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DIP → Assignment 3

Q1. MATLAB Code

Q2. For image sharpening,

sharpened image

$$g(x,y) = f(x,y) + c[\nabla^2 f] \rightarrow \text{laplacian filter}$$

for laplacian filter,

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = f(x+1,y) + f(x-1,y) - 2f(x,y) + f(x,y+1) + f(x,y-1) - 2f(x,y)$$

⇒ Now every term can be written as convolution with impulse function

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = f(x,y) * \delta(x+1,y) + f(x,y) * \delta(x-1,y) - 2f(x,y) * \delta(x,y) + f(x,y) * \delta(x,y+1) + f(x,y) * \delta(x,y-1) - 2f(x,y) * \delta(x,y)$$

$$\Rightarrow \nabla^2 f = \frac{\partial^2 f}{\partial^2 x} + \frac{\partial^2 f}{\partial y^2} = f(x,y) * [\delta(x+1,y) + \delta(x-1,y) - 2\delta(x,y) + \delta(x,y+1) + \delta(x,y-1) - 2\delta(x,y)]$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = f(x,y) * \Big[\underbrace{\delta(x+1,y) + \delta(x-1,y) + \delta(x,y+1) + \delta(x,y-1) - 4\delta(x,y)}_{\text{let us assume this to be } k(x,y)}\Big]$$

We know,

$$g(x,y) = f(x,y) + c\nabla^2 f$$

from previous def^n

$$g(x,y) = f(x+y) + c\left[f(x,y) * k(x,y) \right]$$

$$\boxed{g(x,y) = f(x,y) * \left[\delta(x,y) + ck(x,y) \right]}$$

or

$$\boxed{g(x,y) = f(x,y) * \left[\delta(x,y) + c\left[\delta(x+1,y) + \delta(x,y+1) + \delta(x-1,y) + \delta(x,y-1) - 4\delta(x,y) \right] \right]}$$

$$\Downarrow$$

$$g(x,y) = f(x,y) * w(x,y)$$

where

$$w(x,y) = \delta(x,y) + ck(x,y)$$

or

$$w(x,y) = \delta(x,y) + c\left[\delta(x+1,y) + \delta(x,y+1) + \delta(x-1,y) + \delta(x,y-1) - 4\delta(x,y) \right]$$

Q3. a) MATLAB Code

b) $f(x,y) = \underbrace{\begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}}_{2\times 2}$, $h(x,y) = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}}_{3\times 3}$

Final size of result matrix

$= (2+3-1,\ 2+3-1) = \underline{4\times 4}$

also, our operating matrix will be of the size $2+3\times 2-2,\ 2+3\times 2-2 = \underline{6\times 6}$

initial matrix $\underline{6\times 6}$

0	0	0	0	0	0
0	0 ·1	0	0	0 ·2	0
0	0	6	7	0	0
0	0	8	9	0	0
0	0 ·3	0	0	0 ·4	0
0	0	0	0	0	0

Now we have our result matrix blueprint, what we do is, we take the center of $h(x,y)$ on ·1 and iterate it till ·2 [every column till ·2] for all rows up to ·3.

$* \rightarrow$ center of filter $h(x,y)$

1^{st} iteration :

$$\begin{bmatrix} 0^{0} & 0^{1} & 0^{0} \\ 0^{1} & *0^{-4} & 0^{1} \\ 0^{0} & 0^{1} & 6^{0} \end{bmatrix} \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 7 & 0 & 0 \end{matrix}$$

$$\begin{matrix} 0 & 0 & 8 & 9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

Now we have our bracket 1, we multiply all values in this bracket element-wise and sum it up

$\Rightarrow (0\times0)+(0\times1)+(0\times0)+(0\times1)+(0\times-4)+(0\times1)+(0\times0)$
$+(6\times1)+(6\times0) = \underline{\underline{0}}$ ——— (1)

2^{rd} iteration :

$$\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \begin{bmatrix} 0^{0} & 0^{1} & 0^{0} \\ 0^{1} & 0^{-4} & 0^{1} \\ 0^{0} & 6^{1} & 7^{0} \end{bmatrix} \begin{matrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{matrix}$$

$$\begin{matrix} 0 & 0 & 8 & 9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

Sum of element wise product $=$ $(0\times0)+(1\times0)+(0\times0)+$
$(0\times1)+(-4\times0)+(0\times1)+(0\times0)$
$+(6\times1)+(7\times0) = \underline{\underline{6}}$ ——— (2)

||ly, We keep on sliding this window to the right and perform the same steps repititively.

3rd iteration :

~~Sum of~~ Element wise multiplication

$= 0+0+0+0+0+0+0+0+(7\times1)$

$= \underline{7}$ —— (3)

||ly

4th iteration

Sum of element wise product $= \underline{0}$ —— (4)

||ly 5th iteration (next row)

Sum $= 0+0+0+0+0+0+0+0+0+(6\times1) = \underline{\underline{6}}$ —— (5)

||ly 6th iteration

Sum $= 0+0+0+0+(6\times-4)+(7\times1)+(8\times1)+0+0 =$

$-24+15 = \underline{\underline{-9}}$ —— (6)

||ly 7th iteration

sum $= (7\times-4)+(6\times1)+(9\times1)+0+0$

$= -28+6+9 = \underline{\underline{-13}}$ —— (7)

||ly 8th iteration

Sum = $(7 \times 1) = \underline{7}$ —— (8)

||ly 9th iteration Sum = $(8 \times 1) = 8$ —— (9)

||ly 10th iteration (next row)

Sum = $(8 \times -4) + 9 + 6 = -32 + 15$
$= \underline{\underline{-17}}$ —— (10)

||ly 11th iteration

Sum = $(-9 \times 4) + 8 + 7 = 15 - 36$
$= \underline{\underline{-21}}$ —— (11)

||ly 12th iteration

Sum = $(9 \times 1) = \underline{\underline{9}}$ —— (12)

||ly 13th iteration (next row)

Sum = $\underline{\underline{0}}$ —— (13)

||ly 14th iteration

Sum = $(8 \times 1) = \underline{\underline{8}}$ —— (14)

||ly 15th iteration,

Sum : $(9 \times 1) = \underline{\underline{9}}$ —— (15)

||ly 16th iteration

sum = $\underline{\underline{0}}$ —— (16)

For all equations (1) to (16),

we can construct result matrix as

$$\begin{bmatrix} 0 & 6 & 7 & 0 \\ 6 & -9 & -13 & 7 \\ 8 & -17 & -21 & 9 \\ 0 & 8 & 9 & 0 \end{bmatrix}$$

Q4.

MATLAB Code.