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Quiz #1

COSC 441

2/3/21

1.  $T(n) = 2T(n/3) + 2T(n/4) + 7n^2$

guess  $T(n) \leq cn^2$

$$\leq 2c(n/3)^2 + 2c(n/4)^2 + 7n^2$$

$$= 2cn^2(1/9 + 1/16) + 7n^2 \quad \text{pick } c = 9 \cdot 16 = 144$$

$$= (2 \cdot 16 + 2 \cdot 9)n^2 + 7n^2$$

$$= 57n^2 < cn^2 \quad \text{since } c = 144$$

$T(n)$  is  $O(n^2)$

2.  $T(n) = 2T(n/2) + 3n^2$

$$= 3n^2 + 2 \cdot \left( \frac{2T(n/2) + 3n^2}{2} \right)$$

$$= 3n^2 + 3n^2 + T(n/2)$$

$$= 3n^2 + 3n^2 + \frac{2^2 T(n/2) + 3n^2}{2^2}$$

$$= 3n^2 + 3n^2 + \frac{3n^2}{2^2} + \frac{3n^2}{2^3} + \frac{3n^2}{2^4} \dots \frac{3n^2}{2^k} + \frac{2^k T(n/2)}{2^k}$$

$$= 3n^2 \sum_{i=0}^k \left( \frac{1}{2} \right)^i + T(n/2^{k+1})$$

when is  $\frac{n}{2^{k+1}} = 1$ ?  $\Rightarrow k = \log n - 1$

$$= 3n^2 \sum_{i=0}^{\log n - 1} \left( \frac{1}{2} \right)^i + T(1) < 3n^2 \sum_{i=0}^{\infty} \left( \frac{1}{2} \right)^i + C$$

$$= 3n^2 \cdot \frac{1}{1 - \frac{1}{2}} + C = 6n^2 + C = O(n^2)$$

$$3. T(n) = 9T(n/3) + n^2$$

$$a = 9, b = 3, f(n) = n^2$$

$$n^{\log_b a} = n^{\log_3 9} = n^2 \quad \log_3 9 > 1$$

$$f(n) = n^{\log_3 9}$$

$\Rightarrow$  Case 2 test

$$T(n) = \Theta(n^{\log_3 9} \lg n) = \Theta(n^2 \lg n)$$