HW5: Computational Method for Unconstrained Multivariable Optimization (Random Search and Newton-Raphson's Method) Due: Wed 10/3

A manufacturer of color TV sets is planning the introduction of two new products, a 19–inch LCD flat panel set with a manufacturer's suggested retail price (MSRP) of $339 and a 21–inch LCD flat panel set with an MSRP of $399. The cost to the company is $195 per 19–inch set and $225 per 21–inch set, plus an additional $400,000 in fixed costs. In the competitive market in which these sets will be sold, the number of sales per year will affect the average selling price. It is estimated that for each type of set, the average selling price drops by one cent for each additional unit sold. Furthermore, sales of the 19–inch set will affect sales of the 21–inch set, and vice–versa. It is estimated that the average selling price for the 19–inch set will be reduced by an additional 0.3 cents for each 21–inch set sold, and the price for the 21–inch set will decrease by 0.4 cents for each 19–inch set sold. How many units of each type of set should be manufactured?

Use numerical methods instead of the analytic methods to solve the above problem.

Hints: Since this is an optimization problem, we need to find critical numbers, by setting the gradient of the function equal to zero. This means that we're dealing with the root-finding problem of the gradient: we can use Newton-Raphson's method to identify the locations of these critical numbers. Notice that the method of random search can be utilized to find an initial approximate root for Newton-Raphson's method.

(a) Determine the production levels x_1 and x_2 that maximize the objective function $y = f(x_1, x_2)$. Use the two–variable version of Newton-Raphson's method.

(b) Let "a" denote the price elasticity for 19–inch sets. In part (a) we assumed "a" = 0.01. Now assume that "a" increases by 1%, what will be the new values for each TV set and the profit function?.

Use your results to obtain a numerical estimate of the sensitivities: $S(x_1, a)$, $S(x_2, a)$, and $S(y, a)$. Compare these to the sensitivity results obtained analytically with derivatives.

(c) Compare the analytic methods (Second Derivative Test) to the numerical methods employed in this problem. Which do you prefer? Explain your reason.