23/8/20

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2018137

DIP : Assignment 1

Q1. $\{x, y\} = [\{1,2\}, \{3,5\}, \{-1,-4\}]$

- $XM = Y$
- $X = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ x_3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 3 & 1 \\ -1 & 1 \end{bmatrix}$,
- $Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}$, $M = \begin{bmatrix} m \\ c \end{bmatrix}$

$XM = Y$

$\Rightarrow$ Multiplying by X^{-1} on both sides,

$\Rightarrow (X^{-1}X)M = X^{-1}Y$

$\Rightarrow \boxed{M = X^{-1}Y}$ $\quad \because X^{-1}X = I, \; I \times M = I$

$\hookrightarrow$ pseudoinverse

Can be written as $\rightarrow$

$$M = (X^T X)^{-1} \cdot X^T \cdot Y$$

$$M = \left[\begin{bmatrix} 1 & 3 & -1 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 3 & 1 \\ -1 & 1 \end{bmatrix} \right]^{-1} \cdot \begin{bmatrix} 1 & 3 & -1 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}$$

$$M = \begin{bmatrix} 11 & 3 \\ 3 & 3 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 & 3 & -1 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}$$

$$M = \begin{bmatrix} 0.1250 & -0.1250 \\ -0.1250 & 0.4583 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 & -1 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 0.2500 & -0.2500 \\ 0.3333 & 0.0833 & 0.5833 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}$$

2

$$M = \begin{bmatrix} 2.25 \\ -1.25 \end{bmatrix}$$

we know $M = \begin{bmatrix} m \\ c \end{bmatrix} = \begin{bmatrix} 2.25 \\ -1.25 \end{bmatrix}$

on comparing,

$m = 2.25$ i.e. slope

$c = -1.25$ i.e. intercept

Q2.

$\frac{\partial y^T X M}{\partial M}$, $\quad y^T X M = \begin{bmatrix} 2 & 5 & -4 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 3 & 1 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} m \\ c \end{bmatrix}$

$$y^T X M = \begin{bmatrix} 21 & 3 \end{bmatrix} \begin{bmatrix} m \\ c \end{bmatrix}$$

$$y^T X M = \begin{bmatrix} 21m + 3c \end{bmatrix}$$

$$\Rightarrow \frac{\partial y^T X M}{\partial M} = \begin{bmatrix} \frac{\partial (y^T X M)}{\partial (M(1))} \\ \frac{\partial (y^T X M)}{\partial (M(2))} \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial m}(21m + 3c) \\ \frac{\partial}{\partial c}(21m + 3c) \end{bmatrix}$$

1

$$\Rightarrow \frac{\partial y^T X M}{\partial M} = \begin{bmatrix} 21 \\ 3 \end{bmatrix}$$

if we calculate $X^T y = \begin{bmatrix} 1 & 3 & -1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}$

$$X^T y = \begin{bmatrix} 21 \\ 3 \end{bmatrix}$$

$$\Rightarrow \frac{\partial y^T X M}{\partial M} = x^T y = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

0.5

Show $\neq y^T X$

Q3.

$c = 1$	$c = 2$
Triangle : 2	Triangle : 1
Square : 3	Square : 4

a) for set A which encloses both $c=1$ and $c=2$,

$$P(\text{triangles}) = \frac{2+1}{2+3+1+4} = \frac{3}{10}$$

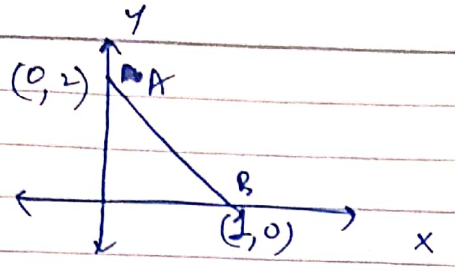
$$P(\text{square}) = \frac{4+3}{10} = \frac{7}{10}$$

b)

$$P(\text{square} \mid c=1) = \frac{P(\text{squares} \cap c=1)}{P(c=1)} = \frac{3/\cancel{10}\,5}{1/2}$$

$$= \frac{3}{5}$$

Q4.



for a probability density function, $Y \geq 0$ which is followed here and also,

area under the graph $y = f(x)$ = $\underline{\underline{1}}$

area under the graph

= area under line AB.

for line AB,

eqⁿ → $\frac{y-2}{x} = \frac{2}{-1}$

$\Rightarrow 2x + y = 2$

$\Rightarrow \boxed{y = 2 - 2x}$

area under $y = f(x) \Rightarrow \int_0^1 f(x)\,dx$

area under $y \Rightarrow \int_0^1 (2-2x)\,dx$

$\Rightarrow \left[2x - x^2\right]_0^1$

$\Rightarrow 2 - 1 - 0$

$\Rightarrow \underline{\underline{1}}$

i.e. fⁿ value ≥ 0 and area under line $f(x) = y = 2-2x$ $\underline{\underline{=1}}$. Hence a probability density function

Q5. 3 bits
i.e. 8 values and 8 quantized levels.

for the values given in list →
-ve valued pixels are taken to be 0
as per convention

→ list → [0, 0.6, 0.7, 3.1, 3.3, 7.4, 10.1, 0, 5.0, 5.5]

minimum value = 0
maximum value = 10.1
range ⇒ 10.1 − 0 = 10.1

interval length of level = $\frac{\text{range}}{\text{no. of levels}}$

length = $\frac{10.1}{8}$ = 1.2625

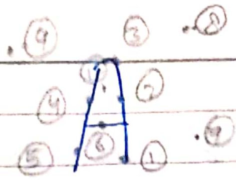
Bits			List element in quantized level
000 → i.e.	interval 1 →	0 − 1.2625	→ {0, 0, 0.6, 0.7}
001 →	interval 2 →	1.2625 − 2.525	
010 →	interval 3 →	2.525 − 3.7875	→ {3.1, 3.3}
011 →	interval 4 →	3.7875 − 5.05	→ {5.0}
100 →	interval 5 →	5.05 − 6.3125	→ {5.5}
101 →	interval 6 →	~~7.5~~ 6.3125 − 7.575	→ {7.4}
110 →	interval 7 →	7.575 − 8.8375	
111 →	interval 8 →	8.8375 − 10.1	→ {10.1}

⇒
−3 → 000
−0.2 → 000
0.6 → 000
0.7 → 000
3.1 → 010
3.3 → 010
5.0 → 011
5.5 → 100
7.4 → 101
10.1 → 111

→ Solⁿ

Values are quantized to 3 bits in the foll. way.

Q6.



A sensor pixel can have only one value ranging from 0 – 2^{n-1} (n is the bits, i.e. 0-255 for 8 bits). If we have only 1 pixel, that pixel will only have one colour, so our image is one-coloured square, and it does not really give us clarity about the stored image, ~~thus~~ we lose all information of the original image (which on observation has 2 colours).

If we have 10x10 pixels (i.e. 100 pixels), that means an array of 100 squares, each of them can hold a different value hence a different colour. In the image, I have marked ~~6~~ 10 points and some of them have black colour and some white, so now we have 100 pixels and we know each one of them can hold a different colour, hence we not only have a higher resolution of the image, but also less information is lost, we can map 100 points to specific values and they will help us to recognise the character 'A'.

So, we favour 10 x 10 pixels over 1 pixel.