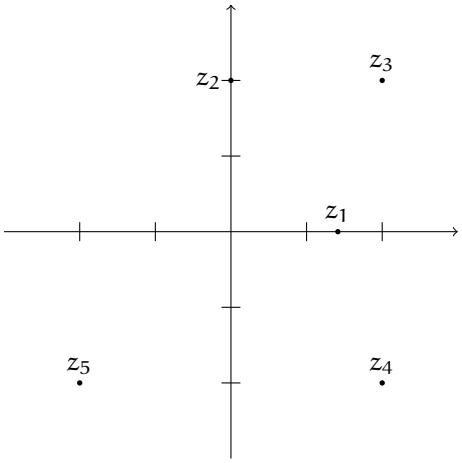
Algèbre et Arithmétique 1

Corrigé

Exercice 1



Exercice 2

(a) $\dfrac{1}{2+i} = \dfrac{1-i}{(1+i)(1-i)} = \dfrac{1-i}{2}$;

(b) $\dfrac{1+i}{1-i} = \dfrac{(1+i)^2}{(1-i)(1+i)} = \dfrac{1+2i-1}{2} = 2$;

(c) $(1+i)^4 = ((1+i)^2)^2 = (2i)^2 = -4$;

(d) $\left(\dfrac{1}{2} - i\dfrac{\sqrt{3}}{2}\right)^3 = \left(\dfrac{1}{2}\right)^3 - 3i\dfrac{\sqrt{3}}{2}\left(\dfrac{1}{2}\right)^2 + 3\left(\dfrac{i\sqrt{3}}{2}\right)^2 - \left(\dfrac{i\sqrt{3}}{2}\right)^3 = \dfrac{1}{8} - \dfrac{3i\sqrt{3}}{8} - \dfrac{9}{8} + \dfrac{3i\sqrt{3}}{8} = -1$;

Exercice 3

Soit $z \in \mathbf{C}$, alors

(a) $z + 2i = iz - 1 \iff z(1-i) = -1 - 2i \iff z = -\dfrac{1+2i}{1-i} = \boxed{\dfrac{1-3i}{2}}$;

(b) $(3+2i)(z-1) = i \iff (3+2i)z = 3+3i \iff z = \dfrac{3+3i}{3+2i} = \boxed{\dfrac{15+3i}{13}}$;

(c) $(2-i)z + 1 = (3+2i)z - i \iff 1+i = (1+3i)z \iff z = \dfrac{1+i}{1+3i} = \boxed{\dfrac{2-i}{5}}$;

(d) $(4-2i)z^2 = (1+5i)z \iff z = 0$ ou $(4-2i)z = 1+5i \iff z = 0$ ou $z = \dfrac{1+5i}{4-2i} = \boxed{\dfrac{-3+11i}{10}}$;

Exercice 5

(a) On a $z^2 = -\dfrac{1}{2} + i\dfrac{\sqrt{3}}{2}$ et $z^3 = -1$.

(b) On a donc $z^4 = z^3 z = -z$ et $z^5 = z^3 z^2 = -z^2$ et $z^6 = (z^3)^2 = 1$.

(c) Comme $z^6 = 1$, l'inverse de z est $z^{-1} = z^5$.

(d) On a $1 + i\sqrt{3} = 2z$, donc

$$(1 + i\sqrt{3})^5 = (2z)^5 = 32z^5 = -32z^2 = 16 - 16i\sqrt{3}$$

(e) On en déduit

$$(1 + i\sqrt{3})^5 + (1 - i\sqrt{3})^5 = 32$$
$$(1 + i\sqrt{3})^5 - (1 - i\sqrt{3})^5 = -32i\sqrt{3}$$