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# DIP Assignment 5 (Theory)

**Q1.** $L \rightarrow$ FFT of Laplacian
$G \rightarrow$ observed image
$F \rightarrow$ true image

Finding $\min \left\{ \sum_k \sum_l \left| W(k,l) G(k,l) - F(k,l) \right|^2 + \gamma \sum_k \sum_l \left| L(k,l) W(k,l) G(k,l) \right|^2 \right\}$ ①

with respect to $W(k,l)$

On differentiating ① with $W(k,l)$, to find min value

$G = HF + N$

~~$2(W \cdot G - F)($~~

$$\Rightarrow \cancel{2}\left(W(HF+N) - F\right)(HF+N)^* + \cancel{2}\gamma (LG)^* (LWG) = 0$$

$$\Rightarrow (WHF + WN - F)\left((HF)^* + N^*\right) + \gamma W |LG|^2 = 0$$

$$\Rightarrow W|HF|^2 + WHFN^* + WN(HF)^* + W|N|^2 - H^*|F|^2 - FN^* + \gamma |LG|^2 W = 0$$

~~Since F and N are uncorrelated,~~ and the $(\cdot *)$ will be 0.

$$\Rightarrow W|HF|^2 + W|N|^2 - H^*|F|^2 + \gamma |LG|^2 W = 0$$

$$\Rightarrow W\left[|HF|^2 + |N|^2 + \gamma |LG|^2\right] = H^* |F|^2$$

$$\Rightarrow W = \frac{H^* |F|^2}{|HF|^2 + |N|^2 + \gamma |LG|^2}$$

$$\Rightarrow \boxed{W = \frac{H^*}{|H|^2 + \frac{|N|^2}{|F|^2} + \frac{\gamma |LG|^2}{|F|^2}}}$$

Hence shown

Q2. Given image pixels

$$f(x,y) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 10 & 10 & 1 \\ 0 & 2 & 3 & 1 \\ 0 & 5 & 15 & 8 \end{bmatrix}$$

Padding it with 0s,

$$f'(x,y) = \begin{bmatrix} 0 & 0_A & 0_B & 0 \\ 1 & 1_E & 1_F & 1 \\ 0 & 10_C & 10_D & 1 \\ 0 & 2 & 3 & 1 \\ 0 & 5 & 15 & 8 \end{bmatrix}$$

Sobel filter for vertical direction

$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

for $G_y$

Sobel filter for horizontal direction

$$\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

for $G_x$

⇒ On taking .* (Hadamard) products we obtain

$$G_y \begin{bmatrix} 10 & -9 \\ 23 & -19 \\ 31 & -8 \end{bmatrix} \qquad G_x \begin{bmatrix} 30 & 31 \\ 3 & 5 \\ -5 & 12 \end{bmatrix}$$

$$\text{Magnitude} = |G_x| + |G_y|$$

$\Rightarrow$ Magnitude of gradient

$$= \begin{bmatrix} |30| & |31| \\ |3| & |5| \\ |-5| & |12| \end{bmatrix} + \begin{bmatrix} |10| & |-9| \\ |23| & |-19| \\ |31| & |-8| \end{bmatrix}$$

$$= \begin{bmatrix} 30 & 31 \\ 3 & 5 \\ 5 & 12 \end{bmatrix} + \begin{bmatrix} 10 & 9 \\ 23 & 19 \\ 31 & 8 \end{bmatrix}$$

$\Rightarrow$ Magnitude of gradient

$$= \underline{\begin{bmatrix} 40 & 40 \\ 26 & 24 \\ 36 & 20 \end{bmatrix}}$$

Direction of gradient

$$= \tan^{-1}\left[\frac{G_y}{G_x}\right]$$

Direction (Angle) of gradient

$$= \tan^{-1} \cancel{\begin{bmatrix} 3 & -4.11 \\ 0.13 & -6.263 \\ -0.161 & -1.5 \end{bmatrix}} \quad \tan^{-1}\begin{bmatrix} 0.333 & -0.29 \\ 7.67 & -3.8 \\ -6.2 & -0.66 \end{bmatrix}$$

Direction (angle) of gradient $= \begin{bmatrix} 18.43 & -16.18 \\ 82.56 & -75.25 \\ -80.82 & -33.70 \end{bmatrix}$

For Non Max Suppression at points E & F in the figure,

→ Angle (E) = $82.56°$

⇒ vertical section suppression

i.e. Mag (E) < Mag (C) < Mag (A)
(26) (36) (40)

⇒ value = 0

→ Angle (F) = $-75.25$

⇒ vertical section suppression

i.e. Mag (D) < Mag (F) < Mag (B)
(20) (24) (40)

⇒ value = 0

Output vector = $\begin{bmatrix} 0 & 0 \end{bmatrix}$