TU4A-4

Non-Foster Circuit for Wideband Matching of High Frequency Helical Antenna

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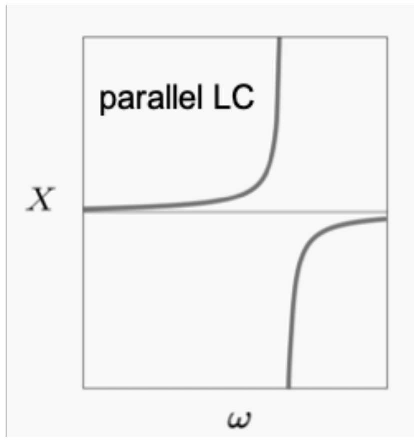
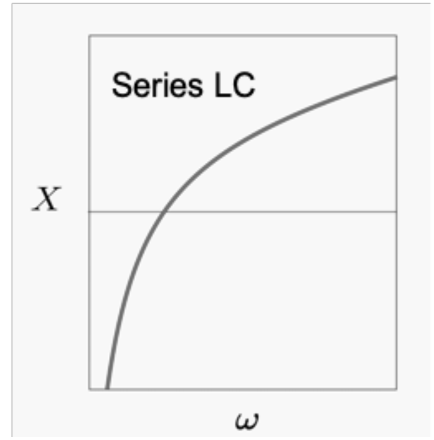
Outline

1. Introduction
2. Stability of Non-Foster system
3. Normalized Determinant Function analysis
4. Match a helical antenna
5. Experiment and results
6. Summary and discussion

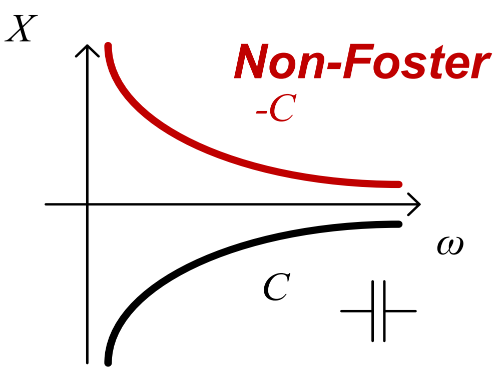
Non-Foster element

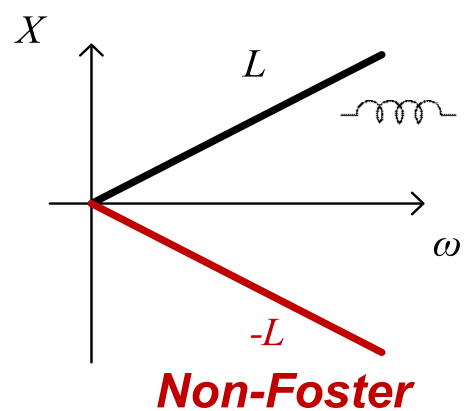
Foster's reactance theorem:





Only positive slope exists





A lossless non-Foster element has to be active.

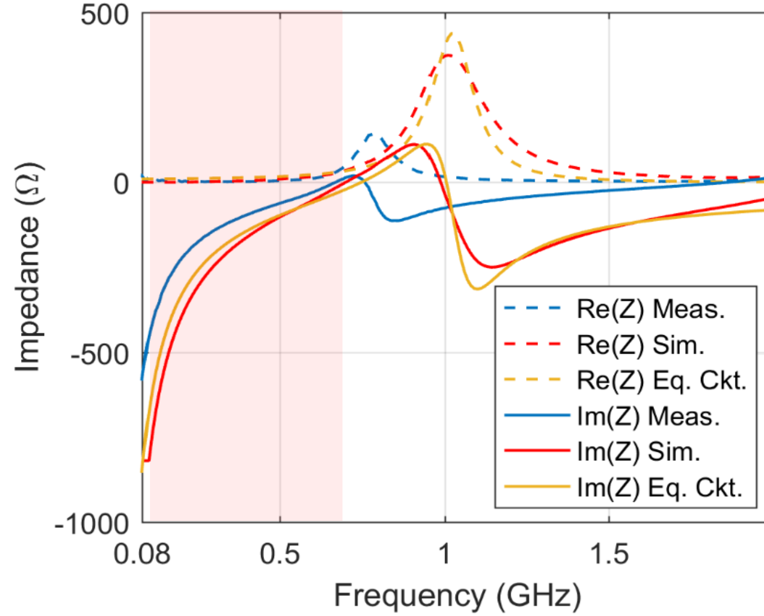
Bandwidth limit of electrically small antenna

- Define electrically small antenna $ka \ll 1,\ k = 2\pi/\lambda$
- **Chu's limit:** the radiation quality factor for a lossless electrically small antenna follows

$$Q_{chu} \geq \frac{1}{k^3a^3} + \frac{1}{ka}$$

$$BW_{3dB,\,\max} = \frac{k^3a^3}{1 + k^2a^2} \approx (ka)^3$$

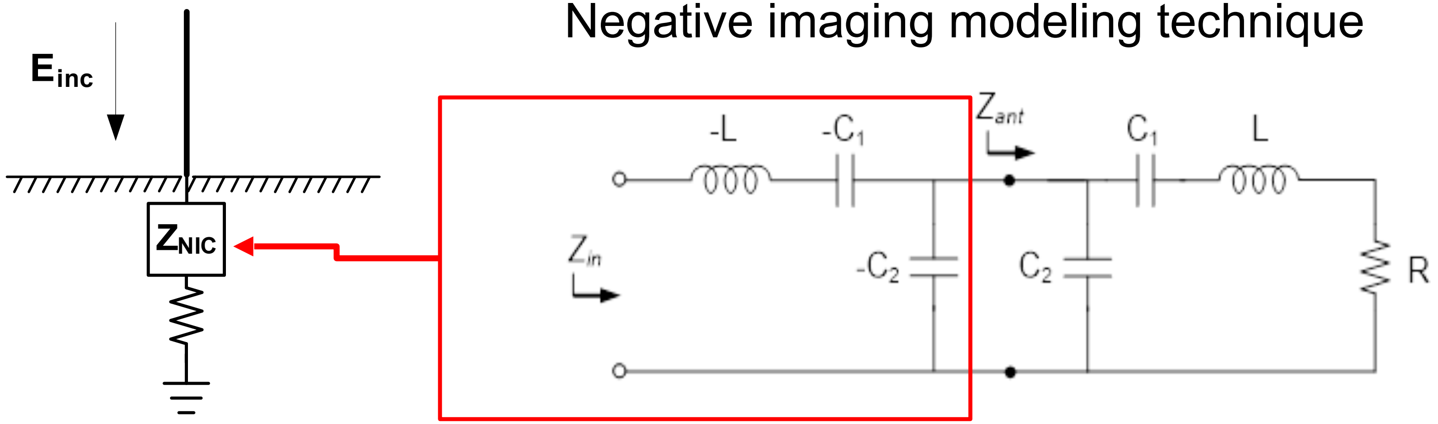
Impedance of a 10 cm monopole antenna

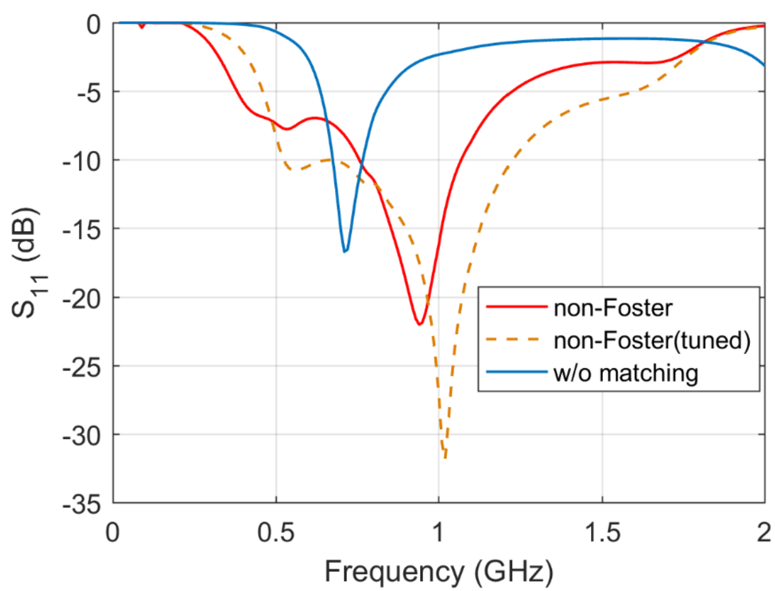


- **Large quality factor, thus suffers from limited bandwidth.**
- **Very low radiation efficiency due to small radiation resistance comparable to conductor resistance.**

Ideal non-Foster antenna matching circuit

Negative imaging modeling technique

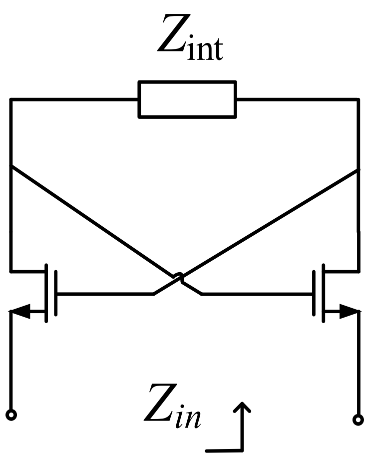




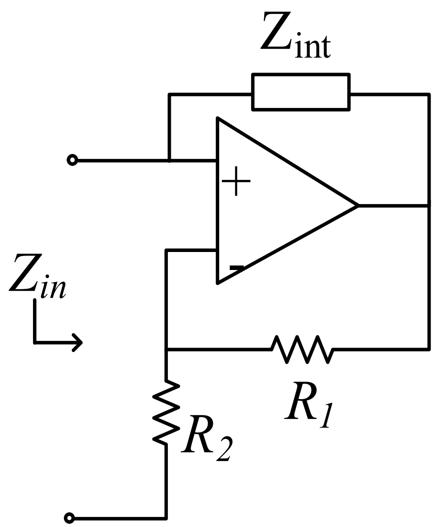
- **6 dB bandwidth: 170 MHz→780 MHz**
- **(Tuned) 10 dB bandwidth: 100 MHz→700 MHz**
- **Practical issues: transistor selection, device parasitics, biasing, noise and most importantly stability.**

Implementation of Non-Foster Element

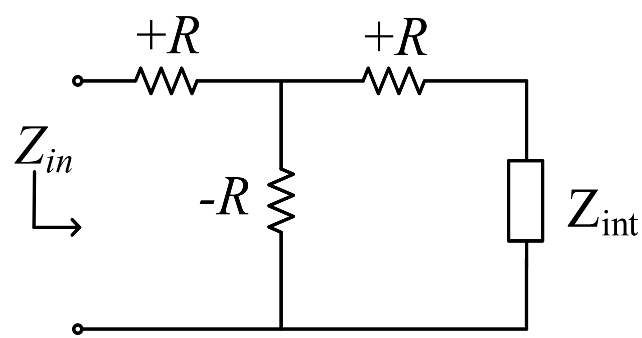
Cross-coupled transistor



Operational amplifier



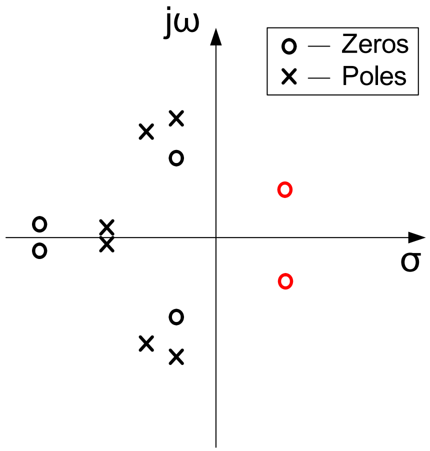
Negative resistor

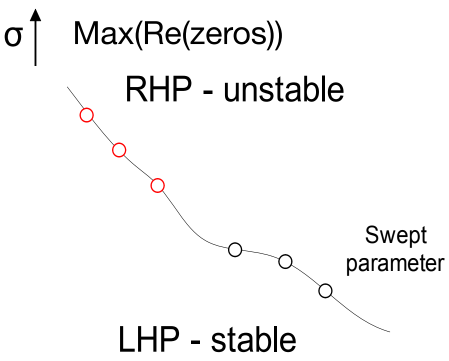


- Convert or invert the positive impedance Z_{int} into negative impedance
- $Z_{in} \propto -Z_{int}$ or $Z_{in} \propto -1 / Z_{int}$

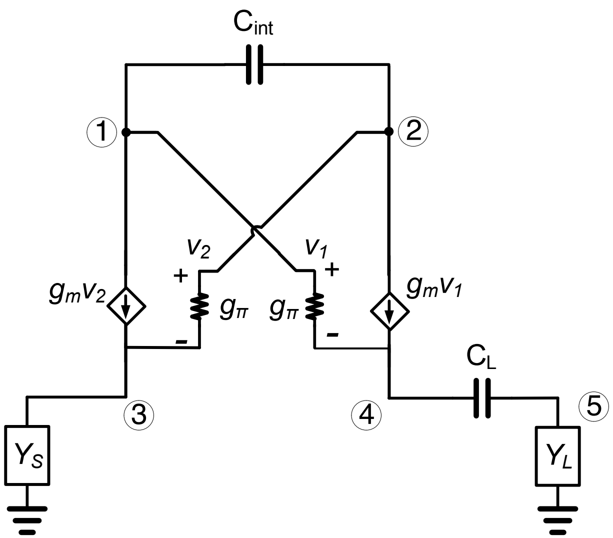
NDF analysis

1. Obtain the circuit model of network, and find the admittance / impedance matrix
2. Calculate the determinant as Δ
3. Deactivate all active devices i.e. let $g_m = 0$
4. Calculate its determinant as Δ_0.
5. $$NDF = \frac{\Delta}{\Delta_0} = \frac{(s - z_1)(s - z_2)(s - z_3)\cdots}{(s - p_1)(s - p_2)(s - p_3)\cdots},$$
6. No RHP zeros ➔ Stable.
   - Directly solve the roots.
   - Routh-Hurwitz criterion.
   - Nyquist plot.





A simple example of NDF



$$z_1 = 0$$

$$z_2 = \frac{C_L - C_{\text{int}}}{C_{\text{int}} C_L (2/g_m + 1/Y_L + 1/Y_S)}$$

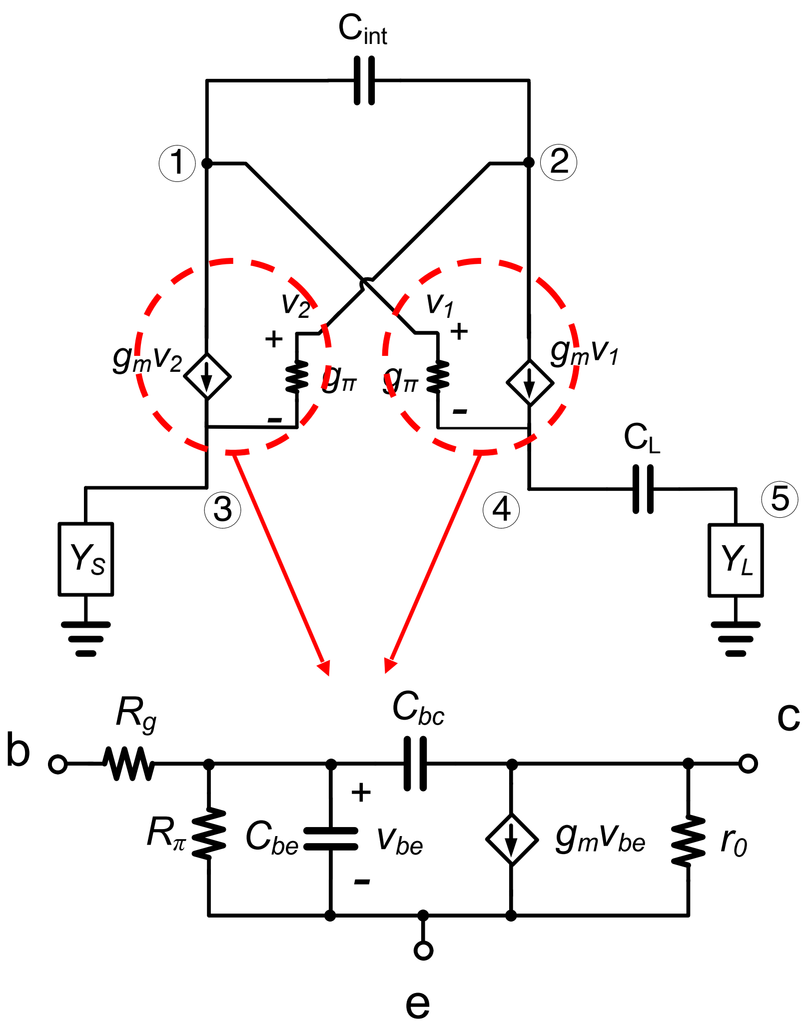
$$\Delta = \begin{vmatrix} sC_{\text{int}} + g_\pi & -sC_{\text{int}} + g_m & -g_m & -g_\pi & 0 \\ -sC_{\text{int}} + g_m & sC_{\text{int}} + g_\pi & -g_\pi & -g_m & 0 \\ 0 & -g_m - g_\pi & g_m + g_\pi + Y_s & 0 & 0 \\ -g_m - g_\pi & 0 & 0 & g_m + g_\pi + sC_L & -sC_L \\ 0 & 0 & 0 & -sC_L & sC_L + Y_L \end{vmatrix}$$

$$\Delta_0 = \begin{vmatrix} sC_{\text{int}} + g_\pi & -sC_{\text{int}} & 0 & -g_\pi & 0 \\ -sC_{\text{int}} & sC_{\text{int}} + g_\pi & -g_\pi & 0 & 0 \\ 0 & -g_\pi & g_\pi + Y_s & 0 & 0 \\ -g_\pi & 0 & 0 & g_\pi + sC_L & -sC_L \\ 0 & 0 & 0 & -sC_L & sC_L + Y_L \end{vmatrix}$$

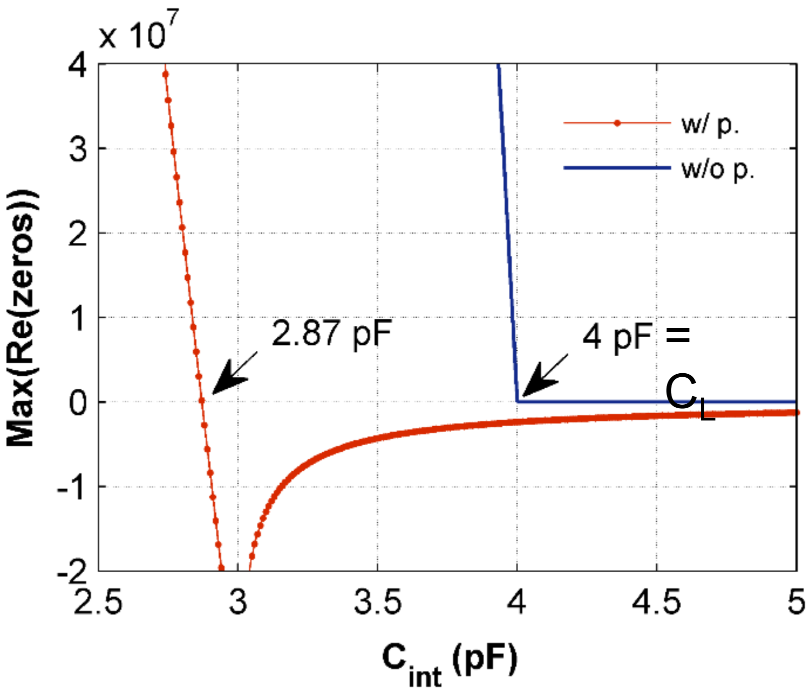
Stable condition is $C_{\text{int}} > C_L$

Practical factors have to taken into account for stability analysis.

The effect of device parasitics

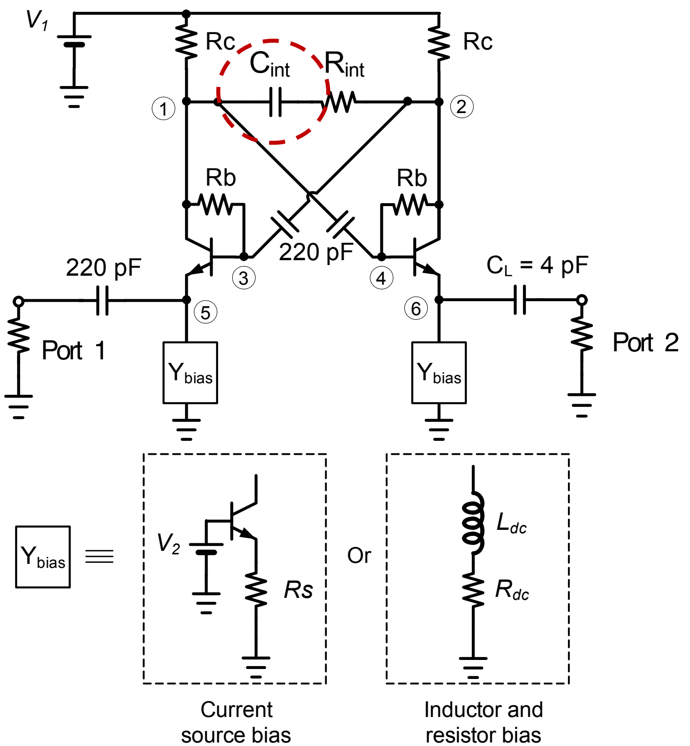


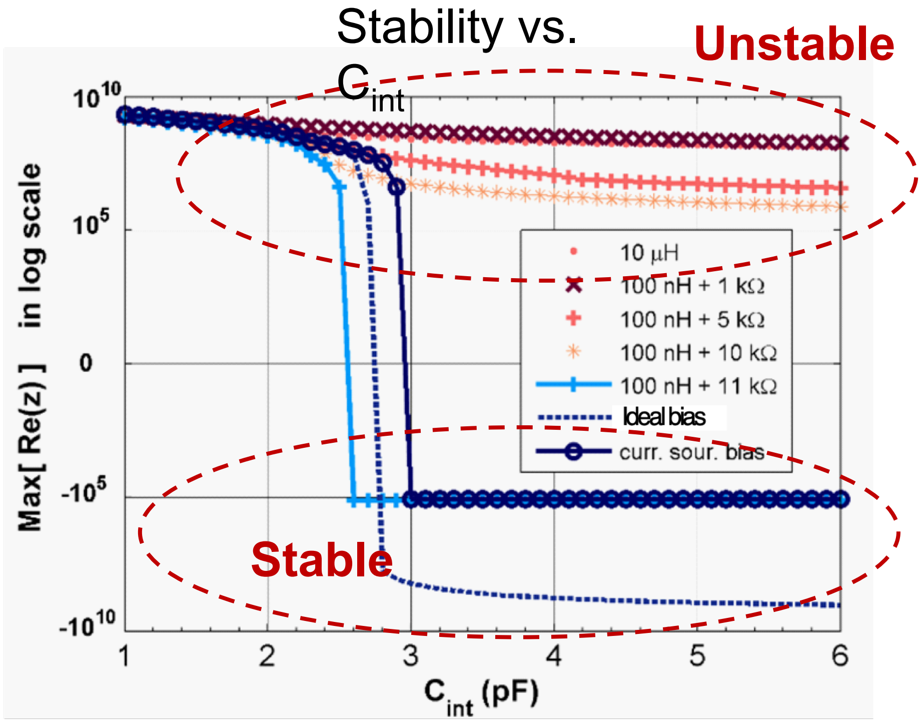
Max(Re(zeros)) vs. parameter C_{int}



The stable condition of $C_{int} > C_L$ is shifted down by 1.13 pF when device parasitics are taken into account.

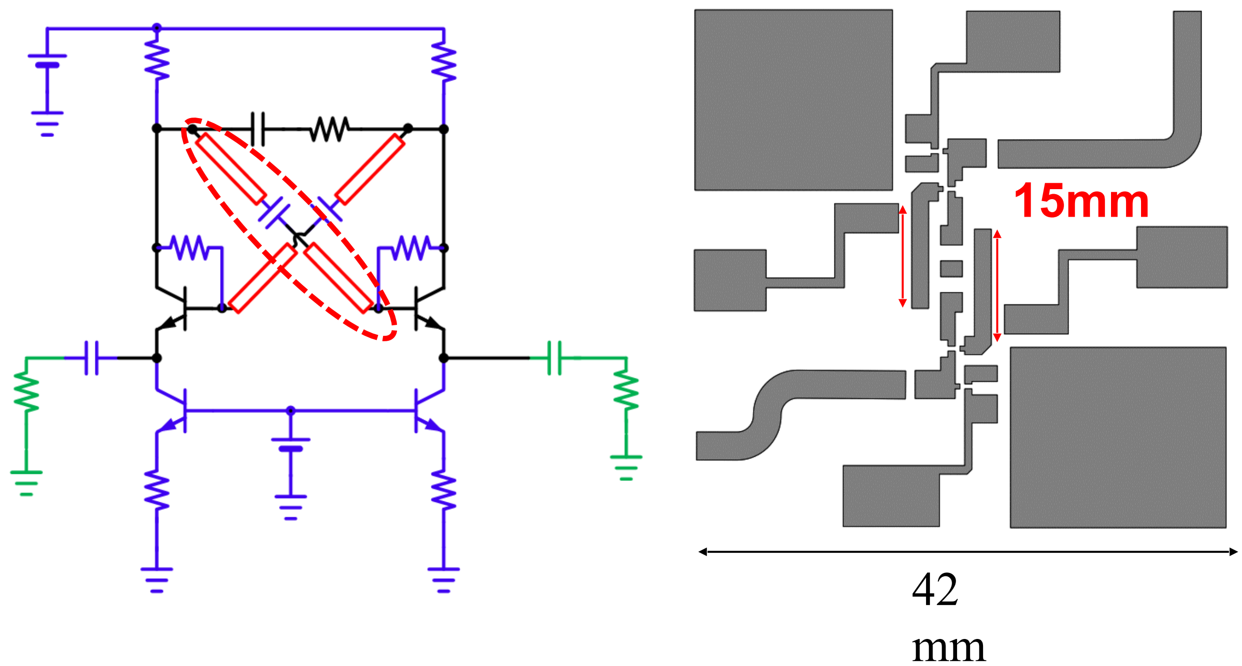
The effect of DC biasing network





- **Always unstable for any values of C_{int} when an inductor-resistor bias is selected (up to 100 nH and 10 kΩ).**
- **A more practical choice of active current source bias is stable.**

The effect of TLs in layout

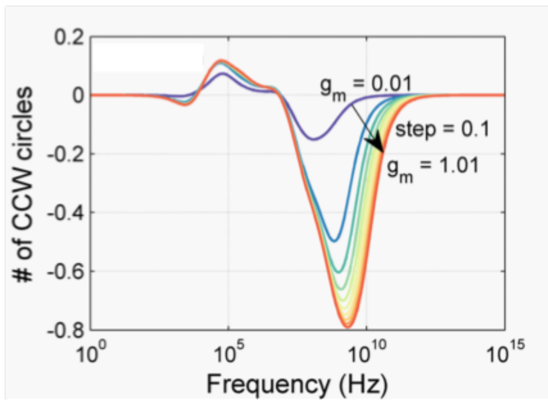


15 mm (~0.01 λ) transmission line in the loop

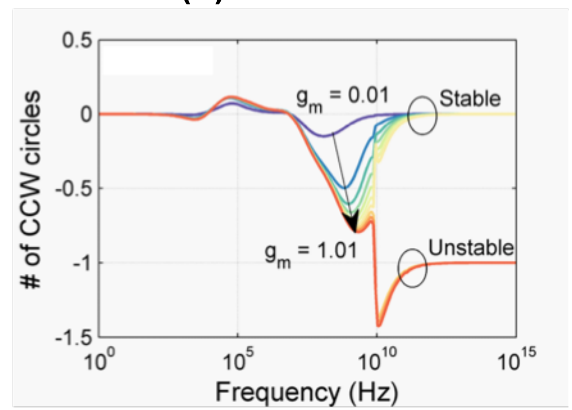
Stability for different length of TL in feedback loop

Sweep the transconductance (g_m) of the transistor

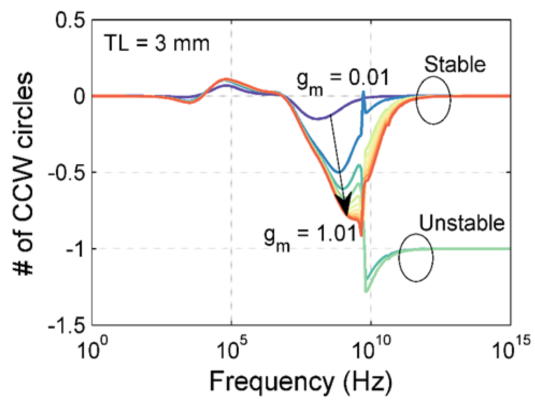
(a) l = 0 mm



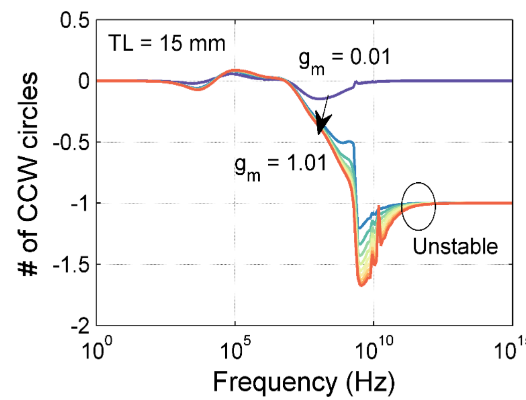
(b) l = 1 mm



(c) l = 3 mm



(d) l = 15 mm

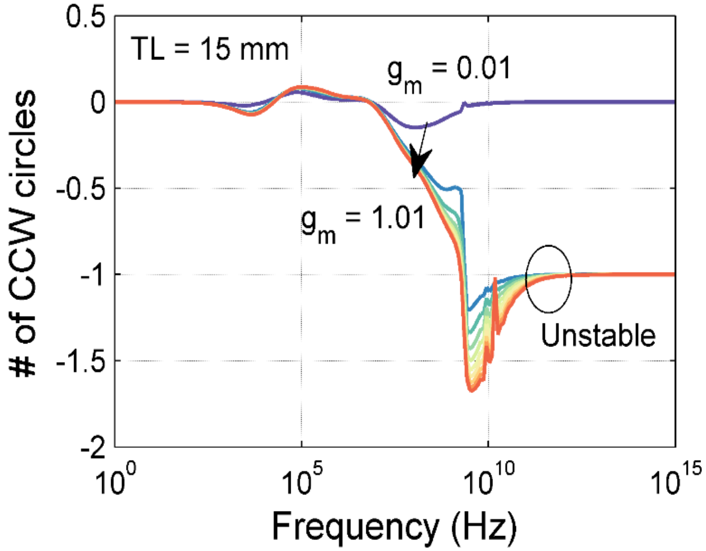


The short (~0.01λ_0 at 300 MHz) TL in the feedback loop has a large impact on the stability.

Adding stabilization resistors

Unstable

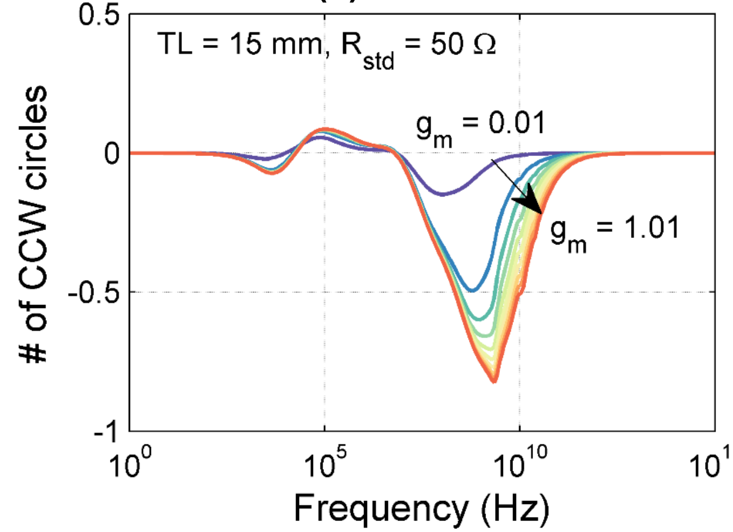
(d) *l* = 15 mm



Without R_{stb}

Stable

(e) *l* = 15 mm

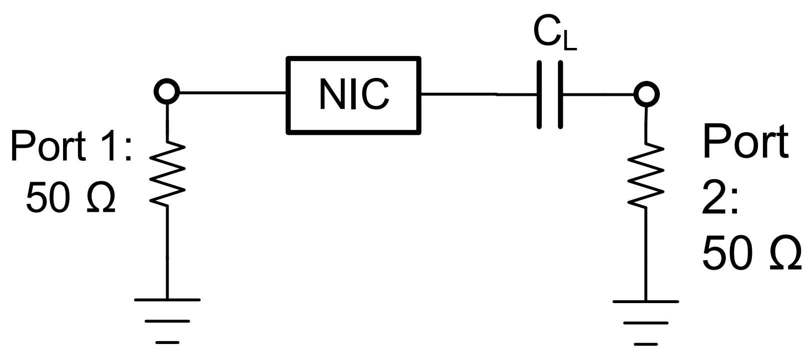


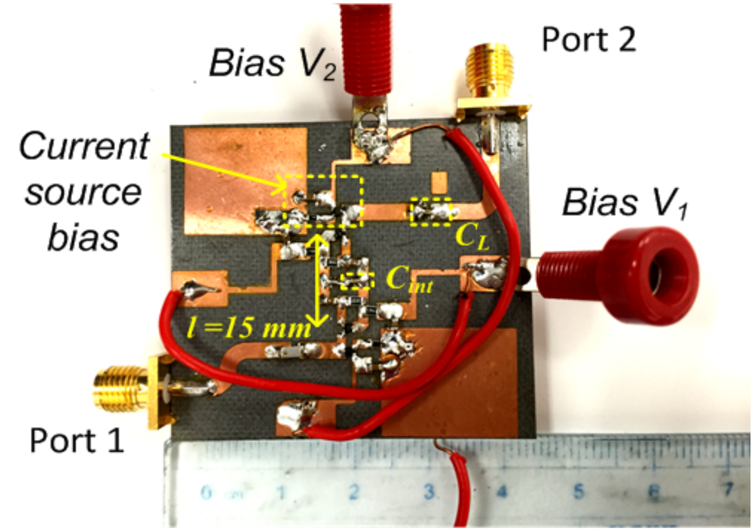
Series R_{stb} = 50 Ω

- **The circuit becomes stable across all values of g_m.**
- **The efficiency of the matching circuit drops by 1.6 dB.**

Experimental setup and fabrication

Experimental setup

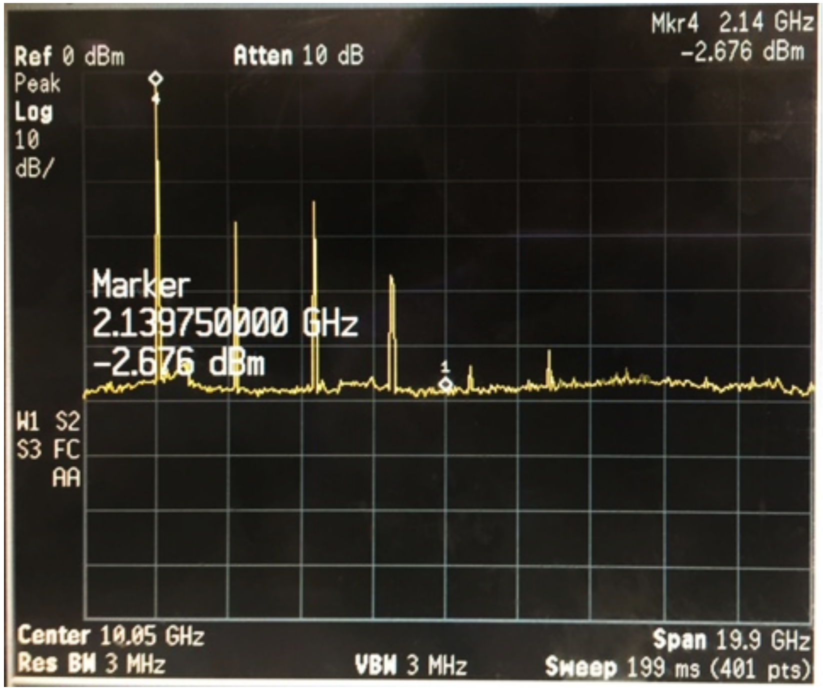




- Non-Foster circuit loaded with a positive capacitor.
- Test S_{11} and S_{21} to evaluate the performance of negative capacitor.
- Deembed C_L and microstrip line to extract the impedance of non-Foster circuit.

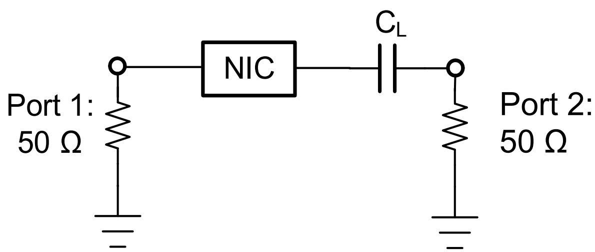
Experimental verification of NDF analysis

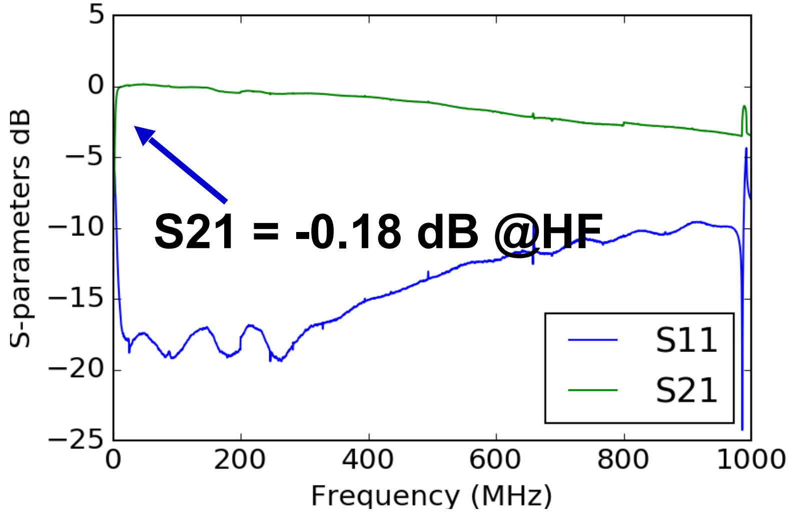
Without R_{stb}



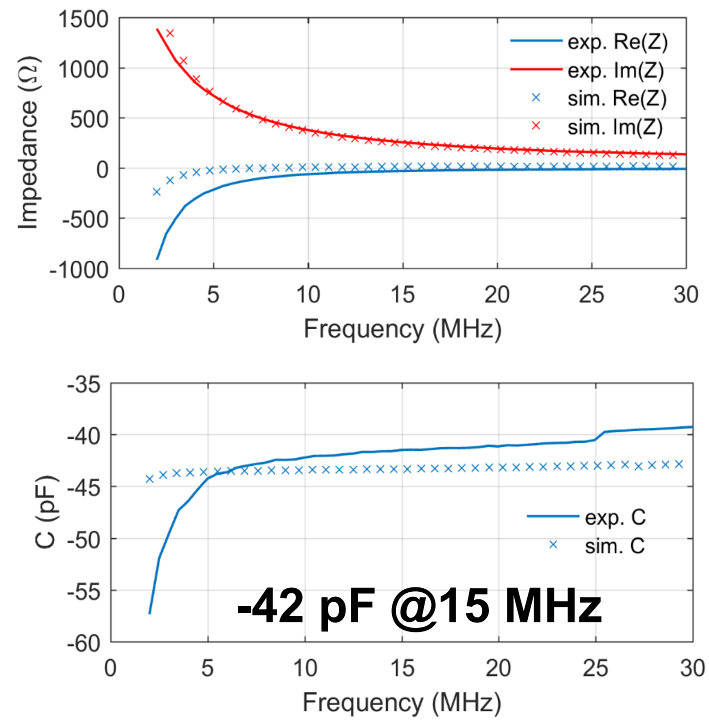
Self-oscillation @ 2.1 GHz

Characterize the negative capacitance





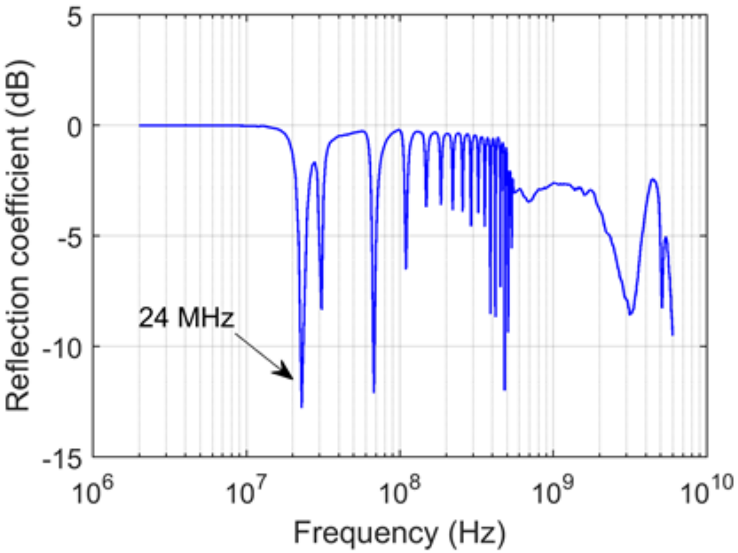
Extracted impedance and -C



- **The circuit loss is largely reduced due to the use of smaller value of R_{stb}.**
- **A -42 pF negative capacitor with low parasitic R is achieved at HF band.**

HF helical antenna with non-Foster matching circuit

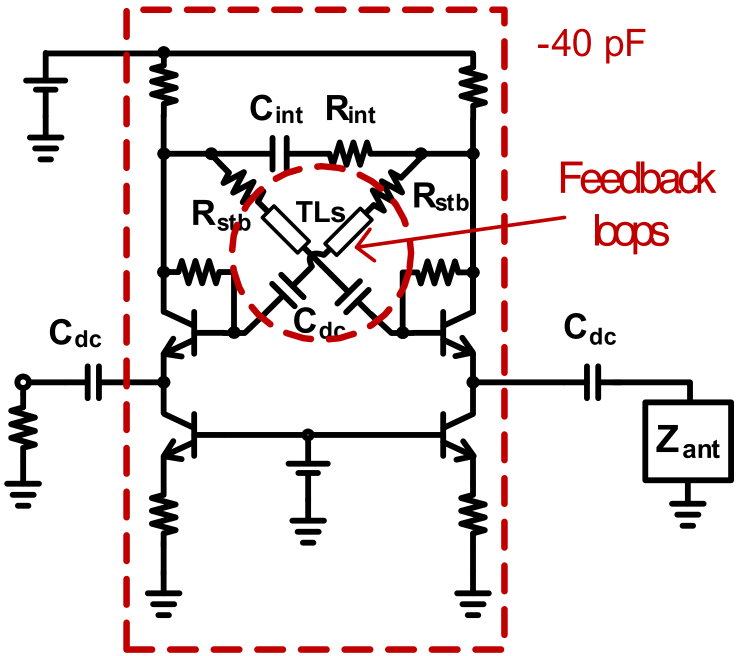
2m

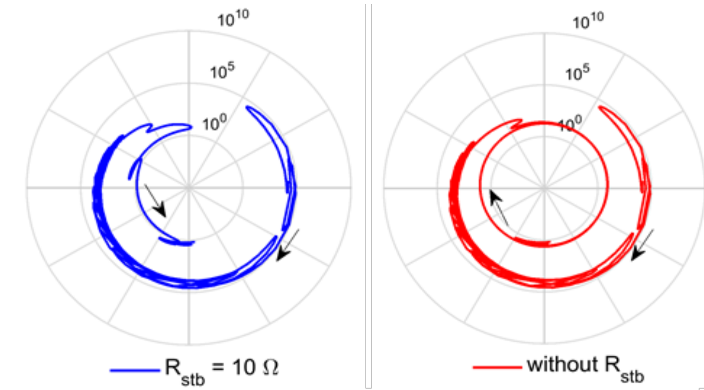


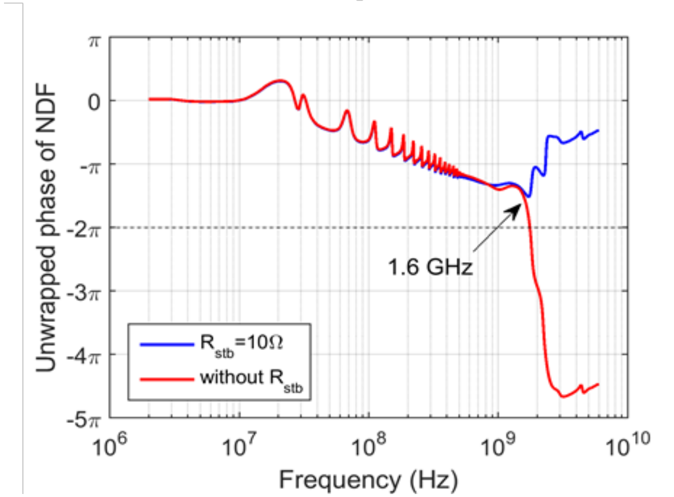
HF band: 3 to 30 MHz

- **The helical antenna has the lowest self-resonant frequency at 24 MHz.**
- **A -40 pF negative capacitor is required to fully cancel the reactance of the helical antenna at HF band.**

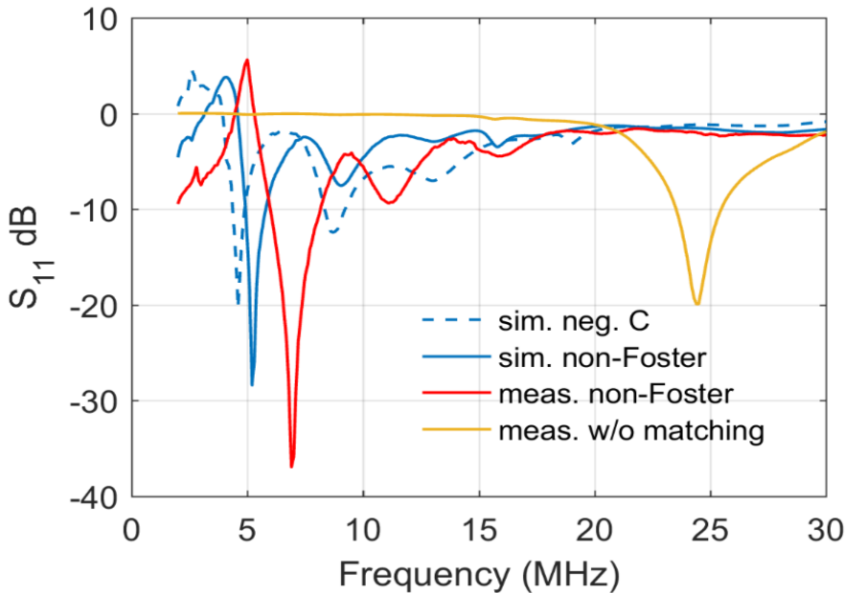
Nyquist plot of the non-Foster helical antenna system





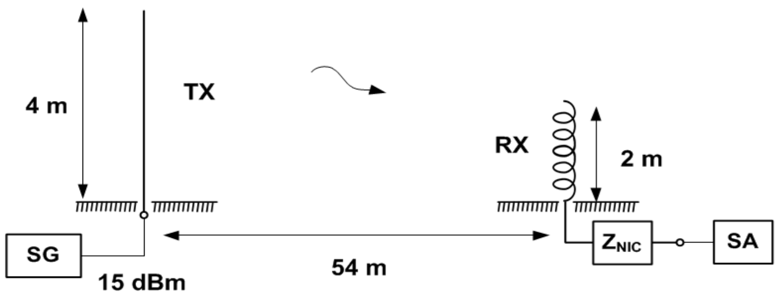


S_{11} of non-Foster matched helical antenna



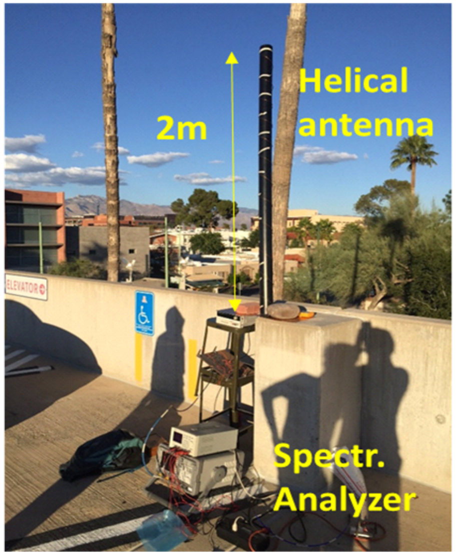
- **Improvement of S_{11} near 7 MHz.**
- **Good consistency between simulation and measurement**
- **Improvement of return loss, however, does not guarantee the increase of the overall gain.**
- **A field test of gain / efficiency is necessary.**

Field test environment



(a)

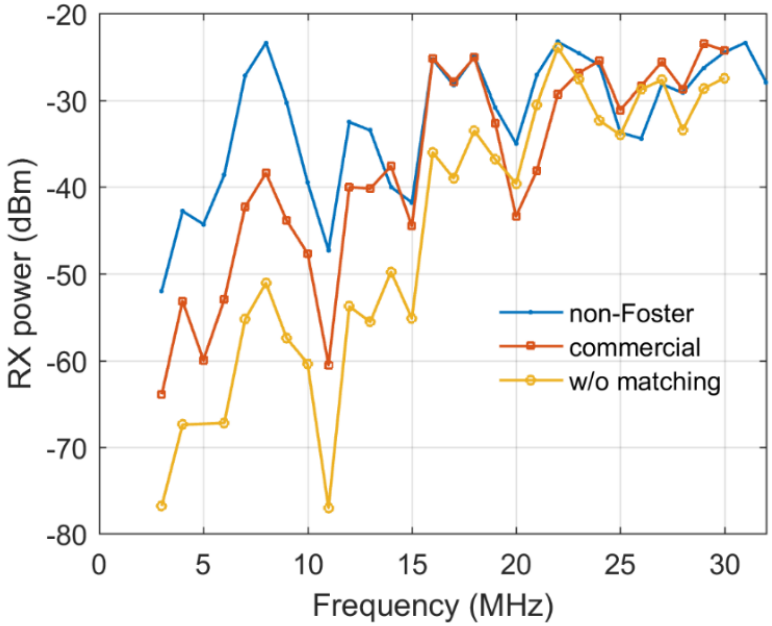
(b)

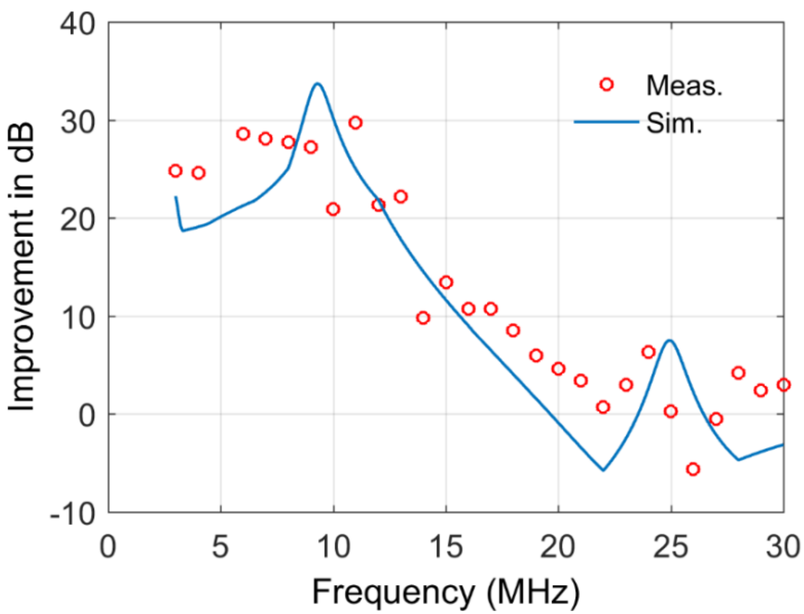


(c)

- The distance of the link is 54 meter (about 0.5 to 5 λ from 3 to 30 MHz).
- Testing cases: 1) helical antenna with non-Foster matching; 2) without matching; 3) a commercial well-matched 4-meter whip antenna.

Received signal enhancement by non-Foster matching circuit





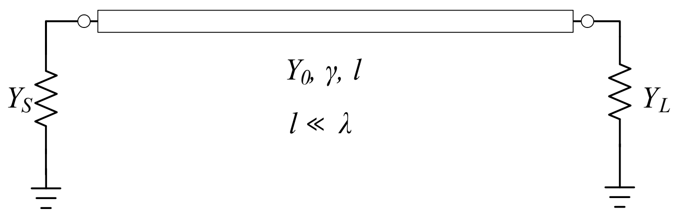
- **20-30 dB received signal power enhancement by non-Foster matching circuit at 3-13 MHz compared to without matching case.**
- **About 15 dB improvement compared to the commercial antenna.**

Conclusion

- Study the practical factors influencing on the stability of a non-Foster antenna system
- The biasing circuit and the layout TL in the feedback loop have great impact on the stability of the Non-Foster system
- Achieve a stable negative capacitor with relatively low parasitic resistance
- Design and test a stable 2-meter-height HF helical antenna with non-Foster matching circuit
- 20-30 dB received signal power gain by non-Foster matching circuit at 3-13 MHz compared to without matching case, and ~15 dB improvement compared to the commercial available antenna

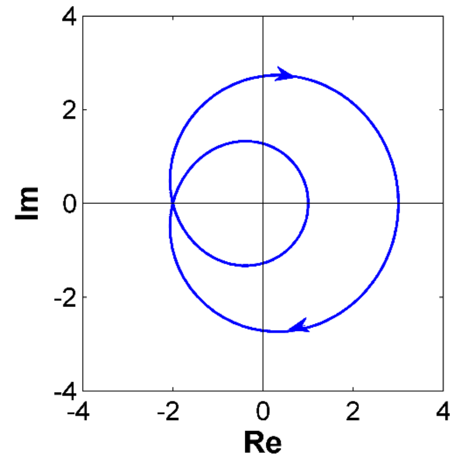
Thank you!

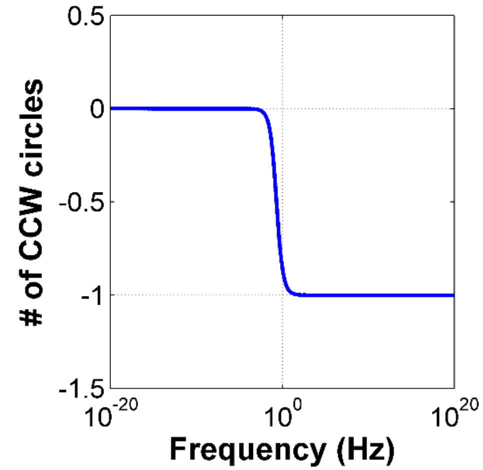
The effect of TLs in layout: Nyquist plot



$$\mathbf{Y}_{\mathbf{TL}} = \begin{bmatrix} Y_S + Y_0 \operatorname{ctgh}(\gamma l) & -Y_0 \operatorname{csch}(\gamma l) \\ -Y_0 \operatorname{csch}(\gamma l) & Y_L + Y_0 \operatorname{ctgh}(\gamma l) \end{bmatrix}$$

$$f(s) = \frac{s^2 - 4s + 3}{s^2 + 2s + 1}$$





- **TLs can be modeled with Y matrix with hyperbolic functions.**
- **Nyquist plot to count the number of RHP zeros without directly solving the equation.**

Stability Analysis Methods

- **Time domain (Transient simulation):**
  - – Inefficient for parametric study
  - – Not predict degrees of stability, where instability comes from
- **Linear frequency domain:**
  - – K-Δ factor, μ factor, Llewellen factors, etc. (Rollet's proviso)
  - – Return ratio, loop gain (easy implemented but not sufficient in more complicated case)
  - – Return difference, Normalized Determinant Function (NDF) method (Complexity ⇔ Accuracy)