

# HW6: Constrained Multivariable Optimization (Lagrange Multipliers)

**Due: Wed 10/10**

A manufacturer of personal computers currently sells 10,000 units per month of a basic model. The cost of manufacture is \$700/unit, and the wholesale price is \$950. During the last quarter the manufacturer lowered the price \$100 in a few test markets, and the result was a 50% increase in sales. The company has been advertising its product nationwide at a cost of \$50,000 per month. The advertising agency claims that increasing the advertising budget by \$10,000/month would result in a sales increase of 200 units/month. Management has agreed to consider an increase in the advertising budget to no more than \$100,000/month.

- (a) Determine the price and the advertising budget that will maximize profit. Use the five-step method. Model as a constrained optimization problem, and solve using the method of Lagrange multipliers.
- (b) Determine the sensitivity of the decision variables (price and advertising) to price elasticity (the 50% number).
- (c) Determine the sensitivity of the decision variables to the advertising agency's estimate of 200 new sales each time the advertising budget is increased by \$10,000 per month.
- (d) What is the value of the multiplier found in part (a)? What is the real-world significance of the multiplier? How could you use this information to convince top management to lift the ceiling on advertising expenditures?
- (e) Draw a graph of the feasible region, the contour plot of the profit function and the gradient of the profit function at the optimum.
- (f) Can we use Matlab "linprog" command to solve this problem?