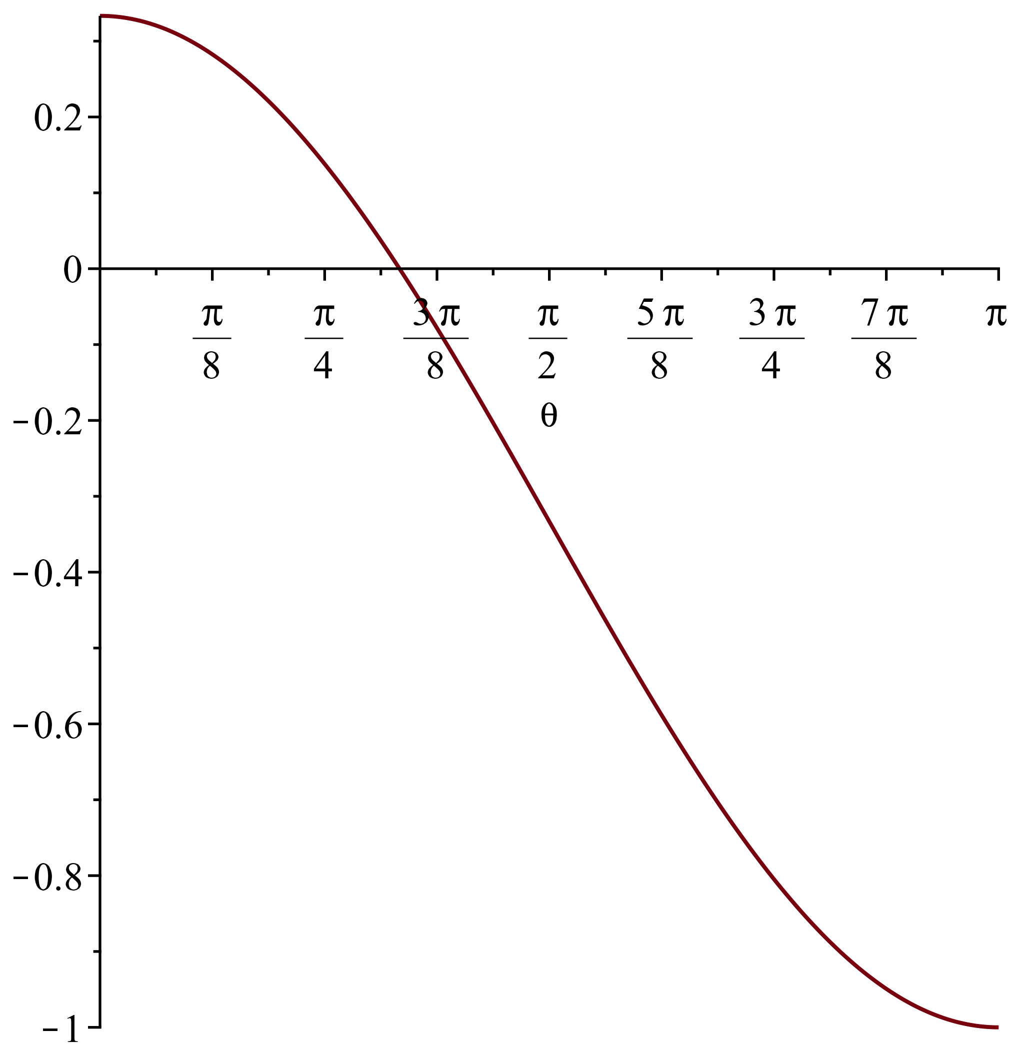
$$vr(v, c, r, R_0, \theta) := \frac{v}{c} \cdot \left(R_0 \cdot \cos(\theta) - r\right)$$

$$\left(v, c, r, R_0, \theta\right) \rightarrow \frac{v\left(R_0 \cos(\theta) - r\right)}{c} \tag{7}$$

$plot\left(vr(1, 3, 1, 2, \theta), \theta = 0 .. \pi\right)$



R_0 ,

R v .

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$$v \tag{8}$$

$$\varphi_{lw}\left(q, v, c, r, R_0\right) := \int_0^{\pi} \frac{2 \cdot \pi \cdot r^2 \cdot \sin(\theta) \cdot \sigma(q, r)}{\sqrt{\left(R_0\right)^2 - 2 \cdot R_0 \cdot r \cdot \cos(\theta) + (r)^2} - \frac{v}{c} \cdot \left(R_0 \cdot \cos(\theta) - r\right)} \, \mathrm{d}\theta$$

$$(q, v, c, r, R_0) \to \int_0^{\pi} \frac{2 \pi r^2 \sin(\theta)\, \sigma(q, r)}{\sqrt{R_0^2 - 2 R_0 r \cos(\theta) + r^2} - \frac{v (R_0 \cos(\theta) - r)}{c}} \, \mathrm{d}\theta \tag{9}$$

$\varphi_{lw}(q, v, c, r, R_0)$

$$\int_0^{\pi} \frac{1}{2} \frac{\sin(\theta)\, q}{\sqrt{R_0^2 - 2 R_0 r \cos(\theta) + r^2} - \frac{v (R_0 \cos(\theta) - r)}{c}} \, \mathrm{d}\theta \tag{10}$$

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$R_+ := 1 :$ $R_- := 2 :$

$c := 3 :$

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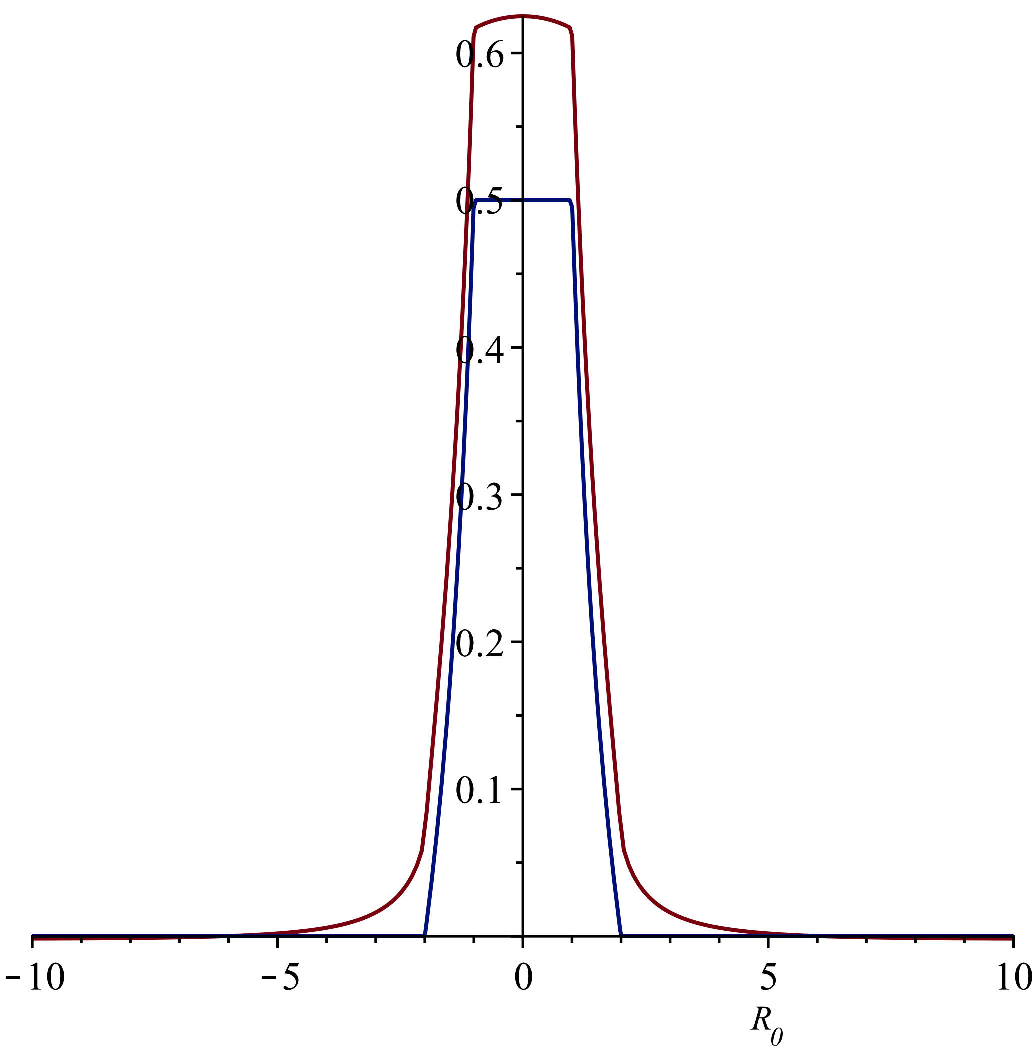
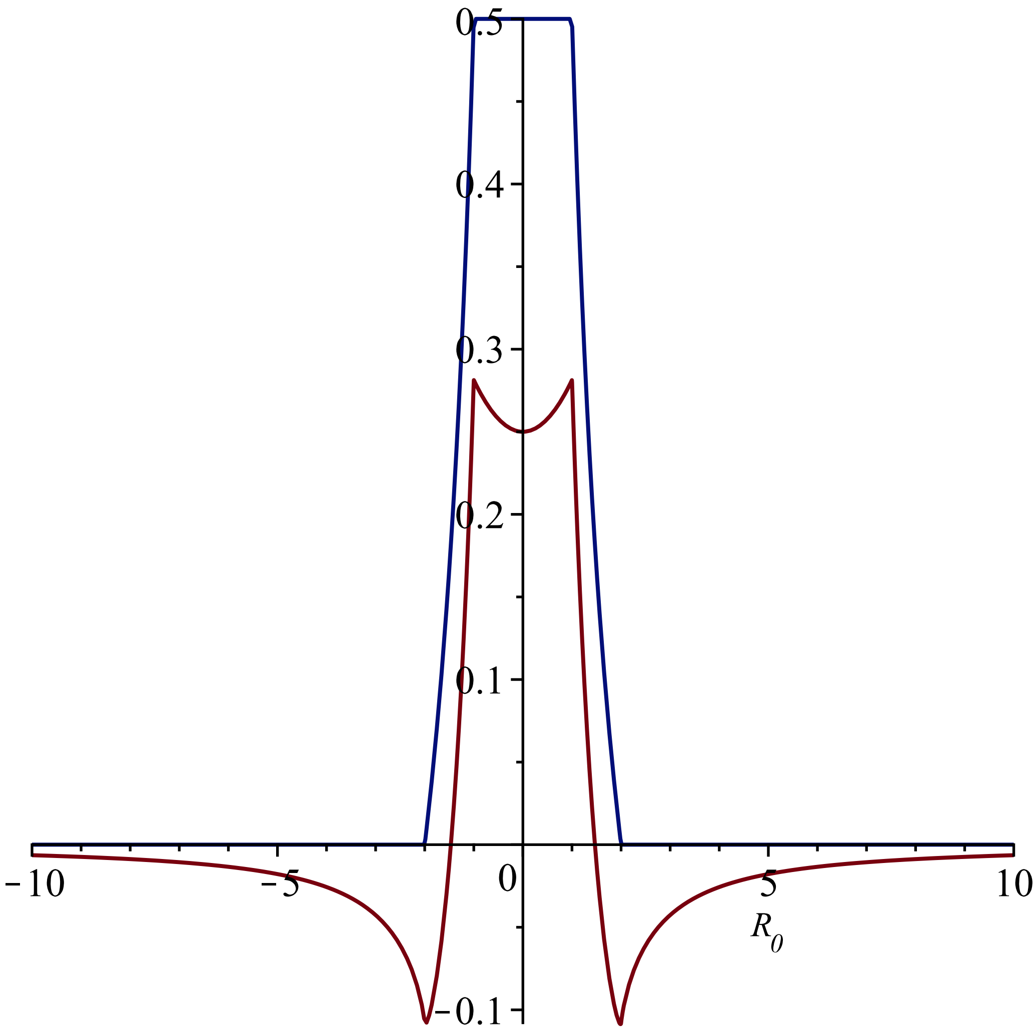
$with(plots) : plot\left(\left[\varphi_R(1, R_+, R_0) + \varphi_{lw}(-1, 1, c, R_-, R_0), \varphi_R(1, R_+, R_0) + \varphi_R(-1, R_-, R_0)\right], R_0 = -10..10\right)$



$with(plots) : plot\left(\left[\varphi_R(1, R_+, R_0) + \varphi_{lw}(-1, -1, c, R_-, R_0), \varphi_R(1, R_+, R_0) + \varphi_R(-1, R_-, R_0)\right], R_0 = -10..10\right)$



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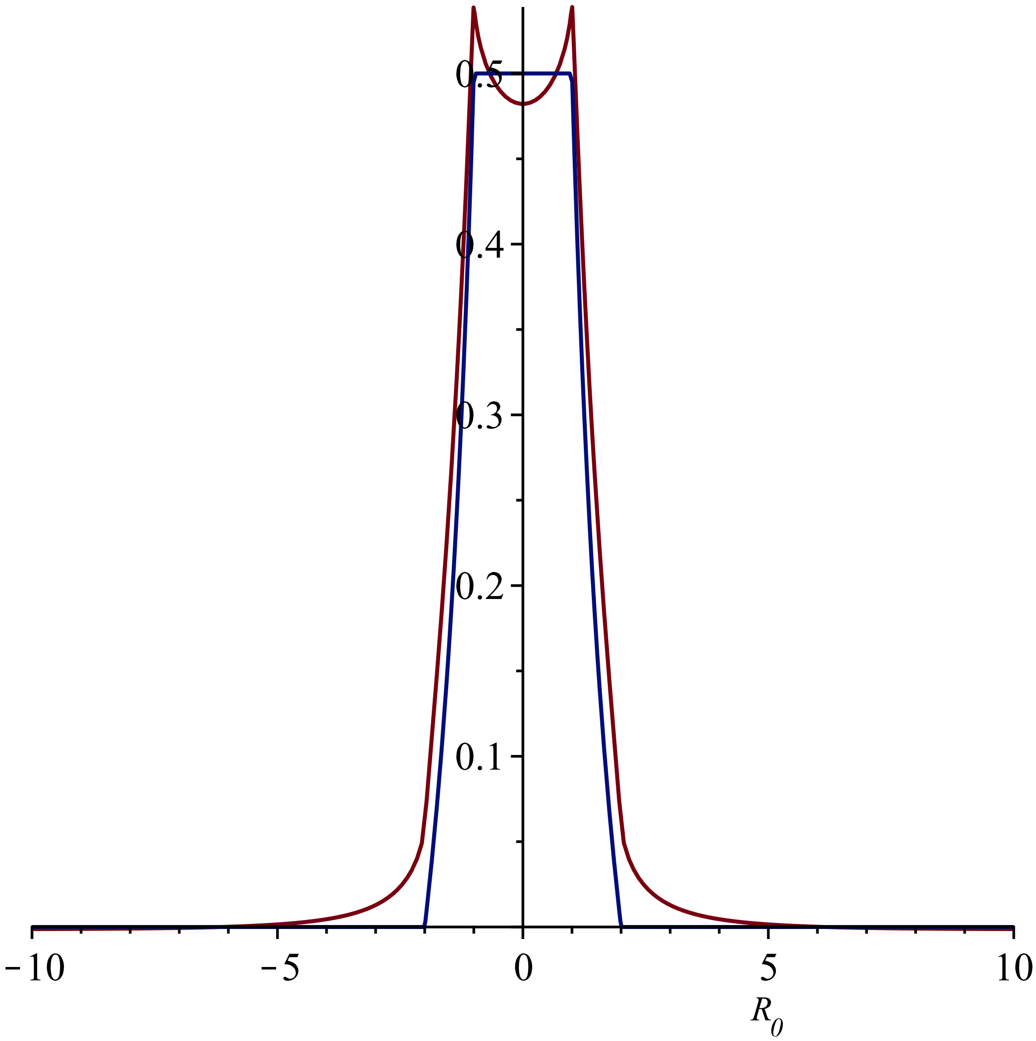
with(plots) : plot$\left(\left[\varphi_{lw}(1, 0.5, c, R_+, R_0) + \varphi_{lw}(-1, 1, c, R_-, R_0), \varphi_R(1, R_+, R_0) + \varphi_R(-1, R_-, R_0)\right], R_0 = -10..10\right)$



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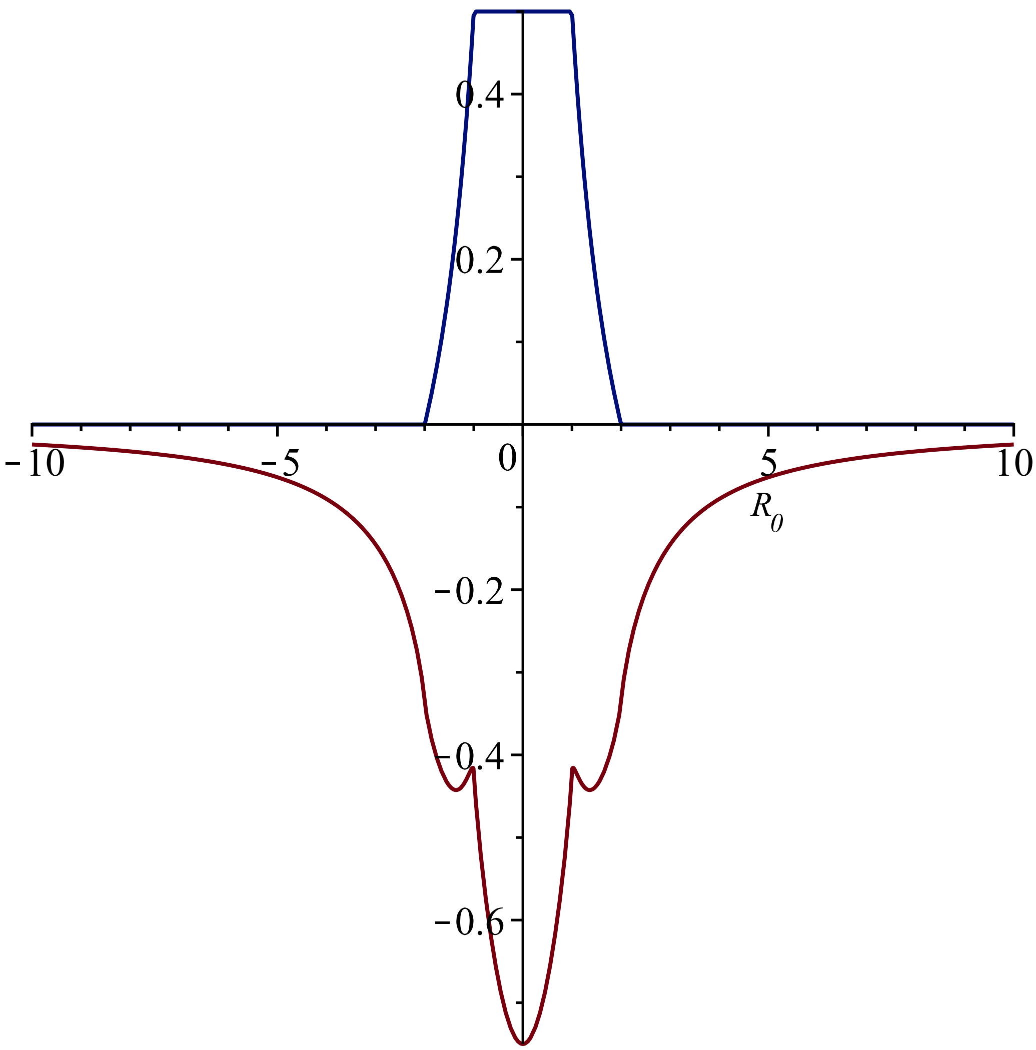
$$with(plots) : plot\left(\left[\varphi_{lw}(1, 1, c, R_+, R_0) + \varphi_{lw}(-1, -2, c, R_-, R_0), \varphi_R(1, R_+, R_0) + \varphi_R(-1, R_-, R_0)\right], R_0 = -10..10\right)$$



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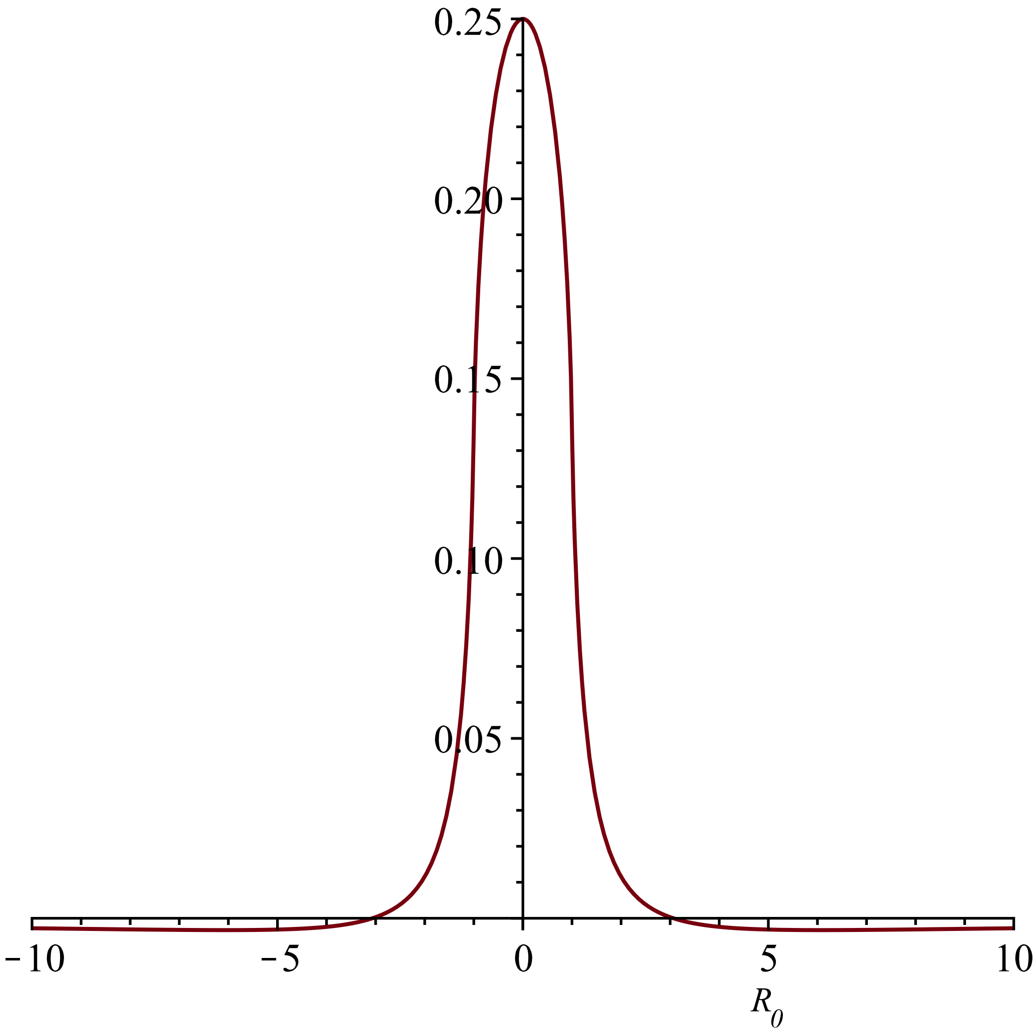
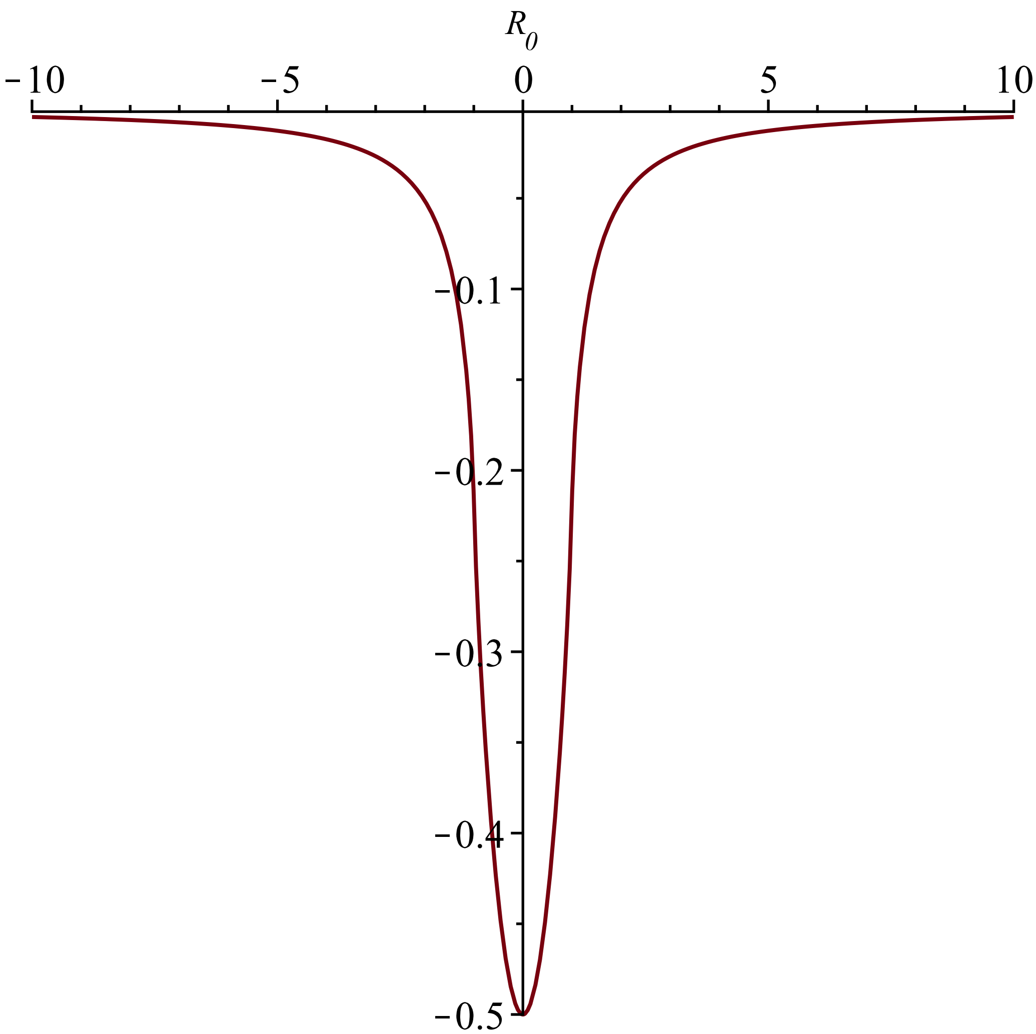
$d\varphi_{lw}(q, v, c, R, R_0) := \varphi_{lw}(q, v, c, R, R_0) + \varphi_{lw}(-q, 0, c, R, R_0)$

$$(q, v, c, R, R_0) \rightarrow \varphi_{lw}(q, v, c, R, R_0) + \varphi_{lw}(-q, 0, c, R, R_0) \tag{11}$$

$with(plots) : plot([d\varphi_{lw}(-1, 1, c, R_+, R_0)], R_0 = -10..10)$



$with(plots) : plot\left(\left[d\varphi_{lw}(-1, -1, c, R_{+}, R_{0})\right], R_{0}=-10\,..10\right)$



:

$v_{0r}(r_0) := v$

$v_{0r}(r_0)$

r_0

$a_{0r}(r_0) := a$

$a_{0r}(r0)$

$$r(t, r_0, v_0, a_0) := r_0 + v_0 \cdot t + \frac{a_0 \cdot t^2}{2}$$

$$(t, r_0, v_0, a_0) \rightarrow r_0 + v_0\, t + \frac{1}{2}\, a_0\, t^2 \tag{12}$$

$$r(t, r_0, v_0, a_0)$$

$$r_0 + v_0\, t + \frac{1}{2}\, a_0\, t^2 \tag{13}$$

$$v_r(t, r_0, v_0, a_0) := v_0 + a_0 \cdot t$$

$$(t, r_0, v_0, a_0) \rightarrow v_0 + a_0\, t \tag{14}$$

$$v_r(t, r_0, v_0, a_0)$$

$$a_0\, t + v_0 \tag{15}$$

$$tzap(t, r_0, v_0, a_0, R_0, \theta) := solve\left(c^2\, (t - t_{zap})^2 = R_0^{\,2} - 2 \cdot R_0 \cdot r(t_{zap}, r_0, v_0, a_0) \cdot \cos(\theta) + r(t_{zap}, r_0, v_0, a_0)^2, t_{zap}\right)$$

$$(t, r_0, v_0, a_0, R_0, \theta) \rightarrow solve\left(c^2\, (t - t_{zap})^2 = R_0^{\,2} - 2\, R_0\, r(t_{zap}, r_0, v_0, a_0)\, \cos(\theta) + r(t_{zap}, r_0, v_0, a_0)^2, t_{zap}\right) \tag{16}$$

$$tzap(t, r_0, v_0, a_0, R_0, \theta)$$

$$RootOf\left(a_0^{\,2}\, \_Z^4 + 4\, v_0\, \_Z^3\, a_0 + \left(-4\, R_0\, a_0 \cos(\theta) + 4\, a_0\, r_0 + 4\, v_0^{\,2} - 36\right) \_Z^2 + \left(-8\, R_0\, v_0 \cos(\theta) + 8\, r_0\, v_0 + 72\, t\right) \_Z - 8\, R_0 \cos(\theta)\, r_0 + 4\, R_0^{\,2} + 4\, r_0^{\,2} - 36\, t^2\right) \tag{17}$$

$$R_{zap}(t_{zap}, r_0, v_0, a_0, R_0, \theta) := \sqrt{R_0^{\,2} - 2 \cdot R_0 \cdot r(t_{zap}, r_0, v_0, a_0) \cdot \cos(\theta) + r(t_{zap}, r_0, v_0, a_0)^2}$$

$$(t_{zap}, r_0, v_0, a_0, R_0, \theta) \rightarrow \sqrt{R_0^{\,2} - 2\, R_0\, r(t_{zap}, r_0, v_0, a_0)\, \cos(\theta) + r(t_{zap}, r_0, v_0, a_0)^2} \tag{18}$$

$$R_{zap}(t_{zap}, r_0, v_0, a_0, R_0, \theta)$$

$$\sqrt{R_0^{\,2} - 2\, R_0 \left(r_0 + v_0\, t_{zap} + \frac{1}{2}\, a_0\, t_{zap}^{\,2}\right) \cos(\theta) + \left(r_0 + v_0\, t_{zap} + \frac{1}{2}\, a_0\, t_{zap}^{\,2}\right)^2} \tag{19}$$

$$R_{zap}(tzap(t, r_0, v_0, a_0, R_0, \theta), r_0, v_0, a_0, R_0, \theta);$$

$$\Big(R_0^{\,2} - 2\, R_0 \Big(r_0 + v_0\, RootOf\big(a_0^{\,2}\, \_Z^4 + 4\, v_0\, \_Z^3\, a_0 + \big(-4\, R_0\, a_0 \cos(\theta) + 4\, a_0\, r_0 + 4\, v_0^{\,2} \tag{20}$$

$$- 36\big)\ \_Z^2 + \big(-8\,R_0\,v_0\cos(\theta) + 8\,r_0\,v_0 + 72\,t\big)\ \_Z - 8\,R_0\cos(\theta)\,r_0 + 4\,R_0^2 + 4\,r_0^2$$

$$- 36\,t^2\big) + \frac{1}{2}\,a_0\,RootOf\Big(a_0^2\ \_Z^4 + 4\,v_0\,\_Z^3\,a_0 + \big(-4\,R_0\,a_0\cos(\theta) + 4\,a_0\,r_0 + 4\,v_0^2$$

$$- 36\big)\ \_Z^2 + \big(-8\,R_0\,v_0\cos(\theta) + 8\,r_0\,v_0 + 72\,t\big)\ \_Z - 8\,R_0\cos(\theta)\,r_0 + 4\,R_0^2 + 4\,r_0^2$$

$$- 36\,t^2\big)^2\Big)\cos(\theta) + \Big(r_0 + v_0\,RootOf\Big(a_0^2\ \_Z^4 + 4\,v_0\,\_Z^3\,a_0 + \big(-4\,R_0\,a_0\cos(\theta)$$

$$+ 4\,a_0\,r_0 + 4\,v_0^2 - 36\big)\ \_Z^2 + \big(-8\,R_0\,v_0\cos(\theta) + 8\,r_0\,v_0 + 72\,t\big)\ \_Z - 8\,R_0\cos(\theta)\,r_0$$

$$+ 4\,R_0^2 + 4\,r_0^2 - 36\,t^2\big) + \frac{1}{2}\,a_0\,RootOf\Big(a_0^2\ \_Z^4 + 4\,v_0\,\_Z^3\,a_0 + \big(-4\,R_0\,a_0\cos(\theta)$$

$$+ 4\,a_0\,r_0 + 4\,v_0^2 - 36\big)\ \_Z^2 + \big(-8\,R_0\,v_0\cos(\theta) + 8\,r_0\,v_0 + 72\,t\big)\ \_Z - 8\,R_0\cos(\theta)\,r_0$$

$$+ 4\,R_0^2 + 4\,r_0^2 - 36\,t^2\big)^2\Big)^2\Big)^{1/2}$$

,

$$K_{zap}(t_{zap}, r_0, v_0, a_0, R_0, \theta) := R_{zap}(t_{zap}, r_0, v_0, a_0, R_0, \theta) - \frac{v_r(t_{zap}, r_0, v_0, a_0)}{c}\cdot\big(R_0\cdot\cos(\theta) - r(t_{zap}, r_0, v_0, a_0)\big)$$

$$(t_{zap}, r_0, v_0, a_0, R_0, \theta) \to R_{zap}(t_{zap}, r_0, v_0, a_0, R_0, \theta) - \frac{v_r(t_{zap}, r_0, v_0, a_0)\,\big(R_0\cos(\theta) - r(t_{zap}, r_0, v_0, a_0)\big)}{c} \tag{21}$$

$$K_{zap}(t_{zap}, r_0, v_0, a_0, R_0, \theta)$$

$$\sqrt{R_0^2 - 2\,R_0\left(r_0 + v_0\,t_{zap} + \frac{1}{2}\,a_0\,t_{zap}^2\right)\cos(\theta) + \left(r_0 + v_0\,t_{zap} + \frac{1}{2}\,a_0\,t_{zap}^2\right)^2} - \frac{1}{3}\,(a_0\,t_{zap} + v_0)\left(R_0\cos(\theta) - r_0 - v_0\,t_{zap} - \frac{1}{2}\,a_0\,t_{zap}^2\right) \tag{22}$$

$$K_{zap}(tzap(t, r_0, v_0, a_0, R_0, \theta), r_0, v_0, a_0, R_0, \theta);$$

$$\Big(R_0^2 - 2\,R_0\Big(r_0 + v_0\,RootOf\Big(a_0^2\ \_Z^4 + 4\,v_0\,\_Z^3\,a_0 + \big(-4\,R_0\,a_0\cos(\theta) + 4\,a_0\,r_0 + 4\,v_0^2 \tag{23}$$

$$- 36\big)\ \_Z^2 + \big(-8\,R_0\,v_0\cos(\theta) + 8\,r_0\,v_0 + 72\,t\big)\ \_Z - 8\,R_0\cos(\theta)\,r_0 + 4\,R_0^2 + 4\,r_0^2$$

$$- 36\,t^2\big) + \frac{1}{2}\,a_0\,RootOf\Big(a_0^2\ \_Z^4 + 4\,v_0\,\_Z^3\,a_0 + \big(-4\,R_0\,a_0\cos(\theta) + 4\,a_0\,r_0 + 4\,v_0^2$$

$$
\begin{aligned}
&-36)\,\_Z^2+\left(-8\,R_0\,v_0\cos(\theta)+8\,r_0\,v_0+72\,t\right)\_Z-8\,R_0\cos(\theta)\,r_0+4\,R_0^2+4\,r_0^2\\
&-36\,t^2\big)^2\Big)\cos(\theta)+\Big(r_0+v_0\,RootOf\big(a_0^2\,\_Z^4+4\,v_0\,\_Z^3\,a_0+\big(-4\,R_0\,a_0\cos(\theta)\\
&+4\,a_0\,r_0+4\,v_0^2-36\big)\,\_Z^2+\left(-8\,R_0\,v_0\cos(\theta)+8\,r_0\,v_0+72\,t\right)\_Z-8\,R_0\cos(\theta)\,r_0\\
&+4\,R_0^2+4\,r_0^2-36\,t^2\big)+\frac{1}{2}\,a_0\,RootOf\big(a_0^2\,\_Z^4+4\,v_0\,\_Z^3\,a_0+\big(-4\,R_0\,a_0\cos(\theta)\\
&+4\,a_0\,r_0+4\,v_0^2-36\big)\,\_Z^2+\left(-8\,R_0\,v_0\cos(\theta)+8\,r_0\,v_0+72\,t\right)\_Z-8\,R_0\cos(\theta)\,r_0\\
&+4\,R_0^2+4\,r_0^2-36\,t^2\big)^2\Big)^2\Big)^{1/2}-\frac{1}{3}\,\big(v_0+a_0\,RootOf\big(a_0^2\,\_Z^4+4\,v_0\,\_Z^3\,a_0+\big(\\
&-4\,R_0\,a_0\cos(\theta)+4\,a_0\,r_0+4\,v_0^2-36\big)\,\_Z^2+\left(-8\,R_0\,v_0\cos(\theta)+8\,r_0\,v_0+72\,t\right)\_Z\\
&-8\,R_0\cos(\theta)\,r_0+4\,R_0^2+4\,r_0^2-36\,t^2\big)\big)\,\Big(R_0\cos(\theta)-r_0-v_0\,RootOf\big(a_0^2\,\_Z^4\\
&+4\,v_0\,\_Z^3\,a_0+\big(-4\,R_0\,a_0\cos(\theta)+4\,a_0\,r_0+4\,v_0^2-36\big)\,\_Z^2+\big(-8\,R_0\,v_0\cos(\theta)+8\,r_0\,v_0\\
&+72\,t\big)\,\_Z-8\,R_0\cos(\theta)\,r_0+4\,R_0^2+4\,r_0^2-36\,t^2\big)-\frac{1}{2}\,a_0\,RootOf\big(a_0^2\,\_Z^4\\
&+4\,v_0\,\_Z^3\,a_0+\big(-4\,R_0\,a_0\cos(\theta)+4\,a_0\,r_0+4\,v_0^2-36\big)\,\_Z^2+\big(-8\,R_0\,v_0\cos(\theta)+8\,r_0\,v_0\\
&+72\,t\big)\,\_Z-8\,R_0\cos(\theta)\,r_0+4\,R_0^2+4\,r_0^2-36\,t^2\big)^2\Big)
\end{aligned}
$$

r , R_0

$$
\varphi(q,t,r_0,v_0,a_0,R_0):=\int_0^{\pi}\frac{2\cdot\pi\cdot r(t,r_0,v_0,a_0)^2\cdot\sin(\theta)\cdot\sigma(q,r_0)}{K_{zap}\left(tzap(t,r_0,v_0,a_0,R_0,\theta),r_0,v_0,a_0,R_0,\theta\right)}\,d\theta;
$$

$$
(q,t,r_0,v_0,a_0,R_0)\rightarrow\int_0^{\pi}\frac{2\,\pi\,r(t,r_0,v_0,a_0)^2\sin(\theta)\,\sigma(q,r_0)}{K_{zap}\left(tzap(t,r_0,v_0,a_0,R_0,\theta),r_0,v_0,a_0,R_0,\theta\right)}\,d\theta \qquad \textbf{(24)}
$$

$$
A_{R_0}(q,t,r_0,v_0,a_0,R_0):=\int_0^{\pi}\frac{2\cdot\pi\cdot r(t,r_0,v_0,a_0)^2\cdot\sin(\theta)\cdot\sigma(q,r_0)\cdot\left(v_r(t_{zap},r_0,v_0,a_0)\cdot\cos(\theta)\right)}{K_{zap}\left(tzap(t,r_0,v_0,a_0,R_0,\theta),r_0,v_0,a_0,R_0,\theta\right)}\,d\theta;
$$

$$(q, t, r_0, v_0, a_0, R_0) \rightarrow \int_0^{\pi} \frac{2\,\pi\, r(t, r_0, v_0, a_0)^2 \sin(\theta)\, \sigma(q, r_0)\, v_r(t_{zap}, r_0, v_0, a_0) \cos(\theta)}{K_{zap}(tzap(t, r_0, v_0, a_0, R_0, \theta), r_0, v_0, a_0, R_0, \theta)}\, \mathrm{d}\theta \tag{25}$$

$evalf([subs(q=-1, r=R_-, v=1, c=3, R_0=2, \varphi_{lw}(q, v, c, r, R_0)), subs(q=-1, t=0, r_0=R_-, v_0=1, a_0=0, R_0=2, \varphi(q, t, r_0, v_0, a_0, R_0))])$

$$[-0.4315231087, -0.3243416113] \tag{26}$$

$Digits := 5$

$$5 \tag{27}$$

c

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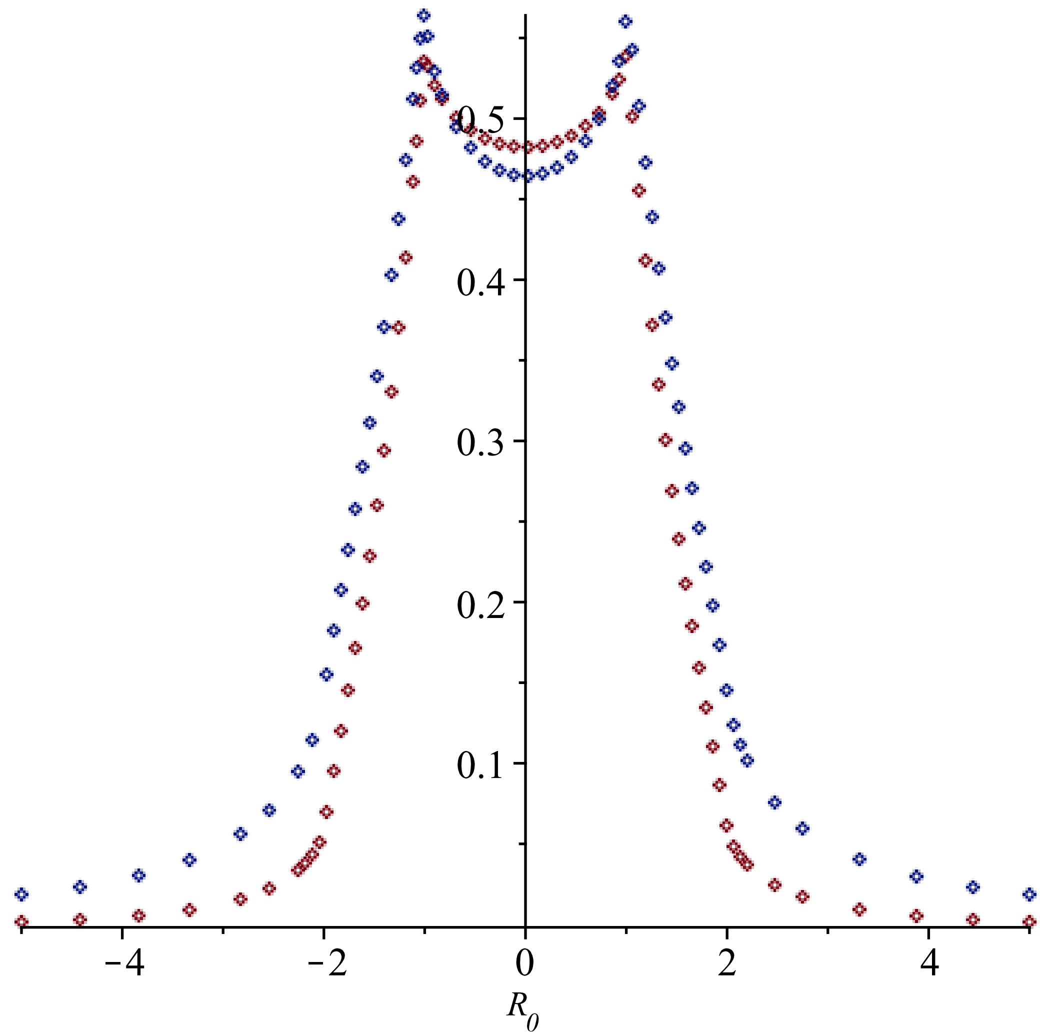
$with(plots) : plot\left(evalf\left(subs\left(q=1, t=0, r_{0e}=R_-, r_{0p}=R_+, v_{0p}=\frac{1}{2}, v_{0e}=1, a_0=0, [\varphi_{lw}(q, v_{0p}, c, r_{0p}, R_0) + \varphi_{lw}(-q, v_{0e}, c, r_{0e}, R_0), \varphi(q, t, r_{0p}, v_{0p}, a_0, R_0) + \varphi(-q, t, r_{0e}, v_{0e}, a_0, R_0)]\right)\right), R_0=-5..5, style=point, numpoints=10\right)$



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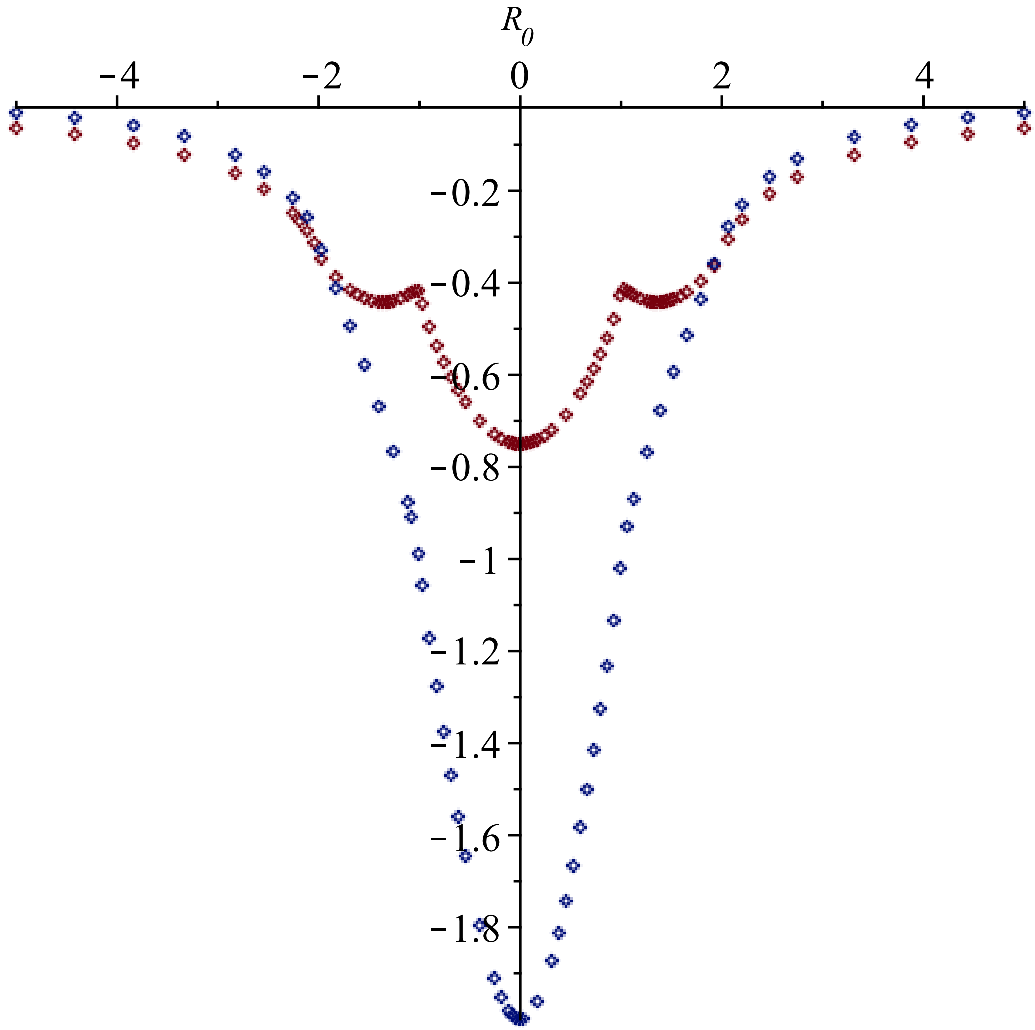
$with(plots) : plot\Big(evalf\Big(subs\Big(q=1, t=0, r_{0e}=R_{-}, r_{0p}=R_{+}, v_{0p}=1, v_{0e}=-2, a_0=0, \big[\,\varphi_{lw}(q, v_{0p}, c, r_{0p}, R_0) + \varphi_{lw}(-q, v_{0e}, c, r_{0e}, R_0), \varphi(q, t, r_{0p}, v_{0p}, a_0, R_0) + \varphi(-q, t, r_{0e}, v_{0e}, a_0, R_0)\,\big]\Big)\Big), R_0=-5..5, style=point, numpoints=10\Big)$



R_0
−4
−2
0
2
4
−0.2
−0.4
−0.6
−0.8
−1
−1.2
−1.4
−1.6
−1.8