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| DOI: |  | ISSN (Print) |

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| --- |
| Original article |
| In print article |
| [https://doi.org/](https://doi.org/\mydoi) |
| UDC 53.09, 621.3, 629.7 |
| PACS numbers: 11.10.Ef, 03.70.+k, 42.50.Lc |

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*E-mail:*

In this paper, the two-dimensional one-body Casimir effect is analyzed on the example of square-shaped nanocells. In the classical one-dimensional two-body Casimir effect a Casimir force appears between two plates as a difference of electromagnetic pressures of zero-point quantum-vacuum oscillation on different sides of each of the plates. The plates are pushed forwards each other by external quantum-vacuum oscillation fields, which in classical configuration exceed internal quantum-vacuum oscillation fields. It is possible to try to create a difference of electromagnetic pressures of quantum-vacuum oscillation on different sides of a single plate due to the difference of the geometry of vacuum resonators on different sides of the plate. For this purpose, it is necessary to grow nanocells on one of surfaces of a smooth metallic plate. As a result, it has been found that the formula for the force per unit area is very similar to the formula of the classical Casimir effect, except for the value of the proportionality coefficient.

The force applied to perfectly conducting honeycombs on a plate as a result of the difference in specific energy density on its different sides can be interpreted as the pressure of the zero-point electromagnetic oscillations. According to the formula presented in this work, for the gold nano-honeycomb with a size of about 2 microns the force should be equal to 8.55 dynes per square meter of the panel, which is quite an acceptable value for the practical use of the expected effect for satellite orbits correction.

Although the effect is small, an experimental confirmation could serve as a critical proof for the existence of Casimir’s virtual quantum photons.

Two dimensional one body Casimir effect, nanocells, nanohoneycomb, Casimir thrust

**Як цитувати:**: . . . [https://doi.org/](https://doi.org/\mydoi).

**In cites**: . . . [https://doi.org/](https://doi.org/\mydoi).

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The introduction of half-quanta in the context of black-body radiation by Planck in 1911 was fundamental for discovering the Casimir effect presented by Casimir in his seminal paper . This is one of the most direct manifestations of a quantum and relativistic phenomenon caused by the zero-point oscillation of quantized fields.

The Casimir effect in its simplest form is the force between a pair of neutral, parallel conducting plates resulting from the modification of the electromagnetic vacuum properties caused by the change in boundary conditions.

The calculation of the Casimir force is a particularly complicated theoretical problem. Remarkably, for closed configurations, i.e., when the Casimir effect manifests for one body instead of two, the Casimir force can be not only attractive but repulsive as well. As it has been shown by Boyer , the latter is true for an ideal metal spherical shell.

Antipin A. V. expects the appearance of the driving force/thrust (due to the Casimir effect on one body), as a result of the difference in the impact of virtual particles (photons) on external and internal reflecting surfaces of pyramidal, conical or objects.

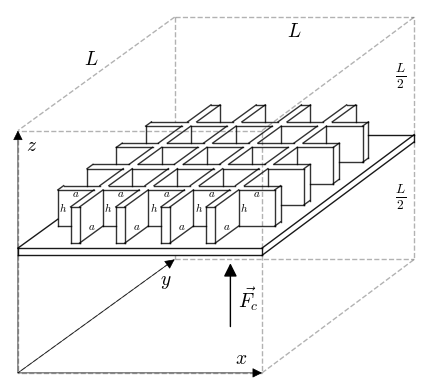
This work is dedicated to one-body Casimir effect for open configuration of perfectly conducting nanohoneycomb.

## 

Let us consider a cubic cavity of volume bounded by perfectly conducting walls where perfectly conducting square plate with side is placed in this cavity parallel to the face, and let the distance between the plate and the face be sufficiently large, , for example. One side of this perfectly conducting square plate is a smooth plane and another is covered with perfectly conducting square-shaped honeycombs with a square side .

On both sides of the plate the expressions $\sum\hbar\omega\big/2$ where the summation extends over all possible resonance frequencies of the cavity (a large cavity: between smooth plane and face) and the cavity (a small cavity, one honeycomb cell: between the bottom of the honeycomb and the opposite face) are divergent and devoid of physical meaning but the difference between these sums on the opposite sides of the plate, $\left(\sum\,\,\hbar\omega\right)\_{I}\big/{\left(2\,V\_{I}\right)} - \left(\sum\,\,\hbar\omega\right)\_{II}\big/{\left(2\,V\_{II}\right)}$, will be shown to have a well-defined value and this value will be interpreted as the interaction between the plate and the both remote faces.

The possible oscillations inside cavities defined by , , (a large cavity between smooth plane and face) and , , (a small cavity, one honeycomb cell) have wave vectors , , (a large cavity between smooth plane and face), and , , (a small cavity, one honeycomb cell), where , , are positive integers; .



[fig:honeycomb\_box\_L]

Let us write the expression for the sum of zero-point energy in general form

Two standing waves correspond to every , , but in case when one of the is zero, there is only one wave.

That is of no importance in case of one honeycomb cell cavity for , since for very large we may regard as continuous variable, replacing summation over with integration. Thus, for a small cavity consisting of one honeycomb, we find

.

Considering we can find the specific energy density , where :

, , where the notation is meant to indicate that the term with and has to be multiplied by $1\big/2$. Thus, for a small cavity consisting of one honeycomb, we have

. That is of no importance in case of a large cavity for , since for very large we may regard , as continuous variables. Thus, for large cavity between smooth plane and face we find

For very large the last summation may also be replaced by an integral and, therefore, it can be seen that energy of a large cavity is given by

, , , ,

Now for the specific energy density for a large cavity, where we can write the following sequence of transformations:

, , . And finally for a large cavity between smooth plane and face we can formulate the energy density as following

,

but for a small cavity, one honeycomb cell we have .

Therefore, it is obvious that the interaction energy is determined by the following energy density difference:

2

This expression is clearly infinite, and to proceed with the calculation, it is convenient to introduce a regulator.

In order to receive a finite result, it is necessary to multiply the integrands by a regularization function which is unity for but tends to zero sufficiently rapidly for . Where may be defined by . The physical meaning is obvious: our plate is hardly an obstacle for very short waves (X-rays e.g.) and, therefore, the zero-point energy of these waves will not be influenced by the position of this plate.

The purpose of regulator is to make the expression finite, and influence of its specific type will be removed by a limit transition in the end.

Introducing the variable , , we have:

2 If and we have so , where the cutting frequency is . Introducing function

we can write

And at least, introducing

we have

Integration of with example of regulator function is presented in Appendix A.

To receive a way of calculating , the Euler-Maclaurin 2D formula could be considered.

## 

According to A.Bikyalis we apply Euler-Maclaurin formula twice on and on . Starting from the following form of this formula

since we are dealing with a very complex mathematical problem of integrating the function, which often oscillates and suffers discontinuities at the points of each integer value of the argument due to the presence of the multiplier , hereafter, we will use the fact that the remainder can also be expressed in the form

We can see that it consists of 4 parts:

the integral ,

the half sum ,

the sum of Bernoulli polynomials

,

and the remainder

.

When applying it to twice on and on we should have the following summands which can be represented as the table:

Taking into account that the function is symmetric on its and arguments, so the two-dimentional Euler-Maclaurin marix presented above is symmetric too.

## 

Taking value of parameter we have:

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Let us consider the expression

Firstly, we can see, that

Therefore, we have

On the other hand, we have found that

$\sum\limits\_{n\_x=0}^{\infty} \sum\limits\_{n\_y=0}^{\infty} G\left(n\_x, n\_y\right) -\int\limits\_{0}^{\infty} \int\limits\_{0}^{\infty} G\left(n\_x, n\_y\right)\,d{n\_x}\,d{n\_y} = \\
\begin{array}{cccc} \, & \int\limits\_{n\_y}^{} \underset{n\_x}{H\_{\sum}}\,G & + \int\limits\_{n\_y}^{}{\sum\limits\_{n\_x}^{}}^{B}\,G & + \int\limits\_{n\_y}^{}\,\underset{n\_x}{R\_{p}}\,G \\
+\underset{n\_y}{H\_{\sum}}\,\int\limits\_{n\_x}^{}\,G & + \underset{n\_y}{H\_{\sum}}\,\underset{n\_x}{H\_{\sum}}\,G & + \underset{n\_y}{H\_{\sum}}\,{\sum\limits\_{n\_x}^{}}^{B}\,G & + \underset{n\_y}{H\_{\sum}}\,\underset{n\_x}{R\_{p}}\,G \\
+ {\sum\limits\_{n\_y}^{}}^{B}\,\int\limits\_{n\_x}^{}\,G & + {\sum\limits\_{n\_y}^{}}^{B}\,\underset{n\_x}{H\_{\sum}}\,G & + {\sum\limits\_{n\_y}^{}}^{B}\,{\sum\limits\_{n\_x}^{}}^{B}\,G & + {\sum\limits\_{n\_y}^{}}^{B}\,\underset{n\_x}{R\_{p}}\,G \\
+ \underset{n\_y}{R\_{p}}\,\int\limits\_{n\_x}^{}\,G & + \underset{n\_y}{R\_{p}}\,\underset{n\_x}{H\_{\sum}}\,G & + \underset{n\_y}{R\_{p}}\,{\sum\limits\_{n\_x}^{}}^{B}\,G & + \underset{n\_y}{R\_{p}}\,\underset{n\_x}{R\_{p}}\,G, \end{array}$

where , and are:

Now using 1D Euler-Maclaurin formula in the form ([eq:7]) we can see that

will be equal to

Now we can find expression ([eq:12]) by using the following summation

It is easy to see that the sum of all summands without remainder

is 0. And, therefore, any possible non zero result of expression ([eq:12]) should be attributed to the remainder

Or using symmetric properties of , we can rewrite the resulting formula

with the following summands:

;

;

;

.

Considering that should be because derivative with respect to , for is , and gives in summation, we can simplify the result in the form

or in detailed form

2

## 

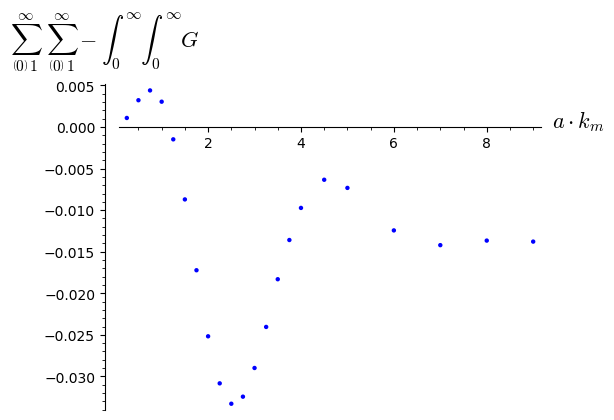
Casimir in his original work has provided his formula in assumption that . But now we can investigate how the expression ([eq:12]) depends on .

Thus, for the energy density difference per we find

|  |
| --- |
| Table 1. The result of evaluating the expression ([eq:12]). |

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| 0.25 | 0.00108989 | 4.99669e-07 | 2 |
| 0.5 | 0.00322377 | 7.99836e-07 | 6 |
| 0.75 | 0.00440392 | 8.72173e-07 | 11 |
| 1.0 | 0.00304837 | 8.49556e-07 | 17 |
| 1.25 | -0.00148115 | 1.77116e-06 | 18 |
| 1.5 | -0.00870823 | 3.67271e-06 | 18 |
| 1.75 | -0.0172279 | 6.80405e-06 | 18 |
| 2.0 | -0.0251746 | 9.98944e-06 | 19 |
| 2.25 | -0.0308341 | 9.37033e-06 | 23 |
| 2.5 | -0.0332867 | 1.42819e-05 | 23 |
| 2.75 | -0.0324363 | 2.09102e-05 | 23 |
| 3.0 | -0.0289864 | 2.96151e-05 | 23 |
| 3.25 | -0.02404795 | 4.07897e-05 | 23 |
| 3.5 | -0.0183092 | 8.56118e-06 | 45 |
| 3.75 | -0.0135918 | 2.185368e-05 | 36 |
| 4 | -0.00973151 | 1.46105e-05 | 45 |
| 4.5 | -0.00634270 | 1.93298e-05 | 48 |
| 5 | -0.00732334 | 2.32423e-05 | 52 |
| 6 | -0.0124360 | 2.37328e-05 | 66 |

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| 7 | -0.0142094 | 3.85332e-05 | 69 |
| 8 | -0.0136610 | 3.29375e-05 | 87 |
| 9 | -0.0137827 | 3.58598e-05 | 99 |



[fig:G\_on\_a\_km]

According to our calculation we can see that

where is a function depended on material properties with well defined limit at . So

For the energy density difference per (in the limit at ) we find that

where is a square side of honeycombs measured in microns.

Can this difference of specific energy density be interpreted as the cause of the force applied to perfectly conducting honeycomb on a plate? For example, my investigations of the configuration used by Casimir have shown that for the geometric configuration of two perfectly conducting plates . But what can be said about honeycomb configuration? Research of this question presented in appendixes B and C shows that .

## 

Therefore, the following conclusions can be drawn: there is a force applied to perfectly conducting honeycombs on a plate as a result of the difference in specific energy density on its different sides. This force depends on the material of the plate. This force depends on the cutoff frequency of the honeycomb plate material at least. This force can be interpreted as the pressure of the zero-point electromagnetic oscillations.

Although the effect is small, an experimental confirmation seems not unfeasible and might be of a certain interest.

Tuo Qu, Fang Liu, Yuechai Lin, Yidong Huang have reported the production of gold nano-honeycomb with a size of about 2 microns. According to the formula presented in this work that honeycomb should have Casimir energy density difference about , that is 8.55 dynes per square meter of the panel, which is quite an acceptable value for the practical use of the expected effect for satellite orbits correction.

It is important to point out, that according to the proposed method, the is calculated not for the total surface area of the honeycombs, but for a part of the panel occupied by cavities (minus the area of the honeycomb walls).

That does not mean, the honeycomb walls should be made as thin as possible, because with a decrease in the wall thickness, the value of will also decrease.

For simplicity of calculations, square-shaped nanocells have been analyzed, but the obtained result can be used, with some correction unknown so far, to estimate the Casimir effect in honeycombs of a different shape (hexagonal, cylindrical, etc.). Such honeycombs are simpler to be manufactured, but require much more complex calculations to estimate the Casimir effect exactly. In addition, the bottom of the honeycomb is not necessarily flat, but spherically concave, for example, which does not fundamentally affect the magnitude of the thrust.

In addition, the appearance of Casimir thrust is expected not only in metal honeycombs, but in dielectric honeycombs as well, because the Casimir effect in dielectrics is well known.

It has been assumed that the height of the the walls of the honeycombs is , while the real height of the walls will be much smaller (say about ).

At the same time, by using the Antipin approach (see Appendix D) it can be shown that the effect should be observed at a smaller height of the ribs, although the dependence of the effect on the height may be the subject of further research.

To answer Hrvoje who considers the Casimir effect as not a consequence of the existence of virtual quantum photons, but as manifestation of the London-Van der Waals dispersion forces, I would like to note that setting up an experiment to measure the thrust produced by nanocells grown on metal plate could serve as a critical experiment to find out which points of view on the nature of the Casimir force corresponds to reality.

## 

2 Let us use the regularization function in the form

Starting from ([eq:4]) and by introducing a variable and , we have

And using this variable, we can make the following substitution

, , .

And now we can rewrite integral ([eq:5]) in form

changing the integration variable from to

because in this form integral can be taken analytically. So, we have the following integrand

and the following limits of integration by : , .

Let us use the Abel substitution:

and the following limits of integration by : , .

Let us denote dependency of from

and derivatives

Now we can rewrite the integrand, making it depending on

Let us extract coefficient near from the denominator.

Now let us move the above coefficient up to the numerator. So, the new numerator will be

Accordingly, the new denominator will be

Now we should convert this denominator to the following form

So, we have the following equation

and its solution

After the conversion determined above, the integrand can be presented as

Let us check determinant by using the expression of and found above.

The determinant is negative and the integral can be easily calculated:

## 

2

Let us consider a rectangular resonator with size .

**For the electric mode**

we have the following solution

and

with

by using , we have

Field energy density $\left(\int \frac{E\_x^2+E\_y^2+E\_z^2}{8 \pi}dV\right)\big/{V}$ is

Full energy density $\left(\int \frac{|\vec{E}|^2}{8 \pi}dV + \int \frac{|\vec{H}|^2}{8 \pi}dV\right)\big/{V}$ is

Electromagnetic pressure $\left({\int \frac {H\_x^2+H\_y^2}{8 \pi} dS}\right)\big/{S}$ on plate is

Their relation is

2

Considering solution with wave propagation in -direction we have which gives

and

In this case the relation of electromagnetic pressure per field energy density is equal to ,

Considering solution with wave propagation in -direction we have which gives

In this case the relation of electromagnetic pressure per field energy density is

Considering solution with wave propagation in -direction, , as in -direction

**For the magnetic mode**

, we have the following solution

and

with

by using , we have

Magnetic field energy density $\left(\int \frac{H\_x^2+H\_y^2+H\_z^2}{8 \pi}dV\right)\big/{V}$ is

Full energy density $\left(\int \frac{|\vec{E}|^2}{8 \pi}dV + \int \frac{|\vec{H}|^2}{8 \pi}dV\right)\big/{V}$ is

Electromagnetic pressure $\left({\int \frac {H\_x^2+H\_y^2}{8 \pi} dS}\right)\big/{S}$ on plate is

Their relation is

Considering solution with wave propagation in -direction, we have which gives

and

In this case the relation of electromagnetic pressure per field energy density is

Considering solution with wave propagation in -direction, we have which gives

In this case the relation of electromagnetic pressure per field energy density is

Considering solution with wave propagation in -direction as in -direction

So, we can see that task of electromagnetic force calculation in the nanohoneycomb configuration is quit easy, because

We can see that if we decrease then also decreases and that leads to

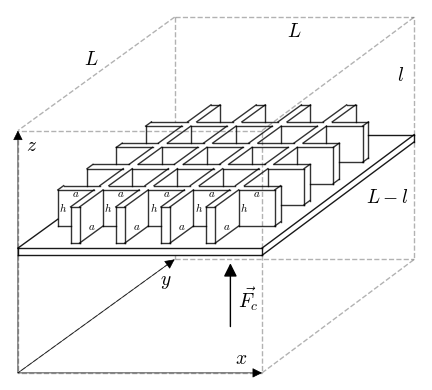
On the other hand, the same result can be shown by using Hamiltonian mechanics approach.

## 

Let us consider a cubic cavity of volume bounded by perfectly conducting walls where perfectly conducting square plate with side is placed in this cavity parallel to the face, and let the distance between the plate and face be sufficiently large, say , for example.

One side of this perfectly conducting square plate is a smooth plane and another is covered by perfectly conducting square-shaped honeycomb a square side .

On both sided of the the plate the expressions $1\big/2\sum\,\hbar\omega$ where the summation extends over all possible resonance frequencies of the cavity (a large cavity between smooth plane and face) and the cavity (a small cavity, one honeycomb cell) are divergent and devoid of physical meaning, but it will be shown that for the both opposite sides the derivative ${d\left<0|\hat{\mathcal{H}}|0\right>}\big/{dl}$ of the vacuums Hamiltonian of the whole system for these sums $\left<0|\hat{\mathcal{H}}|0\right> = 1\big/2\,\left(\sum\,\,\hbar\omega\right)\_{I} + 1\big/2\,\left(\sum\,\,\hbar\omega\right)\_{II}$, has a well-defined value and this value will be interpreted as the interaction between the plate and the both faces.



[fig:honeycomb\_box\_H]

The possible oscillations of the cavities defined by , , (a large cavity between smooth plane and face) and , , (a small cavity, one honeycomb cell) have the wave vectors , , (a large cavity between smooth plane and face), and , , (a small cavity, one honeycomb cell), where . , are positive integers; .

Let us write the expression for the sum of zero-point energy in general form

Two standing waves correspond to every , , , but in case when one of the is zero, there is only one wave. That is of no importance in case of one honeycomb cell cavity for , since for very large we may regard as continuous variable, replacing summation over with integration. Thus, for a small cavity consisting of one honeycomb, we find

.

Considering we can find the specific energy density , where :

,

.

That is of no importance in case of a large cavity for , since for very large we may regard , as continuous variables. Thus, for large cavity between smooth plane and face we find

For very large the last summation may be replaced by an integral and, therefore, it can be seen that energy of a large cavity is given by

where , , .

Now for the specific (per area) energy density for a large cavity, where we can write the following sequence of transformations:

, , . Therefore, it can be seen that specific (per area) vacuum Hamiltonian $\frac{\left<0|\hat{\mathcal{H}}|0\right>}{S}$ of the whole system is given by

$$\frac{\left<0|\hat{\mathcal{H}}|0\right>}{S} = \frac{\hbar\,c}{a^2\,\pi}
\cdot \Bigg\{l\sum\limits\_{n\_x=(0)1}^{\infty}\sum\limits\_{n\_y=(0)1}^{\infty}\left[\,\int\limits\_{0}^{\infty}\sqrt{n\_x^2\frac{\pi^2}{a^2}+n\_y^2\frac{\pi^2}{a^2}+k\_z^2}\,dk\_z\right] +
(L-l)\int\limits\_{0}^{\infty}\int\limits\_{0}^{\infty}\left[\,\int\limits\_{0}^{\infty}\sqrt{k\_x^2+k\_y^2+k\_z^2}\,dk\_z\right]\,\left(\frac{a}{\pi}dk\_x\right)\,\left(\frac{a}{\pi}dk\_y\right)\Bigg\}$$

and interaction $\frac{F}{S} = -\frac{\partial }{\partial z} \,\frac{\left<0|\hat{\mathcal{H}}|0\right>}{S} = \frac{\partial}{\partial l} \,\frac{\left<0|\hat{\mathcal{H}}|0\right>}{S}$ is

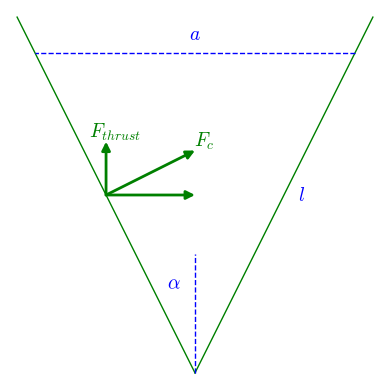
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Then, the formula for the force acting on a perfectly conducting honeycomb on a plate obtained by using Hamiltonian mechanics approach is the same as formula ([eq:2]) received for the difference of specific energy density on its different sides .

2

## 

Antipin gives an estimated calculation of the thrust of the angle by using the Casimir formula, “with the most general and natural approximations known as PFA (Proximity Force Approximation) or PAA (Pairwise Additive Approximation) calculation method , ".



[fig:Antipins\_angle]

Casimir’s interaction energy is given by

Casimir’s force is

Thrust of the metal angle is

Let us make a substitution

The following substitution can be made:

Thus, the formula for the thrust for the angle is derived basing on the length of its wings.

Antipin indicates that the value of is limited from below by the cutoff level, which is determined technologically:

* by the accuracy of plate manufacturing (their roughness, degree of flatness), as well as
* by the the minimum wavelength of photons that can effectively reflect the substance from which the angle is made (by value).

Investigating the dependence of the coefficient in the angle thrust formula, which depends on the half angle , it can be seen that for a given length of the sides, it is more efficient to make the angle as small as possible. However, for the purposes of this work (studying the possibility of obtaining thrust by using nanocells), it is important to note that for a angle with a right angle , the coefficient $\left({cos\, \alpha}\right)\big/\left({\left(sin\, \alpha\right)^4}\right) = 2\sqrt{2}$. Thus, by composing a honeycomb structure from many rectangular angles, it can be shown that the thrust of the panel consisting of rectangular honeycombs is not zero.

Indeed, a rectangular honeycomb with a cell size of and with the same edge height equal to can be imagined as a combination of four angles where a half-angle is equal to . The thrust of the every angle

directed along the bisector of each angle must be multiplied by $sin\left({\pi}/{4}\right)={\sqrt{2}}\big/{2}$ and, when multiplied by 4, the thrust of such a honeycomb cell will be equal to

So, the formula for the specific thrust of cells obtained by using the PFA (Proximity Force Approximation) method or PAA (Pairwise Additive Approximation) method

is to some extent similar to the formula for the magnitude of the two-dimensional Casimir effect on honeycombs presented in the first part of this work. At least the value of the exponent in the denominator is the same

It should be noted, that this formula is received without taking into account the finiteness of the cell edge height (i.e., in the approximation of the infinite edge height), in contrast to the PFA version of the formula for which the edge height is assumed to be equal to the cell width.

The approximate agreement of these formulas indicates that the effect should also be observed at finite height of the ribs, although the dependence of the effect on this height may be the object of further research.

Theoretically it is possible to achieve a greater value of thrust with a panel produced from acute-angled angles, but producing panels from honeycombs seems to be technologically simpler than from angles.

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@articleTuo2019, Author = Tuo Qu, Fang Liu, Yuechai Lin, Yidong Huang, Title = Metal nano-honeycomb fabricated by colloidal assembly and femtosecond-laser annealing, Year = 30 January 2019, Journal = Proc. SPIE 10841, 9th International Symposium on Advanced Optical Manufacturing and Testing Technologies: Meta-Surface-Wave and Planar Optics, volume = 10841, booktitle = 9th International Symposium on Advanced Optical Manufacturing and Testing Technologies: Meta-Surface-Wave and Planar Optics, editor = Mingbo Pu and Xiaoliang Ma and Xiong Li and Minghui Hong and Changtao Wang and Xiangang Luo, organization = International Society for Optics and Photonics, publisher = SPIE, abstract = Bioinspired nanostructures have attracted increasing attentions and found widespread applications in various fields including material, chemical, mechanical and optical engineering because of their unparalleled physical advantages<sup>1</sup>. Honeycomb, a kind of porous structure, owns unique structure features, which enable its properties of low density, high mechanical strength, and high-energy-storage capacity<sup>2</sup>. The high quality of metal honeycomb structure with high uniformity, smooth metal surface, high-aspect-ratio sidewall and sharp corners of the triple junction is useful for plasmonic functional devices. Inspired by the building process of natural honeybee combs, we proposed an unconventional nanofabrication technique to produce high-quality gold nano-honeycombs with high-aspect-ratio (&gt;10:1) and thin (&lt;20 nm) sidewalls. As one of the important applications, the refractive index (RI) sensing behavior of the gold nano-honeycomb arrays was modeled and investigated numerically based on the surface plasmon polariton effect. The simulation results show that, in near-infrared region, the RI sensitivity is about 850 nm/RIU, which is approaching the theoretical limit<sup>3</sup>., keywords = nano-honeycomb, self-assembly, femtosecond-laser annealing, refractive index sensor, year = 2019, url = https://doi.org/10.1117/12.2508593,

@articleHrvoje2016, Author = Hrvoje Nikolić, Title = Proof that Casimir force does not originate from vacuum energy, year = 2016, Journal = Physics Letters B, volume = 761, pages = 197, issn = 0370-2693, url = https://doi.org/10.1016/j.physletb.2016.08.036, abstract = We present a simple general proof that Casimir force cannot originate from the vacuum energy of electromagnetic (EM) field. The full QED Hamiltonian consists of 3 terms: the pure electromagnetic term Hem, the pure matter term Hmatt and the interaction term Hint. The Hem-term commutes with all matter fields because it does not have any explicit dependence on matter fields. As a consequence, Hem cannot generate any forces on matter. Since it is precisely this term that generates the vacuum energy of EM field, it follows that the vacuum energy does not generate the forces. The misleading statements in the literature that vacuum energy generates Casimir force can be boiled down to the fact that Hem attains an implicit dependence on matter fields by the use of the equations of motion and to the illegitimate treatment of the implicit dependence as if it was explicit. The true origin of the Casimir force is van der Waals force generated by Hint.

@articleIntravaia2013, Author = F. Intravaia, Title = Strong Casimir force reduction through metallic surface nanostructuring, Year = 2013, month = Sep, Journal = Nature Communications, volume = 2515, 4, pages = 1, url = https://doi.org/10.1038/ncomms3515, abstract = The Casimir force between bodies in vacuum can be understood as arising from their interaction with an infinite number of fluctuating electromagnetic quantum vacuum modes, resulting in a complex dependence on the shape and material of the interacting objects. Becoming dominant at small separations, the force has a significant role in nanomechanics and object manipulation at the nanoscale, leading to a considerable interest in identifying structures where the Casimir interaction behaves significantly different from the well-known attractive force between parallel plates. Here we experimentally demonstrate that by nanostructuring one of the interacting metal surfaces at scales below the plasma wavelength, an unexpected regime in the Casimir force can be observed. Replacing a flat surface with a deep metallic lamellar grating with sub-100 nm features strongly suppresses the Casimir force and for large inter-surfaces separations reduces it beyond what would be expected by any existing theoretical prediction.,

@articleRodriguez2011, Author = A.W. Rodriguez, F. Capasso, S.G. Johnson, Title = The Casimir effect in microstructured geometries, Year = 2011, month = Apr, Journal = Nature Photonics, volume = V.5, pages = 211, url = https://doi.org/10.1038/nphoton.2011.39, abstract = In 1948, Hendrik Casimir predicted that a generalized version of van der Waals forces would arise between two metal plates due to quantum fluctuations of the electromagnetic field. These forces become significant in micromechanical systems at submicrometre scales, such as in the adhesion between movable parts. The Casimir force, through a close connection to classical photonics, can depend strongly on the shapes and compositions of the objects, stimulating a decades-long search for geometries in which the force behaves very differently from the monotonic attractive force first predicted by Casimir. Recent theoretical and experimental developments have led to a new understanding of the force in complex microstructured geometries, including through recent theoretical predictions of Casimir repulsion between vacuum-separated metals, the stable suspension of objects and unusual non-additive and temperature effects, as well as experimental observations of repulsion in fluids, non-additive forces in nanotrench surfaces and the influence of new material choices.,

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У цій статті проаналізовано двовимірний ефект Казимира одного тіла на прикладі наносот квадратної форми. У класичному одновимірному ефекті Казимира двох тіл сила Казимира між двома пластинами виникає як різниця електромагнітних тисків квантово-вакуумних флуктуацій нульової точки по різні боки кожної з пластин. Пластини штовхаються одна до одної зовнішніми полями квантово-вакуумних осциляцій, щільність яких в класичній конфігурації перевищує щільність внутрішніх. Можна спробувати створити різницю електромагнітних тисків квантово-вакуумних осциляцій по різні боки однієї пластини за рахунок різниці геометрії вакуумних резонаторів на різних сторонах пластини. Для цього необхідно виростити нанокомірки на одній з поверхонь гладкої металевої пластини. В результаті було виявлено, що формула для сили на одиницю площі дуже схожа на формулу класичного ефекту Казимира, за винятком значення коефіцієнта пропорційності.

Силу, прикладену до ідеально провідних сот на пластині в результаті різниці питомої густини енергії на різних її сторонах, можна інтерпретувати як тиск електромагнітних флуктуацій нульової точки. Згідно з формулою, представленою в цій роботі, для золотих наносот розміром близько 2 мкм сила має дорівнювати 8,55 дин на квадратний метр панелі, що є цілком прийнятним значенням для практичного використання очікуваного ефекту для корекції орбіт супутників.

Хоча ефект невеликий, експериментальне підтвердження могло б слугувати вирішальним доказом існування віртуальних квантових фотонів Казимира.

Двовимірний ефект Казимира одного тіла, нанокомірки, наносоти, тяга Казимира