Sonoluminescence and Black Holes as Sources of Squeezed Light

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Abstract. The phenomenon of coherent sonoluminescence is considered as a physical vacuum excitation. The state of the light emitted by sonoluminescence and by black hole evaporation is investigated. It is shown that the light state is a squeezed vacuum state. The question is discussed: is the black hole radiation a thermal radiation?

> My work always tried to unite the true with the beautiful, but when I had to choose one over the other I usually chose the beautiful.
> H.Weyl, (Gross (1988)).

We don't know, does God exist, but nevertheless we know that if He exists, He created the Universe as a pure mathematician. So we believe that without mathematics we can not to understand the Divine Mind in creating the Universe. We believe that abstract patterns, born in the minds of mathematicians, beautifully mesh with the physical structure of the Universe. D.V.Volkov, as you and I do, liked mathematics, but he was very distressed when theoretical physics was replaced by mathematics and pure mathematics in the sense of B.Russel (Bell (1989)): "Mathematics may be defined as a subject in which we newer know what we are talking about, nor whether what we are saying is true." And: "Pure mathematics consists entirely of such asseverations as that, if such proposition is true of *anything*, then such and such another proposition is true. It is essential not to discuss whether the first proposition is really true, and not to mention what the anything is of which it supposed to be true." D.V.Volkov liked another mathematics, mathematics of C.F.Gauss and G.F.B.Riemann, E.Cartan and E.A.Noeter, that possesses great truth and supreme beauty, about which H.Hertz once remarked: "One cannot escape the feeling that these mathematical formulas have an independent existence and intelligence of their own, that they are wiser than we are, wiser even than their discoverers, that we get more out of them than was originally put into them" (Bell (1937)).

Unfortunately my report will not contain much mathematics but I hope that according to uncertainties relation (Gross (1988))

$$\Delta\text{Mathematics} \times \Delta\text{Physics} \geq \text{Constant} \tag{1}$$

it will contain some physics of any interest to you.

Sonoluminescence as a physical vacuum excitation. Sonoluminescence is a convertation of sound energy, in the form of a beam of ultrasonic waves, into light energy. Light is emitted when acoustical energy is focused on a bubble of air trapped in water. Sound having a Mach number of order 10^{-5} is able to create photons with energies of several eV. Radiation occurs only during an interval of less than 50 ps within each cycle of the sound field, which has a period of a few tens of microseconds. The light's spectrum implies that the source of the radiation is similar to a black body at a temperature of tens of thousands of Kelvin (Hiller et al. (1992), Crum (1994), Putterman (1995)).

A beautiful explanation of remarkable phenomenon of sonoluminescence was proposed by J.Schwinger (Schwinger (1992), (1993), (1994)).On putting the question: "How does a macroscopic, classical, hydromechanical system, driven by a macroscopic acoustical force, generate an astonishingly short time scale and an accompanying high electromagnetic frequency, one that is at atomic level?" Schwinger suggested entirely unexpected solution of the problem: sonoluminescence is a result of quantum mechanical excitation of physical vacuum due to the abrupt slowing of the collapse of the bubble of air on which acoustical energy is focused. But why the collapse of the bubble is abruptly slowed? Schwinger suggested: "*The collapse of the cavity is slowed abruptly by the pressure of the light that is created by the abrupt slowing of the collapse.*" Schwinger's theory of sonoluminescence is based on the following phenomenon: the variation in the time of dielectric constant ε of medium is accompanied by emission of photons. Schwinger obtained simple formulas qualitatively describing experimental data. Schwinger's explanation of sonoluminescence was recently discussed by C.Eberlein (Eberlein (1996)), who pointed out on similarity of sonoluminescence and black hole evaporation: both are due to excitation of physical vacuum.

Sonoluminescence as a source of squeezed light. Let scalar field $\Phi(t)$ be the solution of the following equation

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{1}{\varepsilon}\Delta\right)\Phi(t) = 0 \tag{2}$$

where ε is dielectric constant of medium. Disregarding the spin of photons we suppose that the field $\Phi(t)$ describes the light quanta. Let the solution of the equation (2) be the plane wave

$$\Phi(t) = \mathrm{e}^{\mathbf{ikx}}\psi(t) \tag{3}$$

The function $\psi(t)$ satisfy the equation

$$\ddot{\psi} + \omega^2(t)\psi = 0 \tag{4}$$

where

$$\omega^2(t) = \frac{c^2\mathbf{k}^2}{\varepsilon(t)} \tag{5}$$

According to Schwinger let us assume that the $\varepsilon(t)$ is jump function of time with the jump from $\varepsilon \neq 1$ to $\varepsilon = 1$. The jump of dielectric constant leads to the transition

$$e^{-i\omega_1 t} \to \frac{\sqrt{\varepsilon}+1}{2} e^{-i\omega_2 t} + \frac{\sqrt{\varepsilon}-1}{2} e^{i\omega_2 t} \ . \tag{6}$$

Any transition of positive frequency solution to negative frequency solution

$$e^{-i\omega_1 t} \to \alpha e^{-i\omega_2 t} + \beta e^{i\omega_2 t} \tag{7}$$

means the creation of particles, because the quantum field

$$\psi = \frac{1}{\sqrt{\omega_1}} (a_1 e^{-i\omega_1 t} + a_1^+ e^{i\omega_1 t}) \tag{8}$$

will get over in new quantum field

$$\begin{aligned} &\tfrac{1}{\sqrt{\omega_1}} (a_1 e^{-i\omega_1 t} + a_1^+ e^{i\omega_1 t}) \to \\ &\to \tfrac{1}{\sqrt{\omega_2}} \left(\sqrt{\tfrac{\omega_2}{\omega_1}} (\alpha a_1 + \beta^* a_1^+) e^{-i\omega_2 t} + \sqrt{\tfrac{\omega_2}{\omega_1}} (\alpha^* a_1^+ + \beta a_1) e^{i\omega_2 t}\right) = \\ &= \tfrac{1}{\sqrt{\omega_2}} (a_2 e^{-i\omega_2 t} + a_2^+ e^{i\omega_2 t}) \end{aligned} \tag{9}$$

where a_2^+ and a_2 are new creation and annihilation operators.

An overdetermination of creation and annihilation operators means that vacuum with zero number of photons n_1 creates the photons in amounts n_2,

$$n_1 = \langle 0 | a_1^+ a_1 | 0 \rangle = 0 \ , \tag{10}$$

$$n_2 = \langle 0 | a_2^+ a_2 | 0 \rangle = \frac{\omega_2}{\omega_1} |\beta|^2 = \frac{(\sqrt{\varepsilon}-1)^2}{4\sqrt{\varepsilon}} \ . \tag{11}$$

Formula (11) gives the number of photons with definite wave vector $\mathbf{k}$. The total number of photons created in the volume V according to Schwinger (Schwinger (1992), (1993), (1994)) is presented as

$$N = \int \frac{(\sqrt{\varepsilon}-1)^2}{4\sqrt{\varepsilon}} \frac{d^3 k}{(2\pi)^3} V \ . \tag{12}$$

Disregarding the dispersion Schwinger obtained the following expression for the energy emitted in unite volume,

$$\frac{E}{V} = \frac{(\sqrt{\varepsilon}-1)^2}{4\sqrt{\varepsilon}} \int^{k_{max}} \hbar\omega \frac{d^3 k}{(2\pi)^3} = 2\pi^2 \frac{(\sqrt{\varepsilon}-1)^2}{4\sqrt{\varepsilon}} \frac{\hbar c}{\lambda_{\min}^4} \tag{13}$$

where k_{max} is cut-off wave-number and $\lambda_{min} = 2 \times 10^{-5}$ cm for temperature 3^0C. As remarked Schwinger, this value is strikingly close to the edge of transparency of the water.

Let us assume now that $\varepsilon(t)$ varies in time as

$$\varepsilon(t) = \frac{2\varepsilon}{\varepsilon + 1 + (\varepsilon - 1)\mathrm{th}(t/\tau)} \ . \tag{14}$$

In this case the transition from $\varepsilon \neq 1$ to $\varepsilon = 1$ takes place in the course of the characteristic time τ. For the number n_2 of created photons using (Eckart (1930), Flügge (1971)) we obtain the exact result,

$$n_2 = \frac{1}{\rho - 1} \tag{15}$$

where

$$\rho = \frac{\mathrm{sh}^2(\pi\omega\tau(1 + \frac{1}{\sqrt{\varepsilon}}))}{\mathrm{sh}^2(\pi\omega\tau(1 - \frac{1}{\sqrt{\varepsilon}}))}, \qquad \omega = ck \ . \tag{16}$$

If $\omega\tau \to 0$ from (15) and (16) we obtain (11). If $\omega\tau \gg 1$,

$$n_2 = \frac{1}{\mathrm{e}^{\hbar\omega/kT} - 1} \tag{17}$$

where

$$kT = \frac{\hbar\sqrt{\varepsilon}}{4\pi\tau} \ . \tag{18}$$

Thus we see that our radiation is characterized by Plank's spectrum (17) with a temperature defined by (18). But this radiation is not black radiation. It is well known in quantum optics as a squeezed vacuum (Fabre(1992)). The state of squeezed vacuum is defined by following relation,

$$(\alpha^* a_2 - \beta^* a_2^+)|\psi\rangle = 0 \ , \tag{19}$$

and is according to (9) ordinary vacuum state if we use creation and annihilation operators a_1^+ and a_1,

$$a_1|\psi\rangle = 0 \ . \tag{20}$$

The bilinear combinations of creation and annihilation operators are infinitesimal operators of O(2,1)-Lorentz group (Schwinger (1960)). This property of operators a_1^+ and a_1 helps to find easily the solution of the equation (19) in the form

$$|\psi\rangle = \mathrm{e}^{\gamma(a_2^+)^2}|0\rangle \tag{21}$$

where γ is some constant (Fabre(1992)). Thus we see that possible photon numbers n for the state of squeezed vacuum are 0, 2, 4, 6, ... and drastically differ from photon numbers for the black radiation state 0, 1, 2, 3, ...

Black hole radiation, is it black? Similar problems arise in the case of black hole radiation. As was shown by S.Hawking in his classical paper (Hawking (1975)) the black hole radiation is characterized by Plank's spectrum with some temperature. But Hawking's calculation, been technically more complicated, fundamentally is the same as our calculation. Existence of horizon, although very important for understanding the quantum theory of the black hole radiation, doesn't alter neither relations between creation and annihilation operators a_1^+, a_1 and a_2^+, a_2 nor formula for number of emitted photons n_2. So the photon state in the theory of black hole radiation is the squeezed vacuum as in the case of sonoluminescence.

We know that, when S.Hawking found that nonrotating black holes should create and emit a radiation, at first he thought that his calculation was not valid. But what finally convinced him that the radiation was real was that the spectrum of radiation was exactly that which would be emitted by a black body (Hawking (1988)). It was so beautiful! And just it disturbs the judgement! In spite of black holes emit the radiation with Plank spectrum, this radiation is squeezed radiation to be distinguished in quantum optics from thermal radiation. But maybe we need to overdeterminate what black radiation is, who knows.

Alas!

Vita brevis, ars longa, tempus praeceps, experimentum periculosum, iudicium difficile! (Senecae (1981))

Life is brief, art is long, time is precipitous, experiment is perilous, judgment is difficult!

Last year D.V.Volkov passed away, a great scientist and a great man, and we all are the poorer for this loss. Dmitrij Vasiljevich was a hero to us and we are grateful to our good fortune to associate with him. The memory of D.V.Volkov makes us to try to be better, than we are, and helps us to resist when we are forced now to drag out a life, worthless and disgraceful,

Lives of great men all remind us
We can make our lives sublime,
And, departing, leave behind us
Footprints on the sands of time...
H.W.Longfellow, Psalm of Life (Longfellow (1910))

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