MASTER THEOREM

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1-1) $T(n) = 3T(n/2) + n^2$

$a = 3$, $b = 2$, $c = 2$

$c\text{-crit} = \log_2(3) = \sim 1.58$

$\because$ c-crit $< c$, case 3

$\therefore$ check regularity : $a f(\frac{n}{b}) \leq k f(n)$ for some $k < 1$

$3\left(\frac{n^2}{4}\right) \leq k n^2$ for some $k < 1$

choosing $3/4$ k ✓

$\therefore$ regularity holds

$\therefore$ case 3, $\boxed{T(n) = \Theta(n^2)}$

1-2) $T(n) = 7T(\frac{n}{2}) + n^2$

$a = 7$, $b = 2$, $c = 2$

$c\text{-crit} = \log_2(7) = \sim 2.81$

$\because$ c-crit $> c$

$\therefore$ case 1 , $\boxed{T(n) = \Theta(n^{2.81})}$ $\cancel{\Theta(n^{\log_2(7)})}$

1-3) $T(n) = 4T(\frac{n}{2}) + n^2$

$a = 4$, $b = 2$, $c = 2$, $c\text{-crit} = \log_2(4) = 2$

$\because$ c-crit $= c$

$\therefore$ case 2, $\boxed{T(n) = \Theta(n^2 \log(n))}$

1-4) $T(n) = 3T(\frac{n}{4}) + n\log(n)$

$a = 3$, $b = 4$, $c = 1$, $k = 1$, c-crit ≈ 0.79

$\because$ c-crit $< c$, case 3

$\therefore$ regularity $af(\frac{n}{b}) \leq cf(n)$

$\therefore$ $3\left(\frac{n}{4}\log\frac{n}{4}\right) \leq \frac{3}{4} n \log n$ let $c = \frac{3}{4}$

$\because \frac{3}{4} < 1$ $\therefore$ regularity holds, = case 3, $T(n) = \Theta(n \log n)$

1-5) $T(n) = 4T(\frac{n}{2}) + \log n$

$a = 4$, $b = 2$, $c = 0$, $k = 1$, c-crit $= 2$

$\because$ c-crit $> c$, $\therefore$ case 1, $\boxed{T(n) = \theta(n^2)}$

1-6) $T(n) = T(n-1) + n$

↳ this is $\boxed{\text{iteration}}$. Basically a for loop that moves through things.

$T(n) = \theta(n^2)$

1-7) $T(n) = 4T(\frac{n}{2}) + n^2 \lg(n)$

$a = 4$, $b = 2$, $c = 2$, $k = 1$, c-crit $= 2$

$\because$ c-crit $= c$ $\therefore$ case 2, $\boxed{T(n) = \theta((n^2) \times (\log(n))^2)}$

1-8) $T(n) = 5T(\frac{n}{2}) + n^2 \lg(n)$

$a = 5$, $b, 2$, $c = 2$, $k = 1$, c-crit $= 2.322$

$\because$ c-crit $> c$, $\therefore$ case 1, $\boxed{T(n) = \theta(n^{2.322})}$

1-9) $T(n) = 3T(\frac{n}{3}) + n/\log(n)$

$a = 3$, $b = 3$, $c = 2$, $k = 1$, c-crit $= 1$

$\because$ ccrit $< c$, $\therefore$ case 3 $\therefore$ check regularity

$a f(\frac{n}{b}) \leq k f(n)$ where $k < 1$

$3 \cdot \frac{n}{\log(n)} \cdot \frac{1}{3} \leq \frac{n}{\log(n)}$ Let $k = 1$

$\therefore$ regularity holds = case 3 = $\boxed{T(n) = \theta(n^2 \log n)}$

1-10) $T(n) = 2T(\frac{n}{4}) + c$

$a = 2$, $b = 4$, $c = 0$, $k = 0$, c_crit $= 0.5$

$\because$ c-crit $> c$ $\therefore$ case 1, $\boxed{T(n) = \theta(n^{\frac{1}{2}})}$

1-11) $T(n) = T(\frac{n}{4}) + \log(n)$

$a = 1$, $b = 4$, $c = 0$, $k = 1$, c-crit $= 0$

$\because$ ccrit $= c$ $\therefore$ case 2, $\boxed{T(n) = \theta((\log n)^2)}$

MASTER THEOREM

1-12) $T(n) = T(\frac{n}{2}) + T(\frac{n}{4}) + n^2 = \boxed{\text{Does Not Apply}}$

1-13) $T(n) = 2T(\frac{n}{4}) + \log(n)$

$a = 2,\ b = 4,\ c = 0,\ k = 1,\ c_{crit} = 0.5$

$\because c_{crit} > c$, $\therefore$ case 1, $\boxed{T(n) = \theta(n^{0.5})}$

1-14) $T(n) = 3T(\frac{n}{3}) + n \lg n$

$a = 3,\ b = 3,\ c = 1,\ c_{crit} = 1$

$\because c = c_{crit}$ $\therefore$ case 2, $\boxed{T(n) = \theta(n(\log n)^2)}$

1-15) $T(n) = 8T(\frac{n - \sqrt{n}}{4}) + n^2 = \boxed{\text{Does Not Apply}}$

1-16) $T(n) = 2T(\frac{n}{4}) + \sqrt{n}$

$a = 2,\ b = 4,\ c = 1/2,\ c_{crit} = \frac{1}{2}$

$\because c = c_{crit}$ $\therefore$ case 2, $\boxed{T(n) = \theta(n^{\frac{1}{2}} \log n)}$

1-17) $T(n) = 2T(\frac{n}{4}) + n^{0.51}$

$a = 2,\ b = 4,\ C = 0.51,\ c_{crit} = 0.5$

$\because c_{crit} < c$, $\therefore$ case 3 $\rightarrow$ regularity check

$2\frac{n^{0.51}}{4} \leq k\ n^{0.51}$ for $k \leq \frac{1}{2}$ which is $k < 1$

$\therefore$ regularity holds $\therefore$ $\boxed{T(n) = \theta(n^{0.51})}$

1-18) $T(n) = 16T(\frac{n}{4}) + n!$

$a = 16,\ b = 4,\ c_{crit} = 2,\ c \ldots$ grows like crazy

$\therefore$ case 3 $[c_{crit} < c]$, regularity

$$\frac{16}{(\frac{n}{4}+1)(\frac{n}{4}+2)\ldots n} \leq \frac{16}{(\frac{n}{4}+1)^{(\frac{3n}{4})}} \ [n \longrightarrow \infty]$$

never ending = big

for large n, any fixed $c < 1$ works

$\therefore T = \boxed{\theta(n!)}$

1-19) $T(n) = 3T(\frac{n}{2}) + n$

$a=3$, $b=2$, $c=1$, $c_{crit} = 1.58$

$\because$ $c_{crit} > c$ $\therefore$ $\boxed{T(n) = \theta(n^{1.58})}$

1-20) $T(n) = 4T(\frac{n}{2}) + cn$

$a=4$, $b=2$, $c=1$, $c_{crit} = 2$

$\because$ $c_{crit} > c$ $\therefore$ $\boxed{T(n) = \theta(n^2)}$

1-21) $T(n) = 3T(\frac{n}{3}) + \frac{n}{2}$

$a=3$, $b=3$, $c=1$, $c_{crit}=1$

$\because$ $c_{crit} = c$ $\therefore$ case 2, $\boxed{T(n) = \theta(n \log n)}$

1-22) $T(n) = 4T(\frac{n}{2}) + n/\log n$

$a=4$, $b=2$, $c_{crit}=2$, $c=1$

$\because$ $c_{crit} > c$, $\therefore$ case 1, $\boxed{T(n) = \theta(n^2)}$

1-23) $T(n) = \cancel{8}\ 7T(\frac{n}{3}) + n^2$

$a=7$, $b=3$, $c=2$, $c_{crit}=1.77$

$\because$ $c_{crit} < c$, $\therefore$ case 3 $\rightarrow$ regularity

$7\left(\frac{n}{3}\right)^2 \leq kn^2$

$\frac{7}{9}n^2 \leq kn^2 \rightarrow \frac{7}{9} \geq k$ holds = regularity holds

$\therefore$ $\boxed{T(n) = \theta(n^2)}$

1-24) $T(n) = 8T(\frac{n}{3}) + 2^n$

$a=8$, $b=3$, $f(n) = 2^n$ $c_{crit} = \log_3 8$

$\therefore$ case 3, regularity $8 \cdot 2^{n/3} \leq k2^n \rightarrow$ if $k = \frac{1}{2}$, holds

$\therefore$ holds, $\boxed{T(n) = \theta(2^n)}$

1-25) $T(n) = 16T(\frac{n}{4}) + n$

$a=16$, $b=4$, $c=1$, $c_{crit}=2$

$\because$ $c_{crit} > c$, $\therefore$ case 1

$\therefore$ $\boxed{T(n) = \theta(n^2)}$