clear

$$clear \tag{1}$$

R $\qquad\qquad\qquad\qquad$ R_0

$$\varphi_R(q, R, R_0) := \frac{1}{4\cdot\pi}\int_0^{2\pi}\int_0^{\pi}\frac{q\cdot\sin(\theta)}{\sqrt{(R_0)^2-2\cdot R_0\cdot R\cdot\cos(\theta)+(R)^2}}\,\mathrm{d}\theta\,\mathrm{d}\varphi$$

$$(q, R, R_0) \rightarrow \frac{1}{4}\,\frac{\displaystyle\int_0^{2\pi}\int_0^{\pi}\frac{q\sin(\theta)}{\sqrt{R_0^2-2R_0R\cos(\theta)+R^2}}\,\mathrm{d}\theta\mathrm{d}\varphi}{\pi} \tag{2}$$

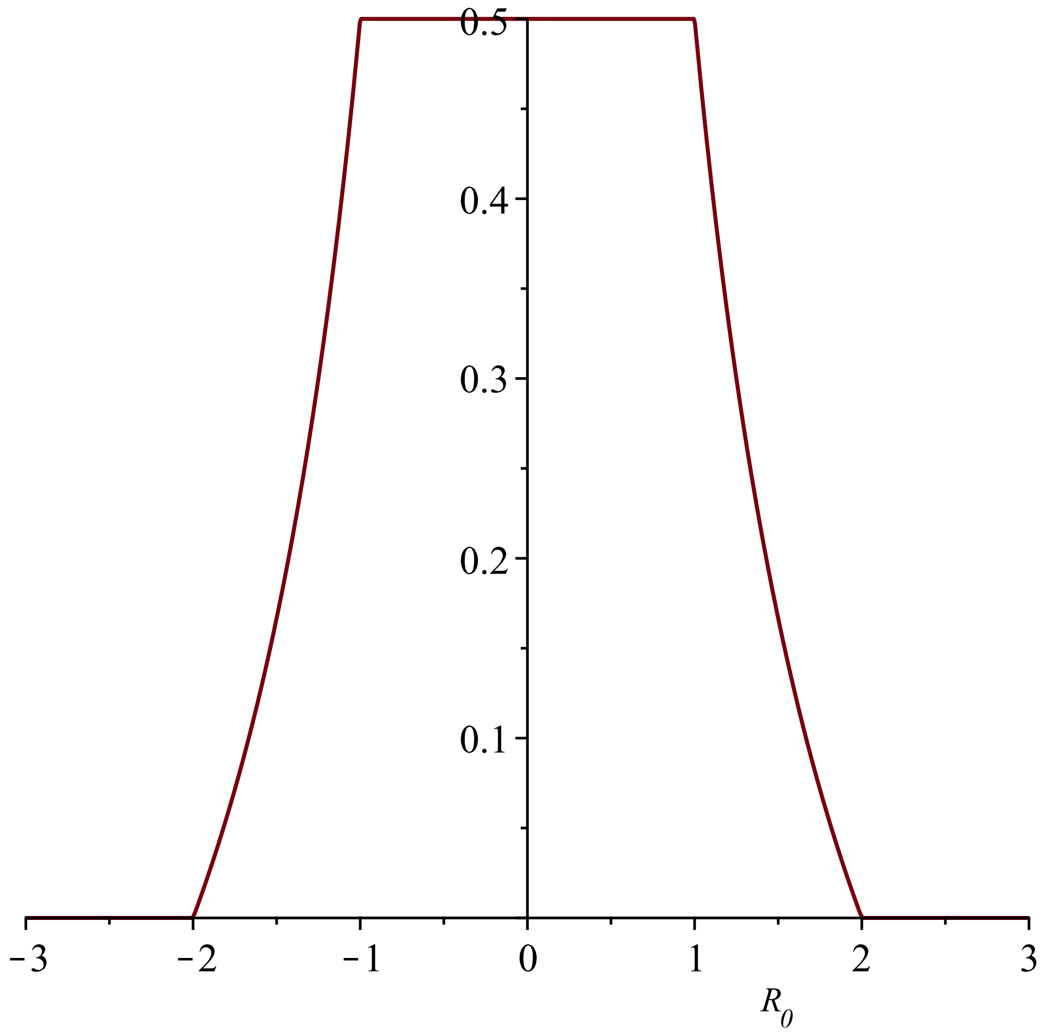
$$\varphi_R(q, 1, R_0) - \varphi_R(q, 2, R_0)$$

$$\frac{1}{4}\,\frac{\displaystyle\int_0^{2\pi}\int_0^{\pi}\frac{q\sin(\theta)}{\sqrt{R_0^2-2R_0\cos(\theta)+1}}\,\mathrm{d}\theta\,\mathrm{d}\varphi}{\pi} - \frac{1}{4}\,\frac{\displaystyle\int_0^{2\pi}\int_0^{\pi}\frac{q\sin(\theta)}{\sqrt{R_0^2-4R_0\cos(\theta)+4}}\,\mathrm{d}\theta\,\mathrm{d}\varphi}{\pi} \tag{3}$$

$$evalf\left(\varphi_R(1, 1, R_0) - \varphi_R(1, 2, R_0)\right)$$

$$0.07957747152\left(\int_{0.}^{6.283185308}\int_{0.}^{3.141592654}\frac{\sin(\theta)}{\sqrt{R_0^2-2.\,R_0\cos(\theta)+1.}}\,\mathrm{d}\theta\,\mathrm{d}\varphi\right) - 0.07957747152\left(\int_{0.}^{6.283185308}\int_{0.}^{3.141592654}\frac{\sin(\theta)}{\sqrt{R_0^2-4.\,R_0\cos(\theta)+4.}}\,\mathrm{d}\theta\,\mathrm{d}\varphi\right) \tag{4}$$

$$with(plots) : plot\left(\varphi_R(1, 1, R_0) - \varphi_R(1, 2, R_0), R_0 = -3\,..3\right)$$



,

$$vr(v, c, R, R_0, \theta) := \frac{v}{c} \cdot (R_0 \cdot \cos(\theta) - R)$$

$$(v, c, R, R_0, \theta) \rightarrow \frac{v\,(R_0 \cos(\theta) - R)}{c} \tag{5}$$

$$cos\_\beta(R, R_0, \theta) := \cos\left(\arcsin\left(\frac{R_0 \cdot \sin(\theta)}{\sqrt{(R_0)^2 - 2 \cdot R_0 \cdot R \cdot \cos(\theta) + (R)^2}}\right)\right)$$

$$(R, R_0, \theta) \rightarrow \cos\left(\arcsin\left(\frac{R_0 \sin(\theta)}{\sqrt{R_0^2 - 2\,R_0\,R\cos(\theta) + R^2}}\right)\right) \tag{6}$$

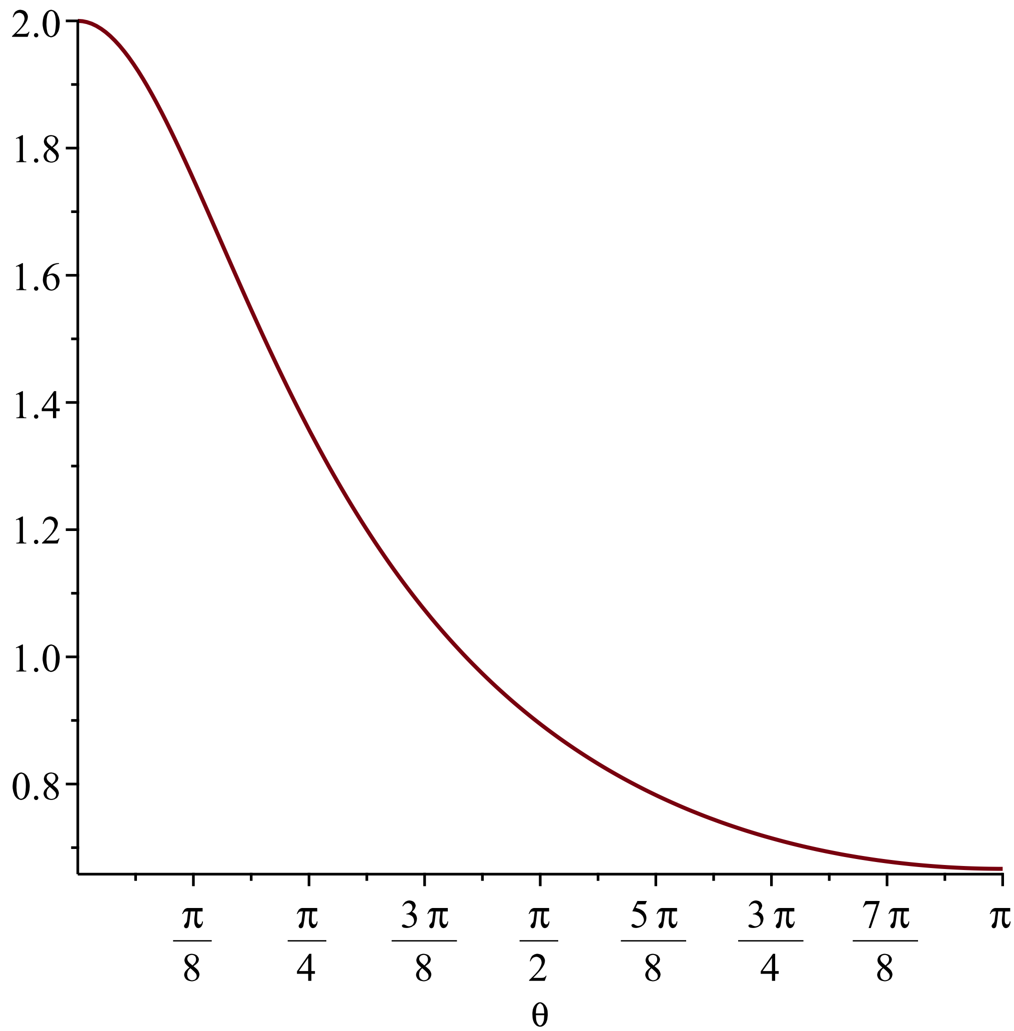
$$\beta(R, R_0, \theta) := \arcsin\left(\frac{R_0 \cdot \sin(\theta)}{\sqrt{(R_0)^2 - 2 \cdot R_0 \cdot R \cdot \cos(\theta) + (R)^2}}\right)$$

$$(R, R_0, \theta) \rightarrow \arcsin\left(\frac{R_0 \sin(\theta)}{\sqrt{R_0^2 - 2 R_0 R \cos(\theta) + R^2}}\right) \tag{7}$$

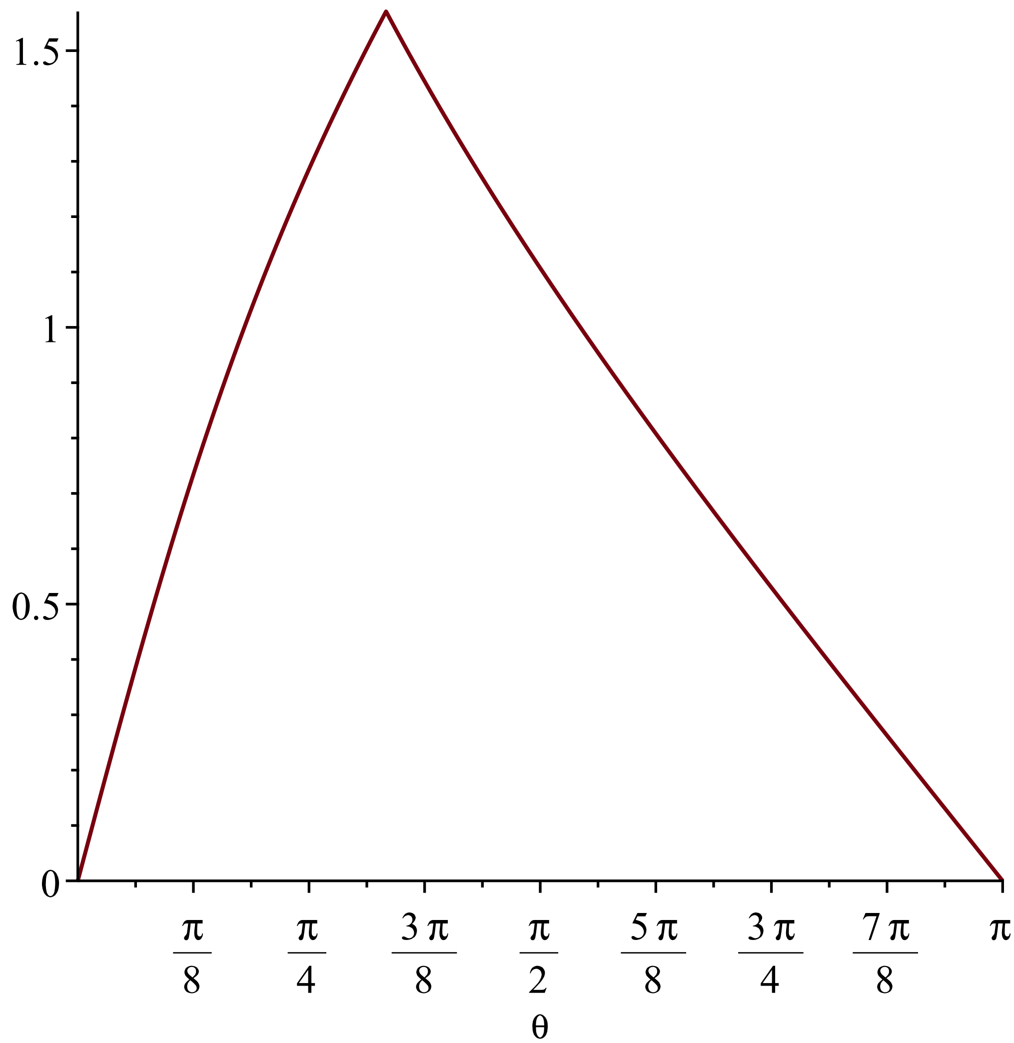
$$R0r(R, R_0, \theta) := \left(\frac{R_0}{\sqrt{(R_0)^2 - 2 \cdot R_0 \cdot R \cdot \cos(\theta) + (R)^2}}\right)$$

$$(R, R_0, \theta) \rightarrow \frac{R_0}{\sqrt{R_0^2 - 2 R_0 R \cos(\theta) + R^2}} \tag{8}$$

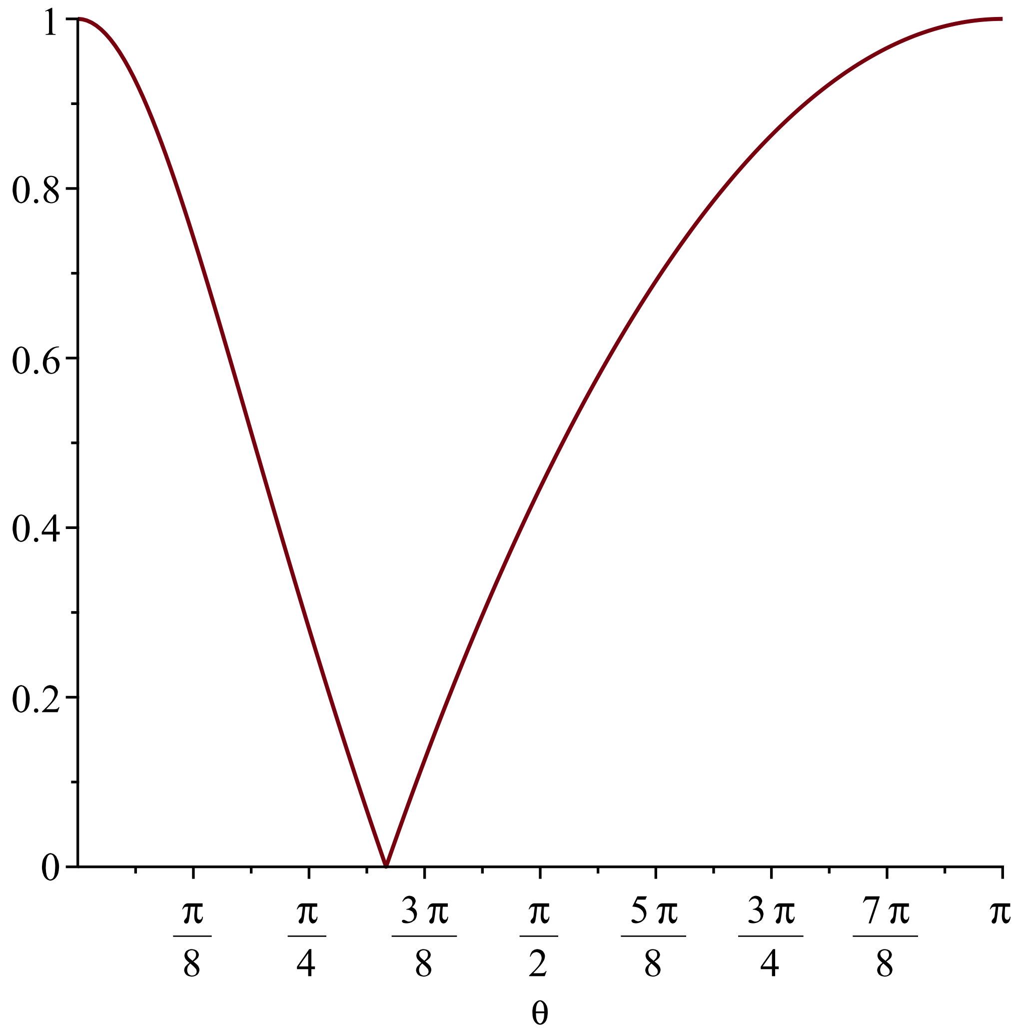
$plot(R0r(1, 2, \theta), \theta = 0 .. \pi)$



$plot(\beta(1, 2, \theta), \theta = 0 .. \pi)$



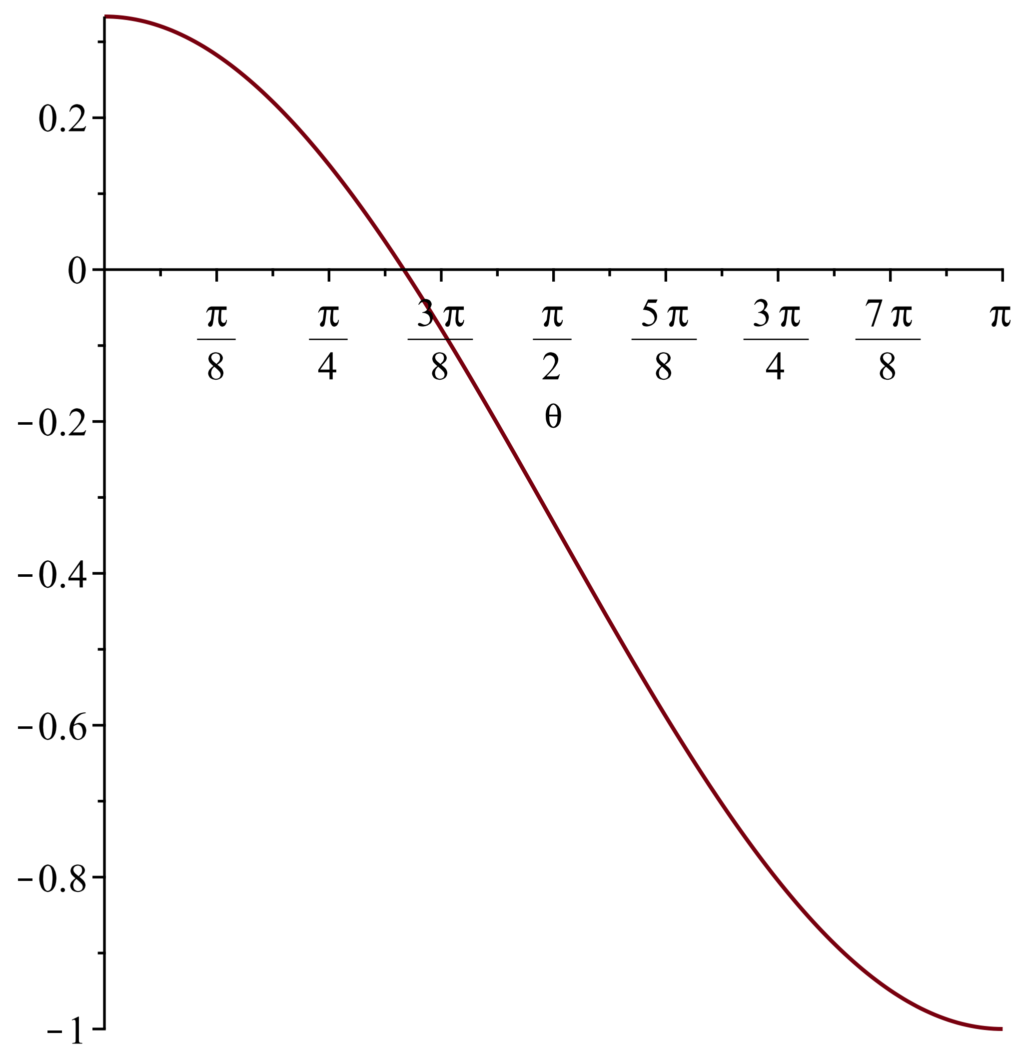
$plot(cos\_\beta(1, 2, \theta), \theta = 0..\pi)$



$\cos(\pi)$

$$-1 \tag{9}$$

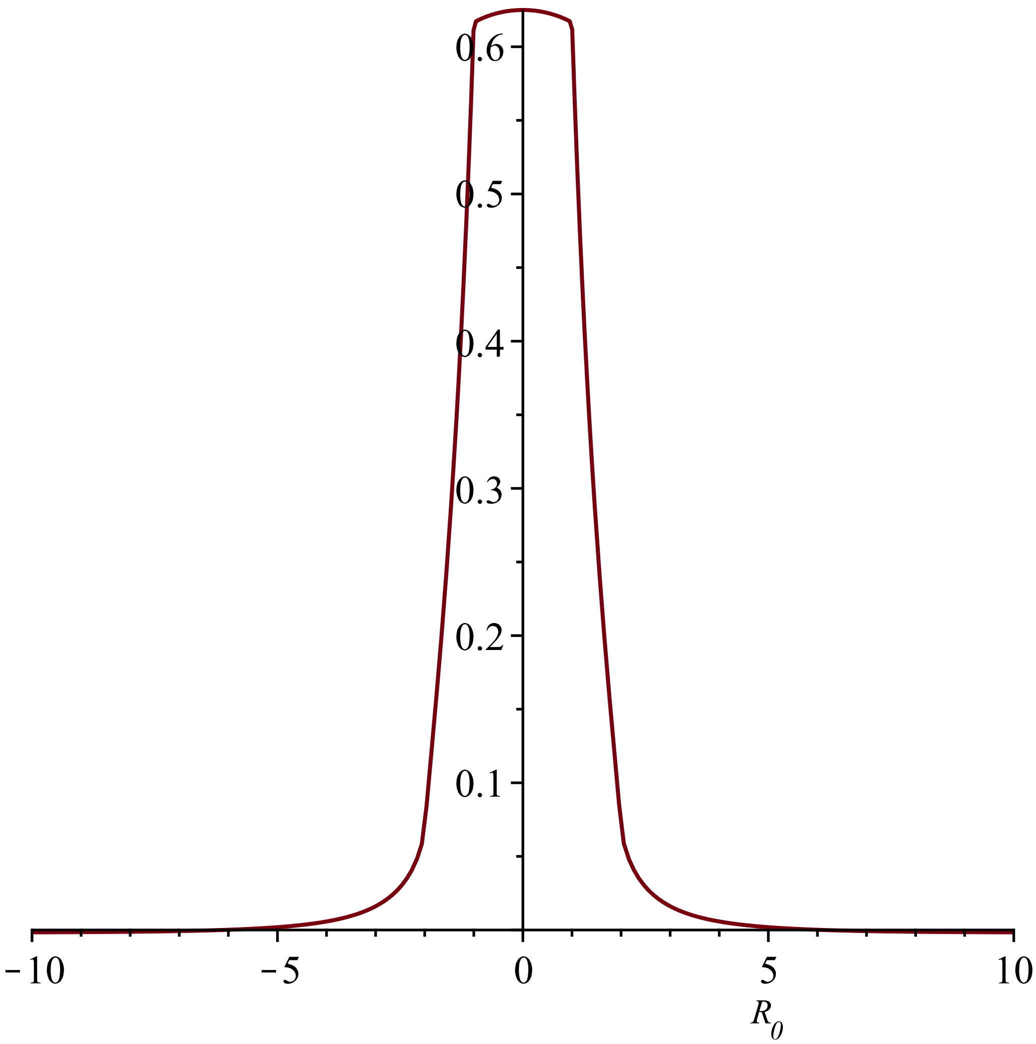
$plot(vr(1, 3, 1, 2, \theta), \theta = 0 .. \pi)$



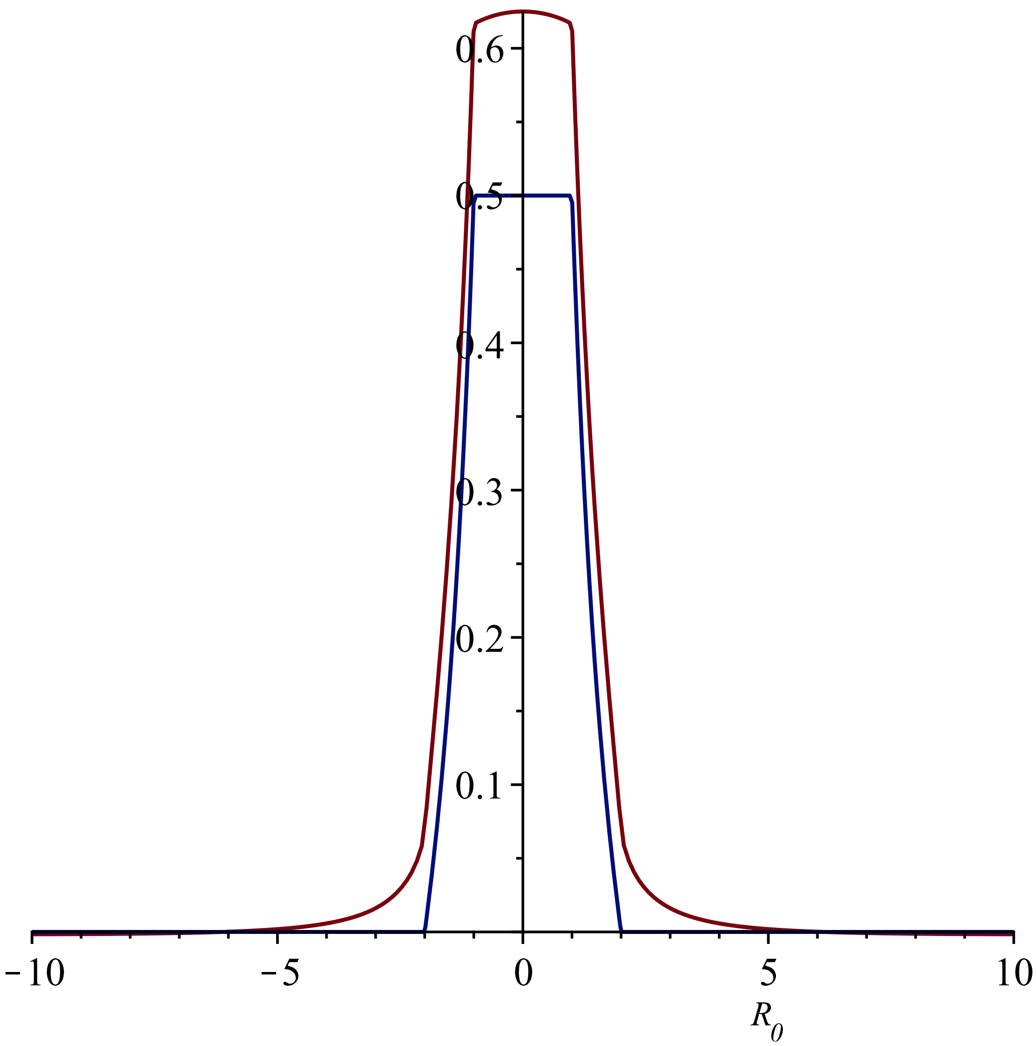
$$\varphi_{lw}(q, v, c, R, R_0) := \frac{1}{4\cdot\pi}\int_0^{2\pi}\int_0^{\pi}\frac{q\cdot\sin(\theta)}{\sqrt{(R_0)^2 - 2\cdot R_0\cdot R\cdot\cos(\theta) + (R)^2} - \frac{v}{c}\cdot\left(R_0\cdot\cos(\theta) - R\right)}\,\mathrm{d}\theta\,\mathrm{d}\varphi$$

$$(q, v, c, R, R_0) \rightarrow \frac{1}{4}\,\frac{\int_0^{2\pi}\int_0^{\pi}\frac{q\sin(\theta)}{\sqrt{R_0^2 - 2R_0R\cos(\theta) + R^2} - \frac{v\left(R_0\cos(\theta) - R\right)}{c}}\,\mathrm{d}\theta\mathrm{d}\varphi}{\pi} \tag{10}$$

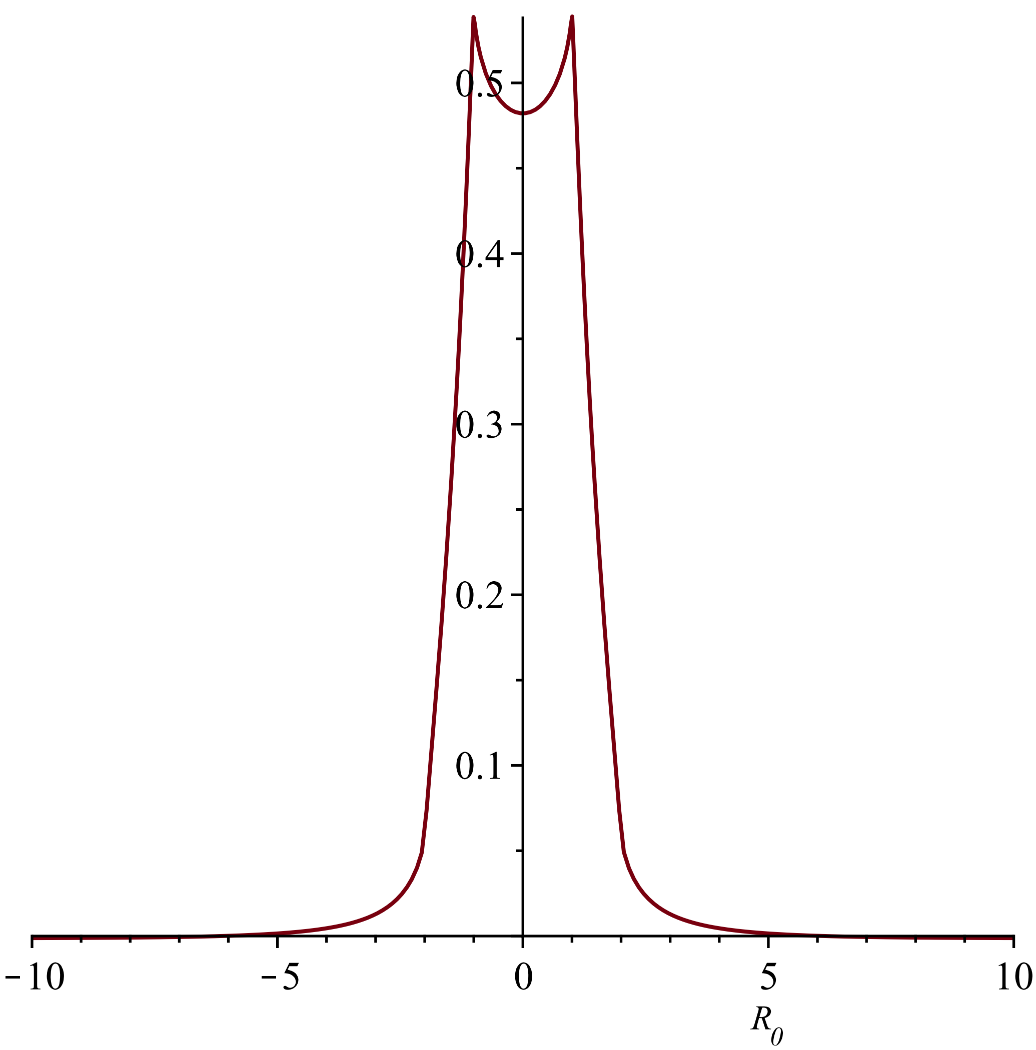
$with(plots) : plot\left(\left[\varphi_R(1, 1, R_0) - \varphi_{lw}(1, 1, 3, 2, R_0)\right], R_0 = -10 ..10\right)$



$with(plots) : plot\left(\left[\varphi_R(1, 1, R_0) - \varphi_{lw}(1, 1, 3, 2, R_0), \varphi_R(1, 1, R_0) - \varphi_R(1, 2, R_0)\right], R_0 = -10 ..10\right)$



$with(plots) : plot\left(\varphi_{lw}(1, 0.5, 3, 1, R_0) - \varphi_{lw}(1, 1, 3, 2, R_0), R_0 = -10 .. 10\right)$



$with(plots) : plot3d\left(\varphi_R(1, R, R_+) - \varphi_{lw}(1, 1, 3, R, 2), R = -10..10, R_+ = 1..2\right)$

$E(q, R_+, R_-, R_0) := -\frac{\partial}{\partial R_0}\left(\varphi_R(q, R_+, R_0) + \varphi_R(-q, R_-, R_0)\right)$

$$(q, R_+, R_-, R_0) \rightarrow -\left(\frac{\partial}{\partial R_0}\left(\varphi_R(q, R_+, R_0) + \varphi_R(-q, R_-, R_0)\right)\right) \tag{11}$$

$E(q, R_+, R_-, R_0)$

$$-\frac{1}{4}\,\frac{\int_0^{2\pi}\int_0^{\pi}\left(-\frac{1}{2}\,\frac{q\sin(\theta)\,(2R_0 - 2R_+\cos(\theta))}{\left(R_0^2 - 2R_0R_+\cos(\theta) + R_+^2\right)^{3/2}}\right)\mathrm{d}\theta\,\mathrm{d}\varphi}{\pi} \tag{12}$$

$$-\frac{1}{4}\,\frac{\displaystyle\int_{0}^{2\pi}\int_{0}^{\pi}\frac{1}{2}\,\frac{q\sin(\theta)\,\left(2R_0-2R_{-}\cos(\theta)\right)}{\left(R_0^{\,2}-2R_0R_{-}\cos(\theta)+R_{-}^{2}\right)^{3/2}}\,\mathrm{d}\theta\,\mathrm{d}\varphi}{\pi}$$

$$E_{lw}(q,v_{+},v_{-},c,R_{+},R_{-},R_0):=-\frac{\partial}{\partial R_0}\left(\varphi_{lw}(q,v_{+},c,R_{+},R_0)+\varphi_{lw}(-q,v_{-},c,R_{-},R_0)\right)$$

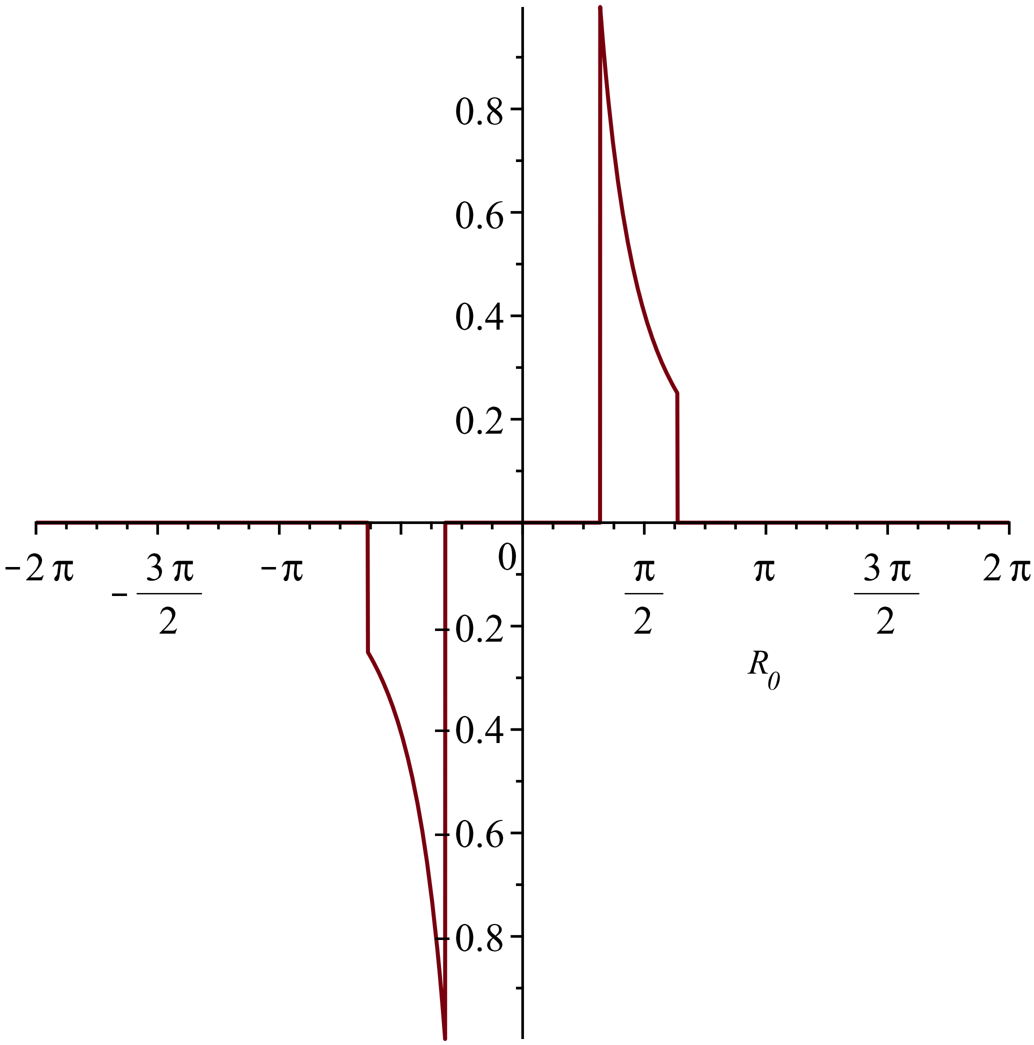
$$(q,v_{+},v_{-},c,R_{+},R_{-},R_0)\rightarrow-\left(\frac{\partial}{\partial R_0}\left(\varphi_{lw}(q,v_{+},c,R_{+},R_0)+\varphi_{lw}(-q,v_{-},c,R_{-},R_0)\right)\right) \tag{13}$$

$$E_{lw}(q,v_{+},v_{-},c,R_{+},R_{-},R_0)$$

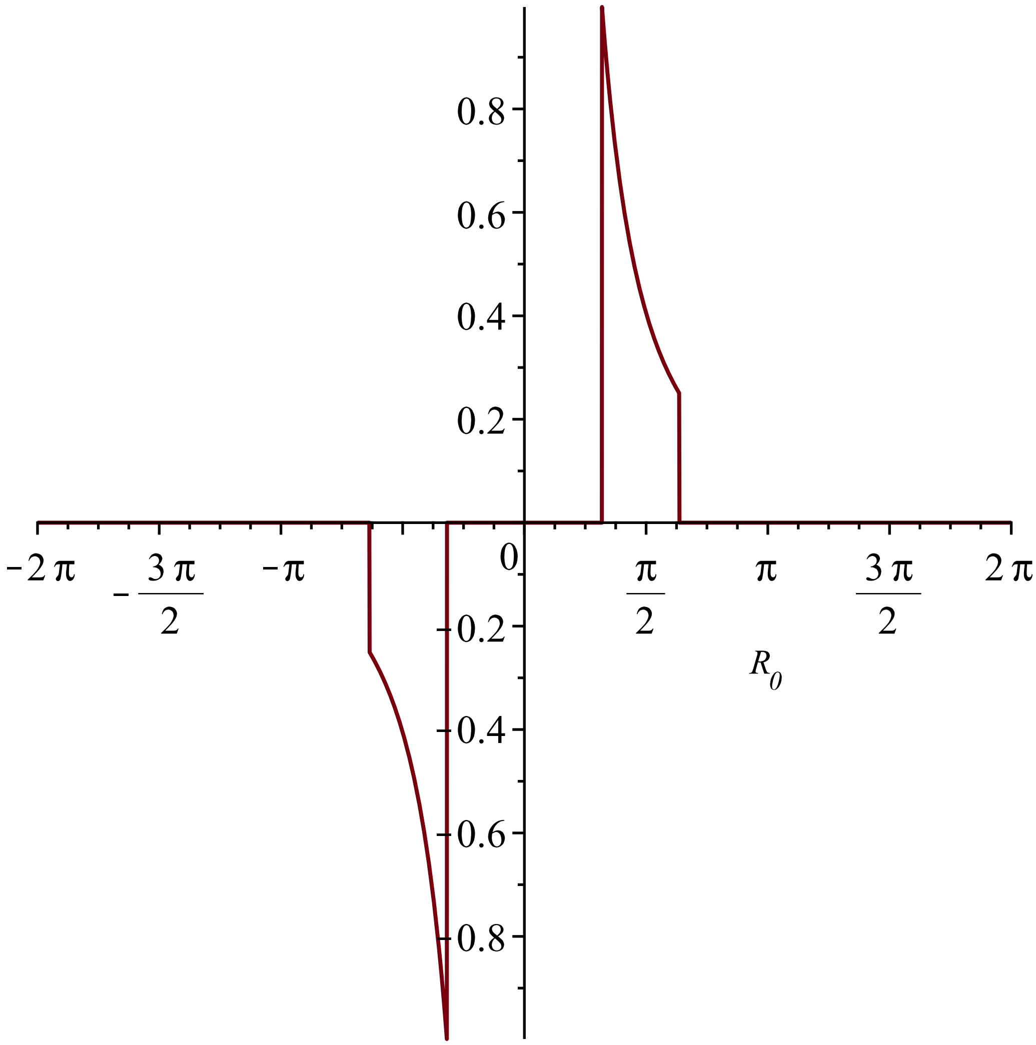
$$-\frac{1}{4}\,\frac{\displaystyle\int_{0}^{2\pi}\int_{0}^{\pi}\left(-\frac{q\sin(\theta)\left(\frac{1}{2}\,\frac{2R_0-2R_{+}\cos(\theta)}{\sqrt{R_0^{\,2}-2R_0R_{+}\cos(\theta)+R_{+}^{\,2}}}-\frac{v_{+}\cos(\theta)}{c}\right)}{\left(\sqrt{R_0^{\,2}-2R_0R_{+}\cos(\theta)+R_{+}^{\,2}}-\frac{v_{+}\left(R_0\cos(\theta)-R_{+}\right)}{c}\right)^{2}}\right)\mathrm{d}\theta\,\mathrm{d}\varphi}{\pi} \tag{14}$$

$$-\frac{1}{4}\,\frac{\displaystyle\int_{0}^{2\pi}\int_{0}^{\pi}\frac{q\sin(\theta)\left(\frac{1}{2}\,\frac{2R_0-2R_{-}\cos(\theta)}{\sqrt{R_0^{\,2}-2R_0R_{-}\cos(\theta)+R_{-}^{2}}}-\frac{v_{-}\cos(\theta)}{c}\right)}{\left(\sqrt{R_0^{\,2}-2R_0R_{-}\cos(\theta)+R_{-}^{2}}-\frac{v_{-}\left(R_0\cos(\theta)-R_{-}\right)}{c}\right)^{2}}\,\mathrm{d}\theta\,\mathrm{d}\varphi}{\pi}$$

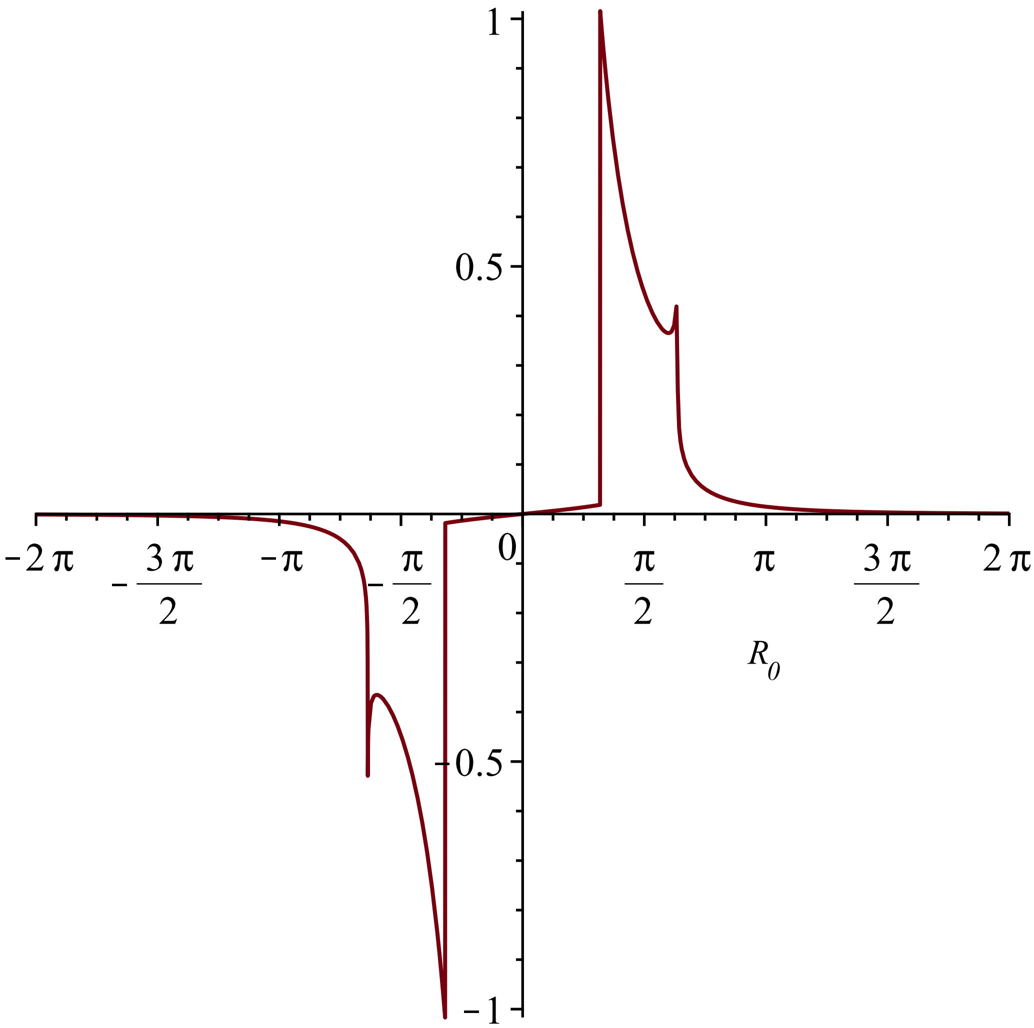
$with(plots):plot(E(1,1,2,R_0),R_0)$



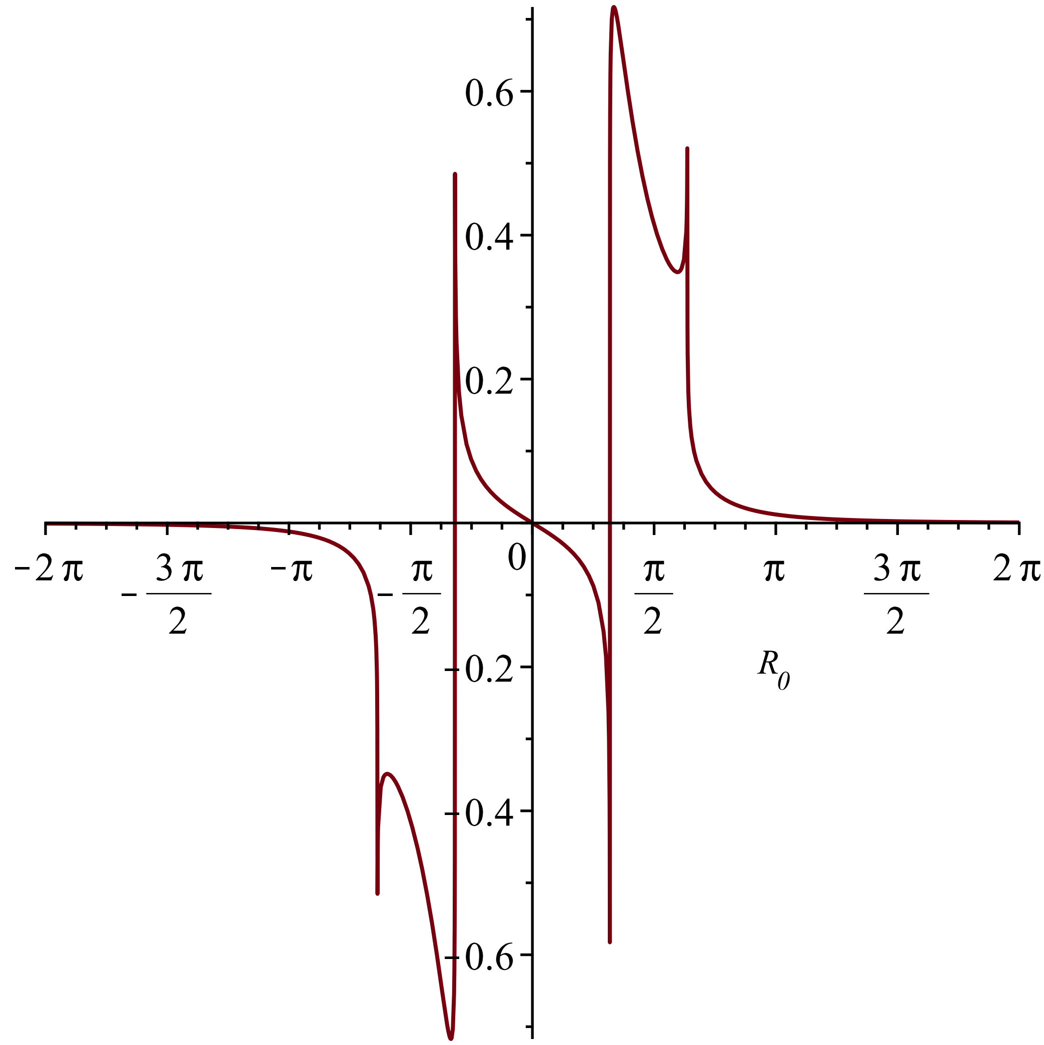
with(*plots*) : *plot*($E_{lw}(1, 0, 0, 3, 1, 2, R_0), R_0$)



$with(plots) : plot\left(E_{lw}\left(1, 0, 1, 3, 1, 2, R_0\right), R_0\right)$



$with(plots) : plot\left(E_{lw}\left(1, 0.5, 1, 3, 1, 2, R_0\right), R_0\right)$


0.6
0.4
0.2
−0.2
−0.4
−0.6
−2 π
−3π/2
−π
−π/2
0
π/2
π
3π/2
2 π
R_0