HW8: Dynamical System (Initial Value Problem)

Due at the beginning of the lecture on Wednesday 11/7

In the whale problem of Example 4.2 we used a logistic model of population growth, where the growth rate of population P in the absence of interspecies competition is

$$g(P) = rP\left(1 - \frac{P}{K}\right).$$

In this problem we will be using a more complex model,

$$g(P) = rP\left(\frac{P-c}{P+c}\right)\left(1 - \frac{P}{K}\right),$$

in which the parameter c represents a minimum viable population level below which the growth rate is negative. Assume that $\alpha = 10^{-8}$ and that the minimum viable population level is 3,000 for blue whales and 15,000 for fin whales.

(a) Can the two species of whales coexist? Use the five-step method, and model as a dynamical system in steady state.

(b) Sketch the vector field for this model. Classify each equilibrium point as stable or unstable.

(c) Assuming that there are currently 5,000 blue whales and 70,000 fin whales, what does this model predict about the future of the two populations?

(d) Suppose that we have underestimated the minimum viable population for the blue whale, and that it is actually closer to 10,000. Now what happens to the two species?

(e) Use ode45 in Matlab to simulate the populations with respect to time in Part (c) and Part (d), respectively.

(f) Implement the forward and modified Euler methods to solve the model numerically.

(g) Analyze the sensitivity of the whale populations to the effect of competition.

***Note: Example 4.2 is the in-class problem for Dynamical Systems.

The mathematical modeling approach to problem solving consists of five steps:

1. Ask the question.
2. Select the modeling approach.
3. Formulate the model.
4. Solve the model.
5. Answer the question.