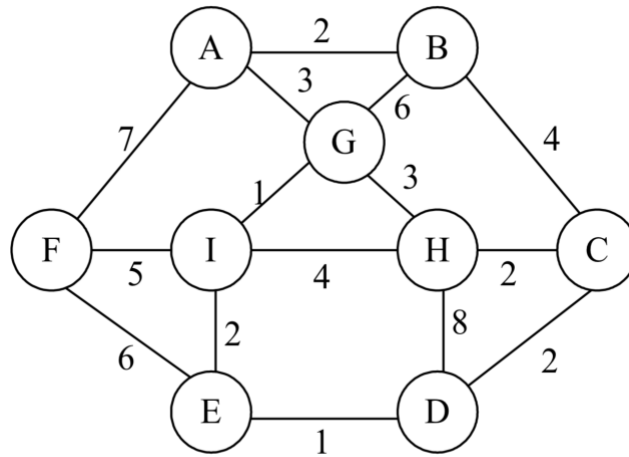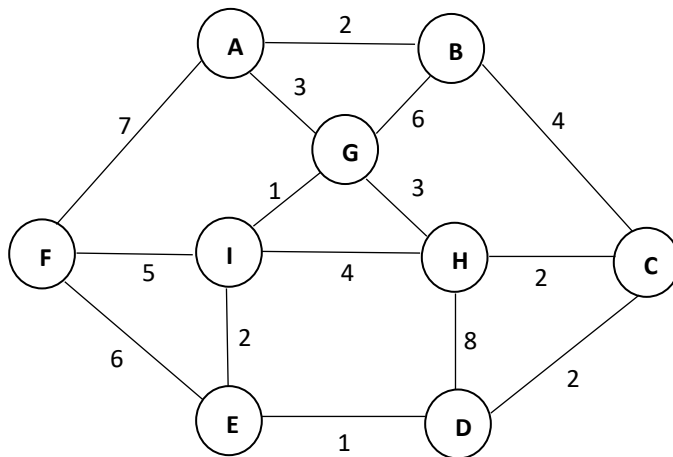**CMSC 451 Homework 4**

1. Execute Prim's minimum spanning tree algorithm by hand on the graph below showing how the data structures evolve specifically indicating when the distance from a fringe vertex to the tree is updated. Clearly indicate which edges become part of the minimum spanning tree and in which order. Start at vertex A.
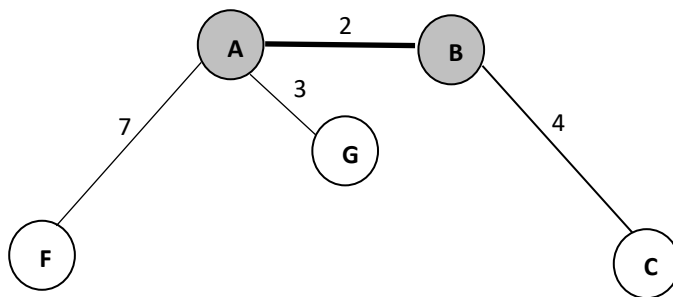

Solution:

A	F	0	minimum
B	U		
C	U		
D	U		
E	U		
F	U		
G	U		
H	U		
I	U		
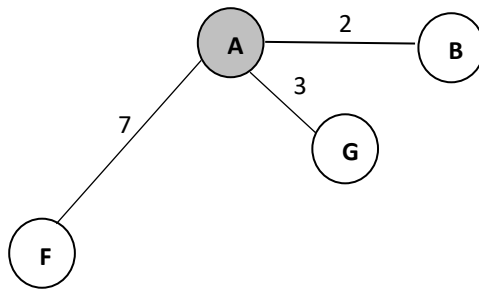
A	T		
B	F	2	minimum
C	U		
D	U		
E	U		
F	F	7	
G	F	3	
H	U		
I	U		

A	T		
B	T		
C	F	4	
D	U		
E	U		
F	F	7	
G	F	3	minimum
H	U		
I	U		
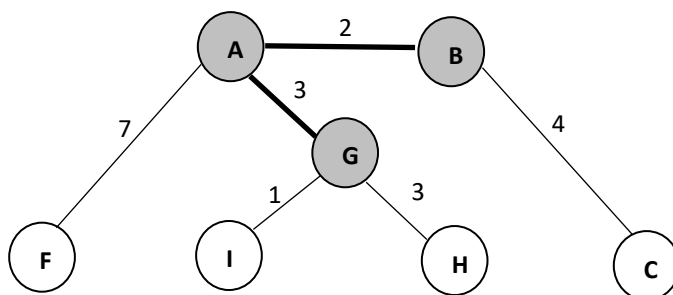
A	T		
B	T		
C	F	4	
D	U		
E	U		
F	F	7	
G	T		
H	F	3	
I	F	1	minimum
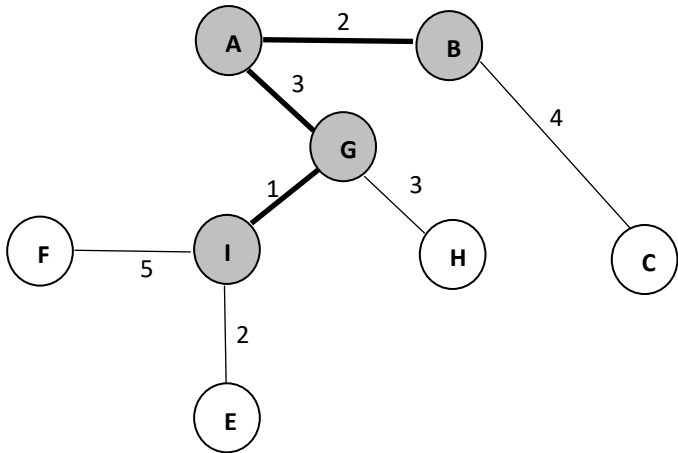
A	T	
B	T	
C	F	4
D	U	
E	F	2
F	F	5
G	T	
H	F	3
I	T	

minimum

updated
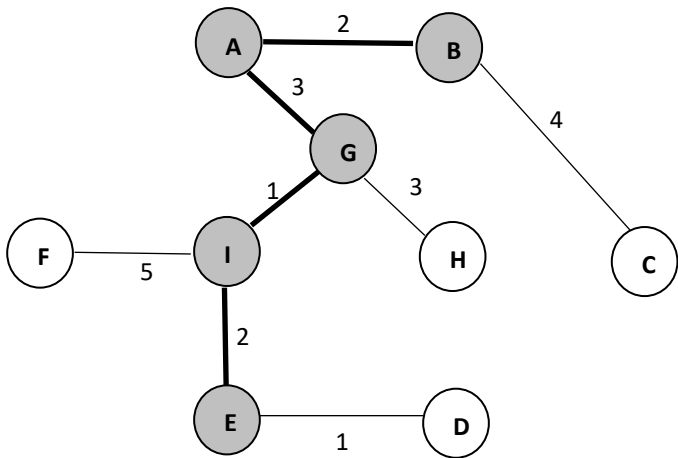
A	T	
B	T	
C	F	4
D	F	1
E	T	
F	F	5
G	T	
H	F	3
I	T	

minimum
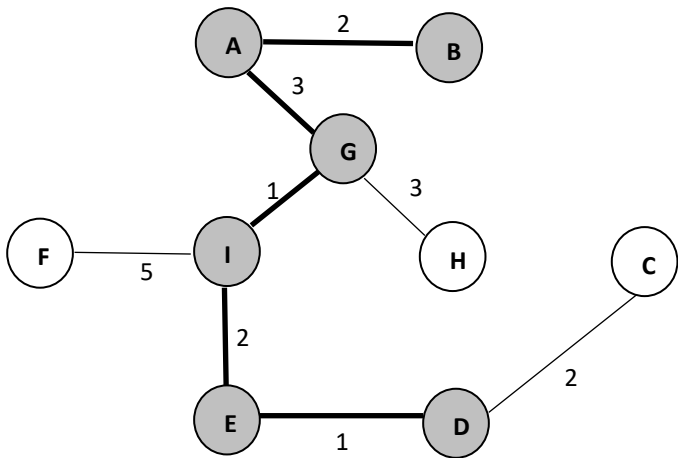
A	T	
B	T	
C	F	2
D	T	
E	T	
F	F	5
G	T	
H	F	3
I	T	

minimum
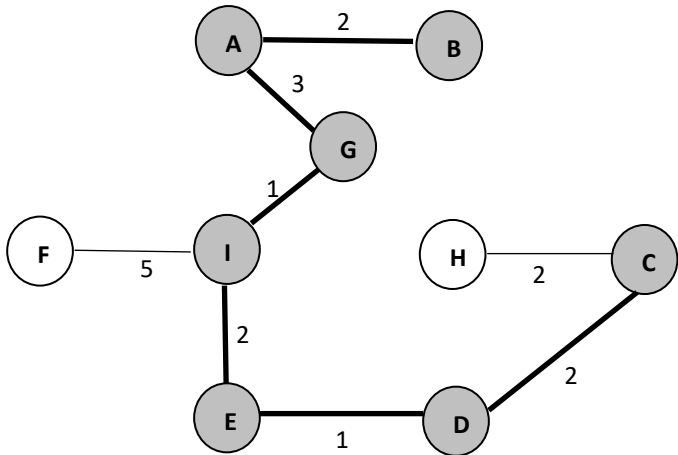
A	T	
B	T	
C	T	
D	T	
E	T	
F	F	5
G	T	
H	F	2
I	T	

updated, minimum
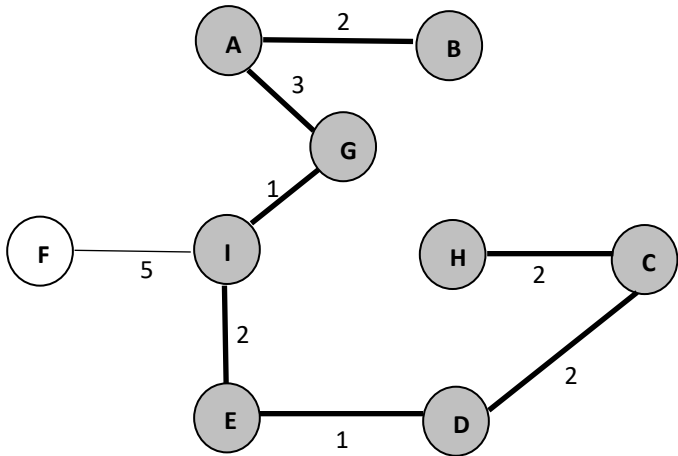
A	T	
B	T	
C	T	
D	T	
E	T	
F	F	5
G	T	
H	T	
I	T	

minimum
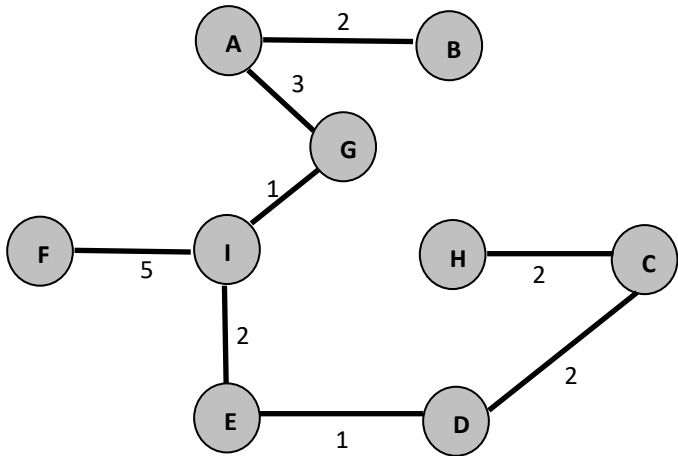
| A	T	
B	T	
C	T	
D	T	
E	T	
F	T	
G	T	
H	T	
I	T	

2. Execute Kruskal's algorithm on the weighted tree shown below. Assume that edges of equal weight will be in the priority queue in alphabetical order. Clearly show what happens each time an edge is removed from the priority queue and how the dynamic equivalence relation changes on each step and show the final minimum spanning tree that is generated
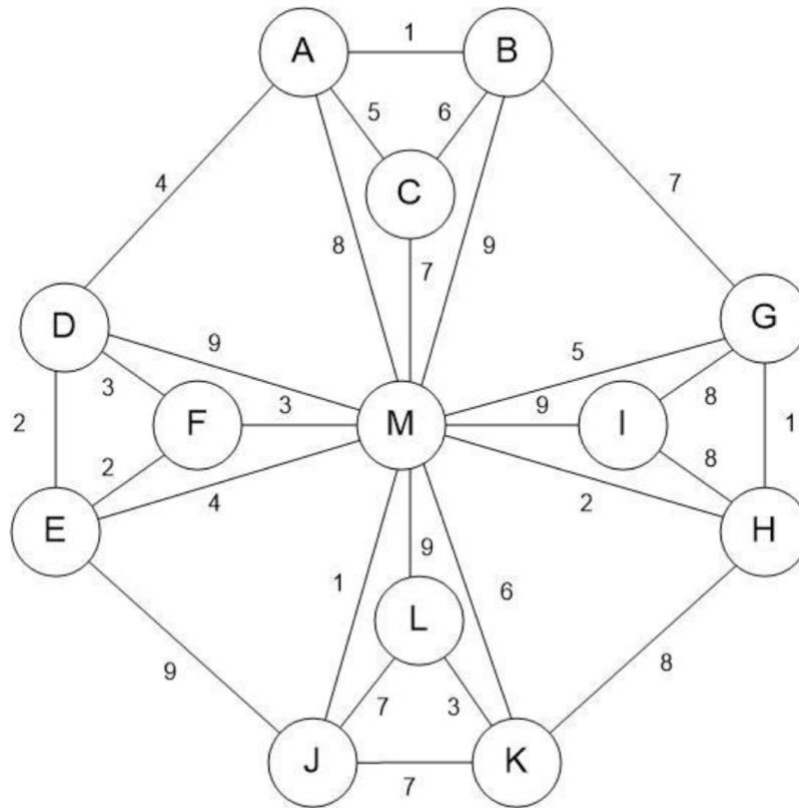
Solution:

Choose AB   1   (A, B)
Choose GH   1   (A, B) (G, H)
Choose JM   1   (A, B) (G, H, J, M)
Choose DE   2   (A, B) (D, E) (G, H, J, M)
Choose EF   2   (A, B) (D, E, F) (G, H, J, M)
Choose FM   3   (A, B) (D, E, F, G, H, J, M)
Choose KL   3   (A, B) (D, E, F, G, H, J, M) (K, L)
Skip   DF 3
Choose AD 4   (A, B, D, E, F, G, H, J, M) (K, L)
Skip   EM 4
Choose AC 5   (A, B, C, D, E, F, G, H, J, M) (K, L)
Skip   GM 5
Choose MK 6   (A, B, C, D, E, F, G, H, J, K, L, M)
Skip   BC 6
Skip   BG 7
Skip   CM 7
Skip   JK 7
Skip   JL 7
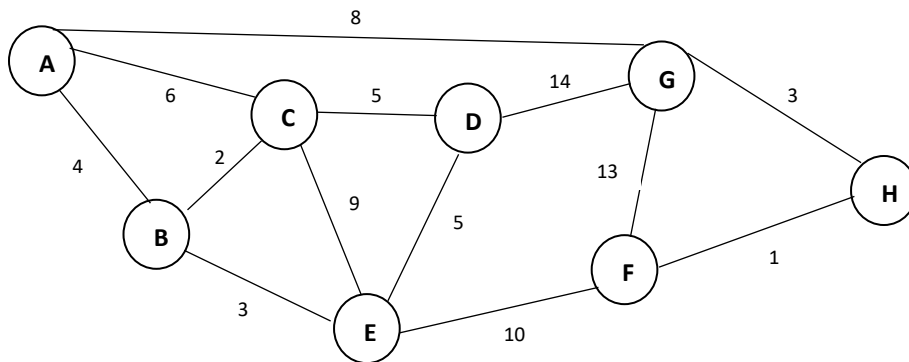Choose GI   8   (A, B, C, D, E, F, G, H, I, J, K, L, M)
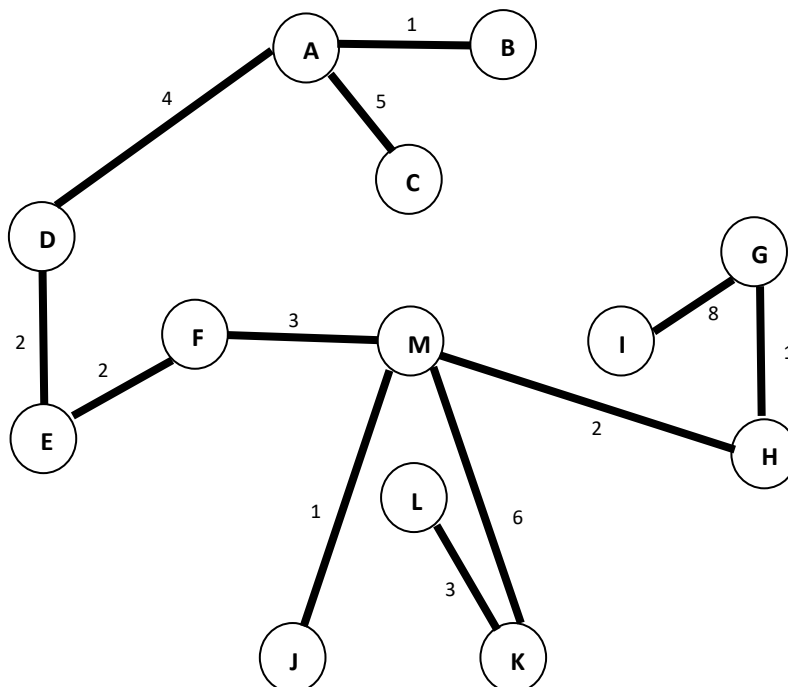
3a. Give an example of a weighted graph for which the minimum spanning tree is unique. Indicate what the minimum spanning tree is for that graph.
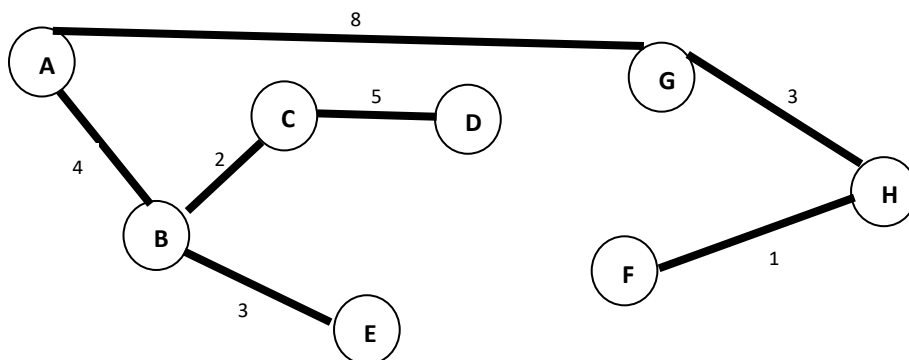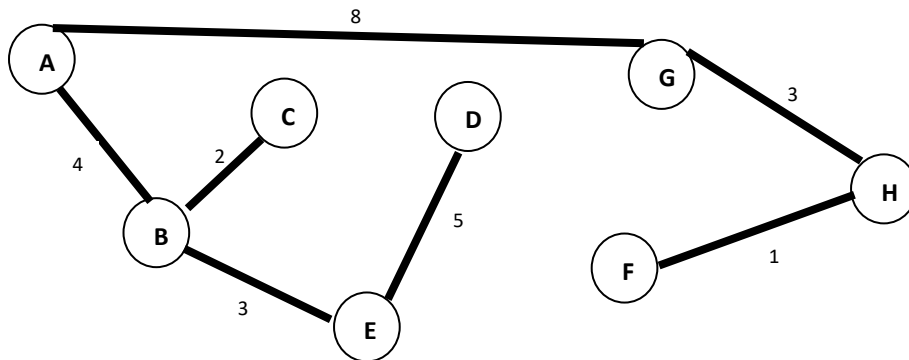
Solution:

Unique MST
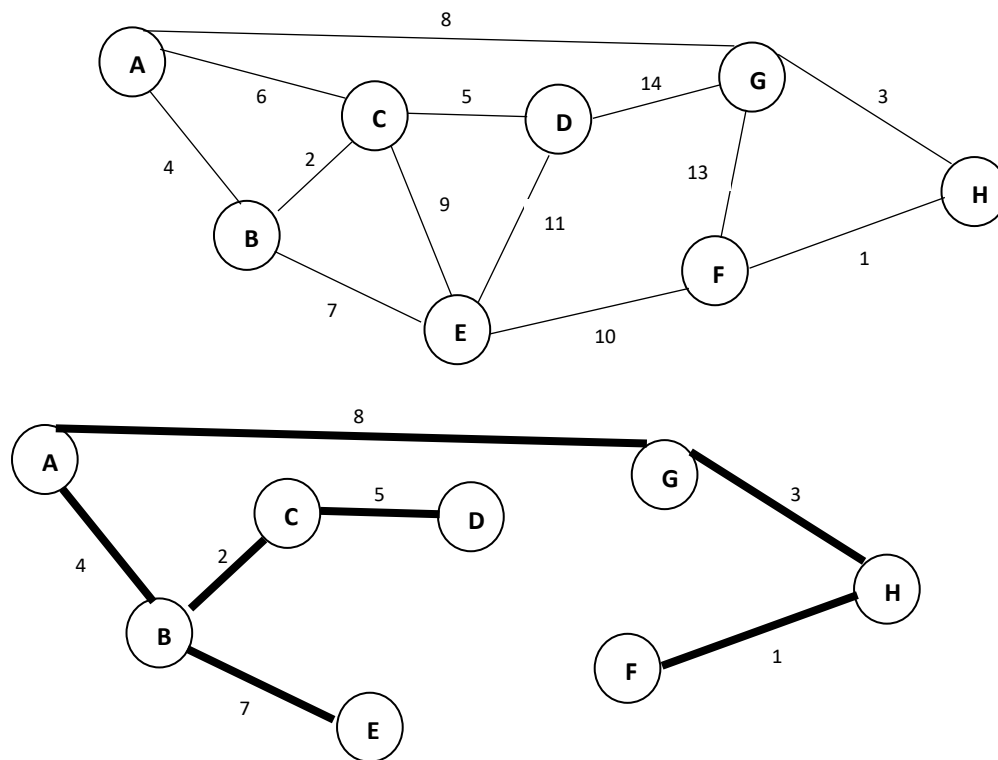
3b. Give another example of a weighted graph that has more than one minimum spanning tree.

3c. Show two different minimum spanning trees for that graph.


3d. What determines whether a graph has more than one minimum spanning tree?

If any of the edge weights are NOT distinct. A graph that has all distinct edge weights will always result in a unique minimum spanning tree.

4. Given the following adjacency lists (with edge weights in parentheses) for a directed graph:

A: B(5), C(3), D(1)
B: C(1), D(3)
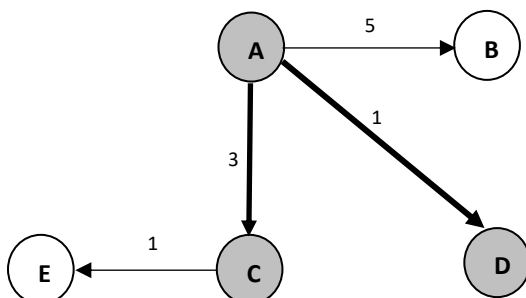C: B(3), D(7), E(1)
D: A(6), C(3)
E: F(5)
F: D(3), A(4)

Execute Dijkstra's shortest-path algorithm by hand on this graph, showing how the data structures evolve, with *A* as the starting vertex. Clearly indicate which edges become part of the shortest path and in which order.
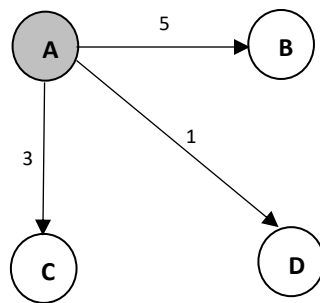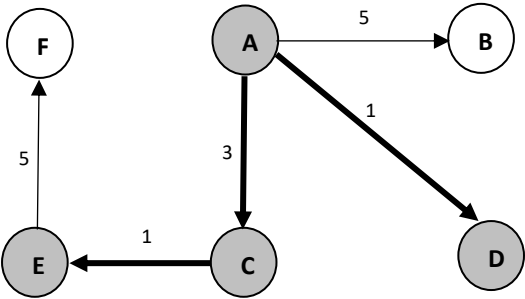

A	T	
B	F	5
C	F	3
D	F	1 minimum
E	U	
F	U	
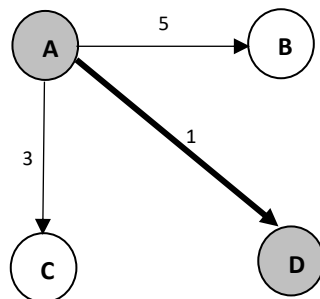
A	T	
B	F	5
C	F	3 minimum
D	T	
E	U	
F	U	

A	T	
B	F	5
C	T	
D	T	
E	F	4 minimum
F	U	

A	T	
B	F	5 minimum
C	T	
D	T	
E	T	
F	F	9
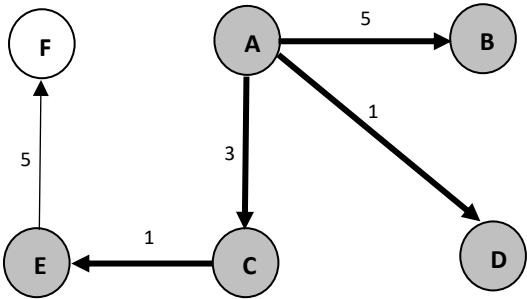
A	T	
B	T	
C	T	
D	T	
E	T	
F	F	9 minimum

A	T	
B	T	
C	T	
D	T	
E	T	
F	T	