Using 3D direct manipulation for real-time structural design exploration

D. Åkesson [A]
C. Mueller [B]

A. PhD Student, Lund University, Lund, Sweden
B. Assistant Professor, Massachusetts Institute of Technology, Boston, MA, USA

Abstract

The impact of decisions on the design process is initially high and declines as the design matures. However, few computational tools are available for the early design phase; thus, an opportunity exists to create such tools. New technology opens up new possibilities to create new and novel computational tools. In this work, an existing application is adapted for a new three-dimensional (3D) input device called the Leap Motion Controller. The controller allows the user to interact with 3D objects on a screen using his/her fingers and hands. The result of this work is a conceptual design application that enables very direct manipulation of 3D objects on the screen to a level that has not been achieved before for this type of application in 3D. An improved human-computer interaction can potentially improve the user's understanding of the structural behavior of a model, cognitive engagement in the design task, and encourage further design exploration. Three different cases are implemented to enable the user to explore different design options with emphasis on geometrical form, as this has the greatest potential to improve structural performance. These case studies demonstrate a new potential for building engineering intuition and improving design space exploration through very direct manipulation in 3D.

Keywords

Interactive structural analysis; Interactive structural form finding; 3D input device; Dynamic relaxation; Conceptual structural design

1 Introduction

The object of this paper is to demonstrate how an interactive three-dimensional (3D) environment can improve conceptual structural design. The goal is to improve the user's understanding of the structural behavior of a model and illustrate how geometrical modifications can change its structural performance.

1.1 Conceptual design tools

The earliest phase of the design process is referred to as the conceptual design phase. Decisions made in this phase have the highest impact of all the decisions made throughout the design process [1]; the impact of decisions then declines as the design matures. The importance of the conceptual design phase is often overlooked and structural aspects are often only considered at a late design stage [2]. A factor contributing to this situation is that very few computational tools are available for conceptual design. The challenge of developing such computational tools is the fuzzy nature of the problem: the knowledge and constraints of the problem are imprecise and incomplete [3]. Conventional advanced structural analysis software requires precise knowledge of the problem and is not agile enough to follow a designer's iterative workflow. It has been shown that premature use of such software can negatively affect the quality of the conceptual design [4]. Conventional structural analysis software has been developed for use in the late design stage, when the major design decisions have already been made, as a tool for the engineer to verify the form.

Many different geometric modeling tools are available for architects today. These geometric modeling tools have, since their introduction in the 1980s, grown increasingly sophisticated, and have, together with the widespread perception of the benefits of technological innovation, created a more intimate relationship between technology and design. This relationship has resulted in parametric design and scripting methods that can generate complex shapes and forms [5]. The distinct separation in design that means that architects use geometric modeling tools and engineers use analysis tools further reinforces the architects role as form-giver and the engineer as form-verifier [6]. To move away from this separation, the term "designer" is used in this paper to represent either an engineer or architect.

The type of design tool that is used to generate and represent ideas also affects the quality and quantity of early prototypes. It was shown in [7] that physical prototyping generates a higher quantity of prototypes within a limited amount of time. The developed prototypes were also perceived as more novel compared to the prototypes that where developed using computer aided design (CAD) or conventional sketches. However, the prototypes that were perceived as more novel tended to fare poorly with respect to all other measureable qualities [7]. As conceptual design is important and few conceptual design tools are available, an opportunity exists to improve the design process by developing such tools.

With new technology such as novel input devices and increased computational power comes new possibilities. The present work makes use of these possibilities to create a new way to interact with and create digital prototypes. The prototypes in this work are structural models. As a computational model is used, its measureable performance can be computed and presented in real-time to the user to potentially improve the quality of the structural models. The measureable performance and guidance in this work emphasizes the geometrical form of the structure, as this has the greatest potential to improve the structural performance. The result of the present work is a computational tool for conceptual structural design with a novel human-computer interaction interface.

1.2 Human-computer interaction

Direct manipulation is a style of human-computer interaction that has a continuous representation of the objects of interest with rapid, reversible, and incremental feedback [8]. Users can directly manipulate objects on the screen using real-world metaphors, which makes the users more engaged with their task and encouraged to explore further [9]. This is achieved by reducing the perceptual and cognitive resources required to understand and use the user interface [10].

The introduction of different input devices such as the mouse and joystick significantly improved the human-computer interaction of user interfaces, which adapted accordingly [10]. Later, when the touch screen was introduced, it had the advantage over these previous devices of a very direct method of inputting information [10]. It closed the gap between the human and computer, and the user could literally touch objects on the screen to manipulate them.

There is a wide repertoire of interaction techniques to create direct manipulation user interfaces for 3D applications using 2D input devices such as the mouse [11]. However, because these types of input devices have one degree of freedom less than the 3D user interface, there will always be a need for gestures or similar methods.

Computer games have seen an increase in the number of novel input devices along with new styles of games to address some of the limitations of conventional systems [12], e.g., the Wii Remote [13], Microsoft's Kinect for Xbox [14], and PlayStation Move [15]. These novel input devices move away from the conventional human-computer interaction to invoke an intuitive interaction that supports the natural human method of working. Games have for a long time been perceived as fun and engaging, and it has been investigated in many different disciplines whether gaming methods can improve human-computer interaction in order to create more effective, immersive, and engaging learning or training [12]. In computer aided design tools, the user experience has been compromised by the engineering design system's step-by-step evolution. The present work moves away from the step-by-step user interface to create an interactive, gaming-like experience using a novel 3D input device.

Although beyond the scope of this paper, the interest in and development of virtual reality glasses such as the Oculus Rift [16] and PlayStation's Project Morpheus [17] have recently increased. These types of virtual reality glasses have primarily been developed for games, but other fields have also shown interest, e.g., in [18], virtual reality is used to help students understand complex structural behavior.

2 Related work

Multiple software tools for conceptual structural design that deploy truss models have previously been developed. The first two such tools, PointSketch [19] and Arcade [20], were developed in parallel and released in 2006. In both software tools, the user can create a computational model using mouse and keyboard input. Forces can then be applied to the model and the results from the computations are visualized. The two software tools were both developed in academia, but industry has shown interest in the concept.

The commercial finite element software SAP2000 launched a "model alive" feature in 2012 that enables real-time feedback for deformations and forces [21]. The software was developed for use with truss-structures. However, geometrical modifications are made through text input. Autodesk launched a new application in 2011 named ForceEffect [22], which is available both as a tablet and web application. The application was developed for designers to analyze and visualize two-dimensional truss structures. The tablet application utilizes a direct manipulation user interface style, where the user can make changes to the model by directly touching the objects. A similar application that further developed the direct manipulation of the interaction is a tablet application named Sketch-a-Frame [23]. This application updates the visualizations of the computational result in real-time as the user makes changes to the geometry, creating a very direct manipulation. Another conceptual design tool that can generate structures through interactive optimization is called StructureFIT [24,25]. This software tool also has a direct manipulation mode, where the user can further explore a generated structure by moving nodes in real-time see how a relative performance index is updated.

Recently, an interactive physics engine was developed to create a user experience inspired by games for design and education [26]. The developed physics engine has been used to create an interactive game called Catastrophe, which aims to teach users which elements are critical to system stability through play.

A popular design tool in practice is the parametric modeler Grasshopper [27], which allows the user to manipulate parametric models through the use of sliders. This can be combined with Karamba [28], which is a structural analysis tool that can also give real-time feedback as to how the structural performance changes when the input parameters are altered. A tool that implements similar functions is Autodesk's Dynamo [29].

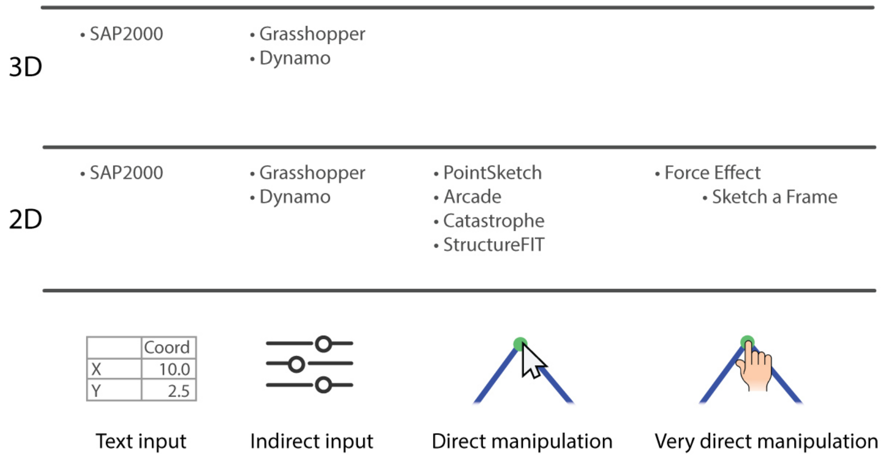

Figure 1: Summary of previous work

A summary of the evaluated conceptual design tools can be seen in Figure 1. The tools are grouped according to number of dimensions and how directly the manipulation can be experienced. Prior to the work presented here, no conceptual structural design tool has before been able to achieve very direct manipulation for 3D. A very direct manipulation user interface has so far only been achieved for 2D conceptual structural design tools. This is because of limitations of the traditional mouse and keyboard human-computer interaction with which the 3D applications were developed.

3 Methodology

The Leap Motion Controller [30] is a novel 3D input device. It can, with the use of infrared cameras, create a virtual model of a user's hands that are in the field of vision of the controller, as shown in Figure 2. This makes it possible to allow users to very directly manipulate representations of objects in 3D—which has the potential to improve the human-computer interaction of structural conceptual design applications. Improved interaction in such applications could potentially improve the user's understanding of the structural behavior of a model, cognitive engagement in the design task, and encourage further design exploration. The present work shows an implementation with a very direct manipulation for 3D that has not been achieved before for conceptual structural design tools.

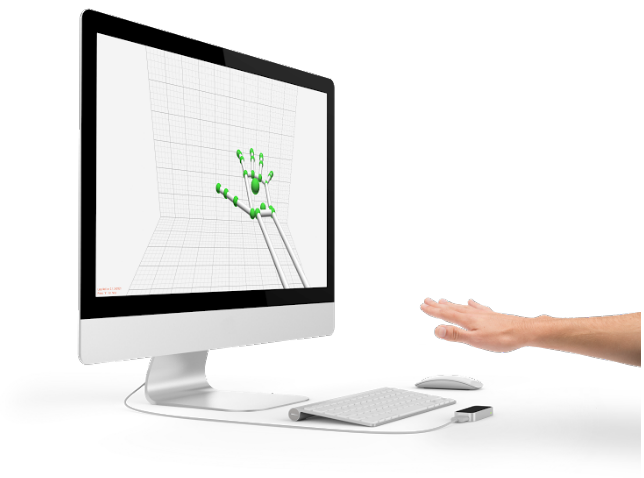

Figure 2: Leap Motion Controller in use

3.1 Leap Motion Controller

A new 3D input device for computers was launched in July 2013 called the Leap Motion Controller [30]. By connecting the controller to a computer and placing it in front of the user, it can track the movement of fingers, hands, and tools with sub millimeter precision without any visible latency [30]. A software development kit (SDK) is available and supports most common programming languages [31], which can be used to implement support for the controller in existing applications.

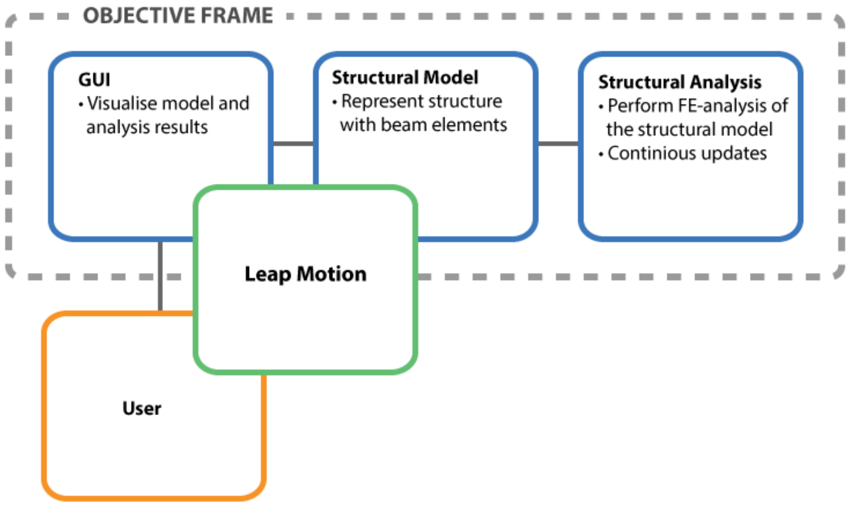

Figure 3: Conceptual diagram of how the Leap Motion Controller integrates into the existing conceptual design application ObjectiveFrame.

The SDK interprets the data from the controller's cameras and creates computational models of any hands or tools (e.g., a pen) that are in the field of vision of the controller.

Using the SDK to integrate it with an application, the computational model can be accessed through the application programming interface (API), which has functions to return the positions and direction-vectors of hands, fingers, and tools in 3D. This can be used to build and visualize a virtual model of a hand or tool that can directly interact with the 3D objects on the screen. The SDK also has built-in support for gestures that can be used to call actions. This creates the potential to create very direct user interfaces for 3D, similar to the possibilities that the multi-touch user interface created for 2D.

However, new input devices require that software adapt accordingly in order to be successful [10]; thus, new 3D input devices are heavily dependent on software developers that can create new novel user interfaces.

3.2 Detailed implementation

The work in this paper builds on an existing application that was developed for conceptual structural design named ObjectiveFrame [32]. The application was developed in the C++ programming language and uses the FLTK toolkit [33] to create the graphical user interface (GUI) and the Newmat matrix library [34] to perform matrix operations.

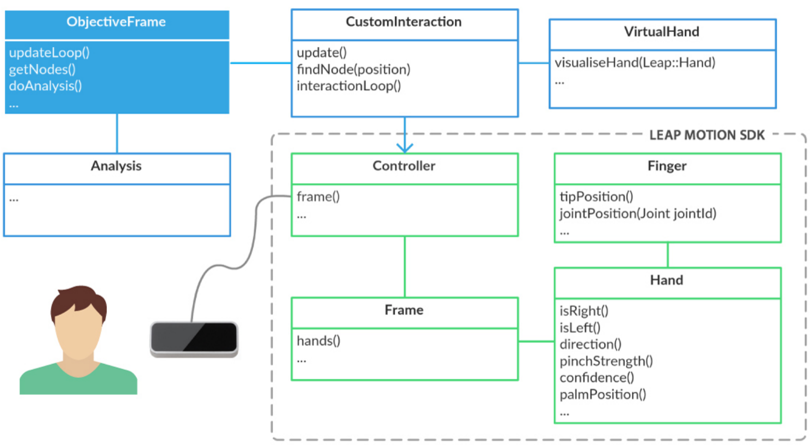

Figure 4: Diagram of how the Leap Motion SDK is integrated with the existing application.

When an instance of the class *Controller* is instantiated, the Leap Motion SDK creates a computational model of the objects in its vision field. The ObjectiveFrame application has an update cycle in which the objects of representation are continuously updated. The class *Controller* is queried at each update to obtain an updated *Frame* that contains information regarding the computational models.

Three different cases have been implemented in the existing application to demonstrate how the Leap Motion Controller can be used in the conceptual design phase.

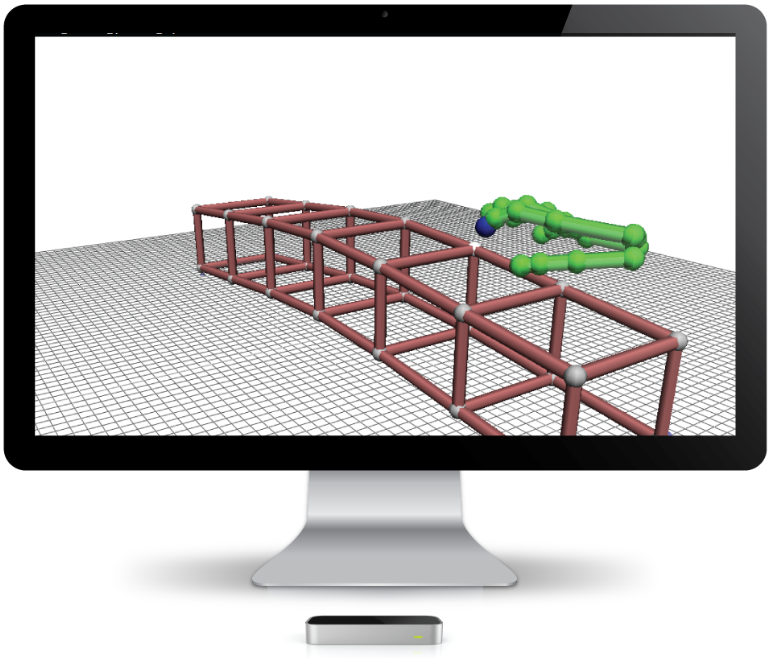

Figure 5: System setup with screenshot from the developed application.

3.3 Direct manipulation interaction model

The data from the Leap Motion SDK is used in the developed application to visualize a virtual model of the user's right hand. The tip of the index finger on the virtual hand is colored blue, as shown in Figure 6—this fingertip is used to select nodes in the 3D space. When the index fingertip position is within a set distance of a node, the node is highlighted. The user can then perform a pinch gesture (see Figure 6) to interact with the highlighted node.

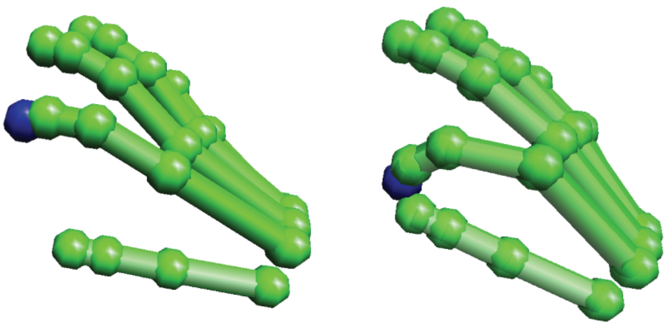

Figure 6: Pinch gesture visualized using the virtual hand model.

The right hand is used to interact with the structure while the left hand simultaneously controls the point of view, i.e., the location and angle at which the model is observed. Moving the palm closer or further away from the controller zooms out or in, respectively. This is a direct manipulation metaphor for pushing the model away. The palm position can also be used to rotate the point of view around the model by moving the palm sideways. The angle of the hand is used to control the angle at which the model is observed.

The position of the virtual hand is relative to the point of the view. Thus, when the user rotates the view, the virtual hand is rotated accordingly around the center of the view. This is in line with the metaphor in which the user reaches into the monitor to interact with the structural model. The Leap Motion SDK also has a confidence parameter. This parameter is lowered if the view is obstructed, e.g., if the left hand obstructs the view of the right hand. This is visualized using the opacity of the right hand to indicate the user that the view is obstructed.

4 Structural analysis for conceptual design

The *Analysis* block in Figure 4 shows the numerical analysis methods that have been implemented in the application to enable structural performance feedback and guidance in the application. In conceptual structural design, a simple mathematical model such as a truss or frame is advantageous for representing a structure [35]. The objective is a general understanding of the structural behavior and numerical precision is of less importance. In later design stages, the simple mathematical model can be substituted with a more advanced mathematical model for improved accuracy.

4.1 Structural analysis

To enable the application to visualize deformations and normalized structural performance, finite element analysis was implemented in the application. The application uses 3D beam finite elements that can be used to create frames or trusses to represent structures. Every beam element has two nodes—each has six degrees of freedom: three rotational and three translational ones [36]. The structural response is computed using

$$\boldsymbol{K}u = f$$

where K is the global stiffness matrix, u the displacement vector, and f the force vector. The compliance of the structure is the work (or strain energy) performed by the truss, and is the sum of the forces multiplied by the displacements:

$$W = \frac{1}{2} f^T u$$

This can be rewritten as

$$W = \frac{1}{2} u^T \boldsymbol{K} u$$

Ideally, the strain energy is a small number that corresponds to an efficient structure. To convey how geometrical modifications change the structural performance—the strain energy of the modified geometry is normalized in the developed application with the initial strain energy. The result is the normalized strain energy, a measure of how well the structure is performing.

4.2 Form finding

The process of designing form-found shapes is called form finding. Either physical models or numerical simulations can be used in this process, where the aim is to find the form for a structure under a load such that static equilibrium is satisfied. The static equilibrium corresponds to a structure that can support the applied load using only compression, and thus is a very efficient structure. For physical models, a hanging chain or cloth can be used the find the static equilibrium.

A number of different numerical methods exist to computationally find the form where static equilibrium is satisfied. The numerical method used in this work is dynamic relaxation, which is a method to solve a set of non-linear equations invented by Alistair Day in 1965 [37]. The method computes the movement of a structure over time to find equilibrium between the internal and external forces.

At each time step Δt, the internal forces for all elements are computed from nodal displacements u. The residual R can be computed using

$$R = f_{ext.} - f_{int.}$$

Using Newton's second law, the acceleration (derivative of the velocity with respect to time) can be computed as follows (at node i, in the x-direction, at time t)

$$R_{ix}^{t} = M_i \dot{v}_{ix}^{t}$$

where M_i is a lumped, fictitious mass at node i. To enforce boundary conditions, the residual is set to zero for the corresponding degrees of freedom. With a known time step, the velocity of node i in the x-direction can be computed using the finite difference method

$$v_{ix}^{t+\Delta t} = v_{ix}^{t-\Delta t/2} + \frac{\Delta t}{M_i} R_{ix}^{t}$$

With the velocity known, the updated geometry can now be updated using

$$x_i^{t+\Delta t} = x_i^t + \Delta t \cdot v_{ix}^{t-\Delta t/2}$$

After the geometry is updated, an iteration is complete and the computations start over, by again computing the residual. The geometry is modified at each iteration until equilibrium between the external and internal forces has been reached. Different types of damping such as kinetic or viscous damping can be used to get the solution to converge. In the present work, the velocities are set to zero in even intervals to obtain the desired dynamic behavior.

5 Case studies

The material parameters are not important in the following conceptual design case studies; however, they have a minor impact of the results and are therefore presented in Table 1.

Young's modulus	$2{,}1 \cdot 10^{9}$
Area	$3{,}5 \cdot 10^{-3}$
Second moment of inertia	$5{,}3 \cdot 10^{-6}$

Table 1: Material properties (representing a steel tube) used in this study

5.1 Case I: Structural response

In this case study, a point load can be applied to a structure, and the resulting deformations are then visualized in real-time. When a pinch gesture is performed on a node, a point load is applied. Then, as the user moves the pinched fingers away from the node, the applied force changes direction and magnitude accordingly. Thus, the further the user moves the hand from the start position, the larger the magnitude of the applied force. The structural deformations are continuously computed and visualized for the user. As the pinch gesture is released, the applied force is removed and the structure is visualized in its undeformed shape.

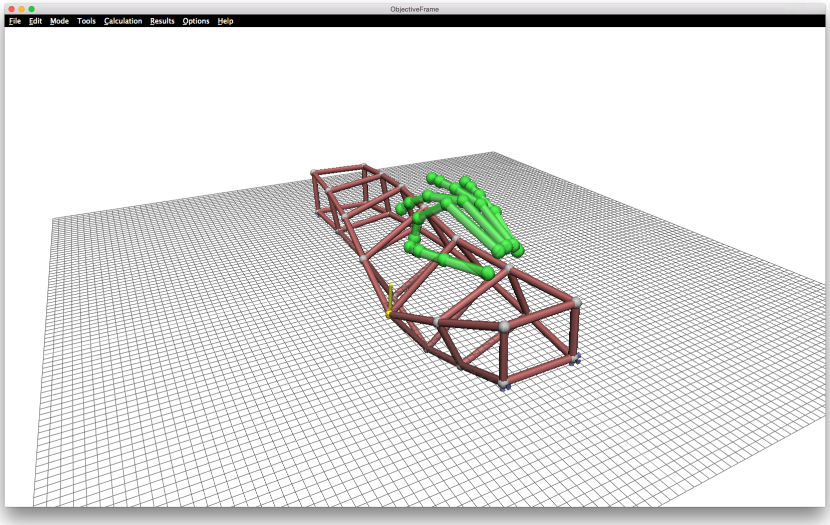

Figure 7: Structural response visualized in real-time

The scaling of the structural response is automatic and relative to the stiffness in the node where the force is applied. The applied force for node I in the x-direction is computed as follows:

$$f_{ix} = P_{currentx} - P_{startx}$$

where P_{startx} is the location of the index finger when the pinch gesture is performed and $P_{currentx}$ is the current position of the index finger.

5.2 Case II: Performance feedback

In this case, the user can move the nodes of a structural model using the pinch gesture. The strain energy of the structural model is continuously computed, normalized, and visualized on the screen for the user. The strain energy is normalized with the initial strain energy, thus the normalized strain energy is initially 1.0, where a lower value corresponds to a better performing structure.

An example of the design exploration process is shown in Figure 8; in this example, symmetry along the depth and width of the model is enforced. Thus, when a node is moved, three other nodes move to enforce symmetry. Enforcing symmetry makes the modeling process faster and different designs can easily be evaluated.

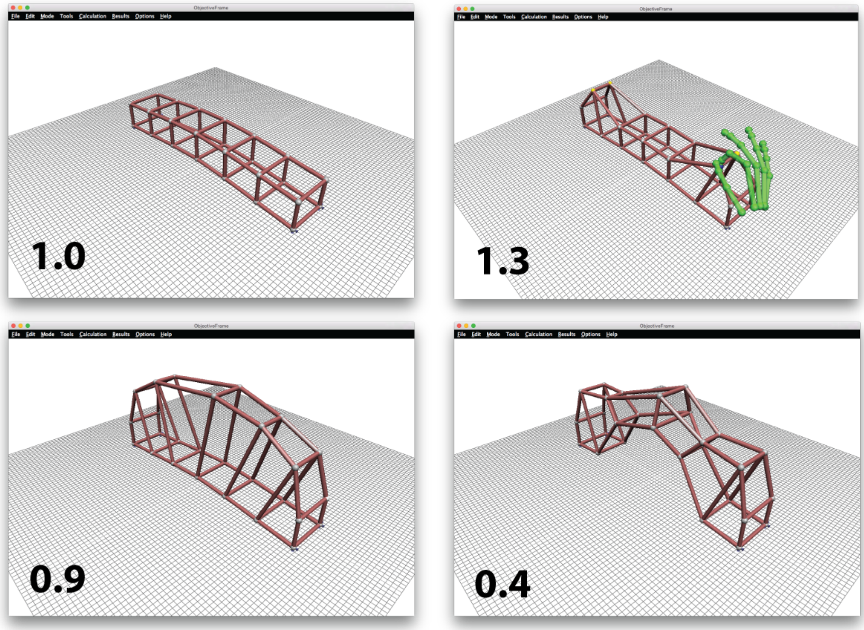

Figure 8: Design process with normalized strain energy

5.3 Case III: Dynamic relaxation

A dynamic relaxation method has also been implemented in the application. In this case, the geometry is computed and updated once every time the application refreshes. This results in an interactive application where the user can observe how the geometry is updated and modified until static equilibrium is reached.

In the following examples, a negative gravity load has been applied to the nodes. The user can then, by using the same interaction as described in Case I, apply a point load to any node. The benefit of using dynamic relaxation is that the geometry will start to converge towards the new static equilibrium even if the static equilibrium was not found for the parent load. This improves the interactivity of the application by following the direct manipulation guideline of continuously updating the object of interest. In Figure 9, an example is shown that corresponds to the physical prototype of a hanging chain.

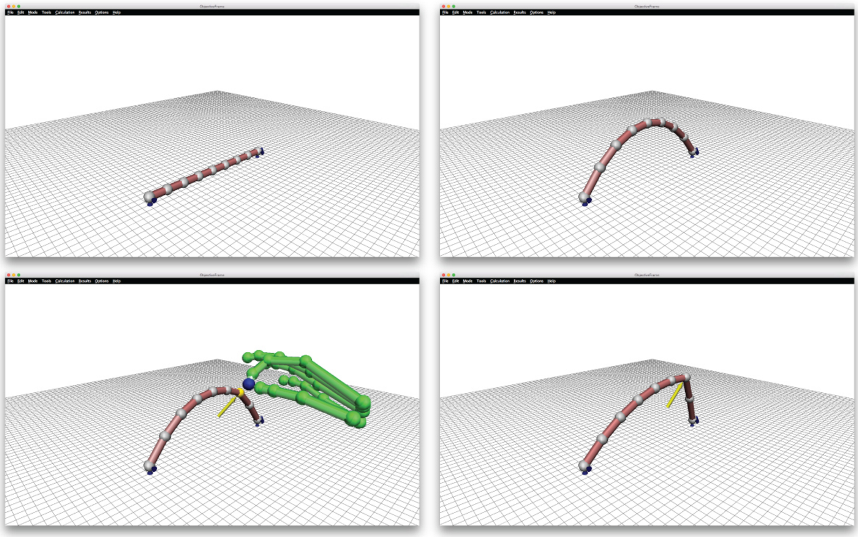

Figure 9: Educational example—the upper left image shows the initial geometry, and the upper right image shows the static equilibrium under a negative gravity load. In addition, the lower left image shows a point load being applied, and the lower right image shows the static equilibrium for a negative gravity load and point load.

As the designer applies a point load to the structure, the 3D dynamic response is interactively visualized. This can be used for educational purposes to improve the user's understanding of the structural behavior of the model.

In Figure 10, an example is shown where the dynamic relaxation is used to find the form of a structure. The point load can here be used to move away from the optimal solution [38] in order to find other interesting sub optimal solutions that might be more aesthetically attractive to the designer. The point load can also, for example, represent a supporting column or a hanging installation in the structure.

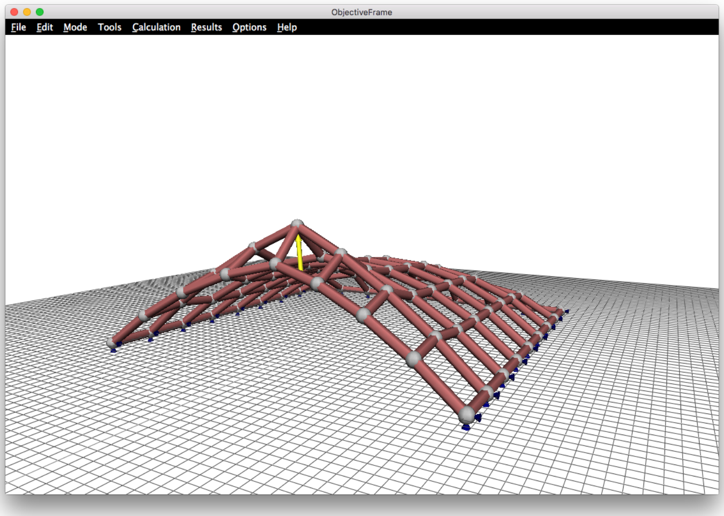

Figure 10: Example of a form-found structure using the developed application.

6 Discussion

This work shows that new 3D input devices have the potential to create very direct manipulation user interfaces for conceptual structural design, previously only achieved for 2D. The human-computer interaction in an existing application is improved, which improves the user's ability to explore different designs. The conceptual design environment is important, and it can have a big impact on the final design. This work shows an alternative user interface for the development of such tools, which in the future could potentially have a big impact on how we design structures.

The implemented methods aim to improve the designer's understanding of both the static and dynamic behavior. The implemented methods also allow the designer to navigate the design space and balance structural performance and aesthetics.

Hand gestures are used in the application to manipulate the location and angle from which the structure is observed. This could easily be implemented in existing CAD software to allow the user to more easily navigate the 3D space.

6.1 Summary of intellectual contributions

- This paper includes critical review of existing tools and techniques for design manipulation in conceptual CAD.
- It is the first to propose very direct manipulation as human-computer interaction mode for 3D structures, thanks to a new type of 3D input device such as the Leap Motion Controller
- It introduces an implemented design tool that allows users to interact with 3D structures through very direct manipulation.
- It demonstrates potential applications through three case studies.

6.2 Future work

The Leap Motion Controller's SDK supports virtual reality glasses such as the Oculus Rift. Combining the two can create a virtual reality in which the user can experience the structure in 3D, interact, and make changes to it using his/her hands.

This could potentially be developed in a game environment where the structure could be visualized in its intended context, i.e., a building site. A game engine could enable real-time renderings of the structure in its context. The designer would then be able to make manipulations, guided by performance feedback, to the rendered structure. This could improve the conceptual design process, and by doing so, have a big impact on the design process and final structure.

6.3 Concluding remarks

- This study responds to need for new, more intuitive, and natural interaction modes in computational design and analysis
- Very direct manipulation significantly improves existing direct manipulation paradigms prevalent in CAD.
- New technologies like the Leap Motion Controller open up unprecedented possibilities for engaging users in the exploration and design of 3D structures, leading to improved understanding of design options and performance in the built environment.

7 Acknowledgements

This work was supported by the Swedish strategic research program eSSENCE. The author would also like to thank Jonas Lindemann for sharing the source code for ObjectiveFrame.

8 References

[1] Wang L, Shen W, Xie H, Neelamkavil J, Pardasani A. Collaborative conceptual design—state of the art and future trends. Comput Des 2002;34:981–96. doi:10.1016/S0010-4485(01)00157-9.

[2] Schlaich M. Challenges in Education–Conceptual and Structural Design. vol. 92. 31st ed., International Association for Bridge and Structural Engineering; 2006, p. 20–6.

[3] Hsu W, Liu B. Conceptual design: issues and challenges. Comput Des 2000;32:849–50. doi:10.1016/S0010-4485(00)00074-9.

[4] Fröderberg M, Crocetti R. Engineers in need of an improved conceptual design toolbox. vol. 102. 37th ed., International Association for Bridge and Structural Engineering; 2014, p. 515–21.

[5] Sakamoto T, Ferré A. Intro. From Control to Des. Parametr. Archit., Actar-D; 2008, p. 5.

[6] Mueller C, Ochsendorf J. An Integrated Computational Approach for Creative Conceptual Structural Design. In: Obrębski JB, editor. IASS, 2013.

[7] Häggman A, Tsai G, Elsen C, Honda T, Yang MC. Connections Between the Design Tool, Design Attributes, and User Preferences in Early Stage Design. J Mech Des 2015;137:71408. doi:10.1115/1.4030181.

[8] Shneiderman B. The future of interactive systems and the emergence of direct manipulation. Behav Inf Technol 1982;1:237–56.

[9] Shneiderman B. Direct Manipulation for Comprehensible, Predictable and Controllable User Interfaces. Proc. 2Nd Int. Conf. Intell. User Interfaces, New York, NY, USA: ACM; 1997, p. 33–9. doi:10.1145/238218.238281.

[10] Sears A, Plaisant C, Shneiderman B. A new era for high precision touchscreens. Adv Human-Computer Interact 1990;3.

[11] Nielson GM, Olsen Jr. DR. Direct Manipulation Techniques for 3D Objects Using 2D Locator Devices. Proc. 1986 Work. Interact. 3D Graph., New York, NY, USA: ACM; 1987, p. 175–82. doi:10.1145/319120.319134.

[12] Kosmadoudi Z, Lim T, Ritchie J, Louchart S, Liu Y, Sung R. Engineering design using game-enhanced CAD: The potential to augment the user experience with game elements. Comput Des 2013;45:777–95. doi:10.1016/j.cad.2012.08.001.

[13] Nintendo. Nintendo Wii n.d. http://www.nintendo.com/wiiu (accessed October 13, 2015).

[14] Microsoft. Kinect n.d. http://www.xbox.com/en-US/xbox-360/accessories/kinect (accessed October 13, 2015).

[15] Playstation. Playstation Move n.d. https://www.playstation.com/en-us/explore/accessories/playstation-move/ (accessed October 14, 2015).

[16] Oculus Rift n.d. https://www.oculus.com/ (accessed October 23, 2015).

[17] Playstation. Project Morpheus n.d. https://www.playstation.com/en-us/explore/project-morpheus/ (accessed October 23, 2015).

[18] Fogarty J, El-Tawil S. Exploring Complex Spatial Arrangements and Deformations in Virtual Reality. Struct. Congr. 2014, n.d., p. 1089–96. doi:http://dx.doi.org/10.1061/9780784413357.097.

[19] Olsson P. Conceptual studies in structural design: pointSketch-a computer based approach for use in early stages of the architectural process. Chalmers University, 2006.

[20] Martini K. A new kind of software for teaching structural behavior and design. In: Oakley D, Smit R, editors. 2006 Build. Technol. Educ. Symp. Proc., University of Maryland; 2006, p. 279.

[21] Clune R, Connor JJ, Ochsendorf JA, Kelliher D. An object-oriented architecture for extensible structural design software. Comput Struct 2012;100-101:1–17. doi:10.1016/j.compstruc.2012.02.002.

[22] Autodesk. Industry First: Autodesk Takes Simulation Mobile with New ForceEffect App for iPad 2011. http://news.autodesk.com/press-release/industry-first-autodesk-takes-simulation-mobile-new-forceeffect-app-ipad (accessed October 22, 2015).

[23] Akesson D, Lindemann J. A tablet computer application for conceptual design. Proc ICE - Eng Comput Mech 2015:1–8. doi:10.1680/eacm.14.00020.

[24] Mueller C. StructureFIT n.d. http://www.caitlinmueller.com/structurefit/ (accessed October 16, 2015).

[25] Mueller CT, Ochsendorf JA. Combining structural performance and designer preferences in evolutionary design space exploration. Autom Constr 2015;52:70–82. doi:10.1016/j.autcon.2015.02.011.

[26] Senatore G, Piker D. Interactive real-time physics. Comput Des 2015;61:32–41. doi:10.1016/j.cad.2014.02.007.

[27] Grasshopper n.d. http://www.grasshopper3d.com/ (accessed October 5, 2015).

[28] Karamba n.d. http://www.karamba3d.com/ (accessed October 5, 2015).

[29] Dynamo n.d. http://www.autodesk.com/products/dynamo-studio/overview (accessed November 5, 2015).

[30] Leap Motion. Leap Motion Launches World's Most Accurate 3-D Motion Control Technology for Computin 2013. https://www.leapmotion.com/news/leap-motion-launches-world-s-most-accurate-3-d-motion-control-technology-for-computing (accessed October 22, 2015).

[31] Leap Motion SDK n.d. https://developer.leapmotion.com/ (accessed September 24, 2015).

[32] Lindemann J, Dahlblom O, Sandberg G. ObjectiveFrame-An educational tool for understanding the behaviour of structures. Appl. Virtual Real. Eng. Constr. Appl. Virtual Real. Curr. Initiat. Futur. Challenges AVR II CONVR, 2001.

[33] Fast Light Toolkit n.d. http://www.fltk.org/ (accessed October 19, 2015).

[34] Robert D. Newmat C++ matrix library. Newmat Doc n.d. http://www.robertnz.net/index.html (accessed October 22, 2015).

[35] Schlaich J, Schafer K. Design and detailing of structural concrete using strut-and-tie models. Struct Eng 1991;69:113–25.

[36] Bathe K-J. Finite element procedures. Klaus-Jurgen Bathe; 2006.

[37] Day AS. An introduction to Dynamic Relaxation. Eng 1965;219.

[38] Kilian A. Steering of form. Shell Struct. Archit. form Find. Optim., Routledge; 2014, p. 131–8.