A Review of Active Metamaterials Incorporating Gain Device / Medium

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Outline

- Background
- Inherent loss and dispersion of passive MTMs
- Active MTMs incorporating gain
- “Non-Foster” broadband active MTMs
- Stability analysis
- Summary

Background of Metamaterials (MTMs)

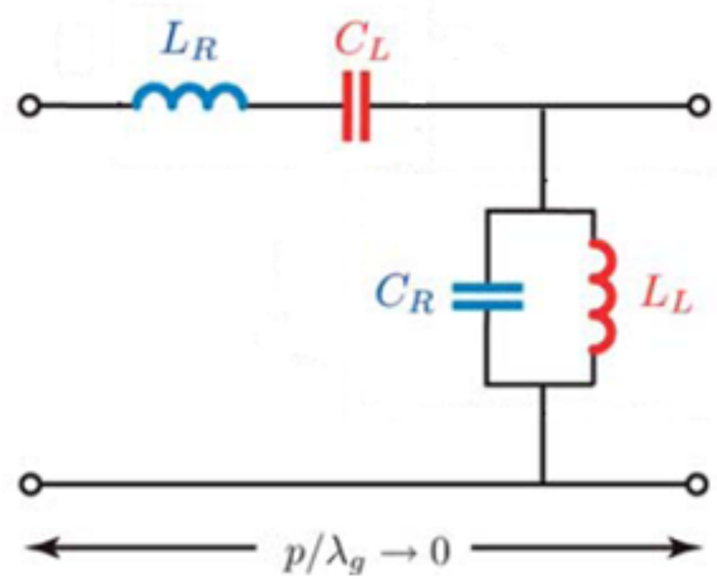
- **Artificial effective medium with unique properties**
- **Negative refractive index medium (NIM)**

3D

Volumetric MTM

SRRs + metal wires

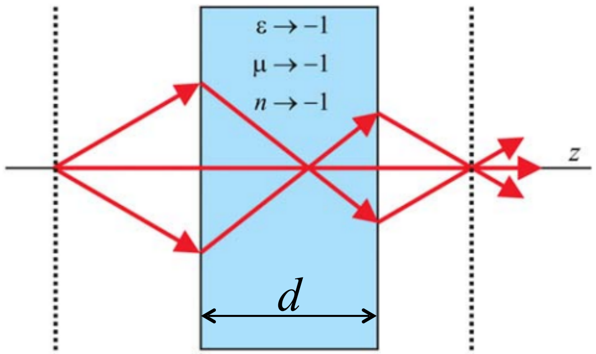
1D Transmission Line Version



- **Numerous potential applications: cloaking, perfect lens, antennas, circuits, etc.**

Bottlenecks of conventional passive MTMs

Example 1: (Pendry's Perfect lens)



beyond diffraction limit

J. B. Pendry, Phys. Rev. Lett., 85, 2000.

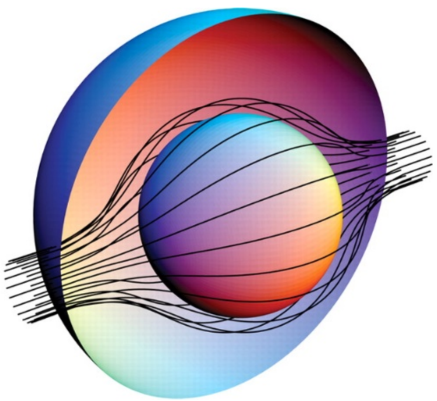
Suppose $\varepsilon = -1$, $\mu = -1 + \delta\mu$,

Resolution enhancement: $R = -\frac{1}{2\pi}\ln\left|\frac{\delta\mu}{2}\right|\frac{\lambda}{d}$

λ/d	R	$\delta\mu$
1.5	10	$< \sim 6 \times 10^{-19}$

D. R. Smith, et. al., *Appl. Phys. Lett.*, 82, 2003

Example 2: (Invisibility cloaking)



J. B. Pendry, et. al., Science, 312, 2006

Suppose d is the thickness of cloak, h is the size of the object

$$\frac{\mathrm{Im}\, n}{\mathrm{Re}\, n} \ll \frac{1}{4\pi}\frac{\lambda}{h+d}.$$

- Delay-loss and delay-bandwidth limit

H. Hashemi, et. al., *Phys. Rev. Lett.*, 104, 2010

Inherent loss and narrow band ☹

Loss and dispersion bound

For a linear time-invariant causal passive medium:

a) Re(n) ≥ 1, no bound on loss and dispersion

b) Re(n) < 1

- If Re(n) is constant in [ω_1, ω_2], the loss within the bandwidth is lower bounded by

$$n'' > |n' - 1| \frac{\omega_2^2 - \omega_1^2}{2\omega_1\omega_2}$$

- If a medium is lossless in a finite bandwidth [ω_1, ω_2], the minimum variation of Re(n) in this bandwidth is

$$\min \delta n' > |n'(\omega_1) - 1| (2\Delta - \Delta^2), \text{ for } n'(\omega_1) < 1,$$

$$\Delta = (\omega_2 - \omega_1)/\omega_2$$

B. Nistad, et. al., Phys. Rev. E, 78, 2008; *M. Gustafsson, et. al., New J. Phys., 12, 2010.*

Loss in practice

- **Achievability**: a passive NIM with negligible loss is theoretically achievable within a finite bandwidth, if *either a large enough loss or exponentially steep variation of the loss* is below observation frequency.
- **From practical point of view**
  - Loss is inevitable due to metal, substrate, and/or radiation loss, especially at optical frequency.
  - The best measured passive NIM at optical frequency has

$$FOM = \frac{|\mathrm{Re}(n)|}{\mathrm{Im}(n)} = 3$$

  - For a planar self-resonant loop the minimum permeability loss tangent is

$$\min(|\tan\delta|) \approx \frac{2}{\pi m}\frac{R}{Z_0}\frac{\lambda_0}{l_{\mathrm{cell}}}.$$

C. García-Meca, Phys. Rev. Lett., 106, 2011; S. A. Cummer, et. al., IEEE Trans. Anten. Propag., 56, 2008.

Active (Gain) Metamaterial Attractive

- **Compensate Loss / Provide Gain**
  - **Introduce external energy source**
- **Potential to Trade Gain with Bandwidth**
- **A New Design Degree of Freedom**

  -σ in addition to (-ε, -μ)
- **Physics is not completely clear: sign of index of refraction, nonlinearity, stability**

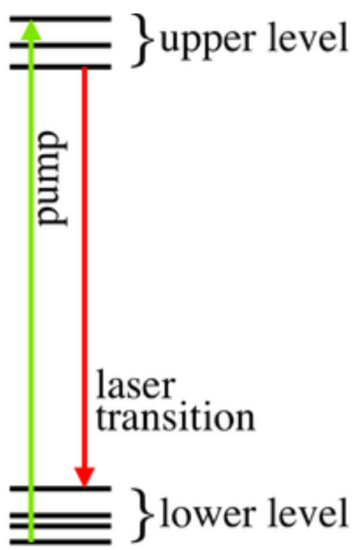
❑ **Understand fundamental physics: debate on whether –n and lossless / gain can exist**

❑ **Demonstrate active microwave / THz metamaterials**

❑ **Explore / develop interesting applications**

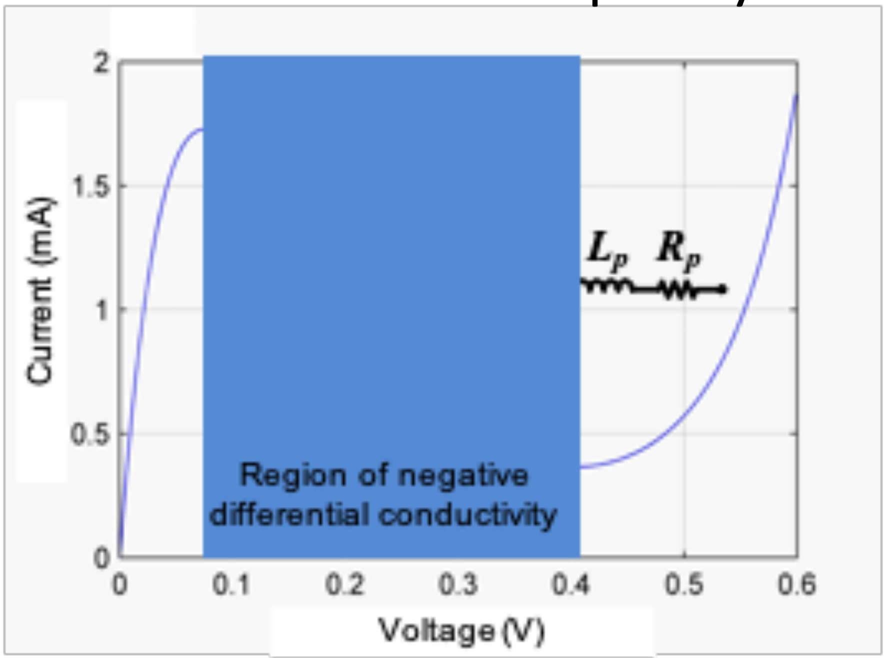
Practical implementation

At optical frequency:



- **Ion-doped crystals**
- **Semiconductor: quantum wells/dots**
- **Organic dye molecule solution, e.g. Rhodamine 800 (Rh800)**

At microwave frequency:



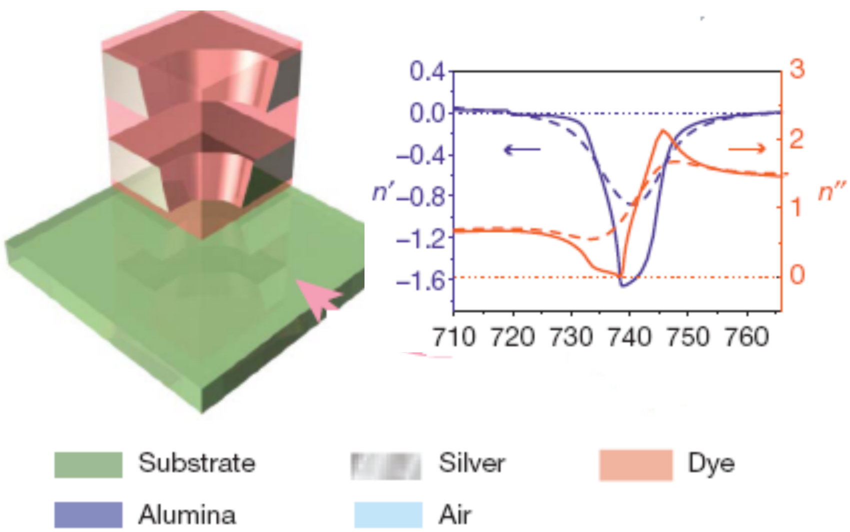
- **Transistor based -R circuits**

Y. Yuan, et. al., Opt. Express, 17, 2009.

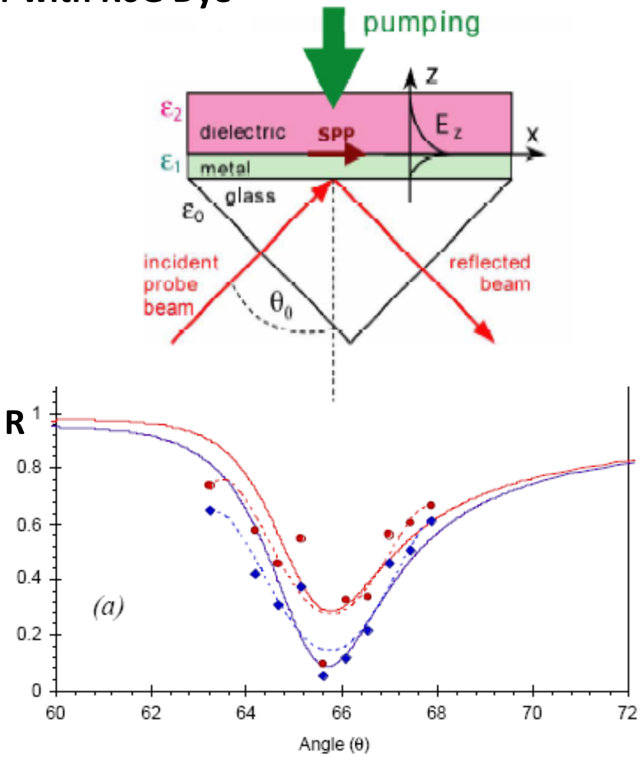
- **Tunnel diodes (TDs) selected**
- **One device, easier biasing**
- **Parasitic elements important**
- **Nonlinear design techniques necessary**

Several Examples – Optical Frequency

Fishnet with Rh800 Dye[1]



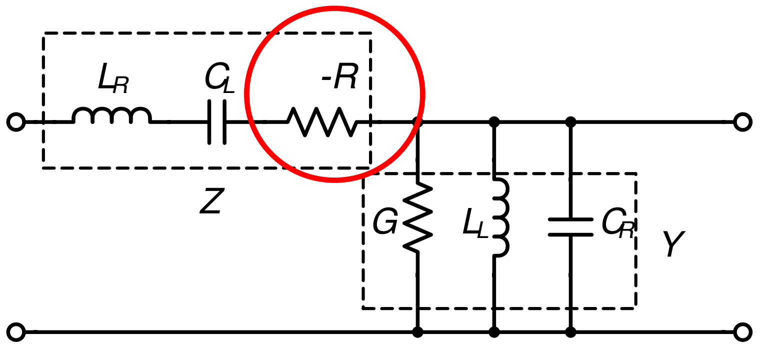
Compensation of Loss in Surface Plasmon Polariton[2]
Polymer with R6G Dye

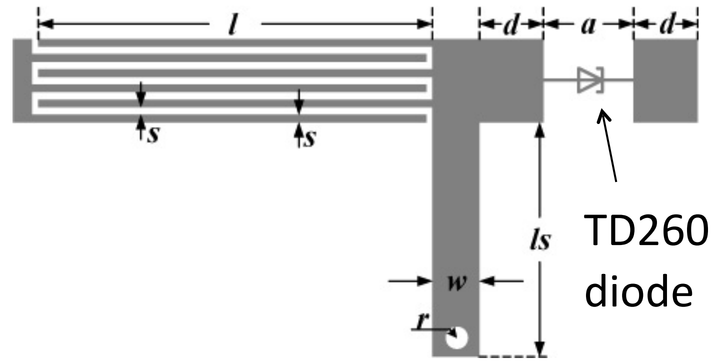


Optical pumping reduces loss

1. S. Xiao, et al., Nature 466, 735-738 (2010).
2. M. Noginov, et al., OE 16, 1385-1393 (2008).

Active CLRH-Transmission Line Design





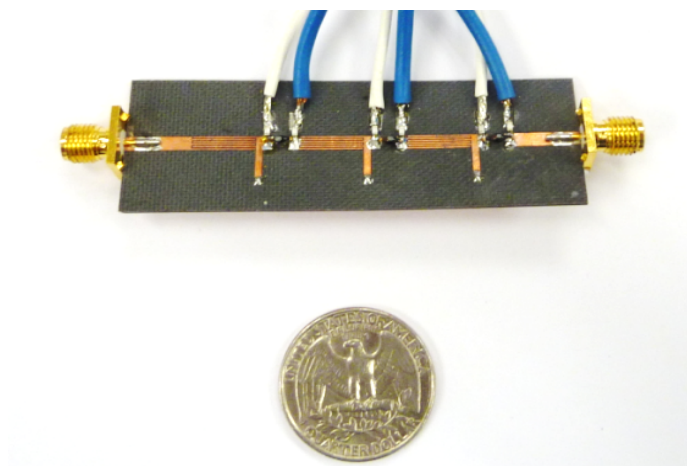
$$\varepsilon_{eff} = Y/(j\omega)$$

$$\mu_{eff} = Z/(j\omega)$$

$$Z = R + j\left(\omega L_R - \frac{1}{\omega C_L}\right)$$

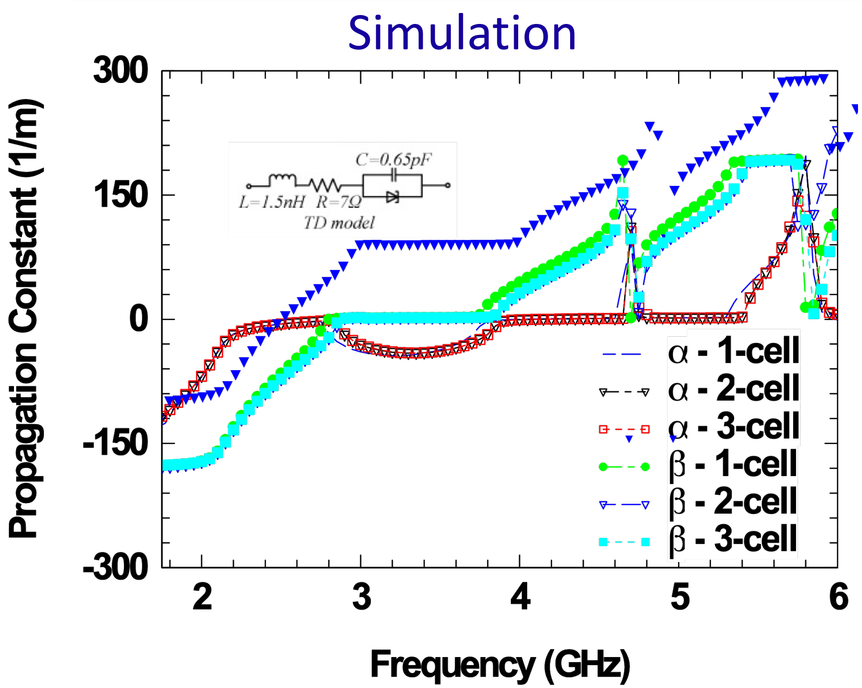
$$Y = G + j\left(\omega C_R - \frac{1}{\omega L_L}\right)$$

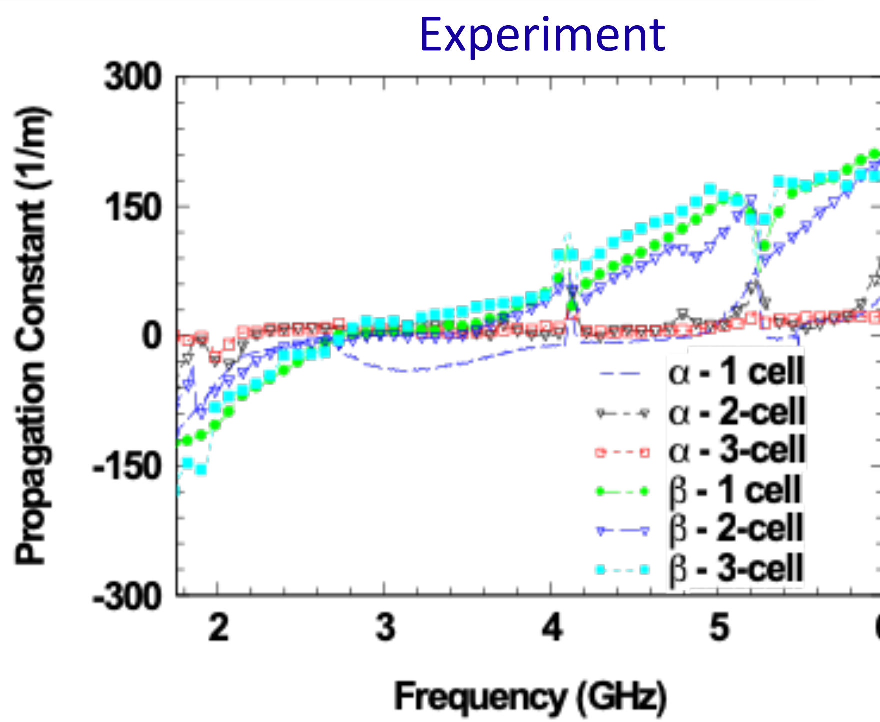
$$\gamma = \alpha + j\beta = \pm\sqrt{ZY}$$

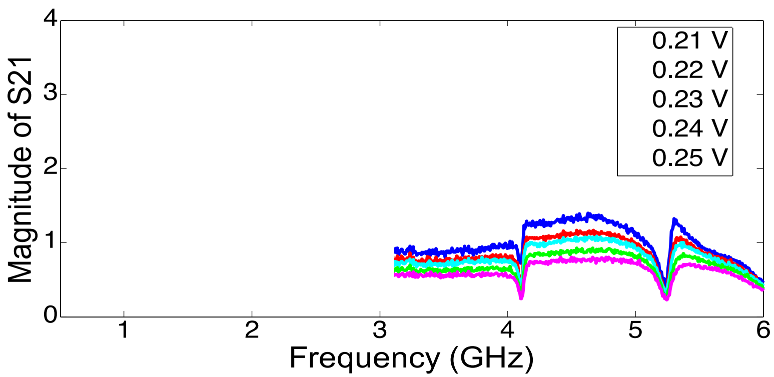


T. Jiang, et.al., Phys. Rev. Lett., 107, 20, 2011.

Simulation and measurement



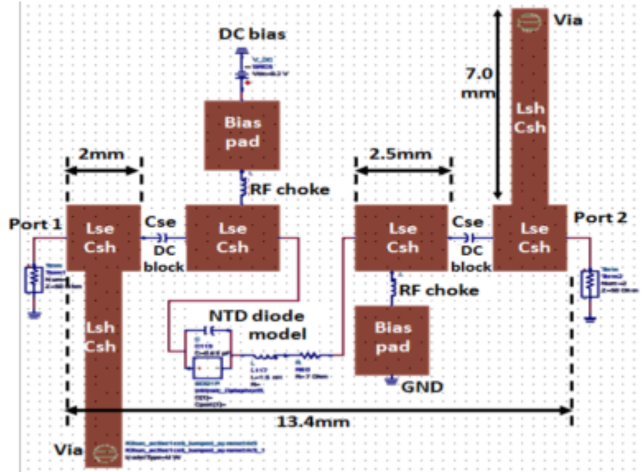


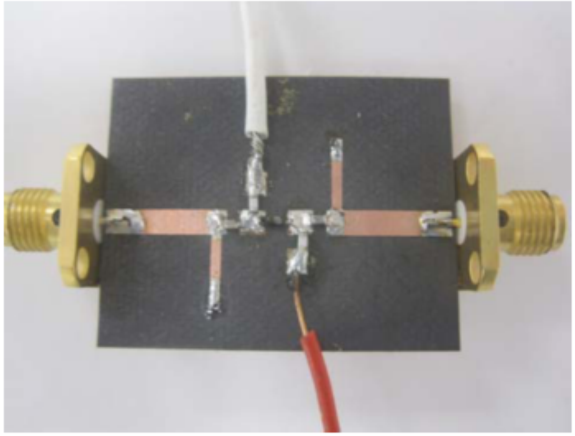


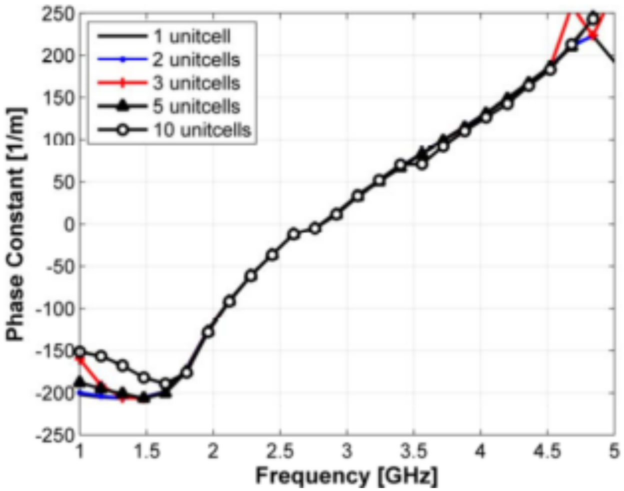
- **Firstly CW gain NIM**
- **Negative α and β at the same time**
- **1, 2, 3 unit cells are measured and remain stable**
- **Gain changed by sweep the DC bias voltage**

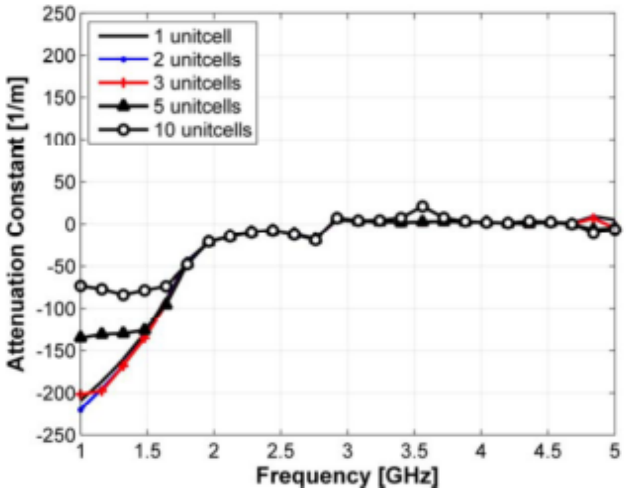
T. Jiang, et.al., Phys. Rev. Lett., 107, 20, 2011.

A balanced active CLRH design





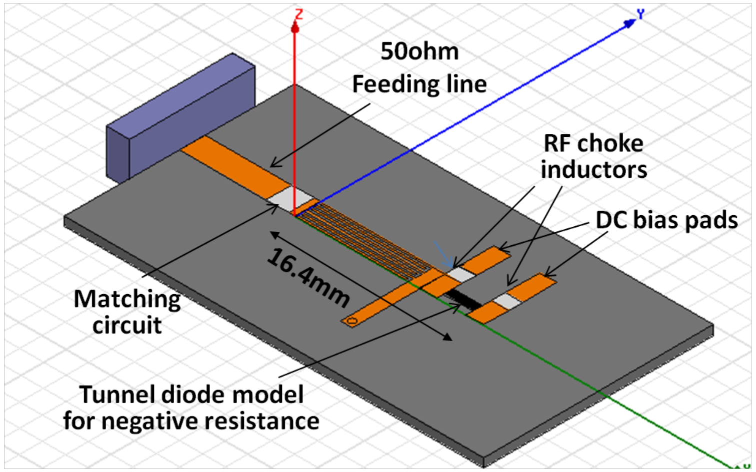


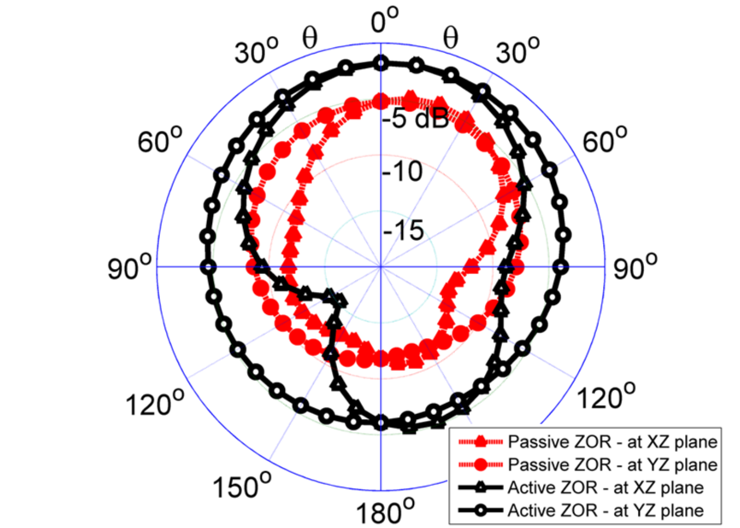


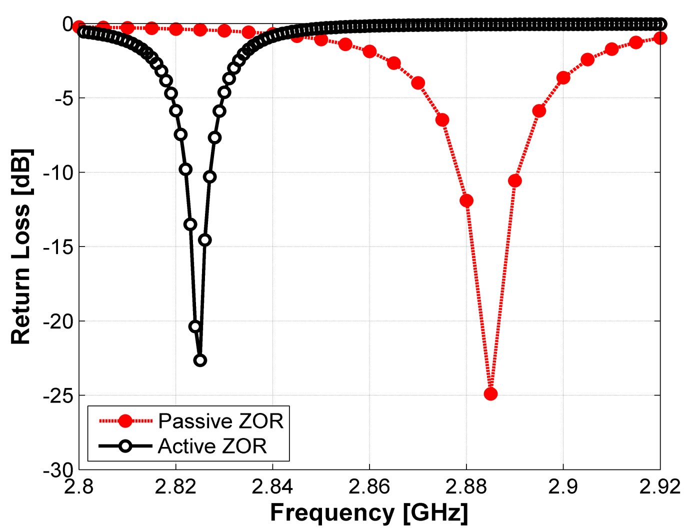
- **No bandgap between the LH and RH propagation**

K. Chang, et. al., IEEE Antennas Wirel. Propag. Lett., 11, 2012.

Active zeroth order antenna





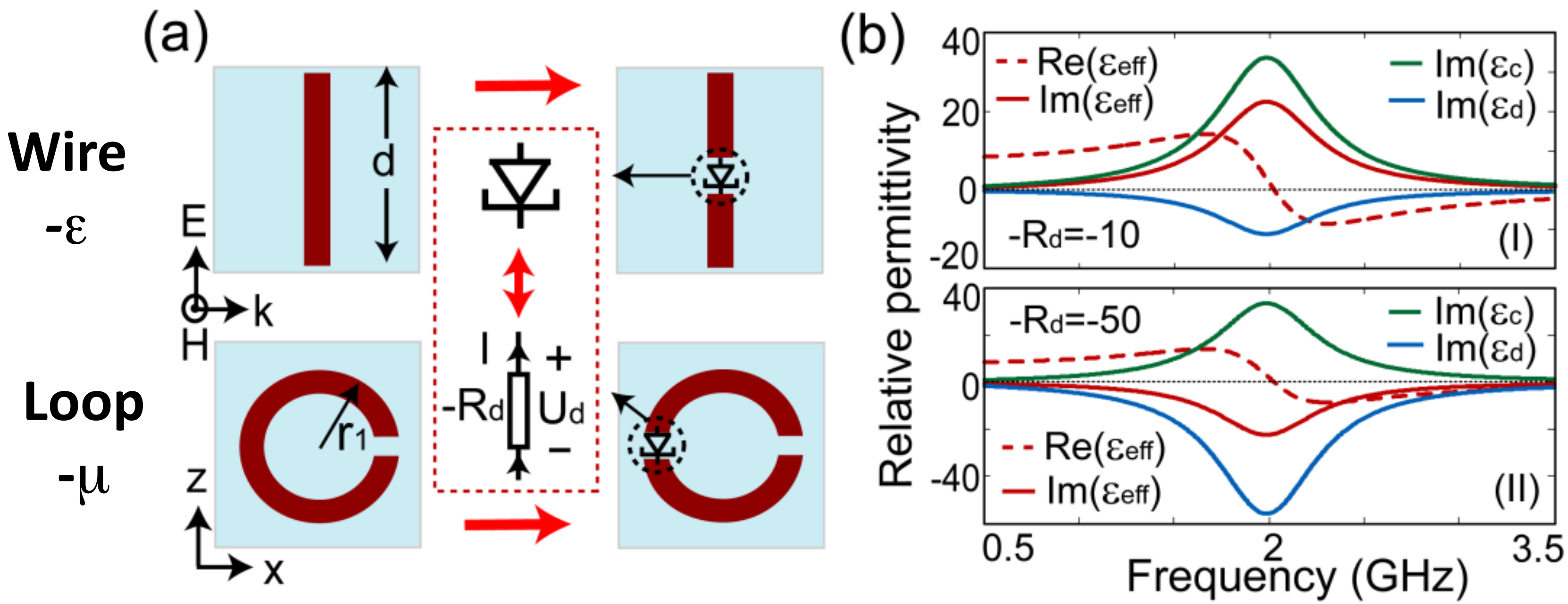


Active ZOR antenna simulation vs. Passive ZOR antenna

- **Resonant frequency: slightly shifted**
- **Effective radiation efficiency: 3dB increased from 26% to 51% (under ideal bias condition)**
  ➔ Total radiated power increased

K. Chang, et. al., IEEE Antennas Wirel. Propag. Lett., 11, 2012.

Volumetric gain-assisted MTM



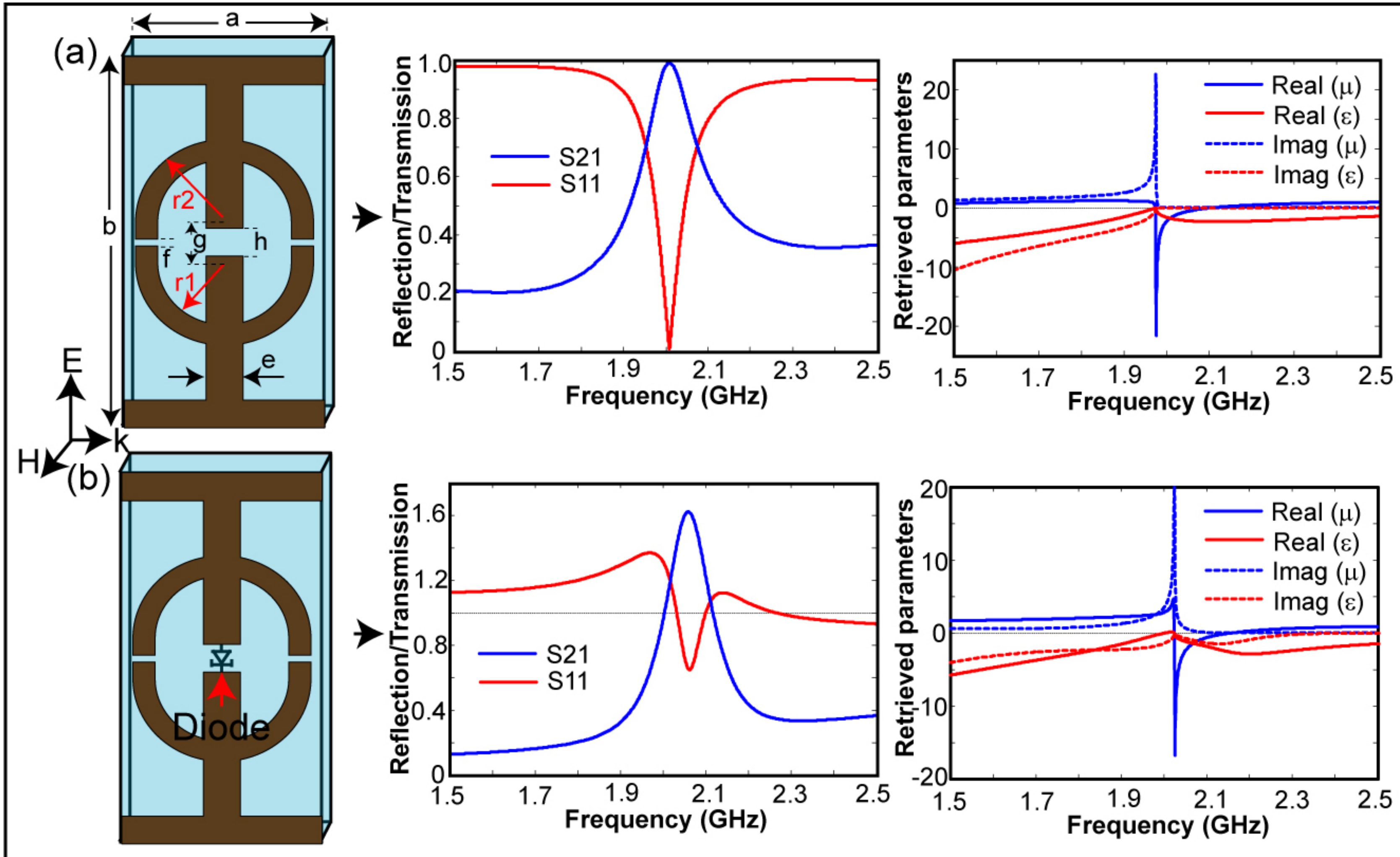
$$\varepsilon_c = \frac{-[\omega^2 - 1/(LC)]/(dL\varepsilon_0)}{[\omega^2 - 1/(LC)]^2 + [\omega(R-R_d)/L]^2} + i\frac{\omega R/(dL^2\varepsilon_0)}{[\omega^2 - 1/(LC)]^2 + [\omega(R-R_d)/L]^2}$$

$$\varepsilon_d = i\frac{-\omega R_d/(dL^2\varepsilon_0)}{[\omega^2 - 1/(LC)]^2 + [\omega(R-R_d)/L]^2}$$

- **Value of –R controls the loss compensation level**
- **Control ε and μ independently**

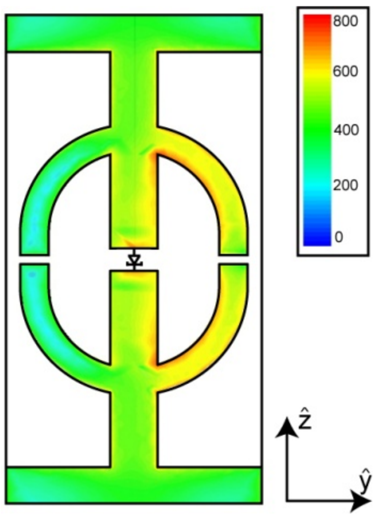
D. Ye, et. al., Nat. Commun., 5, Dec. 2014.

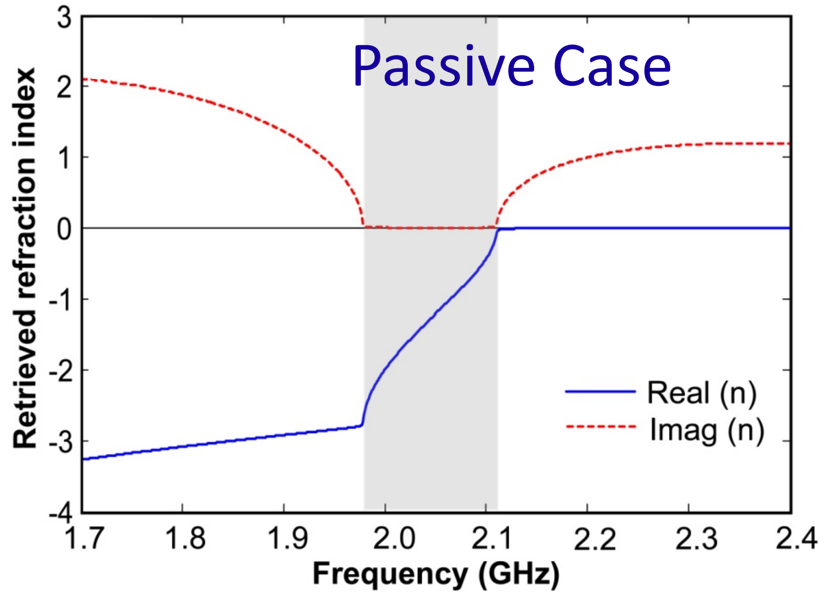
Unit cell design and simulation

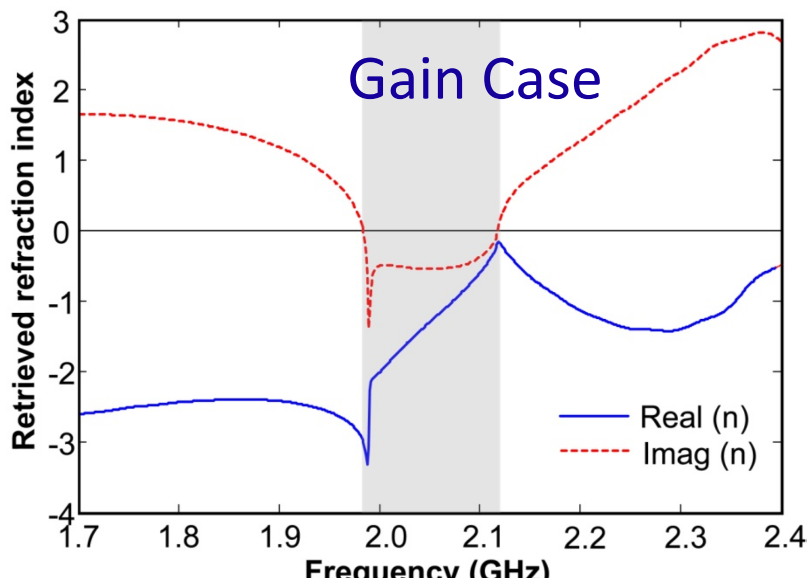


D. Ye, et. al., Nat. Commun., 5, Dec. 2014.

Unit cell design and simulation



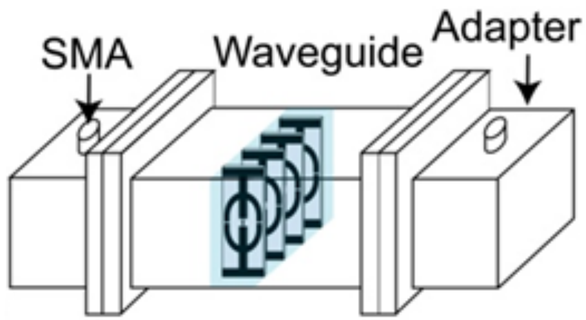


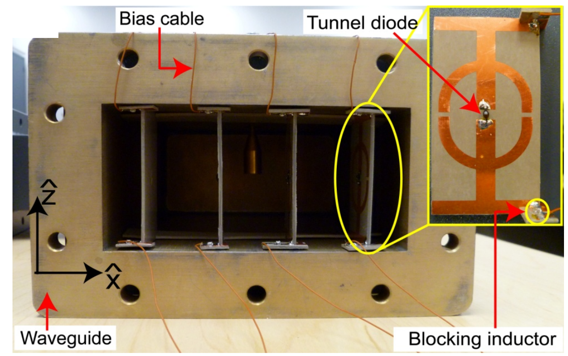


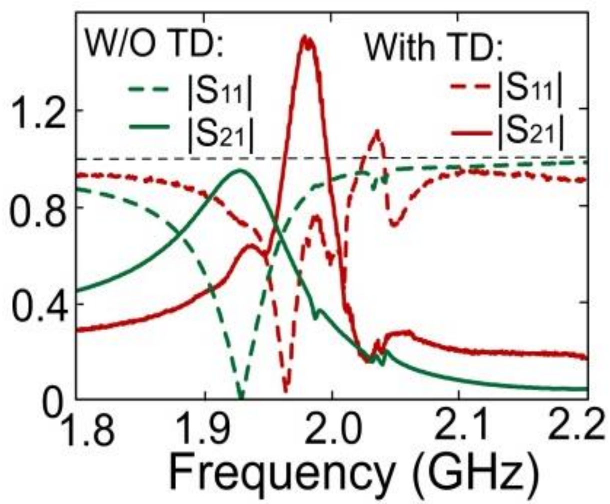
- **Negative Index with gain can be achieved**

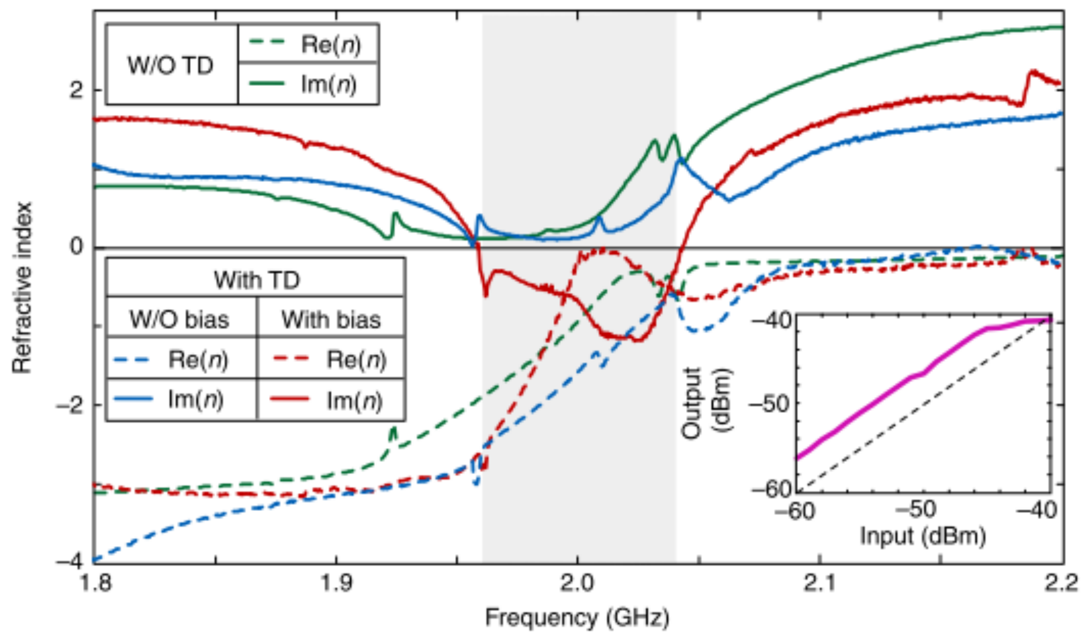
D. Ye, et. al., Nat. Commun., 5, Dec. 2014.

Experimental Demonstration







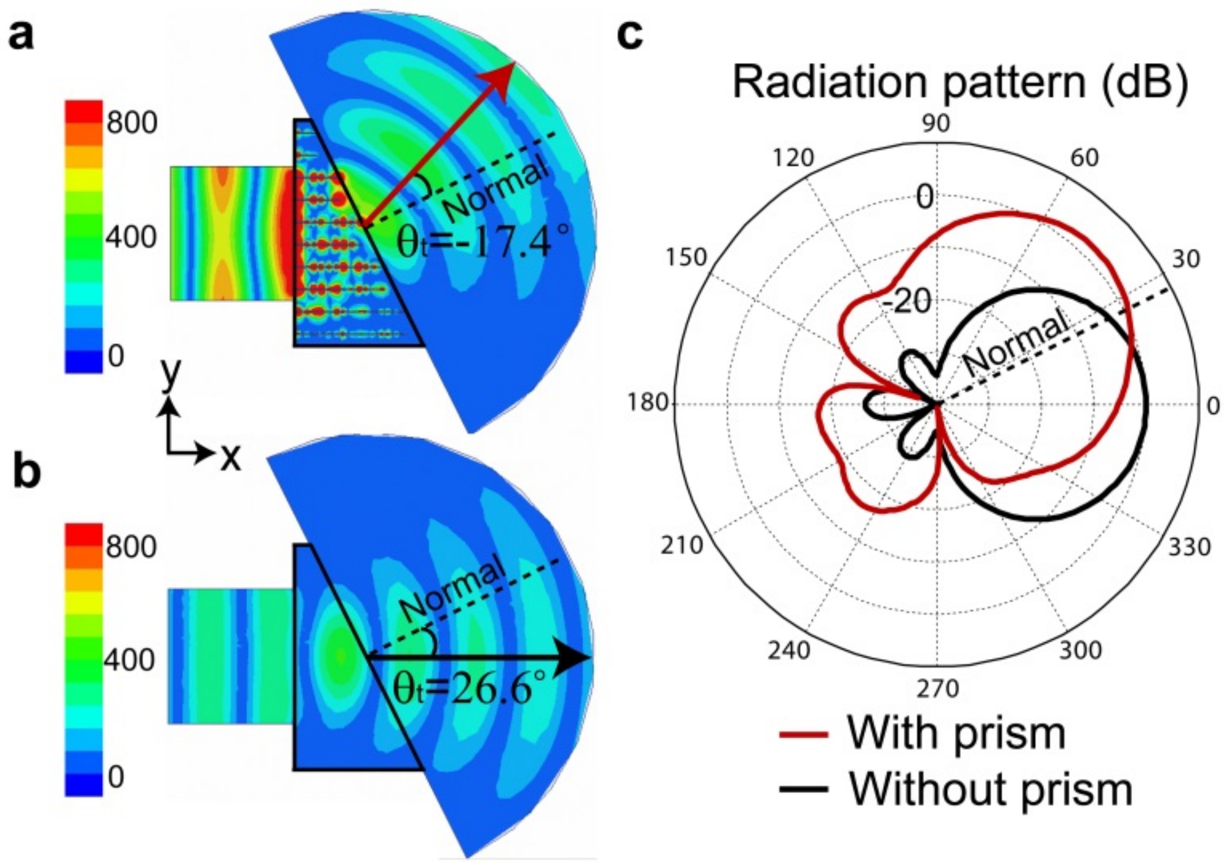


- **Measurements confirm -n’, -n” (gain)**

D. Ye, et. al., *Nat. Commun.*, 5, Dec. 2014.

A Prism Example

1.95 GHz

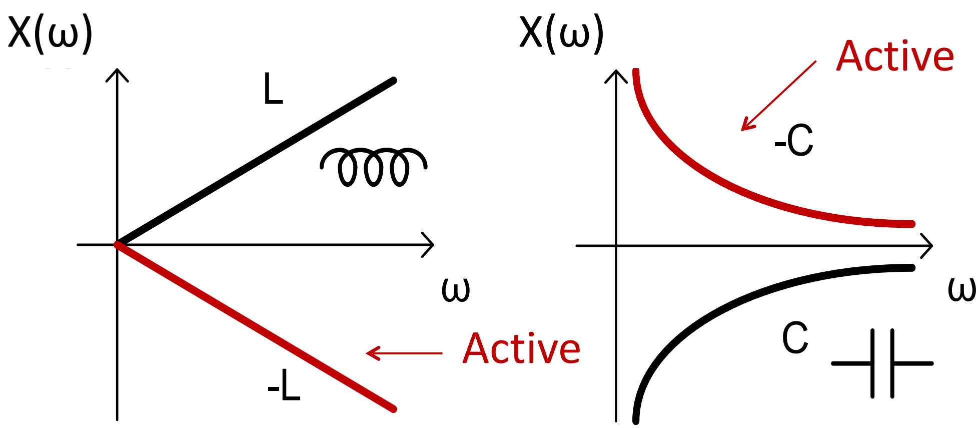


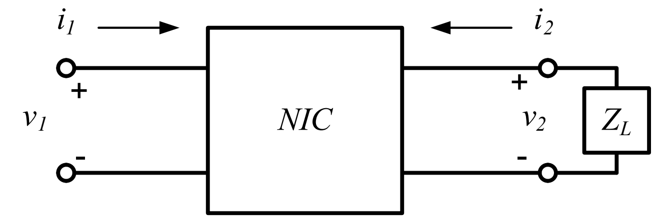
- **Prism based on the same unit cell designed**
- **θ_{in} = 26.6°, θ_t = -17.4°, Gain = 2.37 dB**

D. Ye, et. al., *Nat. Commun.,* 5, Dec. 2014.

Non-Foster elements

Foster's theorem: a passive lossless system

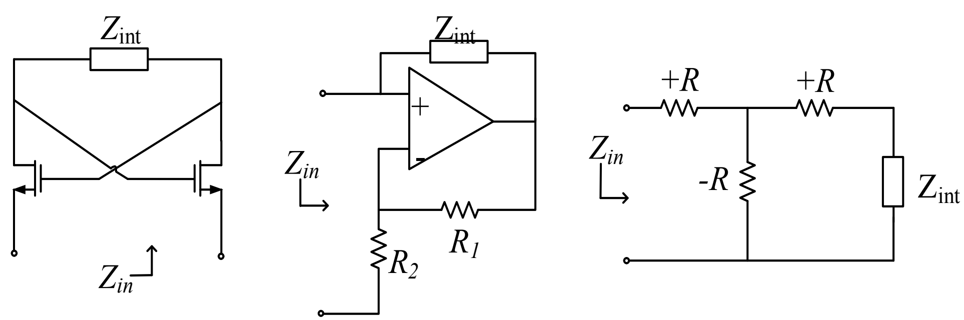




$$Z_{in} = h_{11} - \frac{h_{12}h_{21}Z_L}{1 + h_{22}Z_L}$$

$$\mathrm{h}_{11} = \mathrm{h}_{22} = 0,\ \mathrm{h}_{12}\mathrm{h}_{21} = \mathrm{k},\ \mathrm{k} > 0$$

$$Z_{in} = -kZ_L$$

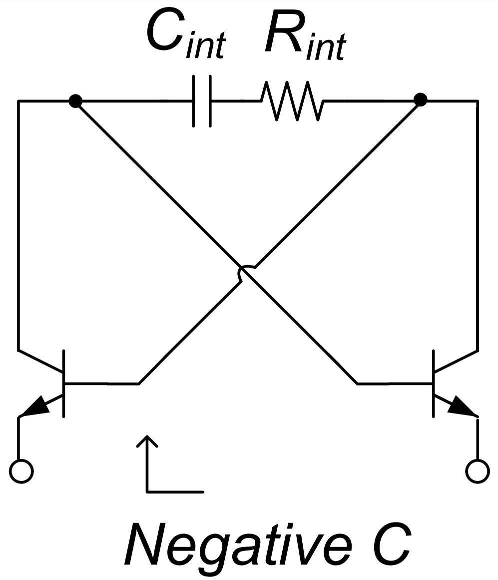


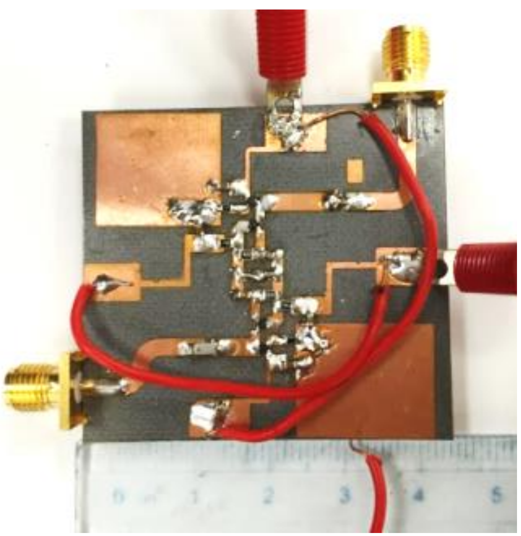
(a) (b) (c)

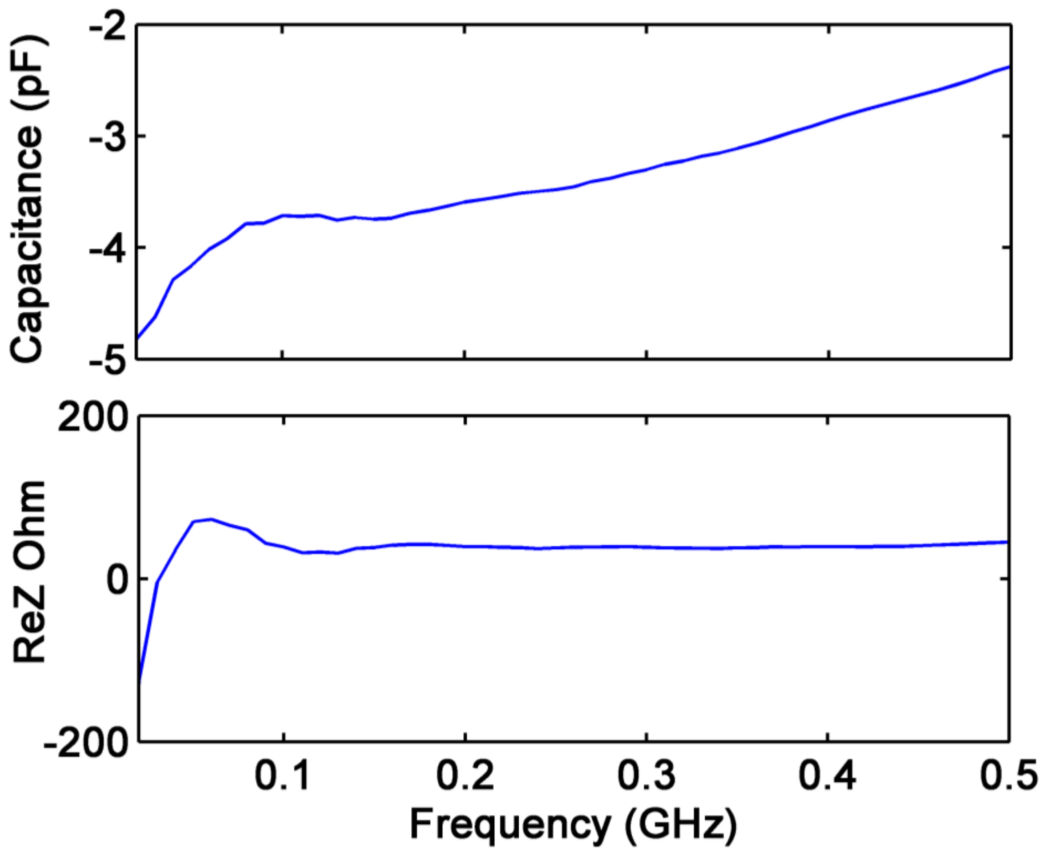
Difficulties:

- **Limited bandwidth – Parasitics**
- **Unwanted resistance part – Lower Q**
- **Potentially unstable – due to many positive feedbacks (SCS or OCS)**

A Linvill type negative capacitor





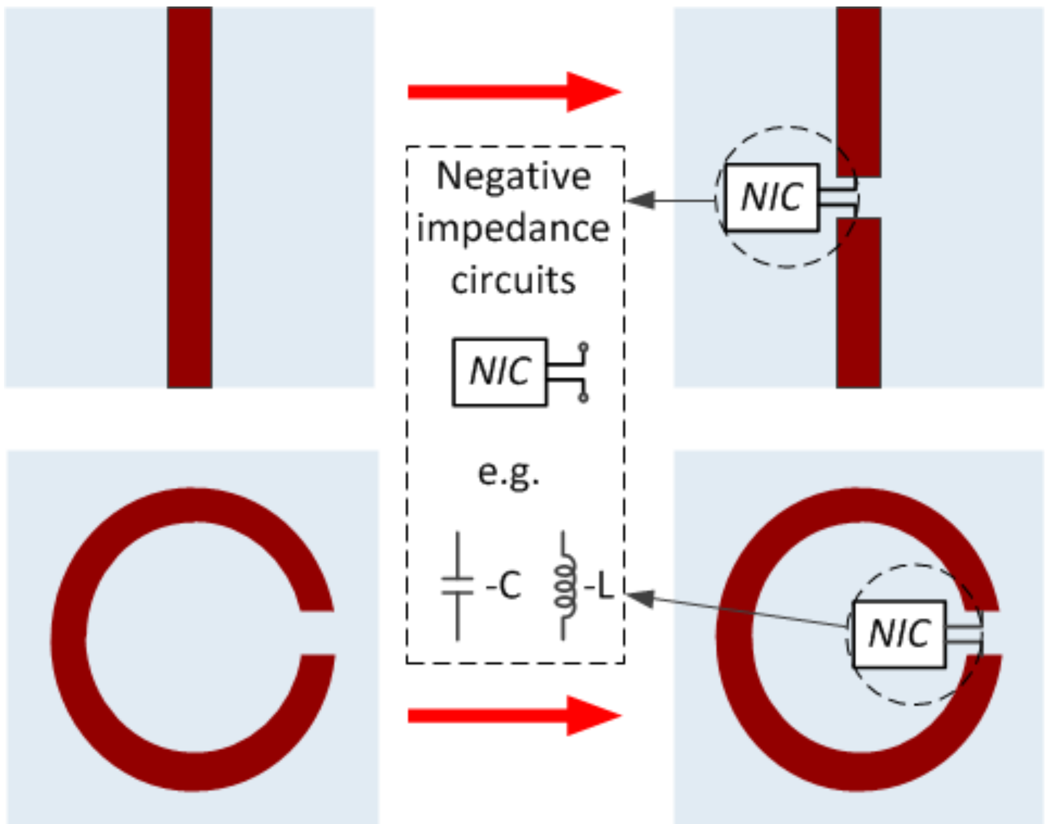


Qi Tang, et. al., Int. Microw. Sympos., Phoenix, 2015

Broadband active MTMs

Wire
-ε

Loop
-μ



$$Z = -Z_w - \frac{Nl_{\text{eff}}^2}{j\omega\epsilon_0(1+|\epsilon|)}.$$

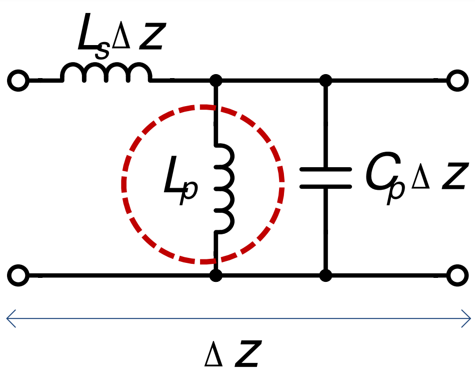
$$Z = -Z_l + j\omega\frac{\mu_0 NS^2}{1+|\mu_r|}.$$

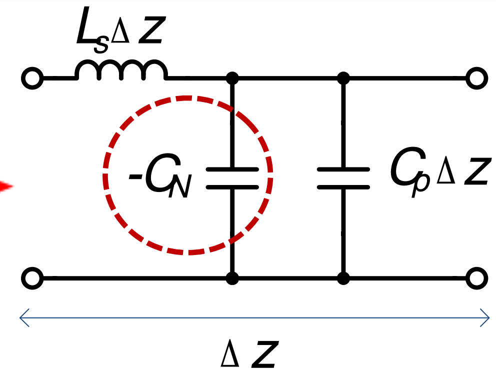
S. Tretyakov, *Microw. Opt. Technol. Lett.*, 31, 3, 2001.

- Stability and nonlinearity

•*G. Fu, Appl. Phys. Lett., 106, 2015.*
•*Y. Fan, et. al., J. Appl. Phys., 113, 2013.*
•*S. Saadat, et. al., IEEE Trans. Antennas Propag., 61, 2013.*
•*K. Z. Rajab, et. al., J. Opt., 14, 2012.*
•*T. P. Weldon, Proc. of IEEE Southeastcon, 2012.*

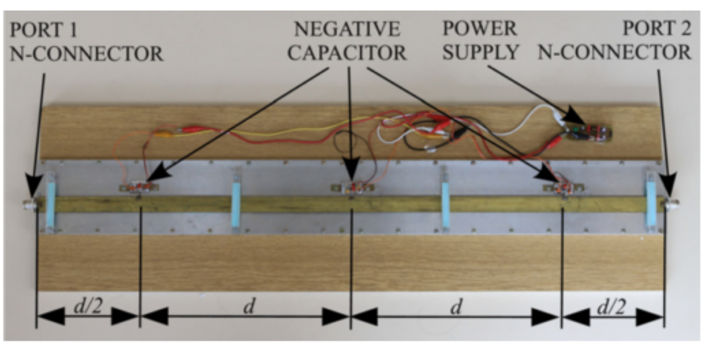
Broadband ε-near-zero MTM



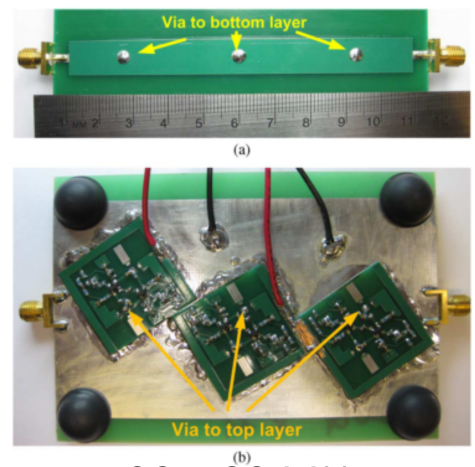


$$\varepsilon_r(\omega) = \frac{1}{\varepsilon_0}\left(C_p - \frac{1}{\omega^2 L_p \Delta z}\right)$$

$$\varepsilon_r(\omega) = \frac{1}{\varepsilon_0}\left(C_p - \frac{C_N}{\Delta z}\right)$$



ε ~0.3, 2 – 40 MHz



20 – 90 MHz

S. Hrabar, *et. al., Appl. Phys. Lett.*, 99, 2011.

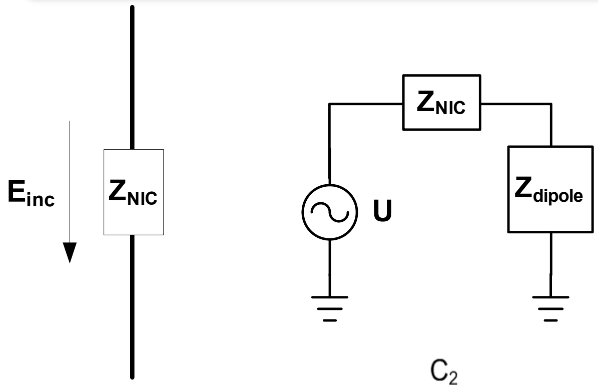
J. Long, *et. al., IEEE MTT trans.*, 62, 2014.

Practical stability test

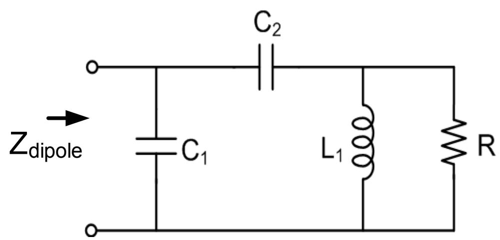
- **Time domain (Transient simulation):**
  - Inefficient for parametric study in a design phase
  - Not predict degrees of stability, where instability comes from
- **Linear frequency domain:**
  - Transfer the distributed MTM structures into equivalent circuit model
  - Use return difference (NDF) method for a general bilateral multiple feedback network
- **Could be extended to include non-linearity**

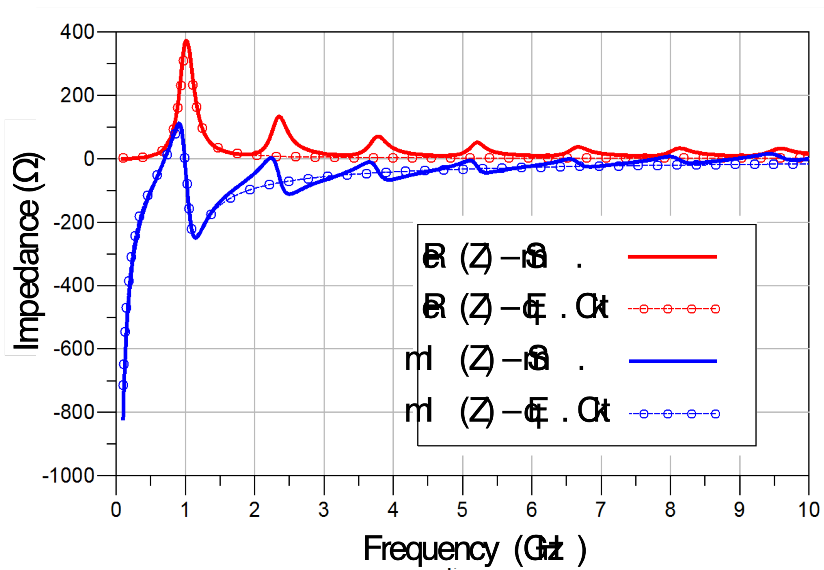
Qi Tang, et. al., Int. Microw. Sympos., Phoenix, 2015

Example of NDF analysis: -C loaded small dipole

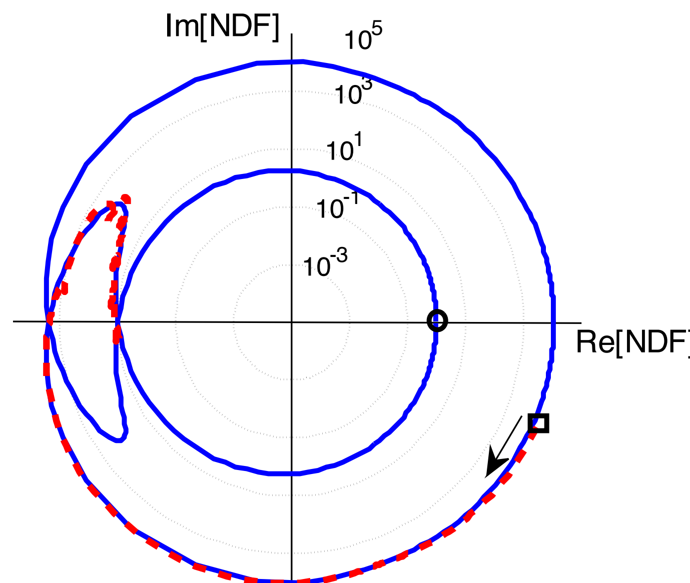


1. equivalent circuit model
2. write down the admittance matrix
3. Normalized by the de-activated matrix
4. calculate the # of RHP zeros by Nyquist plot





A Nyquist plot of NDF



Qi Tang, et. al., Int. Microw. Sympos., Phoenix, 2015

Summary

- The existing theoretical limitations on passive MTMs
- Review of active MTMs: split rings, fishnet structures, or 1D CLRH TL, incorporating gain medium, TDs, or non-foster elements
- Experimental proofs of loss-free, amplifying NIM and ultra-broadband ε-near-zero MTM
- Challenges include instability, nonlinearity, noise, design methodology, etc.
- Potential applications: broadband cloaking, super lens, various antenna applications, etc.

Acknowledgement

Tao Jiang, Kihun Zhang, Dexin Ye, and Lixin Ran

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Thank you for your attention!

Physical constraints of passive media

- **For a linear time-invariant causal passive medium:**
  1. **Symmetry** (real function in time):

     $$\varepsilon(-\omega) = \varepsilon^*(\omega)$$

  2. **Kramers–Kronig relations** (causality):

     $$\varepsilon_r'(\omega) = 1 - \frac{2}{\pi}\int_0^{\infty} \frac{\varepsilon_r''(\omega')\omega'}{-\omega'^2 + \omega^2} d\omega', \quad \varepsilon_r''(\omega) = \frac{2\omega}{\pi}\int_0^{\infty} \frac{\varepsilon_r'(\omega') - 1}{\omega^2 - \omega'^2} d\omega'.$$

  3. **High-frequency asymptote**:

     $$\varepsilon(\omega) \rightarrow 1, \text{ for } \omega \rightarrow \infty$$

  4. **Passivity**:

     $$\varepsilon''(\omega) > 0, \text{ for } \omega > 0$$

- **These constraints lead to intrinsic loss and dispersion in passive MTM**