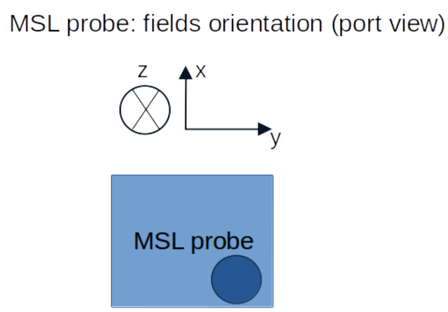
Magnetic Field Measurements Using 3D MSL Probe: Report and Analysis

Session Conducted on April 26, 2024, Covering Discharges #44658 to #44684

This report summarizes the results obtained from measurements of the magnetic fields generated (primarily) by poloidal field coils on GOLEM. The magnetic fields were measured with the MSL probe [2] and a set of four Mirnov coils.

1 Comparison of Magnetic Fields: MSL Probe vs. Mirnov Coils



1.1 Toroidal Magnetic Field

The value measured by MSL was compared with the value of B_t measured with a standard diagnostic (MC coil). See Figures ....

1.2 Horizontal Magnetic Field Generated by Vertical Stablization

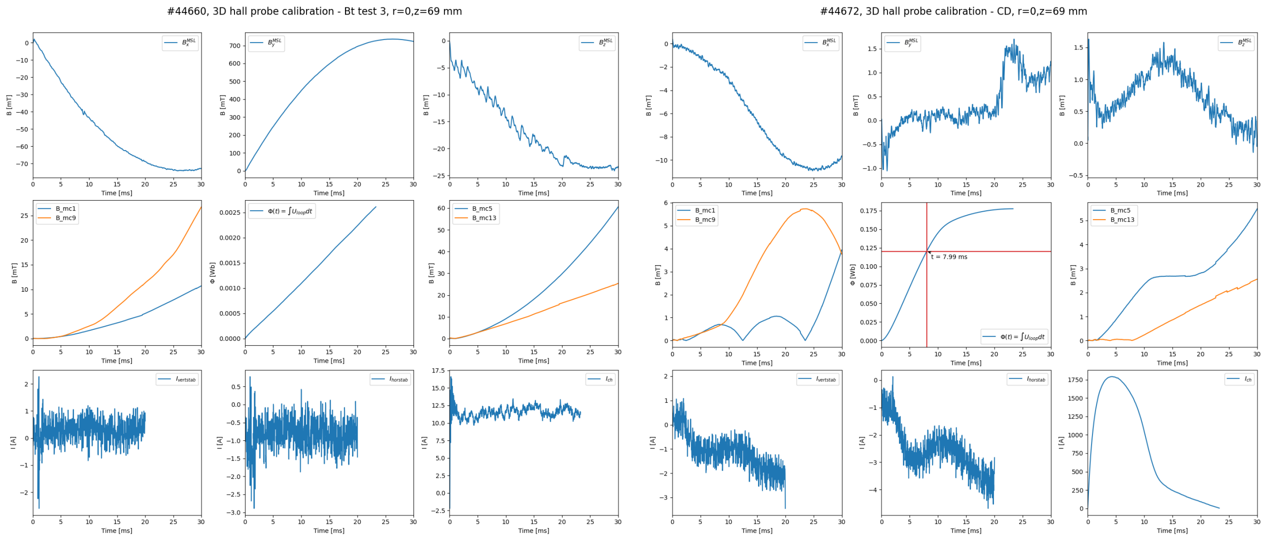
1.3 Vertical Magnetic Field generated by Horizontal Stablization

1.4 Stray Magnetic Fields

Interestingly, the measurements show a larger magnitude of stray fields generated by TF coils than those generated by chamber currents, compare B_x^{MSL} in Figure 1a and 1b. In both cases, the vertical component dominates. TODO: Proc ma B_x u 44684 jinou orientaci nez u 44660, 44672

Comparing the absolute values of stray fields the one from TF coils is almost 10 times higher. This field is probably compensated by the field generated by the plasma current. (otherwise, it wouldn't work, I guess)...

So, the stray field from TF coils may play a nonnegligible role in the plasma position!! This would suggest that since B_t increases during the discharge $\Longrightarrow$ mag. field from PFC should also increase to compensate for this...



(a) #44660, only B_{t} (b) #44672, only E_{CD} - saturated iron core

Figure 1: Measurements focused on stray magnetic fields

1.5 Fields Orientation

Based on the measured values of the magnetic field, the waveform sent to the PFC and the behaviour of the plasma in the experiments, we can conclude on the orientation of the currently used fields:

Field	**Default Orientation**
B_t	ACW
E_t	CW
B_r^{vertStab}	toward LFS
B_z^{horStab}	upwards

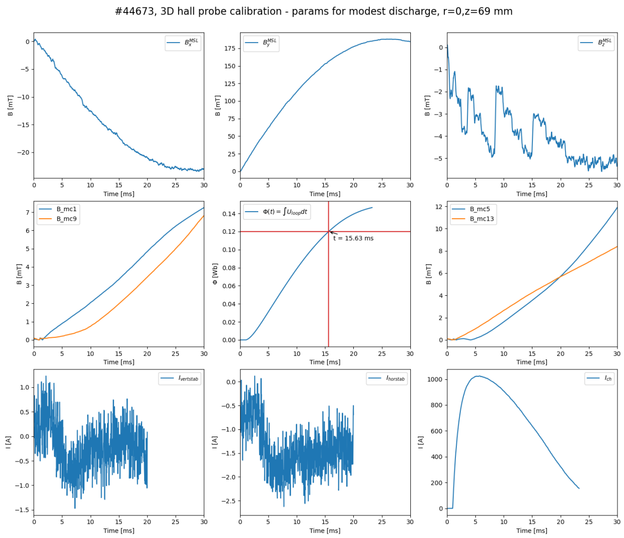
Table 1: Fields orientation

2 Comparison of Measurements with Models

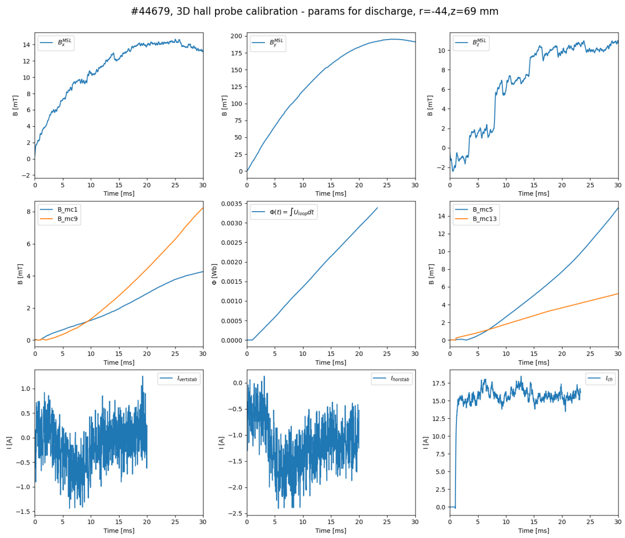
The signal of current in PF coils, measured by Rogowski coils, was recorded only for the first 20 ms after the trigger. Unfortunately, this does not cover the entire pulse duration, making modeling difficult, as the current in the PF coils serves as the input for the models. Nevertheless, in the following subsections, a comparison of the measured magnetic field with the outputs of different modelling approaches is presented.

2.1 Biot-Savart Model

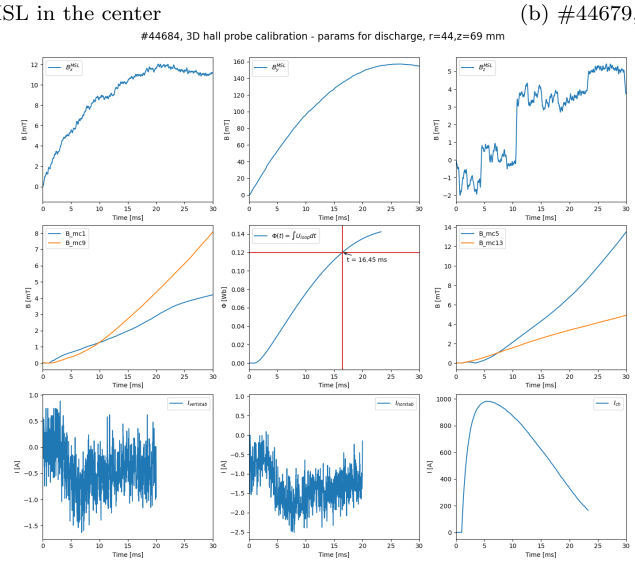
$$\vec{B} = \frac{\mu_0}{4\pi} I \int \frac{d\vec{I} \times (\vec{r} - \vec{r_l})}{|\vec{r} - \vec{r_l}|^3} \tag{1}$$



(a) #44673, MSL in the center



(b) #44679, MSL on HFS



(c) #44684, MSL on LFS,with both B_t and E_t fields

Figure 2: with both B_t and E_t fields (modest discharge)

2.1.1 Numerically solved

[1]

$$
\begin{aligned}
\vec{r} &= (R_1, 0, Z_1) \\
\vec{r_l} &= (R_0 \cos\phi_0, R_0 \sin\phi_0, Z_0) \\
d\vec{l} &= dl(-\sin\phi_0, \cos\phi_0, 0) = \frac{2\pi R_0}{N}(-\sin\phi_0, \cos\phi_0, 0) \\
\phi_{0,i} &= \frac{2\pi i}{N} \\
D_{R,i}^3 &= |\vec{r} - \vec{r_l}^3| = \left((R_1 - R_0\cos\phi_{0,i})^2 + (R_0 \sin\phi_{0,i})^2 + (Z_1 - Z_0)^2\right)^{3/2} \\
B_R &= 10^{-7} \cdot I \cdot dl \cdot (Z_1 - Z_0) \cdot \sum_{i=0}^{N-1} \frac{\cos\phi_{0,i}}{D_{R_i}^3} \\
B_z &= 10^{-7} \cdot I \cdot dl \cdot \sum_{i=0}^{N-1} \frac{R_0 \sin^2\phi_{0,i} - \cos\phi_{0,i} \cdot (R_1 - R_0\cos\phi_{0,i})}{D_{R_i}^3}
\end{aligned}
$$

2.1.2 'Analytically' solved - using Elliptic Integrals

An alternative method to speed up the calculation of the magnetic field from single turn toroidal loop is to use Legendre's complete elliptic integrals of the first and second kind $K(k)$ and $E(k)$ for parameter k to solve the Biot-Savart law. The numerical solution is then:

$$
\begin{aligned}
B_r &= \frac{\mu_0 I}{4\pi} \frac{(Z_0 - Z_1) f_1(k)}{R_1\sqrt{R_0 R_1}} \\
B_z &= \frac{\mu_0 I}{4\pi} \frac{R_1 f_1(k) + R_0 f_2(k)}{R_1\sqrt{R_0 R_1}} \\
k^2 &= \frac{4R_0R_1}{(R_0 + R_1)^2 + (Z_1 - Z_0)^2} \\
f_1(k) &= k(K(k) - \frac{2-k^2}{2(1-k^2)}E(k)) \\
f_2(k) &= \frac{k^3}{2(1-k^2)}E(k)
\end{aligned}
$$

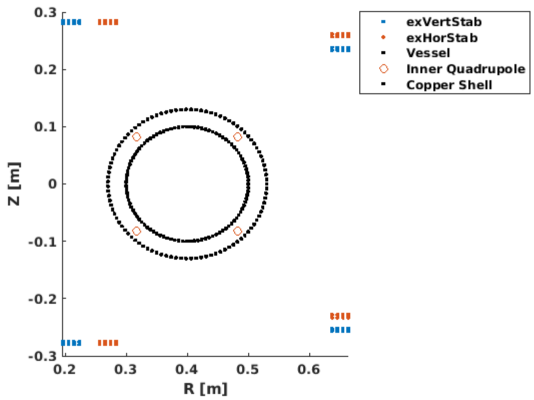
There are two ways to numerically calculate the elliptic integral. One option is to use the iterative method described in Calrslon, Numerical Algorithms. Another possibility are polynomial approximations for the additional parameter m1 from Abrahamowitz, Mathematical Functions with Formulas (Page 232, Chapter 17). In this work, the second method is used.

Note that the matrix of the magnetic inductance of the coils winding m, let us denote as $\mathbf{F_m}$, for the given matrix of coordinates is fixed and thus computed once and stored. The magnetic inductance generated by the current I in the coils winding m is then given as: $B_m(x_{ij}) = \mathbf{F_m}(x_{i,j}) \cdot I$.

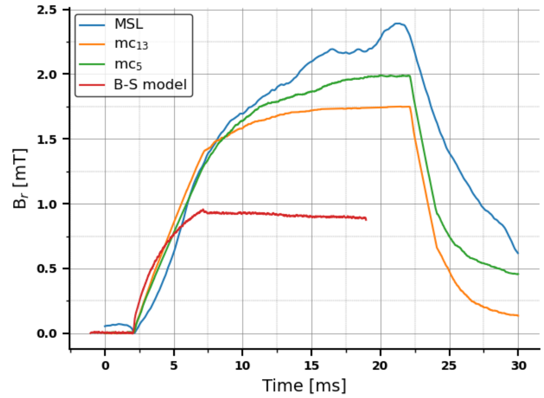
2.1.3 Application

2.2 Convolution Model

[4] Maybe next time....



(a) Position of the coils used in the B-S model



(b) Comparison of measurements with B-S model; # 44464

Figure 3: B-S model validation

2.3 State-Space Model

$$\begin{pmatrix} \dot{\xi}_{\mathrm{cs}} \\ \dot{\xi}_{\mathrm{vv}} \end{pmatrix} = \begin{pmatrix} -\frac{1}{\tau_{\mathrm{cs}}} & M \\ M & -\frac{1}{\tau_{\mathrm{cs}}} \end{pmatrix} \begin{pmatrix} \xi_{\mathrm{cs}} \\ \xi_{\mathrm{vv}} \end{pmatrix} + \begin{pmatrix} b_{\mathrm{cs}} \\ b_{\mathrm{vv}} \end{pmatrix} I_{\mathrm{PFC}} \tag{2}$$

$$B = \begin{pmatrix} k_{\mathrm{cs}} & k_{\mathrm{vv}} \end{pmatrix} \begin{pmatrix} \xi_{\mathrm{cs}} \\ \xi_{\mathrm{vv}} \end{pmatrix} + k_{\mathrm{PFC}} I_{\mathrm{PFC}} \tag{3}$$

[3]

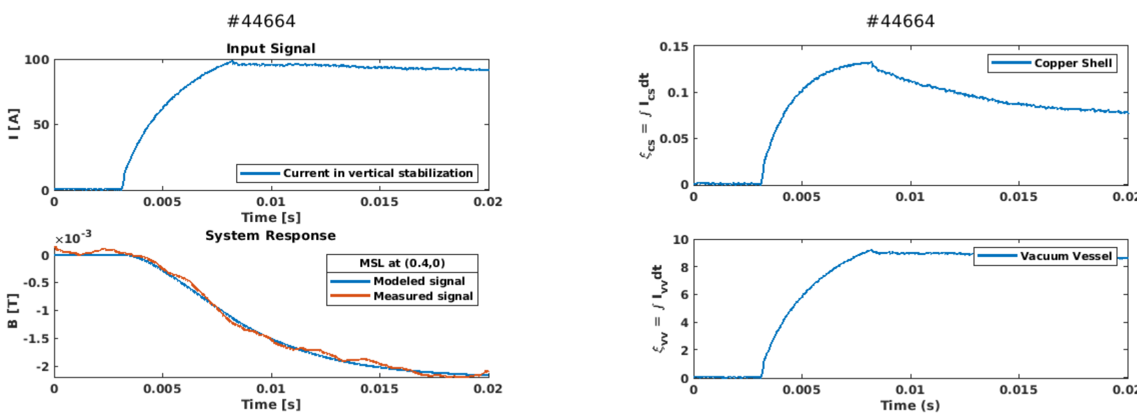


Figure 4: State-Space model identification for #44464

$$\begin{pmatrix} \dot{\xi}_{\mathrm{cs}} \\ \dot{\xi}_{\mathrm{vv}} \end{pmatrix} = \begin{pmatrix} -900 & -1.74 \cdot 10^4 \\ -3.4 \cdot 10^4 & 8.77 \cdot 10^5 \end{pmatrix} \begin{pmatrix} \xi_{\mathrm{cs}} \\ \xi_{\mathrm{vv}} \end{pmatrix} + \begin{pmatrix} 1630 \\ 82398 \end{pmatrix} I_{\mathrm{PFC}} \tag{4}$$

$$B = \begin{pmatrix} 0.0243 & -0.0005 \end{pmatrix} \begin{pmatrix} \xi_{\mathrm{cs}} \\ \xi_{\mathrm{vv}} \end{pmatrix} + 0 \cdot I_{\mathrm{PFC}} \tag{5}$$

The values in A, B, C matrices are too high and, moreover, D matrix is identified as zero $\implies$ model identification is wrong. (Although, the obtained fit looks reasonable.)

2.4 Summary

3 Proposal for the Next Session

- Check that the current in the coils is recorded for at least 40 ms (*should already be by default)*.
- Inner quadrupole; (Saddle coil - just to compare data obtained in [3])
- Impact on breakdown

References

[1] J. Havlíček. *Study of Equilibrium Magnetic Configuration in Tokamak Type Devices*. PhD thesis, Charles University in Prague, 2015.

[2] K. Kovařík, I. Ďuran, I. Boshakova, R. Holyaka, and V. Erashok. Measurement of safety factor using hall probes on castor tokamak. *Czechoslovak Journal of Physics*, 56:B104–B110, Oct 2006.

[3] A. Kubincová. Advanced plasma vertical position reconstruction on the GOLEM tokamak, 2021.

[4] M. Valovič. Control of plasma position in the castor tokamak. *Czechoslovak Journal of Physics B*, 39:1111–1119, Oct 1989.